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Quality Competition and a Demand Spillover Effect: A Case of Product Differentiated Duopoly

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Abstract

Employing the price-quality competition model in a horizontally differentiated products market, we analyze how a demand spillover effect associated with upgrading the quality level of a product affects the strategic relationship between firms and the property of a subgame perfect Nash equilibrium. In particular, we show that the strategic relationship depends on the degree of a demand spillover effect. Then, we consider the cases of second-best policy and cooperative quality choice. Furthermore, we illustrate that there exists a natural Stackelberg equilibrium under asymmetric demand spillover effects that is Pareto superior to other equilibria. Finally, we examine an optimal policy with international R&D rivalry.

Keywords: demand spillover effect; quality choice; product differentiation; Bertrand duopoly; a natural Stackelberg equilibrium; cooperative investment; optimal investment policy

JEL classification: L12, L13

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1. Introduction

Introducing a demand spillover effect into a model of quality choice competition, we analyze the strategic relationship between firms and the property of a subgame perfect Nash equilibrium (SPNE, hereafter) in a two-stage game in which firms compete in price and quality in a horizontally differentiated products market.

With respect to a demand spillover effect introduced in this paper, for example, we suppose that the development of an eco-vehicle equipped with an electric motor or a hybrid engine would enhance the willingness to pay of some customers who are sensitive to environmental issues but do not consider the car design, i.e., the brand name to be very important. This, in turn, implies that some of them (or new customers) may purchase an eco-vehicle regardless of the brand name of automobile makers. In other words, the development of a new product and/or the quality upgrade of a product may expand the market size of the industry. In this case, the effect of one firm's quality-improving investment on the rival firm's market share may not be necessarily negative.

Our model is characterized by the following two aspects: (i) R&D investment competition, in particular, quality choice; and (ii) a demand spillover effect. Related to the first aspect, there is an extensive body of literature in which researchers analyze the market competition and process and/or product R&D investments: firms invest to reduce the marginal cost of production, to develop a new product, or to upgrade the quality level of a product in the first stage, and then compete in price or quantity in the second stage. Consequently, they show how the R&D investments affect the environment of market competition. In addition, they consider a strategic R&D policy. As seminal papers, we take Brander and Spencer (1983), Spencer and Brander (1983), and others. Assuming duopolistic and oligopolistic markets for a homogenous product, they deal with Cournot quantity competition and a process (i.e., cost-reducing) R&D investment, and examine a strategic R&D investment subsidy/tax policy with international rivalry. It will be shown later that an optimal R&D investment policy depends on the strength of a demand spillover effect.

Furthermore, based on a vertical differentiation model, some researchers consider the nature of the equilibrium in product R&D (i.e., quality-improving) investment competition, and show the optimal industrial policies. For example, we have Aoki and Prusa (1996), Park (2001), Zhou et al. (2002), Aoki (2003), and others. In particular, they prove that the strategic

relationship between the firms (i.e., one firm producing a high-quality product and the other producing a low-quality product) and the effect of an increase in the quality level of one firm on the rival firm's profit depend on the mode of market competition (i.e., Bertrand price competition and Cournot quantity competition). However, it will be shown later that the strategic relationship and the effect on the rival firm's profit depend on the strength of a demand spillover effect.

With respect to the second aspect, many researchers have already considered a technological spillover effect in the context of an R&D investment competition. For example, d'Aspremont and Jacquemin (1988) analyze noncooperative and cooperative process R&D investments in the presence of a spillover effect on the marginal cost of production. Furthermore, Symeonidis (2003) considers a spillover effect of quality improvement on the investment cost in the cases of quantity and price competition.

Related to the analysis of a product R&D investment, Ornaghi (2006) states in his empirical analysis that a firm that enhances the quality of its product by learning from technological innovations receives a positive externality that can be estimated only by shifting attention to the demand side. Furthermore, Levin and Reiss (1988) empirically analyze the relationships between cost-reducing (i.e., process) and demand-creating (i.e., product) R&D investments with spillover effects. That is, they show that an increase in the extent of process (product) spillovers will lead to an increase in product (process) R&D, based on the estimation of the model using manufacturing line-of-business data and other data. We focus on a spillover effect on the demand side, and do not address the relationship between the process and product R&D innovations.

Furthermore, Foros et al. (2002) is closely related to ours. They analyze roaming policy in the market for mobile telecommunications and competing mobile telephony providers' incentives to invest in, and share infrastructure. That is, they focus on the interaction between roaming policy and investment incentives in the third generation mobile networks. The mechanism deriving their results is that in the presence of roaming agreements, investments carried out by one firm increase the value of the services provided by other firms. The roaming agreement is proportionate to a demand spillover effect in our model.

Foros et al. (2002) is an extension of d'Aspremont and Jacquemin (1988) and others, which analyze strategic R&D investment. In their model, infrastructure investments give rise to spillover effects, through the roaming agreements, similar to those considered in models

of strategic R&D investments. For example, they show that infrastructure investments in the noncooperative and cooperative cases depend on the roaming quality level.

Although the roaming quality level is endogenously given in the model of Foros et al. (2002), the parameters describing demand spillover effects are assumed to be exogenously given, but asymmetry in our model. Furthermore, we mainly deal with the case of a Bertrand price competition[1].

As illustrated below, we focus on a spillover effect that increases the quality level for a consumer's utility, and therefore on the demand side. In particular, we show that there are both *constructive* and *combative* effects of an increase in the quality level[2]. That is, an increase in the quality level of one firm takes some customers from the other firm, and thus increases the firm's individual demand while the other firm's demand is reduced. For example, using a Hotelling spatial model of duopoly with a quality dimension of product differentiation, Ishibashi and Kaneko (2008) show the effect of privatization on welfare in the case of a mixed duopoly. In their model, an increase in the quality level of one firm reduces the other firm's profit, so that the strategic substitute relationship between the firms holds. In this case, the effect of an increase in the quality level on the rival firm is *combative*. However, as mentioned above, an increase in the quality level of one firm can call new customers into the market, and thus expand market size. As a result, the quality improvement may increase the other firm's demand. In this case, we state that the effect of an increase in the quality level on the rival firm and on the industry is *constructive*.

To sum up, the novelty of this paper, if any, is its examination of the nature of duopolistic competition with a quality dimension of product differentiation associated with a demand spillover effect.

The remainder of the paper is composed as follows. Section 2 presents a model, in which we assume a quasi-linear utility function including a demand spillover effect associated with quality upgrading. Section 3 considers the strategic relationship between firms and the property of SPNE in price–quality duopolistic competition. Section 4 addresses issues such as the cases of second-best quality choice and cooperative quality choice. In this section, we also

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[1] In Appendix A, we analyze the case of a Cournot quantity competition. Our results in that case are basically similar to Lemma 1, 2, and 3 of Foros, et al. (2002).

[2] The terminology is due to Marshall (1919, pp. 304–307), by which he explains the effects of advertisements. We understand that the effect of a quality-upgrading investment on the demand side is similar to that of advertisements.

examine the existence of a natural Stackelberg equilibrium and an optimal R&D investment subsidy/tax policy in the case of international R&D rivalry. Finally, Section 5 summarizes the results and presents some remaining issues.

2. The model

We assume a duopolistic competition in a horizontally differentiated products market. That is, the firms compete in a two-stage game: in stage 1, a firm simultaneously chooses the quality level, q_i, and, in stage 2, a firm simultaneously chooses the price, $p_i, i = 1,2$. We confine our attention to SPNE in the two-stage game by solving the model backwards.

