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Network Connectivity and R&D Competition in a Hotelling Model: Market Coverage, Consumer Expectations, and Asymmetric Firms

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Network Connectivity and R&D Competition in a Hotelling Model: Market Coverage, Consumer Expectations, and Asymmetric Firms

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Abstract
Network connectivity is an important function in network industries. Based on the framework of a Hotelling model, we consider the impact of connectivity between network goods on incentives to innovate product R&D activities and profits. To explore the problems, we focus on the following three perspectives: market coverage (i.e., full and partial coverage), consumer expectations (i.e., rational and active expectations), and asymmetric firms (i.e., a high- and low-quality firm). Our findings are as follows. In the full market coverage case, the impact of connectivity on product R&D activities and profits depends on the type of consumer expectations and the difference in the quality of the firms. However, in the partial market coverage case, as connectivity improves, product R&D activities and profits increase, irrespective of the type of consumer expectations and the difference in the quality of the firms.

Keywords
Network externality, Connectivity, Compatibility, Horizontal interoperability, R&D competition, Market coverage, Consumers' expectations, Firms' heterogeneity, Quality

JEL Classifications
L13, L15, L31, L32, D43

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1. Introduction

Episode 1: An exchange of emails with a colleague who recently went on a family trip to see the aurora:

> "I took some pictures of the aurora with my A-phone, which has the latest camera, and I'd like to transfer them directly to you, but yours didn't work with an A-phone."
>
> "I have a G-phone, so that might not be possible, but I have an A-pad, which is a little old."
>
> "The image quality might be a little poor, but next time we meet, I'll transfer the pictures from my A-phone to your A-pad."

Episode 2: A conversation with a colleague who commutes between university and downtown:

> "It seems like next week; it'll be easier to transfer between the A and B lines at station C on the S-train."
>
> "It'll be more convenient, because I had to walk a bit to transfer every time."
>
> "I used to take the bus to university, too, but from now on I'll start taking the S-train."

1.1 Background and research questions

In a modern digital society, networks are not only spreading to all economic activities, but also to every aspect of our daily lives. Network connectivity, compatibility, and "horizontal interoperability" are important functions in a network economy.[1] These functions are not limited to current information and communication technology industries, and play an important role in transportation such as railways and airlines (e.g., mutual access, alliances, and seamless operation) and banking systems such as automatic teller machines, and so on. Thus, an improvement in the quality of connectivity (compatibility and horizontal interoperability) can be beneficial to consumers who use goods and services with such functions.

However, these trends will surely intensify competition among firms providing network goods and services. Consequently, intense competition occurs at various levels and stages of the production and sale process, including product (service) and process research and development (R&D) investments, prices and sales (quantity) competition, among many others.

In this regard, Heywood et al. (2022, pp. 355–356) views compatibility as:

> "The extent to which one firm's R&D may allow it to lower costs and capture customers can be limited by the lack of compatibility. In addition, it is recognized that the extent of compatibility can influence the introduction of new technology [and that] reflecting this interconnection, firm compatibility decisions by network firms raise public policy issues regarding both anti-competitive behavior and reduced technological progress."[2]

The main problem examined in this paper is how network connectivity affects

[1] "Horizontal interoperability" is the form of interconnection between users in the terminology of Çavuş (2024).

[2] Economides and White (1994) discuss the economic and legal implications of compatibility and networks. They argue that compatibility is equivalent to the more general concept of complementarity and conclude that network arrangements bring benefits to firms, whereas compatibility may have anti-competitive consequences.

incentives to undertake R&D activities and profits.[3] That is, does an increase in the degree of network connectivity improve or reduce incentives to innovate? We are mainly interested in the conditions under which it is possible for network connectivity to reduce incentives for firms to innovate. For example, one might consider the following in our *Episode 1*: will the G-phone company conduct R&D activities to equip its mobile telephones with high-quality cameras to compete with the A-phone company? If the degree of compatibility (connectivity) between the A- and G-phones were to increase, would the G-phone company dare to undertake such R&D activities? Conversely, in that case, will the A-phone company develop mobile telephones equipped with higher-quality cameras in the future? Second, in our *Episode 2*, if the number of passengers using the S-train increases because of convenient transfers, that is, by the improvement in connectivity, will the S-train introduce new trains or improve passenger service?

In considering the problem, as will be explained in detail below, we focus on three perspectives: market coverage, consumer expectations, and heterogeneity of firms. Before doing so, we briefly review the related literature.

1.2 Literature review

Since the seminal studies by Farrell and Saloner (1985) and Katz and Shapiro (1985), there has been an increasing number of studies analyzing R&D investment competition in the presence of network externalities and compatibility (connectivity). Focusing on the characteristics of market structure, especially, demand functions, assumed in the models, we review some related literature.

[3] We do not consider firms' investments to improve network connectivity among internet services (e.g., Crémer et al., 2000; Foros and Hansen, 2001; Ji and Daitoh, 2008).

First, based on the backbone market model as in Crémer et al. (2000), which is an extension of Katz and Shapiro (1985), Knauff and Karbowski (2021) and Heywood, et al. (2022) consider cost-reducing R&D investment competition in the presence of network externalities. Both introduce technological knowledge spillover a là d'Aspremont and Jacquemin (1988) and compare noncooperative R&D with cooperative R&D investments. Relating to our problem, if we assume no technological spillover and no installed based consumers (i.e., no initial network sizes) in the model of Knauff and Karbowski (2021), an increase in compatibility (connectivity) reduces cost-reducing R&D activities in the case of noncooperative R&D. However, in the case of cooperative R&D, an increase in compatibility improves cost-reducing R&D activities.[4]

Second, the following papers apply the utility function of Hoernig (2012), in which a representative (homogeneous) consumer has a quasi-linear (e.g., quadratic) utility function including network effects, and purchases all the network goods provided in the market. For example, see Naskar and Pal (2020), Shrivastav (2021), and Buccella et al. (2022).[5] Using a horizontally differentiated duopoly model with network externalities, Shrivastav (2021) demonstrates the ranking of cost-reducing R&D investments for Bertrand and Cournot duopolistic competition. Furthermore, Shrivastav (2021, Appendix B) also finds the effects of compatibility on R&D investments, and argues that the following results hold in both Bertrand and Cournot competition: (i) if R&D investments are strategic complements, as compatibility increases, R&D investments increase; and (ii)

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[4] We do not examine the case of cooperative R&D. However, as will be shown below, we obtain the same result in the case of partial market coverage.

[5] Naskar and Pal (2020) assume that the degree of compatibility is equal to that of product differentiation. However, relaxing this assumption, Shrivastav (2021) examines more general cases.

if R&D investments are strategic substitutes, as compatibility increases, R&D investments first decrease, and then increase.

Buccella et al. (2022) assume a homogeneous product with network externalities and technological spillover effects. They compare the investments, quantities, and profits in the full compatibility case with those in the incompatibility case, declaring that if there are no technological spillover effects, the level of investment in the incompatibility case is higher than in the full compatibility case.[6]

As Roson (2002) points out, what the demand functions in the first and second models have in common is that an increase in compatibility (connectivity) leads to an increase in the overall market size. However, in the following studies, an increase in connectivity does not necessarily lead to market expansion.

Third, Foros and Hansen (2001) assume that each consumer in a unit-linear market of a Hotelling model has an individual preference for the goods (i.e., heterogeneity) and then purchases either one or none of the goods. Regarding this location demand model, Kim (2000) assumes quality-improving innovation and considers the effect of compatibility on incentives to innovate. Kim (2000, Theorem 5) shows that the effect of an increase in compatibility on the profit of the innovative firm is ambiguous, whereas the profit of the non-innovative rival firm is increased. In this case, the assumption is made that the innovative firm is a high-quality firm, whereas the non-innovative firm is a low-quality firm. This is because an increase in compatibility raises the prices of the innovative firm, leading it to lose market share, which implies that the effect of compatibility on innovation can be negative.

[6] Regarding this point, see Remark 1 below.

Applying a linear Hotelling market model, the approach of Sääskilahti (2006), which is very close to ours, considers cost-reducing innovation and shows that network compatibility is neutralized in the decision regarding cost-reducing investment given "symmetric quality" (i.e., identical strength of network externalities).[7] However, Sääskilahti (2006, Proposition 3) demonstrates that in the case of "asymmetric quality" (i.e., different strengths of network externalities), the effect of an increase in compatibility on the investment of the high (low) "quality" firm is negative (positive). As shown below, assuming symmetric network externalities and different stand-alone values expressing quality in our model, we obtain the same result.

1.3 The purpose and the implication of three perspectives

We assume that there are heterogeneous consumers with individual preferences for network products, as in Kim (2000), Foros and Hansen (2001), and Sääskilahti (2006). Based on a Hotelling linear market model, we consider the impact of an improvement in connectivity between network products on R&D activities (i.e., cost-reducing investments) and on profits, noticing three perspectives: market coverage, consumer expectations for network sizes, and heterogeneity of firms.

(1) Following Crémer et al. (2000), who adopt the well-known model in Katz and Shapiro (1985), and Foros and Hansen (2001), who adopt a unit-linear market following a conventional Hotelling location model, Roson (2002) claims that the difference in their

[7] Sääskilahti (2006) assumes a technological knowledge spillover a là d'Aspremont and Jacquemin (1988). We do not assume technological spillover effects, which are supply-side externalities, but assume symmetric network (consumption) externalities, which implies demand-side externalities.

results depends on alternative hypotheses on the overall market size. That is, the market size is variable in the first paper, whereas it is fixed in the second paper. In particular, an improvement in connectivity expands the overall market size in the first paper, that is, market expansion effects.

Regarding this point, although we adapt the framework of a Hotelling model, we address the cases of full and partial market coverage. In the partial market coverage case, competing firms can expand their market share because there are potential consumers in an uncovered market. Thus, as mentioned in the literature review, while the market expansion effect does not occur in the full market coverage case, there is capacity for this effect to occur in the partial market coverage case. In this case, the impact of connectivity on incentives to innovate depends on the difference in market coverage. For example, Kim (2000), Foros and Hansen (2001), and Sääskilahti (2006) assume the full market coverage case. We suppose that the full (partial) market coverage case in our model corresponds to the stage of a matured (an immature) network products market.

