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Abstract

This paper empirically examines collateral constraints in the Kiyotaki and Moore [1997. *Credit cycles. Journal of Political Economy* 105(2), 211–248] model using land price data from three major prefectures in Japan: Tokyo, Osaka, and Hyogo. After confirming the stationarity of land prices, we estimate their dynamic equations and show that they follow a second-order autoregressive (AR(2)) process, consistent with the presence of binding collateral constraints. We further apply the supremum Wald test and identify structural breaks at the onset of the early 1990s asset price bubble collapse. These results suggest that financial frictions played a critical role in shaping land price dynamics in Japan's regional economies. Overall, our findings demonstrate that the Kiyotaki–Moore framework provides a useful tool for capturing the dynamic behavior of financially constrained economies. By providing new regional evidence, this study contributes to the literature on macroeconomics and financial market imperfections.

Keywords: Collateral Constraints, Financial Frictions, Land Price Dynamics, Kiyotaki-Moore Model, Credit Cycles, Regional Economies.

JEL Classification: G12, E32; E44; R30

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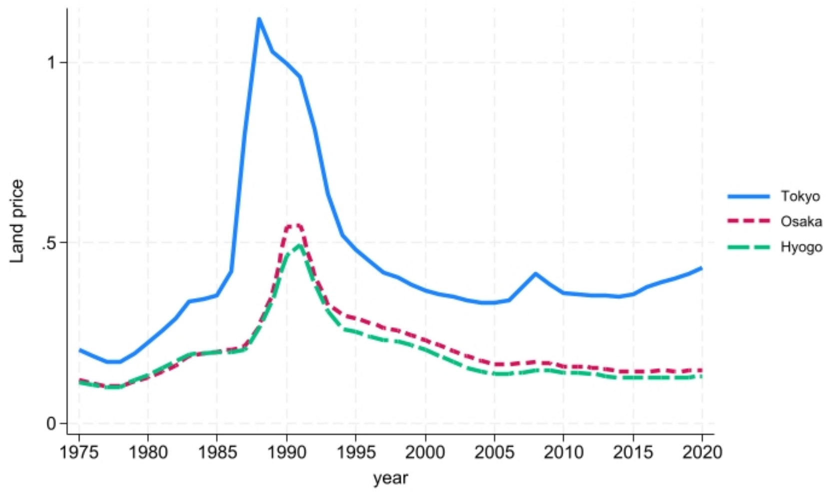
1 Introduction

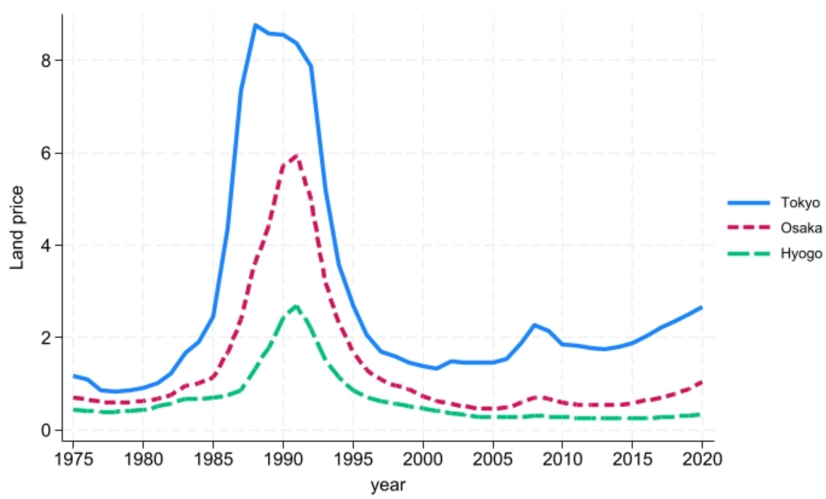
Over the past decades, financial frictions have been incorporated into macroeconomic models to study business cycles and economic growth, attracting considerable scholarly attention. Among these, the dynamic general equilibrium model developed by Kiyotaki and Moore (1997) has provided an influential framework for analyzing business cycles. In their model, collateral constraints amplify initial production shocks, causing downturns and booms to persist longer than in the absence of such constraints. While these theoretical implications are profound, empirical evidence on whether real economies are subject to collateral constraints remains scarce, with only a few exceptions (e.g., Kasa, 1998). To the best of our knowledge, no study has examined the presence of financial frictions at the regional level. To fill this gap, we investigate whether Japanese regional economies have experienced such frictions by directly estimating a closed-form equation for land price dynamics derived from a small open-economy version of the Kiyotaki–Moore model.

In the Kiyotaki–Moore framework, borrowers face financial constraints tied to land values (collateral constraints), whereas savers do not. Consider the adjustment mechanism of land prices. In the absence of collateral constraints, the land price would equal its fundamental value, determined by production technologies. However, collateral constraints distort the efficient allocation of land in production and thereby make land prices sticky. More concretely, suppose that agents perfectly anticipate an increase in the land price one period ahead, in a setting where borrowers face collateral constraints. In this case, the constraint is relaxed today, raising borrowers' demand for land. Consequently, today's land price rises, creating a positive serial correlation between today's and tomorrow's land prices. In this process, allocative inefficiency in land use is partially resolved, tomorrow's total output increases, and tomorrow's total net worth rises. Accordingly, tomorrow's demand for land also increases. To accommodate this heightened demand, the market raises the user cost of land, which leads to a decline in the land price two periods ahead (generating a capital loss for holding land). Hence, there is a negative serial correlation between today's and the day-after-tomorrow's land prices. As a result, the land price follows a second-order autoregressive (AR(2)) process. By directly estimating this AR(2) process, we test whether regional economies in Japan are subject to collateral constraints.

We examine whether collateral constraints affected three representative prefectures in Japan: Tokyo, Osaka, and Hyogo. Tokyo, the nation's capital, is the largest prefecture in terms of economic size, followed by Osaka. These two regions were most severely impacted by the bursting of the asset price bubble in the early 1990s, motivating their inclusion in



residential



commercial

Figure 1: Time Series of Land Price

Notes. The time series processes of land prices from 1975 to 2020 in Tokyo, Osaka, and Hyogo are presented. The upper and lower panels show the residential- and commercial-use land prices, respectively. The data source is explained in section 4.

our analysis. Hyogo, which borders Osaka, is also examined because it experienced a major earthquake in 1995, only a few years after the bubble burst. Figure 1 presents the time series of land prices in these three prefectures. Land prices—both residential and commercial—in Tokyo are consistently higher than in the other two regions. While the trajectories of residential land prices in Osaka and Hyogo are broadly similar, commercial land prices in Osaka are persistently higher than those in Hyogo. A common feature across all three prefectures is that land prices rose rapidly until the early 1990s and then declined sharply beginning around 1991. We investigate whether this pronounced boom–bust pattern can be accounted for by the Kiyotaki–Moore model.

