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# On Environmental Subsidy/Tax Policy with Heterogeneous Consumers: An Application of an Environmentally Differentiated Duopoly Model

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# On Environmental Subsidy/Tax Policy with Heterogeneous Consumers: An Application of an Environmentally Differentiated Duopoly Model

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Abstract

We apply a model of an environmentally differentiated duopoly to the analysis of environmental policy in the form of a subsidy/tax on consumers based on emission levels of products. More specifically, we consider environmental and welfare effects of subsidizing consumers who purchase environmental-friendly goods such as hybrid vehicles. Focusing on types of market coverage by heterogeneous consumers, we examine the issue in the cases of a Bertrand and a Cournot duopoly. In the case of full market coverage with a Bertrand duopoly, an environmental subsidy improves the environment and is socially optimal. However, in the case of partial market coverage, irrespective of mode of competition, the optimal policy depends on the magnitude of the marginal social valuation of environmental damage. That is, if the marginal social valuation of environmental damage is sufficiently large (small), an environmental tax (subsidy) is optimal. Furthermore, in the Bertrand duopoly case, the effect of subsidy on the environment is ambiguous, whereas in the Cournot duopoly case, the subsidy degrades the environment.

JEL classification: D43; H23; L13
Keywords: Environmentally differentiated product; Environmental subsidy/tax; Green market; Bertrand and Cournot duopoly

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# 1. Introduction

A growing demand for environmental care has been observed. That is, recently, consumers purchase goods and services that are environment-friendly or produced with environment-friendly techniques. There is also sufficient evidence to suggest that firms are aware of consumers' behavior in this regard, and that they invest funds in environment-friendly goods, product lines and facilities. Governments in many advanced countries also regulate polluting emissions, environmental waste, and global warming using various environmental policies, including taxes/subsidies, emission standards, tradable emission permits, and eco-labeling, amongst others. Moreover, even local governments in many advanced countries, e.g., the Tokyo Metropolitan Government, the Osaka Prefectural Government, and others, are currently addressing environmental problems such as air pollution.

Previously, many seminal works in the field of environmental economics (Baumol and Oates, 1988, and others) have primarily dealt with polluting wastes or environmental effluents which are the by-products of production process, such as those found in chemical industries. Accordingly, they have mostly considered environmental policies associated with producers. In this paper, however, we focus on products with environmental characteristics in a green market where effluents and noises are the by-products of consumption of heterogeneous 'consumers' who differ in terms of their willingness to pay for the products according to a product's environmental quality.[1] For example, from the point of view of the life cycle of manufacturing products such as vehicles, the volume of $CO_2$ gas produced in the process of consumption is likely to be larger than that in the process of production. Furthermore, the environmental damage caused by the polluting wastes and effluents associated with consumption of the products seems to be external for individual consumers, i.e., environmental negative externalities. However, some consumers who are very conscious of environmental degradation may purchase an environment-friendly product, even if its price is substantially higher than that of an environment-unfriendly product, while other consumers who are not concerned about the environment may purchase a lower priced product, even if it is environment-unfriendly. That is, consumers differ in their degree of consciousness about the environment (Scherhorn, 1993). For example, in the context of car exhaust fumes, the emission level of a hybrid engine motor vehicle such as a Toyota Prius is much lower than that of a wholly gasoline engine motor

[1] In this paper, 'consumers' not only implies households driving cars but also companies using vehicles for transportation. In a sense, 'consumers' are 'users' of environmentally differentiated products.

vehicle. Hence, consumers who care about the environment may prefer the hybrid vehicle, whereas others do not. In addition, more extreme environmentalists may not purchase any type of car, but instead would ride a bicycle or use public transportation such as a train (Kahn, 2007).

A number of papers employ a model of vertically differentiated products to examine environmental subsidy/tax policies with heterogeneous consumers. Cremer et al. (1998, 2003) theoretically and empirically analyze the optimal tax design in the presence of environmental externalities. In realty, governments in many advanced countries allocate tax credits, i.e., a kind of subsidy, to consumers who purchase environment-friendly goods. For example, in Japan the Ministry of the Environment enforces a taxation courtesy system for the introduction of low-emission vehicles. That is, owners of an eco-car receive tax credits, while owners of diesel vehicles incur heavier taxes. Furthermore, the Tokyo Metropolitan Government encourages owners of small-sized companies to purchase low-emission vehicles through the provision of subsidies and financing.[2]

The purpose of this paper is to examine how emission regulations by subsidies/taxes levied on consumers and/or producers of an environment-friendly good, i.e., a cleaner product, affect the environment and social welfare, by applying the model of environmentally differentiated products presented by Moraga-González and Pandrón-Fumero (2002). We address this issue in the cases of a Bertrand and a Cournot duopoly, focusing on types of market coverage by heterogeneous consumers. That is, we deal with the full market coverage type, in which all consumers in the market necessarily purchase either product, and the partial market coverage type, in which some consumers do not purchase any products in the market.

A number of previous studies are closely related to the analysis undertaken here. Bansal and Gangopadhyay (2003) examine ad valorem commodity subsidies/taxes on environmentally differentiated products in the case of a Bertrand duopoly with partial market coverage.[3] They show that discriminating commodity subsidy policy is welfare-superior to discriminating commodity tax policy. However, they did not analyze the Cournot duopoly case. Also, in the context of the linkage between tariff policy and the environment, Toshimitsu (2008) analyzes

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[2] On the environmental policy of the Tokyo Metropolitan Government, see
http://www.metro.tokyo.jp/ENGLISH/POLICY/environment.htm
Furthermore, on recent studies of air pollution regulation in urban transportation, for example, see Nash et al. (2001), Proost and Van Dender (2001), Ferrara (2007), Parry (2007).