As mentioned above, we focus on a spillover effect associated with upgrading quality on the demand side. In particular, we assume that an increase in quality level of one product increases the willingness to pay of a consumer, and thus expands the market size. This, in turn, may increase the rival firm's demand. To highlight this aspect, we assume that the utility of a representative consumer is given by

$$V = U[x_1, x_2; q_1, q_2] + x_0, \text{ and}$$

$$U = \left\{ \alpha(x_1 + x_2) - \frac{1}{2}\left(x_1^2 + x_2^2\right) - \theta x_1 x_2 \right\} + \{\Omega(x_1 + x_2) + \omega_1 x_1 + \omega_2 x_2\}, \quad (1)$$

where x_0 is a numeral good and $p_0 = 1$. Also, $\alpha > 0$ and $1 > \theta > 0$. θ stands for a parameter indicating substitutability between the products.

In the second parenthesis composed of quality arguments as in eq. (1), we assume that $\Omega = \Omega[q_1, q_2]$ is an increasing function of the qualities preexisting in the industry, i.e., $\frac{\partial \Omega}{\partial q_i} > 0$, which is associated with the market size. This illustrates a demand spillover effect. Furthermore, $\omega_i = \omega_i[q_i], i = 1,2$ is an increasing function of its own quality, i.e., $\frac{\partial \omega_i}{\partial q_i} > 0$, which is associated with product *i*'s individual demand. For tractability, we assume as follows.

Assumption 1: Linearity of a demand spillover effect

(i) $\Omega[q_1, q_2] = \varepsilon_1 q_1 + \varepsilon_2 q_2$, *where* $\varepsilon_i (> 0, i = 1,2)$ *is the marginal coefficient of the effect of an increase in the quality level of product i on the market demand*[3].

(ii) $\omega_i[q_i] = \beta_i q_i$, *where* $\beta_i (> 0, i = 1,2)$ *is the marginal coefficient of the effect of an increase in the quality level of product i on its own individual demand.*

The budget constraint is given by $I \geq p_1 x_1 + p_2 x_2 + x_0$, where I is a given income of a representative consumer. Thus, we obtain the optimal behavior of a representative consumer as follows.

$$\frac{\partial U}{\partial x_i} = \{\alpha - x_i - \theta x_j\} + \{(\varepsilon_i + \beta_i) q_i + \varepsilon_j q_j\} = p_i.$$

In this case, the marginal utility of product *i*, in other words, a willingness to pay for it depends on the quality level of product *j* , which is a substitute for product *i*, as well as the quality of its own product. This implies that an increase in the quality level of product *j* increases the marginal utility of product *i*. This illustrates a demand spillover effect[4]. Therefore, we derive the demand function of product *i* as follows.

$$x_i = \frac{\alpha(1-\theta) + \{\varepsilon_i(1-\theta) + \beta_i\} q_i + \{\varepsilon_j(1-\theta) - \beta_j \theta\} q_j - p_i + \theta p_j}{(1-\theta)(1+\theta)}, \qquad (2)$$

where $i, j = 1,2, i \neq j$. For eq. (2), we have

$$\frac{\partial x_i}{\partial q_j} = \frac{\varepsilon_j(1-\theta) - \beta_j \theta}{(1-\theta)(1+\theta)} \geq (<) 0 \Leftrightarrow \mathrm{E}_j \geq (<) \theta, \quad i, j = 1,2, i \neq j, \qquad (3)$$

where $\mathrm{E}_j \equiv \dfrac{\varepsilon_j}{\varepsilon_j + \beta_j} < 1$ stands for the degree of a demand spillover effect. Hereafter, we omit the indexes of the firms in each equation, i.e., $i, j = 1,2, i \neq j$, unless we refer to them especially.

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[3] If $\Omega[q_1, q_2] = 0$, the utility function is formally similar to that of Häckner (2000). See also the appendix in Symeonidis (2003). In this case, the effect of improving the quality level is *combative* only.

[4] With regard to the effect of product R&D investments, Levin and Reiss (1988) present a multiplicative marginal utility function. Contrarily, the marginal utility function in our model is an additive type.

As shown in eq. (3), there are two effects of an increase in the quality level of product *j* on the demand of firm *i* as follows. The first is a positive effect whereby an increase in the quality level of product *j* increases the total market demand for the products, and this, in turn, increases firm *i*'s individual demand, i.e., a *constructive* effect. On the contrary, the second is a negative effect whereby an increase in the quality level of product *j* increases its own demand, and this, in turn, decreases firm *i*'s individual demand because of substitutability between the products, i.e., a *combative* effect. Thus, if the *constructive* effect is larger (smaller) than the *combative* effect, then an increase in the quality level of product *j* increases (reduces) the demand of firm *i*.

3. Quality competition in the presence of a demand spillover effect

3.1 Quality competition

In stage 2, firm *i* decides the price to maximize profit, $\Pi_i = p_i x_i - F_i[q_i]$. Hence, we assume that the investment cost to improve the quality level is given by $F_i[q_i]$, where $F_i[0] = F_i'[0] = 0$, $F_i'[q_i] > 0$, $F_i''[q_i] > 0$, for $q_i > 0$, $\lim_{z_i \to \infty} F_i'[q_i] = \infty$, and $F_i'''[q_i] \geq 0$, $i = 1,2$. Furthermore, to simplify the analysis, we assume the marginal cost of production is zero.

The first-order condition (FOC, hereafter) for maximizing the profit of firm *i* is given by $\frac{\partial \Pi_i}{\partial p_i} = x_i - \Lambda p_i = 0$, where $\Lambda \equiv \frac{1}{(1-\theta)(1+\theta)} > 0$. Taking eq. (3) into account, we obtain

$$\alpha(1-\theta) + \{\varepsilon_i(1-\theta) + \beta_i\}q_i + \{\varepsilon_j(1-\theta) - \beta_j\theta\}q_j - 2p_i + \theta p_j = 0. \quad (4)$$

Therefore, the price of product *i* is given by

$$p_i = \frac{\mathrm{A} + \Phi_i q_i + \Gamma_j q_j}{\Delta}, \quad \Delta \equiv (2-\theta)(2+\theta) > 0, \quad (5)$$

where $\mathrm{A} \equiv \alpha(1-\theta)(2+\theta) > 0$ and $\Phi_i \equiv \varepsilon_i(1-\theta)(2+\theta) + \beta_i(2-\theta^2) > 0$. Furthermore, we have $\Gamma_j \equiv \varepsilon_j(1-\theta)(2+\theta) - \beta_j\theta$. In this case, for eq. (5), it holds that

$$\frac{\partial p_i}{\partial q_j} \geq (<) 0 \Leftrightarrow \Gamma_j \geq (<) 0 \Leftrightarrow \Theta[\mathrm{E}_j] \geq (<) \theta, \tag{6}$$

where $\Theta[\mathrm{E}_j] \equiv \frac{\sqrt{\mathrm{E}_j^{-2} + 8} - \mathrm{E}_j^{-1}}{2} < 1$ and $\Theta'[\mathrm{E}_j] > 0, \quad j = 1,2.$

Eq. (6) shows that if the parameter composed of the degree of a demand spillover effect of product *j*, i.e., $\Theta[\mathrm{E}_j]$, is larger (smaller) than a certain value of substitutability, θ, then an increase in the quality level of product *j* raises (reduces) the price of product *i*. That is, firm *i* has to raise (cut) the price, because the *constructive* effect on product *i*'s demand through an increase in the total market demand is larger (smaller) than the *combative* effect on it through a substitutability between the products.

