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# F.Y. Edgeworth's *Mathematical Psychics* and his Utilitarianism: The Derivation from the 'Sidwick-Barratt Controversy'

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# F.Y. Edgeworth's *Mathematical Psychics* and his Utilitarianism: The Derivation from the 'Sidgwick-Barratt Controversy'*

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Abstract

The purpose of this article is to demonstrate that Edgeworth's *Mathematical Psychics* (1881) has been influenced by various intellectual contemporaries through the 'Sidgwick-Barratt Controversy'. Under the influence of Barratt, Edgeworth admitted the physical methods of ethics; which is clear from his adoption of the 'Fechner's Law' to measure the quantity of pleasure. Through the analysis of the contract between egoistic agents, Edgeworth also attempted to prove the need of utilitarianism as the solution to Sicgwick's 'Dualism of Practical Reasons'. Since Edgeworth asserted that the capacity for pleasure is different among people, criticizing 'equality' tacitly implied in utilitarianism, he admitted 'exact Utilitarianism' which allowed unequal distribution as the 'distributive justice' for the greatest happiness of the society. Thus Edgeworth's *Mathematical Psychics* is not only the economic but also ethical work influenced by 'Sidgwick-Barratt Controversy'.

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Keywords: F.Y. Edgeworth, *Mathematical Psychics*, Utilitarianism, the 'Sidgwick-Barratt Controversy'

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## I. Introduction

Francis Ysidro Edgeworth (1845-1926) published a book entitled *Mathematical Psychics* (1881), which is well known for the 'Box Diagram'. This work, as J.M. Keynes described it, was not only 'a very eccentric book', but also 'the strange but charming amalgam of poetry and pedantry, science and art, wit and learning' (Keynes 1933, 258). 'In order to place [Edgeworth] for us', including this 'very eccentric book', Schumpeter noted as follows: 'first[ly], I mention his utilitarianism, which strongly asserted itself from the beginning (*New and Old Methods of Ethics*, 1877) and looked so incongruous in a man whose mind was nothing if not "cultured"' (Schumpeter 1954, 830).

From the point of 'Utilitarianism', some researchers have attempted to recognize the existence of indeterminacy and its arbitration through Edgeworth's contract theory in *Mathematical Psychics*; for example, by Collard (1975), Creedy (1986) and Newman (1987); as well as by Negishi (1985), Nakano (1995), and Matsushima (2005). According to Nakano, Edgeworth's argument was 'to justify the principle of utilitarianism', and this justification has been accomplished by two arguments; first by 'justifying utilitarianism as the issue of choice under the indeterminacy through contract theory' and secondly by 'justifying it based on altruism' (Nakano 1995, 176). The former is adopted by Creedy, Negishi and Matsushima, and the latter by Collard and Nakano.[1] Matsushima, for example, insists Edgeworth described the prototype of utilitarianism which does not identify 'self interest' with 'public interest' (Matsushima 2005, 53), and Nakano states 'the argument of Edgeworth was an attempt to correct the image of man and bring the element of altruism into utilitarianism' (Nakano 1995, 178). Although these papers focused on Edgeworth's utilitarianism, they dealt with it only in

[1] Creedy, Negishi and Matsushima regard Edgeworth's argument as a pioneer of 'neo-utilitarianism'. However, Nakano (1995) points out that 'indeterminacy' is used differently by Edgeworth and the neo-utilitarians.

terms of economic theory or history of economic doctrine because their observations are limited to the first half of Part II in *Mathematical Psychics*, called 'Economical Calculus'. For this reason, we cannot deny that they ignore Edgeworth's utilitarianism shown in the latter half of the same part, dealing with 'Utilitarian Calculus'.[2]

According to Edgeworth, Part II of *Mathematical Psychics* is composed of 'Economical Calculus', which 'investigates the equilibrium of a system of hedonic forces each tending to maximum individual utility', and 'Utilitarian Calculus, the equilibrium of a system in which each and all tend to maximum universal utility' (Edgeworth 1881, 15-16). His purpose for Part II is to suggest 'an addition to Mr. Sidgwick's "ethical methods"', because Sidgwick's 'division of Hedonism – the class of "Method" whose principle of action may be generically defined *maximizing happiness* – is not exhaustive' (*ibid*., v, 16). This purpose gives us some important suggestions for tracing the whole picture of his utilitarianism in *Mathematical Psychics*; first of all, we must deal with these two 'Calculus[es]' together, and secondly, we have to consider the close relationship between Edgeworth's *Mathematical Psychics* and Sidgwick's *Methods of Ethics* (1874). Undoubtedly, the connection between Edgeworth and Sidgwick started long before *Mathematical Psychics*. In fact, Edgeworth had already published his idea on this subject. His first book *New and Old Methods of Ethics or "Physical Ethics" and "Methods of Ethics"* (1877) was an 'attempt to compare….the "Physical Ethics" of Mr. Alfred Barratt,[3] and the "Methods of Ethics" of Mr. Henry

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[2] Peart and Levy(2005) pay attention to the capacity of pleasure argued in 'Utilitarian Calculus', even though they give fewer paragraphs to and omit the relation between these two 'Calculus[es]'.

[3] Alfred Barratt (1844-81) was educated at Rugby, before entering Balliol College, Oxford in 1862. He became a fellow of Brasenose College in 1868 and was called to the bar in 1872. His *Physical Ethics*, published in 1869, was admired as 'a most remarkable performance for a youth of twenty-four, showing wide reading and marked literary power' (Stephen and Lee 1917, vol.1, 1194-95).