[3] Bansal and Gangopadhyay's (2003) model is related to Cremer and Thisse's (1999) endogenous quality choice model used to analyze an ad valorem commodity tax/subsidy policy in oligopolistic price competition with full market coverage. However, it is assumed in the model of Cremer and Thisse that a firm incurs marginal costs of production as an increasing function of its quality level.

how ad valorem import tariffs levied on a cleaner and a dirtier product affect the environment and welfare in the cases of a Bertrand and a Cournot duopoly with partial market coverage. He finds that the effect of an ad valorem tariff policy on the environment and welfare depends on the mode of competition and the degree of social valuation of environmental damage. As shown below, we deal with an environmental subsidy/tax policy with consumers and/or a firm, associated with the difference in the emission levels of products, but not a commodity subsidy/tax policy, in the cases of a Bertrand and a Cournot duopoly.

Lombardini-Riipinen (2005) considers a mixed policy with a uniform ad valorem commodity tax and an emission (or environmental) subsidy/tax with full market coverage, assuming that a firm incurs marginal costs of production as an increasing function of its abatement efforts. He discusses the impact of optimal first-best policies in the context, including ad valorem commodity or emission taxes on firms, or ad valorem commodity taxes on firms and emission subsidies on consumers. In addition, he argues that the second-best subsidy for consumers and the second-best emission tax on a firm should be set equal to the marginal social value of the environmental damage.

Although our model is closely related to Lombardini-Riipinen (2005), we instead assume a fixed cost associated with a unit emission level, i.e., environmental quality, and analyze both types of full and partial market coverage. In addition, we mainly address a second-best environmental subsidy/tax policy with consumers and/or a firm in cases of a Bertrand and a Cournot duopoly. Furthermore, if we discuss a mixed policy with a commodity tax/subsidy and an emission tax/subsidy, we should consider that the government authority choosing a commodity tax might not necessarily be identical to the authority choosing an emission (or environmental) tax. For example, in Japan, responsibility for the former falls under the control of the Ministry of Economy, Trade and Industry, whereas the Ministry of the Environment administers the latter. Although the analysis of potentially conflicting choices of different authorities within a government is an interesting issue, it can also be complex. Therefore, in this paper, we deal with the environmental subsidy/tax policy of a single government.

The remainder of this paper comprises the following three sections. Section 2 sets the basic model. Section 3 examines the effect of environmental subsidies on aggregate emissions and social welfare in the case of full market coverage with a Bertrand duopoly. We show that an environmental subsidy is socially optimal. Furthermore, in the cases of partial market coverage with a Bertrand and a Cournot duopoly, we show that the optimal policy depends on the magnitude of the marginal social valuation of environmental damage, regardless of the mode of competition. Finally, Section 4 summarizes the results and raises some outstanding issues.

## 2. The Model

*(a) Market and Demands*

We begin by describing a green market in which consumers have heterogeneous preferences for the quality of environmentally differentiated products. That is, a continuum of heterogeneous consumers exists who differ in their marginal valuations $\theta$ of the green features of products. To simplify, we assume that the consumer-matching value is uniformly distributed with density one and falls in the range $\theta \in \left[0, \bar{\theta}\right]$. That is, consumer $\theta$ close to $\bar{\theta}(0)$ is very sensitive (insensitive) to the environmental qualities of products.

Let $e$ denote the observable unit emission level associated with the product $e \in (0, \infty)$. Without losing generality, we assume that firm *C* (*D*) supplies a cleaner (dirtier) product with a per unit emission of $e_C$ ($e_D$) at a price of $p_C$ ($p_D$) and $e_D \geq e_C > 0$. A consumer purchases at most either one or zero unit of the product. Hence, the surplus of consumer $\theta$ who acquires the variant $e$ at a price of $p$ is given by[4]:

$$u = \max\{v - e\theta - p + dS, 0\}, \tag{1}$$

where $v$ is the intrinsic utility obtained from a single unit of the product, irrespective of the variant's unit emission level, and $d$ is a dummy variable with $d = 1$ if $e = e_C$ and $d = 0$ if $e = e_D$. Furthermore, $S = s(e_D - e_C)$ is a subsidy paid to the consumer purchasing a cleaner product. If $s < 0,$ then the government levies emission taxes on the consumer.

Because we focus on how an environmental subsidy/tax policy has an impact on the behavior of firm *C* producing a cleaner product, let us normalize the unit emission level of a dirtier product to unity: $e_D = 1$ and $e_C = \varepsilon \leq 1$.

