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# International asymmetries in population aging and their consequences for the technology gap and global growth

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# International asymmetries in population aging and their consequences for the technology gap and global growth*

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**Abstract**

This paper studies how international asymmetries in population aging shape cross-country technology gaps and global growth. I develop a two-country, two-sector overlapping-generations model with endogenous technological progress free from scale effects. The analysis shows that, in the long-run equilibrium, the faster-aging country's relative technology declines through two mechanisms: reduced per capita labor supply and a reallocation of employment toward the non-tradable sector. Consequently, policies aimed solely at increasing labor-force participation are insufficient to prevent such relative technological decline, because the latter mechanism persists. Numerical simulations confirm these mechanisms and reveal potentially non-monotonic effects on global growth under large demographic asymmetries. I also quantify Japan's relative technological decline due solely to differential aging by calibrating the two countries to Japan and the United States.

*Keywords:* International asymmetries in population aging; Overlapping generations; Endogenous technological progress without scale effects; Non-tradable goods; Sectoral reallocation

*JEL classification:* F43; J11; O30; O41

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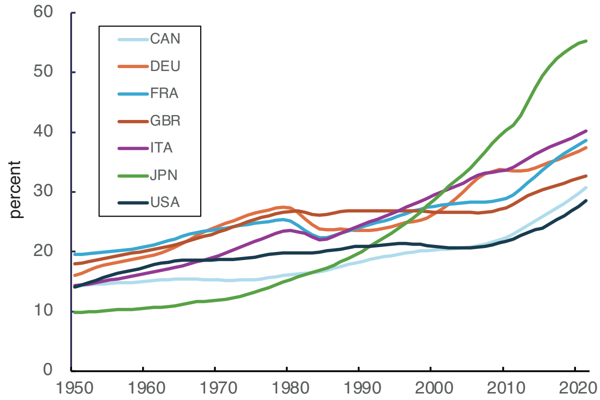
(a) OADRs in G7 member countries




(b) Projection of OADRs in rapidly aging countries and the U.S.

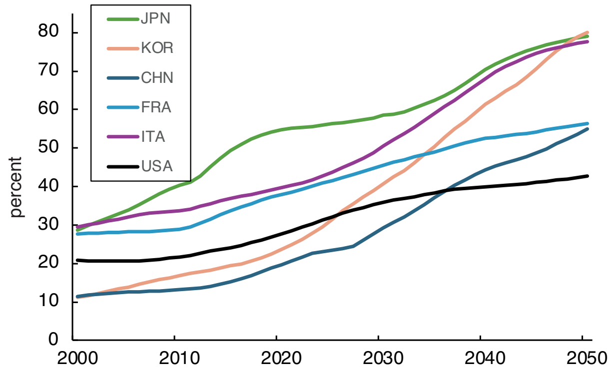



Figure 1: Population aging in developed countries. Source: United Nations, *World Population Prospects 2022*.

## 1 Introduction

Virtually all advanced economies are now experiencing population aging. It is widely regarded as a potential drag on economic growth because it reduces labor supply, a fundamental factor of production. Reflecting this concern, numerous studies have examined the relationship between aging and growth, as reviewed later. Nevertheless, two important perspectives remain largely underexplored. First, the pace of aging differs markedly across countries, yet they coexist in a globally integrated economy. Second, countries undergoing rapid aging tend to be large, so their demographic transitions inevitably affect the rest of the world. Much of the existing literature assumes closed or small open economies, which provides limited insight into how rapid aging in one country spills over internationally. Does a country that undergoes more rapid aging than others fall behind technologically in the global economy? Japan, for example, is one of the most rapidly aging countries and has simultaneously seen a marked slowdown in productivity growth. Indeed, in December 2023, the Japan Productivity Center reported that Japan's labor productivity per worker ranked 31st among the 38 OECD countries in 2022, which is the lowest position since 1970. How does this outcome affect other countries and then global growth? The purpose of this study is to address these questions.

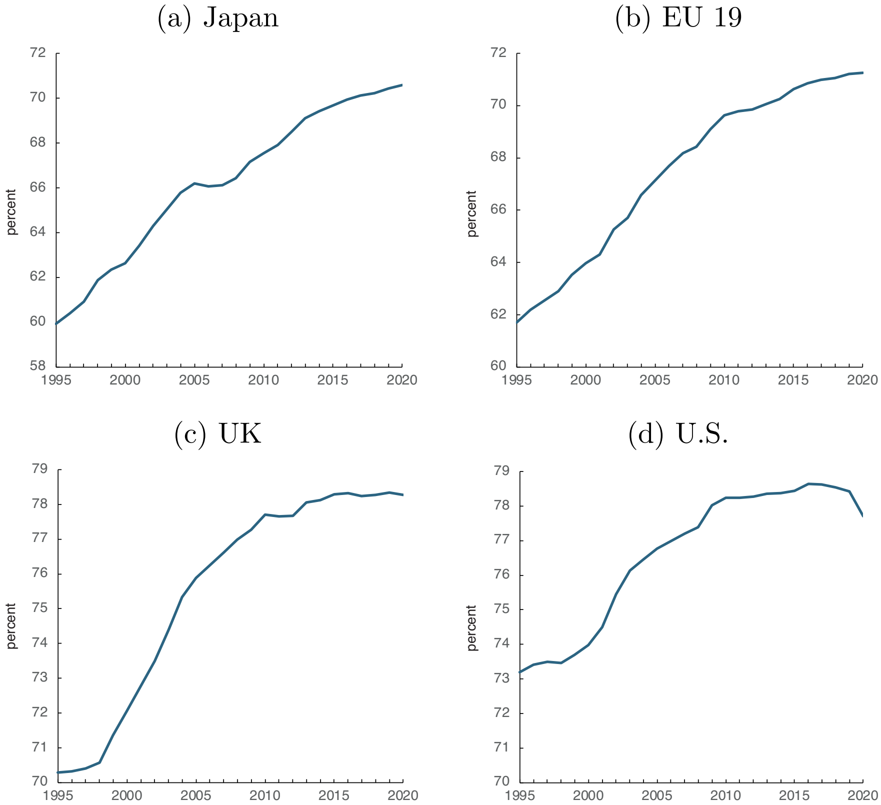



Figure 2: Employment share of the non-tradable goods sector. Source: EU KLEMS database.

There are significant international differences in the progression of aging even among advanced economies. Figure 1 illustrates these differences. Panel (a) plots the old-age dependency ratios (OADRs) from 1950 to 2021 for the G7 countries, where the OADR is defined as the ratio of the population aged 65 and above to the population aged 20–64. Japan had the lowest OADR in 1950, but it has had the highest ratio among G7 members since 2005. Panel (b) shows projections for: (i) representative countries whose OADRs are expected to exceed 50 by 2050; (ii) the top three G7 countries in 2020 (Japan, Italy, and France); and (iii) the U.S. as a benchmark. China's OADR was only 11.5 in 2020 but is projected to surpass that of the U.S. within about 15 years. South Korea's increase will be even faster, overtaking Japan by 2048. Although France and Italy had nearly identical OADRs in 2020, their trajectories are expected to diverge: aging will accelerate in Italy but proceed more gradually in France. Thus, the phenomenon of asymmetric aging across countries is already underway and will intensify in the decades ahead.

For the purpose of this study, I develop a two-country overlapping generations (OLG) model with endogenous technological progress. To aggregate optimization across different generations for analytical tractability, I adopt the perpetual-youth continuous-time OLG framework. In addition, the model features three key elements. First, the economy has two sectors. In the tradable goods sector, differentiated goods are produced under monopolistic competition. In the non-tradable goods sector, goods are produced under perfect competition. Second, research and development (R&D) generates technological progress that improves labor productivity in the

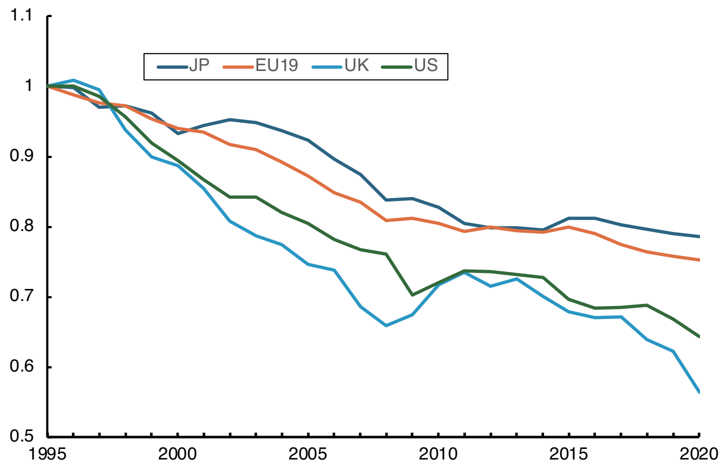



Figure 3: Relative price of tradable to non-tradable goods. Source: EU KLEMS database.

tradable sector, while productivity in the non-tradable sector remains stagnant. I later show that the main results are robust to allowing modest productivity improvements in the non-tradable sector. Third, the model incorporates fully endogenous growth, thereby eliminating scale effects, pioneered by Smulders and van de Klundert (1995), Dinopoulos and Thompson (1998), Peretto (1998), and Young (1998).

Each feature is motivated by empirical evidence. In many countries, employment in the non-tradable sector is both substantial and rising. Figure 2 plots the share of employment in non-tradables relative to total employment in Japan, the EU 19, the UK, and the U.S.[1] The classification of goods as non-tradable follows Cardi and Restout (2015).[2] Figure 3 shows the relative price of tradables to non-tradables, which has been declining. This suggests that technological progress has been slower in non-tradables than in tradables. Taken together, the rising employment share in non-tradables and their stagnant productivity imply that sectoral reallocation itself can be a drag on technological progress. Recent empirical studies also document that aging populations contribute to the shift toward service employment. For instance, using the U.S. household-level data, Cravino et al. (2022) provide the evidence that population aging has been a significant driver of structural transformation, particularly the shift toward services. To capture this effect, the model explicitly includes a non-tradable sector with no technological progress. The elimination of scale effects is also important, as there has long been criticism that it

[1]The EU 19 includes Austria, Belgium, Croatia, Cyprus, Estonia, Finland, France, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, the Netherlands, Portugal, Slovakia, Slovenia, and Spain.

[2]Tradable industries include: A: Agriculture, Forestry, and Fishing; B: Mining and Quarrying; C: Manufacturing; H: Transportation and Storage; J: Information and Communication; K: Financial and Insurance Activities. Non-tradable industries include: D: Electricity, Gas, Steam and Air Conditioning Supply; E: Water Supply; F: Construction; G: Wholesale and Retail Trade; I: Accommodation and Food Services; L: Real Estate Activities; M: Professional, Scientific, and Technical Activities; N: Administrative and Support Service Activities; O: Public Administration and Defense; P: Education; Q: Human Health and Social Work Activities; R: Arts and Entertainment.

is not realistic for population size to influence per capita growth. Since this study focuses on the long-run effects of internationally asymmetric aging, I employ a fully endogenous growth model rather than a semi-endogenous one.

With this framework, I first analytically examine how relative aging affects a country's relative technology, modeling relative aging as internationally asymmetric longevity. Starting from a symmetric steady state, I show that a more rapidly aging country inevitably experiences a long-run decline in its relative technology. At first glance, this result may appear straightforward, but it is not. In fact, asymmetric aging has no effect on a country's relative technology if the non-tradable sector is absent and individual labor supply does not decline with age. The intuition is simple. Longer life expectancy induces households to increase their savings. In a closed economy, this affects the country's R&D investment because domestic savings are perfectly tied to domestic investment. In an open economy with integrated asset markets, however, changes in savings do not translate one-to-one into domestic investment. Under full integration, where interest rates are equalized, such changes influence R&D in both countries equally. Thus, merely extending the model to two countries is insufficient to generate the results described above. This result also implies that caution is needed when applying findings from closed-economy models to the relationship between aging and relative technology, since advanced economies are, in reality, financially integrated.

In practice, the model in this study incorporates a non-tradable sector as an essential feature. In addition, it incorporates an age-related decline in individual labor supply to approximate retirement. Then, the more rapidly aging country loses its technological leadership through two channels: (i) a decline in per capita labor supply, and (ii) a sectoral shift of employment toward the non-tradable goods sector. Longer life expectancy reduces per capita labor input as the elderly share rises, directly lowering R&D employment and thereby slowing productivity growth in the tradable goods sector. In addition, households with longer lives save more, accumulating assets for future consumption. While future consumption of tradables can be partly met by imports, non-tradables must be produced domestically. This induces a long-run reallocation of labor toward non-tradables, which exacerbates the technological disadvantage of the aging country.

These findings carry clear policy implications. For example, in Japan, many firms have extended retirement age or introduced continued-employment schemes. The analysis shows that such measures directly address the first channel identified in this study, but they cannot counteract the second channel, namely the reallocation of employment toward non-tradables. Indeed, when the model is applied to a counterfactual case in which aging does not reduce per capita labor supply, a decline in relative technology still emerges due to sectoral reallocation. Thus, extending working lives may slow but cannot fully stop the technological disadvantage of more rapidly aging countries. Furthermore, in the neighborhood of a symmetric steady state, the analysis shows that relative aging in one country lowers the global rate of technological progress, which in turn depresses the growth rates of per capita variables worldwide.

To complement these analytical results, I conduct numerical simulations. The simulations confirm that many of the theoretical findings extend beyond the immediate neighborhood of a symmetric steady state. However, they also reveal that the effects of aging on the global rate of technological progress can be non-monotonic when demographic asymmetries are large. This result is consistent with the mixed empirical evidence in the literature, where the impact of aging on growth has not been robustly established. Finally, I calibrate the model so that Japan represents the more rapidly aging economy and the United States the less aging one. I quantify the extent to which differential aging can account for Japan's relative technological decline over the past three decades.

### 1.1 Related literature

Although this study analyzes the implications of international asymmetries in population aging under an open economy, it naturally builds on the existing body of research developed within closed-economy frameworks. Prior studies examining the impact of longer life expectancy and the resulting aging on economic growth have emphasized how longer life expectancy induces individuals to increase savings in anticipation of longer life spans.[3] Futagami and Nakajima (2001), for example, showed in a model with capital accumulation and learning-by-doing that population aging can raise the growth rate. Subsequent research advanced the analysis by incorporating R&D-driven technological progress into OLG models, including Prettner (2013), Prettner and Canning (2014), Prettner and Trimborn (2017), Baldanzi et al. (2019), and Kuhn and Prettner (2023). Their studies are consistently based on semi-endogenous growth models, or employ them alongside first-generation models in which scale effects arise.

Davis et al. (2022) make an exceptional contribution by employing a two-country endogenous growth model without scale effects to study how international differences in aging affect technological progress and the spatial distribution of firms. Although this study is closely related to their research, there are two crucial differences in assumptions. First, in their model, the international difference in aging reduces solely to that in per capita labor supply, so internationally observed heterogeneity in consumption and saving patterns does not arise. Second, because their model includes only a single tradable-goods sector, intersectoral employment shifts, which may influence technological progress, are excluded. Although not directly addressing aging, this study is also closely related to van de Klundert and Smulders (2001). They developed a two-country, two-sector endogenous growth model with non-tradable goods and representative agents, and analyzed how international asymmetries determine relative technologies. Their focus is on initial wealth asymmetries: in the long run, a country with greater initial wealth employs more workers in the non-tradable sector, and its relative technology eventually lags behind. Thus, the mechanism generating their results is fundamentally distinct from that of the present study.

Although they are based on neither R&D-based growth models nor two-country models, the

[3] Although not addressed in this study, another major factor driving aging, alongside longevity, is declining fertility. A recent analysis of this factor is provided by Jones (2022). Moreover, Bloom et al. (2024) offer a comprehensive survey on the relationship between declining fertility in advanced economies and economic growth.

following studies are related to this study in that they consider two sectors: general goods and health care services consumed in old age. Using a two-period OLG model in which labor productivity in the goods sector improves with this sector's employment share, van Groezen et al. (2005) show that longer life expectancy lowers the growth rate in a small open economy through an employment shift toward the health care services sector. In a closed economy, however, they show that the growth effect of higher longevity depends on the elasticity of substitution between capital and labor: the growth rate decreases (increases) if the elasticity is greater (smaller) than one. Hashimoto and Tabata (2010), using a small-open, two-period OLG model, also show that higher longevity shifts employment into the health care services sector, yielding similar results. Momota (2012) extends the framework of van Groezen et al. (2005) by introducing non-homothetic preferences over goods and health care services, proving that even in a small open economy, longer life expectancy may reduce employment in the care sector and raise the growth rate, if the elasticity of the marginal utility of health care services is lower than a threshold level.