(2) Regarding the formation of consumer expectations for network sizes, as Suleymanova and Wey (2012) mention, the formation of expectations becomes a critical determinant of market performance in many software and digital markets where network externalities play an important role. So, following the approach of Katz and Shapiro (1985), we consider two types of expectations (i.e., "rational" and "active" expectations).[8]

"Rational" expectations follow the concept of a fulfilled expected equilibrium. That

[8] In the terminology of Suleymanova and Wey (2012), "rational" corresponds to "strong" and "actual" to "weak." Additionally, following Hurkens and López (2014), "rational" corresponds to "passive" and "active" to "responsive."

is, consumers can "rationally" form their expectations of network sizes at the equilibrium. Accordingly, under such expectations, consumers do not believe that the announcements of network sizes in advance by firms are credible, in other words, firms cannot commit to their announcements of network sizes. Conversely, under "active" expectations, consumers believe the announcements of actual sizes (i.e., outputs or number of consumers) are equal to expected network sizes, and thus the firms can commit to their actual output.[9] For example, Kim (2000), Foros and Hansen (2001) and Knauff and Karbowski (2021) assume "rational" expectations, whereas Crémer et al. (2000), Sääskilahti (2006), and Heywood et al. (2022) assume "active" expectations.

(3) As a third perspective, we consider asymmetric firms (or heterogeneity of firms). For example, using an install bases model, Heywood et al. (2022) assume an "incumbent-entrant" model, in which the incumbent firm has already installed bases but the entrant does not. Sääskilahti (2006) assumes that the strength of network externalities is not the same (i.e., asymmetric "quality").

Using the concept of vertically differentiated services assumed by Foros and Hansen (2001), we assume the difference in the level of quality of network products as an index of asymmetric firms. The quality in our model corresponds to a reservation price and an intrinsic stand-alone value in a standard Hotelling model. As shown below, the difference in quality implies the difference in the market share of firms, and the difference in the size of firms.[10] In particular, a firm providing a high-(low-)quality network product

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[9] Following Shy (2001, Definition 2.4, p. 20), "actual" expectations imply that consumers have perfect foresight if, at the time of purchase, they can correctly anticipate how many consumers will buy each brand.

[10] Empirical research on the relationship between firm size and innovation points out the

becomes a big (small) company with a large (small) market share. We will demonstrate that the sensitivity to R&D activities due to an improvement in connectivity depends on the difference in the level of quality.

To sum up, in this paper, we examine the impact of connectivity on R&D activities and profits of high-quality and low-quality firms in the four cases made by the combinations of the two perspectives, market coverage and consumer expectations.

The rest of the paper is organized as follows. In Section 2, using the framework of a Hotelling linear market with network externalities, we derive demand functions in the cases of full and partial market coverage and present cost and profit functions of R&D activity. In Sections 3, given that consumer expectations are rational, we derive the equilibrium in R&D competition in the cases of full and partial market coverage, and consider how connectivity affects asymmetric firms' R&D activities and profits, and then compare the results in the cases of full and partial market coverage. In Section 4, by relaxing the assumption of consumer expectations, in particular, with respect to the full market coverage case, we examine the impact of connectivity on asymmetric firms' R&D activities and profits, given that consumer expectations are active (or responsive), and then compare the equilibrium outcomes with those under rational expectations derived in Section 3. In addition, we note that our model can apply to the case of product (quality-improving) R&D activity. Finally, in Section 5, we summarize our findings and discuss some remaining problems.

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relative advantage of small and medium-sized enterprises in innovation. For example, see Zoltan and Audretsch (1987, 1988), Audretsch and Zoltan (1991), and Rogers (2004).

2. Model

2.1 Demand functions: market coverage and a transportation cost

We introduce network externalities associated with connectivity (compatibility and horizontal interoperability) into a Hotelling linear market model.[11] *Firm i*, which is located at both ends of a unit-linear market, provides *product i*, $i = 0,1$. Consumers are uniformly distributed over a unit line of the closed interval $[0,1]$ according to their subjective taste preferences. That is, consumer $\theta \in [0,1]$ has the following surplus (net utility) function:

$$U_\theta = \begin{cases} v_0 - t\theta - p_0 + N_0 & \textit{if buying} \quad \textit{product} \quad 0 \\ v_1 - t(1-\theta) - p_1 + N_1 & \textit{if buying} \quad \textit{product} \quad 1 \\ 0 & \textit{if buying} \quad \textit{nothing} \end{cases}, \qquad (1)$$

where v_i is the intrinsic (stand-alone) value of *product i*, implying the level of quality of *product i*,[12] t is a transportation cost, expressing a disutility, p_i is price of *product i*, and N_i is a network effect of *product i*, which is explicitly specified below.

Looking at a transportation cost, we derive demand functions in the cases of full and partial market coverage. A transportation cost implies product substitutability. The smaller (larger) a transportation cost, the higher (lower) production substitutability.

[11] We use the terms connectivity, compatibility, and interoperability interchangeably throughout the paper because we assume that they have an identical role and function in our model.

[12] As an example, we consider the intrinsic function of a personal computer (PC) disconnected in networks and the quality of a mobile phone camera. See Griva and Vettas (2011), in which they assume horizontally and vertically differentiated network products in a simple Hotelling model.

Therefore, the type of market coverage depends on the size of a transportation cost.

Using Equation (1), the consumer indexed θ^*, whose surplus is indifferent between *products 0* and *1*, is given by $\theta^* = \frac{1}{2} + \frac{v_0 - v_1 - p_0 + p_1 + N_0 - N_1}{2t}$.[13]

First, for the case of full market coverage to hold, the following conditions must be met: $U_{\theta=\theta^*} > 0$ and $U_{\theta=0}$ $(U_{\theta=1})$ > 0. Taking Equation (1), we obtain the following conditions.

FMC: $t \le v_0 - p_0 + N_0 + v_1 - p_1 + N_1 \equiv T$ and $v_i - p_i + N_i > 0,\ \ i = 0,1.$

See Appendix A (1), in which we demonstrate the conditions under which *FMC* holds in equilibrium and discuss the implications.

Namely, the transportation cost must be lower than the total net values of two network products. In other words, there exists an upper bound of the transportation cost. That is, not too high transportation cost implies that the degree of product substitutability is relatively large. Accordingly, most consumers in the market have access to either of the network products.

Given the conditions, all consumers in the market purchase either of two network products. Thus, the demand function of *firm 0* is given by:

$$x_0 = \theta^* = \frac{t + v_0 - v_1 - p_0 + p_1 + N_0 - N_1}{2t}, \tag{2}$$

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13 Because we consider duopolistic competition in our model, we assume as follows:

$$0 < \theta^* < 1 \Leftrightarrow \left|(v_0 - p_0 + N_0) - (v_1 - p_1 + N_1)\right| < t \Leftrightarrow \left|U_{\theta=0} - U_{\theta=1}\right| < t.$$

Otherwise, either of the firms could become a monopolist.

where $N_i \equiv n\left(x_i^e + \phi x_j^e\right), \quad i, j = 0,1, \quad i \neq j.$ Parameter $n(>0)$ is the strength of network externalities, $\phi \in [0,1]$ is the degree of network connectivity (hereinafter, connectivity), and x_i^e is the expected network size of *product i*, which expresses the expected number of consumers purchasing *product i*. Thus, $x_i^e + \phi x_j^e$ is the total expected network size of the network products, in which $n x_i^e$ expresses the "within-group" (direct) network effects for consumers purchasing *product i* from themselves, and $n\phi x_j^e$ expresses the "cross-group" (indirect) network effects for consumers purchasing *product i* from consumers purchasing *product j*.[14] Regarding the demand function of *product 1*, based on Equation (2), we have $x_1 = 1 - x_0$.

Second, using the same procedure as in the full market coverage case, for the case of partial market coverage to hold, the following conditions must be met: $U_{\theta=\theta^*} < 0$ and $U_{\theta=0}$ $\left(U_{\theta=1}\right)$ > 0. Hence, we obtain the following conditions.

PMC: $t > v_0 - p_0 + N_0 + v_1 - p_1 + N_1 \equiv T$ and $v_i - p_i + N_i > 0, \quad i = 0,1.$

See Appendix A (2), in which we demonstrate the conditions under which the *PMC* holds in equilibrium and discuss the implications.

In this case, the transportation cost must be higher than the total values of the two network products. Namely, there exists a lower bound of the transportation cost. The high

[14] Following the terminology of Sääskilahti (2006), the "within-group" network effect corresponds to a "home" network, and the utility is labelled as an "intra-network utility". Similarly, the "cross-group" network effect corresponds to a "rival" network and the utility as an "inter-network utility".

transportation cost (large disutility) implies that the network products are sufficiently differentiated, and thus, there are some consumers who cannot have access to the network products. In other words, there are some potential consumers who will purchase the network products by changing the degree of connectivity.

Given the conditions, using Equation (1), i.e., $U_\theta = 0$, the marginal consumer purchasing *product 0* is given by $\theta_0 = \frac{v_0 - p_0 + N_0}{t} \equiv z_0$. Similarly, for *product 1*. Thus, we obtain the following demand function:

$$z_i = \frac{v_i - p_i + N_i}{t}, \quad i = 0,1, \tag{3}$$

where $N_i = n\left(z_i^e + \phi z_j^e\right), \quad i,j = 0,1, \quad i \neq j$. Using Equation (3), it also holds that $z_0 + z_1 < 1 \Leftrightarrow t > T$. Thus, $1 - z_0 - z_1 > 0$ expresses that there are some potential consumers not purchasing network products.

FMC and *PMC* imply the following situations. A lower (higher) transportation cost shows that the level of substitutability of a network product is relatively high (low). For example, based on the data of the Statistics Bureau of Japan regarding information and communications industries, the ownership rate per household in 2023 will be around 90% for smartphones and 60% for personal computers, while wearable devices will account for less than 10%. Thus, the smartphones (wearable devices) market may correspond to the full (partial) market coverage case.

2.2 Profit function, process R&D activities, and game structure

Using Equations (1) and (2), in the cases of full (partial) market coverage, the gross profit function of *firm i* is expressed as $\pi_i^f = (p_i - c_i)x_i$ $\left(\pi_i^p = (p_i - c_i)z_i\right)$, where c_i is the marginal cost of *firm i*, $i=0,1$. Superscript f (p) denotes the full (partial) market coverage case under rational expectations.

To consider process (cost-reducing) R&D activities, regarding the variable expressing a marginal cost, we assume $c_i = \overline{c}_i - \alpha_i \left(\geq 0\right)$, $i=0,1$, where α_i denotes the degree of cost-reducing R&D activity and $\overline{c}_i$ is the initial level of marginal cost before implementing R&D activity. Furthermore, we express the variable as follows: $\overline{v}_i \equiv v_i - \overline{c}_i > 0$, $i=0,1$. The variable $\overline{v}_i$ denotes the level of quality net of marginal cost of *firm i* before R&D activities (hereinafter, the net quality). As we will explain below, this parameter expresses the heterogeneity (or asymmetric size) of firms.[15]

The firms incur fixed costs for their R&D activities. We assume the following R&D activity (investment) cost function: $F\left(\alpha_i\right) = \frac{k}{2}\left(\alpha_i\right)^2$, $k>0$. Thus, the net profit function of *firm i* is expressed as $\Pi_i^m = \pi_i^m - F\left(\alpha_i\right)$, $m=f,p$, $i=0,1$.

