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Strategic Investment Timing and Consumer Savviness for Add-On Services

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Strategic Investment Timing and Consumer Savviness for Add-On Services*

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Abstract

We examine duopolistic competition within a real options framework, where two firms provide horizontally differentiated core goods along with vertically differentiated add-on services. Focusing on consumer savviness regarding the quality of add-on services, we analyze its impact on prices, investment timing, and social welfare. Our analysis yields three main findings. First, the price of a core good with high-quality add-on services is lower (resp. higher) than that of a core good with low-quality add-on services when the proportion of non-savvy consumers in the population is high (resp. low). Second, both firms are incentivized to invest preemptively when the proportion of non-savvy consumers is high or the quality difference in add-on services is small; however, when the proportion of non-savvy consumers is low or the quality gap is significant, only the firm offering high-quality add-on services retains this incentive. Third, as the proportion of non-savvy consumers decreases, the flow of social surplus increases; nonetheless, the overall expected social welfare may decline due to a delay in the follower's investment and a prolonged monopoly duration. This suggests a need for a policy mix that combines consumer protection with the encouragement of market entry.

Keywords: add-on service, consumer savviness, real options, investment timing, delay.

JEL classification: D90, L22, L24, M21, O31.

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1 Introduction

Firms across various markets often bundle their core products with additional services. Examples of additional services include extended warranties for home appliances, maintenance packages for automobiles, toner cartridges for printers, overdraft protection in banking, and optional services such as nuisance call blocking and voicemail management for mobile telecommunication services. These services, known as *add-on services*, can enhance the perceived value of a product, especially for "savvy" consumers who recognize their relevance and quality. However, not all consumers are well-informed or attentive to the costs and benefits of these services. The presence of "non-savvy" consumers introduces a behavioral aspect to market outcomes, influencing not only how firms set prices but also when they make investments, ultimately affecting social welfare. Nevertheless, the interaction between firms' investment timing decisions and social welfare through consumer savviness has received limited attention in the literature, despite its significance.

To address this gap, we examine a duopolistic market where firms offer horizontally differentiated *core goods* bundled with vertically differentiated *add-on services*. By focusing on consumer savviness regarding the quality of these add-on services, we investigate the strategic investment decisions made by firms under uncertainty, applying a real options framework à la Dixit and Pindyck (1994). Our goal is to understand how the composition of consumer types affects pricing, investment timing, and expected social welfare.

The analytical procedure and main findings are as follows. First, we analyze the pricing of a core good and its associated add-on services set by firms. We show that when the proportion of non-savvy consumers is high, the price of a core good with high-quality add-on services is lower than that of a core good with low-quality add-on services, and vice versa when the proportion of non-savvy consumers is low. Moreover, we confirm that the price of add-on services increases as the proportion of non-savvy consumers decreases.

Next, we characterize the market equilibria. We establish that a firm providing a core good with high-quality add-on services, referred to as a "high-quality firm," makes investments earlier than a firm providing low-quality add-on services, referred to as a "low-quality firm," regardless of the proportion of non-savvy consumers. We then differentiate between two types of equilibria: a *preemptive equilibrium* and a *semi-preemptive equilibrium*. The configuration of these equilibria depends on various factors, such as firms' profit flows, the proportion of savvy consumers, and the quality difference in add-on services. From the perspective of quality difference, we demonstrate that when the quality difference in add-on services is small, a preemptive equilibrium occurs, where both firms have a preemptive incentive. By contrast, when the quality difference is large, a semi-preemptive equilibrium occurs, where only the high-quality firm has a preemptive incentive. For an intermediate range of quality difference, both equilibria coexist, where a preemptive equilibrium occurs when the proportion of non-savvy consumers is high, while a semi-preemptive equilibrium occurs when it is low. This implies that regarding the proportion of non-savvy consumers, a preemptive (resp. a semi-preemptive) equilibrium is likely to occur when the proportion of non-savvy consumers is high (resp. low).

Finally, we conduct a welfare analysis of the equilibria identified in our model. We confirm that social surplus flow (an instantaneous social surplus) increases as the proportion of non-savvy consumers decreases, regardless of the equilibrium type. As a benchmark, we show that if add-on services are homogeneous, the expected social welfare in equilibrium rises as the proportion of non-savvy consumers declines. However, when add-on services are vertically differentiated, the expected social welfare may decline in any type of equilibrium as the proportion of non-savvy consumers diminishes. We identify that the key factor driving this result is consumers' perceptions of add-on service quality, which lead the follower (the low-quality firm) to delay its investment timing and extend the duration of the leader's monopoly.[1] This suggests that consumers' savviness regarding add-on service quality can negatively impact social welfare by delaying firms' investment

[1] While the follower's delay is present in any type of equilibrium, the leader's investment timing can delay in a certain case of preemptive equilibrium. This delay also negatively affects the expected social welfare.

timings.

The findings of this paper recommend a policy mix of consumer protection and market-entry promotion. Consumer protection policies, such as information disclosure and consumer education, help consumers recognize the value of add-on services. These policies can enhance product value by enabling consumers to fully understand the costs and benefits of these services. However, our results indicate that consumers' perceptions of add-on services may deter firms from entering the market. Therefore, it is crucial to implement entry-promotion policies concurrently to prevent a reduction in social welfare.