Empirical studies have produced mixed results on the relationship between aging and growth. Lindh and Malmberg (1999), using OECD panel data from 1950 to 1990, found a negative correlation between the elderly share in 1950 and the subsequent growth rate of GDP per worker. Aksoy et al. (2019) reported that aging reduces innovation and slows investment and output growth between 1970 and 2014. By contrast, Acemoglu and Restrepo (2017), using panel data for 169 countries from 1990 to 2015, found not a negative effect but rather a significantly positive relationship between aging and growth. In follow-up work, they argued that automation technology explains this finding (Acemoglu and Restrepo, 2022). Recently, however, Eggertsson et al. (2019) showed that when the sample is restricted to the post-financial-crisis period, these results are reversed. Gehringer and Prettner (2019) argue that reductions in mortality across OECD countries contributed positively to labor productivity and TFP growth over the period 1960–2011. Using panel data on OECD countries for 1975–2014, Emerson et al. (2024) found a negative relationship between aging and GDP per worker growth. They also provide a theoretical framework to explain this finding, emphasizing physical and human capital as the engines of growth, along with health care expenditures. Thus, the relationship between aging and growth remains unsettled, with diverse empirical findings. Numerical simulations in this study reveal that aging can have non-monotonic effects on per capita growth, depending on the scale of demographic asymmetries. This theoretical prediction resonates with the mixed empirical findings in the literature, suggesting that the diverse results observed across countries and periods may not reflect inconsistency, but rather an inherent feature of how aging interacts with growth in a globally integrated economy.

### 1.2 Organization of the paper

The remainder of the paper is structured as follows. Section 2 sets up the two-country OLG model with tradable and non-tradable sectors. Section 3 derives the equilibrium conditions. Section 4 presents analytical results around a symmetric steady state, showing that relative aging disadvantages the more rapidly aging country and slows global growth. Section 5 complements the analytical results with numerical simulations. It also calibrates the model to Japan and the United States to quantify technology gaps and wage differentials due to asymmetric aging. Section 6 explores the robustness of the results to alternative assumptions. Section 7 concludes this paper.

# 2 Model

The world economy consists of two countries, called country 1 and country 2. Variables and parameters for country 2 are marked with an asterisk "$*$" whenever their values differ from those in country 1. The economy produces two types of goods: non-tradable and tradable goods. Asset markets are integrated across two countries, while labor, which is the primary factor of production, is not internationally mobile.

## 2.1 Households

Consider households in country 1. The demographic structure follows a variant of Yaari (1965) and Blanchard (1985), in which lives end stochastically. The initial population is exogenously given as $L > 0$. Each individual faces a probability of death, where $\mu > 0$ denotes the Poisson arrival rate of death. At each instant, a new generation of size $\mu L$ is born without bequests, ensuring that the population remains constant at $L$. In Section 6, I relax this assumption to allow for population growth.

The expected lifetime utility of an individual born at time $s$ is

$$U(s) = \int_s^{\infty} e^{-(\rho+\mu)(t-s)} \log C_t(s) dt,$$

where $C_t(s)$ denotes consumption and $\rho > 0$ is the subjective discount rate. The argument "$(s)$" represents the time of birth. Consumption has a two-tier structure. At the upper level, $C(s)$ is a Cobb–Douglas composite of the two goods:

$$C_t(s) = (C_{N,t}(s))^{\beta} (C_{T,t}(s))^{1-\beta},$$

where $C_N(s)$ and $C_T(s)$ are consumption of non-tradable and tradable goods, respectively. The parameter $\beta \in (0, 1)$ is the expenditure share of non-tradables. At the lower level, consumption of the tradable goods is given by the CES composite of horizontally differentiated varieties:

$$C_{T,t}(s) = \left( \int_0^{M_t} x_t(i,s)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_0^{M_t^*} x_t(i^*,s)^{\frac{\varepsilon-1}{\varepsilon}} di^* \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $x(i, s)$ and $M$ denote demand and the number of domestic varieties, while $x(i^*, s)$ and $M^*$ denote demand and the number of foreign varieties. The parameter $\varepsilon > 1$ is the elasticity of substitution between varieties.

Let $A_t(s)$ denote the financial wealth of an individual of cohort $s$. Individuals are born without assets ($A_s(s) = 0$) except for the initial generation. Labor supply declines with age: the labor supply of age $t - s$ is $e^{-\phi(t-s)}$ where $\phi \geq 0$. This parameter serves as an indicator of old-age labor force participation, with smaller values corresponding to higher participation. In general, the decline in labor supply with age may stem from two sources. One is physiological deterioration, which implies that $\phi$ could depend on mortality $\mu$. The other is institutional, such as mandatory retirement at a certain age, regardless of health status. Following Blanchard (1985, Sec.3) and Blanchard and Fischer (1989, Ch.3), this study abstracts from the first cause and emphasizes the second; this age-related declining schedule is interpreted as an approximation of retirement.[4] Accordingly, like them, I assume that $\phi$ is independent of $\mu$. Then, assets evolve according to the following budget constraint:

$$\frac{dA_t(s)}{dt} = (r_t + \mu)A_t(s) + w_t e^{-\phi(t-s)} - E_t(s),$$

$$E_t(s) \equiv P_{N,t}C_{N,t}(s) + \int_0^{M_t} p_t(i)x_t(i,s)di + \int_0^{M_t^*} p_t^*(i^*)x_t(i^*,s)di,$$

where $r$, $w$, $P_N$, $p(i)$, and $p^*(i^*)$ are the interest rate, the wage rate, the price of non-tradables, the price of tradables of domestic variety $i$, and the price of foreign tradables of variety $i^*$, respectively. The term $\mu A_t(s)$ represents the insurance premium received from the insurance company.

The household optimization problem is standard; details are provided in Appendix A.1. Define average (or per capita) assets and expenditure as

$$A_t \equiv \mu \int_{-\infty}^{t} e^{-\mu(t-s)}A_t(s)ds, \quad E_t \equiv \mu \int_{-\infty}^{t} e^{-\mu(t-s)}E_t(s)ds.$$

The average demand for each good is defined analogously. Then, household behavior can be summarized as

$$\dot{A}_t = r_t A_t + w_t \frac{\mu}{\mu + \phi} - E_t, \tag{1}$$

$$\dot{E}_t/E_t = r_t - \rho + \phi - (\rho + \mu)(\mu + \phi)A_t/E_t, \tag{2}$$

$$C_{N,t} = \beta E_t/P_{N,t},$$

$$x_t(i) = (p_t(i)/P_{T,t})^{-\varepsilon}(1-\beta)E_t/P_{T,t},$$

$$x_t(i^*) = (p_t^*(i^*)/P_{T,t})^{-\varepsilon}(1-\beta)E_t/P_{T,t},$$

where a dot denotes time differentiation. $P_T$ is the price index of the tradables:

$$P_{T,t} \equiv \left(\int_0^{M_t} p_t(i)^{1-\varepsilon}di + \int_0^{M_t^*} p_t^*(i^*)^{1-\varepsilon}di^*\right)^{1/(1-\varepsilon)}. \tag{3}$$

Equation (1) is the per capita budget constraint, where the term $\mu/(\mu + \phi)$ represents the per

[4] I formulate worker retirement in this way for analytical tractability. Alternatively, Kuhn and Prettner (2023) assume that each individual works for a finite length of time.

capita labor supply.[5] Provided that $\phi \neq 0$, the per capita labor supply declines as mortality decreases (i.e., as $\mu$ becomes smaller), since lower mortality reduces the share of young individuals whose labor supply is relatively large. Equation (2) highlights that average expenditure growth differs from individual expenditure growth, $\dot{E}_t(s)/E_t(s) = r_t - \rho$, due to cohort turnover. Newborns contribute higher labor income but no wealth, creating opposing effects on average expenditure growth. Specifically, the third term in (2) reflects the labor-supply effect of newborns, while the fourth term reflects their zero initial wealth. Thus, (2) captures not only intertemporal choices of individuals but also the demographic composition of cohorts.

This study allows for cross-country differences in three dimensions: life expectancy ($\mu \neq \mu^*$), old-age labor participation ($\phi \neq \phi^*$), and population size ($L \neq L^*$). Households in country 2 face an analogous problem, yielding

$$\dot{A}_t^* = r_t A_t^* + w_t^* \frac{\mu^*}{\mu^* + \phi^*} - E_t^*, \tag{1*}$$

$$\dot{E}_t^*/E_t^* = r_t - \rho + \phi^* - (\rho + \mu^*)(\mu^* + \phi^*)A_t^*/E_t^*, \tag{2*}$$

$$C_{N,t}^* = \beta E_t^*/P_{N,t}^*,$$

$$x_t^*(i) = (p_t(i)/P_{T,t})^{-\varepsilon}(1-\beta)E_t^*/P_{T,t},$$

$$x_t^*(i^*) = (p_t^*(i^*)/P_{T,t})^{-\varepsilon}(1-\beta)E_t^*/P_{T,t}.$$

## 2.2 Non-tradable goods sector

As stated in the introduction, the relative price of non-tradables to tradables has continued to rise, mainly because technological progress has been slower in the non-tradable sector. To incorporate this fact in the simplest way, I assume that no technological progress occurs in the non-tradable sector. The marginal productivity of labor is constant at $\chi > 0$. The good is produced and supplied under perfect competition. Profit maximization implies

$$P_{N,t} = w_t/\chi, \quad P_{N,t}^* = w_t^*/\chi.$$

In Section 6, I relax this assumption and consider the case in which labor productivity improves also in this sector.

## 2.3 Tradable goods sector

Each variety is produced under monopolistic competition. Consider a firm producing variety $i \in [0, M_t]$ in country 1. Its technology is

$$Y_t(i) = (Z_t(i))^\theta L_t^Y(i),$$

[5] Even if retirement would occur stochastically with hazard $\phi$ instead of deterministic declining, we can obtain the same per capita budget constraint as (1) given perfect insurance markets for retirement. To see this, suppose each agent supplies one unit of labor until a Poisson retirement shock with hazard $\phi$. Let $d_t$ and $b_t$ denote contributions to and benefits from retirement insurance, respectively. With actuarially fair retirement insurance, $w_t - d_t = b_t$ holds, and the zero-profit condition of the insurance company is $\int_{-\infty}^t e^{-\mu(t-s)}(1-e^{-\phi(t-s)})b_t ds = \int_{-\infty}^t e^{-(\mu+\phi)(t-s)} d_t ds$. Solving these equations yields $b_t(= w_t - d_t) = w_t\mu/(\mu+\phi)$; that is, the expected labor income equals $w_t\mu/(\mu+\phi)$.

where $Y(i)$ is output, $L^Y(i)$ is employment for production, and $Z(i)$ is the technology level. Here, $Z(i)^\theta$ captures marginal labor productivity, and $\theta$ measures its elasticity with respect to $Z(i)$. To ensure that the firm's optimization problem is well-behaved, I impose the following assumption, which guarantees that the first-order conditions are not only necessary but also sufficient for a maximum.

**Assumption 1.** *$\theta < 1/(\varepsilon - 1)$.*

Since $Y(i) = Lx(i) + L^*x^*(i)$, the firm's gross profit (i.e., profit before paying R&D costs) is

$$\Pi_t(i) = \left(p_t(i) - w_t Z_t(i)^{-\theta}\right)\left(\frac{p_t(i)}{P_{T,t}}\right)^{-\varepsilon} \frac{(1-\beta)(LE_t + L^*E_t^*)}{P_{T,t}}. \tag{4}$$

The firm can improve technology by allocating labor to R&D:

$$\dot{Z}_t(i) = K_t L_t^R(i), \tag{5}$$

where $L^R(i)$ is employment for R&D and $K$ is the marginal productivity of labor in innovation. $K$ is independent of $i$, reflecting the fact that the marginal productivity depends on public knowledge available to all firms. Following the literature like Baldwin and Forslid (2000), I assume that public knowledge in a country incorporates international spillovers:

$$K_t = \frac{1}{M_t + M_t^*}\left(\int_0^{M_t} Z_t(i)di + \zeta \int_0^{M_t^*} Z_t^*(i^*)di^*\right),$$

where $\zeta \in (0,1)$ is the degree of spillover. This specification implies that firms in country 1 benefit not only from domestic knowledge but also from foreign technology, weighted by $\zeta$. The public knowledge $K^*$ for country 2 is defined analogously:

$$K_t^* = \frac{1}{M_t^* + M_t}\left(\int_0^{M_t^*} Z_t^*(i^*)di^* + \zeta \int_0^{M_t} Z_t(i)di\right).$$

The firm chooses the time paths of its price $p(i)$, R&D labor $L^R(i)$, and technology $Z(i)$ to maximize

$$V_t(i) = \int_t^\infty e^{-\int_t^\tau r_u du}\left(\Pi_\tau(i) - w_\tau L_\tau^R(i)\right) d\tau,$$

subject to (4) and (5), taking other variables as given. The current-value Hamiltonian corresponding to this problem is $H(i) = \Pi(i) - wL^R(i) + q(i)KL^R(i)$, where $q(i)$ is the costate variable. The first-order conditions are

$$p_t(i) = \frac{\varepsilon}{\varepsilon - 1}\frac{w_t}{(Z_t(i))^\theta}, \tag{6}$$

$$w_t = q_t(i)K_t, \tag{7}$$

$$\dot{q}_t(i) = r_t q_t(i) - \theta(\varepsilon - 1)\frac{\Pi_t(i)}{Z_t(i)}. \tag{8}$$

Appendix A.2 shows that the maximized Hamiltonian is strictly concave in a firm's state variable $Z(i)$ if Assumption 1 is satisfied. That is, the first-order conditions (6)–(8) are not only necessary but also sufficient conditions for optimization.[6] The costate variable $q(i)$ corresponds to the shadow price of a firm's technology $Z(i)$, that is, $q(i)$ is the implicit price of technology if it could be fairly traded in a market.

I assume $\theta^* = \theta$. The first-order conditions of a firm producing variety $i^*$ in country 2 are

$$p_t^*(i^*) = \frac{\varepsilon}{\varepsilon - 1}\frac{w_t^*}{(Z_t^*(i^*))^\theta}, \tag{6*}$$

$$w_t^* = q_t^*(i^*)K_t^*, \tag{7*}$$

$$\dot{q}_t^*(i^*) = r_t q_t^*(i^*) - \theta(\varepsilon - 1)\frac{\Pi_t^*(i^*)}{Z_t^*(i^*)}. \tag{8*}$$

Following the literature, I henceforth impose symmetry within each country and drop the variety index, e.g., $Z_t(i) = Z_t$. Let $S$ denote country 1's sales share:

$$S_t \equiv \frac{M_t p_t Y_t}{(1-\beta)(LE_t + L^* E_t^*)} = \frac{M_t p_t^{1-\varepsilon}}{P_T^{1-\varepsilon}},$$

which implies that country 2's sales share is $S_t^* \equiv 1 - S_t$. From (3), (6), and (6*),

$$S_t = \frac{M_t\left(w_t Z_t^{-\theta}\right)^{1-\varepsilon}}{M_t\left(w_t Z_t^{-\theta}\right)^{1-\varepsilon} + M_t^*\left(w_t^* Z_t^{*-\theta}\right)^{1-\varepsilon}}.$$

Each potential entrant can establish a new firm by inventing a new variety. Following Peretto (1998), I assume away knowledge externalities in this product innovation: inventing one variety requires a fixed $1/\eta > 0$ units of labor.[7] The free-entry condition for positive entry is

$$V_t = \frac{w_t}{\eta}. \tag{9}$$

Let $L^E$ denote the labor employed in developing new varieties:

$$\dot{M}_t = \eta L_t^E. \tag{10}$$

Similarly, $V_t^* = w_t^*/\eta$ and $\dot{M}_t^* = \eta L_t^{E*}$. From the definition of $V$ and integrated asset markets,

$$r_t = \frac{\Pi_t - w_t L_t^R + \dot{V}_t}{V_t} = \frac{\Pi_t^* - w_t^* L_t^{R*} + \dot{V}_t^*}{V_t^*}. \tag{11}$$

[6] See Acemoglu (2009, Ch.7) for the maximized Hamiltonian. As stated in Theorems 7.6 and 7.13 in this book, Arrow's sufficiency theorem implies that if the maximized Hamiltonian is concave, then the first-order conditions are sufficient. Thus, even if the concavity of the maximized Hamiltonian would fail to hold, (6)–(8) might be sufficient conditions for firms' value maximization. Nonetheless, I assume $\theta < 1/(\varepsilon - 1)$ since it is hard to verify the sufficiency of the first-order conditions if this inequality is not satisfied.