Our estimations indicate that all three prefectures were subject to collateral constraints during the period 1975–2020, and that the Kiyotaki–Moore model provides a suitable framework for describing these regional economies over this horizon. We further test for structural change using the supremum Wald test, which endogenously determines break points. The results suggest that Tokyo experienced a structural change in 1988–1989, while Osaka and Hyogo did so in 1991–1992. As shown in Figure 1, these break points closely coincide with the peaks of land prices during the bubble economy. It is therefore highly plausible that the bursting of the bubble economy triggered the structural changes observed in all three prefectures.

Financial frictions are widely recognized in the literature as a crucial factor in understanding macroeconomic phenomena such as economic growth and business cycles. Galor and Zeira (1993) and Aghion et al. (2005) demonstrate theoretically that alleviating financial frictions promotes economic growth. Regarding business cycles, Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Pintus and Wen (2013) analyze financial accelerator mechanisms and examine how productivity shocks propagate through macroeconomic variables. Matsuyama (2007) derives endogenous business fluctuations driven by financial frictions. Furthermore, recent research has examined more closely the implications of collateral constraints for macroeconomic dynamics. Cordoba and Ripoll (2004) refine the mechanism by which asset prices amplify credit cycles, while Kiyotaki and Moore (2019) extend the original framework to incorporate liquidity and monetary policy considerations. Iacoviello (2005) and Iacoviello and Neri (2010) explicitly introduce housing and real estate prices into dynamic stochastic general equilibrium (DSGE) models, demonstrating the role of borrowing constraints in shaping business cycle fluctuations. At the international level, Mendoza (2010) develops a model with collateral constraints that accounts for sudden stops and financial crises in emerging economies. Collectively, these contributions establish a broad theoretical foundation for understanding how land and housing prices interact with financial frictions to generate cyclical fluctuations. Yet they do not empirically examine whether economies in fact confront financial frictions. In contrast, Kasa (1998) derives dynamic equations for the current account and land prices from a small open-economy version of the Kiyotaki–Moore model and empirically investigates their dynamics. Following this approach, Kunieda and Shibata (2005) and Kunieda et al. (2016) obtain closed-form solutions for current account dynamics from the Kiyotaki–Moore model. Using these solutions, they directly test whether the Japanese economy (Kunieda and Shibata, 2005) and representative Asian economies (Kunieda et al., 2016) are subject to collateral constraints. Building on this line of research, the present study advances the literature by deriving an AR(2) process for land prices from the Kiyotaki–Moore model. This novel approach enables us to test for collateral constraints relying solely on land price data.

The rest of the paper is organized as follows. Section 2 presents a small open-economy version of the Kiyotaki–Moore model. Section 3 characterizes the competitive equilibrium and derives a testable dynamic equation for land prices. Section 4 conducts the empirical analysis, examining whether Japanese prefectures faced collateral constraints during 1975–2020. Section 5 concludes.

2 Model

A local economy is considered a small open economy from the perspective of the rest of the world. Therefore, we apply a small open-economy version of the Kiyotaki–Moore model (Kiyotaki and Moore, 1997) to investigate regional economies. The basic structure of our model is similar to that of Kasa (1998). By introducing the population ratio between savers and borrowers, we derive an AR(2) process for land prices, which enables us to empirically test whether financially constrained borrowers exist in the economy.

The economy consists of savers and borrowers. While savers do not face collateral constraints, borrowers are subject to collateral constraints. The total population is normalized to one, with the ratio of borrowers to savers given by $\lambda : 1 - \lambda$ $(0 \leq \lambda \leq 1)$. All borrowers are identical in preferences and technology, and likewise, all savers are identical in the same respects. The period utility functions of savers and borrowers are given by $\ln c_t^*$ and $\ln \tilde{c}_t$, respectively, where c_t^* and $\tilde{c}_t$ denote the consumption of a saver and a borrower. In what follows, we assume that savers exist, $\lambda \neq 1$, unless stated otherwise.

2.1 Savers

Savers employ a production technology given by

$$y_{t+1}^* = G(x_t^*),$$

where y_{t+1}^* and x_t^* denote output and land input, respectively. The function $G(\cdot)$ satisfies $G''(\cdot) < 0 < G'(\cdot)$ and the Inada conditions: $\lim_{x^* \to 0} G'(\cdot) = \infty$, $\lim_{x^* \to \infty} G'(\cdot) = 0$, and $G(0) = 0$.

Each saver solves the lifetime utility maximization problem

$$\max \sum_{t=0}^{\infty} \beta^t \ln c_t^*$$
$$\text{s.t.} \quad c_t^* + q_t(x_t^* - x_{t-1}^*) + Rb_{t-1}^* = G(x_{t-1}^*) + b_t^* \tag{1}$$

with x_0^* given. Eq. (1) is the flow budget constraint, where $\beta \in (0, 1)$ is the subjective discount factor, b_t^* represents debt if positive and assets if negative, q_t is the land price, and $R > 1$ is the constant gross world interest rate. Because production requires one period of gestation, x_0^* is predetermined.

The first-order conditions of the lifetime utility maximization problem are

$$c_{t+1}^* = \beta R c_t^* \tag{2}$$

and

$$\frac{G'(x_t^*)}{u_t} = R \tag{3}$$

where $u_t := q_t - q_{t+1}/R$. Eq. (2) is the Euler equation, and Eq. (3) is the intratemporal optimality condition with respect to land, where u_t is interpreted as the land user cost. Together with the transversality condition, Eqs. (2) and (3) constitute the necessary and sufficient conditions for optimality.

2.2 Borrowers

Borrowers use a linear production technology, $\tilde{y}_{t+1} = a\tilde{x}_t$, where $\tilde{x}_t$ denotes land input, a denotes productivity, and $\tilde{y}_{t+1}$ denotes output. Borrowers face financial constraints in borrowing from the financial market, which depend on the collateral value in each period (collateral constraints). Following Kiyotaki and Moore (1997), we impose the following technical conditions on the parameters:

$$a > R\beta a > G'\left(\frac{\bar{X}}{1-\lambda}\right), \tag{4}$$

where $\bar{X}$ is the total land endowment in the economy. Inequalities (4) ensure the existence of a unique steady state in the economy. Each borrower solves the following lifetime utility maximization problem:

$$\begin{aligned} &\max \sum_{t=0}^{\infty} \beta^t \ln \tilde{c}_t \\ \text{s.t.}\ \ &\tilde{c}_t + q_t(\tilde{x}_t - \tilde{x}_{t-1}) + R\tilde{b}_{t-1} = a\tilde{x}_{t-1} + \tilde{b}_t, &(5)\\ &\tilde{b}_t \leq \frac{q_{t+1}\tilde{x}_t}{R}, &(6)\end{aligned}$$

with $\tilde{x}_0$ given. Eq. (5) is the flow budget constraint and Eq. (6) is the collateral constraint, respectively. Again, production requires one gestation period, so $\tilde{x}_0$ is predetermined.