In stage 1, firm *i* decides the quality level to maximize profit, which is given by $\Pi_i = R_i[q_i, q_j] - F_i[q_i]$, where $R_i[q_i, q_j] = \Lambda p_i^2$. The FOC is given by

$$\frac{\partial \Pi_i}{\partial q_i} = \frac{\partial R_i}{\partial q_i} - \frac{\partial F_i}{\partial q_i} = 2\Lambda p_i \frac{\Phi_i}{\Delta} - \frac{\partial F_i}{\partial q_i} = 0. \tag{7}$$

Also, the second-order condition (SOC, hereafter) is given by

$$\frac{\partial^2 \Pi_i}{\partial q_i^2} = \frac{\partial^2 R_i}{\partial q_i^2} - \frac{\partial^2 F_i}{\partial q_i^2} = 2\Lambda \left\{ \frac{\Phi_i}{\Delta} \right\}^2 - \frac{\partial^2 F_i}{\partial q_i^2} < 0. \tag{8}$$

Furthermore, the cross effect is given by

$$\frac{\partial^2 \Pi_i}{\partial q_i \partial q_j} = \frac{\partial^2 R_i}{\partial q_i \partial q_j} = 2\Lambda \left\{ \frac{\Phi_i}{\Delta} \right\} \left\{ \frac{\Gamma_j}{\Delta} \right\} \geq (<) 0 \Leftrightarrow \Gamma_j \geq (<) 0. \tag{9}$$

With respect to the effect on the profit (i.e., revenue) of the rival firm, we obtain

$$\frac{\partial \Pi_i}{\partial q_j} = \frac{\partial R_i}{\partial q_j} = 2\Lambda p_i \frac{\Gamma_j}{\Delta} \geq (<) 0 \Leftrightarrow \Gamma_j \geq (<) 0. \tag{10}$$

For the following analysis, we assume as follows.

Assumption 2: Asymmetric demand spillover effects

The degree of a demand spillover effect of product 1 is larger than that of product 2, i.e., $\mathrm{E}_1 > \mathrm{E}_2$. *In this case, it holds that* $\Theta_1 \equiv \Theta[\mathrm{E}_1] > \Theta_2 \equiv \Theta[\mathrm{E}_2]$.

Taking into account eq. (9), eq. (10), and *Assumption 2*, we derive the following lemma.

Lemma 1

(i) *If* $\Theta_1 > \Theta_2 > \theta$, *each firm's reaction curve is sloping upward. Hence an increase in the quality of product 2 (1) increases the revenue of firm 1 (2).*

(ii) *If* $\theta > \Theta_1 > \Theta_2$, *each firm's reaction curve is sloping downward. Hence an increase in the quality of product 2 (1) decreases the revenue of firm 1 (2).*

(iii) *If* $\Theta_1 > \theta > \Theta_2$, *firm 1's reaction curve is sloping downward, whereas firm 2's is sloping upward. Hence an increase in the quality of product 2 (1) decreases (increases) the revenue of firm 1 (2).*

As analyzed in d'Aspremont and Jacquemin (1988) and others, the strategic relationship between firms in the case of cost-reducing R&D investment competition is one of substitutes. Furthermore, in the case of quality competition, e.g., Aoki and Prusa (1996), Park (2001), Zhou et al. (2002), Aoki (2003), and others, the strategic relationship between firms depends on the mode of market competition. However, in our model, the strategic relationship depends on the degree of a demand spillover effect. Accordingly, *Lemma 1* holds in the case of a Cournot duopoly (see Appendix A).

For *Lemma 1 (i)* (*Lemma 1 (ii)*), if the degree of demand spillover effects of both firms' products is larger (smaller) than a certain value of substitutability, an increase in the quality level of one firm expands (reduces) the rival firm's demand, and this, in turn, increases (decreases) the rival firm's price and quality. As a result, the rival firm's profit increases (decreases). Similarly, an increase in the rival firm's quality increases (decreases) the firm's profits. In this case, the strategic complement (substitute) relationship between the firms is confirmed. See Figure 1 (Figure 2), in which we illustrate Pareto superior sets as the area surrounded by the iso-profit curves of both firms, i.e., a real line of $\Pi_i^N, i = 1,2$.

Place Figure 1 and Figure 2 approximately here

For *Lemma 1 (iii)*, if the degree of a demand spillover effect is asymmetric between the

firms, i.e., the degree of a demand spillover effect of product *1* is larger and that of product *2* is smaller than a certain value of substitutability, an increase in product *1* expands firm *2*'s demand, while an increase in product *2* reduces firm *1*'s demand. In this case, firm *2* increases its price and quality, while firm *1* decreases them. As a result, firm *2*'s profit increases, while firm *1*'s falls. Thus, a strategic substitute relationship holds for firm *1*, while a strategic complement relationship holds for firm *2*. See Figure 3, in which, similarly, we illustrate Pareto superior sets as the area surrounded by the iso-profit curves (drawing a real line) of both firms.

Place Figure 3 approximately here

3.2 The existence and the property of SPNE

Based on eq. (5) and eq. (7), for example, assuming a simple quadratic cost function of quality, i.e., $F_i[q_i] = \frac{f}{2} q_i^2, i = 1,2, f > 0,$ we prove that there exists an SPNE in a price–quality competition (see Appendix B). Here, in view of *Lemma 1*, we consider the properties of the SPNE and the implications of demand spillover effects.

Let us deal with the case in which the degree of demand spillover effects of both products is large, i.e., $\Theta_1 > \Theta_2 > \theta$. As mentioned above, both reaction curves are upward sloping and exist in the Pareto superior sets (see Figure 1). Formally, the nature of an SPNE in quality competition is the same as that of Bertrand–Nash equilibrium. In this case, neither firm will take an initiative to improve their quality level. This is because an increase in the quality level helps the rival firm to increase its profit according to a *constructive* effect. Thus, as will be analyzed later, this implies that each firm will be a follower (or a free rider).

By contrast, when the degree of demand spillover effects of both products is small, i.e., $\theta > \Theta_1 > \Theta_2$, both reaction curves are downward sloping and do not exist in the Pareto superior sets (see Figure 2). Formally, the nature of an SPNE in quality competition is the same as that of Cournot–Nash equilibrium. In this case, each firm has an incentive to increase its quality level to take market share from the rival firm due to a *combative* effect. Thus, this implies that both firms will be a leader.

We now consider the case of asymmetric demand spillover effects, i.e., $\Theta_1 > \theta > \Theta_2$. As illustrated in Figure 3, the reaction curve of firm *1* is downward sloping, while that of firm

2 is upward sloping. Hence, the reaction curve of firm *1* exists in the Pareto superior sets. Firm *1* has an incentive to increase its quality level to take market share from firm *2*, just as in a Cournot quantity competition. On the other hand, because firm *2* knows that the degree of a demand spillover effect from firm *1* is large, firm *2* chooses to reduce the quality level, and this, in turn, makes firm *1* increase the quality level. As a result, the market size expands, and thus the profits of both firms increase. This suggests the possibility of a cooperative quality choice between the firms, i.e., semi-collusion.

4. Discussion and application

4.1 Second-best qualities

We assume that a public authority, i.e., a government, can decide the quality levels of the products to maximize social welfare in the first stage, although the firms noncooperatively decide prices in the second stage.

The social welfare function is expressed as

$$W \equiv W[q_1, q_2] = CS[q_1, q_2] + PS[q_1, q_2], \tag{11}$$

where $CS[q_1, q_2] \equiv U[x_1, x_2; q_1, q_2] - p_1 x_1 - p_2 x_2 = \frac{1}{2}\left(x_1^2 + x_2^2\right) + \theta x_1 x_2$ is consumer surplus and $PS[q_1, q_2] \equiv \Pi_1[q_1, q_2] + \Pi_2[q_1, q_2]$ is producer surplus, in other words, joint profits.

Given eq. (11), the FOC to maximize social welfare with respect to the qualities is given by $\frac{\partial W}{\partial q_i} = \frac{\partial CS}{\partial q_i} + \frac{\partial \Pi_i}{\partial q_i} + \frac{\partial \Pi_j}{\partial q_i} = 0.$ When evaluating the FOC at $\frac{\partial \Pi_i}{\partial q_i} = 0,$ it holds that $\left.\frac{\partial W}{\partial q_i}\right|_{\frac{\partial \Pi_i}{\partial q_i}=0} > 0, i = 1,2.$ See Appendix C. Thus, we derive the following result.

Result 1

Regardless of the degree of a demand spillover effect, the firms provide the products associated with lower qualities rather than those of the second-best case.