We now consider the demands for the environmentally differentiated products. The index of

[4] The utility function as in (1) implies that individual consumers do not consider environmental damage to the whole economy such as acid rain, global warming, and air pollution, but are sensitive to the environmental qualities of the relevant products. However, we introduce the social valuation of environmental damage into the utility function of an individual consumer as follows: $u = \max\{v - e\theta - p + dS - \gamma E, -\gamma E\}$. Hence, consumers who do not purchase any products in the market only suffer from environmental damage caused by aggregate emissions. Thus, the reservation utility is expressed as $-\gamma E$, not zero. Regardless, our results are not substantially changed.

the marginal consumer who is indifferent between the surplus given by purchasing a dirtier and a cleaner product is characterized by $\tilde{\theta} = \frac{p_C - p_D}{1-\varepsilon} - s$. Furthermore, the index of the marginal consumer who is indifferent between the surplus given by purchasing a cleaner product and nothing is characterized by $\hat{\theta} = \frac{v - p_C + s(1-\varepsilon)}{\varepsilon}$. Thus, consumer $\theta$ falling into $0 \le \theta < \tilde{\theta}$ $(\tilde{\theta} < \theta < \hat{\theta})$ purchases a dirtier (cleaner) product. Accordingly, there exist two types of the market if the following conditions hold.

Case FMC: Full market coverage, if $\hat{\theta} \ge \overline{\theta}$.

Case PMC: Partial market coverage, if $\hat{\theta} < \overline{\theta}$.

With respect to Case FMC, if $v$ is sufficiently large, consumers are willing to pay, even for a dirtier product. This is because the direct intrinsic utility is large enough to compensate for the loss of utility caused by emissions of the product. Thus, the type of full market coverage holds. This is where all consumers necessarily purchase either product. In this case, let $q_D$ represent the quantity demanded for a dirtier product. Given a uniform distribution, demand is given by $q_D = \tilde{\theta}$. Also, the quantity demanded for a cleaner product is given by $q_C = \overline{\theta} - \tilde{\theta}$. Therefore, the direct demand functions are given by:

$$q_D = \frac{P_C - P_D}{1-\varepsilon} - s \text{ and } q_C = \overline{\theta} + s - \frac{P_C - P_D}{1-\varepsilon}. \quad (2)$$

Because the corresponding inverse demand functions are not derived from (2), we do not deal with the case of full market coverage with a Cournot duopoly.

On the other hand, with regard to Case PMC, consumer $\theta$ falling into $\hat{\theta} < \theta \le \overline{\theta}$ does not purchase anything in the market. The condition holds if $v$ is not large. That is, consumers are not willing to pay, even for a cleaner product, because the direct intrinsic utility is not large enough to compensate for the loss of utility caused by emissions of the product. Therefore, in the case of PCM, the direct demand functions for both products are given by:

$$q_D = \frac{P_C - P_D}{1-\varepsilon} - s \text{ and } q_C = \frac{(1-\varepsilon)v - P_C + \varepsilon P_D}{\varepsilon(1-\varepsilon)} + \frac{s}{\varepsilon}. \quad (3)$$

Furthermore, in view of (3), the corresponding inverse demand functions are given by:

$$p_D = v - q_D - \varepsilon q_C \text{ and } p_C = v - \varepsilon(q_D + q_C) + s(1-\varepsilon). \quad (4)$$

*(b) Firms and Profits*

Let us explain the firms producing the products in the market. The firms have to decide on a unit emission level, i.e., environmental quality, of the product before competition in price or in quantity. That is, they invest in environmental research and development (R&D) or build a product line associated with the environmental qualities. As discussed earlier, because we focus on the decision of a unit emission level by firm *C*, we assume that the cost function is given by: $F_C = F_C(\varepsilon),\ F_C'(\varepsilon) < 0,\ F_C''(\varepsilon) \geq 0.$ We also assume that the cost of firm *D* is constant with respect to the unit emission level: $\overline{F}_D = F_D(1).$ Furthermore, for simplicity, the marginal costs of production are independent of a unit emission level, and assumed to be zero.

We assume that the government subsidizes firm *C* an amount of $\Sigma = \sigma(1-\varepsilon)$ per output in order to help firm *C* produce a much cleaner product. If $\sigma < 0,$ then the government levies emission taxes on firm *C*. Therefore, the profit functions of the firms are expressed by: $\Pi_D = p_D q_D - \overline{F}_D$ and $\Pi_C = (p_C + \Sigma)q_C - F_C(\varepsilon).$

*(c) Government and Social Welfare*

We proceed to the composition of social welfare. First, aggregate emissions that cause environmental degradation are expressed as:

$$E = q_D + \varepsilon q_C. \tag{5}$$

Second, under the full market coverage type, aggregate consumer surplus is expressed as:

$$CS = \int_0^{\tilde{\theta}} (v-\theta)d\theta - p_D q_D + \int_{\tilde{\theta}}^{\overline{\theta}} \{v - \varepsilon\theta + s(1-\varepsilon)\}d\theta - p_C q_C. \tag{6}$$

Furthermore, because there are three kinds of consumers under the partial market coverage type, aggregate consumer surplus can be represented by:

$$CS = \int_0^{\tilde{\theta}} (v-\theta)d\theta - p_D q_D + \int_{\tilde{\theta}}^{\hat{\theta}} \{v - \varepsilon\theta + s(1-\varepsilon)\}d\theta - p_C q_C + 0. \tag{7}$$

Third, the producer surplus that implies aggregate industrial profits is expressed as:

$$PS = \Pi_D + \Pi_C. \tag{8}$$

Therefore, given (5), (6) or (7), and (8), the net social surplus included in the social valuation of the environmental damage is defined by:

$$W \equiv CS - \gamma E + PS - \Omega, \tag{9}$$

where $\gamma(\geq 0)$ denotes the marginal social valuation of environmental damage. Furthermore, $\Omega = (S + \Sigma)q_C = (s + \sigma)(1-\varepsilon)q_C$ represents incurred budget deficits of the government subsidizing consumers and firm *C*.