The structure of the game consists of two stages. At the first stage, the firms simultaneously decide the level of R&D activities, and at the second stage, the firms compete on prices. We assume that consumers have rational expectations for network sizes of the products and form their expectations before the second stage (or after the first stage). We employ the concept of a fulfilled expectation equilibrium presented by Katz and Shapiro (1985) and derive a subgame perfect Nash equilibrium by backward induction.

[15] Following the terminology of Hashizume and Nariu (2020), $\overline{v}_i$ represents the value of *firm i*'s efficiency at the initial situation (before R&D activities) in accordance with Zanchettine (2006) and implies a positive primary markup.

3. Analysis: Market Coverage and R&D Competition

3.1 Equilibrium and the impact of connectivity in the full market coverage case

In the second stage, the firm decides the price to maximize profit, given the expected network sizes. Taking Equation (2), the first-order condition (FOC) of profit maximization of *firm i* is given by $\frac{\partial \pi_i^f}{\partial p_i} = x_i - \frac{p_i - c_i}{2t} = 0, \quad i = 0,1.$ At the fulfilled expectation equilibrium, i.e., $x_i^e = x_i = \frac{p_i - c_i}{2t}$, we obtain the following price-cost margin and output:

$$p_i^f - c_i = \frac{\{3t - n(1-\phi) + \Delta_i + \alpha_i - \alpha_j\} t}{3t - n(1-\phi)}, \tag{4}$$

$$x_i^f = \frac{p_i^f - c_i}{2t} = \frac{3t - n(1-\phi) + \Delta_i + \alpha_i - \alpha_j}{2\{3t - n(1-\phi)\}}, \tag{5}$$

where $t > \frac{n}{3}\left(\geq \frac{n(1-\phi)}{3} \right)$ and $\Delta_i \equiv \overline{v}_i - \overline{v}_j, \quad i, j = 0,1, \quad i \neq j.$

$\Delta_i \left(= -\Delta_j\right)$ expresses the difference in the net quality between the firms at the initial situation.[16] If $\Delta_i > 0$, i.e., $\overline{v}_i > \overline{v}_j$, then *firm i* (*j*) is a high- (low-)quality firm, and, correspondingly, the market share of *firm i* is larger than that of *firm j*. Assuming Δ_i as the indicator of firm size, in some cases, we call *firm i* (*j*) a big (small) company.[17]

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[16] Following the terminology of Foros and Hansen (2001), the difference in the qualities corresponds to that in vertically differentiated services.

[17] We observe that even relatively small and medium-sized enterprises produce high-quality products with a high level of technology. However, in this paper, we simply refer to a high (low) quality firm as a big (small) company in the sense that the firm has a

In the first stage of competition for R&D activities, the net profit function of *firm i* is expressed as $\Pi_i^f = \left(p_i^f - c_i\right)x_i^f - F\left(\alpha_i\right) = \frac{\left(p_i^f - c_i\right)^2}{2t} - \frac{k}{2}\left(\alpha_i\right)^2, \quad i = 0,1$. The FOC with respect to R&D activity is given by:

$$\frac{\partial \Pi_i^f}{\partial \alpha_i} = \frac{p_i^f - c_i}{3t - n(1-\phi)} - k\alpha_i = \frac{\left\{3t - n(1-\phi) + \Delta_i + \alpha_i - \alpha_j\right\}t}{\left\{3t - n(1-\phi)\right\}^2} - k\alpha_i = 0. \qquad (6)$$

Additionally, we derive the following second-order condition (SOC) and cross effect:

$$\frac{\partial^2 \Pi_i^f}{\partial \alpha_i^2} = \frac{t}{\left\{3t - n(1-\phi)\right\}^2} - k < 0 \text{ and } \frac{\partial^2 \Pi_i^f}{\partial \alpha_i \partial \alpha_j} = \frac{-t}{\left\{3t - n(1-\phi)\right\}^2} < 0.$$

Because the cross effect is negative, the firms' R&D activities are strategic substitutes.

Based on Equation (6), it holds in the equilibrium that

$$\alpha_i^f\left(\phi, \Delta_i\right) = \alpha^{f*}\left(\phi\right) + \frac{t\Delta_i}{k\left\{3t - n(1-\phi)\right\}^2 - 2t}, \quad i = 0,1. \qquad (7)$$

where $\alpha^{f*}\left(\phi\right) \equiv \frac{t}{k\left\{3t - n(1-\phi)\right\}}$ is the average level of process R&D activities of the firms, i.e., $\alpha^{f*}\left(\phi\right) = \frac{\alpha_0^f + \alpha_1^f}{2}$.[18] It is obvious that an increase in connectivity reduces the average R&D activity.

In view of Equation (7), if $\Delta_i \geq 0$, the equilibrium R&D activity is positive.

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relatively large (small) market share.

[18] It also expresses the level of R&D activities when the firms are symmetric, i.e., $\Delta_i = 0, \quad i = 0,1$.

However, if $\Delta_i < 0$, that is, *firm i* (*j*) is a small (big) company, for equilibrium R&D activity to be positive, the following condition must hold.[19]

Condition F: $\dfrac{k\{3t-n(1-\phi)\}^2-2t}{k\{3t-n(1-\phi)\}} \equiv \hat{\Delta}^f > -\Delta_i = \Delta_j, \quad i,j=0,1, \quad i \neq j.$

Condition F implies that the difference in the net quality between the firms must not be large in the equilibrium.[20] Using Equations (5) and (7), we obtain the following relationship: $x_i^f > (<) x_j^f \Leftrightarrow \Delta_i > (<) 0, \quad i,j=0,1, \quad i \neq j.$ As mentioned above, if $\Delta_i > 0,$ the market share (or the size) of *firm i* is larger than that of *firm j*. It also holds that $\alpha_i^f > (<) \alpha_j^f \Leftrightarrow \Delta_i > (<) 0.$ The level of R&D activity of the big company is larger than that of the small company. However, with respect to the sensitivity of network connectivity on R&D activity, using Equation (7), we obtain:

$$\frac{d\alpha_i^f(\phi,\Delta_i)}{d\phi} = -ntA(\phi,\Delta_i), \tag{8}$$

where $A(\phi,\Delta_i) \equiv \dfrac{1}{k\{3t-n(1-\phi)\}^2} + \dfrac{2k\{3t-n(1-\phi)\}\Delta_i}{\left[k\{3t-n(1-\phi)\}^2-2t\right]^2}, \qquad i=0,1.$ Thus, we summarize the effect of an increase in connectivity on process R&D activity as follows.

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[19] Because the sign of the determinant of the Hessian matrix is positive, the local maximum value condition is $k\{3t-n(1-\phi)\}^2-2t>0.$

[20] Given *Condition F*, regarding the FOC of *firm i*, we obtain the following: $\left.\dfrac{\partial \Pi_i^f}{\partial \alpha_i}\right|_{\substack{\alpha_i=0\\ \alpha_j>0}} > 0,\ i,j=0,1,\ i \neq j.$

Proposition 1. In the full market coverage case:

(1) if $\Delta_i \geq 0,$ an increase in connectivity reduces R&D activity, and

(2) if $\Delta_i < 0,$ the following relationship holds:

$$\frac{d\alpha_i^f(\phi,\Delta_i)}{d\phi} > (<) 0 \Leftrightarrow -\Delta_i > (<) \Delta'^f \equiv \frac{\left[k\left\{3t-n(1-\phi)\right\}^2-2t\right]^2}{2k^2\left\{3t-n(1-\phi)\right\}^3}\left(<\hat{\Delta}^f\right), \quad i=0,1.$$

Proposition 1 (1) shows that an increase in connectivity reduces the incentive to innovate process R&D of a big company. Conversely, in Proposition 1 (2), when the difference in the net quality (or the difference in the size of firms) is large, i.e., $|\overline{v}_0-\overline{v}_1| > \Delta'^f$, an increase in connectivity improves the incentive to innovate process R&D of a small company. Otherwise, it reduces the incentive.

Namely, unless the difference in the sizes of both firms is large, an improvement in connectivity reduces incentives to innovate their R&D activities. However, if the difference is sufficiently large, the sensitivity of R&D activities to improving network connectivity has a negative effect on a big company but a positive effect on a small company.

Why does an improvement in connectivity between network products suppress or improves incentives to innovate? Using Equation (6), the effect of connectivity on the marginal net profit of *firm i* is given by:

$$\frac{\partial^2 \Pi_i^f}{\partial \alpha_i \partial \phi} = -\frac{n\left\{3t-n(1-\phi)+2\Delta_i+2(\alpha_i-\alpha_j)\right\}t}{\left\{3t-n(1-\phi)\right\}^3}, \quad i,j=0,1, \quad i \neq j.$$

Substituting the equilibrium outcome given in Equation (7) into the above equation, we obtain:

$$\frac{\partial^2 \Pi_i^f}{\partial \alpha_i \partial \phi} = -\frac{nt}{\{3t - n(1-\phi)\}^2}\left[1 + \frac{2k\{3t - n(1-\phi)\}\Delta_i}{k\{3t - n(1-\phi)\}^2 - 2t}\right].$$

If $\Delta_i \geq 0,$ then the sign is negative. Because an increase in connectivity reduces the marginal net profit, it decreases incentives to innovate. *Firm i* is a big company with a large market share, and then, an increase in connectivity spills over a part of its market share to the rival *firm j* (i.e., a small company) through the cross-group network effects.

Conversely, if $\Delta_i < 0,$ that is, the difference in the size of firms is sufficiently large, and *firm i* is a small company, then the effect on the marginal profit is positive.[21] In this case, the benefit (i.e., a part of the market share) of the rival *firm j* (i.e., a big company) spills over to *firm i* by the cross-group network effects. As a result, an increase in connectivity improves the small company's incentive to innovate process R&D activity.

