DISCUSSION PAPER SERIES

Discussion paper No. 77

Voracity, growth and welfare

Kenji Fujiwara

School of Economics, Kwansei Gakuin University

September, 2011

MASTERY FOR SERVICE

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho

Nishinomiya 662-8501, Japan

Voracity, growth and welfare

Kenji Fujiwara*
School of Economics, Kwansei Gakuin University, 662-8501
Nishinomiya, Hyogo, Japan

Abstract

This paper explores some implications of the comparison between feedback Nash and Stackelberg equilibria for growth and welfare in a 'voracity' model. We show that as compared to the Nash equilibrium, the Stackelberg equilibrium involves a lower growth rate while it leaves both the leaders and the followers better off, i.e., the Stackelberg equilibrium is Pareto superior to the Nash equilibrium.

JEL classification: C73, O41

Keywords: Dynamic game, Growth, Welfare, Feedback Nash equilibrium, Feedback Stackelberg equilibrium

*Tel: +81-798-54-7066 Fax: +81-798-51-0944. E-mail: kenjifujiwara@kwansei.ac.jp.

1 Introduction

The last decades have witnessed rapid growth of an individual country and the world. Reflecting this observation, there has been a large literature of theories and evidences of economic growth.[1] Most of the previous works in this field have commonly adopted an assumption that there is no strategic interaction among agents. Relaxing this assumption, Tornell and Velasco (1992) make clear some interesting implications of a feedback Nash equilibrium of an AK model of endogenous growth, finding a possibility that a technological progress can reduce both the growth rate and welfare. This is because the technological progress boosts over-extraction of each consumer, and hence accelerates the tragedy of the commons. Tornell and Velasco (1992) call this perverse effect of a technological progress a voracity effect. This result indicates that conventional wisdom that rests on the assumption of no strategic interactions may be invalid in a strategic setting.

This paper seeks more about growth and welfare in a Tornell-Velasco (1992) model.[2] In particular, we pay special attention to the role of a leadership by deriving a feedback Stackelberg equilibrium of the Tornell-Velasco model. There exists a certain literature that extends the Tornell-Velasco model. Introducing a private asset, Tornell and Lane (1995) and Tornell (1998) make richer arguments on the voracity effect. Taking into account endogenous labor supply, Mino (2006) demonstrates that the growth rate is affected by the interplay between the voracity effect and the scale effect, the former of which has a negative impact on growth and the latter of which has a positive impact. None of these works examines a leader-follower model by focusing on the Nash equilibrium.

However, the heterogeneity of agents is profoundly observed and their action is far from simultaneous, which requires us to allow for a hierarchical play. To our knowledge, Shimomura (1991) is the first to formally characterize the feedback Nash and Stackelberg equilibria in a capitalistic game a la Lancaster (1973). While his focus is on the characterization of equilibria, we discuss the implications of the presence of a leadership for growth and welfare. After deriving the feedback Stackelberg equilibrium where there are an arbitrary number of leaders and followers, we show three main results. First, the

[1] See, for instance, Barro and Sala-i-Martin (2004) and Acemoglu (2007) for a comprehensive review.

[2] There is another strand of literature that introduces strategic interactions into an endogenous growth model, e.g., Vencatachellum (1998), Shibata (2002), and Dockner and Nishimura (2004, 2005). These papers are, however, mainly interested in the comparison among efficient, open-loop Nash, and feedback Nash equilibria, which differs much from this paper in scope.

voracity effect is larger in the Stackelberg equilibrium than in the Nash equilibrium. Second, the Stackelberg equilibrium involves a lower growth rate than the Nash equilibrium. Third, the Stackelberg equilibrium leaves both the leaders and the followers better off relative to the Nash equilibrium.

This paper is organized as follows. Section 2 briefly reviews the Tornell-Velasco (1992) model and the feedback Nash equilibrium. Section 3 turns to the feedback Stackelberg equilibrium. Section 4 compares these two equilibria and proves our main results. Section 5 concludes.