Here we should consider the characteristics of the government. That is, it is plausible for a central government such as the Japanese government to determine environmental subsidies/taxes to maximize the net social surplus given by (9). However, suppose, for example, that the consumers are Tokyo residents, but the firms are not located in Tokyo; rather, they are in Osaka or abroad. In this case, the purpose of the local government such as the Tokyo Metropolitan Government is to maximize the net social welfare of the Tokyo residents represented by $W^L \equiv CS - \gamma E - Sq_C$.

Accordingly, the local government would decide the optimal subsidies with the residential consumers, but might never subsidize firm *C*. This is because the local government, as well as the residents, would not bear the burden of the lump-sum tax needed to subsidize the firm. Thus, we are able to assume that the local government determines an environmental subsidy/tax on the residential consumers to maximize $W^L$, whereas the central government decides an environmental subsidy/tax on firm *C* to maximize (9). However, as mentioned earlier, we do not consider the issues of policy games of an environmental subsidy/tax between local and central governments and of policy conflict or cooperation between them. Therefore, in what follows, we deal with an environmental subsidy/tax policy of a single government maximizing the net social surplus given by (9).

We present a three-stage game as follows. In the first stage, the government subsidizes/taxes consumers and/or firm *C*. In the second stage, firm *C* decides a unit emission level of the product, given the subsidy/tax. In the final stage, the firms compete in price or in quantity in the market, given the subsidy/tax and the unit emission level. The solution of this game is the subgame perfect Nash equilibrium.

# 3. The Effects of Environmental Subsidy/Tax Policy

## *3.1 The Full Market Coverage Type*

Because the derivation of the Bertrand-Nash equilibrium in the final stage is simple, we only present the equilibrium outcomes as follows.

$$p_D = \frac{(1-\varepsilon)\{\bar{\theta} - (s+\sigma)\}}{3} \text{ and } p_C = \frac{(1-\varepsilon)(2\bar{\theta} + s - 2\sigma)}{3}. \tag{10}$$

Substituting (10) into (2), the equilibrium quantities are given by:

$$q_D = \frac{\bar{\theta} - (s+\sigma)}{3} \text{ and } q_C = \frac{2\bar{\theta} + (s+\sigma)}{3}, \tag{11}$$

where $\frac{\partial q_D}{\partial \omega} = -\frac{\partial q_C}{\partial \omega} < 0,\ \omega = s, \sigma.$ That is, the subsidy has an impact on market share between the products. Furthermore, an increase in the unit emission level of a cleaner product reduces prices, but does not affect quantities.

In the second stage, the first-order profit maximization condition for firm *C* is given by:

$$\frac{\partial \Pi_C}{\partial \varepsilon} = -\frac{(2\bar{\theta} + s + \sigma)^2}{9} - \frac{\partial F_C}{\partial \varepsilon} = 0, \tag{12}$$

where the second-order condition always holds. We consider the effect of an environmental subsidy on a unit emission level:

$$\frac{d\varepsilon}{d\omega} = -\frac{\partial^2 \Pi_C / \partial\varepsilon\partial\omega}{\partial^2 \Pi_C / \partial\varepsilon^2} > (\leq) 0 \Leftrightarrow \frac{\partial^2 \Pi_C}{\partial\varepsilon\partial\omega} > (\leq) 0, \omega = s, \sigma. \tag{13}$$

Given the right-hand side expression in (13), we derive $\frac{\partial^2 \Pi_C}{\partial\varepsilon\partial\omega} = -\frac{2(2\bar{\theta} + s + \sigma)}{9} < 0.$ Thus, we have $\frac{d\varepsilon}{d\omega} < 0, \omega = s, \sigma.$ That is, an environmental subsidy policy with consumers and/or firm *C* reduces a unit emission level of a cleaner product.

In order to analyze welfare effects of an environmental subsidy/tax policy, we first examine the effect on aggregate emissions. By substituting (11) into (5), aggregate emissions are given by:

$$E = \frac{\{(1+2\varepsilon)\bar{\theta} - (1-\varepsilon)(s+\sigma)\}}{3}. \tag{14}$$

Because it holds that $\frac{\partial E}{\partial \omega} = -\frac{1-\varepsilon}{3} < 0,\ \frac{\partial E}{\partial \varepsilon} = q_C > 0,$ and $\frac{\partial \varepsilon}{\partial \omega} < 0,$ we obtain

$$\frac{dE}{d\omega} = \frac{\partial E}{\partial \omega} + \frac{\partial E}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \omega} < 0, \omega = s, \sigma. \tag{15}$$

In view of (15), an environmental subsidy reduces aggregate emissions in the case of full market coverage. Put differently, given a unit emission level of a cleaner product in the short-run, some consumers change from purchasing a dirtier product to purchasing a cleaner product by a subsidy. In the long-run, firm *C* reduces the unit emission level of the product, with even a subsidy for consumers as well as a subsidy for itself.