4.2 Cooperative investment equilibrium: joint profit maximization

We consider the cooperative case in which the firms decide the quality level to maximize the joint profits (i.e., producer surplus), although they noncooperatively compete in price in the market.

Let us define the joint profits as follows: $PS[q_1, q_2] \equiv \Pi_1[q_1, q_2] + \Pi_2[q_1, q_2]$. Thus, the FOC to maximize the joint profits with respect to the quality level of firm *1's* product is given by $\frac{\partial PS}{\partial q_1} = \frac{\partial \Pi_1}{\partial q_1} + \frac{\partial \Pi_2}{\partial q_1} = 0$. In this case, when evaluating the FOC at $\frac{\partial \Pi_1}{\partial q_1} = 0$, in view of eq. (10), it holds that $sign\left(\left. \frac{\partial PS}{\partial q_1} \right|_{\frac{\partial \Pi_1}{\partial q_1}=0} \right) = sign\left(\frac{\partial \Pi_2}{\partial q_1} \right) = sign(\Gamma_1)$.

Furthermore, from the FOC of firm *2*, we obtain the same outcome, i.e., $sign\left(\left. \frac{\partial PS}{\partial q_2} \right|_{\frac{\partial \Pi_2}{\partial q_2}=0} \right) = sign(\Gamma_2)$.

Accordingly, based on *Lemma 1*, we derive the following result.

Result 2

(i) *If* $\Theta_1 > \Theta_2 > \theta$, *the cooperative quality level of both firms' products is higher than the noncooperative one.*

(ii) *If* $\theta > \Theta_1 > \Theta_2$, *the cooperative quality level of both firms' products is lower than the noncooperative one.*

(iii) *If* $\Theta_1 > \theta > \Theta_2$, *firm 1's product quality level is higher and firm 2's product quality level is lower than the noncooperative one, respectively.*

When the degree of a demand spillover effect is large, upgrading the quality level of one firm expands market size, and thus increases the other firm's profit as well as that firm's profit. Thus, in the case of the cooperative quality decision, both firms have incentives to improve

the quality level compared with the noncooperative case. Furthermore, because an increase in the quality level enhances consumer surplus, the cooperative quality decision is preferable from a social welfare point of view.

On the other hand, when the degree of a demand spillover effect is small, upgrading the quality level of one firm takes consumers from the other firm, and thus, reduces the other firm's profit. Thus, in the cooperative case, both firms decrease quality levels compared with the noncooperative case. Thus, because a decrease in the quality level reduces consumer surplus, the cooperative quality decision is not preferable from a social welfare point of view.

Under asymmetric demand spillover effects, in the case of the cooperative quality decision, firm *1* (*2*) providing the product with a high (low) demand spillover effect increases (decreases) the quality level, because an increase in the quality level of firm *1's* (*2's*) product increases joint profits. In other words, an increase in the quality level of the product with a high demand spillover effect promotes a *constructive* effect, whereas a decrease in the quality level of the product with a low demand spillover effect reduces a *combative* effect. From a social welfare point of view, whether the cooperative quality decision under an asymmetric spillover effect is preferable or not is unidirectional.

4.3 A natural Stackelberg equilibrium

Related to the analysis in Sections 3.2 and 4.2, here, applying the extended endogenous timing game of an observable delay framework in Hamilton and Slutsky (1990) and following the definition of Albaek (1990), we analyze the existence of a natural Stackelberg equilibrium. That is, the players determine the timing of the action as well as the action itself. If the players choose the actions at different times, then the player choosing a later time observes the action chosen by the initiating player, giving rise to a sequential play subgame, and thus Stackelberg equilibrium holds in the game. If the players instead choose actions at the same time, then a simultaneous play subgame occurs, and thus Nash equilibrium holds in the game.

Consequently, by using the extended endogenous timing game of observable delay, we provide comparisons between the payoffs in the simultaneous play game and the two sequential play games. In this case, if one firm prefers being a leader to being a follower and prefers not to play a simultaneous Nash game, while the other firm prefers being a follower to being a leader and also prefers not to play a simultaneous Nash game, then a natural Stackelberg equilibrium is attained.

Although we omit the complicated calculations, taking *Lemma 1* into account, we can easily derive the following relationships between the profits (see Figures 1, 2 , and 3).

Lemma 2

(i) *If* $\Theta_1 > \Theta_2 > \theta$, *it holds that* $\Pi_i^L > \Pi_i^N$ *and* $\Pi_i^F > \Pi_i^N$, $i = 1,2.$

(ii) *If* $\theta > \Theta_1 > \Theta_2$, *it holds that* $\Pi_i^L > \Pi_i^N > \Pi_i^F$, $i = 1,2.$

(iii) *If* $\Theta_1 > \theta > \Theta_2$, *it holds that* $\Pi_1^L > \Pi_1^N$ *and* $\Pi_1^F > \Pi_1^N$, *and that* $\Pi_2^L > \Pi_2^N > \Pi_2^F$.

Under *Lemma 2 (i)*, just as in a Bertrand price competition, both firms prefer to being either a leader or a follower to playing a simultaneous Nash game. However, each firm prefers being a follower to being a leader, because it holds that $\Pi_i^F > \Pi_i^L$, $i = 1,2.$ Furthermore, two Stackelberg equilibria in which either firm is a leader exist in the Pareto superior sets (see S_1 and S_2 in Figure 1). Similarly, under *Lemma 2 (ii)*, just as in a Cournot quantity competition, both firms prefer to being a leader to being a follower and playing a simultaneous Nash game. See Theorem V (A) in Hamilton and Slutsky (1990). Furthermore, two Stackelberg equilibria in which either firm is a leader do not exist in the Pareto superior sets (see S_1 and S_2 in Figure 2).

Under *Lemma 2 (iii)*, firm *1* prefers being either a leader or a follower to playing a simultaneous Nash game. On the other hand, firm *2* prefers being a leader to being a follower and playing a simultaneous Nash game. In this case, because firm *1* expects that firm *2* will be a leader, firm *1* is to be a follower. Therefore, based on Hamilton and Slutsky (1990, Theorem V (B)) and Albaek (1990, *Definition*), we derive as follows.

Result 3

There exists a natural Stackelberg situation under asymmetric demand spillover effects in which firm 2 providing the product that has a low demand spillover effect is a leader and firm 1 providing the product that has a high demand spillover effect is a follower.

In addition, we show that the natural Stackelberg equilibrium exists in the Pareto superior sets. Furthermore, compared with the Nash equilibrium, in the natural Stackelberg equilibrium, firm *2* providing the product that has a low demand spillover effect reduces the quality level, whereas firm *1* providing the product that has a high demand spillover effect increases the quality level[5]. See point S_2 in Figure 3.

A minor firm that does not have a brand value and provides a product with only a small effect on market demand decides in advance to choose the lower quality level. This makes the major firm that has a big brand name and provides the product with a large effect on market demand increase its quality level. Thus, the market expands, and both firms can raise their prices. As a result, both firms would make more profits than otherwise.

4.4 Optimal quality-upgrading R&D investment policy with international rivalry

We analyze an optimal policy, using a standard framework of a third-country market presented by Spencer and Brander (1983). That is, there are two countries, home and foreign, in each of which a firm exits. Each firm sells the product to the third-country market, but does not sell it to markets in the home and foreign countries. Hereafter, the firm located in the home (foreign) country is firm *1* (*2*).

We assume that the home and foreign governments commit to a quality-upgrading R&D investment subsidy/tax policy in stage 0, prior to the two-stage game played by the firms.