2 A model

The model is an extension of an AK model of Tornell and Velasco (1992). There are $m \geq 1$ leaders and $n \geq 1$ followers, both of which extract a renewable resource for their consumption. Thus, the problem of player j is formulated as

$$\begin{aligned} \max_{c_j} \quad & \int_0^\infty e^{-rt} \frac{c_j^{1-\theta}}{1-\theta} dt, \quad \theta \in (0,1) \\ \text{s.t.} \quad & \dot{k} = ak - c_j - \sum_{i \neq j} c_i, \quad a > 0. \end{aligned}$$

where c_j is consumption of player j, $r > 0$ is a discount rate, and k is a stock of a renewable resource. Tornell and Velasco (1992) compute the feedback Nash equilibrium in linear strategies in this model, according to which the equilibrium consumption and welfare per player are obtained as follows.[3]

$$c(k) = \left[\frac{r - (1-\theta)a}{1 - (m+n)(1-\theta)} \right] k \tag{1}$$

$$V(k_0) = \frac{k_0^{1-\theta}}{1-\theta} \left[\frac{r - (1-\theta)a}{1 - (m+n)(1-\theta)} \right]^{-\theta}, \tag{2}$$

where $V(\cdot)$ is a value function of each player. Following Tornell and Velasco (1992), let us make

Assumption 1. $r - (1-\theta)a < 0$ and $1 - (m+n)(1-\theta) < 0$.

Under this assumption, equilibrium consumption in (1) is positive, and welfare in (2) is bounded.

[3]Subscript j signifying players is dropped since we focus on the symmetric equilibrium.

3 A feedback Stackelberg equilibrium

This section turns to the leader-follower model in which each of m leaders announce a linear feedback strategy $\omega_i k$ before the n followers move. The game is solved with backward induction and the problem of a representative follower j is given by

$$\begin{aligned} \max_{c_j} \quad & \int_0^\infty e^{-rt} \frac{c_j^{1-\theta}}{1-\theta} dt \\ \text{s.t.} \quad & \dot{k} = ak - c_j - \sum_{i=1}^m \omega_i k - \sum_{l \neq j}^n c_l. \end{aligned}$$

To solve this problem, let us construct a Hamilton-Jacobi-Bellman equation of player j:

$$rV^j(k) = \max_{c_j} \left\{ \frac{c_j^{1-\theta}}{1-\theta} + V_k^j(k) \left(ak - c_j - \sum_{i=1}^m \omega_i k - \sum_{l \neq j}^n c_l \right) \right\}, \tag{3}$$

where $V_k^j(\cdot) \equiv dV^j(\cdot)/dk$. The first-order condition for maximizing the right-hand side is

$$c_j = \left[V_k^j(k) \right]^{-\frac{1}{\theta}}. \tag{4}$$

Guessing that $V^j(k) = Ak^{1-\theta}/(1-\theta)$, (4) becomes $c_j = A^{-1/\theta}k$. Substituting these into (3), we have an identity in k:

$$\frac{rAk^{1-\theta}}{1-\theta} = \frac{A^{\frac{\theta-1}{\theta}} k^{1-\theta}}{1-\theta} + Ak^{1-\theta} \left(a - nA^{-\frac{1}{\theta}} - \sum_{i=1}^m \omega_i \right).$$

The undetermined coefficient A that satisfies this identity is obtained as

$$A = \left[\frac{r - (1-\theta)\left(a - \sum_{i=1}^m \omega_i\right)}{1 - n(1-\theta)} \right]^{-\theta}, \tag{5}$$

and the equilibrium consumption of the followers is

$$c_j(k) = \left[\frac{r - (1-\theta)\left(a - \sum_{i=1}^m \omega_i\right)}{1 - n(1-\theta)} \right] k. \tag{6}$$