Second, we examine the effect on aggregate consumer surplus given by (6). The direct effect is given by $\frac{\partial CS}{\partial \omega} = \frac{1-\varepsilon}{3}(q_D + 2q_C) > 0.$ As the indirect effect is $\frac{\partial CS}{\partial \varepsilon} = q_D\bar{\theta} - \frac{1}{2}(q_C)^2,$

however, the sign is not unidirectional. For example, it holds that $\lim_{\omega \to 0} \frac{\partial CS}{\partial \varepsilon} > 0$. But, if $(\bar{\theta} >)\omega \geq (3\sqrt{3} - 5)\bar{\theta} \approx 0.2\bar{\theta}$, we obtain $\frac{\partial CS}{\partial \varepsilon} \leq 0$. Thus, unless an environmental subsidy is sufficiently large, the effect on aggregate consumer surplus is given by $\frac{dCS}{d\omega} > 0, \omega = s, \sigma$.

Third, with regard to the effect on producer surplus, we obtain:

$$\frac{\partial PS}{\partial \omega} = \frac{2(1-\varepsilon)}{3}\left\{\bar{\theta} + 2(s+\sigma)\right\} > 0 \text{ and } \frac{\partial PS}{\partial \varepsilon} = \frac{\partial \Pi_D}{\partial \varepsilon} = -\frac{\{\bar{\theta} - (s+\sigma)\}^2}{9} < 0.$$

Given $\frac{\partial \varepsilon}{\partial \omega} < 0$, the sign of the effect on producer surplus is positive, $\frac{dPS}{d\omega} > 0, \omega = s, \sigma$. Thus, an environmental subsidy increases producer surplus.

Fourth, we should consider the effect on the government's budget deficit. We derive $\frac{\partial \Omega}{\partial \omega} = (1-\varepsilon)\left\{\frac{2[\bar{\theta} + (s+\sigma)]}{3}\right\} > 0$ and $\frac{\partial \Omega}{\partial \varepsilon} = -(s+\sigma)q_C < 0$. Thus, given $\frac{\partial \varepsilon}{\partial \omega} < 0$, an environmental subsidy increases the budget deficit, i.e., $\frac{d\Omega}{d\omega} > 0, \omega = s, \sigma$.

Taking the results derived above into account, we conclude that an environmental subsidy policy is socially optimal under the full market coverage type.

## *3.2 The Partial Market Coverage Type*

### *3.2.1 The Bertrand Duopoly Case*

With respect to the partial market coverage type, in which there are some consumers not purchasing any products, we need to confirm the results under the full market coverage type shown earlier.

We can easily derive the equilibrium prices in the final stage as follows:

$$p_D = \frac{(1-\varepsilon)\{v - (s+\sigma)\}}{4-\varepsilon} \text{ and } p_C = \frac{(1-\varepsilon)\{2v + (2-\varepsilon)s - 2\sigma\}}{4-\varepsilon}. \tag{16}$$

Given (16), subsidizing firm *C* reduces the prices of both products, whereas subsidizing consumers raises the price of a cleaner product, but decreases that of a dirtier product. Substituting (16) into (3), the equilibrium quantities are given by:

$$q_D = \frac{v - (s+\sigma)}{4-\varepsilon} \text{ and } q_C = \frac{2v + (2-\varepsilon)(s+\sigma)}{\varepsilon(4-\varepsilon)}, \tag{17}$$

where $\frac{\partial q_D}{\partial \varepsilon} > 0$, $\frac{\partial q_C}{\partial \varepsilon} < 0$, $\frac{\partial q_D}{\partial \omega} < 0$, and $\frac{\partial q_C}{\partial \omega} > 0, \omega = s, \sigma$. Because a decrease in the unit emission level of a cleaner product infers an increase in environmental quality, it increases (reduces) the quantity demanded of a cleaner (dirtier) product. Furthermore, it is clear that a subsidy with consumers and/or firm *C* increases (reduces) the quantity demanded of a cleaner (dirtier) product. Hence, the magnitude of the effect of subsidizing consumers on the quantity demanded is equal to that of subsidizing firm *C*.

In the second stage, firm *C* chooses the unit emission level of the product to maximize profits. The first-order profit maximization condition for firm *C* is given by:

$$\frac{\partial \Pi_C}{\partial \varepsilon} = q_C \left\{ (1-2\varepsilon) q_C + 2\varepsilon(1-\varepsilon) \frac{\partial q_C}{\partial \varepsilon} \right\} - \frac{\partial F_C}{\partial \varepsilon} = 0, \tag{18}$$

where $\{\bullet\} < 0$. Assuming the second-order condition is satisfied, we derive the effect of an environmental subsidy on the unit emission level:

$$\frac{d\varepsilon}{d\omega} = -\frac{\partial^2 \Pi_C / \partial \varepsilon \partial \omega}{\partial^2 \Pi_C / \partial \varepsilon^2} > (\leq) 0 \Leftrightarrow \frac{\partial^2 \Pi_C}{\partial \varepsilon \partial \omega} > (\leq) 0, \omega = s, \sigma. \tag{19}$$

We calculate the right-hand side expression in (19) as follows.

$$\frac{\partial^2 \Pi_C}{\partial \varepsilon \partial \omega} = -\frac{2(2-\varepsilon)}{\varepsilon(4-\varepsilon)^3} \left\{ 6v + (10 - 8\varepsilon + \varepsilon^2)(s+\sigma) \right\} - \frac{2(1-\varepsilon)(8-4\varepsilon+\varepsilon^2)}{\varepsilon(4-\varepsilon)^2} q_C < 0.$$

Thus, we have $\frac{d\varepsilon}{d\omega} < 0$. That is, an environmental subsidy with consumers and/or firm *C* reduces the unit emission level of a cleaner product.