Sidgwick' (Edgeworth 1877, 1).[4] About that time, Barratt and Sidgwick had a 'Controversy' (*ibid*., ii) in *Mind* about the details of *Methods of Ethics*. In addition, Edgeworth's second article entitled 'Hedonical Calculus' (1879) was later reprinted in Part II of *Mathematical Psychics* under the new title of 'Utilitarian Calculus'.[5] This shows that Sidgwick's *Methods of Ethics*, as well as the 'Controversy' between Sidgwick and Barratt, was very important for Edgeworth before the publication of *Mathematical Psychics*. Therefore, we also need to examine this point carefully in order to clarify how *Mathematical Psychics* had been composed.[6]

The purpose of this article is to demonstrate that Edgeworth's *Mathematical Psychics* was influenced by various intellectual sources from his contemporaries through this Sidgwick-Barratt Controversy, which has been ignored by existing papers. For this purpose, in the second section, I am going to deal with the substance of the Sidgwick-Barratt Controversy and Edgeworth's ideas, before dealing with *Mathematical Psychics*. In the third section, I consider how *Mathematical Psychics* is related to this controversy, and trace Edgeworth's utilitarianism. Finally, in the fourth section, I will make my conclusions.

## II. The Sidgwick-Barratt Controversy and Edgeworth

In this section, I summarize the Sidgwick-Barratt Controversy, which can be regarded as the main source of Edgeworth's thought. Firstly, I give the outline of the 'Controversy',

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[4] Since *New and Old Methods of Ethics* was published soon after the second edition of *Methods of Ethics*, Edgeworth referred to the first edition. However, he referred to the second edition in *Mathematical Psychics*.

[5] As Newman states, there are no essential changes between the two (Newman 1987, 88). As to the corrections, see 'Notes on *Mathematical Psychics*' in Creedy (1986, 135-150).

[6] Edgeworth's first publication was the article entitled 'Mr. Matthew Arnold on Bishop Butler's Doctrine of Self-Love' in *Mind* 1876. This was a review of Arnold's interpretation of Butler's self-love. Although the author signed as T. Y. Edgeworth, it is regarded as F.Y. Edgeworth (Edgeworth 1876, 571).

and then deal with its influence on Edgeworth before *Mathematical Psychics*.

## 1. *The Methods of Ethics* and 'The "Suppression" of Egoism'

As Sidgwick confessed in the preface to the sixth edition of the *Methods of Ethics*, he doubted the consistency between two Hedonisms, namely that 'each man does seek his own Happiness" and that "each man ought to seek the general Happiness' (Sidgwick 1906, xvii), because we have to accept that it is right to sacrifice our own happiness for the good of society.

Prompted by this problem, he tried to analyze the three practical reasons from a neutral position, namely 'Egoism', taking this term 'as implying the adoption of his own greatest happiness as the ultimate end of each individual's action' (*ibid*, 119), 'Intuitionalism', which 'regards as the practically ultimate end of moral actions conformity to certain rules or dictates of Duty unconditionally prescribed' (*ibid*., 96), and 'Utilitarianism', which means 'that the conduct which, under any given circumstances, is objectively right, is that which will produce the greatest amount of happiness on the whole' (*ibid*., 411). Furthermore, he aimed 'to point out their mutual relations and their conflictions' (Sidgwick 1877a, 13).

When we simplify Sidgwick's framework, we see that he examined the relationship between intuitionalism and utilitarianism, and also that between egoism and utilitarianism. Regarding the former relationship, there is no conflict between these two practical reasons, because intuition allows us to recognize that universal happiness is the good and it makes the universe happier. Thus he stated that intuitionalism is not only compatible with utilitarianism, but also gives 'a rational basis for such a system' (*ibid*., 456). However, regarding the latter relationship, Sidgwick concluded 'that the inseparable connexion between utilitarian duty and the greatest happiness of the individual who conforms to it, cannot be satisfactorily demonstrated on empirical

grounds' (*ibid*., 463-64), since he could not confirm the existence of a God who assures everyone's interests by promoting universal happiness. Sidgwick, then, considered the possibility of consistency between 'Egoism' and 'Utilitarianism' from two points of view, of 'Sympathy' and 'Religion,' but never arrived at 'a complete synthesis'. Thus, his system, which has to admit that these two practical reasons are respectively independent principles, is named the 'Dualism of Practical Reasons'.

Sidgwick, as mentioned above, declared neutrality in the preface to the first edition of the *Methods of Ethics*. Nevertheless, due to its composition, the book was apt to be misunderstood as a work that affirmed utilitarianism while denying intuitionalism and egoism.[7] Various criticisms of the first edition of *Methods of Ethics* were introduced in the preface to the second edition, and this preface includes the following passage. 'Another [critic] has constructed an article on the supposition that my principal object is the "suppression of Egoism": has gone to the length of a pamphlet under the impression (apparently) that the "main argument" of my treatise is a demonstration of Universalistic Hedonism' (*ibid*., xi). Though the name of the writer was not made clear, this article was written by Barratt under the title 'The "Suppression" of Egoism' printed in *Mind*, vol. 2, in 1877.[8]

In this article, Barratt criticized Sidgwick for establishing 'Intuitional proof' and as a result suppressing egoism, omitting 'Physical proof' and disproving 'Introspective proof' (Barratt 1877, 167). According to Barratt, Sidgwick's *Methods of Ethics* premised the existence of 'Moral Faculty,' which acknowledges objective good

[7] *Methods of Ethics* dealt with egoism, intuitionalism and then utilitarianism in succession. This order gave the impression that utilitarianism was the most important method among these three (Okuno 1999, 26). 'I find that more than one critic….has consequently supposed me to be writing as an assailant of two of the methods which I chiefly examine, and a defender of the third' (Sidgwick 1977a, xi).

[8] The other authors who attacked Sidgwick were, for instance, Bain, Bradley and T.H. Green (Schneewind 1977, 192; Yukiyasu 1992, 196).

intuitively, and only classified axioms or principles which 'Reasons' dictates through this 'Faculty'. Barratt called this method 'Intuitional proof' and complained that it is only a treatise of a single method of ethics, although the work was entitled 'Methods of Ethics'.