The borrower's first-order conditions are

$$\frac{1}{\tilde{c}_t} - \frac{\beta R}{\tilde{c}_{t+1}} - \phi_t = 0, \tag{7}$$

$$-\frac{q_t}{\tilde{c}_t} + \frac{\beta(a + q_{t+1})}{\tilde{c}_{t+1}} + \frac{q_{t+1}}{R}\phi_t = 0, \tag{8}$$

where ϕ_t is the Lagrange multiplier associated with the collateral constraint in period t. Together with the transversality condition, Eqs. (7) and (8) constitute the necessary and sufficient conditions for this maximization problem. Kunieda and Shibata (2005) and Kunieda et al. (2016) show that there exists a period T such that from period T onward, the collateral constraint in Eq. (6) is always binding under the parameter conditions in inequalities (4). Focusing on the case in which the collateral constraint is always binding, Eqs. (5) and (6) yield

$$\tilde{c}_t + u_t\tilde{x}_t = a\tilde{x}_{t-1}. \tag{9}$$

Furthermore, from Eqs. (7) and (8), it follows that

$$\tilde{c}_{t+1} = \frac{\beta a}{u_t}\tilde{c}_t. \tag{10}$$

3 Equilibrium

A competitive equilibrium in this small open economy with the world interest rate R is represented by sequences of the land price $\{q_{t+1}\}$ and allocations $\{(c_t^*, c_t), (x_t^*, x_t), (b_t^*, b_t)\}$ for $t \geq 0$, such that savers and borrowers solve their respective optimization problems and the land market clears.

Since the period utility is logarithmic, Eqs. (9) and (10), together with the transversality condition, yield

$$u_t \tilde{x}_t = \beta a \tilde{x}_{t-1}, \tag{11}$$

the derivation of which is put in the appendix. Combining Eq. (3) with the market-clearing condition $(1 - \lambda)x_t^* + \lambda \tilde{x}_t = \bar{X}$ and Eq. (11) gives the following dynamic equation with respect to the borrower's land:

$$G'\left(\frac{\bar{X} - \lambda \tilde{x}_t}{1 - \lambda}\right) \tilde{x}_t = R\beta a \tilde{x}_{t-1}. \tag{12}$$

Thus, the dynamical system with respect to $(\tilde{x}_t, u_t)$ consists of Eqs. (11) and (12).

3.1 Steady state

The non-trivial steady state of the dynamical system $(\hat{x}, \hat{u})$ is well defined due to the second inequality in (4), which is given by

$$\hat{x} := \frac{\bar{X} - (1 - \lambda)G'^{-1}(\beta R a)}{\lambda}, \tag{13}$$

and

$$\hat{u} = \beta a. \tag{14}$$

From Eq. (14), the steady-state land price is

$$\hat{q} = \frac{R\beta a}{R - 1}. \tag{15}$$

From Eq. (12) and the market-clearing condition $(1 - \lambda)x_t^* + \lambda \tilde{x}_t = \bar{X}$, the steady state of the saver's land is

$$\hat{x}^* = G'^{-1}(\beta R a). \tag{16}$$

Although Remark 1 below is not central to our main analysis, it provides a theoretical insight into the land distribution between savers and borrowers in the steady state.

Remark 1. *Consider the land distribution between savers and borrowers in the steady state, measured by* $\hat{x}/\hat{x}^* = (1/\lambda)\left[\bar{X}/G'^{-1}(\beta R a) - (1-\lambda)\right]$. *The following results hold:*

1. $\partial(\hat{x}/\hat{x}^*)/\partial R > 0$. *An increase in the world interest rate raises the marginal product of savers' technology in equilibrium. Consequently, savers reduce their land use, while borrowers increase theirs.*

2. $\partial(\hat{x}/\hat{x}^*)/\partial a > 0$. *A rise in the marginal product of borrowers' technology increases their land use and reduces savers' land use.*

3. $\partial(\hat{x}/\hat{x}^*)/\partial \beta > 0$. *In the steady state, the marginal product of borrowers' technology exceeds that of savers' technology due to the presence of the land user cost. Hence, an increase in the propensity to save (for both savers and borrowers) reallocates more financial resources to borrowers through the financial market.*

3.2 Stability

By linearizing Eqs. (11) and (12), we obtain the following matrix representation:

$$\begin{pmatrix} \hat{x} & \hat{u} \\ 0 & R\beta a - \frac{\lambda G''(\hat{x}^*)\hat{x}}{1-\lambda} \end{pmatrix} \begin{pmatrix} u_t - \hat{u} \\ \tilde{x}_t - \hat{x} \end{pmatrix} = \begin{pmatrix} 0 & \beta a \\ 0 & R\beta a \end{pmatrix} \begin{pmatrix} u_{t-1} - \hat{u} \\ \tilde{x}_{t-1} - \hat{x} \end{pmatrix},$$

or equivalently,

$$\begin{pmatrix} u_t - \hat{u} \\ \tilde{x}_t - \hat{x} \end{pmatrix} = J \begin{pmatrix} u_{t-1} - \hat{u} \\ \tilde{x}_{t-1} - \hat{x} \end{pmatrix}, \tag{17}$$

where

$$J = \frac{1}{\hat{x}\left(R\beta a - \frac{\lambda G''(\hat{x}^*)\hat{x}}{1-\lambda}\right)} \begin{pmatrix} 0 & -\frac{\lambda \beta a G''(\hat{x}^*)\hat{x}}{1-\lambda} \\ 0 & R\beta a \hat{x} \end{pmatrix}.$$

Eq. (17) describes the local dynamics in the neighborhood of the steady state. The eigenvalues of the Jacobian matrix are 0 and $R\beta a/[R\beta a - \lambda G''(\hat{x}^*)/(1-\lambda)]$, which lies in $(0,1)$. Thus, the steady state is locally stable. Since $\tilde{x}_0$ and x_0^* are predetermined, it follows from Eq. (3) that u_0 is also predetermined. Therefore, equilibrium is uniquely determined in the neighborhood of the steady state.

Figure 2 illustrates the phase diagram of the difference equation (12), where $\{\tilde{x}_t\}$ converges monotonically to the steady state. Accordingly, u_t also converges monotonically to the steady state following Eq. (11). Hence, equilibrium is uniquely determined globally as well.