[7] As shown by Peretto and Connolly (2007), even when knowledge spillovers are at work in entry activities, introducing flow fixed costs to the incumbent firms can eliminate scale effects.

## 3 Equilibrium

In country 1, labor demand for producing the non-tradable goods is $\beta E_t L/(P_{N,t}\chi) = \beta E_t L/w_t$. The total labor demand in the tradable sector is $M_t(L_t^Y + L_t^R) + L_t^E$. Thus, the labor market equilibrium in country 1 is

$$\frac{\mu}{\mu+\phi}L = \frac{\beta L E_t}{w_t} + M_t\left(\frac{Y_t}{Z_t^{\theta}} + \frac{\dot{Z}_t}{K_t}\right) + \frac{\dot{M}_t}{\eta}. \tag{12}$$

Similarly, the labor market equilibrium in country 2 is

$$\frac{\mu^*}{\mu^*+\phi^*}L^* = \frac{\beta L^* E_t^*}{w_t^*} + M_t^*\left(\frac{Y_t^*}{Z_t^{*\theta}} + \frac{\dot{Z}_t^*}{K_t^*}\right) + \frac{\dot{M}_t^*}{\eta}. \tag{13}$$

Hereafter, labor in country 2 is set as the numeraire:

$$w_t^* = 1 \forall t.$$

Thus, $w_t$ also captures the international wage gap.

Let $z_t \equiv Z_t/Z_t^*$ denote country 1's relative technology or the international technology gap. Similarly, let $m_t \equiv M_t/M_t^*$ denote the relative number of varieties produced in country 1. The knowledge stock defined in Section 2.3 can be rewritten as

$$\frac{K_t}{Z_t} = \frac{m_t + \zeta/z_t}{m_t+1}, \quad \frac{K_t^*}{Z_t^*} = \frac{1+\zeta m_t z_t}{m_t+1}. \tag{14}$$

Using (14), (7) and (7*) are rewritten as

$$w_t = q_t Z_t \frac{m_t + \zeta/z_t}{m_t+1}, \quad 1 = q_t^* Z_t^* \frac{1+\zeta m_t z_t}{m_t+1}. \tag{15}$$

With $z$ and $m$, country 1's sales share simplifies to

$$S_t = \frac{m_t}{m_t + \left(w_t z_t^{-\theta}\right)^{\varepsilon-1}}. \tag{16}$$

Henceforth, I characterize the steady-state equilibrium in which $z_t$, $m_t$, and $w_t$ are constant over time. In such an equilibrium, $Z$ and $Z^*$ continue to grow at a constant rate, denoted by $g_Z$.

### 3.1 Relative wage, market share, and entry given the relative technology

In the steady state, $qZ$ and $q^*Z^*$ are constant by (15). This implies that $q$ and $q^*$ decline over time in proportion to $1/Z$. Then, (8) and (8*) with $\dot{q}/q = \dot{q}^*/q^*$ imply

$$\frac{\Pi}{\Pi^*} = \frac{qZ}{q^*Z^*}. \tag{17}$$

Thus, the ratio of profit flows aligns with the ratio of their stock values of $Z$. Using the definition of $\Pi$ and (15), (17) can be rewritten as

$$\frac{S}{m(1-S)} = w\frac{1+\zeta m z}{m+\zeta/z}. \tag{18}$$

From the choice of the numeraire, $V_t^*$ is always given by $1/\eta$. In addition, (9) shows that $V$ is stationary in the steady state. Then, from (11), $r = (\Pi - wL^R)/V = (\Pi^* - w^* L^{R*})/V^*$. In Appendix A.3, I show that these equations are respectively rewritten as

$$\frac{\dot{Z}}{Z} = g_Z = \frac{1}{\varepsilon}\frac{S(1-\beta)(LE+L^*E^*)}{Mw}\frac{m+\zeta/z}{m+1} - \frac{r}{\eta}\frac{m+\zeta/z}{m+1}, \tag{19}$$

$$\frac{\dot{Z}^*}{Z^*} = g_Z = \frac{1}{\varepsilon}\frac{(1-S)(1-\beta)(LE+L^*E^*)}{M^*}\frac{1+\zeta mz}{m+1} - \frac{r}{\eta}\frac{1+\zeta mz}{m+1}. \tag{20}$$

Taking into account (18), we find that the first terms on the right-hand sides of (19) and (20) are equal. This implies that the second terms also must be equal:

$$m + \zeta/z = 1 + \zeta mz, \tag{21}$$

which determines the relative number of varieties $m$ as a function of $z$:

$$m = m(z) \equiv \frac{1-\zeta/z}{1-\zeta z}.$$

Thus, the steady-state value of $z$ must satisfy $\zeta < z < 1/\zeta$.

Substituting (16) into (18) and using $m = m(z)$ and (21), we obtain the relative wage as an increasing and concave function of $z$:

$$w = w(z) \equiv z^{\theta(\varepsilon-1)/\varepsilon}.$$

Then, substituting $m = m(z)$ and $w = w(z)$ back into (16),

$$S = S(z) \equiv \frac{1}{1 + \frac{1-\zeta z}{1-\zeta/z} z^{-\theta(\varepsilon-1)/\varepsilon}},$$

which lies in $(0,1)$ if $\zeta < z < 1/\zeta$. Equations (18) and (21) also show that international differences in the market share stem from differences in wages and varieties:

$$\frac{S(z)}{1-S(z)} = w(z)m(z).$$

**Lemma 1.** *Suppose that $z$ increases, that is, country 1 becomes more productive relative to country 2 in the tradable goods sector. Then, country 1's relative mass of varieties, $m$, increases, its relative wage, $w$, rises, and its market share, $S$, expands.*

*Proof.* This follows immediately since $m(z)$, $w(z)$, and $S(z)$ are all increasing in $z$. □

From (14), it follows that if (21) holds, then $K/Z = K^*/Z^*$. Using $m(z)$, they are given by

$$\frac{K_t}{Z_t} = \frac{K_t^*}{Z_t^*} = k(z) \equiv \frac{1-\zeta^2}{2-\zeta/z-\zeta z}.$$

Rearranging (8) yields

$$\frac{\dot{q}}{q} = -g_Z = r - \theta\frac{\varepsilon-1}{\varepsilon}\frac{S(1-\beta)(LE+L^*E^*)}{Mw}\frac{m+\zeta/z}{m+1}, \tag{22}$$

From (19), (22), and $M/M^* = m(z)$, we can express $r$, $M$, and $M^*$ as

$$r = \frac{\eta[1-\theta(\varepsilon-1)]}{k(z)\theta(\varepsilon-1)-\eta} g_Z, \tag{23}$$

$$M = \frac{\varepsilon-1}{\varepsilon} \frac{k(z)\gamma(z)S(z)(1-\beta)(LE+L^*E^*)}{wg_Z}, \tag{24}$$

$$M^* = \frac{\varepsilon-1}{\varepsilon} \frac{k(z)\gamma(z)(1-S(z))(1-\beta)(LE+L^*E^*)}{g_Z}, \tag{25}$$

where

$$\gamma(z) \equiv \frac{1}{\varepsilon-1} \frac{k(z)\theta(\varepsilon-1)-\eta}{k(z)-\eta}.$$

In (23), the numerator is positive under Assumption 1. The denominator depends on $k(z)$. From its definition, $k(z)$ attains a minimum of $(1+\zeta)/2$ at $z=1$, so that $k'(1)=0$ and $k(z)$ is U-shaped in $z$. Intuitively, an increase in $z$ generates two opposing effects: it reduces the relative usefulness of foreign knowledge but at the same time raises the weight placed on domestic knowledge through variety creation. Which effect dominates depends on the level of $z$.[8] If $k(z)\theta(\varepsilon-1) > \eta$ holds at $z=1$, this inequality is satisfied for all $z \in (\zeta, 1/\zeta)$. Therefore, I assume the following inequality which implies that the marginal productivity of labor in creating new varieties cannot be too high. As long as Assumption 2 holds, $\gamma(z) > 0$ automatically follows since $\theta(\varepsilon-1) < 1$. Assumptions 1 and 2 jointly require $2\eta/(1+\zeta) < \theta(\varepsilon-1) < 1$. This corresponds to Condition (a) of Proposition 2 in Peretto (1998, p.290).

**Assumption 2.** $(1+\zeta)\theta(\varepsilon-1)/2 > \eta$.

The following lemma is the well-known property in the fully endogenous growth models eliminating scale effects.

**Lemma 2.** *In the steady state with $z$ and $g_Z$ constant, $M$ and $M^*$ grow at the same rate as the population size. Thus, if the population is constant over time, so are $M$ and $M^*$.*

*Proof.* Labor demand to produce the non-tradable goods is $\beta EL/w(z)$, which cannot exceed labor supply $\mu L/(\mu+\phi)$. Hence, $E$ is constant in the steady state, and similarly $E^*$. From (24) and (25), $M$ and $M^*$ must then be proportional to population. □

[8] More specifically, since $k(z) = \frac{m(z)+\zeta/z}{m(z)+1}$,

$$k'(z) = \frac{1}{(m+1)^2}\left[-(m+1)\zeta/z^2 + (1-\zeta/z)m'(z)\right].$$

Inside the brackets, the first term reflects the loss of foreign spillovers as $z$ rises, while the second reflects the greater relevance of domestic knowledge through additional entry. We can arrange this equation as

$$k'(z) = \frac{(1-\zeta^2)\zeta(1-1/z^2)}{(2-\zeta/z-\zeta z)^2} \gtreqless 0 \Leftrightarrow z \gtreqless 1.$$

## 3.2 Conditions for the steady state

From Lemma 2, $\dot{M} = \dot{M}^* = 0$ in the steady state. Appendix A.3 shows that the market-clearing conditions for labor, (12) and (13), reduce to

$$\frac{\mu}{\mu+\phi}L = \frac{\beta LE}{w(z)} + m(z)\frac{\varepsilon-1}{\varepsilon}(1+\gamma(z))(1-S(z))(1-\beta)(LE+L^*E^*), \tag{26}$$

$$\frac{\mu^*}{\mu^*+\phi^*}L^* = \beta L^*E^* + \frac{\varepsilon-1}{\varepsilon}(1+\gamma(z))(1-S(z))(1-\beta)(LE+L^*E^*). \tag{27}$$

From these, we obtain

$$L\left(\frac{\mu}{\mu+\phi} - \frac{\beta E}{w(z)}\right) = m(z)L^*\left(\frac{\mu^*}{\mu^*+\phi^*} - \beta E^*\right). \tag{28}$$

The left-hand side is employment in the tradable sector in country 1, while $L^*(\frac{\mu^*}{\mu^*+\phi^*} - \beta E^*)$ in the right-hand side is that in country 2. Equation (28) thus equates their ratio to the relative number of varieties. Since one of (26)–(28) is redundant, I henceforth use (27) and (28).

If the economy had no non-tradable sector ($\beta = 0$), (28) is simplified to

$$m(z) = \frac{L}{L^*}\frac{\frac{\mu}{\mu+\phi}}{\frac{\mu^*}{\mu^*+\phi^*}}.$$

In this hypothetical case, country 1's relative technology is pinned down so that its relative number of varieties equals its relative labor supply. The relative labor supply, in turn, depends only on relative population size and relative per capita labor supply. Thus, when $\beta = 0$, relative aging in country 1 affects its relative technology solely through a decline in per capita labor supply. This counterfactual highlights the key role of the non-tradable sector. Let us further assume no retirement ($\phi = \phi^* = 0$), which yields $m(z) = L/L^*$. Thus, in this case, regardless of which country undergoes relatively faster aging, such demographic change has no impact on the aging country's relative technology. Then, the following lemma holds.

**Lemma 3.** *If the non-tradable sector is absent ($\beta = 0$) and individual labor supply does not decline with age ($\phi = \phi^* = 0$), relative aging of a country has no effect on its relative technology.*

Henceforth, I focus on the case of $\beta \neq 0$. Because (27) and (28) involve three endogenous variables, $z$, $E$, and $E^*$, one more condition is needed. This is provided by the international equalization of interest rates. From (2) and (2*) with $\dot{E} = \dot{E}^* = 0$,

$$r = \rho - \phi + (\rho+\mu)(\mu+\phi)\frac{A}{E} = \rho - \phi^* + (\rho+\mu^*)(\mu^*+\phi^*)\frac{A^*}{E^*}. \tag{29}$$

Thus, $A$ and $A^*$ are also constant in the steady state. From (1) and (2) with $\dot{A} = \dot{E} = 0$,

$$0 = r + w(z)\frac{\mu}{\mu+\phi}\frac{1}{A} - \frac{E}{A},$$

$$0 = r - \rho + \phi - (\rho+\mu)(\mu+\phi)\frac{A}{E}.$$

From these two equations,

$$(\rho+\mu)(\mu+\phi)(A/E)^2+(\rho-\phi)(A/E)-\left[1-w(z)\frac{\mu}{\mu+\phi}\frac{1}{E}\right]=0.$$

Focusing on steady states with $A>0$ and $r>0$, $E>w(z)\mu/(\mu+\phi)$. Solving for $A/E$,

$$\frac{A}{E}=\frac{-(\rho-\phi)+\sqrt{(\rho-\phi)^2+4(\rho+\mu)[(\mu+\phi)-\mu w(z)/E]}}{2(\rho+\mu)(\mu+\phi)}. \tag{30}$$

Similarly,

$$\frac{A^*}{E^*}=\frac{-(\rho-\phi^*)+\sqrt{(\rho-\phi^*)^2+4(\rho+\mu^*)[(\mu^*+\phi^*)-\mu^*/E^*]}}{2(\rho+\mu^*)(\mu^*+\phi^*)}. \tag{31}$$

Substituting (30) and (31) into (29).

$$\begin{aligned}2r&=\rho-\phi+\sqrt{(\rho-\phi)^2+4(\rho+\mu)\left(\mu+\phi-\frac{w(z)\mu}{E}\right)}\\&=\rho-\phi^*+\sqrt{(\rho-\phi^*)^2+4(\rho+\mu^*)\left(\mu^*+\phi^*-\frac{\mu^*}{E^*}\right)}.\end{aligned} \tag{32}$$

Note that (27), (28), and (32) involve only $z$, $E$, and $E^*$. In the steady-state equilibrium, $z$, $E$, and $E^*$ are jointly determined from (27), (28), and (32).

Once $z$ is determined, $w(z)$, $m(z)$, $k(z)$, and $S(z)$ are determined accordingly. $M$ and $M^*$ are determined from (24) and (25), $A$ and $A^*$ are determined from (30) and (31), and $r$ and $g_Z$ are determined from (23) and (32):

$$g_Z=\frac{k(z)\theta(\varepsilon-1)-\eta}{\eta[1-\theta(\varepsilon-1)]}\frac{1}{2}\left[\rho-\phi^*+\sqrt{(\rho-\phi^*)^2+4(\rho+\mu^*)\left(\mu^*+\phi^*-\frac{\mu^*}{E^*}\right)}\right]. \tag{33}$$

The levels of real consumption are given by $C_t=E/P_t$ and $C_t^*=E^*/P_t^*$, where

$$P_t=\left(\frac{w(z)}{\beta\chi}\right)^{\beta}\left(\frac{P_{T,t}}{1-\beta}\right)^{1-\beta},\quad P_t^*=\left(\frac{1}{\beta\chi}\right)^{\beta}\left(\frac{P_{T,t}}{1-\beta}\right)^{1-\beta}.$$

From (3), (6), and (6*), $P_T$ in the steady state is

$$P_{T,t}=\frac{\varepsilon}{\varepsilon-1}Z_t^{*-\theta}\left[M\left(w(z)z^{-\theta}\right)^{1-\varepsilon}+M^*\right]^{1/(1-\varepsilon)}.$$

Since the term in brackets is constant in the steady state, $P_T$ continues to decline at the rate of $\theta g_Z$. Therefore, in the steady state, the growth rate of consumption is $\dot{C}_t/C_t=\dot{C}_t^*/C_t^*=g_C\equiv(1-\beta)\theta g_Z$.