This policy mix is clearly evident in mobile telecommunications markets. The OECD provides guidelines that address consumer protection issues, such as payment security and information disclosure, while also promoting competition and innovation.[2] Similarly, the EU Electronic Communications Code emphasizes the need for "better consumer protection" by promoting tariff transparency and facilitating the transfer of service providers without requiring changes to phone numbers, while also highlighting the importance of competition and innovation.[3] In Japan, consumer protection policies, which include the requirement of informative customer contracts and their detailed explanations, have been implemented alongside of measures to promote market entry, such as number portability and access rule setting, since the mid-2000s during the 3G network era.[4] As a result of this policy mix, mobile virtual network operators (MVNOs) have successfully entered the Japanese mobile telecommunications market and now operate along with mobile network operators (MNOs).[5]

The remainder of this paper is structured as follows. The next section discusses the related literature. Section 3 presents the model. Section 4 derives firms' profit flows

[2] See OECD (2014) for consumer protection policies in mobile telecommunications.

[3] See European Commission (2018).

[4] The Japanese mobile telecommunications market transitioned to the 5G network era around 2020. Since the 1G network era in the 1980s, consumer perception of add-on services was low until the early 2000s (2G network era). During that time, firms providing low-quality services could easily enter the market if they had their own telecom facilities, such as wireless base stations or towers.

[5] As of December 2023, there are 1,890 MVNOs and 4 MNOs operating in the Japanese mobile telecom market.

in a duopoly and in a monopoly. Section 5 describes the equilibria and shows how the proportion of non-savvy consumers affects the characterization of the equilibria. We conduct welfare analysis in Section 6. Section 7 offers concluding remarks.

2 Related Literature

This study belongs to two strands of literature: behavioral industrial organization and real options analysis.

Among the extensive literature on behavioral industrial organization, our model is based on Armstrong (2015).[6] Armstrong (2015) surveys a number of market models in which savvy and non-savvy consumers interact. Although our Hotelling duopoly model follows one of the models in Armstrong (2015) in which two symmetric sellers supply a core product and an add-on service, we substantially extend it in two ways. First, we introduce vertical quality differentiation in add-on services to focus on the effects of consumer savviness on competition among firms and its economic consequences, such as pricing and equilibrium conditions. Second, we extend Armstrong's model into a real options framework. This extension enables us to investigate how consumer savviness affects firms' investment timing decisions.[7]

Regarding the second extension, our model is closely related to Pawlina and Kort (2006), who analyze a duopoly investment game in which firms are asymmetric in terms of investment technology and characterize three kinds of entry equilibria: preemptive, sequential, and simultaneous equilibria. Instead of focusing on asymmetric technology, we demonstrate the influence of consumer savviness regarding the quality of add-on services on the equilibria and discuss its welfare implications. Similarly, Huisman and Kort (2015) also employ a real options model to study entry deterrence by considering firms'

[6] Heidhues and Köszegi (2018) provide a comprehensive survey of the literature.

[7] Investment timing by firms in various stochastic environments is analyzed in the real options literature. See Dixit and Pindyck (1994) for a systematic treatment of the real options approach and Smit and Trigeorgis (2004) for real options analyses in game-theoretic environments.

investment decisions in terms of timing and capacity level. They show that the entry deterrence domain increases with uncertainty because when uncertainty rises, a follower prefers to wait for information acquisition before investing, which implies an entrant's delay of investment. In their model, a critical factor in making the entry deterrence strategy profitable is the longer duration of a leader's monopoly. Although the logic and source of such long duration differ from our study, the prolonged duration of a leader's monopoly is also crucial to their welfare results. In our study, a high proportion of savvy consumers induces the prolonged duration of a leader's monopoly, which in turn leads to a decline in social welfare.

The findings regarding the relationship between the proportion of savvy consumers and social welfare relate to the literature on consumer protection policies, such as consumer education and mandatory disclosure. For example, Kosfeld and Schüwer (2017) analyze the effects of consumer education policies on prices and welfare in retail financial markets, particularly when some consumers are naive about shrouded add-on prices. In their model, a portion of naive consumers becomes informed through a bank's unshrouding activity; they then demonstrate that consumer education can have a negative impact on welfare.[8] Additionally, Ispano and Schwardmann (2023) analyze a model in which firms can decide whether to disclose quality. They find that while three consumer protection policies (i.e., mandatory disclosure, third-party disclosure, and consumer education) can enhance welfare under monopoly conditions, they may actually reduce welfare when vertically differentiated firms exploit disadvantaged consumers.

In contrast to these existing studies on consumer protection policies, our research does not focus on a firm's strategic decision regarding quality disclosure. Instead, we introduce a firm's investment decision in a dynamic environment and highlight a potential drawback of consumer protection policies from a welfare perspective, which arises from the delay in

[8] Kosfeld and Schüwer also show that if firms can price discriminate based on levels of consumer sophistication, educating naive consumers may lead to greater exploitation in a model with shrouded attributes. For more details on shrouded attribute models, see Gabaix and Laibson (2006) and Heidhues et al. (2017).

firms' investment timing. Consequently, we advocate for the effectiveness of a consumer protection policy combined with entry promotion to enhance social welfare, as commonly observed in mobile telecommunications markets.