As Barratt published *Physical Ethics* (1869) and suggested the introduction of physical scientific method into ethics, he questioned the lack of objectivity in 'Moral Faculty' which Sidgwick premised. If 'Moral Faculty' has no objectivity, then the duties for each person intuitionally recognized must be 'what he thinks [is] his duty', in other words, they must be 'subjective only'(*ibid*., 169). To know whether the duty shown by the moral faculty is right or not, therefore, the objective good must be indispensable. Barratt strengthened importance of the objective good by using the following acoustics analogy: 'from hearing alone how can we know that sound means the same, that is, stands in the same relation, to all men?' (*ibid*., 169). In acoustics, by the physical criterion of vibration, it is possible to distinguish sounds in a given situation. Barratt stated that the objective good of 'pleasure' plays the same role in ethics as vibration does in acoustics. He previously insisted, in *Physical Ethics* that, physical stimulus (or 'irritation') can be divided into two classes: that which preserves or develops sensory tissue may be pleasure, and that which injures or deranges it may be pain (negative pleasure) (Barratt 1869, 288, 290). Stimulus of pleasure can be observed from a physical point of view, and, here, the objective criterion might be found. This was the reason that he regarded 'pleasure' as an objective good. While Barratt regarded this objective proof based on physical scientific methods as 'Physical proof', Sidgwick denied the physical scientific ethics. He printed a reply to Barratt's article in *Mind* in 1877, noting that 'as regards the "Physical Method" of ethics, it is enough to say that there cannot possibly be any such "method"' and that 'ethical conclusions can only be logically reached by starting with ethical premises: how the latter are got, it was no part

of my plan to consider' (Sidgwick 1877b, 412).

After pointing out the omission of 'Physical proof' in the *Methods of Ethics*, Barratt observed the fact that Sidgwick had disproved the 'Introspective proof'. In Barratt's understanding, although Sidgwick admitted pleasure as the universal motive, his idea, which set great store on duty, had made the proof imperfect. To make this proof perfect, Barratt stated, Sidgwick had to 'refute…that self-examination shows us that pleasures and pains are as a matter of fact the only motives to voluntary action, and act in proportion to their intensity' (Barratt 1877, 173). Thus Barratt regarded the methods in the *Methods of Ethics* critically as 'not different "Methods of Ethics" but different results of the same method' (*ibid*., 168). At the same time, he developed his own doctrine against the 'Dualism of Practical Reasons'.

Barratt emphasized that pleasure must be the end of all action, and that actors belonging to any organization have internal and external relation to their acts. Therefore, all actions depend on two principles: internal and external. As he explained, the former is the principle for maximizing the pleasure of an individual as a unit of an organization, that is, 'Egoism'. On the other hand, the external principle is simply 'Utilitarianism', which is the principle for the organization aiming to maximize universal happiness. Barratt admitted that practical reasons were a mixture of these two principles. However their relative importance is not the same. As the organizations become more advanced and more complicated, actors for the harmony of the organizations regard utilitarianism as more important than egoism. In other words, in the simplest organizations, there is little room for utilitarianism. Thus, he insisted that egoism was the essential nature of men and that utilitarianism was an acquired principle. He therefore suggested the motives for utilitarianism were yielded by altering one's belief or consequences of his actions for rewards, punishment and so on. His conclusion was 'the *Ethical* value of

Utilitarianism of whatever kind could only be as a *method* of Egoism' (*ibid*., 185).[9] Sidgwick did not clearly refute this opinion. Before long, the 'Controversy' between them came to an end with the death of Barratt in 1881.

As mentioned above, in addition to the difference in the interpretation of 'methods' of ethics, there was a conflict between Sidgwick, who had to admit the 'Dualism of Practical Reasons', and Barratt, who insisted that utilitarianism can be reduced to the fundamental principles of egoism. This was the Sidgwick-Barratt Controversy to which Edgeworth paid great attention.

## 2. Edgeworth before *Mathematical Psychics*

Edgeworth, who had shown his extraordinary talents in classics and ethics since his college days,[10] dealt with the above-mentioned Sidgwick-Barratt Controversy in *New and Old Methods of Ethics*, published in 1877 (Keynes 1933, 256). In this work, he firstly agreed with Barratt's opinion of introducing physical scientific methods to ethics; no doubt Edgeworth knew the objection to that kind of approach on the grounds that 'between physics and morals there is a great gulph', but he emphasized that 'the whole scope of deductive science, and especially of applied mathematics, is to deduce from laws about one set of phenomena propositions about phenomena quite disparate' (Edgeworth 1877, 18). As Barratt connected air vibrations and sound impressions in

[9] Barratt published 'Ethics and Psychogony' in *Mind* 1878. In this article, while admitting that *Methods of Ethics* had been a good deal altered, he insisted Sidgwick still excludes Psychogony from Psychology and Ethics. Barratt, in addition, took up Edgeworth's *New and Old Methods of Ethics* in the same article, but there is no room here for a full explanation of it.

[10] Edgeworth entered Trinity College, Dublin in July 1861, and graduated in December 1865. His tutor was J.K. Ingram, but during this period there is no official record for his study of economics in classes. He came at the top on Examination for Honors of Classics and of Ethics during this period (TCD MUN/V/28/2, TCD MUN/V/30/21-24). Then, he moved to Oxford in January 1867, and mainly studied at Balliol College. Keynes indicates the possibility of his studying on Political Economy through Benjamin Jowett, his tutor in Balliol (Keynes 1933, 255).

acoustics, Edgeworth concentrated on a rule which mediated between physical and ethical science. This rule was 'Fechner's Law' in experimental psychology (or psychophysics),[11] which deduced intensity of sensations from intensity of stimulus.[12]

Edgeworth replaced these two intensities respectively with quantity of pleasure and pleasurable stimulus, and expressed a formula (or 'a quasi-Fechnerian law'); the first differential on pleasurable stimulus is positive, and the second negative (*ibid*., 42).[13] This formula embodies his idea that any pleasure can be measured objectively, as well as his solution to that kind of utilitarianism, like J.S. Mill's, which takes into consideration the difference of the quality of pleasure. In short, what is thought to be the difference of the quality of pleasures was, in effect, the difference of the quantity of pleasures. Edgeworth maintained every pleasure can be measured using the same criterion.