Next, we examine the impact of connectivity on net profit, which is expressed as $\Pi_i^f = \Pi_i^f\left[\alpha_i^f(\phi,\Delta_i), \alpha_j^f(\phi,\Delta_j), \phi\right], \quad i,j = 0,1, \quad i \neq j.$ The total effect of an increase in connectivity on the net profit of *firm i* is given by:

$$\frac{d\Pi_i^f}{d\phi} = \frac{\partial \Pi_i^f}{\partial \alpha_i^f}\frac{d\alpha_i^f}{d\phi} + \frac{\partial \Pi_i^f}{\partial \alpha_j^f}\frac{d\alpha_j^f}{d\phi} + \frac{\partial \Pi_i^f}{\partial \phi} = \underbrace{\frac{\partial \Pi_i^f}{\partial \alpha_j^f}\frac{d\alpha_j^f}{d\phi}}_{\text{indirect (strategic) effect}} + \underbrace{\frac{\partial \Pi_i^f}{\partial \phi}}_{\text{direct effect}}, \quad i,j = 0,1, \quad i \neq j,$$

where $\frac{\partial \Pi_i^f}{\partial \alpha_i^f} = 0$ by the FOC. Using Equation (4), we obtain the following direct effect:

$$\frac{\partial \Pi_i^f}{\partial \phi} = \frac{-n(p_i^f - c_i)}{3t - n(1-\phi)}\frac{\left(\Delta_i + \alpha_i^f - \alpha_j^f\right)}{3t - n(1-\phi)} = \frac{-n(p_i^f - c_i)}{3t - n(1-\phi)}\frac{k\{3t - n(1-\phi)\}\Delta_i}{k\{3t - n(1-\phi)\}^2 - 2t}.$$

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[21] It holds that $\left(\hat{\Delta}^f >\right) - \Delta_i > \frac{k\{3t - n(1-\phi)\}^2 - 2t}{2k\{3t - n(1-\phi)\}} > \Delta'^f.$

Then, we have the following relationship: $\Delta_i > (<) 0 \Leftrightarrow \frac{\partial \Pi_i^f}{\partial \phi} < (>) 0, \quad i = 0,1.$ If *firm i* is a high-(low-)quality firm, i.e., a big (small) company, the direct effect on its profit is negative (positive). For example, if *firm i* is a big company, the direct effect is negative. This is because a part of the benefits (*firm i*'s customer or its market share) spills over to the rival small company through the cross-group network effects.[22] Based on Equation (5), in fact, the direct effect of an increase in connectivity on the market share is given by

$$\frac{\partial x_i^f}{\partial \phi} = \frac{-n}{2\{3t - n(1-\phi)\}} \frac{k\{3t - n(1-\phi)\}\Delta_i}{k\{3t - n(1-\phi)\}^2 - 2t} < 0, \quad \text{if} \quad \Delta_i > 0.$$

Furthermore, the effect of the rival firm's R&D activity on the net profit of *firm i* is $\frac{\partial \Pi_i^f}{\partial \alpha_j^f} = -\frac{p_i^f - c_i}{3t - n(1-\phi)} < 0.$ Thus, regarding strategic (indirect) effects, we derive:

$$\frac{\partial \Pi_i^f}{\partial \alpha_j^f} \frac{d\alpha_j^f}{d\phi} = \frac{-(p_i^f - c_i)}{3t - n(1-\phi)} \frac{d\alpha_j^f}{d\phi} = \frac{n(p_i^f - c_i)}{3t - n(1-\phi)} \left[\frac{t}{k\{3t - n(1-\phi)\}^2} - \frac{2tk\{3t - n(1-\phi)\}\Delta_i}{\left[k\{3t - n(1-\phi)\}^2 - 2t\right]^2} \right].$$

If $\Delta_i \leq 0,$ the strategic effect is positive. However, if $\Delta_i > 0,$ the following relationship holds: $\Delta'^f > (<) \Delta_i \Leftrightarrow \frac{\partial \Pi_i^f}{\partial \alpha_j^f} \frac{d\alpha_j^f}{d\phi} > (<) 0, \quad i, j = 0,1, \quad i \neq j.$ For example, if *firm i* is a big company, which implies that the difference in the net quality of the firms is sufficiently large, i.e., $\Delta_i > \Delta'^f$, and thus, its market share is also sufficiently large, then the strategic effect is negative. Otherwise, the strategic effect is positive.

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[22] If the firms are symmetric, i.e., $\Delta_i = 0,$ it holds that $\left. \frac{\partial \Pi_i^f}{\partial \phi} \right|_{\alpha_0^f = \alpha_1^f} = 0.$ The direct effect is zero. This implies the spillover effects between two firms offset each other.

Based on the direct and strategic effects, the total effect of an increase in connectivity on the net profit is given by:

$$\frac{d\Pi_i^f}{d\phi} = \frac{\partial \Pi_i^f}{\partial \alpha_j^f}\frac{d\alpha_j^f}{d\phi} + \frac{\partial \Pi_i^f}{\partial \phi}$$

$$= \frac{-n(p_i^f - c_i)}{3t - n(1-\phi)}\left[\frac{k^2\{3t - n(1-\phi)\}^3 \Delta_i}{\left[k\{3t - n(1-\phi)\}^2 - 2t\right]^2} - \frac{t}{k\{3t - n(1-\phi)\}^2}\right].$$

Thus, if $\Delta_i \le 0,$ the total effect is positive. However, if $\Delta_i > 0,$ we have the following relationship: $\Delta''^f > (<)\Delta_i \Leftrightarrow \frac{d\Pi_i^f}{d\phi} > (<)0, \quad i, j = 0,1, \quad i \ne j,$ where $\Delta''^f \equiv \frac{t\left[k\{3t - n(1-\phi)\}^2 - 2t\right]^2}{k^3\{3t - n(1-\phi)\}^5}\left(< \Delta'^f\right).$ Thus, we summarize the result as follows.

Proposition 2. In the full market coverage case:

(1) if $\Delta_i \le 0,$ an increase in connectivity increases net profit, and

(2) if $\Delta_i > 0,$ the following relationship holds: $\Delta''^f > (<)\Delta_i \Leftrightarrow \frac{d\Pi_i^f}{d\phi} > (<)0, \quad i = 0,1.$

Proposition 2 (1) demonstrates that if *firm i* is a small company, the direct and strategic effects are positive. Thus, an increase in connectivity improves its net profit. In Proposition 2 (2), unless the difference in the net quality of the firms is sufficiently large, the total effect on net profit is positive. This is because the strategic effect is positive, although the direct effect is negative. In this case, the magnitude of the negative direct effect is not very large. Accordingly, the positive strategic effect dominates the negative direct effect. However, if the difference is sufficiently large, as network connectivity improves, the net profit of a high-quality firm (i.e., a big company) decreases, while that of a low-quality firm (i.e., a small company) increases. This is because the direct and strategic effects become negative for the net profit of a big company.

Proposition 2 provides the following policy implication. For example, we suppose that a government promotes construction of a common network system in the market. Then, the R&D activity of a big company will be suppressed, while that of a small company will be promoted. Furthermore, the net profit of a big company will decrease, while that of a small company will increase. However, if either the difference in the firms is not large or the firms are symmetric, then the policy reduces incentives to innovate, whereas it increases the profits.

Remark 1. Symmetric case: $\Delta_i = 0, \quad i = 0,1.$

Based on Propositions 1 and 2, it holds that $\alpha^{f*}(\phi = 0) > \alpha^{f*}(\phi = 1)$ and $\Pi_i^{f*}(\phi = 0) < \Pi_i^{f*}(\phi = 1)$. This demonstrates that providing network products with perfect connectivity is desirable for the firms, although the level of R&D activities is the lowest. In addition, in view of Equation (4), the equilibrium price is expressed as $p_i^f(\phi) = t + c_i(\phi)$, where $c_i(\phi) \equiv \overline{c}_i - \alpha^{f*}(\phi)$, so that we obtain $p_i^f(\phi = 1) > p_i^f(\phi = 0)$. That is, the firms provide their network products with lower quality and higher price (i.e., degraded network products), compared with the case of imperfect connectivity.[23] As addressed in the introduction, this implies that improving connectivity (compatibility) is

[23] As will be discussed in section 4.3, we can examine the case of quality-improving R&D competition. In this case, under perfect (imperfect) connectivity, the level of quality-improving R&D activities becomes the lowest (highest).

likely to be anti-competitive and increases the collusive behavior of firms.

3.2 Equilibrium and the impact of connectivity in the partial market coverage case

By relaxing the assumption of market coverage, we consider the case of partial market coverage, in which there are some potential consumers not purchasing network products.

Following the approach in the previous section, and using Equation (3), we derive the following outcomes at the equilibrium in price competition:

$$p_i^p - c_i = \frac{\left[(2t-n)\overline{v}_i + n\phi\overline{v}_j + (2t-n)\alpha_i + n\phi\alpha_j\right]t}{D^p}, \tag{9}$$

$$z_i^p = \frac{p_i^p - c_i}{t} = \frac{\left[(2t-n)\overline{v}_i + n\phi\overline{v}_j + (2t-n)\alpha_i + n\phi\alpha_j\right]}{D^p}, \tag{10}$$

where $D^p \equiv \{2t - n(1+\phi)\}\{2t - n(1-\phi)\} > 0$ for $t > n\left(\geq \frac{n(1+\phi)}{2}\right)$, $i, j = 0,1,\ i \neq j.$

The net profit function is $\Pi_i^p = \left(p_i^p - c_i\right)z_i^p - F(\alpha_i) = \frac{\left(p_i^p - c_i\right)^2}{t} - \frac{k}{2}\left(\alpha_i\right)^2$. Thus, the FOC of *firm i* in R&D competition is given by:

$$\frac{\partial \Pi_i^p}{\partial \alpha_i} = \frac{2(2t-n)\left(p_i^p - c_i\right)}{D^p} - k\alpha_i = 0, \quad i = 0,1. \tag{11}$$

The SOC, the cross effect, and the effect of an increase in the rival firm's R&D activity on the net profit are respectively given by:

$\frac{\partial^2 \Pi_i^p}{\partial \alpha_i^2} = 2t\left(\frac{2t-n}{D^p}\right)^2 - k < 0,$ $\frac{\partial^2 \Pi_i^p}{\partial \alpha_i \partial \alpha_j} = \frac{2t(2t-n)n\phi}{(D^p)^2} \geq 0 \Leftrightarrow \phi \geq 0,$ and

$\frac{\partial \Pi_i^p}{\partial \alpha_j} = \frac{2n\phi}{D^p}\left(p_i^p - c_i\right) \geq 0 \Leftrightarrow \phi \geq 0.$

The second equation implies strategic complements in R&D competition. Here, we should consider the outcomes shown in the second and third equations, which differ from those in the full market coverage case. In view of Equation (5), when the market is fully covered, the rival firm's R&D activity takes away the firm's market share. Conversely, in view of Equation (10), we have $\frac{\partial z_i^p}{\partial \phi} = \frac{n\left[\left(\overline{v}_j + \alpha_j\right)D^p + Q\right]}{(D^p)^2} > 0,$ where $Q \equiv n\phi\left\{(2t-n)(\overline{v}_i + \alpha_i) + n\phi(\overline{v}_j + \alpha_j)\right\} > 0,\ i, j = 0,1,\ i \neq j.$ That is, the firm's market share increases through the cross-group network effects (i.e., market expansion effects). In addition, the direct effect of an increase in connectivity on net profit is positive.[24]

Using Equations (9) and (11), we obtain the following R&D activity in equilibrium:

$$\alpha_i^p\left(\phi, \Delta_i\right) = \alpha^{p*}\left(\phi\right) + \frac{2t(2t-n)}{R^p}\frac{\Delta_i}{2},\quad i = 0,1. \tag{12}$$

where $R^p \equiv kD^p\left\{2t - n(1-\phi)\right\} - 2t(2t-n) > 0.$ $\alpha^{p*}\left(\phi\right) \equiv \frac{2t(2t-n)\overline{v}}{U^p}$ is the average level of R&D activities of the firms, $U^p \equiv kD^p\left\{2t - n(1+\phi)\right\} - 2t(2t-n) > 0,$ and $\overline{v} \equiv \frac{\overline{v}_0 + \overline{v}_1}{2}$ is the average level of the net quality of the firms at the initial situation.