We now turn to the problem of leaders. Given the announced strategy of leader i, $\omega_i k$, and the followers' strategy (6), the resource dynamics is rewritten as

$$\begin{aligned} \dot{k} &= ak - \sum_{i=1}^m \omega_i k - n \cdot \frac{r - (1-\theta)\left(a - \sum_{i=1}^m \omega_i\right)}{1 - n(1-\theta)} \\ &= \frac{a - nr - \sum_{i=1}^m \omega_i}{1 - n(1-\theta)}, \end{aligned}$$

the solution of which is explicitly computed:

$$k(t) = k_0 e^{\frac{a-nr-\sum_{i=1}^{m}\omega_i}{1-n(1-\theta)}t}.$$

Substituting this into the strategy of leader i and its discounted stream of utility, we have

$$\begin{aligned}
& \int_0^\infty e^{-rt}\frac{(\omega_i k)^{1-\theta}}{1-\theta}dt \\
= & \int_0^\infty e^{-rt}\frac{\omega_i^{1-\theta}k_0^{1-\theta}}{1-\theta}e^{\frac{(1-\theta)\left(a-nr-\sum_{i=1}^{m}\omega_i\right)}{1-n(1-\theta)}t}dt \\
= & \frac{\omega_i^{1-\theta}k_0^{1-\theta}}{1-\theta}\int_0^\infty e^{\frac{-r[1-n(1-\theta)]+(1-\theta)\left(a-nr-\sum_{i=1}^{m}\omega_i\right)}{1-n(1-\theta)}t}dt \\
= & k_0^{1-\theta}\cdot\frac{1-n(1-\theta)}{1-\theta}\cdot\frac{\omega_i^{1-\theta}}{r-(1-\theta)\left(a-\sum_{k=1}^{m}\omega_i\right)}, \qquad (7)
\end{aligned}$$

which is to be maximized by player i.

Each leader chooses ω_i to maximize (7), which involves the first-order condition:

$$\frac{(1-\theta)\omega_i^{-\theta}\left[r-(1-\theta)\left(a-\sum_{k=1}^{m}\omega_k\right)-\omega_i\right]}{\left[r-(1-\theta)\left(a-\sum_{k=1}^{m}\omega_k\right)\right]^2}=0.$$

From this equation, and focusing on a non-zero strategy, any leader i chooses $\omega_i = r-(1-\theta)\left(a-\sum_{k=1}^{m}\omega_k\right)$, which implies that all the leaders choose the same strategy. In the symmetric equilibrium where $\omega_i=\omega_k$, we have

$$c_i(k) = \omega_i k = \frac{r-(1-\theta)a}{1-m(1-\theta)}k. \qquad (8)$$

Substituting this into (6), the follower's strategy becomes

$$c_j(k) = \frac{r-(1-\theta)(a-m\omega_i)}{1-n(1-\theta)}k = \frac{r-(1-\theta)a}{[1-n(1-\theta)][1-m(1-\theta)]}k. \qquad (9)$$

As in the Nash case, we introduce the following restrictions on the parameters.

Assumption 2. $1-m(1-\theta)<0$ and $1-n(1-\theta)>0$, i.e., $m>1/(1-\theta)>n$.

The inequality $1-m(1-\theta)<0$ is used to ensure the positivity of ω_i in (8) and the inequality $1-n(1-\theta)>0$ is analogously adopted to guarantee $c_j(k)>0$.

4 Comparison of Nash and Stackelberg equilibria

Having derived the Nash and Stackelberg equilibria in linear feedback strategies, we readily compare how the two regimes affect the growth rate and welfare.