In this case, the profit function of firm *i* in stage 1 is revised as follows.

$$\Pi_i[q_i, q_j] = R_i[q_i, q_j] - (1 - s_i)F_i[q_i], i, j = 1,2, i \neq j,$$

where $s_i(<1)$ is an investment subsidy from the government of country *i*. A negative s_i implies an investment tax. Thus, taking the FOC into account, we derive the effects of a subsidy on the quality level of the firms as follows.

$$\frac{\partial q_i}{\partial s_i} = -\frac{F_i'}{\Delta}\frac{\partial^2 \Pi_j}{\partial q_j{}^2} > 0 \text{ and } \frac{\partial q_j}{\partial s_i} = \frac{F_i'}{\Delta}\frac{\partial^2 \Pi_j}{\partial q_j \partial q_i} \geq (<) 0 \Leftrightarrow \frac{\partial^2 \Pi_j}{\partial q_j \partial q_i} \geq (<) 0,$$

where $i, j = 1,2, i \neq j$. It is clear that a government *i* subsidy always increases firm *i*'s investment level *i*, whereas the effect on firm *j*'s level depends on the sign of the cross effect

[5] See *Result 2 (iii)*. That is, the behavior of the firms in the natural Stackelberg equilibrium is similar to that in the case of cooperative quality choice.

of its profit function.

We first consider the unilateral policy at stage 0. That is, noncooperatively, government i chooses a subsidy/tax on its domestic firm, i.e., s_i, to maximize domestic social welfare, W_i, which is given by

$$W_i[s_i] = \Pi_i[q_i, q_j] - s_i F_i[q_i] = R_i[q_i, q_j] - F_i[q_i],$$

where $q_i = q_i[s_i, s_j], i, j = 1,2, i \neq j$. From the FOC to maximize the social welfare of country i, i.e., $\frac{\partial W_i}{\partial s_i} = 0$, given s_j, the optimal quality-upgrading R&D investment policy in the noncooperative case is given by $s_i^* = \frac{1}{F_i'}\left(\frac{\partial R_i}{\partial q_j}\right)\left(\frac{\partial q_j / \partial s_i}{\partial q_i / \partial s_i}\right)$.

In view of the effects on the quality levels derived above, this optimal policy is rewritten as:

$$s_i^* = \frac{1}{F_i'}\left(\frac{\partial R_i}{\partial q_j}\right)\left(-\frac{\frac{\partial^2 \Pi_j}{\partial q_j \partial q_i}}{\frac{\partial^2 \Pi_j}{\partial q_j^2}}\right) = \frac{1}{F_i'}\left(\frac{\partial R_i}{\partial q_j}\right)\left(\frac{dq_j}{dq_i}\right).$$

In this case, we obtain $sign(s_i^*) = sign\left\{\left(\frac{\partial R_i}{\partial q_j}\right)\left(\frac{dq_j}{dq_i}\right)\right\}$. Furthermore, because it holds that $sign\left(\frac{dq_j}{dq_i}\right) = sign\left(\frac{\partial^2 \Pi_j}{\partial q_j \partial q_i}\right)$, we derive as follows.

$$sign(s_i^*) = sign\left\{\left(\frac{\partial R_i}{\partial q_j}\right)\left(\frac{\partial^2 \Pi_j}{\partial q_j \partial q_i}\right)\right\}, \tag{12}$$

where $i, j = 1,2, i \neq j$. Therefore, as already shown in Spencer and Brander (1983), Park (2001), Toshimitsu and Jinji (2008), and others, in view of eq. (12), we summarize as follows.

Lemma 3

An optimal quality-upgrading R&D investment policy of the home government, i.e., country i, depends on

(i) *the sign of the externality of the foreign firm's R&D activities toward the home firm, i.e.,* $\frac{\partial R_i}{\partial q_j}$, $i, j = 1,2, i \neq j$, *and*

(ii) *the sign of the slope of the foreign firm's R&D investment reaction curves, i.e.,* $\frac{dq_j}{dq_i}$, *in other words, the sign of the cross effect of the foreign firm's profit function, i.e.,* $\frac{\partial^2 \Pi_j}{\partial q_j \partial q_i} \left(= \frac{\partial^2 R_j}{\partial q_j \partial q_i} \right)$, $i, j = 1,2, i \neq j$,

where firm i (j) is the home (foreign) firm.

Lemma 3 (ii) means that the strategic relationship between firms is either substitute or complement, i.e., $\frac{dq_j}{dq_i} < 0$ or $\frac{dq_j}{dq_i} > 0$, $i, j = 1,2, i \neq j$. Taking into account *Lemmas 1* and *3*, we present the optimal quality-upgrading R&D investment policy in the noncooperative case as follows.

Result 4

(i) *If either* $\Theta_1 > \Theta_2 > \theta$ *or* $\theta > \Theta_1 > \Theta_2$, *an optimal R&D investment policy is a subsidy, i.e.,* $s_i^* > 0$, $i = 1,2$.

(ii) *If* $\Theta_1 > \theta > \Theta_2$, *an optimal R&D investment policy is a tax, i.e.,* $s_i^* < 0$, $i = 1,2$.

Furthermore, based on *Result 4*, we directly derive as follows.

Corollary 1

(i) *A noncooperative quality-upgrading R&D investment subsidy policy is Pareto improving (beggar-thy-neighbor) if* $\Theta_1 > \Theta_2 > \theta$ $(\theta > \Theta_1 > \Theta_2)$.

(ii) A noncooperative quality-upgrading R&D investment tax policy on firm 2 (1) is Pareto improving (beggar-thy-neighbor) if $\Theta_1 > \theta > \Theta_2$.

As analyzed in Section 4.3, an optimal R&D investment policy in the noncooperative case is one in which the government makes the domestic firm the leader in a Stackelberg game. This aspect has already been addressed in the context of strategic trade and industrial policies. Thus, if the degree of a demand spillover effect of both firms is either larger or smaller than a certain value of substitutability of the products, each government gives the domestic firm a subsidy to upgrade their quality level. Incidentally, in the case of a large enough demand spillover effect, subsidizing the domestic firm increases the foreign firm's quality level as well as that of the domestic firm. This, in turn, leads to an increase in the profits of both firms, and thus the welfare of both countries. Thus, the R&D investment subsidy is a Pareto improving policy. However, in the case of a smaller demand spillover effect, subsidizing the domestic firm reduces the foreign firm's quality level, but increases that of the domestic firm. This, in turn, leads to the profit of the foreign firm falling, and an increase in the profit of the domestic firm. In this case, the R&D investment subsidy is a beggar-thy-neighbor policy.

In the case of asymmetric spillover effects, each government taxes the domestic firm to reduce the quality level. That is, a decrease in the quality level of firm *1* (*2*) reduces (increases) the quality level of firm *2* (*1*). In this case, the profit of firm *1* (*2*) increases by the negative (positive) externality effect on the revenue of firm *1* (*2*). However, a country's R&D investment tax policy, in which the firm produces the product associated with a larger (smaller) demand spillover effect, reduces (increases) the profit of the other firm, and thus the welfare of the other country. Therefore, the R&D investment tax policy is a beggar-thy-neighbor (Pareto improving) policy.

Second, we proceed to analyze the optimal R&D investment policy in the cooperative case, in which the government institutes an R&D investment policy to maximize the joint social welfare of both countries:

$$W[s_1, s_2] \equiv W_1[s_1, s_2] + W_2[s_1, s_2] = \Pi_1[q_1, q_2] + \Pi_2[q_1, q_2],$$

where $q_i = q_i[s_1, s_2]$, $i = 1,2$. Hence, the FOC to maximize the joint social welfare with respect to the cooperative R&D investment policy of the home government is represented as

$$\frac{\partial W}{\partial s_1} = \left(\frac{\partial R_2}{\partial q_1} - s_1 F_1{}'\right)\left(\frac{\partial q_1}{\partial s_1}\right) + \left(\frac{\partial R_1}{\partial q_2} - s_2 F_2{}'\right)\left(\frac{\partial q_2}{\partial s_1}\right) = 0.$$

Similarly, for the cooperative R&D investment policy of the foreign government, we have

$$\frac{\partial W}{\partial s_2} = \left(\frac{\partial R_2}{\partial q_1} - s_1 F_1{}'\right)\left(\frac{\partial q_1}{\partial s_2}\right) + \left(\frac{\partial R_1}{\partial q_2} - s_2 F_2{}'\right)\left(\frac{\partial q_2}{\partial s_2}\right) = 0.$$

From the two equations above, an optimal cooperative R&D investment policy is given by

$$s_i{}^C = \frac{1}{F_i{}'}\left(\frac{\partial R_j}{\partial q_i}\right), \quad i, j = 1,2, i \neq j,$$

where superscript C denotes the cooperative case. Therefore, we sum up as follows.