3 The Model

Two firms, firm l and firm h, have an investment opportunity to enter a market of core goods along with add-on services. The core goods provided by the two firms are horizontally differentiated. In addition to core goods, consumers need add-on services such as technical support, maintenance services, spare parts provision, and optional services. We assume that the add-on services offered by the two firms are vertically differentiated: firm l provides low-quality services, while firm h provides high-quality services.

We consider a continuous-time model. At each time, consumers exist and they are distributed uniformly on the unit interval of $[0, 1]$ as described in a Hotelling model. After investing at any time, firm l is located at the extreme point 0, whereas firm h is located at the extreme point 1 of the unit interval. Each consumer is assumed to buy one unit of core goods. A consumer located at x ($\in [0, 1]$) obtains the basic willingness to pay, R, from a core good and incurs a disutility from traveling to buy the core good of firm l (resp. firm h), denoted by tx (resp. $t(1-x)$). All consumers recognize their willingness to pay for core goods.

After purchasing a core good, a consumer obtains an add-on service provided by the firm from which he/she buys the core good. At the price p of add-on services provided by firm l (resp. firm h), a consumer demands $q_l(p)$ (resp. $q_h(p)$) and gains the consumer surplus of $s_l(p) \equiv \int_p q_l(x)\,dx$ (resp. $s_h(p) \equiv \int_p q_h(x)\,dx$) from the add-on services. Because the add-on services are vertically differentiated, we assume that for any p, $q_l(p) < q_h(p)$ and $s_l(p) < s_h(p)$. To derive a firm's profit flow explicitly in the analysis below, we use a linear demand formulation for add-on services, i.e., $q_l(p) = a_l - p$ and $q_h(p) = a_h - p$, where $a_l < a_h$.

There are two kinds of consumers, *savvy* and *non-savvy*. We assume that a savvy consumer recognizes the existence and quality of add-on services, whereas a non-savvy consumer does not do. The proportion of savvy (resp. non-savvy) consumers in the population is θ (resp. $1-\theta$). Two firms do not know which consumers are savvy or non-savvy in the unit line. Thus, θ is interpreted as the probability that a consumer located at a point on the unit line is savvy.

Denoting the price of firm i's core good by P_i, $i = l, h$, the indirect utility of a consumer located at x is represented as follows:[9]

$$\begin{aligned} U_l(x) &= R - P_l + s_l(p_l) - tx \text{ when buying from firm } l, \\ U_h(x) &= R - P_h + s_h(p_h) - t(1-x) \text{ when buying from firm } h. \end{aligned}$$

We should note that a non-savvy consumer makes a purchasing decision of a core good without recognizing the existence and quality of the add-on services derived from the core good.

For analytical simplicity, a firm's unit production costs for core goods and add-on services are assumed to be zero.[10] We also assume that the investment cost to enter the market is the same for both firms, denoted by I.

Firm i's profit flow at each time, $\varphi_{it}(\mathbf{P}, \mathbf{p})$, where $\mathbf{P} \equiv (P_l, P_h)$ and $\mathbf{p} \equiv (p_l, p_h)$, consists of two parts: $\varphi_{it}(\mathbf{P}, \mathbf{p}) \equiv Y_t \Pi_i(\mathbf{P}, \mathbf{p})$, where Y_t represents the stochastic part of profit, while $\Pi_i(\mathbf{P}, \mathbf{p})$ represents the deterministic part. $\Pi_i(\mathbf{P}, \mathbf{p})$ depends on the market structure, which takes $\Pi_i^m(\mathbf{P}, \mathbf{p})$ (resp. $\Pi_i^d(\mathbf{P}, \mathbf{p})$) when firm i is a monopoly (resp. a duopoly). We interpret Y_t as the industry-wide common shock between the two firms,

[9] In our formulation, when core goods are durable goods, P_i should be interpreted as the prices paid by installment or their rental prices.

[10] Even when we introduce cost differentiation for add-on services between the two firms such that $c_l < c_h$, where c_i is the cost for firm i, the qualitative results of our analysis remain unchanged as long as $a_l - c_l < a_h - c_h$ holds.

and it follows a geometric Brownian motion:

$$dY_t = \alpha Y_t dt + \sigma Y_t dZ_t$$

where $dZ_t \sim N(0,\ dt)$. Here, $\alpha\ (> 0)$ is a drift parameter that represents the industry-wide growth, σ is an instantaneous standard deviation parameter, and Z_t is the Wiener increment that is normally distributed with mean zero and variance dt.

We should note that in our formulation, firm i's profit flow $\varphi_{it}(\mathbf{P}, \mathbf{p})$ takes a multiplicative form of the industry-wide common shock Y_t. Appendix E provides an example where firm i's profit flow takes this form and shows that both the consumer surplus flow and the social surplus flow are also expressed as a multiplicative form of Y_t.

The timing of events is as follows. First, given the proportion of savvy consumers θ, each of the two firms simultaneously determines its investment timing. Second, given firm i's position (i.e., whether it is a leader or a follower) determined by its investment timing, firm i sets the prices of the core good and the add-on service, and consumers buy the goods from one of the two firms, which determines the firms' profits.