4.1 Growth rates

Making use of (1), the growth rate in the feedback Nash equilibrium is

$$g^N \equiv \left(\frac{\dot{k}}{k}\right)^N = a - (m+n)\frac{r-(1-\theta)a}{1-(m+n)(1-\theta)} = \frac{a-r(m+n)}{1-(m+n)(1-\theta)}, \tag{10}$$

where superscript N refers to the Nash equilibrium. In a similar way, Eqs. (8) and (9) allow us to compute the growth rate in the feedback Stackelberg equilibrium:

$$\begin{aligned} g^S &\equiv \left(\frac{\dot{k}}{k}\right)^S = a - m\frac{r-(1-\theta)a}{1-m(1-\theta)} - n\frac{r-(1-\theta)a}{[1-n(1-\theta)][1-m(1-\theta)]} \\ &= \frac{a-r[m+n-mn(1-\theta)]}{[1-m(1-\theta)][1-n(1-\theta)]}. \end{aligned} \tag{11}$$

where superscript S stands for the Stackelberg equilibrium. Given Assumptions 1 and 2, we can confirm the voracity effect of a technological progress both in the Nash and in the Stackelberg equilibrium, namely, $\partial g^N/\partial a < 0$ and $\partial g^S/\partial a < 0$. In addition, we can prove:

Lemma 1. The voracity effect is stronger in the feedback Stackelberg equilibrium than in the feedback Nash equilibrium.

Proof. Under Assumptions 1 and 2, the effect of an increase in a on the growth rate in each equilibrium is

$$\begin{aligned} \frac{\partial g^N}{\partial a} &= \frac{1}{1-(m+n)(1-\theta)} < 0 \\ \frac{\partial g^S}{\partial a} &= \frac{1}{[1-m(1-\theta)][1-n(1-\theta)]} < 0. \end{aligned}$$

Subtracting the former from the latter yields

$$\frac{\partial g^S}{\partial a} - \frac{\partial g^N}{\partial a} = -\frac{mn(1-\theta)^2}{[1-m(1-\theta)][1-n(1-\theta)][1-(m+n)(1-\theta)]} < 0.$$

that is, it follows that $\partial g^S/\partial a < \partial g^N/\partial a < 0$. Thus, the detrimental effect of a rise in a on the growth rate is stronger in the Stackelberg case than in the Nash case. **Q.E.D.**

The reason behind this result is obvious in view of the property of the strategic complement between the strategy of leaders and followers. In the Stackelberg equilibrium, both class of players opt for higher consumption than in the Nash equilibrium. A technological progress in the form of a rise in a accelerates this tendency for the tragedy of the commons and hence its detrimental effect on the growth rate is also enhanced. Let us turn to the comparison between g^N and g^S. Since subtracting g^N from g^S yields

$$g^S - g^N = \frac{mn(1-\theta)[r-(1-\theta)a]}{[1-m(1-\theta)][1-n(1-\theta)][1-(m+n)(1-\theta)]} < 0,$$

by noting Assumptions 1 and 2, we have established:

Proposition 1. The growth rate is higher in the feedback Nash equilibrium than in the feedback Stackelberg equilibrium.

(Figure 1 around here)

The intuition behind this result is well understood by using Figure 1. This figure depicts the relationship between ω_i and ω_j in a two-player case. From (6) and the assumption that $1 - n(1-\theta) > 0$, there is a strategic complement between ω_i and ω_j. In the Nash game, the equilibrium is determined by the intersection of the two reaction curves and given by N. If, on the other hand, player i is a leader, it chooses ω_i such that its iso-welfare curve is tangent to the reaction curve of the follower (player j). Therefore, the Stackelberg equilibrium is obtained by S. As is clear from the figure, both players consume more in the Stackelberg equilibrium than in the Nash equilibrium. That is, the Stackelberg equilibrium makes the tragedy of the commons stronger and hence the associated growth rate is smaller than the growth rate in the Nash equilibrium.