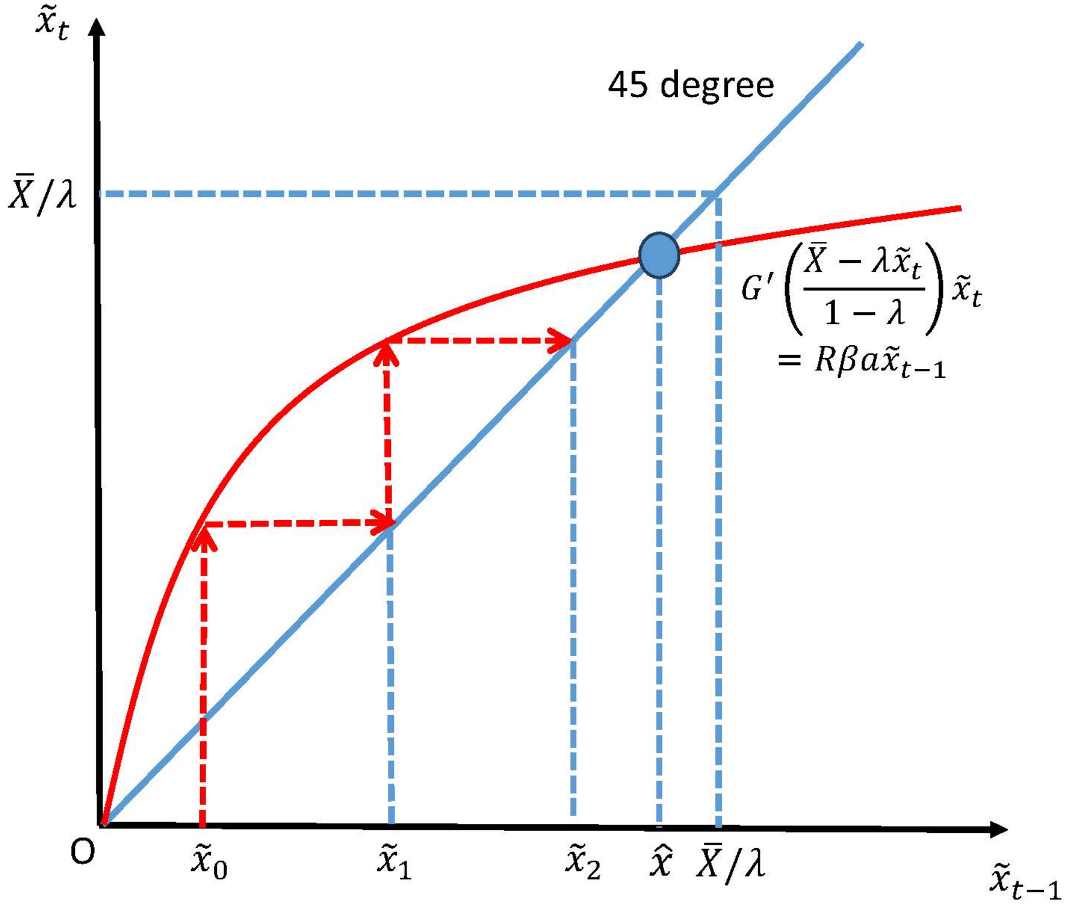


Figure 2: Land dynamics

Notes. The phase diagram of the difference equation (12) is illustrated. $\bar{X}/\lambda$ is the upper limit of $\tilde{x}_t$. $\{\tilde{x}_t\}$ converges monotonically to the steady state. Accordingly, u_t also converges monotonically to the steady state following Eq. (11). Hence, equilibrium is uniquely determined globally as well.

3.3 Testable equation

Thus far, we have assumed that savers are present in the economy, i.e., $\lambda \neq 1$. Before deriving a land price dynamic equation that enables us to empirically examine whether an economy is subject to collateral constraints, we first consider the case in which only borrowers exist, i.e., $\lambda = 1$. Aggregating Eq. (11) across all borrowers when $\lambda = 1$, we obtain $u_t\bar{X} = \beta a\bar{X}$, which implies that the land user cost is $u_t = \beta a$ for all $t \geq 0$. This leads to the difference equation $q_{t+1} = Rq_t - R\beta a$. Although any solution of this equation is consistent with the individual transversality condition, we show in the Appendix that only the particular solution $q_t = R\beta a/(R-1)$ for all $t \geq 0$ satisfies the no-Ponzi-game condition in the small open economy. This result implies that the land price is constant and uniquely determined in equilibrium if only borrowers are present in the economy. In contrast, if borrowers do not exist ($\lambda = 0$), Eq. (3), together with the transversality condition, implies that the land price is uniquely determined as $q_t = G'(\bar{X})/(R-1)$. Overall, when $\lambda = 1$ or $\lambda = 0$, the land price is constant in equilibrium.

For the case in which $0 < \lambda < 1$, linearizing Eq. (3) in the neighborhood of the steady state yields

$$-Ru_t + R\beta a = \frac{\lambda G''(\hat{x}^*)}{1-\lambda}(\tilde{x}_t - \hat{x}). \tag{18}$$

Using Eq. (18) to eliminate $\tilde{x}_t$ and $\tilde{x}_{t-1}$ from Eq. (11), we obtain

$$u_t\left[\hat{x} + \frac{R(1-\lambda)}{\lambda G''(\hat{x}^*)}(-u_t + \beta a)\right] = \beta a\left[\hat{x} + \frac{R(1-\lambda)}{\lambda G''(\hat{x}^*)}(-u_{t-1} + \beta a)\right]. \tag{19}$$

Proposition 1. *Suppose that the economy is in the neighborhood of the steady state. The dynamic equation for the logarithm of the land price* $\log(q_t)$ *is given by*

$$\log(q_{t+1}) = \alpha + \beta_1 \log(q_t) + \beta_2 \log(q_{t-1}), \tag{20}$$

where

$$\alpha := \frac{(1-R)\hat{x}\log\left(\frac{R\beta a}{R-1}\right)}{\hat{x} - \frac{R(1-\lambda)\beta a}{\lambda G''(\hat{x}^*)}},$$

$$\beta_1 := \frac{R\left[\hat{x} - \frac{(R+1)(1-\lambda)\beta a}{\lambda G''(\hat{x}^*)}\right]}{\hat{x} - \frac{R(1-\lambda)\beta a}{\lambda G''(\hat{x}^*)}} > 1,$$

and

$$\beta_2 := \frac{\frac{R^2(1-\lambda)\beta a}{\lambda G''(\hat{x}^*)}}{\hat{x} - \frac{R(1-\lambda)\beta a}{\lambda G''(\hat{x}^*)}} < 0.$$

Proof. See the Appendix.

If both savers and borrowers exist ($0 < \lambda < 1$) in the economy and borrowers face binding credit constraints, the land price q_t follows the second-order autoregressive process (AR(2)) as shown in Eq. (20) in the neighborhood of the steady state. The intuition behind the positive relation between q_t and q_{t+1} and the negative relation between q_{t-1} and q_{t+1} is as follows. If agents perfectly anticipate an increase in the land price one period ahead, the constraint is relaxed today and borrowers' demand for land increases (see Eqs. (5) and (6)). Then, today's land price rises, which generates a positive serial correlation between today's and tomorrow's land prices. Relaxing the collateral constraint partially corrects allocative inefficiency in land use, tomorrow's total output in the economy increases, and tomorrow's total net worth rises. As a result, tomorrow's demand for land also increases. To accommodate this increased demand, the market raises the user cost of land, which leads to a decline in the land price two periods ahead, generating a capital loss for holding land. Hence, a negative serial correlation between today's and the day-after-tomorrow's land prices occurs.

4 Estimation

Based on the testable equation in the previous section, we specify the following empirical model for estimation:

$$\log(q_t) = \alpha + \beta_1 \log(q_{t-1}) + \beta_2 \log(q_{t-2}) + \epsilon_t, \tag{21}$$

where ϵ_t is an error term. The theoretical prediction is if both savers and borrowers exist and borrowers face binding collateral constraints, Eq. (21) follows the AR(2) process with $\beta_1 > 1$ and $\beta_2 < 0$. We examine this theoretical prediction by estimating Eq. (21) with the land price dada.