Lemma 4

An optimal cooperative R&D investment policy by the home government depends on the sign of the externality of the home firm's R&D activities toward the foreign firm, i.e., $\frac{\partial R_j}{\partial q_i}$, $i, j = 1,2, i \neq j$, *where firm i (j) is the home (foreign) firm.*

Based on *Lemma 4*, we directly derive an optimal policy in the case of cooperation between home and foreign governments as follows.

Result 5

(i) If $\Theta_1 > \Theta_2 > \theta$ $(\theta > \Theta_1 > \Theta_2)$, *then a cooperative R&D investment subsidy (tax) is optimal, i.e.,* $s_i^C > 0$ $(s_i^C < 0)$, $i = 1,2.$

(ii) If $\Theta_1 > \theta > \Theta_2$, *then a cooperative R&D investment subsidy on firm 1 and tax on firm 2 are optimal, i.e.,* $s_1^C > 0$ *and* $s_2^C < 0.$

As already addressed in Section 4.2, the analysis of the cooperative R&D investment policy is similar to that of a cooperative quality decision game to maximize the joint profits of the firms. Because, in the case of a large (small) demand spillover effect, an increase (decrease) in the quality level of the two firms increases joint profits, the government

subsidizes (taxes) the domestic firm.

In the model of vertical product differentiation, Zhou et al. (2002, Proposition 3) show that jointly optimal policies involve an investment subsidy in the developed country and an investment tax in the less developed country. In their model, the firm producing the product with a high (low) quality level locates in the developed (less developed) country. That is, the jointly optimal policies expand the difference in quality levels, and thus mitigate price competition. This, in turn, increases the joint profits of the firms, and thus the joint welfare of their countries.

In our model, under asymmetric demand spillover effects, subsidizing firm *1* producing the product with a large demand spillover effect and taxing firm *2* producing the product with a small demand spillover effect increases (reduces) the quality level of firm *1* (*2*). This, in turn, locates the combination of these quality levels in the Pareto superior sets. Thus, the joint profits of the firms and the joint welfare of the countries increase. However, in this case, these increases are due to both an increase in the *constructive* effect by upgrading the quality level of the product with a large demand spillover effect and a decrease in the *combative* effect by downgrading the quality level of the product with a small demand spillover effect.

5. Conclusions

Assuming a demand spillover effect associated with upgrading the quality level of a product, we have analyzed the strategic relationship between firms and the property of an SPNE in a price–quality duopolistic competition in a horizontally differentiated products market. In particular, upgrading the quality level can increase the rival firm's demand. Hence, we have shown that if the degree of a demand spillover effect is larger (smaller) than a certain value of substitutability between the products, a strategic complement (substitute) relationship between the firms is obtained. Accordingly, both firms have incentives to increase (decrease) their quality level. Furthermore, in the case of asymmetric demand spillover effects between firms, the strategic complement and substitute relationships hold. That is, if the rival firm produces the product associated with a large (small) demand spillover effect, the firm has an incentive to increase (reduce) the quality level.

Furthermore, we have analyzed some issues as follows. First, we have compared the

quality levels of the SPNE with those of the second-best case. That is, the quality levels in the former are lower than those in the latter in spite of the degree of the demand spillover effect. Second, we have discussed the case of a cooperative quality choice. That is, comparing the noncooperative case with the cooperative one, we have found that if a demand spillover effect is larger (smaller) than a certain value of substitutability of the products, the cooperative quality level is higher (lower) than the noncooperative quality level. Furthermore, if the firm produces the product associated with a large (small) demand spillover effect, the cooperative quality level is higher (lower) than the noncooperative quality level. Third, we have illustrated that there is a natural Stackelberg equilibrium under asymmetric demand spillover effects. That is, the firm producing the product associated with a large (small) demand spillover effect is a follower (leader). In addition, a natural Stackelberg equilibrium is Pareto superior to the other equilibria. Finally, we have addressed an optimal R&D investment policy in the noncooperative and cooperative cases. If the degree of a demand spillover effect is either sufficiently large or small, then an optimal R&D investment policy is a subsidy in the noncooperative case. However, under asymmetric demand spillover effects, an optimal R&D investment policy is a tax. In the case of a cooperative R&D investment policy, a subsidy (tax) is optimal, with a large (small) demand spillover effect. Also, under an asymmetric demand spillover effect, a cooperative optimal policy is a subsidy (tax) on the firm producing the product with a large (small) demand spillover effect. This result is similar to that in the cooperative quality decision game to maximize the joint profit.

We have assumed the external effect of upgrading quality on the demand side. Although we usually understand the *combative* effect of quality improvement on a rival firm, we rarely consider the *constructive* effect. However, we may examine the demand side effects of advertising investment as an example. That is, a persuasive and/or informative advertising investment of one firm may increase demand from the rival firm's customers as well as its own customers because the market's size expands due to new consumers entering the market. Furthermore, as discussed in Foros et al. (2002), as examples to justify a demand spillover effect in our model, we take information and communication technology industries such as mobile telecommunications, computer hardware and software, cloud computing systems, and broadcasting.

We have simplified the model to make the analysis tractable; for example, a quasi-linear demand function and the linearity of the demand spillover effect function. Were we to assume

more general functions, we would have to reconsider the model.

Appendix A: the Cournot duopoly case

From the optimal behavior of a representative consumer, we obtain the inverse demand function as follows[6].

$$p_i = \alpha + (\varepsilon_i + \beta_i)q_i + \varepsilon_j q_j - x_i - \theta x_j, i, j = 1,2, i \neq j. \tag{A.1}$$

In this case, in stage 2, Cournot–Nash equilibrium is given by

$$x_i = \frac{\mathrm{A}^C + \Phi_i{}^C q_i + \Gamma_j{}^C q_j}{\Delta}, i, j = 1,2, i \neq j, \tag{A.2}$$

where superscript C denotes the Cournot competition. Also, $\mathrm{A}^C \equiv \alpha(2-\theta) > 0$, $\Phi_i{}^C \equiv \varepsilon_i(2-\theta) + 2\beta_i > 0$, and $\Gamma_j{}^C \equiv \varepsilon_j(2-\theta) - \beta_j\theta$. Given (A.2), it holds that

$$\frac{\partial x_i}{\partial q_j} \geq (<) 0 \Leftrightarrow \Gamma_j{}^C \geq (<) 0 \Leftrightarrow E_j \geq (<) \frac{\theta}{2}, i, j = 1,2, i \neq j. \tag{A.3}$$

In stage 1, the profit function is expressed as

$$\Pi_i{}^C = p_i x_i - F_i(q_i) = x_i[q_i, q_j]^2 - F_i(q_i), i, j = 1,2, i \neq j.$$

In this case, the FOC and the SOC are given by

$$\frac{\partial \Pi_i{}^C}{\partial q_i} = 2x_i \frac{\Phi_i{}^C}{\Delta} - \frac{\partial F_i}{\partial q_i} = 0 \text{ and } \frac{\partial^2 \Pi_i{}^C}{\partial q_i{}^2} = 2\left\{\frac{\Phi_i{}^C}{\Delta}\right\}^2 - \frac{\partial^2 F_i}{\partial q_i{}^2} < 0.$$

Furthermore, the cross effect is given by

$$\frac{\partial^2 \Pi_i{}^C}{\partial q_i \partial q_j} = 2\left\{\frac{\Phi_i{}^C}{\Delta}\right\}\left\{\frac{\Gamma_j{}^C}{\Delta}\right\} \geq (<) 0 \Leftrightarrow \Gamma_j{}^C \geq (<) 0. \tag{A.4}$$

With respect to the effect on profit (revenue), we obtain

$$\frac{\partial \Pi_i{}^C}{\partial q_j} = 2x_i \frac{\Gamma_j{}^C}{\Delta} \geq (<) 0 \Leftrightarrow \Gamma_j{}^C \geq (<) 0. \tag{A.5}$$

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[6] If we assume $\varepsilon_i = \varepsilon, \beta_i = \beta, \varepsilon + \beta = 1,$ and $\theta = 1,$ then we can derive the inverse demand function of Foros, et al. (2002).