## 4 Effects of relative aging in a country

In general, (27), (28), and (32) are highly nonlinear and cannot be solved analytically. I first show the existence of a steady state under symmetry:

$$\mu=\mu^*=\overline{\mu},\quad \phi=\phi^*=\overline{\phi},\quad L=L^*.$$

Then, (32) is simplified to $E/w(z) = E^*$. Substituting this result into (28) and using the symmetry assumption,

$$m(z) \equiv \frac{1 - \zeta/z}{1 - \zeta z} = 1,$$

which implies

$$z = 1, \quad w(1) = 1, \quad S(1) = 1/2.$$

Expenditures are equal across countries. From this and (27),

$$E = E^* = \overline{E} \equiv \frac{1}{\alpha} \frac{\overline{\mu}}{\overline{\mu} + \overline{\phi}}, \tag{34}$$

where

$$\alpha \equiv \beta + \frac{\varepsilon - 1}{\varepsilon}(1 - \beta)(1 + \gamma(1)),$$
$$\gamma(1) \equiv \frac{1}{\varepsilon - 1} \frac{(1 + \zeta)\theta(\varepsilon - 1) - 2\eta}{1 + \zeta - 2\eta} > 0.$$

Since $A > 0$, per capita expenditure must exceed per capita wage income. At the same time, the employment in the non-tradable sector cannot exceed the per capita labor supply. Hence, under symmetry, the inequality $\beta\overline{E} < \frac{\overline{\mu}}{\overline{\mu}+\overline{\phi}} < \overline{E}$ must hold. Using (34), this inequality is simplified to $\beta < \alpha < 1$. The following lemma shows that this inequality indeed holds.

**Lemma 4.** *Under symmetry ($\mu = \mu^* = \overline{\mu}$, $\phi = \phi^* = \overline{\phi}$, and $L = L^* = \overline{L}$), there exists a unique steady state.*

*Proof.* From the definition of $\alpha$, $\beta < \alpha$ is straightforward. It remains to show $\alpha < 1$, which holds if $\gamma(1) < 1/(\varepsilon - 1)$. We can rewrite $\alpha$ as

$$\alpha = 1 - \frac{1 - \beta}{\varepsilon}[1 - (\varepsilon - 1)\gamma(1)].$$

From the definition of $\gamma(z)$,

$$\gamma(z) \equiv \frac{1}{\varepsilon - 1} \frac{k(z)\theta(\varepsilon - 1) - \eta}{k(z) - \eta}.$$

Under Assumptions 1 and 2, both numerator and denominator are positive. Because $\theta(\varepsilon - 1) < 1$ from Assumption 1, $\gamma(z)$ is less than $1/(\varepsilon - 1)$ for all $z \in (\zeta, 1/\zeta)$. □

I now analyze the long-run effects of one country's relative aging, starting from a symmetric steady state. In Section 5, the analysis is extended numerically to more general settings. Here, I focus on the case where life expectancy in country 1 increases relative to that in country 2:

$$d\mu < d\mu^* \leq 0,$$

while keeping other parameters symmetric.

Differentiating (27), (28), and (32) and evaluating all derivatives at the initial symmetric steady state ($z = 1$, $w = 1$, $m = 1$, $S = 1/2$, and $E = E^* = \overline{E}$),

$$\frac{\overline{\phi}}{(\overline{\mu}+\overline{\phi})^2}d\mu^* = \beta dE^* + \frac{\alpha-\beta}{2}(dE+dE^*) + \left[\frac{(1-\beta)(\varepsilon-1)}{\varepsilon}\gamma'(1) - (\alpha-\beta)2S'(1)\right]\overline{E}dz, \quad (35)$$

$$\left[m'(1)\left(\frac{\overline{\mu}}{\overline{\mu}+\overline{\phi}} - \beta\overline{E}\right) - \beta\overline{E}w'(1)\right]dz = \frac{\overline{\phi}}{(\overline{\mu}+\overline{\phi})^2}(d\mu - d\mu^*) - \beta(dE - dE^*), \quad (36)$$

$$dE - dE^* = \overline{E}w'(1)dz - \overline{E}^2\frac{\overline{\phi} + (\rho+2\overline{\mu})(1-1/\overline{E})}{(\rho+\overline{\mu})\overline{\mu}}(d\mu - d\mu^*). \quad (37)$$

Equations (36) and (37) imply the following relationship between changes in country 1's relative technology level $z$ and its relative aging:

$$dz = \frac{\Omega}{m'(1)\left(\frac{\overline{\mu}}{\overline{\mu}+\overline{\phi}} - \beta\overline{E}\right)}(d\mu - d\mu^*), \quad (38)$$

where

$$\Omega \equiv \frac{\overline{\phi}}{(\overline{\mu}+\overline{\phi})^2} + \beta\overline{E}^2\frac{\overline{\phi} + (\rho+2\overline{\mu})(1-1/\overline{E})}{(\rho+\overline{\mu})\overline{\mu}}. \quad (39)$$

It has already been established that $m'(z) > 0$ and $\beta\overline{E} < \overline{\mu}/(\overline{\mu}+\overline{\phi})$. Thus, the sign of $\Omega$ determines the impact of country 1's relative aging on its relative technology. The first term in (39) reflects the reduction in per capita labor supply in country 1: as individuals live longer, the elderly population rises, thereby reducing per capita labor supply. This directly decreases the amount of labor allocated to R&D and, in turn, relatively slows productivity improvements in the tradable sector. The second term captures the effect of the change in savings in country 1. With longer life expectancy, households postpone consumption, raising asset accumulation for future consumption. While future consumption of tradable goods can be partially met through imports, non-tradable goods must be produced domestically. This mechanism, therefore, shifts employment toward the non-tradable sector in the long run.

If $\overline{\phi} = 0$, aging does not reduce per capita labor supply, and only the savings effect remains. From (34), we have $\overline{E} = 1/\alpha > 1$. Thus, aging in country 1 induces higher savings in this country and reallocates employment toward the non-tradable sector. In this case,

$$\Omega = \frac{\beta}{\alpha^2}\frac{(\rho+2\overline{\mu})(1-\alpha)}{(\rho+\overline{\mu})\overline{\mu}} > 0.$$

Hence, country 1's relative aging makes it technologically less advanced than country 2, solely due to the employment shift. This finding highlights that even if aging does not reduce per capita labor supply, the resulting sectoral reallocation can still generate a technological disadvantage.

When $\overline{\phi} > 0$, $\overline{E}$ can fall below 1, depending on $\overline{\mu}$ and $\overline{\phi}$. This implies that the second term can be positive or negative. This ambiguity reflects the dual nature of aging: while longer life expectancy increases savings incentives, it simultaneously reduces wage income (the source of savings). However, Appendix A.4 shows that (39) can be rewritten as

$$\Omega = \frac{[\alpha(\alpha-\beta)(\rho+\overline{\mu}) + \beta\overline{\mu}(1-\alpha)]\,\overline{\phi} + \beta\overline{\mu}(\rho+2\overline{\mu})(1-\alpha)}{(\overline{\mu}+\overline{\phi})^2\alpha^2(\rho+\overline{\mu})} > 0.$$

Thus, regardless of parameter values, country 1 necessarily becomes less technologically advanced than country 2. Since the choice of which country ages more rapidly is arbitrary, the following general result holds:

**Proposition 1.** *If one country experiences more rapid aging from a symmetric steady state, its technology becomes less advanced than that of the other country in the long run. This result holds even when aging does not reduce per capita labor supply.*

*Proof.* See Appendix A.4. □

This result implies that the technological disadvantage caused by aging is not merely a consequence of reduced labor supply. Even if elderly labor participation were maintained, the sectoral reallocation induced by demographic change would still inevitably erode a country's relative technological position.

Having established this robust channel through which aging affects relative technology, I next examine its broader consequences for other macroeconomic variables.

**Proposition 2.** *If one country experiences more rapid aging from a symmetric steady state, its relative mass of varieties decreases, its relative wage declines, and its market share contracts in the long run.*

*Proof.* This follows directly from Lemma 1 and Proposition 1. □

A decline in the relative wage also reduces the relative price of non-tradables, consistent with the Balassa–Samuelson effect. Specifically, the real exchange rate $P^*/P = (P_N^*/P_N)^\beta = w(z)^{-\beta}$ deteriorates as relative aging slows technological progress in the tradable sector.

Finally, I analyze how relative aging affects the long-run growth rate of the world economy. Noting that $k'(1) = 0$, differentiation of (33) yields

$$dg_Z = \frac{k(1)\theta(\varepsilon-1)-\eta}{\eta[1-\theta(\varepsilon-1)]}\overline{D}^{-1/2}\left\{\left[\overline{\phi}+(\rho+2\overline{\mu})(1-1/\overline{E})\right]d\mu^* + \frac{(\rho+\overline{\mu})\overline{\mu}}{\overline{E}^2}dE^*\right\}, \tag{40}$$

where $\overline{D} > 0$ is the expression under the square root in (33) evaluated at the symmetric steady state. Given $1-\theta(\varepsilon-1) > 0$ and $k(1)\theta(\varepsilon-1)-\eta > 0$, the effect on the rate of technological progress depends on two channels: the direct demographic effect (via $d\mu^*$); and the indirect effect through expenditure (via $dE^*$). Using (35) and (37), $dE^*$ can be identified. Appendix A.5 shows that

$$dE^* = \frac{1}{\alpha(\overline{\mu}+\overline{\phi})^2}\left\{\frac{\overline{\mu}(1-\alpha)(\rho+\overline{\phi}+2\overline{\mu})}{2\alpha(\rho+\overline{\mu})}(d\mu-d\mu^*)+\overline{\phi}d\mu^*\right\}.$$

Substituting this result into (40) and rearranging,

$$dg_Z = \frac{k(1)\theta(\varepsilon-1)-\eta}{\eta[1-\theta(\varepsilon-1)]}\overline{D}^{-1/2}(1-\alpha)(\rho+\overline{\phi}+2\overline{\mu})\frac{d\mu+d\mu^*}{2}, \tag{41}$$

derivation of which is also given in Appendix A.5.

**Proposition 3.** *If one country experiences more rapid aging from a symmetric steady state, the common long-run growth rate of the two countries declines.*

*Proof.* See Appendix A.5. □

Table 1: Benchmark values of parameters

| Parameter | Description | Value | Target/Reference |
|---|---|---|---|
| $\rho$ | Discount rate | 0.02 | Standard |
| $\varepsilon$ | Elasticity of substitution | 5 | Markup = 1.25 |
| $\chi$ | Productivity in sector $N$ | 1 | Normalization |
| $\theta$ | Elasticity of quality w.r.t technology | 0.19 | $\theta(\varepsilon-1)<1$ is satisfied |
| $\zeta$ | International knowledge spillovers | 0.15 | Davis et al. (2022) |
| $L=L^*$ | Population | 1 | Normalization |
| $\mu=\mu^*=\overline{\mu}$ | Arrival rate of death | 0.0365 | $e^{-45\overline{\mu}}/(1-e^{-45\overline{\mu}})=0.24$ |
| $\phi=\phi^*=\overline{\phi}$ | Decline rate of labor supply | (i) 0 | No-retirement case |
| | | (ii) 0.0154 | $e^{-45\overline{\phi}}=0.5$ |
| $\beta$ | Expenditure share on non-tradables | (i) 0.644 | Employment share = 0.65 |
| | | (ii) 0.647 | Same as above |
| $\eta$ | Productivity in entry | (i) 0.381 | $g_C=0.02$ |
| | | (ii) 0.228 | Same as above |

# 5 Numerical analysis

Although Propositions 1–3 clearly state the effects of country 1's relative aging, they may hold only in the neighborhood of the symmetric steady-state equilibrium. Therefore, in this section, I conduct a numerical analysis to examine the effects in more general cases.

## 5.1 Calibration for benchmark values of parameters

The parameters in this model are $\rho,\varepsilon,\chi,\beta,\zeta,\theta,\eta,\mu,\mu^*,\phi,\phi^*,L$, and $L^*$. I set $\rho=0.02$, following the standard practice in the macroeconomics literature. The elasticity parameter $\varepsilon$ is set to 5, implying a markup of 1.25, and the value of $\chi$ is normalized to 1 for simplicity. The value of $\theta$ is set to 0.19 so that Assumption 1 is satisfied, given the value of $\varepsilon$. Regarding the degree of international knowledge spillovers, $\zeta$, I follow Davis et al. (2022) and set it to 0.15.

If parameter values differ between the two countries even before the demographic shift, it becomes difficult to assess the effect of relative aging in country 1. Therefore, the baseline value of each parameter is assumed to be identical across the two countries. The population is normalized as $L=L^*=1$. In Section 5.3, I relax this symmetry assumption and consider an initially asymmetric case.

I adopt 1995 as the baseline year because the OADR in that year was relatively similar across G7 countries, as shown in Figure 1. Japan, which later experienced rapid aging, had an OADR of 23.89%, not significantly different from that of the other six G7 countries (24.19%). Since individuals are assumed to start working at age zero in this model, I interpret age 20 in the data as age zero in the model. Accordingly, the OADR in the model is given by $e^{-45\overline{\mu}}/(1-e^{-45\overline{\mu}})$. I calibrate $\overline{\mu}$ so that the OADR equals 0.24, which yields $\overline{\mu}\simeq 0.0365$.

Another key parameter is $\overline{\phi}$, which governs the labor participation of the elderly. I consider two scenarios. In the first, I set $\overline{\phi} = 0$, in which per capita labor supply remains unchanged in response to aging. The purpose of this scenario is to isolate the effect of the employment shift toward the non-tradable sector. In the second scenario, I incorporate retirement. In this model, individual labor supply is assumed to decline exponentially with age, initially falling rapidly and then tapering off. Therefore, if $\overline{\phi}$ is chosen so that the labor supply at age 65 is nearly zero, the implied overall labor supply is smaller than in reality. Instead, I set $\overline{\phi}$ such that $e^{-45\overline{\phi}} = 0.50$, which yields $\overline{\phi} \simeq 0.0154$. With this value of $\overline{\phi}$, an individual is expected to have supplied more than 90% of their lifetime expected labor by age 65: $\int_0^{45} e^{-(\overline{\mu}+\overline{\phi})a}da / \int_0^{\infty} e^{-(\overline{\mu}+\overline{\phi})a}da \equiv 1 - e^{-45(\overline{\mu}+\overline{\phi})} \simeq 0.903$. Therefore, setting $e^{-45\overline{\phi}} = 0.50$ is a reasonable approximation.

The remaining parameters are $\beta$ and $\eta$. The parameter $\beta$ represents the expenditure share of non-tradable goods and is closely related to the employment share in that sector. The parameter $\eta$ denotes the marginal productivity of labor in new product development. While variety expansion itself does not affect the long-run growth rate in this model, this parameter influences the rate of long-run technological progress, as shown in (33). Consequently, it affects the long-run per capita growth rate. I therefore use the expenditure share of non-tradables and the per capita growth rate as targets to calibrate $\beta$ and $\eta$. According to EU KLEMS data, Japan's employment share of the non-tradable sector in 1995 was 59.9%, which is relatively close to the 61.6% average across 19 EU countries. In contrast, the share was higher in the UK and the US. Taking an intermediate value, I set the target share at 65%. Finally, I set the target value of the per capita growth rate at 2%. $\beta$ and $\eta$ are determined such that $\beta\overline{E} = 0.65 \times \overline{\mu}/(\overline{\mu} + \overline{\phi})$ and $\overline{g}_C = 0.02$.