Our analysis focuses on three representative prefectures in Japan: Tokyo, Osaka, and Hyogo. Tokyo is the capital, and Osaka is the second largest prefecture after Tokyo. We selected these two regions because they were most severely affected by the bubble burst in 1990. Hyogo, located adjacent to Osaka, was also chosen because it experienced a major earthquake in 1995, only a few years after the bubble burst.

Table 1: Augmented Dickey-Fuller test

Variable	Observations	Lags	p-value
$\ln(q)$ (Tokyo, residential)	44	1	0.0567
$\ln(q)$ (Tokyo, commercial)	44	1	0.0289
$\ln(q)$ (Tokyo, average)	44	1	0.0264
$\ln(q)$ (Osaka, residential)	44	1	0.0851
$\ln(q)$ (Osaka, commercial)	44	1	0.0326
$\ln(q)$ (Osaka, average)	44	1	0.0290
$\ln(q)$ (Hyogo, residential)	44	1	0.0814
$\ln(q)$ (Hyogo, commercial)	44	1	0.0714
$\ln(q)$ (Hyogo, average)	44	1	0.0974
$\ln(q)$ (Japan, residential)	43	2	0.0903
$\ln(q)$ (Japan, commercial)	44	1	0.0426
$\ln(q)$ (Japan, average)	44	1	0.0331

Notes. The augmented Dickey–Fuller test is performed with one-period or two-period lags, depending on the case. The test for Hyogo (commercial) includes a trend term. The null hypothesis (H0) assumes the presence of a unit root. MacKinnon approximate p-values are reported. The null hypothesis is rejected at conventional significance levels for all land prices.

4.1 Data and Stationarity

We compile raw data on land prices for Tokyo, Osaka, Hyogo, and Japan as a whole (National Land price) over the period 1975–2020 from the database of the "time series chart of fluctuation rate and average land price" provided by the Ministry of Land, Infrastructure, Transport and Tourism. To obtain real land prices, we deflate the nominal series using the consumer price index (2020 base) available on e-Stat, the portal site for Japanese Government Statistics. For robustness, we prepare three types of datasets for each region: residential use, commercial use, and their average. In addition to the prefectural data, we construct national-level series for Japan, which are later used as instrumental variables in the regression analysis.

Table 1 reports the results of the augmented Dickey–Fuller (ADF) test (all variables in logarithms) for the prefectural and national land price series. Depending on the case, the test is conducted with one- or two-period lags, and in the case of Hyogo (commercial), a trend term is included. As the table indicates, the null hypothesis of a unit root is rejected at conventional significance levels for all series. Therefore, in the subsequent analysis, we assume that all land prices follow stationary processes. This assumption is also consistent with theory; otherwise, the transversality condition would not hold.

Table 2: Regression Results

	Tokyo		
variable	residential	commercial	average
$\ln(q_{t-1})$	1.541*** (0.114)	1.721*** (0.090)	1.720*** (0.090)
$\ln(q_{t-1})$'s 95% conf. interval	[1.309, 1.772]	[1.538, 1.904]	[1.537, 1.902]
$\ln(q_{t-2})$	-0.641*** (0.111)	-0.801*** (0.089)	-0.800*** (0.089)
constant	0.377*** (0.131)	0.435*** (0.140)	0.394*** (0.125)
observations	44	44	44
Durbin's alternative test p-value	0.236	0.734	0.551

	Osaka		
variable	residential	commercial	average
$\ln(q_{t-1})$	1.609*** (0.108)	1.805*** (0.079)	1.802*** (0.079)
$\ln(q_{t-1})$'s 95% conf. interval	[1.390, 1.829]	[1.644, 1.966]	[1.641, 1.963]
$\ln(q_{t-2})$	-0.693*** (0.107)	-0.865*** (0.079)	-0.862*** (0.079)
constant	0.250** (0.094)	0.280*** (0.091)	0.253*** (0.081)
observations	44	44	44
Durbin's alternative test p-value	0.006	0.776	0.723

	Hyogo		
variable	residential	commercial	average
$\ln(q_{t-1})$	1.656*** (0.102)	1.796*** (0.084)	1.782*** (0.086)
$\ln(q_{t-1})$'s 95% conf. interval	[1.449, 1.864]	[1.625, 1.968]	[1.607, 1.957]
$\ln(q_{t-2})$	-0.733*** (0.102)	-0.845*** (0.085)	-0.835*** (0.087)
constant	0.222** (0.083)	0.193** (0.076)	0.190** (0.073)
observations	44	44	44
Durbin's alternative test p-value	0.052	0.156	0.088

Notes. All regressions are estimated by OLS. Standard errors are reported in parentheses. 95% confidence intervals are shown below the corresponding coefficients. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively. The null hypothesis of Durbin's alternative test is no first-order autocorrelation in the residuals.

Table 3: Prais-Winsten Regression

variable	Osaka residential	Hyogo residential	Hyogo average
$\ln(q_{t-1})$	1.394*** (0.131)	1.455*** (0.128)	1.625*** (0.113)
$\ln(q_{t-1})$'s 95% conf. interval	[1.128, 1.661]	[1.196, 1.713]	[1.395, 1.854]
$\ln(q_{t-2})$	-0.514*** (0.131)	-0.552*** (0.127)	-0.686*** (0.114)
constant	0.357** (0.149)	0.282** (0.125)	0.220* (0.110)
observations	44	44	44

Notes. The results of Prais–Winsten regressions are presented for Osaka (residential) and Hyogo (residential and average), where Durbin's alternative test detected serial correlation in the OLS residuals (see Table 2). Standard errors are reported in parentheses. 95% confidence intervals are shown below the corresponding coefficients. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively. These results confirm that the coefficients remain statistically significant and their signs consistent with the theoretical predictions, even after correcting for serial correlation.

4.2 Results

Table 2 reports the OLS estimation results for Eq. (21). The estimated coefficients of $\ln(q_{t-1})$ are positive and statistically significant at the 1% level for all datasets—residential use, commercial use, and their average—across the three prefectures. Moreover, all estimates of this coefficient exceed one. The coefficients of $\ln(q_{t-2})$ are negative and statistically significant at the 1% level for all datasets in all three prefectures. These results are in line with the theoretical predictions of the Kiyotaki–Moore model.

We next examine whether first-order autocorrelation exists in the error terms using Durbin's alternative test, which is robust in the presence of lagged dependent variables among the regressors. The p-values of this test indicate that the null hypothesis of no autocorrelation is not rejected for Tokyo and for Osaka (commercial and average) as well as Hyogo (commercial). However, the null is rejected for Osaka (residential) and for Hyogo (residential and average). Table 3 presents the Prais–Winsten regression results for these three cases. These results confirm the robustness of our findings: while the absolute magnitudes of the coefficients of $\ln(q_{t-1})$ and $\ln(q_{t-2})$ are somewhat smaller than in Table 2, both coefficients remain statistically significant, and their signs are consistent with theoretical predictions.