4 Price Setting and Profit Flows

In this section, we derive the deterministic part of a firm i's profit flow Π_i, $i = l, h$, in a duopoly and a monopoly. To restrict our attention to an interior solution in an equilibrium, we make an assumption as follows:

Assumption 1 $t > \frac{5-4\theta}{6}\left[\left(\frac{a_h}{2-\theta}\right)^2 - \left(\frac{a_l}{2-\theta}\right)^2\right]$.

We also assume that the willingness to pay for core goods, R, is large enough for all consumers to buy a core good in equilibrium (full coverage).

4.1 Duopoly

First of all, we obtain the profit flow when the market is a duopoly. A savvy consumer (resp. a non-savvy consumer) that is indifferent between firms l and h, denoted by $\widehat{x}$ (resp. $\widehat{\widehat{x}}$), is represented by

$$\begin{aligned}\widehat{x} &= \frac{1}{2}+\frac{1}{2t}\left[\left(s_l\left(p_l\right)-P_l\right)-\left(s_h\left(p_h\right)-P_h\right)\right],\\ \widehat{\widehat{x}} &= \frac{1}{2}+\frac{1}{2t}\left(P_h-P_l\right).\end{aligned}$$

Then, firm l's price-decision problem is the following.

$$\max_{p_l,P_l} \Pi_l^d \equiv \left(P_l+\pi_l\left(p_l\right)\right)\left[\theta\widehat{x}+\left(1-\theta\right)\widehat{\widehat{x}}\right],$$

where $\pi_l\left(p_l\right)\equiv p_l q_l\left(p_l\right)$. From the first-order conditions with respect to p_l and P_l, we have

$$p_l q_l'\left(p_l\right)+\left(1-\theta\right)q_l\left(p_l\right)=0, \tag{1}$$

$$P_l+\pi_l\left(p_l\right)=2t\left[\theta\widehat{x}+\left(1-\theta\right)\widehat{\widehat{x}}\right]. \tag{2}$$

Similarly, firm h's price-decision problem is represented by

$$\max_{P_h,\ p_h} \Pi_h^d \equiv \left(P_h+\pi_h\left(p_h\right)\right)\left[\theta\left(1-\widehat{x}\right)+\left(1-\theta\right)\left(1-\widehat{\widehat{x}}\right)\right],$$

where $\pi_h\left(p_h\right)\equiv p_h q_h\left(p_h\right)$. From the two first-order conditions, we obtain the followings.

$$p_h q_h'\left(p_h\right)+\left(1-\theta\right)q_h\left(p_h\right)=0, \tag{3}$$

$$P_h+\pi_h\left(p_h\right)=2t\left[\theta\left(1-\widehat{x}\right)+\left(1-\theta\right)\left(1-\widehat{\widehat{x}}\right)\right]. \tag{4}$$

From (1) and (3), the equilibrium prices of add-on services are determined as follows:

$$p_l^* = \frac{(1-\theta)\,a_l}{2-\theta} \text{ and } p_h^* = \frac{(1-\theta)\,a_h}{2-\theta}.$$

Because $a_l < a_h$, $p_l^* < p_h^*$ holds. This implies that the price of high-quality add-on services is higher than that of low-quality add-on services, irrespective of the proportion of savvy consumers in the population θ.

Note that firm i's equilibrium price of add-on services depends only on θ, despite the price competition in a duopoly. In particular, as θ increases, p_i^* monotonically decreases. More specifically, at $\theta = 0$, the price of add-on services becomes a monopoly price, i.e., $p_i^* = \frac{a_i}{2}\,(\equiv \arg\max \pi_i\,(p_i))$. By contrast, at $\theta = 1$, the price of add-on services is a competitive (or efficient) price, i.e., $p_i^* = 0$.

We next derive the equilibrium prices of core goods. Substituting p_l^* and p_h^* into $\widehat{x}$ of (2) and (4), we obtain the simultaneous equations system with respect to P_l and P_h. Solving the system, we obtain the equilibrium prices of core goods as follows:

$$\begin{aligned} P_l^* &= t - \frac{1}{6}\left((4-5\theta)\,A_l(\theta) + (2-\theta)\,A_h(\theta)\right), \\ P_h^* &= t - \frac{1}{6}\left((2-\theta)\,A_l(\theta) + (4-5\theta)\,A_h(\theta)\right), \end{aligned}$$

where $A_i(\theta) \equiv \left(\frac{a_i}{2-\theta}\right)^2$, $i = l,\ h$. In contrast to the equilibrium prices of add-on services p_l^* and p_h^*, those of core goods reflect price competition in a duopoly; they are affected by the rival firm's pricing decision because each firm would like to grab as many consumers as it can.

Substituting the equilibrium prices into a firm's profit, we obtain the equilibrium profit of firm i $(= l,\ h)$ under duopoly, Π_i^{d*}, as follows:

$$\Pi_l^{d*} = \frac{1}{2t}\left(t - \frac{1}{6}(2-\theta)(A_h(\theta) - A_l(\theta))\right)^2 \equiv \Pi_l^{d*}(\theta), \tag{5}$$

$$\Pi_h^{d*} = \frac{1}{2t}\left(t + \frac{1}{6}(2-\theta)(A_h(\theta) - A_l(\theta))\right)^2 \equiv \Pi_h^{d*}(\theta). \tag{6}$$

From (5) and (6), the equilibrium profit of firm h is higher than that of firm l, regardless of θ. Differentiating (5) and (6) with respect to θ, we find the effect of the changes in θ on the profit flows of firms l and h as follows:

$$\frac{d\Pi_l^{d*}(\theta)}{d\theta} < 0,\ \frac{d\Pi_l^{d*2}(\theta)}{d\theta^2} > 0,\ \frac{d\Pi_h^{d*}(\theta)}{d\theta} > 0, \text{ and } \frac{d\Pi_h^{d*2}(\theta)}{d\theta^2} > 0.$$

(Insert Figure 1 around here.)