Table 1 summarizes the calibrated parameter values. Since I consider two scenarios for $\overline{\phi}$, two corresponding sets of values are computed for $\beta$ and $\eta$. In this model, the real interest rate is given by $r - \dot{P}/P$. As the real interest rate is not targeted in the calibration, it is necessary to check whether its value lies within a plausible range. In case (i), it is 0.0210, and in case (ii), it is 0.0267. Both values are close to the real interest rates observed in practice and widely used in the macroeconomic literature. In both calibration scenarios, $(1 + \zeta)\theta(\varepsilon - 1)/2 \simeq 0.437$, which is greater than $\eta$. Thus, Assumption 2 is satisfied. Finally, $\zeta < z < 1/\zeta$ is satisfied in all cases.

Before proceeding with the analysis of relative aging, one important caveat should be noted. The benchmark value $\overline{\mu}$ is calibrated to match the OADR. With $\overline{\mu} \simeq 0.0365$, however, the implied life expectancy at age 20 in the model deviates from the actual figure. Specifically, the model implies a life expectancy of only $1/\overline{\mu} \simeq 27.4$, which is much shorter than the observed life expectancy at age 20. The OADR is given by $e^{-45\overline{\mu}}/(1 - e^{-45\overline{\mu}})$ and life expectancy is given by $1/\overline{\mu}$. Thus, if we calibrate the model to match the actual OADR, we cannot simultaneously match the actual life expectancy, and vice versa. Therefore, in Section 6, I conduct a comparative statics exercise in which $\overline{\mu}$ is calibrated so that $1/\overline{\mu}$ directly matches the actual life expectancy at age 20.

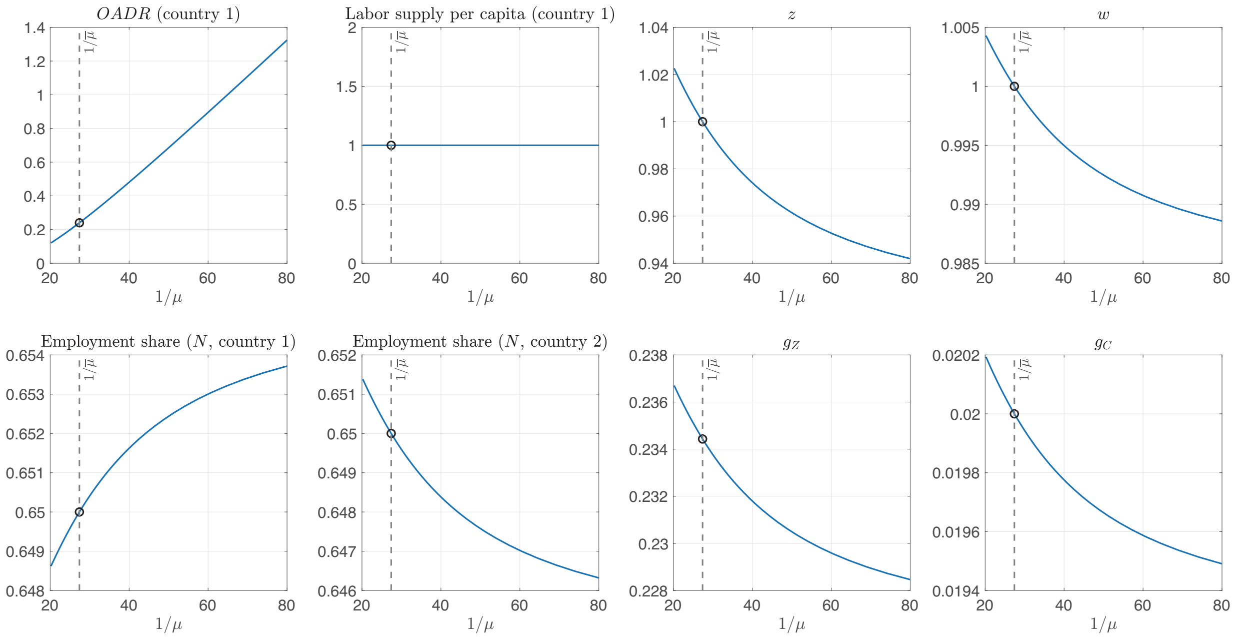



Figure 4: Comparative statics (Case of $\overline{\phi} = 0$ : model with no retirement)

### 5.2 Effects of country 1's aging

I now examine the long-run effects of a change in $\mu$. To show the results graphically, I hold $\mu^*$ and all other parameters unchanged. Figures 4 and 5 show the results under calibration scenarios (i) and (ii), respectively. In all panels of these figures, the horizontal axis represents $1/\mu$, which corresponds to life expectancy at age 20 in country 1. A higher value of $1/\mu$ therefore indicates a greater degree of population aging in country 1. The vertical dashed line in each panel marks the baseline value of $\mu$.

I begin with Figure 4. The first panel shows that the OADR in country 1 rises monotonically and at an increasing rate as aging progresses. The second panel confirms that, under the assumption $\overline{\phi} = 0$, per capita labor supply remains unchanged despite rising longevity. The third panel presents changes in country 1's relative technology, a key variable of interest. This panel indicates that Proposition 1 holds not only near the symmetric steady state but globally as well. In other words, a country that experiences rapid aging tends to lose its technological leadership in the global economy. As shown in the first panel, when $1/\mu = 60$, the OADR slightly exceeds 80%, which corresponds to the projection for Japan in 2050. At this benchmark, the third panel shows that country 1's relative technology has declined by about 5%. In this case, the loss stems solely from the employment shift to the non-tradable sector.

Since the comparative statics for $z$ hold globally, the conclusions of Proposition 2 also apply globally. For example, the fourth panel shows that the relative decline in technology anchors a relative decline in wages in the aging country. At the 2050 benchmark ($1/\mu = 60$), this corresponds to about a 1% decline. The fifth and sixth panels illustrate the sectoral shift: the share of employment in the non-tradable sector rises in country 1, while it falls in country 2.

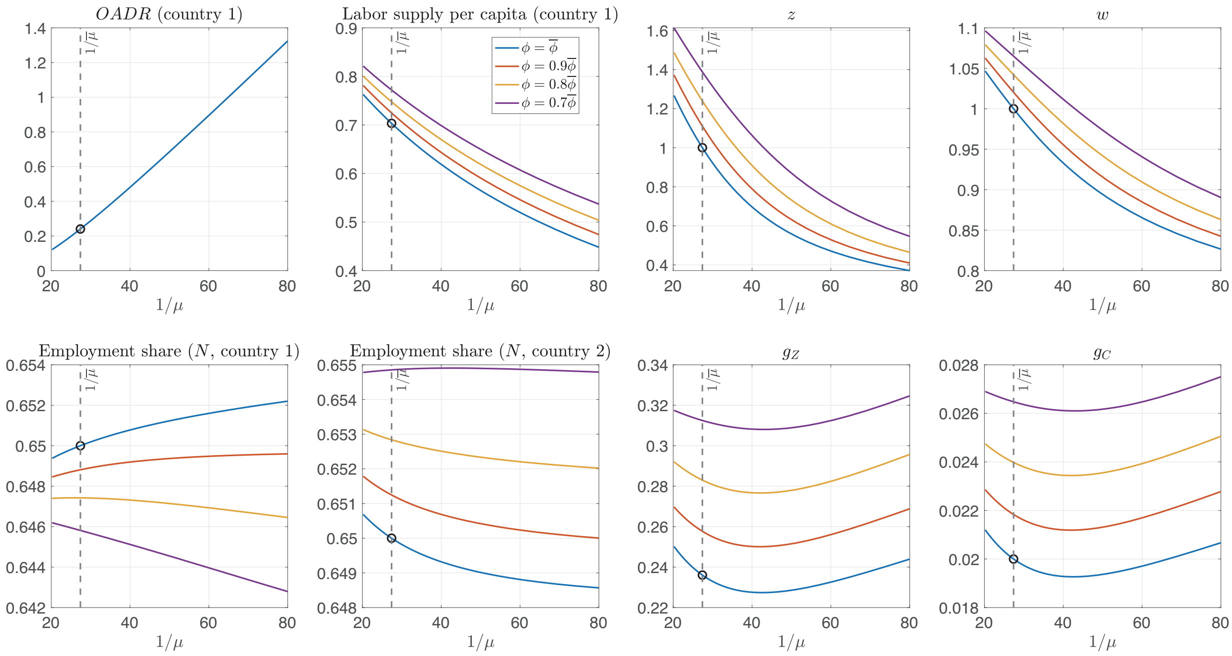



Figure 5: Comparative statics (Case of $\overline{\phi} \neq 0$ : model with retirement)

Finally, the seventh and eighth panels illustrate changes in the long-run rate of technological progress and the long-run growth rate of per capita consumption, respectively. These panels confirm that the result stated in Proposition 3 holds more generally.

Next, I turn to Figure 5, which presents the comparative statics results for case (ii), where mortality risk affects per capita labor supply. The first panel is exactly the same as that in Figure 4, showing how the OADR of country 1 increases. The second panel demonstrates that, in this case, rising longevity decreases the per capita labor supply. From the second panel onward, the blue curve shows the outcomes under the baseline value $\overline{\phi}$. In addition, to assess the mitigation effects of higher old-age labor participation, I consider three cases in which $\phi$ takes smaller values than the benchmark, namely $0.9\overline{\phi} \simeq 0.0139$, $0.8\overline{\phi} \simeq 0.0123$, and $0.7\overline{\phi} \simeq 0.0108$. The value of $\phi^*$ is fixed at $\overline{\phi}$, because if it were to also change, it would induce an additional effect of an increase in per capita labor supply in country 2.

As established in Section 4, the decline in per capita labor supply itself contributes to the decline in country 1's relative technology, $z$. The third panel shows the result. Compared with Figure 4, the magnitude of decline is larger. At $1/\mu = 60$, country 1's relative technology falls to about half the level of country 2, a far more severe deterioration than the first scenario. Accordingly, the fourth panel shows a sharper wage decline. Importantly, even when elderly labor participation is higher (lower $\phi$), the decline is not mitigated in any substantial way. These findings carry clear policy implications. Even if higher elderly labor participation is promoted, the adverse effects of aging on relative technology and wages are only marginally mitigated. This suggests that policies aimed solely at extending working lives may not be sufficient to offset the technological disadvantages caused by demographic change.

Table 2: Recalibration

| Parameter | Value | Target/Reference |
|---|---|---|
| $L$ | 1 | Normalization |
| $L^*$ | 2.363 | Population ratio of the U.S. to Japan (1990–2020 average) |
| $\phi = \phi^* = \overline{\phi}$ | 0.0154 | $e^{-45\overline{\phi}} = 0.5$ |
| $\beta$ | 0.647 | Employment share $= 0.65$ |
| $\eta$ | 0.284 | $g_C = 0.02$ |

Note: For $\rho$, $\varepsilon$, $\chi$, $\theta$, $\zeta$, and $\overline{\mu}$, the same values as in the benchmark calibration are applied.

The fifth and sixth panels again show the sectoral shift of employment. When $\phi = \overline{\phi}$, these changes align with the changes in the case of $\phi = 0$. This implies that even under this scenario, the employment shift to the non-tradable sector continues to contribute to technological deterioration in country 1. However, as $\phi$ becomes smaller, the pattern diverges. In particular, at $\phi = 0.7\overline{\phi}$, the effect is reversed: the share decreases in country 1, while it increases in country 2. This reversal reflects lower savings incentives in country 1, as individuals expect to work at older ages. If both $\phi$ and $\phi^*$ fall equally, this effect cancels out, since savings behavior adjusts symmetrically across countries.

Finally, the seventh and eighth panels reveal a non-monotonic pattern of the long-run rate of technological progress and the long-run growth rate of per capita consumption. They decline near the baseline, which is consistent with Proposition 3, but eventually turn upward, producing a U-shaped response. This arises because $g_Z$ depends on $k(z)$, the ratio of public knowledge to the level of technology: $K/Z(= K^*/Z^*)$. As shown in Section 3.1, $k(z)$ has a U-shaped profile with $k'(1) = 0$. Therefore, close to the symmetric steady state, the effect of $k(z)$ does not appear, but as the equilibrium moves further away, it dominates the effect on $g_Z$. The non-monotonic responses observed in the seventh and eighth panels also connect to the empirical literature. Empirical studies examining the link between aging and growth have reported mixed results, ranging from negative to positive, and in some cases non-monotonic effects. The U-shaped patterns generated by my model are partly consistent with these diverse findings.

### 5.3 Quantifying the technology gap

To provide a more detailed quantitative assessment, I recalibrate the model by identifying country 1 with Japan, where aging has advanced the most, and country 2 with the U.S., its largest trading partner. Table 2 reports the recalibration results. Because the two countries' population sizes differ substantially, I allow $L \neq L^*$ and set their values so that the ratio matches the actual U.S.–Japan population ratio. Although absolute population size is irrelevant for the key variables in this fully endogenous growth model, the ratio matters because it determines the relative labor resources allocated to R&D. I apply case (ii) for $\phi$ and $\phi^*$, since retirement exists in the actual economy. Because $L \neq L^*$, the calibrated value of $\eta$ differs from that in the benchmark case (ii).

In the previous section, I analyzed the effects of a change in $\mu$ alone to illustrate the results

Table 3: Technology gap

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\mu$ | 0.0397 | 0.0366 | 0.0334 | 0.0305 | 0.0277 | 0.0246 | 0.0231 |
| $\mu^*$ | 0.0391 | 0.0386 | 0.0393 | 0.0392 | 0.0383 | 0.0365 | 0.0340 |
| $OADR$ | 0.201 | 0.239 | 0.287 | 0.339 | 0.404 | 0.494 | 0.548 |
| $OADR^*$ | 0.208 | 0.214 | 0.205 | 0.207 | 0.217 | 0.240 | 0.276 |
| $z$ | 0.254 | 0.250 | 0.245 | 0.241 | 0.237 | 0.233 | 0.232 |
| $w$ | 0.787 | 0.767 | 0.763 | 0.761 | 0.758 | 0.756 | 0.755 |
| Employment share ($N$, 1) | 0.6499 | 0.6502 | 0.6505 | 0.6508 | 0.6510 | 0.6513 | 0.6514 |
| Employment share ($N$, 2) | 0.6500 | 0.6499 | 0.6497 | 0.6496 | 0.6496 | 0.6496 | 0.6496 |
| Per capita growth rate ($\times 10^{-2}$) | 2.043 | 2.041 | 2.065 | 2.072 | 2.068 | 2.052 | 2.017 |

of comparative statics graphically. Here, by contrast, both $\mu$ and $\mu^*$ are allowed to evolve in order to track the actual OADR paths of Japan and the U.S. Table 3 reports the international asymmetries in aging and their implications for the technology gap and other key variables. The first two rows display the paths of $\mu$ and $\mu^*$, respectively. The third and fourth rows then show the implied OADRs, which correspond to the actual OADRs of Japan and the U.S. in 1990, 1995, 2000, 2005, 2010, 2015, and 2020. The subsequent rows report relative technology $z$, relative wages $w$, employment shares in the non-tradable sector in both countries, and the per capita consumption growth rate.

The results clearly underscore the quantitative significance of demographic divergence. Between 1990 and 2020, Japan's OADR rose from 0.201 to 0.548, whereas the U.S. OADR increased only modestly, from 0.208 to 0.276. This divergence translates into a steady decline in Japan's relative technology level $z$, from 0.254 to 0.232, representing about a 9% fall over three decades. Relative wages $w$ follow this pattern, falling from 0.787 to 0.755, reflecting the downward pressure exerted by the widening technology gap. Taken together, these results suggest that demographic asymmetry alone can generate a sizable and persistent erosion of technological leadership. These findings are consistent with actual data: Japan's total factor productivity growth has slowed markedly, and international comparisons consistently place Japan near the bottom among advanced economies in labor productivity.