Edgeworth then turned to the 'Dualism of Practical Reasons', another subject in the 'Controversy'. He basically accepted Sidgwick's definition of Practical Reasons and

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[11] G.T. Fechner was a pioneer of German experimental psychology and a founder of psychophysics. His works, represented by his *Elemente der Psychophysik* (1860), influenced German psychologist Wundt, who set up one of the first psychological laboratories in the world at the University of Leipzig (Boring 1929, 272; Imada 1962, 180-84).

[12] Fechner's Law is expressed as $\gamma = k(\log y - \log \beta)$: where the symbols $\gamma$, $k$, $y$ and $\beta$ denote respectively, the quantity of sensation, the just noticeable difference, the quantity of stimulus (intensity of stimulus), and the stimulus threshold (Boring 1929, 278-79; Imada 1962, 186-87): while '$k$ and $\beta$ are constants' (Edgeworth 1877, 40).

[13] The formula, 'quasi-Fechnerian law', can be expressed as $\pi = k|f(y) - f(\beta)|$: where the symbols $\pi$, $k$, $f$, $y$ and $\beta$ respectively denote, 'the pleasure of a sentient element', 'capacity for pleasure', a function which the first differential is positive and the second is negative, the quantity of pleasure for stimulus and 'the "threshold", the lowest value of stimulus for which there is sense of pleasure at all' : while '$\beta$ and $k$ are co-efficients' (Edgeworth 1877, 42) .

tried to unite them with 'Utilitarianism' by adopting the expectation of the 'evolution' of moral sentience: 'the egoist, then, may have the power and the motive to cultivate a desire for the general good, not, indeed, to the pitch of pure utilitarianism, but to a degree asymptotically approaching it, in the progress of evolution, in the course of generations' (*ibid*., 33). Here, what his evolution of moral sentience meant, was the extension of altruistic feelings with the social development, or advance of the members from the lower classes to the upper classes, because he thought universal happiness grew among the 'χαρίεντες (charientes)' and 'σπουδαῖοι (spoudaioi)' ('persons of refinement' and 'good, virtuous persons') (Edgeworth 1877, 33; Newman 2003, 251). This idea could be accepted only as his expectation for evolution rather than the solution to the 'Dualism of Practical Reasons'. The solution was redesigned with other tools in the later publication *Mathematical Psychics*, mentioned below. Thus Edgeworth, in *New and Old Methods of Ethics*, agreed with Barratt on the introduction of physical methods into Ethics, but on the 'Dualism of Practical Reasons' he arrived at a different conclusion from Sidgwick or Barratt.

As mentioned above, after *New and Old Methods of Ethics*, Edgeworth published 'Hedonical Calculus' which was reprinted as the 'Utilitarian Calculus' in Part II of *Mathematical Psychics*. Soon after the contribution of this article to *Mind*, Edgeworth became acquainted with Jevons, and read his *Theory of Political Economy* (second edition, 1879) and also Marshall's 'The Pure Theory of Foreign Trade' (1879) or *Economics of Industry* (1879) at Jevons' recommendation.[14] This must have been essentially his first study of Economics. Indeed, for example, he owes his notion of utility expressed with two dimensions, 'intensity' and 'time', or his fundamental function of utility both to Jevons (Edgeworth 1881, 7; 20).[15] It was not until Edgeworth

[14] See Edgeworth (1881, 34n1), Black (1962, 215) and Creedy (1986, 43).
[15] As to the relationship between utility functions of Jevons and that of Edgeworth, see

came under Jevons' influence that he acquired the new tool of 'economical approach grounded on marginal utility theory', in addition to the 'philosophical or ethical approach' and the 'psychophysical approach' (Fukuoka 1999, 189).

## III. Edgeworth's Utilitarianism in *Mathematical Psychics*

This section discusses how Edgeworth's *Mathematical Psychics* and the Sidgwick-Barratt Controversy are related. In particular, the first half is devoted to the explanation of his solution to the 'Dualism of Practical Reasons' and the second to the characteristics of his utilitarianism.

### 1. The Diagram and Utilitarian arrangement

Edgeworth, as mentioned above, consciously gave an account of the nature of Part II of *Mathematical Psychics* and its relation to Sidgwick's *Methods of Ethics*. He has stated in the preface that 'the Calculus of Pleasure (Part II) may be divided into two species — the Economical and the Utilitarian; the principle of division suggesting an addition to Mr. Sidgwick's "ethical methods"' (Edgeworth 1881, v). In the introduction to Part II, he indeed regards Sidgwick's division of Hedonism as not being exhaustive. Edgeworth indicates that an indefinite number of impure methods can exist between 'Pure Egoistic' and 'Pure Universalistic'. Therefore, it is possible that the moral structure of the actor may be 'μικτήτις' (miktetis, which means 'a mixed kind') of these two (Newman 2003, 161), 'and even the *Deductive Egoist* might need Utilitarian Calculus' (Edgeworth 1881, 16). To explain these points was the aim of 'Economical Calculus'.

'Economical Calculus' is the analysis of egoism based on mathematical methods. This is because Edgeworth, in the *New and Old Methods of Ethics*, had already agreed

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Edgeworth (1881, 104-5), as well as Uemiya (2005, 127-28).

with Barratt in that he supported the introduction of physical scientific methods to the moral sciences. Such an idea is strengthened in Part I of *Mathematical Psychics*, which points out the resemblance between the mathematical and moral sciences and, furthermore, regards the role of energy in physics and that of pleasure in moral sciences as the same.[16]

In this Calculus, the following assumptions are made: firstly that 'every agent is actuated only by self-interest' (the first principle of Economics),[17] i.e. the agents are egoists; secondly that 'there is free communication throughout *a normal* competitive field'; thirdly that four conditions for the field of perfect competition are satisfied (as shown in *Mathematical Psychics*, 18-19).[18]

Based on these assumptions, Edgeworth directs his attention to two problems, namely, '*how far contract is indeterminate*', and if the indeterminacy exists 'in what direction an escape from its evils is to be sought' (*ibid*., 20). To analyze the above problems, he drew the 'Diagram' on page 28 to illustrate the contract between Robinson Crusoe and Friday. In this diagram, he starts the transaction between one Crusoe and one Friday. Then equal-natured agents (Clones of Crusoe, or of Friday) are respectively introduced one after the other, and the transaction between Crusoes and Fridays is supposed to hold.