Figure 1 depicts the effect of a change in θ on the profit flows of firms l and h. To obtain the intuitive explanation of Figure 1, we report the impact of θ on the prices of add-on services and core goods, as well as on the equilibrium demand for firm l's good.

$$\begin{aligned}
\frac{dp_l^*}{d\theta} &< 0 \text{ and } \frac{dp_h^*}{d\theta} < 0,\\
\frac{dP_l^*}{d\theta} &= -\frac{1}{6(2-\theta)^3}\left[-(2+5\theta)(a_l)^2 + (2-\theta)(a_h)^2\right] \gtreqless 0,\\
\frac{dP_h^*}{d\theta} &= -\frac{1}{6(2-\theta)^3}\left[(2-\theta)(a_l)^2 - (2+5\theta)(a_h)^2\right] > 0,\\
D_l^* &\equiv \theta\widehat{x}^* + (1-\theta)\widehat{\widehat{x}}^*\\
&= \frac{1}{2} - \frac{2-\theta}{12t}(A_h(\theta) - A_l(\theta)), \qquad (7)\\
\text{where } \widehat{x}^* &= \frac{1}{2} - \frac{5-4\theta}{12t}(A_h(\theta) - A_l(\theta)), \text{ and}\\
\widehat{\widehat{x}}^* &= \frac{1}{2} + \frac{2\theta-1}{12t}(A_h(\theta) - A_l(\theta)).
\end{aligned}$$

Due to Assumption 1, $0 < \widehat{x}^* < 1$ and $0 < \widehat{\widehat{x}}^* < 1$ hold. From (7), we also confirm that $D_l^* < \frac{1}{2}$ for all $\theta \in [0,\ 1]$. This means that the market demand for firm h's good is larger than that for firm l's good, regardless of the proportion of savvy consumers θ. In addition, we obtain

$$\frac{\partial D_l^*}{\partial \theta} = -\frac{1}{12t}\left(A_h\left(\theta\right) - A_l\left(\theta\right)\right) < 0.$$

Thus, as the proportion of savvy consumers increases, the market demand for firm l's good decreases.

We then provide the intuitive explanation of Figure 1. As an extreme case, we consider the case in which all consumers are non-savvy, i.e., $\theta = 0$. Any consumer recognizes neither the existence nor the quality of add-on services, allowing both firms to set a monopoly price for their add-on services. Then, firm h can set a higher monopoly price for add-on services and achieve greater monopoly profit from the add-on services than firm l. Achieving the greater monopoly profit from add-on services incentivizes firm h to set a lower price for core good, thereby attracting more demand than firm l.

As θ increases from $\theta = 0$, a portion of consumers begins to recognize the existence and quality of add-on services. Then, firm h's profitable strategy is to increase the price of its core good and decrease the price of add-on services. At the same time, the price of firm l's core good increases due to the property of strategic complements.

As θ increases further, savvy consumers are still willing to accept a higher price for firm h's core good compared to that of firm l's. This is because firm h's add-on services exhibit high quality, and the price of firm h's add-on services is low enough to compensate for the high price of its core good. In fact, in the extreme case of $\theta = 1$, although $P_h^* > P_l^*$, firm h can attract a larger share of savvy consumers than firm l (i.e., $D_l^* < \frac{1}{2}$) with the competitive add-on service price (i.e., $p_h^* = p_l^* = 0$). As a result, the equilibrium profit of firm h is higher than that of firm l, regardless of the proportion of savvy consumers θ.

We summarize the results regarding firms' pricing strategies in the following proposition.

Proposition 1 *(i) When the proportion of non-savvy consumers in the population is high (resp. low), the price of the core good offered by a firm with high-quality add-on services is lower (resp. higher) than that offered by a firm with low-quality add-on services. (ii) As the proportion of non-savvy consumers in the population decreases, the price of add-on services declines. Specifically, the add-on price is the monopoly price when all consumers are non-savvy, whereas it is the competitive (or efficient) price when all consumers are savvy.*

4.2 Monopoly

We assume that in the case of monopoly, a firm that enters the market locates at the extreme point 0 of the unit interval irrespective of the quality of add-on services.

When a consumer located at x buys the monopolist firm i's core good, his/her indirect utility is represented by

$$U_i(x) = R - P_i + s_i(p_i) - tx, \quad i = l,\ h.$$

Assuming that the firm can maximize its profit when it sets prices such that all consumers buy its good, its profit-maximization problem is formulated as follows:

$$\max_{P_i,\ p_i} \Pi_i^m \equiv P_i + \pi_i(p_i)$$

$$s.t.\ \ R - P_i + s_i(p_i) - t \geq 0 \text{ and } R - P_i - t \geq 0, \quad i = l,\ h.$$

It is obvious that the participation constraint for non-savvy consumers is only binding.