In order to analyze aggregate industrial profits afterward, we here present the effect of a decrease in the unit emission level of a cleaner product on the profit of firm *D*:

$$\frac{\partial \Pi_D}{\partial \varepsilon} = -\frac{2+\varepsilon}{4-\varepsilon} (q_D)^2 < 0. \tag{20}$$

In view of (20), a decrease in the unit emission level of a cleaner product extends the degree of differentiation between the environmental qualities of the products. This, in turn, mitigates price competition. Thus, an environmental subsidy increases the profits of firm *D* because it reduces the unit emission level of a cleaner product.

We are now in a position to examine the effect of an environmental subsidy/tax policy on social welfare. First, substituting (17) into (5), we obtain aggregate emissions in the partial market coverage type

$$E = \frac{3v + (1-\varepsilon)(s+\sigma)}{4-\varepsilon}. \tag{21}$$

Let us use the effect on aggregate emissions expressed as $\frac{dE}{d\omega} = \frac{\partial E}{\partial \omega} + \frac{\partial E}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \omega}, \omega = s, \sigma.$ The first term on the right-hand side is the direct effect of an environmental subsidy, where the sign is positive, i.e., $\frac{\partial E}{\partial \omega} = \frac{1-\varepsilon}{4-\varepsilon} > 0.$ A subsidy policy directly increases aggregate emissions. Put differently, given the unit emission level in the short-run, the subsidy increases the quantity demanded of a cleaner product more than it reduces the quantity demanded of a dirtier product. This implies that some consumers, who would not buy any products in the market without the subsidy, purchase the product due to the improvement in environmental quality brought about by the subsidy. Thus, a subsidy directly or in the short-run degrades the environment. However, the second term on the right-hand side expresses the indirect effect of the change in the unit emission level. The sign of the indirect effect is negative, as $\frac{\partial E}{\partial \varepsilon} = \frac{3}{4-\varepsilon} q_D > 0$ and $\frac{\partial \varepsilon}{\partial \omega} < 0.$ Thus, in the long-run, the subsidy reduces aggregate emissions. Consequently, the total effect of an environmental subsidy policy on aggregate emissions is ambiguous. If the magnitude of the indirect (or long-run) effect is larger than that of the direct (or short-run) effect, an environmental subsidy decreases aggregate emissions. Otherwise, it increases them.

Second, we consider the effect of an environmental subsidy on aggregate consumer surplus in (7). Hence, the effect can be expressed by $\frac{dCS}{d\omega} = \frac{\partial CS}{\partial \omega} + \frac{\partial CS}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \omega}, \omega = s, \sigma.$ With respect to the first term on the right-hand side, the sign of the direct effect on aggregate consumer surplus is positive, as $\frac{\partial CS}{\partial \omega} = \frac{1-\varepsilon}{4-\varepsilon}(q_D + 2q_C) > 0.$ However, as to the indirect effect in the second term, we obtain $\frac{\partial CS}{\partial \varepsilon} = \frac{q_D\{3q_D + (2+\varepsilon)q_C\}}{4-\varepsilon} - \frac{(q_C)^2}{2}.$ The sign of the equation is not necessarily negative. For example, it holds that $\lim_{\varepsilon \to 1} \frac{\partial CS}{\partial \varepsilon} > 0.$ But, if $\varepsilon \leq \frac{4}{5},$ then we obtain $\frac{\partial CS}{\partial \varepsilon} < 0.$ It may not be intuitively unusual to assume that the sign of the effect on aggregate consumer surplus is negative when the environmental quality of a cleaner product deteriorates. Thus, an environmental subsidy increases aggregate consumer surplus.

Third, with regard to the effect of the subsidy policy on producer surplus, we obtain $\frac{\partial PS}{\partial \omega} = \frac{2(1-\varepsilon)}{4-\varepsilon}\{(2-\varepsilon)q_C - q_D\} + \frac{8-4\varepsilon+\varepsilon^2}{\varepsilon^2(4-\varepsilon)^2} > 0$ and $\frac{\partial PS}{\partial \varepsilon} = \frac{\partial \Pi_D}{\partial \varepsilon}.$ Thus, taking $\frac{\partial \varepsilon}{\partial \omega} < 0$

and (20) into account, we obtain $\frac{dPS}{d\omega}=\frac{\partial PS}{\partial \omega}+\frac{\partial PS}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \omega}>0, \omega=s,\sigma.$ As a result, an environmental subsidy increases producer surplus. In particular, we note that subsidizing consumers increases the quantity demanded of a cleaner product and extends the difference in emission levels of both products. Given (20), this leads to an increase in profits of both firms.

Fourth, we should consider the effect of an environmental subsidy on the government's budget deficit, $\Omega=(s+\sigma)(1-\varepsilon)q_C$. We derive the followings:

$$\frac{\partial \Omega}{\partial \omega}=(1-\varepsilon)\left\{q_C+\frac{(2-\varepsilon)(s+\sigma)}{\varepsilon(4-\varepsilon)}\right\}>0, \text{ and}$$

$$\frac{\partial \Omega}{\partial \varepsilon}=-(s+\sigma)\left\{q_C+\frac{(1-\varepsilon)[4(2-\varepsilon)v+(8-4\varepsilon+\varepsilon^2)(s+\sigma)]}{\varepsilon^2(4-\varepsilon)^2}\right\}<0.$$

Thus, taking (19) into account, an environmental subsidy increases the budget deficit, i.e., $\frac{d\Omega}{d\omega}=\frac{\partial \Omega}{\partial \omega}+\frac{\partial \Omega}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \omega}>0, \omega=s,\sigma.$

Using the results derived above, we are able to consider an optimal environmental subsidy/tax policy in the case of the partial market coverage model. As already shown, the welfare effects of subsiding consumers are equivalent to those of subsidizing firm *C*. Accordingly, we mainly address hereafter the optimal environmental subsidy/tax policy with consumers, i.e., $\omega=s$.