Similarly for *Condition F*, based on Equation (12), if $\Delta_i \geq 0,$ the equilibrium R&D activity is positive. However, if $\Delta_i < 0,$ for equilibrium R&D activity to be positive, the following condition must hold:[25]

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[24] Regarding the direct effect on net profit of *firm i*, we derive:

$$\frac{\partial \Pi_i^p}{\partial \phi} = \frac{2\left(p_i^p - c_i\right)n\left[\left(\overline{v}_j + \alpha_j\right)D^p + 2Q\right]}{(D^p)^2} > 0,\quad i, j = 0,1,\ i \neq j.$$

[25] *Condition P* can be rewritten as:

$$\frac{kD^p\left\{2t - n(1-\phi)\right\} - 2t(2t-n)}{kD^p\left\{2t - n(1+\phi)\right\} - 2t(2t-n)} = \frac{R^p}{U^p} > 1 > \frac{\overline{v}_j - \overline{v}_i}{\overline{v}_j + \overline{v}_i}\left(> 0\right),$$

Condition P: $\frac{kD^p\{2t-n(1-\phi)\}-2t(2t-n)}{kD^p\{2t-n(1+\phi)\}-2t(2t-n)}(2\overline{v})\equiv\hat{\Delta}^P>-\Delta_i=\Delta_j,\quad i,j=0,1,\quad i\neq j.$

Using Equation (12), the effect of an increase in connectivity on R&D activity is given by: $\frac{d\alpha_i^p(\phi,\Delta_i)}{d\phi}=\frac{d\alpha^{p*}(\phi)}{d\phi}+\frac{2t(2t-n)}{R^p}\frac{\Delta_i}{2}\left(-\frac{dR^p}{d\phi}\frac{1}{R^p}\right),$

where $\frac{d\alpha^{p*}(\phi)}{d\phi}=\frac{2t(2t-n)\overline{v}}{U^p}\left(-\frac{dU^p}{d\phi}\frac{1}{U^p}\right)>0$ because $-\frac{dU^p}{d\phi}=nk\{2t-n(1+\phi)\}\{2t-n(1-3\phi)\}>0.$ That is, an increase in connectivity improves the average R&D activities. This result is opposite to the full market coverage case.

If $\Delta_i\leq 0,$ an increase in connectivity improves the equilibrium R&D activity.[26] That is, an increase in connectivity is preferable for the R&D activity of a small company. Conversely, if $\Delta_i>0,$ could an increase in connectivity reduce the R&D activity of *firm i* (i.e., a big company)? The answer is No. Given *Condition P*, we derive as follows: $\frac{d\alpha_i^p(\phi,\Delta_i)}{d\phi}>0.$ Namely, the R&D activity function of connectivity is monotonically increasing in the partial market coverage case (see Appendix B for the proof). Therefore,

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where $i,j=0,1,\quad i\neq j.$

[26] Based on *Condition PMC*, in the case of partial market coverage, we assume that a transportation cost is sufficiently large, i.e., $2t-n(1+3\phi)>0\Leftrightarrow t>2n\geq\frac{n(1+3\phi)}{2}.$ Thus, we have the following: $-\frac{dR^p}{d\phi}=-nk\{2t-n(1-\phi)\}\{2t-n(1+3\phi)\}<0.$

we summarize the results derived above as follows.

Proposition 3. In the case of partial market coverage, an increase in connectivity increases process R&D activity, irrespective of the difference in the net quality of the firms.

We address the implication of Proposition 3. Using Equation (11), we derive $\frac{\partial^2 \Pi_i^p}{\partial \alpha_i \partial \phi} > 0, \quad i = 0,1.$ That is, an improvement in connectivity increases marginal net profit, irrespective of the difference in the net quality. This result differs from the full market coverage case.

Second, we can express net profit as follows: $\Pi_i^p = \Pi_i^p \left[\alpha_i^p(\phi, \Delta_i), \alpha_j^p(\phi, \Delta_j), \phi \right],$ $i, j = 0,1, \quad i \neq j.$ Hence, taking the FOC, the total effects of an increase in connectivity on the net profit of *firm i* is given by $\frac{d\Pi_i^p}{d\phi} = \frac{\partial \Pi_i^p}{\partial \alpha_j^p}\frac{d\alpha_j^p}{d\phi} + \frac{\partial \Pi_i^p}{\partial \phi}, \quad i, j = 0,1, \quad i \neq j.$ As mentioned above, because the first and second terms of the right-hand side of the equation are positive, it holds that $\frac{d\Pi_i^p}{d\phi} > 0.$

Therefore, we summarize the results as follows.

Proposition 4. In the partial market coverage case, an increase in connectivity increases net profit, irrespective of the difference in the net quality of the firms.

Remark 2.[27]

Based on Propositions 3 and 4, because the process R&D activity and net profit are monotonically increasing functions of connectivity, we have $\alpha_i^p(\phi=0)<\alpha_i^p(\phi=1)$ and $\Pi_i^p(\phi=0)<\Pi_i^p(\phi=1)$, $i=0,1$. The results imply that providing network products with perfect connectivity is preferable for the firms, and the level of process R&D activities is the highest. The results differ from those in the case of full market coverage. For example, under a common network environment (e.g., identical operating system and perfect compatibility), the firms will provide the most upgraded network products and services for consumers.

3.3 Market coverage matters: market expansion effects and technological progress

Considering the difference in market coverage, for a while, we suppose that the difference in the net quality of the firms is not sufficiently large or that the firms are symmetric. In these cases, Proposition 3 is opposite to Proposition 1 (1). This depends on the assumption of market coverage. As mentioned above, in the full market coverage case, an increase in connectivity induces the benefits from the market share spillovers to a rival firm through cross-group network effects, and thus, reduces incentives to innovate. However, in the partial market coverage case, an increase in connectivity expands market share (i.e., market expansion effects); as a result, it improves incentives to innovate. Therefore, the impact of connectivity on incentives to innovate (i.e., technological progress) depends on market structure; that is, whether the market is fully covered or not. If the market is not completely covered, in other words, if there is an opportunity for firms to expand their

[27] The results in Remark 2 hold in the symmetric case.

market shares, an improvement in connectivity between the firms (network products) promotes incentives to innovate by market expansion effects.

Furthermore, examining the results alone, Propositions 2 (1) and 4 are formally the same, but the effects on net profits are completely different. In the full market coverage case, the firms earn more profits by degrading their network products, while in the partial market coverage case, they can earn more profits by upgrading their network products. This is because an increase in connectivity intensifies competition for a limited market share in the full market coverage case, while it has the effect of expanding the market size of each firm in the partial market coverage case.

We now consider the case in which the difference in the net qualities of the firms is sufficiently large, i.e., a high-quality (big) company and a low-quality (small) company compete on prices and process R&D activities in a network products market. In this case, when examining the effects of an increase in connectivity, we need to focus on the cross-group network effects. That is, the impacts of an increase in connectivity on R&D activities and net profits are greater for a small company than for a big company. This is because the cross-group network effect is larger for a small company than for a big company. Therefore, an improvement in connectivity increases the R&D activity and net profit of a small company whereas it reduces the R&D activity and net profit of a big company.

4. Discussion

So far, we have assumed rational expectations, such that consumers expect the network sizes of the products before the firms decide their prices. Relaxing this assumption, we examine the R&D incentive problem under the case of active (responsive) expectations. That is, as discussed in the introduction, firms can commit to the announcement of the network sizes of their products and consumers believe that the announcements are credible. Accordingly, consumers form expectations of network sizes using the announcements.

In this section, we consider the full market coverage case. Regarding the partial market coverage case (see Appendix C), in which we demonstrate that the results are not significantly different from those under rational expectations.

4.1 Equilibrium and the impact of connectivity under active expectations

Under the assumption of active expectations, consumers believe firms' announcement of output (i.e., the number of consumers) in advance, and form expectations of network sizes of the products, based on the announcements and prices. Thus, it holds that $x_i^e = x_i[p_i, p_j, x_i^e, x_j^e], \ \ i, j = 0,1, \ \ i \neq j.$ Taking Equation (2), we derive the following direct demand function, which the firms face:

$$x_i = \frac{t - n(1-\phi) + v_i - v_j - p_i + p_j}{2\{t - n(1-\phi)\}}, \quad i, j = 0,1, \quad i \neq j. \tag{13}$$

With respect to price competition in the second stage, based on Equation (13), the FOC is given by $\frac{\partial \pi_i^{fA}}{\partial p_i} = x_i - \frac{p_i - c_i}{2\{t - n(1-\phi)\}} = 0, \ \ i = 0,1,$ where superscript *fA* is the full market coverage case under active expectations. Thus, we obtain the following outcomes at this stage:

$$p_i^{fA} - c_i = \{t - n(1-\phi)\} + \frac{\Delta_i + \alpha_i - \alpha_j}{3}, \tag{14}$$

$$x_i^{fA} = \frac{p_i^{fA} - c_i}{2\{t - n(1-\phi)\}} = \frac{1}{2} + \frac{\Delta_i + \alpha_i - \alpha_j}{6\{t - n(1-\phi)\}}, \quad i, j = 0,1, \quad i \neq j, \tag{15}$$

where we assume $t > n\left(\geq n(1-\phi)\right)$.