Then, the prices set by a firm, $\{P_i^{m*},\ p_i^{m*}\}$, are derived as follows:

$$\begin{aligned} P_i^{m*} &= R - t, \\ p_i^{m*} &= \frac{a_i}{2} \equiv p_i^M \left(= \arg\max \pi_i \left(p_i\right)\right). \end{aligned}$$

We should note that the price of core goods becomes the same between the two firms, irrespective of the quality difference in add-on services.

Substituting these prices into firm i's monopoly profit, the equilibrium profit Π_i^{m*} is represented as follows:

$$\Pi_i^{m*} = R - t + \pi_i \left(p_i^M\right). \tag{8}$$

Equation (8) implies that the equilibrium monopoly profit Π_i^{m*} is not affected by the proportion of savvy consumers. In addition, $\Pi_l^{m*} < \Pi_h^{m*}$ holds.

5 Investment Timing and Equilibria

This section analyzes the investment timing decisions made by the two firms.[11]

5.1 Value Functions

We derive value functions when a firm becomes a follower or a leader.

5.1.1 Follower

We consider a value function when a firm becomes a follower. Supposing that firm h has already invested, firm l's value function as a follower is represented by

$$V_l^F(Y) = \begin{cases} \left[\frac{Y_l^{F*}\Pi_l^{d*}(\theta)}{r-\alpha} - I\right]\left(\frac{Y}{Y_l^{F*}}\right)^{\beta} & \text{if } Y < Y_l^{F*} \\ \frac{Y\Pi_l^{d*}(\theta)}{r-\alpha} - I \ \text{ if } \ Y \geq Y_l^{F*}, \end{cases} \tag{9}$$

[11]The analytical methods in this section are based on Dixit and Pindyck (1994) and Pawlina and Kort (2006).

where $\beta = \frac{1}{2}\left(1 - \frac{2\alpha}{\sigma^2} + \sqrt{\left(1 - \frac{2\alpha}{\sigma^2}\right)^2 + \frac{8r}{\sigma^2}}\right)$. In (9), Y_l^{F*} represents the threshold for firm l's investment timing as a follower. This threshold is defined in terms of the level of industry-wide shock Y, which is characterized by

$$Y_l^{F*}(\theta) = \frac{\beta}{\beta - 1}\frac{r - \alpha}{\Pi_l^{d*}(\theta)}I. \tag{10}$$

Similarly, when firm h becomes a follower, its value function is the following.

$$V_h^F(Y) = \begin{cases} \left[\frac{Y_h^{F*}\Pi_h^{d*}(\theta)}{r-\alpha} - I\right]\left(\frac{Y}{Y_h^{F*}}\right)^\beta & \text{if } Y < Y_h^{F*} \\ \frac{Y\Pi_h^{d*}(\theta)}{r-\alpha} - I \ \text{ if } \ Y \geq Y_h^{F*}, \end{cases} \tag{11}$$

Y_h^{F*} is the threshold of firm h's investment timing as a follower, and it is characterized by

$$Y_h^{F*}(\theta) = \frac{\beta}{\beta - 1}\frac{r - \alpha}{\Pi_h^{d*}(\theta)}I. \tag{12}$$

From (10) and (12), we verify that for all $\theta \in [0,\ 1]$, $Y_h^{F*}(\theta) < Y_l^{F*}(\theta)$ because $\Pi_l^{d*}(\theta) < \Pi_h^{d*}(\theta)$. This means that, as a follower, firm h always enters the market earlier than firm l.

5.1.2 Leader

We characterize a value function when a firm becomes a leader. Suppose that firm l has an incentive to immediately enter the market as a leader (i.e., it has a preemptive incentive). Then, given firm h's threshold of investment timing as a follower Y_h^{F*}, firm l's value function as a leader is the following.

$$V_l^L(Y) = \begin{cases} \frac{Y\Pi_l^{m*}}{r-\alpha} - I + \frac{Y_h^{F*}\left(\Pi_l^{d*}(\theta) - \Pi_l^{m*}\right)}{r-\alpha}\left(\frac{Y}{Y_h^{F*}}\right)^\beta & \text{if } Y < Y_h^{F*} \\ \frac{Y\Pi_l^{d*}(\theta)}{r-\alpha} - I \ \text{ if } \ Y \geq Y_h^{F*}. \end{cases} \tag{13}$$

Note that when $Y \leq Y_h^{F*}$, $V_l^L(Y)$ is a concave function of Y.

Similarly, given firm l's threshold of investment timing as a follower Y_l^{F*}, firm h's value function as a leader when it has a preemptive incentive is as follows:

$$V_h^L(Y) = \begin{cases} \frac{Y\Pi_h^{m*}}{r-\alpha} - I + \frac{Y_l^{F*}\left(\Pi_h^{d*}(\theta)-\Pi_h^{m*}\right)}{r-\alpha}\left(\frac{Y}{Y_l^{F*}}\right)^{\beta} & \text{if } Y < Y_l^{F*} \\ \frac{Y\Pi_h^{d*}(\theta)}{r-\alpha} - I & \text{if } Y \geq Y_l^{F*} \end{cases} \tag{14}$$

As in the case of firm l, $V_h^L(Y)$ is a concave function of Y when $Y < Y_l^{F*}$.