Taken together, the regression results demonstrate that land prices in Tokyo, Osaka, and Hyogo follow AR(2) processes consistent with binding collateral constraints. This supports the applicability of the Kiyotaki–Moore model to regional land price dynamics in Japan during 1975–2020.

4.3 Empirical reality versus theory

The theoretical framework has demonstrated that land prices follow an AR(2) process, which is a necessary condition for the presence of collateral constraints. However, the dynamic characteristics of land prices implied by the theory have not yet been explored. Let the eigenvalues of the difference equation (20) be κ_1 and κ_2 ($\kappa_1 < \kappa_2$). As shown in the Appendix, the theory predicts $0 < \kappa_1 < 1 < \kappa_2$, implying that the dynamic path of land prices is uniquely determined and monotonically converges to the steady state.

By contrast, when the estimated coefficients β_1 and β_2 are applied to Eq. (20), the resulting eigenvalues differ markedly from the theoretical prediction. Table 4 reports the computed eigenvalues across specifications. In all cases, the eigenvalues are complex with moduli less than one. This outcome indicates that land-price dynamics exhibit damped oscillations around the steady state and that equilibrium is not uniquely determined, in contrast with the theoretical model. The implied oscillations have a finite period, typically spanning 6.1–6.3 years, before the deterministic fluctuations die out. While the AR(2) representation is consistent with the presence of collateral constraints, these additional features suggest that other sources of inefficiency—such as externalities or alternative forms of market imperfections—may be necessary to explain the observed indeterminacy and cyclical behavior, which would generate endogenous boom-bust fluctuations. A further investigation of these extensions lies beyond the scope of the present paper.

Table 4: Eigenvalues of AR(2) Processes Estimated in Table 2

prefecture–land type	eigenvalues (r)	modulus ($\lvert r\rvert$)	approx. period (years)
Tokyo, residential	$0.7708 \pm 0.2280i$	0.801	6.2
Tokyo, commercial	$0.8605 \pm 0.4053i$	0.895	6.1
Tokyo, average	$0.8598 \pm 0.4067i$	0.894	6.1
Osaka, residential	$0.7955 \pm 0.3161i$	0.833	6.2
Osaka, commercial	$0.9025 \pm 0.3957i$	0.930	6.1
Osaka, average	$0.9004 \pm 0.3987i$	0.928	6.1
Hyogo, residential	$0.8280 \pm 0.2323i$	0.856	6.3
Hyogo, commercial	$0.8815 \pm 0.3743i$	0.919	6.1
Hyogo, average	$0.8750 \pm 0.3836i$	0.914	6.1

Notes. Eigenvalues are computed from the AR(2) characteristic equation $r^2 - \beta_1 r - \beta_2 = 0$ using the estimates in Table 2. The roots are complex with modulus less than one. The approximate period is computed as $2\pi/\theta$, where $\theta = \arg(r)$.

Table 5: IV Estimation Results (Instrument: National Land Prices)

	Tokyo		
variable	residential	commercial	average
$\ln(q_{t-1})$	1.473*** (0.148)	1.716*** (0.091)	1.713*** (0.098)
$\ln(q_{t-1})$ 95% conf. int.	[1.183, 1.764]	[1.537, 1.896]	[1.520, 1.905]
$\ln(q_{t-2})$	-0.579*** (0.128)	-0.796*** (0.091)	-0.793*** (0.096)
constant	0.396*** (0.121)	0.436*** (0.139)	0.396*** (0.126)
observations	44	44	44
First-stage F-value	30.49	61.66	62.20
Hansen test p-value	0.384	0.841	0.729

	Osaka		
variable	residential	commercial	average
$\ln(q_{t-1})$	1.894*** (0.227)	1.823*** (0.092)	1.822*** (0.098)
$\ln(q_{t-1})$ 95% conf. int.	[1.448, 2.340]	[1.642, 2.003]	[1.629, 2.015]
$\ln(q_{t-2})$	-0.962*** (0.206)	-0.882*** (0.090)	-0.882*** (0.096)
constant	0.204 (0.165)	0.278** (0.107)	0.250** (0.100)
observations	44	44	44
First-stage F-value	9.80	92.04	82.04
Hansen test p-value	0.272	0.781	0.960

	Hyogo		
variable	residential	commercial	average
$\ln(q_{t-1})$	1.808*** (0.202)	1.903*** (0.204)	1.904*** (0.213)
$\ln(q_{t-1})$ 95% conf. int.	[1.412, 2.204]	[1.502, 2.304]	[1.486, 2.323]
$\ln(q_{t-2})$	-0.878*** (0.186)	-0.950*** (0.194)	-0.955*** (0.201)
constant	0.202 (0.126)	0.185* (0.099)	0.180* (0.099)
observations	44	44	44
First-stage F-value	14.25	16.22	14.80
Hansen test p-value	0.679	0.452	0.642

Notes. The results of IV estimations are reported, instrumenting $\ln(q_{t-1})$ with one- and two-period-lagged National Land prices. Robust standard errors are in parentheses. 95% confidence intervals are reported below coefficients. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. Reported statistics include the first-stage F-value for excluded instruments and the Hansen p-value for overidentifying restrictions.

4.4 Robustness: IV estimations

The OLS and Prais–Winsten estimations reported above are broadly consistent with the predictions of the Kiyotaki–Moore model. However, potential endogeneity in the one-period-lagged prefectural land price remains a central concern, although the two-period-lagged land price can be considered exogenous in Eq. (21). From a theoretical perspective, reverse causality may arise: if borrowers anticipate a rise in tomorrow's land price, today's borrowing increases (Eq. (6)), which in turn raises today's land price (Eq. (5)). Moreover, if error terms exhibit first-order autocorrelation, the lagged dependent variable introduces another source of endogeneity. To address these issues, we implement instrumental-variable (IV) estimations as a robustness check. Specifically, the lagged prefectural land price $\log(q_{t-1})$ is instrumented with one-period- and two-period-lagged National Land prices. These instruments capture generic real estate market conditions across Japan, thereby being plausibly correlated with regional land price dynamics in the respective periods while being less affected by prefecture-specific shocks and contemporaneous aggregate shocks and satisfying the exclusion restriction. Table 5 reports the estimation results.

The IV results reported in Table 5 broadly confirm the robustness of our earlier findings. The estimated coefficients of $\ln(q_{t-1})$ remain positive, statistically significant, and greater than one, while those of $\ln(q_{t-2})$ are negative and significant across specifications, consistent with the theoretical predictions of the Kiyotaki–Moore model. Compared with the OLS and Prais–Winsten estimates, the magnitudes of the coefficients are slightly strengthened except for Tokyo (residential), but the overall signs and statistical significance are preserved. The first-stage F statistics for the excluded instruments are generally strong, and the Hansen test p-values provide no evidence against the validity of the overidentifying restrictions. Moreover, the eigenvalues computed from estimated β_1 and β_2 remain complex with moduli less than one. Taken together, these results provide robust evidence that the observed AR(2) dynamics of land prices in regional economies are not driven by spurious endogeneity, reinforcing the interpretation that collateral constraints are binding and the damping cyclical behavior of land prices is present in the Japanese economy.