Recently, Fernandez-Villaverde et al. (2025) argued that in the context of population aging, GDP per working-age person is a more appropriate indicator than GDP per capita. This argument is persuasive, but it does not diminish the importance of the findings of this study. The reason is that the growth rate of GDP per working-age person can be decomposed into the growth rate of GDP per capita minus the growth rate of the working-age population share. Indeed, when the working-age population share declines, the working-age–based indicator can be mechanically boosted even if per capita GDP stagnates. However, as this study demonstrates, population aging leads not only to a decline in the working-age share but also to a slowdown in relative technological progress. As a result, labor productivity decreases and per capita GDP growth stagnates. This analysis focuses precisely on this productivity channel, which remains crucial regardless of how GDP growth is normalized.

The employment share of the non-tradable sector evolves as expected: it rises in the more rapidly aging country and declines in the less rapidly aging one. However, the magnitude of the change in the model is smaller than that observed in reality, because intersectoral shifts are driven solely by cross-country differences in aging. As noted in the Introduction, in practice, employment shares of non-tradables have increased even in countries experiencing slower demographic change. Capturing this broader pattern would require treating intersectoral reallocation as a global trend. One approach would be to aggregate tradable and non-tradable goods using a CES function instead of a Cobb–Douglas function. As shown by Ngai and Pissarides (2007) and Acemoglu and Guerriei (2008), when the elasticity of substitution between the two goods is less than one, employment in the non-tradable sector, where technological progress is limited or absent, tends to rise persistently. This is precisely the effect highlighted by Baumol (1967). In this case, a country experiencing relatively rapid aging would see this employment shift amplified, while one with slower aging would see it mitigated. However, when tradable and non-tradable goods are aggregated using a CES function in this way, employment in the tradable sector may decline to a negligible level. For this reason, the present study does not adopt this assumption. Such extensions are left for future research.

Finally, the last row illustrates the long-run trajectory of per capita consumption growth. As Figure 5 shows, even a unilateral change in $\mu$ triggers a non-monotonic response in the growth rate. When both $\mu$ and $\mu^*$ fluctuate simultaneously, the effects become even more complex to track. In 1990, Japan's OADR was slightly below that of the U.S., but by 1995 the positions had reversed as both countries' OADRs increased, and the growth rate declined. This pattern resembles the case analyzed in Section 4. In 2000, Japan's OADR rose further while that of the U.S. declined, leading to a modest recovery in the growth rate. This corresponds to the mechanism, shown in the final panel of Figure 5, in which a sharp decline in $\mu$ results in an increase in the growth rate. A similar pattern is observed in 2005. After 2010, as population aging advanced in the U.S. as well, the effect on the growth rate once again turned negative.

# 6 Discussion

## 6.1 Productivity improvement in the non-tradable sector

I have derived the main results under the assumption that there is no technological progress in the non-tradable sector. Here, I relax this assumption and consider the case in which the marginal productivity of labor in this sector, $\chi$, now improves over time. At the same time, it is natural to assume that $P_T/P_N$ declines over time, reflecting the observed decline in the relative price of tradable goods. Accordingly, I assume that $\chi_t$ increases due to inter-sectoral knowledge spillovers: $\dot{\chi}_t/\chi_t = \xi\theta g_Z$, where $0 < \xi < 1$. In the steady state, $P_T/P_N$ continues to decline at a rate of $(1-\xi)\theta g_Z$. Since $\chi$ does not appear in Equations (27), (28), or (32), the main results in Section 4 remain intact. The only difference is in the long-run growth rate of consumption,

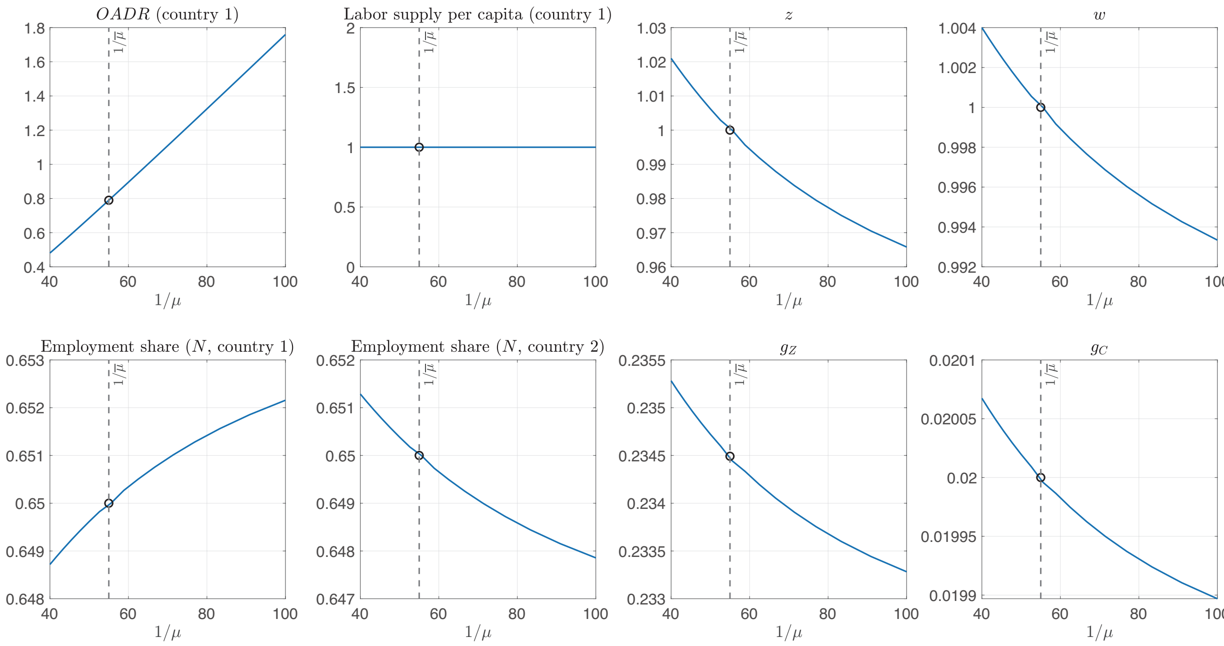



Figure 6: Comparative statics under alternative calibration scenario (Case of $\overline{\phi} = 0$)

which is now given by $g_C = [\beta\xi + (1-\beta)]\theta g_Z$.

There is a growing need to increase labor productivity in the non-tradable goods sector (e.g., health care services). In this model, this corresponds to an increase in the parameter $\chi$. However, an increase in $\chi$ merely reduces the price of the non-tradable good, without reducing labor input in that sector. It is important to note, however, that this robustness relies on the assumption that the expenditure shares across goods remain constant, which stems from the Cobb–Douglas aggregation for consumption. As discussed in Section 5.3, the expenditure share would depend on relative prices and continue to change if we instead assumed a CES aggregator for tradables and non-tradables, which is beyond the scope of this study.

### 6.2 Numerical analysis under alternative calibration scenarios

In this study, the benchmark value $\overline{\mu}$ is determined by targeting the OADR. However, for this value, the resulting life expectancy at age 20 in the model deviates from the actual figure. In this section, I consider an alternative calibration scenario. Specifically, I set $\overline{\mu}$ so that the life expectancy at age 20 is equal to 55 years. This results in $\overline{\mu} \simeq 0.0182$. Accordingly, $\eta \simeq 0.376$ in case (i), while $\eta \simeq 0.206$ in case (ii).

Figures 6 and 7 present the comparative statics results. The results confirm that the direction of change remains the same as in the original calibration scenario.

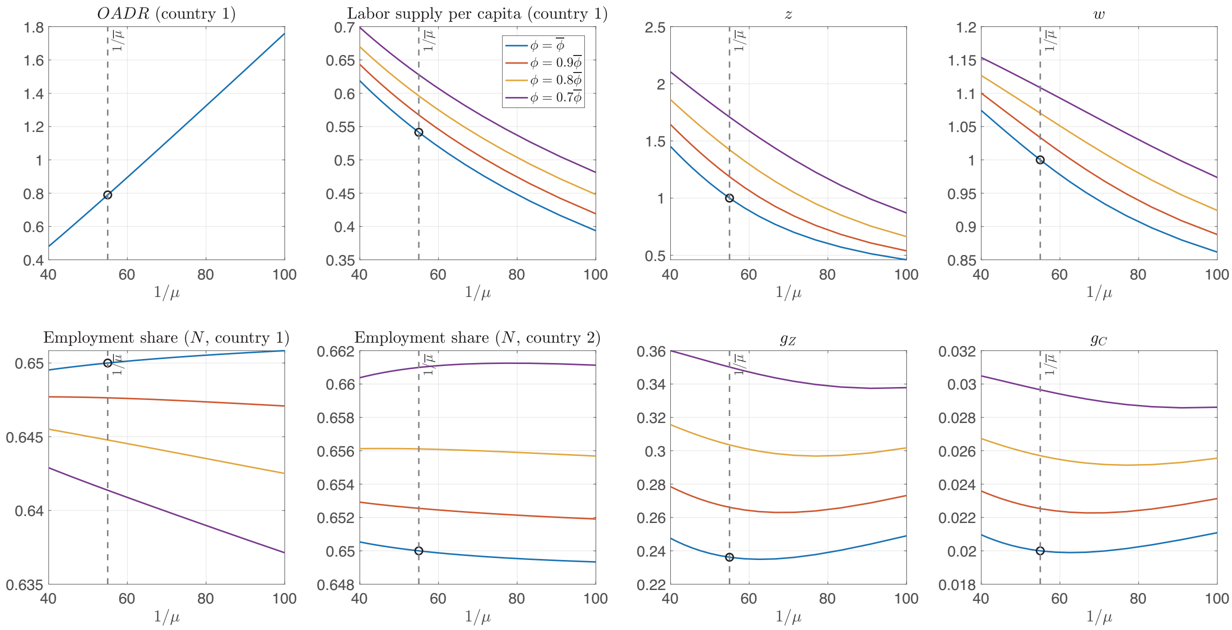



Figure 7: Comparative statics under alternative calibration scenario (Case of $\overline{\phi} \neq 0$)

### 6.3 Model with population growth

This section introduces population growth (or decline) into the model. Let $\lambda$ and $\lambda^*$ denote the population growth rate of countries 1 and 2, respectively. Hence, $\lambda + \mu$ and $\lambda^* + \mu^*$ correspond to the birth rates.

In the real world, there are differences not only in the population size but also in its growth rate across countries. However, incorporating both differences into the model creates a technical problem. If the rate of population growth internationally differs permanently, the larger country alone determines the key variables in the long-run equilibrium. In other words, in the long run, the two-country model behaves like one closed country and one small open economy, lacking international interdependence. To avoid this problem, I assume equal population growth rates: $\lambda = \lambda^*$. Since $L_t = L_0 e^{\lambda t}$ and $L_t^* = L_0^* e^{\lambda t}$, $L_t / L_t^* = L_0 / L_0^*$. Henceforth, I omit the subscript “0.”

With this modification, (27), (28), and (32) are replaced by the following equations, respectively:

$$\frac{\lambda + \mu^*}{\lambda + \mu^* + \phi^*} = \beta E^* + \frac{\varepsilon - 1}{\varepsilon}(1-\beta)(1-S(z))\left(E\frac{L}{L^*} + E^*\right)\left[1 + \gamma(z) + \frac{\lambda k(z)\gamma(z)}{\eta g_Z}\right], \tag{27'}$$

$$\frac{\lambda + \mu}{\lambda + \mu + \phi} - \frac{\beta E}{w(z)} = \frac{L^*}{L} m(z)\left(\frac{\lambda + \mu^*}{\lambda + \mu^* + \phi^*} - \beta E^*\right), \tag{28'}$$

$$-\phi + \sqrt{(\rho - \lambda - \phi)^2 + 4(\rho + \mu)\left(\lambda + \mu + \phi - w(z)\frac{\lambda + \mu}{E}\right)}$$
$$= -\phi^* + \sqrt{(\rho - \lambda - \phi^*)^2 + 4(\rho + \mu^*)\left(\lambda + \mu^* + \phi^* - \frac{\lambda + \mu^*}{E^*}\right)}. \tag{32'}$$

Derivations of these equations are given in Appendix A.6. Notice that if $\lambda = 0$, (27')–(32') becomes exactly the same as (27)–(32), respectively. In (27'), the rate of technological progress $g_Z$ is included, which is given by

$$g_Z = \frac{k(z)\theta(\varepsilon-1)-\eta}{\eta[1-\theta(\varepsilon-1)]}\frac{1}{2}\left[\rho+\lambda-\phi^*+\sqrt{(\rho-\lambda-\phi^*)^2+4(\rho+\mu^*)\left(\lambda+\mu^*+\phi^*-\frac{\lambda+\mu^*}{E^*}\right)}\right]. \tag{42}$$

It is easy to characterize the symmetric steady state by imposing $\mu = \mu^* = \overline{\mu}$, $\phi = \phi^* = \overline{\phi}$, and $L = L^*$. By doing so, $z = 1$, $w = 1$, $m = 1$, and $S = 1/2$. The expenditure is

$$E = E^* = \tilde{E} \equiv \frac{1}{\alpha + \frac{\varepsilon-1}{\varepsilon}\frac{\lambda(1-\beta)k(1)\gamma(1)}{\eta g_Z}}\frac{\lambda+\overline{\mu}}{\lambda+\overline{\mu}+\overline{\phi}}. \tag{43}$$

Thus, $g_Z$ and $\tilde{E}$ are jointly determined from (42) and (43).

I examine the robustness of Proposition 1 in Section 4. To focus on the change in mortality, I hold $\lambda$ constant: the birth rate declines at the same magnitude as the mortality rate. Following the same calculation procedure as that in that section, we obtain the following equation:

$$dz = \frac{-\tilde{\Omega}}{m'(1)\left(\frac{\lambda+\overline{\mu}}{\lambda+\overline{\mu}+\overline{\phi}}-\beta\tilde{E}\right)}(d\mu^*-d\mu),$$

where $\tilde{\Omega}$ is

$$\tilde{\Omega} \equiv \frac{\overline{\phi}}{(\lambda+\overline{\mu}+\overline{\phi})^2}+\beta\tilde{E}^2\frac{\overline{\phi}+(\lambda+\rho+2\overline{\mu})(1-1/\tilde{E})}{(\rho+\overline{\mu})(\lambda+\overline{\mu})}.$$

Derivation is given in Appendix A.6. Thus, when the aging of country 1 progresses relatively, that country's technology becomes less advanced than the other country if $\tilde{E} > 1$. If this is the case, Proposition 2 is also robust.

## 7 Conclusion

This study has examined the long-term technological consequences of asymmetric population aging in the global economy. I developed a two-country OLG model with endogenous growth free of scale effects and introduced realistic demographic asymmetries across countries. The analysis shows that when one country ages more rapidly than the other, it inevitably experiences a relative technological decline in the steady state. Two reinforcing mechanisms drive this outcome: a fall in per capita labor supply and a shift of employment toward the non-tradable sector. The latter mechanism persists even if policies that raise elderly labor force participation mitigate the former. As a result, extending working lives alone cannot prevent technological lag in aging economies. The consequences extend beyond national borders: relative wages fall and market shares in tradables shrink in aging countries. Moreover, relative aging induces a non-monotonic response of the global rate of technological progress.

Several directions remain for future research, among which two stand out. First, this study assumed that expenditure shares across goods are age-invariant. Allowing age-dependent preferences, such as rising demand for healthcare services with aging, would likely amplify sectoral shifts beyond what the present model predicts. Second, because the technological disadvantage stems partly from labor moving into the non-tradable sector, it is crucial to assess the potential of automation and mechanization in this sector. Extending the model to include capital as an additional factor of production would allow us to evaluate how far such technologies can mitigate employment shifts and curb relative technological decline.