In R&D competition, the net profit function of *firm* i is expressed as $\Pi_i^{fA} = \left(p_i^{fA} - c_i\right) x_i^{fA} - F\left(\alpha_i\right) = \frac{\left(p_i^{fA} - c_i\right)^2}{2\{t - n(1-\phi)\}} - \frac{k}{2}\left(\alpha_i\right)^2, \quad i = 0,1.$ The FOC of *firm* i is given by:[28]

$$\frac{\partial \Pi_i^{fA}}{\partial \alpha_i} = \frac{3\{t - n(1-\phi)\} + \Delta_i + \alpha_i - \alpha_j}{9\{t - n(1-\phi)\}} - k\alpha_i = 0, \quad i, j = 0,1, \quad i \neq j. \tag{16}$$

Based on Equation (16), it holds in the equilibrium that

$$\alpha_i^{fA}\left(\phi, \Delta_i\right) = \alpha^{fA*} + \frac{\Delta_i}{9k\{t - n(1-\phi)\} - 2}, \quad i = 0,1, \tag{17}$$

where $\alpha^{fA*} \equiv \frac{1}{3k}$ denotes the average R&D activities of the firms. If $\Delta_i < 0,$ for equilibrium R&D activity to be positive, the following condition must hold:

Condition fA: $\frac{9k\{t - n(1-\phi)\} - 2}{3k} \equiv \hat{\Delta}^{fA} > -\Delta_i = \Delta_j.$

[28] We derive the following SOC and cross effect:

$$\frac{\partial^2 \Pi_i^{fA}}{\partial {\alpha_i}^2} = \frac{1}{9\{t - n(1-\phi)\}} - k < 0 \quad \text{and} \quad \frac{\partial^2 \Pi_i^{fA}}{\partial \alpha_i \partial \alpha_j} = \frac{-1}{9\{t - n(1-\phi)\}} < 0.$$

Because the cross effect is negative, the R&D activities are strategic substitutes. In addition, the condition for the local maximum value is $9k\{t - n(1-\phi)\} - 2 > 0,$ because the sign of the determinant of Hessian matrix is positive.

Condition fA implies that the difference in the net quality between the firms must not be large in the equilibrium.[29]

In view of Equation (17), the effect of an increase in connectivity on the R&D activity is given by $\frac{d\alpha_i^{fA}(\phi,\Delta_i)}{d\phi} = \frac{-9kn\Delta_i}{\left[9k\left\{t-n(1-\phi)\right\}-2\right]^2} \geq (<) 0 \Leftrightarrow \Delta_i \leq (>) 0.$ Thus, we summarize the result as follows.

Proposition 1A. Under active expectations in the full market coverage case,

(1) if $\Delta_i > (<) 0,$ an increase in connectivity reduces (increases) R&D activity, and

(2) if $\Delta_i = 0,$ a change in connectivity does not affect R&D activity.

Proposition 1A (1) implies that an increase in connectivity reduces (improves) the incentive to innovate R&D activity of a high-(low-)quality firm. The results are like those of Propositions 1 (1) and (2). Using Equation (16), we obtain the effect of an increase in connectivity on the marginal net profit in equilibrium:

$$\frac{\partial^2 \Pi_i^{fA}}{\partial \alpha_i \partial \phi} = \frac{-n\Delta_i}{\left\{t-n(1-\phi)\right\}\left[9\left\{t-n(1-\phi)\right\}-2\right]} \leq (>) 0 \Leftrightarrow \Delta_i \geq (<) 0.$$

Thus, under active expectations, the impact of connectivity on the marginal net profit depends on the difference in the net quality of the firms. Namely, as connectivity increases, the degree of cross-group network effects of a low-quality firm (i.e., a small company) becomes larger than that of a high-quality firm (i.e., a big company).

29 Given *Condition fA*, regarding the FOC of *firm i*, we obtain the following: $\left.\frac{\partial \Pi_i^{fA}}{\partial \alpha_i}\right|_{\substack{\alpha_i=0 \\ \alpha_j>0}} > 0,\ i, j = 0,1,\ i \neq j.$

Accordingly, the impact on a small (big) company is positive (negative).

As in Proposition 1 (1), in the symmetric case, an increase in connectivity reduces R&D activity. However, Proposition 1A (2) shows that changing connectivity does not affect R&D activities. As we will explain below, under active expectations, the firms can decide their prices taking consumer expectations of the network sizes into account; that is, internalizing network (consumption) externalities. Thus, changing connectivity does not affect the decision of R&D activities. This implies network neutrality with respect to connectivity, under which consumers have perfect foresight, as mentioned by Shy (2001) and Sääskilahti (2006).

Second, using the FOC, i.e., $\frac{\partial \Pi_i^{fA}}{\partial \alpha_i} = 0,$ the total effect on net profit is expressed as: $\frac{d\Pi_i^{fA}}{d\phi} = \frac{\partial \Pi_i^{fA}}{\partial \alpha_j^{fA}} \frac{d\alpha_j^{fA}}{d\phi} + \frac{\partial \Pi_i^{fA}}{\partial \phi}, \quad i, j = 0,1, \quad i \neq j.$ The direct effect of connectivity on net profit is $\frac{\partial \Pi_i^{fA}}{\partial \phi} = \frac{n\left(p_i^{fA} - c_i\right)\left(p_j^{fA} - c_j\right)}{2\left\{t - n(1-\phi)\right\}^2} > 0.$ In addition, the effect of the rival firm's R&D activity on net profit is $\frac{\partial \Pi_i^{fA}}{\partial \alpha_j} = -\frac{p_i^{fA} - c_i}{3\left\{t - n(1-\phi)\right\}} < 0.$ Thus, in view of Proposition 1A, the total effect of an increase in connectivity on net profit is expressed as:

$$\left.\frac{d\Pi_i^{fA}}{d\phi}\right|_{\alpha_0^{fA} = \alpha_1^{fA} = \alpha^{fA*}} = \underset{-}{\frac{\partial \Pi_i^{fA}}{\partial \alpha_j^{fA}}} \underset{\pm}{\frac{d\alpha_j^{fA}}{d\phi}} + \underset{+}{\frac{\partial \Pi_i^{fA}}{\partial \phi}}. \tag{18}$$

Thus, we have the following result.

Proposition 2A. Under active expectation in the full market coverage case,

(1) if $\Delta_i < 0,$ an increase in connectivity increases net profit, and

(2) if $\Delta_i > 0$, the following relationship holds:

$$\Delta''^{fA} \equiv \frac{\left[9k\{t-n(1-\phi)\}-2\right]^2}{3k\cdot 9k\{t-n(1-\phi)\}} > (<)\Delta_i \Leftrightarrow \frac{d\Pi_i^{fA}}{d\phi} > (<)0, \quad i=0,1,$$

where $\hat{\Delta}^{fA} > \Delta''^{fA}$.

Proposition 2A is the same as Proposition 2 under rational expectations. That is, either if the difference in the net quality is either negative or is small, then net profit increases as connectivity improves. However, if the difference is sufficiently large, the net profit of a big (small) company decreases (increases) by increasing connectivity. This is because the cross-group network effects affect the net profit of a big (small) company, negatively (positively).

Remark 3. Symmetric case

In view of Equation (18) and Proposition 1A (2), we have:

$$\left.\frac{d\Pi_i^{fA}}{d\phi}\right|_{\alpha_0^{fA}=\alpha_1^{fA}} = \underbrace{\frac{\partial\Pi_i^{fA}}{\partial\alpha_j^{fA}}}_{-}\underbrace{\frac{d\alpha_j^{fA}}{d\phi}}_{0} + \underbrace{\frac{\partial\Pi_i^{fA}}{\partial\phi}}_{+} = \underbrace{\frac{\partial\Pi_i^{fA}}{\partial\phi}}_{+} > 0.$$

In this case, the indirect (strategic) effect is zero because R&D activities do not change by increasing connectivity (i.e., network neutrality). Thus, the marginal cost is constant. However, the direct effect on profit is positive because an increase in connectivity increases the price, and thus the price-cost margin increases. As a result, an increase in connectivity increases net profit. This result differs from that under rational expectations, in which the direct effect is zero in the symmetric equilibrium.

4.2 Expectations matter: internalization of network externalities under active expectations

Comparing Proposition 1 with 1A, irrespective of consumer expectations, the impact of connectivity on incentives to innovate process R&D activity depends on the difference in the net quality of the firms. That is, an increase in connectivity reduces (improves) the incentive of a big (small) company. Similarly, as for Propositions 2 with 2A, an increase in connectivity decreases (increases) the net profit of a big (small) company.

To examine the implications of consumer expectations, we focus on the symmetric case, i.e., $\Delta_i = 0,$ which is independent of the difference in the net quality of the firms.

Taking Equations (4), (7), (14), and (17), and considering the outcomes in each symmetric equilibrium, we derive:[30]

$$\alpha^{f*} = \frac{t}{k\{3t - n(1-\phi)\}} \geq \alpha^{fA*} = \frac{1}{3k}, \tag{19}$$

$$p^{f} - c\left(\alpha^{f*}\right) = t \geq p^{fA} - c\left(\alpha^{fA*}\right) = t - n + n\phi. \tag{20}$$

$$\Pi^{f*} = \frac{t}{2} - \frac{1}{2k}\left\{\frac{t}{3t - n(1-\phi)}\right\}^2 \geq \Pi^{fA*} = \frac{t - n + n\phi}{2} - \frac{1}{18k}. \tag{21}$$

In view of Equations (19), (20), and (21), with perfect connectivity, i.e., $\phi = 1,$ these outcomes are the same. In other words, unless there is perfect connectivity, e.g., $\phi = 0,$ the outcomes under rational expectations are larger than those under active expectations.

Based on Equation (19), we obtain the following results. Under active expectations, firms reflect network effects in their pricing, and thus, there is no need to consider

[30] Regarding the outputs, it holds that $x^{f*} = x^{fA*} = \frac{1}{2}.$

network effects when determining the level of R&D activity (i.e., internalizing network externalities), so it remains constant regardless of the change of connectivity, and is kept at the level it would be without network effects. Conversely, under rational expectations, the level of R&D activity is determined by considering network effects. In doing so, the higher the level of connectivity, the more one firm's market share will be lost to the other firm through the cross-group network effects, so the level of R&D activity will be kept low. Under perfect connectivity, network effects cancel each other out, so the level of R&D activity is kept to the level it would be in the case of no network effects and be also the same as under active expectations. Thus, unless connectivity is imperfect, the R&D activity is larger than that under active expectations.