In our model, the value functions as a follower (or a leader) are asymmetric between firm l and firm h due to the difference of profit flows in a duopoly. Then, it is not always the case that each of the two firms has a preemptive incentive when it becomes a leader, as shown in Pawlina and Kort (2006). Hence, the derivation of a leader's investment timing is deferred in the next subsection.

5.2 Two Types of Equilibrium

In our model, depending on the values of each firm as a follower and a leader, two types of equilibrium can occur: a *preemptive equilibrium* and a *semi-preemptive equilibrium*. We characterize both types of equilibrium.

5.2.1 Preemptive equilibrium

The preemptive equilibrium occurs when two firms have an incentive to preempt the market. This situation is characterized by the stipulation that each firm has a certain range of Y that satisfies the following condition before its rival enters the market as a follower.

$$\xi_i(Y) \equiv V_i^L(Y) - V_i^F(Y) > 0, \quad i = l,\ h.$$

This condition indicates that both firms l and h have an incentive to become a leader rather than a follower at Y. We already verify that as a follower, firm h enters the market earlier than firm l (i.e., $Y_h^{F*} < Y_l^{F*}$). Then, since $\Pi_l^{m*} < \Pi_h^{m*}$, $\Pi_l^{d*}(\theta) < \Pi_h^{d*}(\theta)$ for any

θ, and $V_i^L(Y)$ is concave in Y for $Y < Y_j^{F*}$ $(j \neq i)$, we derive the following condition.

$$\xi_h\left(Y_l^P\right) \equiv V_h^L\left(Y_l^P\right) - V_h^F\left(Y_l^P\right) > 0, \tag{15}$$

where Y_l^P is the smallest solution of $\xi_l(Y) = 0$. The condition (15) implies that firm h becomes a leader if both firms have an incentive to preempt the market. Then, firm h's investment timing as a leader, Y_h^{L*}, is characterized by

$$Y_h^{L*} = \min\left\{Y_l^P,\ \widetilde{Y}_h^{L*}\right\},$$

where $\widetilde{Y}_h^{L*}$ is represented by

$$\widetilde{Y}_h^{L*} = \frac{\beta}{\beta-1}\frac{r-\alpha}{\Pi_h^{m*}}I.$$

$\widetilde{Y}_h^{L*}$ is the threshold of investment timing in the situation where firm h is pre-determined as a leader and it takes advantage of the option value as a monopoly. After firm h enters the market as a leader at Y_h^{L*}, firm l enters the market at Y_l^{F*} as a follower. Figure 2 illustrates the preemptive equilibrium in the case where $Y_h^{L*} = Y_l^P (< \widetilde{Y}_h^{L*})$.

(Insert Figure 2 around here.)

Firm h's investment timing as a leader, $Y_h^{L*} = \min\left\{Y_l^P,\ \widetilde{Y}_h^{L*}\right\}$, is explained as follows. First, when $Y_l^P < \widetilde{Y}_h^{L*}$, firm l has an incentive to preempt the market for $Y > Y_l^P$, whereas it does not do so for $Y < Y_l^P$. Hence, firm h's optimal investment timing as a leader is $Y_h^{L*} = Y_l^P$ because $V_h^L(Y)$ is an increasing function of Y. Next, when $Y_l^P > \widetilde{Y}_h^{L*}$, firm h can enjoy the option value of waiting to obtain a monopoly profit because firm l does not have an incentive to enter the market for all $Y < Y_l^P$. As a result, $Y_h^{L*} = \widetilde{Y}_h^{L*}$.

We should also note that Y_l^P is influenced by the proportion of savvy consumers θ, because both of the value functions $V_l^L(.)$ and $V_l^F(.)$ are influenced by θ. Hence, we denote it by $Y_l^P(\theta)$, hereafter. By contrast, $\widetilde{Y}_h^{L*}$ is not influenced by θ. This is because the profit flow in a monopoly Π_h^{m*} is unaffected by θ. These properties are revisited in the welfare analysis of Section 6.

5.2.2 Semi-preemptive equilibrium

Semi-preemptive equilibrium occurs when one of the two firms does not have an incentive to preempt the market. The condition that firm i does not have an incentive to preempt the market is characterized by

$$\xi_i(Y) < 0 \text{ for } Y \in \left[Y_0,\ Y_j^{F*}\right],\ i,\ j = l,\ h, \text{ and } i \neq j,$$

where Y_0 is the initial value of Y.

Because $\Pi_l^{m*} < \Pi_h^{m*}$, $\Pi_l^{d*}(\theta) < \Pi_h^{d*}(\theta)$ for any θ, and $V_i^L(Y)$ is concave in Y for $Y < Y_j^{F*}$ $(j \neq i)$, we can verify that firm h has a preemptive incentive for $Y < Y_l^{F*}$ at any θ, whereas there exists a case in which firm l does not have an incentive to preempt the market. Thus, there exists only the semi-preemptive equilibrium in which firm h is a leader and firm l is a follower. Formally, in the semi-preemptive equilibrium, we have

$$\xi_l(Y) \equiv V_l^L(Y) - V_l^F(Y) < 0 \text{ for any } Y \in \left[Y_0,\ Y_h^{F*}\right]. \tag{16}$$

Thus, in the semi-preemptive equilibrium, firm h's investment timing as a leader is $\widetilde{Y}_h^{L*}$, because the equilibrium is the same as the situation in which firm h's role is predetermined as a leader.