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[16] The following shows that Edgeworth regards the role of Mathematics in social sciences as important: 'He that will not verify his conclusions as far as possible by mathematics, as it were bringing the ingots of common sense to be assayed and coined at the mint of the sovereign science, will hardly realize the full value of what he holds, will want a measure of what it will be worth in however slightly altered circumstances, a means of conveying and making it current' (Edgeworth 1881, 3). He also thinks highly of Cournot as 'the father of mathematical Economics' in 'On Unnumerical Mathematics', appendix I of *Mathematical Psychics* (*ibid*., 83).

[17] Therefore, the term 'economical' can be regarded as being the same as that of 'egoistic'.

[18] The four conditions are defined as (1) 'any individual is free to *recontract* with any out of an indefinite number' : (2) 'any individual is free to *contract* (at the same time) with an indefinite number': (3) 'any individual is free to *recontract* with another independently of, *without the consent* being required of, any third party': (4) 'any individual is free to *contract* with another independently of a third party' (Edgeworth 1881, 18-19).

The diagram leads him to believe that the contract between one agent on each side is apt to meet the '*deadlock*', and that, as the number of agents is increased, the range of contract curve shrinks, which means that the contract gradually gains determinacy. Ultimately, when the number of Crusoes and Fridays becomes infinite, the contract comes to 'perfect competition' and the range of contract curve is uniquely determined. Though Edgeworth adopted his fundamental ideas from Jevons, his criticism of Jevons' theory of exchange is evident. While Jevons held the notion of unique exchange ratio called 'law of indifference', Edgeworth asserts that without perfect competition the exchange ratio cannot be unique.

His conclusion from this is that, in the field of competition, 'to impair, it may be conjectured, the reverence paid to *competition*; in whose results — as if worked out by a play of physical forces, impersonal, impartial — economists have complacently acquiesced' if there is no '*multiety of atoms*' (*ibid*., 50). Insofar as the number of competitors is finite, the contracts are indeterminate and this must be the common case. Furthermore, he emphasizes that such an 'indeterminacy' can be found in 'the whole region of in a wide sense *contract*', for instance, 'in international, in domestic politics; between nations, classes, sexes' (*ibid*., 51).

To avoid 'the evil of indeterminacy' in every contract, Edgeworth indicates that '*competition requires to be supplemented by arbitration, and the basis of arbitration between self-interested contractors is the greatest possible sum-total utility*' (*ibid*., 56). His indication means to take 'utilitarian arrangement', which maximizes the utility of each contractor and that of the society simultaneously. The utilitarian point which this arrangement shows should be achieved by maximizing the sum of one's utility and other utilities weighted by the '*coefficient of effective sympathy*'.[19]

[19] Though Edgeworth does not make it clear, he supposes the coefficient of effective sympathy ranges from 0 to 1 and differs among individuals. Regarding his utility

Thus, Edgeworth reaches his original solution to the 'Dualism of Practical Reasons'. Although Edgeworth himself formerly depended on the notion of evolution in *New and Old Methods of Ethics*, in *Mathematical Psychics* he finds a single solution with 'Sympathy'.[20] This is the opposite of Sidgwick who examined the consistency between 'Egoism' and 'Utilitarianism' from the points of 'Sympathy' or 'Religion'. What Edgeworth succeeds in doing, is uniting the two principles of 'Practical Reasons' by 'Sympathy' led through the contract theory, which cannot be done without Economics. In this sense, Edgeworth's argument can be regarded as 'an attempt to correct the image of man and bring the element of altruism into utilitarianism' (Nakano 1995, 176). At the same time, in opposition to Barratt, he argues that even among the egoists the method of utilitarianism is needed as the 'principle of arbitration'.[21] Thus 'the *economical* leads up to the *utilitarian calculus*' (*ibid*., 56).

We should, however, carefully recognize that Edgeworth's 'utilitarian point' is not a quantitatively equal distribution (in his words, 'quantitative mean'). He admits this quantitative mean can be found in the neighborhood of 'utilitarian point', but they are not exactly the same. In general, these two are treated as the same point, but Edgeworth insists that this misunderstanding comes from the notion that 'equality' is implicitly or

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function with considering this coefficient, see Edgeworth (1881, 53), Collard (1975, 358-60), Nakano (1995, 177) and Uemiya (2005, 120).

[20] 'It is far from the spirit of the philosophy of pleasure to depreciate the importance of religion; but in the present inquiry, and dealing with the lower elements of human nature, we should have to seek a more obvious transition, a more earthy passage, from the principle of self-interest to the principle, or at least the practice, of utilitarianism' (Edgeworth 1881, 52-53).

[21] Matsushima points out that this statement of Edgeworth on the 'principle of arbitration' is influenced by Marshall's *Economics of Industry*, especially chapter VIII of Book III entitled 'Arbitration and Conciliation'(Matsushima 2005, 48). Indeed, *Mathematical Psychics* owes a lot to *Economics of Industry*. However, the greater concern for Edgeworth is to be found in the Sidgwick-Barratt Controversy since the publication of *New and Old Methods of Ethics*. Therefore Edgeworth introduced 'economical approach' for the solution to the 'Dualism of Practical Reasons' by referring to *Theory of Political Economy* as well as *Economics of Industry*.

naturally involved in the word ‘Utilitarianism’. Near the end of ‘Economical Calculus’, Edgeworth indicates that ‘qualitative mean’ is indeed ‘utilitarian equity’. If we compare these two notions of ‘mean’ (quantitative and qualitative), the word of ‘Utilitarianism’ or ‘Utilitarian’ used by Edgeworth undoubtedly involves his original ideas.