Therefore, taking eq. (A.3), eq. (A.4), and eq. (A.5) into account, we can derive the same results as *Lemma 1*. In other words, regardless of the mode of competition, the strategic relationships of quality decisions between the firms depend on the degree of demand spillover effects.

Appendix B: subgame perfect Nash equilibrium

Let us assume the cost function as follows: $F_i[q_i] = \frac{f}{2} q_i^2, i = 1,2, f > 0.$ In this case, taking eq. (5) into account, the FOC shown by eq. (7) is rewritten as

$$\mathrm{X}_i\left(\mathrm{A} + \Phi_i q_i + \Gamma_j q_j\right) - f q_i = 0, i, j = 1,2, i \neq j, \tag{B.1}$$

where $\mathrm{X}_i \equiv 2\Lambda \frac{\Phi_i}{\Delta^2} > 0.$ Thus, the SOC is given by $f > \mathrm{X}_i \Phi_i, i = 1,2.$

Accordingly, we derive the reaction function of firm *i* as follows.

$$q_i = \frac{\mathrm{A}\mathrm{X}_i}{f - \mathrm{X}_i \Phi_i} + \frac{\mathrm{X}_i \Gamma_j}{f - \mathrm{X}_i \Phi_i} q_j, i, j = 1,2, i \neq j. \tag{B.2}$$

Therefore, if the determinant of the equation system given by eq. (B.1) is positive, i.e.,

$$D \equiv (f - \mathrm{X}_1 \Phi_1)(f - \mathrm{X}_2 \Phi_2) - \mathrm{X}_1 \Gamma_2 \mathrm{X}_2 \Gamma_1 > 0,$$

then there exists an SPNE in a price–quality competition game as follows.

$$q_i^* = \frac{\mathrm{A}\mathrm{X}_i}{D} \left\{ f - \mathrm{X}_j \left(\Phi_i - \Gamma_i \right) \right\}, \tag{B.3}$$

where we assume $f > \mathrm{X}_j \left(\Phi_i - \Gamma_i \right), \ i, j = 1,2, i \neq j.$

Appendix C: second-best qualities

Because it holds that $x_i = \Lambda p_i \left\lfloor q_1, q_2 \right\rfloor, i = 1,2,$ the consumer surplus is rewritten as $CS[q_1, q_2] = \Lambda^2 \left\{ \frac{1}{2} \left(p_1^2 + p_2^2 \right) + \theta p_1 p_2 \right\}.$ In this case, the effect of an increase in the quality level of product *i* on consumer surplus is generally expressed by

$$\frac{\partial CS}{\partial q_i} = \Lambda^2 \left\{ p_i \left(\frac{\partial p_i}{\partial q_i} + \theta \frac{\partial p_j}{\partial q_i} \right) + p_j \left(\frac{\partial p_j}{\partial q_i} + \theta \frac{\partial p_i}{\partial q_i} \right) \right\}, i, j = 1,2, i \neq j. \tag{C.1}$$

The first term of eq. (C.1) is given by $\frac{\partial p_i}{\partial q_i} + \theta \frac{\partial p_j}{\partial q_i} = \frac{\varepsilon_i (2+\theta) + 2\beta_i}{\Lambda\Delta} > 0$. The second is given by $\frac{\partial p_j}{\partial q_i} + \theta \frac{\partial p_i}{\partial q_i} = \frac{\varepsilon_i (2+\theta) + \beta_i \theta}{\Lambda\Delta} > 0$. Thus, it holds that $\frac{\partial CS}{\partial q_i} > 0, i = 1,2$.

Furthermore, with respect to the effect on producer surplus, in view of eq. (10), we have $\frac{\partial \Pi_j}{\partial q_i} = 2\Lambda p_j \frac{\partial p_j}{\partial q_i}$. Thus, we derive

$$\left.\frac{\partial W}{\partial q_i}\right|_{\frac{\partial \Pi_i}{\partial q_i}=0} = \Lambda \left\{ \Lambda p_i \left(\frac{\partial p_i}{\partial q_i} + \theta \frac{\partial p_j}{\partial q_i} \right) + p_j \underline{\underline{\left((2+\Lambda) \frac{\partial p_j}{\partial q_i} + \theta \frac{\partial p_i}{\partial q_i} \right)}} \right\}. \tag{C.2}$$

With regard to the second term of eq. (C.2), i.e., the double underline, we obtain

$$(2+\Lambda) \frac{\partial p_j}{\partial q_i} + \theta \frac{\partial p_i}{\partial q_i} = \Lambda \left\{ (1-\theta)(2+\theta) \left[2(1-\theta^2) + \theta \right] \varepsilon_i + \theta^3 \beta_i \right\} > 0.$$

Because the first term mentioned above is positive, it holds that $\left.\frac{\partial W}{\partial q_i}\right|_{\frac{\partial \Pi_i}{\partial q_i}=0} > 0, i = 1,2$.

References

Albaek, S., 1990, 'Stackelberg leadership as a natural solution under cost uncertainty', *Journal of Industrial Economics*, 38, pp. 335–347

Aoki, R., 2003, 'Effect of credible quality investment with Bertrand and Cournot competition', *Economic Theory*, 21, pp. 653−672

Aoki, R. and Prusa, T. J., 1996, 'Sequential versus simultaneous choice with endogenous quality', *International Journal of Industrial Organization*, 15, pp. 103−121

Brander, J. and Spencer, B., 1983, 'Strategic commitment with R&D: The symmetric case', *Bell Journal of Economics*, 14, pp. 225–235

d'Aspremont, C. and Jacquemin, A., 1988, 'Cooperative and noncooperative R&D in duopoly with spillovers', *American Economic Review*, 78, pp. 1133−1137

Foros, Ø., Hansen, B., and Sand, J. Y., 2002, 'Demand-side spillovers and semi-collusion in the mobile communications market', *Journal of Industry, Competition and Trade*, 2, pp. 259–278

Häckner, J., 2000, 'A note on price and quantity competition in differentiated oligopolies', *Journal of Economic Theory*, 93, pp. 233−239

Hamilton, J. H. and Slutsky, S. M., 1990, 'Endogenous timing in duopoly games: Stackelberg or Cournot equilibria', *Games and Economic Behavior*, 2, pp. 29–46

Ishibashi, K. and Kaneko, T., 2008, 'Partial privatization in mixed duopoly with price and quality competition', *Journal of Economics*, 95, pp. 213−231

Levin, R. C. and Reiss, P. C., 1988, 'Cost-reducing and demand-creating R&D with spillover', *RAND Journal of Economics*, 19, pp. 538−556

Marshall, A., 1919, *Industry and Trade: A Study of Industrial Technique and Business Organization, and of their Influences on the Condition of Various Classes and Nations* (MacMillan and Co., London)

Ornaghi, C., 2006, 'Spillovers in product and process innovation: Evidence from manufacturing firms', *International Journal of Industrial Organization*, 24, pp. 349−380

Park, J.-H., 2001, 'Strategic R&D policy under vertically differentiated oligopoly', *Canadian Journal of Economics*, 34, pp. 967−987