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# Appendix

## A.1 Households' behavior

**Conditions for utility maximization** The utility maximization problem of an individual born at $s$ is given by

$$
\begin{aligned}
\max_{\left(A_t(s), C_{N,t}(s), x_t(i,s), x_t(i^*,s)\right)_{t\geq 0}} \quad & U_t(s) = \int_s^\infty e^{-(\rho+\mu)(t-s)} \log C_t(s) dt, \\
\text{s.t.} \quad & C_t(s) = \left(C_{N,t}(s)\right)^\beta \left(C_{T,t}(s)\right)^{1-\beta}, \\
& C_{T,t}(s) = \left( \int_0^{M_t} x_t(i,s)^{\frac{\varepsilon-1}{\varepsilon}} di + \int_0^{M_t^*} x_t(i^*,s)^{\frac{\varepsilon-1}{\varepsilon}} di^* \right)^{\frac{\varepsilon}{\varepsilon-1}}, \\
& \frac{dA_t(s)}{dt} = (r_t + \mu) A_t(s) + w_t e^{-\phi(t-s)} \\
& \qquad - P_{N,t} C_{N,t}(s) - \int_0^{M_t} p_t(i) x_t(i,s) di - \int_0^{M_t^*} p_t^*(i^*) x_t(i^*,s) di^*, \\
& A_s(s) = 0, \quad \lim_{t\to\infty} e^{-\int_s^t (r_\tau+\mu)d\tau} A_t(s) \geq 0.
\end{aligned}
$$

The optimization is divided into three stages. In the first stage, an individual decides the demand for each variety $x_t(i,s)$ and $x_t(i^*,s)$ so as to minimize the expenditure for tradable goods, for the moment taking $C_T(s)$ as given. The results of such expenditure minimization are

$$
x_t(i,s) = \left(\frac{p_t(i)}{P_{T,t}}\right)^{-\varepsilon} C_{T,t}(s), \quad x_t(i^*,s) = \left(\frac{p_t^*(i^*)}{P_{T,t}}\right)^{-\varepsilon} C_{T,t}(s),
$$

where $P_T$ is the price index of tradable goods, given in (3). In the second stage, an individual decides $C_N(s)$ and $C_T(s)$ so as to minimize the expenditure $E(s) = P_N C_N(s) + P_T C_T(s)$, taking $C(s)$ as given. The solution to this problem is given by

$$
C_N(s) = \beta E(s)/P_N, \quad C_T(s) = (1-\beta)E(s)/P_T, \quad P = (P_N/\beta)^\beta (P_T/(1-\beta))^{1-\beta}.
$$

In the final stage, the individual makes intertemporal decisions regarding consumption and savings. The individual chooses the time path of $E_t(s)$ and $A_t(s)$ to solve the following problem:

$$
\begin{aligned}
\max_{(A_t(s), E_t(s))_{t\geq 0}} \quad & U_t(s) = \int_s^\infty e^{-(\rho+\mu)(t-s)} (\log E_t(s) - \log P_t) dt, \\
\text{s.t.} \quad & \frac{dA_t(s)}{dt} = (r_t + \mu) A_t(s) + w_t e^{-\phi(t-s)} - E_t(s), \qquad \text{(A.1)} \\
& A_s(s) = 0, \quad \lim_{t\to\infty} e^{-\int_s^t (r_\tau+\mu)d\tau} A_t(s) \geq 0.
\end{aligned}
$$

As a result of intertemporal optimization, the following Euler equation and the transversality condition hold at the individual level:

$$
\frac{dE_t(s)/dt}{E_t(s)} = r_t - \rho, \tag{A.2}
$$

$$
\lim_{t\to\infty} e^{-\int_s^t (r_\tau+\mu)d\tau} A_t(s) = 0. \tag{A.3}
$$

From this equation and the budget constraint, the consumption expenditure at time $t(\geq s)$ of an individual born at time $s$ is obtained as follows:

$$E_t(s) = (\rho + \mu)\left[A_t(s) + e^{-\phi(t-s)} \int_t^\infty e^{-\int_t^\tau (r_u+\mu+\phi)du} w_\tau d\tau\right],$$

where the second term within the bracket represents the individual's human wealth at time $t$. The term $\rho + \mu$ captures the individual's marginal propensity to consume out of wealth. As life expectancy increases due to a decrease in $\mu$, this propensity decreases and the individual saves more.

**Derivation of (1)** We can derive this equation by taking the same calculation procedure as Blanchard (1985). The average wealth is defined as $A_t \equiv \mu \int_{-\infty}^t e^{-\mu(t-s)} A_t(s)ds$. Differentiating $A_t$ with respect to $t$,

$$\dot{A}_t = -\mu A_t + \mu \int_{-\infty}^t e^{-\mu(t-s)} \frac{dA_t(s)}{dt} ds + \underbrace{\mu A_t(t)}_{=0}.$$

The last term on the right-hand side is zero because each individual is born without bequests. Substituting the individual's budget constraint (A.1) into the second term on the right-hand side yields

$$\begin{aligned}
\dot{A}_t &= -\mu A_t + \mu \int_{-\infty}^t e^{-\mu(t-s)} \left[(r_t + \mu)A_t(s) + w_t e^{-\phi(t-s)} - E_t(s)\right] ds \\
&= -\mu A_t + (r_t + \mu)A_t + \mu \int_{-\infty}^t e^{-(\mu+\phi)(t-s)} w_t ds - E_t \\
&= r_t A_t + w_t \frac{\mu}{\mu + \phi} - E_t.
\end{aligned}$$

This equation is (1).

**Derivation of (2)** From (A.1)–(A.3), an individual's expenditure at $t$ is given by

$$E_t(s) = (\rho + \mu)\left(A_t(s) + e^{-\phi(t-s)} H_t\right), \tag{A.4}$$

where

$$H_t \equiv \int_t^\infty e^{-\int_t^\tau (r_u+\mu+\phi)du} w_\tau d\tau.$$

The average expenditure is defined as $E_t \equiv \mu \int_{-\infty}^t e^{-\mu(t-s)} E_t(s)ds$. Substituting (A.4) into this definition,

$$\begin{aligned}
E_t &= (\rho + \mu)A_t + \mu(\rho + \mu)H_t \int_{-\infty}^t e^{-(\mu+\phi)(t-s)} ds \\
&= (\rho + \mu)\left(A_t + \frac{\mu}{\mu + \phi} H_t\right). \qquad (A.5)
\end{aligned}$$

Differentiating $E_t$ with respect to $t$,

$$\begin{aligned}
\dot{E}_t &= (\rho+\mu)\left(\dot{A}_t + \frac{\mu}{\mu+\phi}\dot{H}_t\right)\\
&= (\rho+\mu)\left\{ r_t A_t + w_t \frac{\mu}{\mu+\phi} - E_t + \frac{\mu}{\mu+\phi}\left[ (r_t+\mu+\phi)\underbrace{\left(\frac{E_t}{\rho+\mu} - A_t\right)\frac{\mu+\phi}{\mu}}_{=H_t} - w_t \right]\right\}\\
&= (r_t - \rho + \phi)E_t - (\rho+\mu)(\mu+\phi)A_t.
\end{aligned}$$

This equation is (2). Since $A_t(t) = 0$,

$$E_t(t) = H_t.$$

Using this fact and (A.5), $A_t$ is expressed as

$$(\rho+\mu)A_t = E_t - \frac{\mu}{\mu+\phi}E_t(t)$$

Therefore, $\dot{E}_t/E_t$ can be also expressed as

$$\begin{aligned}
\frac{\dot{E}_t}{E_t} &= r_t - \rho + \phi - (\mu+\phi) + \mu\frac{E_t(t)}{E_t}\\
&= r_t - \rho - \mu\frac{E_t - E_t(t)}{E_t}.
\end{aligned}$$

## A.2 Sufficiency of firms' optimization

Equations (6) and (7) are respectively the first-order conditions for $p(i)$ and $L^R(i)$. Substituting these equations into the Hamiltonian yields

$$\tilde{H}(Z(i)) \equiv \max_{p(i), L^R(i)} H(i) = \frac{1}{\varepsilon-1}Z(i)^{\theta(\varepsilon-1)}{P_T}^{\varepsilon-1}(1-\beta)(LE + L^*E^*).$$

This is the maximized Hamiltonian. Because of Assumption 1, $\tilde{H}(Z(i))$ is strictly concave with respect to the firm's state variable $Z(i)$. Thus, we can apply Arrow's sufficiency theorem to show that the paths of $p(i)$, $L^R(i)$, and $Z(i)$ satisfying (6)–(8) and the transversality condition: $\lim_{t\to\infty} e^{-\int_0^t r_\tau d\tau} q_t(i) Z_t(i) = 0$ are the solutions to the maximization problem.

## A.3 Derivations of key equations in Section 3

**Derivations of (19) and (20)** The gross profit and the labor demand for R&D in country 1 are respectively expressed as

$$\Pi = \frac{1}{\varepsilon}pY = \frac{1}{\varepsilon}\frac{S(1-\beta)(LE + L^*E^*)}{M}, \tag{A.6}$$

$$L^R = \frac{\dot{Z}}{K} = \frac{m+1}{m+\zeta/z}\frac{\dot{Z}}{Z}. \tag{A.7}$$

Substituting these results into $rV = \Pi - wL^R$,

$$\frac{r}{\eta}w = \frac{1}{\varepsilon}\frac{S(1-\beta)(LE+L^*E^*)}{M} - w\frac{m+1}{m+\zeta/z}\frac{\dot{Z}}{Z}. \tag{A.8}$$

Arranging (A.8) leads to (19).

The gross profit and the labor demand for R&D in country 2 are respectively expressed as

$$\Pi^* = \frac{1}{\varepsilon}p^*Y^* = \frac{1}{\varepsilon}\frac{(1-S)(1-\beta)(LE+L^*E^*)}{M^*}, \tag{A.9}$$

$$L^{R*} = \dot{Z}^*/K^* = \frac{m+1}{1+\zeta mz}\frac{\dot{Z}^*}{Z^*}. \tag{A.10}$$

Substituting these results into $rV^* = \Pi^* - L^{R*}$ (recall that $w^* = 1$),

$$\frac{r}{\eta} = \frac{1}{\varepsilon}\frac{(1-S)(1-\beta)(LE+L^*E^*)}{M^*} - \frac{m+1}{1+\zeta mz}\frac{\dot{Z}^*}{Z^*}. \tag{A.11}$$

Arranging (A.11) leads to (20).

**Derivations of (26) and (27)** The demand for R&D activities $L^R$ is given by (A.7) and $Y$ is given by

$$Y = \frac{1}{p}\frac{S(1-\beta)(LE+L^*E^*)}{M} = \frac{\varepsilon-1}{\varepsilon w}\frac{S(1-\beta)(LE+L^*E^*)}{M}.$$

In the steady state, $\dot{M} = 0$. Using these results, we can rewrite (12) as

$$\frac{\mu}{\mu+\phi}L = \frac{\beta LE}{w(z)} + \frac{\varepsilon-1}{\varepsilon w(z)}S(z)(1-\beta)(LE+L^*E^*) + \frac{Mg_Z}{k(z)}. \tag{A.12}$$

Using the fact that $S(z) = (1-S(z))w(z)m(z)$ and $M = m(z)M^*$, we can further arrange (A.12) as

$$\frac{\mu}{\mu+\phi}L = \frac{\beta LE}{w(z)} + m(z)\left[\frac{\varepsilon-1}{\varepsilon}(1-S(z))(1-\beta)(LE+L^*E^*) + \frac{M^*g_Z}{k(z)}\right].$$

Substituting (25) into (A.12) yields (26).

We can obtain (27) in a similar way. In country 2, the equation corresponding to (A.12) is

$$\frac{\mu^*}{\mu^*+\phi^*}L^* = \beta L^*E^* + \frac{\varepsilon-1}{\varepsilon}(1-S(z))(1-\beta)(LE+L^*E^*) + \frac{M^*g_Z}{k(z)}. \tag{A.13}$$

Substituting (25) into this equation yields (27).

## A.4 Proof of Proposition 1

Showing $\Omega > 0$ completes the proof of this proposition. Substituting $\overline{E} = \overline{\mu}/[(\overline{\mu}+\overline{\phi})\alpha]$ into the definition of $\Omega$ in the main text yields

$$\begin{aligned}\Omega &= \frac{\overline{\phi}}{(\overline{\mu}+\overline{\phi})^2} + \beta\frac{\overline{\mu}^2}{(\overline{\mu}+\overline{\phi})^2\alpha^2}\frac{\overline{\phi} + (\rho+2\overline{\mu})\left(1-\frac{\overline{\mu}+\overline{\phi}}{\overline{\mu}}\alpha\right)}{(\rho+\overline{\mu})\overline{\mu}} \\ &= \frac{\alpha^2(\rho+\overline{\mu})\overline{\phi} + \beta\overline{\mu}\left[\overline{\phi} + (\rho+2\overline{\mu})\left(1-\frac{\overline{\mu}+\overline{\phi}}{\overline{\mu}}\alpha\right)\right]}{(\overline{\mu}+\overline{\phi})^2\alpha^2(\rho+\overline{\mu})}.\end{aligned}$$

The numerator is arranged as

$$\begin{aligned}
&\alpha^2(\rho+\overline{\mu})\overline{\phi}+\beta\overline{\mu}\left[\overline{\phi}+(\rho+2\overline{\mu})\left(1-\alpha-\alpha\overline{\phi}/\overline{\mu}\right)\right] \\
&=\left[\alpha^2(\rho+\overline{\mu})+\beta\overline{\mu}-\alpha\beta(\rho+2\overline{\mu})\right]\overline{\phi}+\beta\overline{\mu}(\rho+2\overline{\mu})(1-\alpha) \\
&=\left[\alpha^2(\rho+\overline{\mu})+\beta\overline{\mu}-\alpha\beta(\rho+\overline{\mu}+\overline{\mu})\right]\overline{\phi}+\beta\overline{\mu}(\rho+2\overline{\mu})(1-\alpha) \\
&=\left[\alpha(\alpha-\beta)(\rho+\overline{\mu})+\beta\overline{\mu}(1-\alpha)\right]\overline{\phi}+\beta\overline{\mu}(\rho+2\overline{\mu})(1-\alpha)>0,
\end{aligned}$$

which completes the proof.

### A.5 Proof of Proposition 3

To prove this proposition, it suffices to derive (41). To that end, I first derive $dE^*$.

**Derivation of $dE^*$** From the definitions of $\gamma(z)$ and $S(z)$,

$$\begin{aligned}
\gamma'(z)&=\frac{1}{\varepsilon-1}\frac{\eta[1-\theta(\varepsilon-1)]}{(k(z)-\eta)^2}k'(z)=\frac{1}{\varepsilon-1}\frac{\eta[1-\theta(\varepsilon-1)]}{(k(z)-\eta)^2}\frac{(1-\zeta^2)\zeta(1-1/z^2)}{(2-\zeta/z-\zeta z)^2}, \\
\Rightarrow\gamma'(1)&=0, \\
S'(z)&=\frac{-z^{-\frac{\theta(\varepsilon-1)}{\varepsilon}}}{\left[1+\frac{1-\zeta z}{1-\zeta/z}z^{-\frac{\theta(\varepsilon-1)}{\varepsilon}}\right]^2}\left[-\frac{\zeta(1-\zeta/z)+(\zeta/z^2)(1-\zeta z)}{(1-\zeta/z)^2}-\frac{1-\zeta z}{1-\zeta/z}\frac{\theta(\varepsilon-1)}{\varepsilon}\frac{1}{z}\right], \\
\Rightarrow S'(1)&=\frac{1}{4}\left(\frac{2\zeta}{1-\zeta}+\theta\frac{\varepsilon-1}{\varepsilon}\right)>0.
\end{aligned}$$

Then, (35) is rewritten as

$$\frac{\overline{\phi}}{(\overline{\mu}+\overline{\phi})^2}d\mu^*=\frac{\alpha-\beta}{2}dE+\frac{\alpha+\beta}{2}dE^*-\frac{\alpha-\beta}{2}\left(\frac{2\zeta}{1-\zeta}+\theta\frac{\varepsilon-1}{\varepsilon}\right)\overline{E}dz.$$

Rearranging gives

$$dE=-\frac{\alpha+\beta}{\alpha-\beta}dE^*+\left(\frac{2\zeta}{1-\zeta}+\theta\frac{\varepsilon-1}{\varepsilon}\right)\overline{E}dz+\frac{2}{\alpha-\beta}\frac{\overline{\phi}}{(\overline{\mu}+\overline{\phi})^2}d\mu^*. \tag{A.14}$$