What Edgeworth means by ‘Utilitarianism’ or ‘Utilitarian’ is dealt with in the ‘Utilitarian Calculus’, the latter section of Part II of *Mathematical Psychics*.

## 2. Edgeworth’s ‘exact Utilitarianism’

In the introduction of *Mathematical Psychics*, Edgeworth briefly summarizes the ‘Utilitarian Calculus’ in order to prove that the ‘Greatest Happiness’ principle is the end of right action. In addition, this ‘Calculus’ critically deals with Sidgwick’s utilitarianism. To put it more concretely, Edgeworth insists ‘if sentients differ in Capacity for happiness — under similar circumstances some classes of sentients experiencing on average more pleasure (*e.g.* of imagination and sympathy) and less pain (*e.g.* of fatigue) than others — there is no presumption that equality of circumstances is the most felicific arrangement’ (*ibid.*, vii). In this passage, he clearly states his rejection of ‘equality’, which utilitarianism tacitly implies, is clearly declared. It is beyond doubt that this view comes from the suspicion which he casts on the utilitarianism of Sidgwick and the Benthamites.

Edgeworth values Bentham highly because the latter applied a quantitatively precise analysis to the end of action (*ibid.*, 117). However, he showes a unique interpretation of Bentham’s famous phrase ‘the Greatest Happiness of the greatest number’. The parable Edgeworth uses demonstrates this; ‘suppose a greater illumination attainable with a smaller number of lamps (supplied with more material), does the criterion in this case give a certain sound?’ (*ibid.*, 117-18). This statement can also be found also on page 74 of *New and Old Methods of Ethics*. What he means by this is that when a certain amount

of ‘distribuend means’ is shared among a certain number of ‘distributees’, the total of happiness in the society can be the greatest even though the minority possess the greater part of the ‘distribuend means’. To support ‘the Greatest Happiness’ principle by such a distribution, it must be taken for granted that the difference of the capacity for happiness among the members should be presupposed. Equal distribution, then, is allowed only under the condition of equal capacity for happiness.

Sidgwick, as Edgeworth indicates, directed his attention to the difference of capacity for happiness in the *Methods of Ethics*.[22] Sidgwick’s observation on the distribution was, indeed, of ‘happiness’, not of ‘means of happiness’. When the difference of capacity is accepted, as in the case of Edgeworth, the equal distribution of happiness as well as means of happiness should not be admitted for the greatest happiness. Sidgwick, however, declared that as hedonistic calculations are all indefinite ‘the Utilitarian formula seems to supply no answer to this question [of distribution]’ (Sidgwick 1877, 384). Finally, he came to accept Benthamism in the form of ‘equal distribution of happiness’ as follows: ‘the principle which most Utilitarians have either tacitly or expressly adopted is that of equality: as given in Bentham’s formula, “everybody to count one, and nobody for more than one”. And this principle is obviously the simplest, and the only one which does not need a special justification: for, as we saw, it must be reasonable to treat any one man in the same way as any other, if there be no reason apparent for treating him differently’ (*ibid*., 385). Edgeworth thinks that this idea of Sidgwick’s is inconsistent, and attempts to make the ‘principle of the greatest happiness’ more precise in ‘Utilitarian Calculus’.

Edgeworth, first of all, gives the following four ‘definitions’ with some explanations:

[22] As evidence for this fact, Edgeworth quotes the following sentence in *Mathematical Psychics* (Edgeworth 1881, 124): ‘equal happiness is not to be attained by equal distribution of objects of desire. For some require more and some less to be equally happy’ (Sidgwick 1877a, 256n).

(1) '*Pleasure* is used for "preferable feeling" in general' (Edgeworth 1881, 56): (2) '*Means* are the distributable proximate means of pleasure'(*ibid*., 57): (3) 'An individual has greater *capacity for happiness* than another, when for the same amount whatsoever of means he obtains a greater amount of pleasure, *and also* for the same increment (to the same amount) whatsoever of means a greater increment of pleasure' (*ibid*., 57): (4) 'An individual has more *capacity for work* than another, when for the same amount whatsoever of work done he incurs a less amount of fatigue, *and also* for the same increment (to the same amount) whatsoever of work done a less increment of fatigue' (*ibid*., 59). The latter two definitions deal respectively with opposite sides of value, namely 'means' and 'work,' or 'pleasure' and 'fatigue (or pain)'. Edgeworth asserts that, as the reality of definition (4) is obvious, definition (3), which takes opposite signs, must be practical.

Then, Edgeworth states the 'axiom' that 'Pleasure is measurable, and all pleasures are commensurable' (*ibid*., 59). This idea was given in *New and Old Methods of Ethics* by adopting 'Fechner's law,' denoted above. In *Mathematical Psychics*, although the formula is not presented again, he clearly and simply expresses the unit of pleasure: 'any individual experiencing a unit of pleasure-intensity during a unit of time is to "count for one"' (*ibid*., 8). Thus, as pleasure might be quantitatively grasped, so naturally interpersonal comparison, or in other words cardinal comparison, of pleasure could be admitted for Edgeworth.

Under such definitions and the 'axiom', as the increment of pleasure may decrease with increase of its means, the first increment of means should be given to the most capable of pleasure, and the second to the next capable. In accordance with this rule of distribution, Edgeworth thinks, even though the 'distribuend means' are occupied by a few, due to their superior capacities for happiness, the society can enjoy the greatest happiness. 'Thus, *the distribution of means as between the equally capable of pleasure*

*is equality; and generally is such that the more capable of pleasure shall have more means and more pleasure*' (*ibid*., 64). Similarly, as the increment of fatigue is thought to increase in proportion to the increase of work, the first increment of work should be given to the most capable of work, and the second to the next capable. Then, Edgeworth insists '*the most capable of work shall do more work*–so much more work, as to suffer more fatigue' (*ibid*., 66).