The key point is the effect on aggregate emissions. If an environmental subsidy reduces aggregate emissions, i.e., $\frac{dE}{ds}<0,$ we can easily conclude that an environmental subsidy with consumers is the optimal policy. However, if the magnitude of the direct effect on aggregate emissions is significantly larger than that of the indirect effect, it holds that $\frac{dE}{ds}>0.$ That is, an environmental subsidy increases aggregate emissions, and thus degrades the environment. Hence, if the marginal social valuation $\gamma$ is sufficiently large, i.e., $\gamma>\frac{\frac{dCS}{ds}+\frac{dPS}{ds}}{\frac{dE}{ds}}\equiv\gamma^{B*},$ because the government gives much weight to the environment, then an environmental tax with consumers is the optimal policy. Otherwise, i.e. $\gamma<\gamma^{B*}$, an environmental subsidy is socially optimal.

*3.2.2 The Cournot Duopoly Case*

To address whether the mode of competition affects the effects of an environmental subsidy/tax policy on the environment and social welfare, we consider the above issues in the case of a Cournot duopoly by following the same procedure as in the previous subsections.

Because the derivation of the Cournot-Nash equilibrium in the final stage is simple, we present the equilibrium outcomes:

$$q_D = \frac{v-(1-\varepsilon)(s+\sigma)}{4-\varepsilon} \text{ and } q_C = \frac{(2-\varepsilon)v+2(1-\varepsilon)(s+\sigma)}{\varepsilon(4-\varepsilon)}, \tag{22}$$

where $\frac{\partial q_D}{\partial \varepsilon} > 0$, $\frac{\partial q_C}{\partial \varepsilon} < 0$, $\frac{\partial q_D}{\partial \omega} < 0$, $\frac{\partial q_C}{\partial \omega} > 0$, and $\frac{\partial^2 q_C}{\partial \varepsilon \partial \omega} < 0, \omega = s, \sigma.$

In the second stage, the first-order profit maximization for firm *C* is given by:

$$\frac{\partial \Pi_C}{\partial \varepsilon} = q_C\left\{q_C + 2\varepsilon\frac{\partial q_C}{\partial \varepsilon}\right\} - \frac{\partial F_C}{\partial \varepsilon} = 0,$$

where $\{\bullet\} < 0$. We assume that the second-order condition is satisfied. Furthermore, the cross derivative is given by:

$$\frac{\partial^2 \Pi_C}{\partial \varepsilon \partial \omega} = 2\frac{\partial q_C}{\partial \omega}\left\{q_C + \varepsilon\frac{\partial q_C}{\partial \varepsilon}\right\} + 2\varepsilon q_C\frac{\partial^2 q_C}{\partial \varepsilon \partial \omega} < 0,$$

where $\{\bullet\} < 0.$ Thus, the effect of an environmental subsidy on a unit emission level is

$$\frac{d\varepsilon}{d\omega} = -\frac{\partial^2 \Pi_C / \partial \varepsilon \partial \omega}{\partial^2 \Pi_C / \partial \varepsilon^2} < 0, \omega = s, \sigma. \tag{23}$$

That is, an environmental subsidy with consumers and/or firm *C* reduces the unit emission level of a cleaner product.

Now, we proceed to the analysis of the effect of an environmental subsidy/tax policy on social welfare. First, substituting (22) into (5), we obtain aggregate emissions as follows.

$$E = \frac{(1-\varepsilon)(v+s+\sigma)}{4-\varepsilon}. \tag{24}$$

Given (24), we have: $\frac{\partial E}{\partial \omega} = \frac{1-\varepsilon}{4-\varepsilon} > 0.$ and $\frac{\partial E}{\partial \varepsilon} = -\frac{\{v+3(s+\sigma)\}}{(4-\varepsilon)^2} < 0.$ Furthermore, taking into account $\frac{\partial \varepsilon}{\partial \omega} < 0,$ paradoxically, an environmental subsidy increases aggregate emissions, i.e., $\frac{dE}{d\omega} > 0, \omega = s, \sigma.$ That is, the subsidy directly increases aggregate emissions, as mentioned in the case of a Bertrand duopoly. In addition, as to the indirect effect, the subsidy policy reduces the unit emission level of a cleaner product, whereas it sufficiently increases market share of the product. In this case, an increase in the market share of a cleaner product leads to an increase in aggregate emissions.

Under the partial market coverage type, as mentioned earlier, an environmental subsidy policy not only raises the environmental quality of a cleaner product, but also increases the willingness to pay for the products of consumers, who would never purchase any products without the policy. That is, some consumers would change from taking public transportation such as trains to driving hybrid vehicles because of the environmental subsidy on the latter.