4.2 Welfare

While Proposition 1 is concerned with how the presence of a leadership influences the equilibrium growth rate, we now consider welfare aspects of it. For this purpose, we compare the payoff level of the follower in the two regimes. Substituting (8) into (5), the follower's welfare in the Stackelberg equilibrium is

$$V^j(k_0) = \frac{k_0^{1-\theta}}{1-\theta}\left[\frac{r-(1-\theta)(a-m\omega_i)}{1-n(1-\theta)}\right]^{-\theta} = \frac{k_0^{1-\theta}}{1-\theta}\left\{\frac{r-(1-\theta)a}{[1-m(1-\theta)][1-n(1-\theta)]}\right\}^{-\theta}. \tag{12}$$

Therefore, (2) and (12) yield the difference in the equilibrium payoff of the follower in the Stackelberg and Nash equilibria:

$$V^j(k_0) - V(k_0) = \frac{k_0^{1-\theta}}{1-\theta}\left[\frac{r-(1-\theta)a}{1-(m+n)(1-\theta)}\right]^{-\theta}\left[\left\{\frac{[1-m(1-\theta)][1-n(1-\theta)]}{1-(m+n)(1-\theta)}\right\}^{\theta} - 1\right] > 0.$$

Thus, the Stackelberg equilibrium leaves the follower better off than the Nash equilibrium. Noting that the Stackelberg leader always enjoys a higher payoff than in the Nash equilibrium, we can state:

Proposition 2. Both the leader and follower are better off in the feedback Stackelberg equilibrium than in the feedback Nash equilibrium.

Figure 1 gives a diagrammatic representation Proposition 2. In the figure, it is clear that both the leader (player i) and the follower (player j) at S achieve higher welfare than at N. The reason for this finding is as follows. In the present model, the leaders have an incentive to extract the common resource to enjoy a higher utility by moving first. In response to this behavior of the leaders, the followers also increase their consumption than the Nash level. This bilateral increase in consumption enhances welfare of both all players as compared to the Nash case because they commit to increased consumption in every point of time.

Finally, we address why the leader-follower game enhances a voracity but improves welfare of both classes of players. To seek the reason, let us consider what happens if we have a technological progress in the form of increased a. This encourages capital accumulation as a direct through the capital accumulation equation. Taking into account the resulting increase in k, all players expand their consumption, which applies to both the Nash game and the Stackelberg game, and leads to the voracity effect on growth. Comparing the two games, the Stackelberg leaders commit to more consumption than the Nash level by taking advantage of their leadership. Observing this commitment of the leaders, the followers also consume more than the Nash level because otherwise their utility decrease as a result of the leaders' aggressive behavior. Accordingly, capital accumulation is slower, namely, the voracity effect is stronger in the Stackelberg game than in the Nash game. What is striking that this stronger voracity raises welfare of not only the leaders but also the followers. This is because the leaders' precommitment to more consumption encourages the followers to consume more in *every* point of time, and improves the followers' utility. That is, the strategic complement property of the model causes a Pareto superiority of the Stackelberg equilibrium over the Nash equilibrium although the former involves a stronger voracity than the latter.

5 Concluding remarks

We have extended a dynamic game model of endogenous growth to accommodate the presence of leaderships to address how it affects the growth rate and welfare. It has been established that the growth rate is lower but welfare of both classes of players is higher in the feedback Stackelberg equilibrium than in the feedback Nash equilibrium. While the above result has a certain novelty, it is admittedly proved in the simplest version of the Tornell-Velasco (1992) model. It is an interesting extension to allow for private assets as in the second model of Tornell and Velasco (1992) and endogenous labor supply as in Mino (2001).

Acknowledgements

I gratefully acknowledge helpful comments from Ryo Arawatari, Yoichi Gokan, Yunfang Hu, Shin Imoto, Kazuo Mino, Keiichi Morimoto, Jumpei Tanaka, and an anonymous referee. Any remaining error is my own responsibility.

References

Acemoglu, D., 2009. Introduction to modern economic growth. Princeton University Press, Princeton.

Barro, R.J., Sala-i-Martin, X., 2003. Economic growth. MIT Press, Cambridge.

Dockner, E.J., Nishimura, K., 2004. Strategic growth. Journal of Difference Equations and Applications 10, 515-527.

Dockner, E.J., Nishimura, K., 2005. Capital accumulation games with a non-concave production function. Journal of Economic Behavior and Organization 57, 408-420.

Lancaster, K., 1973. The dynamic inefficiency of capitalism. Journal of Political Economy 81, 1092-1109.