The problem here is how the individual capacity of happiness or work can be determined. Edgeworth advances that both these capacities increase with 'evolution'. He already adopted the notion of this relationship between capacities and evolution from Spencer and Barratt in his previous work,[23] and he applies it to education in *Mathematical Psychics* as well. 'To advance the whole population by any [evolution] the same degree of evolution is then desirable; but it is probably not the most desirable application, a given quantity of a of *means of education*.[24] For it is probable that the highest in the order of evolution who are most capable of evolution are most *capable of education* and improvement. In the general advance the most advanced should advance most' (*ibid*., 68).

The accompanying problem is the possibility that the capacities for happiness or work *a posteriori* expand (or shrink) with education. Edgeworth disregards this, simply noting '[it] would hardly now be maintained in face of what is known about heredity' (*ibid*., 59). Similar opinions can be found in the same work. He notes that 'in so far as endogamy should not be the rule', for the hereditary selection, the parents' superior capacities are beneficial not only for the next generation, but for all following

[23] See Edgeworth (1877, 72; 1881, 58). As Barratt's 'true mentor was Herbert Spencer' (Matthew and Harrison 2004, vol.4, 32l), it can be said that Edgeworth applied Spencerion concepts to his *New and Old Methods of Ethics*. As to Spencer's influence on Edgeworth, see also Baccini(2007, 80-84).

[24] 'The words, "given quantity of a of means" should read "of a given quantity of *means*"' (Creedy 1986, 143).

generations. In *New and Old Methods of Ethics*, Edgeworth's notion of evolution was the social development or the advance of the member from the lower classes to the upper classes. In *Mathematical Psychics*, this notion develops to include eugenic thought, which is shown by the quotation from Galton's *Hereditary Genius* (1879) (*ibid*., 72; Galton 1879, 415). Though some difference exists among the capacities of each individual in the same class, people who generally tend to inherit superior capacities belong to the higher classes, and these people are capable of advancement. Such was his notion of evolution.

As the capacities for happiness or work differ among the different classes, 'equality' cannot be 'distributive justice'. Rather, Edgeworth maintains that in order to achieve the greatest happiness 'unequal distribution' of means is appropriate for 'distributive justice'.[25] He uses classical metaphor to describe the power of the word 'equality,' likening it to Zeus in the *Iliad*. 'πολλάων πολίων κατέλυσε κάρηνα ήδ' έτι και λύσει τού γάρ κράτος έστϊ μέγιστον', (pollaon polion katelyse karhena ed eti kai lysei tu garh krhatus esti megiston, which means 'he has destroyed the topmost towers of many cities, and will do so again, for his is the greatest power') (Edgeworth 1881, 77; Newman 2003, 167).[26] For Edgeworth, what seems to be destroyed by 'equality' is 'the crust of convention', and 'there may be needed an άξία [axia, which means 'measure'] for the unequal distribution' (Edgeworth 1881, 77 ; Newman 2003, 167).

The convention Edgeworth refers to here is distinction of class and sex. For example, when we think of the existence of classes, the reason the upper classes can receive larger remunerations is that they hold the skill and talent for scientific work which belong to the superior capacity for pleasure. On the other hand the lower classes should be given 'the work of which they seemed most capable', i.e. manual work (*ibid*., 78).

[25] Only the case where each individual holds equal capacity for happiness or work allows exceptionally the equal distribution of means to become 'distributive justice'.
[26] It is well known that young Edgeworth read Homer. See Keynes (1933, 266).

Furthermore, the reason women are exempt from harder work is because the stronger, which means in general men with larger capacity for work, should do much more work.[27] In such conventions, Edgeworth insists, 'we may see a reason deeper than Economics' (*ibid*., 78). According to Edgeworth, who noted 'concerning the classification of future society, common sense anticipates no utopia of equality' (*ibid*., 79), 'the struggle for life' seems to lead people to such 'utilitarian selection'.

Edgeworth calls the utilitarianism explained above 'exact Utilitarianism'. 'Exact Utilitarianism may also …. present the end of Politics' (*ibid*., 80). According to him, 'a political "contract" for the adjustment of conflicting interests should have two qualities': namely, the one which 'should be clear and fixed, universally interpretable in the same sense' and the other which 'should be such that the naturally more powerful class, those who, though fewer, outweigh the more numerous by strength, ability, and capacity to co-operate, should not have reason to think that they would fare better under some other contract' (*ibid*., 80). When these two qualities are considered, Edgeworth states that two kinds of 'Utilitarianism' can be adopted, which he called '*iso*cratical Utilitarianism' and '*aristo*cratical Utilitarianism', or 'exact Utilitarianism'. The former, which regards 'equality' as important, excels in the first quality, and the latter, which admits inequality, excels in the second. That Edgeworth put more weight on the second quality is clear from the following: 'the principle "Every man, and every woman, to count for one", should be very cautiously applied' (*ibid*., 81). Moreover, he notes that 'universal equal suffrage is less likely to be approved than plural votes conferred not only (as Mill thought) upon sagacity, but also upon capacity for happiness' (*ibid*., 81).

[27] However, such considerations should not be due from the stronger to the weaker of the same sex. In this case, 'there is wanting a natural instinct predisposing to the duties of benevolence; there has been wanting also a fixed criterion of strength to fix the associations of duty; and, lastly, competition has interfered, while competition between man and woman has been much less open (and much less obviously useful to the race' (Edgeworth 1881,79).

Thus, when Edgeworth uses the words 'Utilitarianism' or 'Utilitarian', he implies 'exact Utilitarianism', in other words '*aristo*cratical Utilitarianism'; which denies the nominal 'equality' due to the differences of individual capacities.