Second, the effect of an environmental subsidy on aggregate consumer surplus is expressed as $\frac{dCS}{d\omega} = \frac{\partial CS}{\partial \omega} + \frac{\partial CS}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \omega}, \omega = s, \sigma$. Hence, we derive $\frac{\partial CS}{\partial \omega} = \frac{\{q_D + (2 - 2\varepsilon + \varepsilon^2)q_C\}}{4 - \varepsilon} > 0$ and $\frac{\partial CS}{\partial \varepsilon} = -\frac{\{q_D + (s+\sigma)\}\{q_D + (2-\varepsilon)q_C\}}{4-\varepsilon} - \frac{(q_C)^2}{2} < 0$. Given (23), an environmental subsidy necessarily increases aggregate consumer surplus.

Third, as to the effect on producer surplus, we obtain

$$\frac{\partial PS}{\partial \omega} = \frac{2(1-\varepsilon)(2q_C - q_D)}{4-\varepsilon} > 0 \text{ and } \frac{\partial PS}{\partial \varepsilon} = \frac{\partial \Pi_D}{\partial \varepsilon} = 2q_D \frac{\partial q_D}{\partial \varepsilon} > 0.$$

The second equation implies that improving the environmental quality of a cleaner product reduces the profits of firm *D*. Given (23), the sign of the effect on producer surplus, $\frac{dPS}{d\omega} = \frac{\partial PS}{\partial \omega} + \frac{\partial PS}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \omega}, \omega = s, \sigma,$ is not unidirectional. Put differently, if the magnitude of the direct effect is larger (smaller) than the magnitude of the indirect effect, an environmental subsidy increases (decreases) producer surplus.

Fourth, as to the effect of an environmental subsidy on the government's budget deficit, we obtain the followings.

$$\frac{\partial \Omega}{\partial \omega} = (1-\varepsilon)\left\{q_C + \frac{2(1-\varepsilon)}{\varepsilon(4-\varepsilon)}(s+\sigma)\right\} > 0, \text{ and}$$

$$\frac{\partial \Omega}{\partial \varepsilon} = -(s+\sigma)\left\{q_C + \frac{(1-\varepsilon)[(8-4\varepsilon+\varepsilon^2)v + 2(4-2\varepsilon+\varepsilon^2)(s+\sigma)]}{\varepsilon^2(4-\varepsilon)^2}\right\} < 0.$$

Thus, given (23), an environmental subsidy increases the budget deficit, i.e., $\frac{d\Omega}{d\omega} > 0, \omega = s, \sigma.$

Suppose that the government places much weight on the environment. If the marginal social valuation is substantially large, $\gamma > \frac{\frac{dCS}{ds} + \frac{dPS}{ds}}{\frac{dE}{ds}} \equiv \gamma^{C*}$, then an environmental subsidy directed to consumers is not socially optimal. In other words, the government should charge consumers environmental taxes to reduce consumption of the products, even thought the unit emission level of a cleaner product rises. Conversely, if the government does not consider the environment as its priority, i.e., $\gamma < \gamma^{C*}$, an environmental subsidy is an optimal policy.

Therefore, in view of the analysis in 3.2.1 and 3.2.2, we conclude that, under the partial market coverage type, regardless of the mode of competition, if the marginal social valuation of environmental damage is sufficiently large, then an environmental tax directed at consumers is socially optimal. Otherwise, an environmental subsidy is the optimal policy.

# 4. Concluding Remarks

We have analyzed how an environmental subsidy/tax policy associated with consumers and/or a producer of an environmental-friendly (cleaner) product has impacts on the environment and social welfare, and examined whether the environmental subsidy policy is socially optimal with regard to a Bertrand and a Cournot duopoly, looking at two types of market coverage.

Our results are as follows.

(i) The effects on the environment and social welfare of subsidizing consumers purchasing a cleaner product are equivalent to those of subsidizing the firm producing it.

(ii) An environmental subsidy reduces aggregate emissions in the case of full coverage market with a Bertrand duopoly, whereas it increases them in the case of partial coverage market with a Cournot duopoly. Also, in the case of partial coverage market with a Bertrand duopoly, the effect of environmental subsidy on aggregate emissions is ambiguous.

(iii) An environmental subsidy policy is socially optimal in the case of full coverage market with a Bertrand duopoly.

(iv) An environmental subsidy (tax) policy is socially optimal in the case of partial coverage market, regardless of the mode of competition, if the marginal social valuation of environmental damages is sufficiently small (large, respectively).

Let us now discuss some outstanding issues. First, as mentioned in the Introduction, an interesting issue arises in the form of an environmental subsidy/tax policy game between local and central governments. For example, the local government decides an environmental subsidy/tax for consumers by taking into account the welfare improvements for residential consumers included in the valuation of the environment in the local area. However, the central government chooses an environmental subsidy/tax for firms to maximize social welfare included in aggregate industry profits. In this case, as discussed by Lombardini-Riipinen (2005) regarding a commodity tax policy, we can analyze the policy mix, e.g., an environmental subsidy/tax and a commodity tax. Second, we should discuss other environmental policies using direct pollution controls, such as emission standards and quotas, as well as indirect pollution controls, such as tradable emission permits, emission and/or commodity taxes. For example, in future work we intend to address how emission taxes on an environment-unfriendly polluting good, i.e., a dirtier product, affect the environment and social welfare by employing a similar model to that presented in this paper. Hence, we need to examine whether the effects of environmental subsidies, i.e., carrots, on a cleaner product are equivalent to those of emission taxes, i.e., sticks, on a dirtier product.

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