4.5 Structural change

One may suspect that the impacts of $\ln(q_{t-1})$ and $\ln(q_{t-2})$ changed structurally over the estimation period. To examine this possibility, we test for structural breaks in these coefficients by applying the supremum Wald test, which determines the break point endogenously. The results are reported in Table 6. Significant structural breaks are detected in Tokyo (residen-

Table 6: Supremum Wald test

Variable	Test Statistic	p-value	Estimated Break Point
Tokyo, residential	16.675	0.015	1988
Tokyo, commercial	10.170	0.200	1988
Tokyo, average	13.464	0.059	1988
Osaka, residential	49.134	0.000	1991
Osaka, commercial	13.048	0.069	1991
Osaka, average	17.549	0.010	1991
Hyogo, residential	23.682	0.000	1991
Hyogo, commercial	27.565	0.000	1991
Hyogo, average	29.267	0.000	1991

Notes. The supremum Wald test is applied to detect structural breaks in the coefficients of $\ln(q_{t-1})$ and $\ln(q_{t-2})$. Reported p-values correspond to the null hypothesis of no structural break. The estimated break dates are determined endogenously by the test.

tial and average), Osaka (all cases), and Hyogo (all cases), while no statistically significant break is found in Tokyo (commercial). The break dates for Tokyo are identified around 1988–1989, and those for Osaka and Hyogo in 1991–1992, which correspond closely to the bursting of Japan's bubble economy.

Table 7 reports the OLS estimation results, splitting the data at the identified break points. Comparing Tables 2 and 7 highlights both similarities and differences across the prefectures. Before the structural change, the three prefectures exhibit broadly similar patterns. In every case, the coefficient of $\ln(q_{t-1})$ is positive and statistically significant at conventional levels, while the coefficient of $\ln(q_{t-2})$ is negative but insignificant, except for Osaka (commercial and average). Moreover, the estimated coefficient of $\ln(q_{t-1})$ is greater than one in all specifications, but this finding is not robust: the 95% confidence intervals all but Osaka (commercial and average) include values below one. After the structural change, all three prefectures display comparable results. In all prefectures, the coefficient of $\ln(q_{t-1})$ in all the cases is significant, greater than one, and its 95% confidence interval excludes values less than one. Additionally, the coefficient of $\ln(q_{t-2})$ is negative and significant.

Taken together, these results do not imply that the Kiyotaki–Moore model applies only to the periods after the structural break. The estimations with the entire dataset in the previous sections detect binding collateral constraints. Thus, although structural breaks can be statistically identified, the Kiyotaki–Moore model should be understood as capturing collateral-constrained dynamics throughout the estimation period. Furthermore, the point estimate of the coefficient of $\ln(q_{t-1})$ is greater than one and the coefficient of $\ln(q_{t-2})$ is

Table 7: Structural Change

	Tokyo (before 1988)		
variable	residential	commercial	average
$\ln(q_{t-1})$	1.381*** (0.324)	1.751*** (0.347)	1.713*** (0.361)
$\ln(q_{t-1})$ 95% conf. interval	[0.648, 2.113]	[0.966, 2.536]	[0.895, 2.531]
$\ln(q_{t-2})$	-0.211 (0.461)	-0.814 (0.481)	-0.750 (0.504)
	Tokyo (after 1989)		
variable	residential	commercial	average
$\ln(q_{t-1})$	1.495*** (0.144)	1.609*** (0.122)	1.604*** (0.123)
$\ln(q_{t-1})$ 95% conf. interval	[1.200, 1.790]	[1.359, 1.858]	[1.353, 1.855]
$\ln(q_{t-2})$	-0.575*** (0.129)	-0.689*** (0.112)	-0.684*** (0.113)

	Osaka (before 1991)		
variable	residential	commercial	average
$\ln(q_{t-1})$	1.445*** (0.337)	1.719*** (0.272)	1.704*** (0.288)
$\ln(q_{t-1})$ 95% conf. interval	[0.712, 2.179]	[1.126, 2.312]	[1.076, 2.331]
$\ln(q_{t-2})$	-0.454 (0.429)	-0.753** (0.319)	-0.735* (0.339)
	Osaka (after 1992)		
variable	residential	commercial	average
$\ln(q_{t-1})$	1.453*** (0.120)	1.506*** (0.136)	1.457*** (0.144)
$\ln(q_{t-1})$ 95% conf. interval	[1.207, 1.699]	[1.226, 1.787]	[1.161, 1.753]
$\ln(q_{t-2})$	-0.490*** (0.102)	-0.614*** (0.115)	-0.570*** (0.121)

	Hyogo (before 1991)		
variable	residential	commercial	average
$\ln(q_{t-1})$	1.530*** (0.268)	1.584*** (0.308)	1.571*** (0.305)
$\ln(q_{t-1})$ 95% conf. interval	[0.945, 2.114]	[0.912, 2.255]	[0.906, 2.236]
$\ln(q_{t-2})$	-0.556 (0.324)	-0.581 (0.384)	-0.569 (0.380)
	Hyogo (after 1992)		
variable	residential	commercial	average
$\ln(q_{t-1})$	1.461*** (0.153)	1.373*** (0.144)	1.347*** (0.150)
$\ln(q_{t-1})$ 95% conf. interval	[1.146, 1.776]	[1.076, 1.669]	[1.037, 1.657]
$\ln(q_{t-2})$	-0.515*** (0.131)	-0.472*** (0.124)	-0.448*** (0.129)

Notes. The sample is divided into subperiods according to the break points identified by the supremum Wald test (Table 6). Standard errors are in parentheses. 95% confidence intervals are reported below the corresponding coefficients. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

not statistically different from zero before the structural change in Tokyo (all cases), Osaka (residential), and Hyogo (all cases). These outcomes would indicate that the land prices of these prefectures follow an AR(1) process before the structural change and do not satisfy the transversality condition from the theoretical point of view. In other words, the land prices before the structural change enter a divergence process, exhibiting the bubbly aspect of their dynamic behavior. While the Kiyotaki–Moore model is designed to analyze macroeconomic phenomena when an economy faces collateral constraints, the occurrence of structural breaks at points where a boom, contradictory to dynamic general equilibrium, transitions to a bust followed by a return to an equilibrium path is consistent with the Kiyotaki–Moore model. Therefore, given the theoretical concept of the Kiyotaki–Moore model, identifying break points provides valuable insight into the timing and nature of turning points in the Japanese economy.

5 Concluding Remarks

This paper has empirically examined collateral constraints in the Kiyotaki–Moore model using land price data from three major prefectures in Japan: Tokyo, Osaka, and Hyogo. We find that land prices follow AR(2) processes, consistent with the presence of binding collateral constraints. These results are robust across different specifications and estimation methods, including Prais–Winsten regressions to address serial correlation and instrumental-variable (IV) regressions to mitigate potential endogeneity of lagged land prices.