Equation (20) implies that the price-cost margin under rational expectations is larger than that under active expectations. In addition, as connectivity increases, these prices increase: the price under rational expectations increases because of an increase in the marginal cost, whereas the price under active expectations increases because of improving connectivity.

Because the price-cost margin under rational expectations is larger than that under active expectations and the market share of each firm in equilibrium is a half, the gross profit under rational expectations is larger than that under active expectations. However, the investment cost in the former is larger than that in the latter. In this case, the difference in the gross profits is larger than that in the investment cost. As a result, Equation (21) holds.

With respect to the partial market coverage case, the difference in the formation of consumer expectations does not qualitatively change the results for R&D activity and net profit, in view of Propositions 3 and 3A, and 4 and 4A (see Appendix C).

4.3 Application: product (quality-improving) R&D activities[31]

Although we have considered process (cost-reducing) R&D activities in a network products market, we can apply our model to the case of product (quality-improving) R&D activities. That is, we assume that an intrinsic (stand-alone) value of network products expresses the level of quality: $v_i = \underline{v}_i + \beta_i,\ i = 0,1,$ where $\beta_i\,(\geq 0)$ is the degree of quality-improving R&D activity and $\underline{v}_i$ is the initial level of quality before implementing R&D activity. Thus, we express the variable as follows: $\underset{\sim}{v}_i \equiv \underline{v}_i - c_i > 0,$ $i = 0,1,$ which denotes the initial quality net of marginal cost of *firm i*.

By changing these variables, for example, with respect to the price-cost margin in the cases of full and partial market coverage under rational expectations, i.e., Equations (4) and (5), we derive the following equations:

$$p_i^{f[q]} - c_i = \frac{\left\{3t - n(1-\phi) + \underset{\sim}{\Delta}_i + \beta_i - \beta_j\right\} t}{3t - n(1-\phi)} \quad \text{and}$$

$$p_i^{p[q]} - c_i = \frac{\left[(2t-n)\underset{\sim}{v}_i + n\phi \underset{\sim}{v}_j + (2t-n)\beta_i + n\phi\beta_j\right] t}{D^p},$$

where superscript $m[q],\ m = f, p$ denotes quality-improving R&D activities in the full (partial) market coverage case. Furthermore, regarding both market coverage cases under active expectations, we obtain the similar outcomes. Therefore, we can derive the same results as in the case of process R&D activities.

In this applied model, we can investigate the impact of connectivity on the quality

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31 Cheng and Chan (2023), which is different from our model, consider quality competition under a firm-specific network externality without connectivity (compatibility) in a framework of a vertical product differentiation.

corresponds to that in vertically differentiated services (see Foros and Hansen, 2001).[32] For example, in the fully (partially) covered market, an improvement in connectivity may downgrade (upgrade) the quality level of network products and services.

5. Conclusion

5.1 Summary

Introducing network externalities into a standard Hotelling linear market model, we have considered the impact of connectivity between network products on process (cost-reducing) R&D activities and profits. From three perspectives, market coverage (i.e., full and partial), consumer expectations (i.e., rational and active), and asymmetric firms—the heterogeneity of firms (i.e., high and low quality) —we have demonstrated their effects in each case.

First, in the full market coverage case, (i) under rational expectations, an improvement in connectivity reduces incentives to innovate R&D activities and increases net profits unless the difference in the level of net quality of the firms is sufficiently large; otherwise, the improvement increases (reduces) the incentive and net profit of a low-(high-)quality firm: and (ii) under active expectations, the improvement increases (reduces) the incentive and net profit of a low-(high-)quality firm; additionally, if there is no difference in the firms, the change in connectivity does not affect R&D activity. That is, network neutrality arises. This is because firms set their prices considering consumers'

[32] We can newly define the difference in the net quality of the firms at the initial term as follows: $\underset{\sim}{\Delta}_i \equiv \underset{\sim}{v}_i - \underset{\sim}{v}_j$, $i, j = 0,1$, $i \neq j$.

expectations of network seizes (i.e., internalizing consumption externalities).

Second, in the partial market coverage case, irrespective of how consumer expectations are formed and of the difference in the level of net quality of the firms, an improvement in connectivity promotes incentives to innovate process R&D activity and increases net profit. This is because improving connectivity between network products expands the market share of each firm, and thus, induces incentives to innovate (i.e., market expansion effects).

When assuming the difference in the level of quality, i.e., stand-alone value, of the firms as firm's heterogeneity, that difference corresponds to the difference in the market share or in the size of firms in equilibrium. Thus, summarizing the results shown above from the perspective of firm's heterogeneity, irrespective of market coverage and consumer expectations, an improvement in network connectivity will increase a small (low-quality) company's motivation for technological development, thereby increasing its profit. However, the opposite results arise for a big company providing the network product with a high-quality service. That is, because of the asymmetry in cross-group network effects, a small company has a greater incentive for R&D activity than a big company.

5.2 Some remaining problems

There are some remaining problems in this paper. First, we have mainly examined how network connectivity affects incentives to innovate and profits. However, we should consider the effect on consumer and social welfare. For example, as in an information and telecommunications industry, improving quality of network connectivity provides direct benefits for users. However, the improvement is possible to result in increased usage fees.

Thus, we will investigate the impact of improving connectivity on consumer and social surplus. When considering regulation of digital markets, this issue is critical point.

Second, relating to the first, although policy analysis in a network industry is beyond the scope of this paper, we do note some implications of the model. The problems of network connectivity, compatibility, and horizontal interoperability are closely related to standardization and compatibility policies facilitating market competition and promoting R&D activity. In this regard, we should consider an optimal connectivity (compatibility) policy.[33] Thus, these issues including the first and second are our future research.

Third, we have assumed the case of symmetric two-way connectivity including non-connectivity. Relaxing this assumption, we should examine the cases of asymmetric two-way and one-way connectivity. For example, although users of the network product provided by a high-quality firm can connect with that provided by a low-quality firm, users of that provided by a low-quality firm cannot connect with the network product of a high-quality firm. Or, the opposite case. In addition, we have dealt with connectivity as an exogenous parameter. However, as mentioned above, we consider connectivity and compatibility as policy variables. Furthermore, we should examine them as strategic variables between competing firms in a noncooperative game and instruments in negotiation, alliance, and collusion in a cooperative game.[34]

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[33] From legal and regulation perspectives in digital markets, Çavuş (2024) discuss whether European Union competition law and the Digital Markets Act promote innovation.

[34] For example, Buccella et al. (2022) consider Cournot competition with process (cost-reducing) innovation in an endogenous compatibility decision game. They demonstrate that full compatibility is a Nash equilibrium in the compatibility decision game. Related to this issue, Buccella et al. (2024) consider the compatibility decision game in the case of Cournot duopoly. In their model, they assume that the market size of the product

Finally, from the perspectives of platform and multi-sided markets, we have implicitly assumed that the firms providing network products (suppliers) themselves are platforms in a one-sided market. We will extend our model to platform competition in multi-sided markets. For example, we assume that *firms 0* and *1* are operating system (OS) platforms, that application suppliers provide their products and services (e.g., social network service, online games, search engine, and other application software) to the platforms, and that there are users subscribing to the applications through the OS. Furthermore, as an example, we could take electronic-commerce and electronic-marketplaces such as Amazon and Rakuten. In such an environment, we will investigate how an improvement in connectivity between the competing platforms affects application suppliers and the platforms themselves. Additionally, we need to consider how and what administrative agents of platforms and application suppliers should control when competing in digital markets to sustain competitiveness and improve consumer welfare.

Appendix A.

(1) On *Condition FMC*: with low transportation cost (i.e., high substitutability)

We demonstrate the conditions under which the full market coverage case holds in equilibrium. *Condition FMC* is rewritten as:

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represents an index of product quality. Comparing the equilibrium profits in the cases of full compatibility, incompatibility, and asymmetric compatibility and incompatibility, they derive the subgame perfect Nash equilibrium in the game. They discover that the equilibrium outcomes depend on the degree of the relative quality. Based on a similar model (2024), in which they assume the same quality level and a quasi-fixed cost of compatibility, Buccella et al. (2023) consider endogenous product compatibility and market outcomes, which depend on the degree of compatibility and quasi-fixed cost.

$$t < v_0 - p_0 + N_0 + v_1 - p_1 + N_1 \Leftrightarrow 3t - n(1+\phi) - \frac{2t}{k\left\{3t - n(1-\phi)\right\}} < \overline{v}_0 + \overline{v}_1.$$ [35]

We define the following function: $H^f(t) \equiv 3t - n(1+\phi) - \frac{2t}{k\left\{3t - n(1-\phi)\right\}}$ for $t > \underline{t}^f \equiv \frac{n(1-\phi)}{3} \geq 0.$ Then, it holds that $H^f(t) > (<) 0 \Leftrightarrow t > (<) \hat{t}^f$ for $\hat{t}^f \equiv \left\{ t \geq \frac{n(1+\phi)}{3} \middle| H^f(t) = 0 \right\} > \underline{t}^f$ and that $\frac{dH_f(t)}{dt} > 0$ for $t > \underline{t}^f$. Because $H^f(t)$ is a monotonically increasing function of a transportation cost, t, there exists the following critical level of the transportation cost: $\overline{t}^f \equiv \left\{ t > \hat{t}^f \middle| H^f(t) = \overline{v}_0 + \overline{v}_1 \right\}$. This critical level expresses the upper bound of a transportation cost.[36]

Therefore, the condition that the full market coverage case holds in equilibrium is given by: $\overline{t}^f > t\left(> \underline{t}^f\right)$.

Regarding the full market coverage in the case of active expectations, we can prove the condition in a similar way to that for Appendix A (1).

(2) On *Condition PMC*: with high transportation cost (i.e., low substitutability)

Using the same approach as above, we demonstrate the conditions under which the partial market coverage case holds in the equilibrium. Namely, the following conditions are necessary for the transportation cost: $v_i - p_i + N_i > 0,$ $i = 0,1,$ and

[35] If the condition holds, it is plausible that $v_i - p_i + N_i > 0,$ $i = 0,1.$

[36] Regarding the critical level, $\overline{t}^f$, if a fixed cost parameter k decreases, the strength of network externalities n increases, or the degree of connectivity ϕ increases, then the upper bound of a transportation cost increases. Furthermore, the total net stand-alone value $\overline{v}_0 + \overline{v}_1$ increases the upper bound.