Substituting (A.14) into (37) to eliminate $dE$ and noting that $w'(1)=\theta(\varepsilon-1)/\varepsilon$, we can obtain

$$\frac{2\alpha}{\alpha-\beta}dE^*=\overline{E}\frac{2\zeta}{1-\zeta}dz+\overline{E}^2\frac{\overline{\phi}+(\rho+2\overline{\mu})(1-1/\overline{E})}{(\rho+\overline{\mu})\overline{\mu}}(d\mu-d\mu^*)+\frac{2}{\alpha-\beta}\frac{\overline{\phi}}{(\overline{\mu}+\overline{\phi})^2}d\mu^*. \tag{A.15}$$

We can rewrite $dz$ in (38) as

$$\begin{aligned}
dz&=\frac{\Omega}{m'(1)\left(\frac{\overline{\mu}}{\overline{\mu}+\overline{\phi}}-\beta\overline{E}\right)}(d\mu-d\mu^*) \\
&=\frac{\Omega}{\frac{2\zeta}{1-\zeta}\frac{\overline{\mu}}{\overline{\mu}+\overline{\phi}}\frac{\alpha-\beta}{\alpha}}(d\mu-d\mu^*) \\
&=\frac{1}{\frac{2\zeta}{1-\zeta}\frac{\overline{\mu}}{\overline{\mu}+\overline{\phi}}\frac{\alpha-\beta}{\alpha}}\left[\frac{\overline{\phi}}{(\overline{\mu}+\overline{\phi})^2}+\beta\overline{E}^2\frac{\overline{\phi}+(\rho+2\overline{\mu})(1-1/\overline{E})}{(\rho+\overline{\mu})\overline{\mu}}\right](d\mu-d\mu^*)
\end{aligned}$$

Substituting the resulting $dz$ into (A.15),

$$2\alpha dE^* = \underbrace{\left[\frac{\overline{\phi}}{(\overline{\mu}+\overline{\phi})^2} + \alpha\overline{E}^2\frac{\overline{\phi}+(\rho+2\overline{\mu})(1-1/\overline{E})}{(\rho+\overline{\mu})\overline{\mu}}\right]}_{\equiv\Gamma}(d\mu - d\mu^*) + \frac{2\overline{\phi}}{(\overline{\mu}+\overline{\phi})^2}d\mu^*.$$

We can arrange $\Gamma$ as

$$\begin{aligned}
\Gamma &= \frac{\overline{\phi}}{(\overline{\mu}+\overline{\phi})^2} + \alpha\left[\frac{\overline{\mu}}{\alpha(\overline{\mu}+\overline{\phi})}\right]^2\frac{\overline{\phi}+(\rho+2\overline{\mu})\left(1-\frac{\overline{\mu}+\overline{\phi}}{\overline{\mu}}\alpha\right)}{(\rho+\overline{\mu})\overline{\mu}} \\
&= \frac{\overline{\phi}}{(\overline{\mu}+\overline{\phi})^2} + \frac{1}{(\overline{\mu}+\overline{\phi})^2}\frac{\overline{\mu}}{\alpha}\frac{\overline{\phi}+(\rho+2\overline{\mu})\left(1-\frac{\overline{\mu}+\overline{\phi}}{\overline{\mu}}\alpha\right)}{\rho+\overline{\mu}} \\
&= \frac{1}{(\overline{\mu}+\overline{\phi})^2\alpha(\rho+\overline{\mu})}\left\{\overline{\phi}\alpha(\rho+\overline{\mu}) + \overline{\mu}\overline{\phi} + (\rho+2\overline{\mu})\left[\overline{\mu}(1-\alpha)-\overline{\phi}\alpha\right]\right\} \\
&= \frac{1}{(\overline{\mu}+\overline{\phi})^2\alpha(\rho+\overline{\mu})}\left\{\overline{\phi}\alpha(\rho+\overline{\mu}) + \overline{\mu}\overline{\phi} + (\rho+2\overline{\mu})\overline{\mu}(1-\alpha) - (\rho+\overline{\mu})\overline{\phi}\alpha - \overline{\mu}\overline{\phi}\alpha\right\} \\
&= \frac{1}{(\overline{\mu}+\overline{\phi})^2\alpha(\rho+\overline{\mu})}\left\{\overline{\mu}\overline{\phi} + (\rho+2\overline{\mu})\overline{\mu}(1-\alpha) - \overline{\mu}\alpha\overline{\phi}\right\} \\
&= \frac{\overline{\mu}(1-\alpha)(\overline{\phi}+\rho+2\overline{\mu})}{(\overline{\mu}+\overline{\phi})^2\alpha(\rho+\overline{\mu})} > 0.
\end{aligned}$$

Thus, $dE^*$ is obtained as

$$dE^* = \frac{1}{\alpha(\overline{\mu}+\overline{\phi})^2}\left[\frac{\overline{\mu}(1-\alpha)(\rho+\overline{\phi}+2\overline{\mu})}{2\alpha(\rho+\overline{\mu})}(d\mu - d\mu^*) + \overline{\phi}d\mu^*\right]. \tag{A.16}$$

**Derivation of (41) (Proof of Proposition 3)** Now I prove Proposition 3. Substituting (A.16) and $\overline{E} = \frac{\overline{\mu}}{\alpha(\overline{\mu}+\overline{\phi})}$ into (40),

$$dg_Z = \frac{k(1)\theta(\varepsilon-1)-\eta}{\eta[1-\theta(\varepsilon-1)]}\overline{D}^{-1/2}\Psi,$$

where

$$\Psi \equiv \left[\overline{\phi} + (\rho+2\overline{\mu})\left(1-\alpha-\frac{\overline{\phi}\alpha}{\overline{\mu}}\right)\right]d\mu^* + (\rho+\overline{\mu})\frac{\alpha}{\overline{\mu}}\left[\frac{\overline{\mu}(1-\alpha)(\rho+\overline{\phi}+2\overline{\mu})}{2\alpha(\rho+\overline{\mu})}(d\mu-d\mu^*) + \overline{\phi}d\mu^*\right].$$

We can arrange $\Psi$ as follows:

$$\begin{aligned}
\Psi &= \left[\overline{\phi} + (\rho+2\overline{\mu})\left(1-\alpha-\frac{\overline{\phi}\alpha}{\overline{\mu}}\right)\right]d\mu^* + (\rho+\overline{\mu})\frac{\overline{\phi}\alpha}{\overline{\mu}}d\mu^* + \frac{1}{2}(1-\alpha)(\rho+\overline{\phi}+2\overline{\mu})(d\mu-d\mu^*) \\
&= \left[\overline{\phi} + (\rho+2\overline{\mu})(1-\alpha)\right]d\mu^* - (\rho+2\overline{\mu})\frac{\overline{\phi}\alpha}{\overline{\mu}}d\mu^* + (\rho+\overline{\mu})\frac{\overline{\phi}\alpha}{\overline{\mu}}d\mu^* + \frac{1}{2}(1-\alpha)(\rho+\overline{\phi}+2\overline{\mu})(d\mu-d\mu^*) \\
&= \left[\overline{\phi} + (\rho+2\overline{\mu})(1-\alpha)\right]d\mu^* - \overline{\phi}\alpha d\mu^* + \frac{1}{2}(1-\alpha)(\rho+\overline{\phi}+2\overline{\mu})(d\mu-d\mu^*) \\
&= (1-\alpha)\left(\rho+\overline{\phi}+2\overline{\mu}\right)d\mu^* + \frac{1}{2}(1-\alpha)(\rho+\overline{\phi}+2\overline{\mu})(d\mu-d\mu^*) \\
&= \frac{1}{2}(1-\alpha)(\rho+\overline{\phi}+2\overline{\mu})(d\mu+d\mu^*).
\end{aligned}$$

Substituting this result into the equation of $dg_Z$ yields

$$dg_Z = \frac{k(1)\theta(\varepsilon-1)-\eta}{\eta[1-\theta(\varepsilon-1)]}\overline{D}^{-1/2}\frac{1}{2}(1-\alpha)(\rho+\overline{\phi}+2\overline{\mu})(d\mu+d\mu^*),$$

which is indeed Equation (41) in the main body. Because of $d\mu < d\mu^* \leq 0$, we can verify that $dg_Z < 0$.

### A.6 Details of the model with population change in Section 6.3

**The average wealth and the average expenditure** First, I derive the average wealth and the average expenditure with population change. Since the derivations closely parallel those in Appendix A.3, I therefore only sketch the main steps. In the model with population change, the average wealth and the average expenditure are respectively defined as

$$\begin{aligned}
A_t &\equiv \frac{1}{L_t}\int_{-\infty}^{t}(\lambda+\mu)L_s e^{-\mu(t-s)}A_t(s)ds = (\lambda+\mu)\int_{-\infty}^{t}e^{-(\lambda+\mu)(t-s)}A_t(s)ds,\\
E_t &\equiv \frac{1}{L_t}\int_{-\infty}^{t}(\lambda+\mu)L_s e^{-\mu(t-s)}E_t(s)ds = (\lambda+\mu)\int_{-\infty}^{t}e^{-(\lambda+\mu)(t-s)}E_t(s)ds.
\end{aligned}$$

By taking the same calculation procedure as Appendix A.3, one can obtain

$$\dot{A}_t = (r_t-\lambda)A_t + w_t\frac{\lambda+\mu}{\lambda+\mu+\phi} - E_t, \tag{A.17}$$

$$\dot{E}_t = (r_t-\rho+\phi)E_t - (\rho+\mu)(\lambda+\mu+\phi)A_t. \tag{A.18}$$

I continue to choose labor in country 2 as the numeraire: $w^* = 1$. The average wealth and the average expenditure in country 2 are respectively given by

$$\dot{A}_t^* = (r_t-\lambda)A_t^* + w_t^*\frac{\lambda+\mu^*}{\lambda+\mu^*+\phi^*} - E_t^*, \tag{A.19}$$

$$\dot{E}_t^* = (r_t-\rho+\phi^*)E_t^* - (\rho+\mu^*)(\lambda+\mu^*+\phi^*)A_t^*. \tag{A.20}$$

**Derivation of (27'), (28'), and (32')** Note that Lemma 2 in the main text still holds in this model. This is because (25), which shows this lemma, holds in this model. Therefore, in the model with population change, $\dot{M}/M = \dot{M}^*/M^* = \lambda$ in the long run, as shown in Lemma 2. This implies that the labor demand for entry is not zero even in the long run. Specifically, (26) and (27) in the main text are replaced by the following equations, respectively:

$$\frac{\lambda+\mu}{\lambda+\mu+\phi} = \frac{\beta E}{w(z)} + m(z)\frac{\varepsilon-1}{\varepsilon}(1-\beta)(1+\gamma(z))(1-S(z))(E+E^*L_t^*/L_t) + \frac{\lambda}{\eta}\frac{M_t}{L_t}, \tag{A.21}$$

$$\frac{\lambda+\mu^*}{\lambda+\mu^*+\phi^*} = \beta E^* + \frac{\varepsilon-1}{\varepsilon}(1-\beta)(1+\gamma(z))(1-S(z))(EL_t/L_t^*+E^*) + \frac{\lambda}{\eta}\frac{M_t^*}{L_t^*}. \tag{A.22}$$

In each of the two equations above, the last term on the right-hand side is the labor used for new entry. Using (25) and noting $L_t/L_t^* = L/L^*$, (A.22) is rewritten as

$$\frac{\lambda+\mu^*}{\lambda+\mu^*+\phi^*} = \beta E^* + \frac{\varepsilon-1}{\varepsilon}(1-\beta)(1-S(z))\left(E\frac{L}{L^*}+E^*\right)\left(1+\gamma(z)+\frac{\lambda k(z)\gamma(z)}{\eta g_Z}\right),$$

which is essentially the same as (27') in the main text. Since $m(z) = M_t/M_t^*$, (A.21) and (A.22) imply

$$\frac{\lambda+\mu}{\lambda+\mu+\phi} - \frac{\beta E}{w(z)} = \frac{L^*}{L} m(z) \left( \frac{\lambda+\mu^*}{\lambda+\mu^*+\phi^*} - \beta E^* \right),$$

which is essentially the same as (28') in the main text. From (A.17) and (A.18) with $\dot{A}_t = 0$ and $\dot{E}_t = 0$,

$$0 = r - \lambda + w(z) \frac{\lambda+\mu}{\lambda+\mu+\phi} \frac{1}{A} - \frac{E}{A}, \tag{A.23}$$

$$0 = r - \rho + \phi - (\rho+\mu)(\lambda+\mu+\phi) \frac{A}{E}. \tag{A.24}$$

Similarly, from (A.19) and (A.20) with $\dot{A}_t^* = 0$ and $\dot{E}_t^* = 0$,

$$0 = r - \lambda + \frac{\lambda+\mu^*}{\lambda+\mu^*+\phi^*} \frac{1}{A^*} - \frac{E^*}{A^*}, \tag{A.25}$$

$$0 = r - \rho + \phi^* - (\rho+\mu^*)(\lambda+\mu^*+\phi^*) \frac{A^*}{E^*}, \tag{A.26}$$

Then, using (A.23)–(A.26), we can find that (32) is replaced by

$$\begin{aligned} 2r =& \rho - \phi + \lambda + \sqrt{(\rho-\lambda-\phi)^2 + 4(\rho+\mu)\left(\lambda+\mu+\phi - w(z)\frac{\lambda+\mu}{E}\right)} \\ =& \rho - \phi^* + \lambda + \sqrt{(\rho-\lambda-\phi^*)^2 + 4(\rho+\mu^*)\left(\lambda+\mu^*+\phi^* - \frac{\lambda+\mu^*}{E^*}\right)}. \end{aligned}$$

This equation is (32') in the main text.

**The symmetric steady state** Consider the situation of $\mu = \mu^* = \overline{\mu}$, $\phi = \phi^* = \overline{\phi}$, and $L = L^*$. Then, (32') is simplified to $E/w(z) = E^*$ as in the case of no population growth. Substituting this result into (28') and using the symmetry assumption, we can obtain $m(z) = 1$, which in turn implies

$$z = 1, \quad w(1) = 1, \quad S(1) = 1/2.$$

Since $w(1) = 1$, $E = E^*$. Let $\tilde{E}$ denote the value of expenditure in this symmetric steady state. Equation (27') provides the following relationship between $\tilde{E}$ and $g_Z$:

$$E = E^* = \tilde{E} \equiv \frac{1}{\alpha + \frac{\varepsilon-1}{\varepsilon}\frac{\lambda(1-\beta)k(1)\gamma(1)}{\eta g_Z}} \frac{\lambda+\overline{\mu}}{\lambda+\overline{\mu}+\overline{\phi}},$$

which corresponds to (43) in the main text. Therefore, $\tilde{E}$ and $g_Z$ are determined from (42) and (43).

**The effect of relative aging on the relative technology** Differentiating (28') and (32') with respect to $z$, $E$, $E^*$, $\mu$, and $\mu^*$ and evaluating the differential coefficients at the symmetric steady state, one obtains

$$\left[m'(1)\left(\frac{\lambda+\overline{\mu}}{\lambda+\overline{\mu}+\overline{\phi}}-\beta\tilde{E}\right)-\beta\tilde{E}w'(1)\right]dz=-\beta(dE-dE^*)-\frac{\overline{\phi}}{(\lambda+\overline{\mu}+\overline{\phi})^2}(d\mu^*-d\mu),$$
$$dE-dE^*=\tilde{E}w'(1)dz+\tilde{E}^2\frac{\overline{\phi}+(\lambda+\rho+2\overline{\mu})(1-1/\tilde{E})}{(\rho+\overline{\mu})(\lambda+\overline{\mu})}(d\mu^*-d\mu).$$

From these equations, one can obtain

$$dz=\frac{\tilde{\Omega}}{m'(1)\left(\frac{\lambda+\overline{\mu}}{\lambda+\overline{\mu}+\overline{\phi}}-\beta\tilde{E}\right)}(d\mu-d\mu^*),$$

where $\tilde{\Omega}$ is defined as

$$\tilde{\Omega}\equiv\frac{\overline{\phi}}{(\lambda+\overline{\mu}+\overline{\phi})^2}+\beta\tilde{E}^2\frac{\overline{\phi}+(\lambda+\rho+2\overline{\mu})(1-1/\tilde{E})}{(\rho+\overline{\mu})(\lambda+\overline{\mu})}.$$

Thus, when the aging of country 1 progresses relatively, that country's technology becomes less advanced than the other country if $\tilde{E}>1$.