Our empirical findings also show that the bursting of the bubble economy in the early 1990s generically triggered structural changes in land price dynamics for all the prefectures. This pattern underscores the close relationship between collateral values, financial market conditions, and boom–bust cycles in Japan's regional economies. The Kiyotaki–Moore framework thus proves effective in capturing the dynamic behavior of collateral-constrained economies over time, even in the presence of structural breaks.

Although the analysis focuses on three prefectures, the approach can be readily extended to other regions or countries to assess the broader validity of the model. Future research could also incorporate additional macroeconomic variables, such as investment and financial indicators, to enrich the analysis. Overall, this study contributes to the literature by providing regional evidence in support of the Kiyotaki–Moore model and by offering new insights into the role of collateral constraints in shaping economic fluctuations.

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Appendix

Derivation of Eq. (11)

From Eqs. (9) and (10), we obtain

$$\begin{aligned}\frac{u_t\tilde{x}_t}{c_t} &= \beta + \beta\frac{u_{t+1}\tilde{x}_{t+1}}{c_{t+1}} \\ &= \beta + \beta^2 + \cdots + \beta^\tau \frac{u_{t+\tau}\tilde{x}_{t+\tau}}{c_{t+\tau}}. \end{aligned} \tag{A1}$$

The transversality condition is

$$\lim_{\tau\to\infty} \beta^\tau \frac{u_{t+\tau}\tilde{x}_{t+\tau}}{c_{t+\tau}} = 0. \tag{A2}$$

Applying Eq. (A2) to Eq. (A1), Eq. (A1) becomes

$$\begin{aligned}\frac{u_t\tilde{x}_t}{c_t} &= \lim_{\tau\to\infty}\sum_{s=1}^{\tau}\beta^s \\ &= \frac{\beta}{1-\beta}. \end{aligned} \tag{A3}$$

Eqs. (9) and (A3) yield Eq. (11). □

For the case in which $\lambda = 1$

When $\lambda = 1$, only borrowers are inhabitant in the economy. Aggregating Eq. (11) across all borrowers when $\lambda = 1$, we obtain $u_t\bar{X} = \beta a\bar{X}$, and the land user cost becomes $u_t = \beta a$ for all $t \geq 0$, which leads to $q_{t+1} = Rq_t - R\beta a$. Solving this difference equation, it follows that

$$q_t = \frac{R\beta a}{R-1} + \left(q_0 - \frac{R\beta a}{R-1}\right)R^t, \tag{A4}$$

where $q_0 \geq R\beta a/(R-1)$ because $q_t \geq 0$ for all $t \geq 0$. Since the difference equation, $q_{t+1} = Rq_t - R\beta a$, satisfies Eqs. (9) and (11), any solution given by Eq. A4 with $q_0 \geq R\beta a/(R-1)$ is consistent with each borrower's individual transversality condition.

On the other hand, Eq. (10) and $u_t = \beta a$ yield $\tilde{c}_{t+1} = \tilde{c}_t$, and thus, $\tilde{c}_t$ is constant for all $t \geq 0$. Letting $\tilde{c}_t = \tilde{c}$ and aggregating the flow budget constraint (5) across borrowers, we obtain

$$\tilde{c} + Rb_{t-1} = a\bar{X} + b_t, \tag{A5}$$

where Eq. (A5) is the aggregate flow budget constraint that the economy faces and $b_t = \lambda\tilde{b}_t$ is the total debt. From Eq. (A5), we obtain the lifetime budget constraint of the economy as follows:

$$\lim_{t\to\infty} \sum_{s=1}^{t} \frac{\tilde{c}}{R^s} + b_0 = \lim_{t\to\infty} \sum_{s=1}^{t} \frac{a\bar{X}}{R^s} + \lim_{t\to\infty} \frac{b_t}{R^t}. \tag{A6}$$

In Eq. (A6), the no Ponzi game condition of the economy is given by

$$\lim_{t\to\infty} \frac{b_t}{R^t} \leq 0. \tag{A8}$$

Aggregating the collateral constraint (6) across borrowers yields

$$b_t = \frac{q_{t+1}\bar{X}}{R}. \tag{A9}$$

From Eqs. (A4) and (A9), it follows that

$$\frac{b_t}{R^t} = \bar{X}\left[\frac{\beta a}{R^t(R-1)} + \left(q_0 - \frac{R\beta a}{R-1}\right)\left(\frac{1}{R}\right)\right]. \tag{A10}$$

From Eq. (A10), $q_0 = R\beta a/(R-1)$ must hold in order for the no Ponzi game condition (A8) to be satisfied. Therefore, from Eq. (A4), it holds that $q_t = R\beta a/(R-1)$ for all $t \geq 0$. □

Proof of Proposition 1: derivation of Eq. (20)

Taking the logarithm of both sides of Eq. (19) yields

$$\log u_t + \log\left[\hat{x} + \frac{R(1-\lambda)}{\lambda G_1''(\hat{x}^*)}(-u_t + \beta a)\right] = \log(\beta a) + \log\left[\hat{x} + \frac{R(1-\lambda)}{\lambda G_1''(\hat{x}^*)}(-u_{t-1} + \beta a)\right]. \tag{A11}$$

By log-linearizing both sides of Eq. (A11) in the neighborhood of the steady state and using Eqs. (14) and (15), we obtain

$$\left[1-\frac{(1-\lambda)R\beta a}{\lambda G''(\hat{x}^*)\hat{x}}\right]\log(q_{t+1})=(1-R)\log\left(\frac{R\beta a}{R-1}\right)$$
$$+R\left[1-\frac{(1-\lambda)(R+1)\beta a}{\lambda G''(\hat{x}^*)\hat{x}}\right]\log(q_t)+\frac{(1-\lambda)R^2\beta a}{\lambda G''(\hat{x}^*)\hat{x}}\log(q_{t-1}). \tag{A12}$$

Rearranging Eq. (A12) yields Eq. (20). In Eq. (20), $\beta_2<0$ is obvious. Since β_1 is computed as

$$\beta_1=1-\frac{\frac{(1-\lambda)\beta a}{\lambda G''(\hat{x}^*)}}{\hat{x}-\frac{R(1-\lambda)\beta a}{\lambda G''(\hat{x}^*)}},$$

it follows that $\beta_1>1$. $\square$

Proof of $0<\kappa_1<1\kappa_2$

The characteristic equation of Eq. (20) is $r^2-\beta_1 r-\beta_2=0$. Define $f(r):=r^2-\beta_1 r-\beta_2$. Then, the following hold

$$f(0)=-\beta_2=-\frac{\frac{R^2(1-\lambda)\beta a}{\lambda G''(\hat{x}^*)}}{\hat{x}-\frac{R(1-\lambda)\beta a}{\lambda G''(\hat{x}^*)}}>0 \tag{A13}$$

and

$$f(1)=1-\beta_1-\beta_2=\frac{(1-R)\hat{x}}{\hat{x}-\frac{R(1-\lambda)\beta a}{\lambda G''(\hat{x}^*)}}<0. \tag{A14}$$

From Eq. (A6) and (A14), it holds that $0<\kappa_1<1<\kappa_2$. $\square$

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