$t > v_0 - p_0 + N_0 + v_1 - p_1 + N_1$. That is, regarding *condition PMC*, we derive the following relationship:[37]

$$t > \overline{v}_0 + \overline{v}_1 + \alpha_0^p + \alpha_1^p - \left\{ \left(p_0^p - c_0 \right) + \left(p_1^p - c_1 \right) \right\} + n\left(1+\phi\right)\left(z_0^p + z_1^p\right)$$
$$\Leftrightarrow kD^p \left\{2t - n(1+\phi)\right\}^2 > \overline{v}_0 + \overline{v}_1.$$

The above condition can be rewritten as: $H^p(t) > \overline{v}_0 + \overline{v}_1$,

where $$H^p(t) \equiv 16k\left\{t - \frac{n(1-\phi)}{2}\right\}\left\{t - \frac{n(1+\phi)}{2}\right\}^3 > 0 \quad \text{for} \quad t > \underline{t}^p \equiv \frac{n(1+\phi)}{2}.$$

Because it holds that $\frac{dH^p(t)}{dt} > 0$ for $t > \underline{t}^p$, $H^p(t)$ is a monotonically increasing function of a transportation cost, t. Thus, there exists the following critical level of transportation cost: $\overline{t}^p \equiv \left\{ t > \underline{t}^p \middle| H^p(t) = \overline{v}_0 + \overline{v}_1 \right\}$. This critical level expresses the lower bound of a transportation cost.[38]

Therefore, the condition that the partial market coverage case holds in equilibrium is given by: $t > \overline{t}^p \left(> \underline{t}^p \right)$.

Regarding the partial market coverage in the case of active expectations, we can prove the condition in a similar way to that for Appendix A (2).

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[37] In addition, we derive the following relationship:
$$v_i - p_i + N_i > 0 \Leftrightarrow (2t-n)\left(\overline{v}_i + \alpha_i^p\right) + n\phi\left(\overline{v}_j + \alpha_j^p\right) > 0,\ i, j = 0,1,\ i \neq j.$$

[38] Regarding the critical level, $\overline{t}^p$, if a fixed cost parameter k decreases, the strength of network externalities n increases, or the degree of connectivity ϕ increases, then the lower bound of a transportation cost increases. Furthermore, the total net stand-alone value $\overline{v}_0 + \overline{v}_1$ increases the lower bound.

Appendix B. Proof of Proposition 3.

We have:

$$\frac{d\alpha_i^p(\phi,\Delta_i)}{d\phi}=\frac{2t(2t-n)\bar{v}}{U^p}\left(-\frac{dU^p}{d\phi}\frac{1}{U^p}\right)+\frac{2t(2t-n)}{R^p}\frac{\Delta_i}{2}\left(-\frac{dR^p}{d\phi}\frac{1}{R^p}\right). \tag{B.1}$$

In this case, we obtain as follows: $-\frac{dU^p}{d\phi}=nk\{2t-n(1+\phi)\}\{2t-n(1-3\phi)\}>0$ and $-\frac{dR^p}{d\phi}=-nk\{2t-n(1-\phi)\}\{2t-n(1+3\phi)\}<0.$[39]

Using Equation (B.1), we derive the following relationship:

$$\frac{d\alpha_i^p(\phi,\Delta_i)}{d\phi}>(<)0\Leftrightarrow\left(\frac{R^p}{U^p}\right)^2>(<)\frac{\Delta_i/2}{\bar{v}}\left(\frac{dR^p/d\phi}{-dU^p/d\phi}\right). \tag{B.2}$$

Regarding the left-hand side of Equation (B.2), we derive: $LHS\equiv\left(\frac{R^p}{U^p}\right)^2>1,$ based on *Condition P* (see footnote 25). Similarly, the right-side hand is expressed as:

$$RHS\equiv\left(\frac{\Delta_i/2}{\bar{v}}\right)\left(\frac{dR^p/d\phi}{-dU^p/d\phi}\right)=\left(\frac{\bar{v}_i-\bar{v}_j}{\bar{v}_i+\bar{v}_j}\right)\left(\frac{\{2t-n(1-\phi)\}\{2t-n(1+3\phi)\}}{\{2t-n(1+\phi)\}\{2t-n(1-3\phi)\}}\right).$$

Thus, if $\bar{v}_i-\bar{v}_j\le 0\Leftrightarrow\Delta_i\le 0,$ it holds that $LHS>1>0\ge RHS$. Conversely, if $\bar{v}_i-\bar{v}_j>0\Leftrightarrow\Delta_i>0,$ then the followings hold:

$$\left(\frac{\bar{v}_i-\bar{v}_j}{\bar{v}_i+\bar{v}_j}\right)<1 \text{ and } \frac{\{2t-n(1-\phi)\}\{2t-n(1+3\phi)\}}{\{2t-n(1+\phi)\}\{2t-n(1-3\phi)\}}<1.$$

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39 Based on *Condition PMC*, we assume that a transportation cost is sufficiently large, i.e., $2t-n(1+3\phi)>0\Leftrightarrow t>2n\ge\frac{n(1+3\phi)}{2}$. Thus, it holds that $\frac{dR^p}{d\phi}>0$. See also footnote 26.

Because $RHS < 1$, we have $LHS > 1 > RHS$. Therefore, we prove that $\frac{d\alpha_i^p(\phi, \Delta_i)}{d\phi} > 0.$

Appendix C. Active expectations in the partial market coverage case

Under active expectations (i.e., $z_i^e = z_i$), using Equation (3), the following direct demand function is derived.

$$z_i = \frac{(t-n)v_i + n\phi v_j - (t-n)p_i - n\phi p_j}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}}, \quad i, j = 0,1, \quad i \neq j. \tag{C.1}$$

Equation (C.1) implies that the network products in the market are complements through network connectivity unless connectivity is not zero. For example, we can imagine that the local telecommunications companies connect using a long-distance cable. Unless the cable breaks, people can talk on the telephone not only with people living in the same area, but also with people who live far away.

In the stage of price competition, the FOC of profit maximization of *firm i* is given by $\frac{\partial \pi_i^{pA}}{\partial p_i} = z_i - \frac{(t-n)(p_i - c_i)}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}} = 0, \quad i = 0,1$. In this case, because it holds that $\frac{\partial^2 \pi_i^{pA}}{\partial p_i \partial p_j} = \frac{\partial z_i}{\partial p_j} = \frac{-n\phi}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}} < 0$, as mentioned above, strategic substitutes arise in price competition. Using the FOC, we derive the following outcomes in equilibrium in the stage of price competition:

$$p_i^{pA} - c_i = \frac{\{2(t-n)^2 - (n\phi)^2\}(\overline{v}_i + \alpha_i) + (t-n)n\phi(\overline{v}_j + \alpha_j)}{D^{pA}}, \tag{C.2}$$

$$z_i^{pA} = \frac{(t-n)(p_i^{pA} - c_i)}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}}, \tag{C.3}$$

where $D^{pA} \equiv \{2(t-n)+n\phi\}\{2(t-n)-n\phi\} > 0, \quad i, j = 0,1, \quad i \neq j.$ Based on Equations (C.2) and (C.3), the net profit function is expressed as: $\Pi_i^{pA} = \frac{t-n}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}}\left(p_i^{pA} - c_i\right)^2 - \frac{k}{2}(\alpha_i)^2.$

In the first stage of R&D competition, the FOC is given by:

$$\frac{\partial \Pi_i^{pA}}{\partial \alpha_i} = \frac{2(t-n)}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}} \frac{\left\{2(t-n)^2 - (n\phi)^2\right\}}{D^{pA}}\left(p_i^{pA} - c_i\right) - k\alpha_i = 0, \quad i = 0,1. \tag{C.4}$$

The SOC, the cross effect, and the effect of an increase in the rival firm's R&D activity on the net profit are respectively given as:

$$\frac{\partial^2 \Pi_i^{pA}}{\partial \alpha_i^2} = \frac{2(t-n)}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}}\left\{\frac{2(t-n)^2 - (n\phi)^2}{D^{pA}}\right\}^2 - k < 0,$$

$$\frac{\partial^2 \Pi_i^{pA}}{\partial \alpha_i \partial \alpha_j} = \frac{2\{2(t-n)^2 - (n\phi)^2\}}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}}\left\{\frac{t-n}{D^{pA}}\right\}^2 n\phi \geq 0 \Leftrightarrow \phi \geq 0,$$

and

$$\frac{\partial \Pi_i^{pA}}{\partial \alpha_j} = \frac{2(t-n)^2 n\phi}{\{t-n(1-\phi)\}\{t-n(1+\phi)\} D^{pA}}\left(p_i^{pA} - c_i\right) \geq 0 \Leftrightarrow \phi \geq 0.$$

The outcomes expressed in the second and third equations are the same as those under rational expectations in the partial market coverage case. That is, in the partial market coverage case, the differences in expectations do not significantly affect the outcomes.

Using Equations (C.2) and (C.4), we obtain the following R&D activities in equilibrium:

$$\alpha_i^{pA}(\phi, \Delta_i) = \alpha^{pA*}(\phi) + \frac{2(t-n)}{R^{pA}}\frac{\Delta_i}{2}, \quad i = 0,1, \tag{C.5}$$

where $R^{pA} \equiv kD^{pA}\{2(t-n) - n\phi\} - 2(t-n)\{2(t-n)^2 - (n\phi)^2\} > 0.$ $\alpha^{pA*}(\phi) \equiv \frac{2(t-n)\overline{v}}{U^{pA}}$ is the average level of process R&D activities of the firms, where $U^{pA} \equiv kD^{pA}\{2(t-n) + \phi)\} - 2(t-n)\{2(t-n)^2 - (n\phi)^2\} > 0.$

Based on Equation (C.5), by the same process as in the case of rational expectations (i.e., Appendix B), we can derive $\frac{d\alpha_i^{pA}(\phi, \Delta_i)}{d\phi} > 0.$ In addition, regarding the impact on net profit, we can also obtain $\frac{d\Pi_i^{pA}}{d\phi} > 0.$ [40] Therefore, we summarize the results as follows.

Proposition 3A. Under active expectations in the partial market coverage, an increase in connectivity increases R&D activity, irrespective of the difference in the level of net quality of the firms.

Proposition 4A. Under active expectations in the partial market coverage, an increase in connectivity increases net profit, irrespective of the difference in the level of net quality of the firms.

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[40] The total effects of an increase in connectivity on the net profit of *firm i* is given by $\frac{d\Pi_i^{pA}}{d\phi} = \frac{\partial\Pi_i^{pA}}{\partial\alpha_j^{pA}}\frac{d\alpha_j^{pA}}{d\phi} + \frac{\partial\Pi_i^{pA}}{\partial\phi}, \quad i, j = 0,1, \quad i \neq j.$ The first and second terms imply that the indirect effects are positive. In addition, regarding the third term expressing the direct effect, it holds that $\frac{\partial\Pi_i^{pA}}{\partial\phi} > 0.$

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