Mino, K., 2006. Voracity vs. scale effect in a growing economy without secure property rights. Economics Letters 93, 278-284.

Shibata, A., 2002. Strategic interactions in a growth model with public infrastructure. Metroeconomica 53, 434-460.

Shimomura, K., 1991. The feedback equilibria of a differential game of capitalism. Journal of Economic Dynamics and Control 15, 317-338.

Tornell, A., 1999. Voracity and growth in discrete time. Economics Letters 62, 139-145.

Tornell, A., Lane, P.R., 1999. The voracity effect. American Economic Review 89, 22-46.

Tornell, A., Velasco, A., 1992. The tragedy of the commons and economic growth: why does capital flow from poor to rich countries? Journal of Political Economy 100, 1208-1231.

Vencatachellum, D., 1998. A differential R&D game: implications for knowledge-based growth models. Journal of Optimization Theory and Applications 96, 175-189.

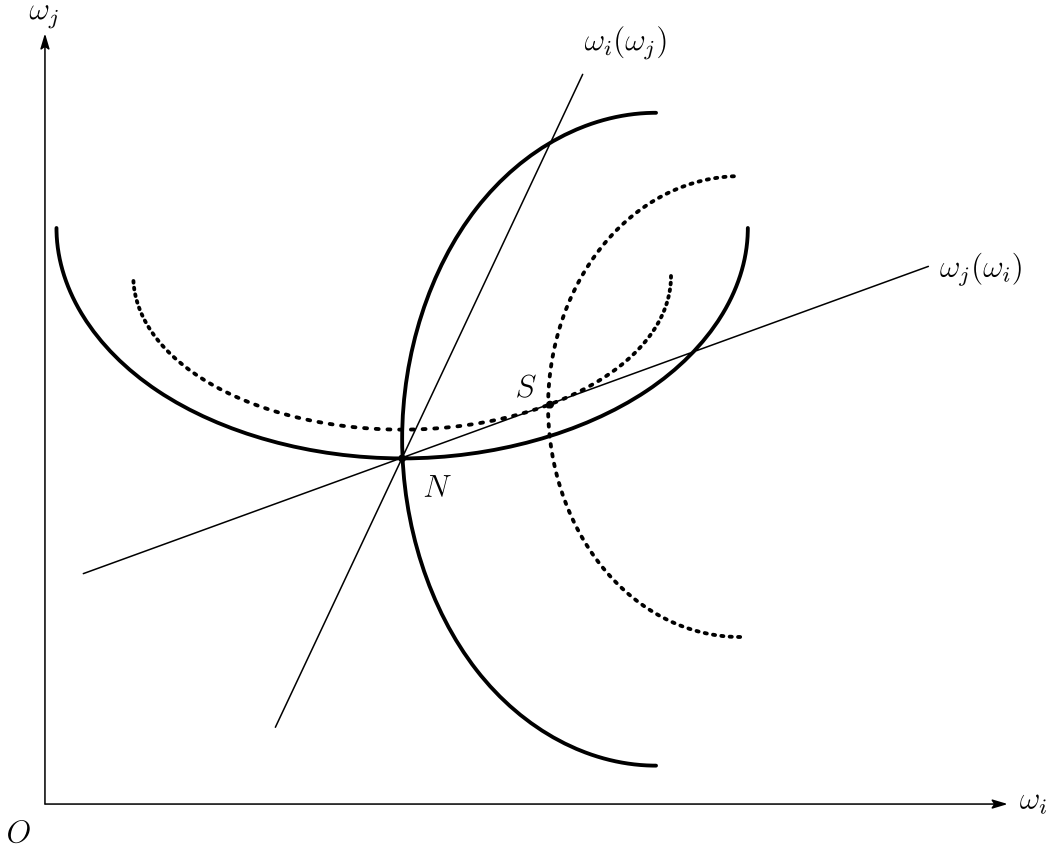

Figure 1: Nash and Stackelberg equilibria