Spencer, B. J. and Brander, J. A., 1983, 'International R & D rivalry and industrial strategy', *Review of Economic Studies*, 50, pp. 707−722

Symeonidis, G., 2003, 'Comparing Cournot and Bertrand equilibria in a differentiated

duopoly with product R&D', *International Journal of Industrial Organization*, 21, pp. 39–55

Toshimitsu, T. and Jinji, N. 2008, 'A note on strategic trade policy and endogenous quality choice', *Review of International Economics*, 16, pp. 173−185

Zhou, D., Spencer, S. M. and Vertinsky, I., 2002, 'Strategic trade policy with endogenous choice of quality and asymmetric cost', *Journal of International Economics*, 56, pp. 205–232

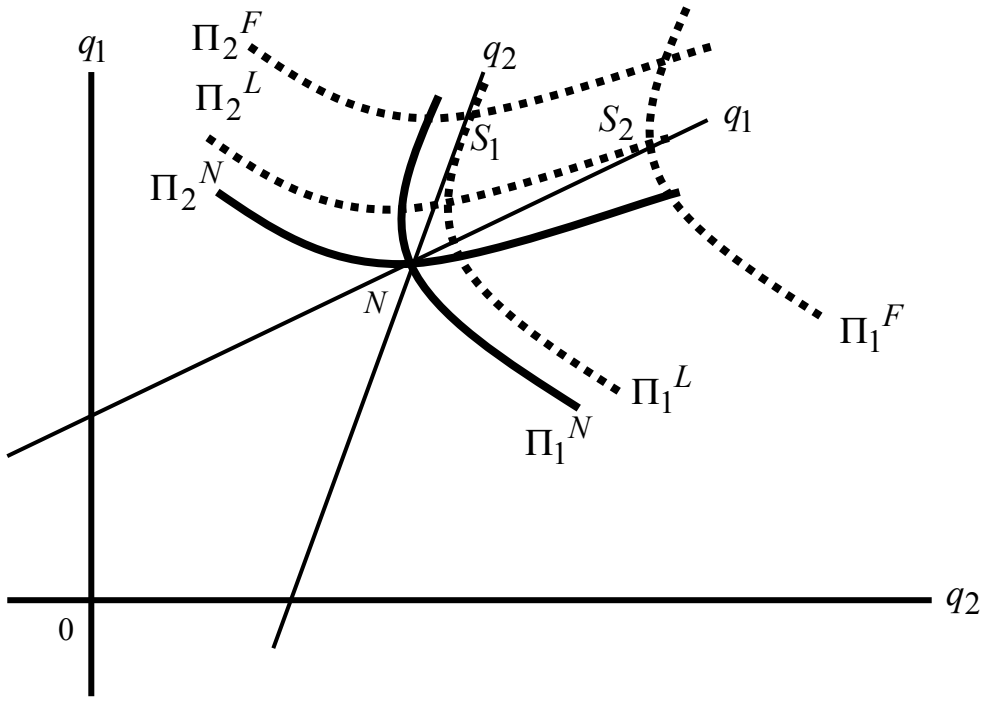

Figure 1 $\Theta_1 > \Theta_2 > \theta$

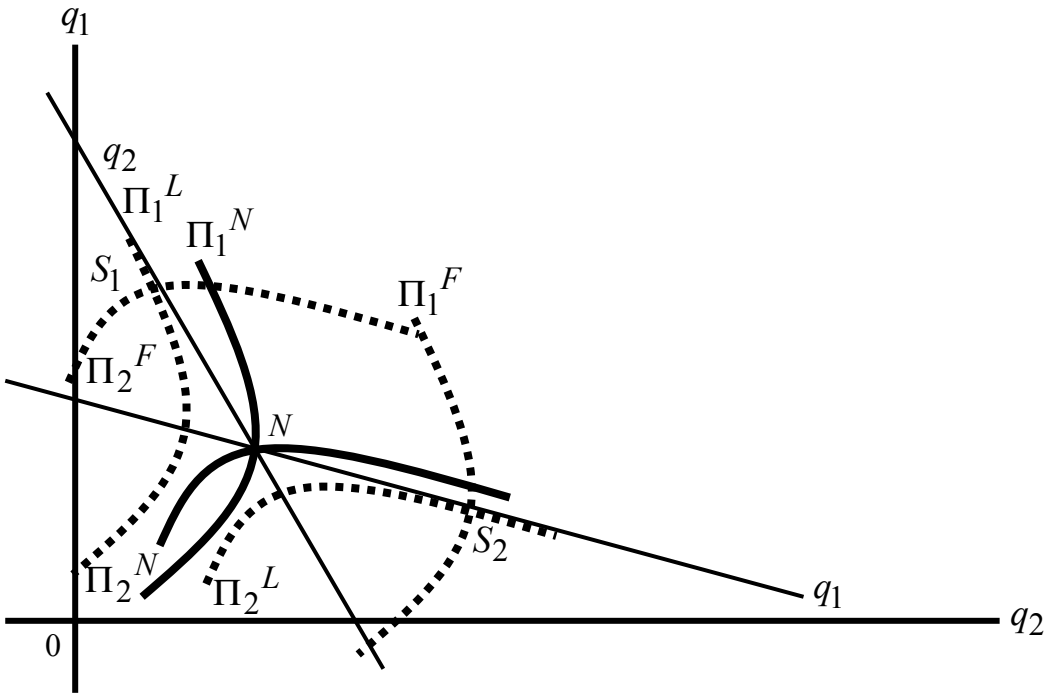

Figure 2 $\theta > \Theta_1 > \Theta_2$

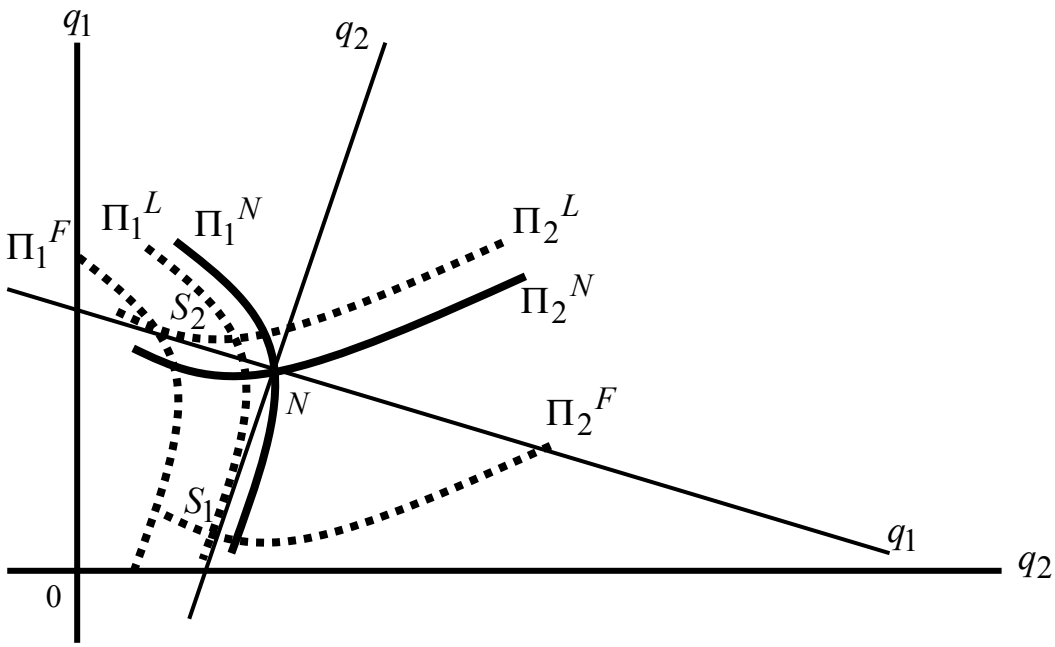

Figure 3 $\Theta_1 > \theta > \Theta_2$