## IV. Conclusion

Edgeworth's *Mathematical Psychics* is a work influenced by various intellectual sources, mainly from Sidgwick and Barratt. From the latter half of the 1870s, Edgeworth may have been attracted to the 'Dualism of Practical Reasons' through the Sidgwick-Barratt Controversy. He insisted in *New and Old Methods of Ethics*, that egoism could be united with Utilitarianism through the evolution of moral sentience. However, his idea, at this stage, seems to be no more than his expectation, and for the solution to the 'Dualism of Practical Reasons' it has less persuasiveness.

In *Mathematical Psychics*, published four years after his first book, absorbing critically Jevons' exchange theory based on the marginal utility theory, Edgeworth developed still more precise discussions. With the 'Diagram', he insists that as indeterminacy may exist in the contract between egoists, utilitarianism is indispensable for its arbitration. However, the range of his contract theory is never confined within the framework of Economics. The indeterminacy inherent in transactions in the market was a simple substitution for the 'Dualism of Practical Reasons'. What Edgeworth showed was, therefore, a solution to the 'Dualism of Practical Reasons' through the explanation of the limits of adopting egoism and its need of utilitarianism in the egoistic world. This solution was opposite to Sidgwick and Barratt.[28]

Maintaining the importance of utilitarianism as practical reason, Edgeworth's point of view shifted from the discussion on homogeneous individuals (Economical Calculus) to

[28] In fact, he agrees with the opinion that 'the concrete nineteenth century man is for the most part an impure egoist, a mixed utilitarian' (Edgeworth 1881, 104).

that on the society consisting of heterogeneous individuals (Utilitarian Calculus). From this shift of discussion he derived his original utilitarianism. Although Edgeworth had admired Sidgwick's distinction between 'Egoism' and 'Utilitarianism' (*ibid*., 102),[29] he was not convinced of the utilitarianism of Sidgwick, mentioned in section III. Edgeworth's utilitarianism was much influenced by Barratt's physical scientific methods of ethics, and premised the commensurability of pleasures with an identical unit. Since Edgeworth indicated the difference of capacities among individuals, he criticized 'equality' on the eugenic notion of evolution, which is a mixture of Spencer's evolutionary theory and Galton's eugenics. For him, 'unequal distribution of means' based on each capacity was 'distributive justice'. As shown in the 'lamps' parable, if the sum of utility reaches maximum, even extremely unequal distributions are admitted.[30] Thus Edgeworth's 'exact Utilitarianism' was a supplement to Sidgwick's, and a refutation not only of the Benthamites, who suppose complete equality, but also of J.S. Mill, who admits the difference of the quality of pleasures.

We can see how contemporary economists take these ideas in representative reviews by Marshall (1881) and Jevons (1881).[31] Marshall's review paid attention to 'Economical Calculus', and admired Edgeworth as a 'genius'; but at the same time criticized him for the complexity of the abstract discussion and for too much use of

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[29] 'The distinction between egoism and utilitarianism has been drawn with matchless skill by Mr. Sidgwick' (Edgeworth 1881, 102).

[30] Edgeworth, at least, assures that 'each individual has and shall retain that minimum of means just sufficient to bring him up to the zero-point of happiness (a conception facilitated by, though not quite identical with, the economical "natural minimum of wages") (Edgeworth 1881, 64). A similar idea can also be found in *New and Old Methods of Ethics* as follows: 'there is then a "threshold" …. consisting of the necessities of life, all that must be pre-supposed before the sentient begins to experience pleasure ("satur est quun clamat Horatius 'Euœ'" ['Horace is well fed when he utters the bacchic cry' (Newman 2003, 253)]). To the necessities of the individual must be added his contribution to the necessities of the social state to which he belongs' (Edgeworth 1877, 76).

[31] There is another unsigned review on page 293 in *Mind* of 1881 (Anon. 1881, 293).

mathematics. In contrast, Jevons, who mainly dealt with 'Utilitarian Calculus,' applauded the work as 'a very remarkable one', but did not completely agree with Edgeworth's argument. This attitude can be seen for example on the division of sex. From his own experience in Australia,[32] Jevons criticized Edgeworth as follows: 'the anthropologists have hardly succeeded as yet in reconciling with theory the unfortunate position of women in primitive society' (Jevons 1881, 582). At least, Jevons seems to have regarded Edgeworth's argument on conventions as 'hierarchy' rather than strict equity in society.

Though there are some authors, like Arrow (1994), who understand Edgeworth as one who insists on the validity of the hierarchy of social scale as well as that of sex difference[33], the contemporary conventions were not at all other than what 'Utilitarianism' affirms for Edgeworth.

In conclusion, Edgeworth's essential ideas in *Mathematical Psychics* originated in the Sidgwick-Barratt Controversy, and absorbed various thoughts from fields such as philosophy, psychology and economics via his *New and Old Methods of Ethics* until it finally became the work connecting ethics and economics. Therefore, recent research confined within the range of economic theory or history of economic doctrine may only permit us to understand *Mathematical Psychics* partially. This is clear from the fact that Edgeworth himself wrote in the recommendation to University College, Liverpool as follows: 'I have written two books, "New and Old [Methods of] Ethics" (published by Parker & Co., 1877) and "Mathematical Psychics" (published by Kegan, Paul & Co., 1881); the former having for its subjects – Morals, and the Logical Methods of the Moral Science; the latter treating equally of Moral Philosophy and Political Economy' (MSS Edgeworth D7/13).

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[32] 'Among the Australian aborigines, for instance, the husband makes the wife carry all the burdens, and knocks her on the head if she declines or flags' (Jevons 1881, 582).

[33] See Arrow (1994, 94).

To sum up, Edgeworth dealt equally with 'Utilitarian Ethics' and 'Economics' in *Mathematical Psychics*. This means his *Mathematical Psychics* can not be brought into existence without utilitarianism, even though Schumpeter noted that 'in his case, as in that of Jevons, we can leave out the utilitarianism from any of his economic writings without affecting their scientific contents' (Schumpeter 1954, 831).

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