Figure 3 illustrates the semi-preemptive equilibrium. From this figure, we can infer that a semi-preemptive equilibrium is likely to occur when there is a significant difference between $\Pi_l^{d*}(\theta)$ and $\Pi_h^{d*}(\theta)$. This suggests that a semi-preemptive equilibrium is more likely to occur when θ is large, as indicated by Figure 1. We will further examine this inference in the analysis of the next subsection.

(Insert Figure 3 around here.)

5.3 Equilibrium Configuration

5.3.1 Threshold between preemptive and semi-preemptive equilibria

In this section, we examine the conditions under which a particular type of equilibrium occurs. To do so, we define *the relative magnitude of duopoly profits*, κ^d, as follows:

$$\kappa^d \equiv \frac{\Pi_l^{d*}}{\Pi_h^{d*}} \left(< 1\right).$$

The relative magnitude of duopoly profits κ^d depends on a_h, a_l, and θ. In particular, by taking a_h and a_l as given and differentiating κ^d with respect to θ, we obtain

$$\frac{d\kappa^d}{d\theta} \left(= \frac{d\kappa^d\left(\theta\right)}{d\theta} \right) = \frac{\Pi_l^{d*\prime}\left(\theta\right)\Pi_h^{d*}\left(\theta\right) - \Pi_l^{d*}\left(\theta\right)\Pi_h^{d*\prime}\left(\theta\right)}{\left(\Pi_h^{d*}\left(\theta\right)\right)^2} < 0,$$

because $\Pi_l^{d*\prime}\left(\theta\right) < 0$ and $\Pi_h^{d*\prime}\left(\theta\right) > 0$.

As described in (16), the condition for the semi-preemptive equilibrium with firm h as the leader is as follows:

$$\xi_l\left(Y\right) \equiv V_l^L\left(Y\right) - V_l^F\left(Y\right) < 0 \text{ for any } Y \in \left[Y_0,\ Y_h^{F*}\right].$$

When (16) does not hold, only the preemptive equilibrium occurs. Hence, to derive the threshold between the two equilibria, we need to identify a pair $\left\{Y^*,\ \kappa^{d*}\right\}$ that meets the following two requirements:[12]

$$\xi_l\left(Y^*; \kappa^{d*}\right) = 0 \quad (17)$$

$$\left.\frac{\partial \xi_l\left(Y; \kappa^{d*}\right)}{\partial Y}\right|_{Y=Y^*} = 0 \quad (18)$$

where κ^{d*} represents the threshold level of the relative magnitude of duopoly profits

[12] We follow the procedure outlined in Appendix B of Pawlina and Kort (2006) for the analysis presented here.

between the preemptive equilibrium and the semi-preemptive equilibrium. From Figures 2 and 3, we verify that a semi-preemptive equilibrium occurs at a given θ when $\kappa^d(\theta) < \kappa^{d*}$, because the profit difference between firm h and firm l is larger than that at the threshold κ^{d*} between the preemptive and semi-preemptive equilibria. On the other hand, a preemptive equilibrium occurs at θ when $\kappa^d(\theta) > \kappa^{d*}$.

Using (17) and (18), we characterize κ^{d*} for any $\theta \in [0,\ 1]$, as shown in the following lemma.

Lemma 1 *Given θ, the threshold between a preemptive equilibrium and a semi-preemptive equilibrium, κ^{d*}, is characterized by*

$$\left(\kappa^{d*}\right)^{\beta} - \beta\kappa^{d*} + \beta\chi(\theta) - (\chi(\theta))^{\beta} = 0, \tag{19}$$

where $\chi(\theta) \equiv \frac{\Pi_l^{m*}}{\Pi_h^{d*}(\theta)}$. Then, $\frac{d\kappa^{d*}}{d\theta} > 0$ holds. In addition, κ^{d*} is uniquely determined for any $\theta \in [0,\ 1]$.

Proof. See Appendix A. ■

In Lemma 1, (19) represents the implicit function form of κ^{d*}. Hence, from (19), we obtain $\kappa^{d*} = \kappa^{d*}(\theta,\ a_h,\ a_l)$. In particular, the threshold κ^{d*} is a function of θ, which implies that $\kappa^{d*}(\theta)$ is the threshold of the relative magnitude of duopoly profits between the preemptive and semi-preemptive equilibria when taking θ as given. We also verify that $\kappa^{d*}(\theta)$ is uniquely determined at any θ. Furthermore, we recognize that the threshold $\kappa^{d*}(\theta)$ is increasing in θ. Then, the threshold $\kappa^{d*}(\theta)$ is not generically identical to $\kappa^d(\theta)$ at a given θ, as stated above. Hence, we need to check the positioning of $\kappa^{d*}(\theta)$ relative to $\kappa^d(\theta)$ for any $\theta \in [0,\ 1]$, which is investigated in the next subsection.

5.3.2 Conditions for the existence of preemptive and semi-preemptive equilibria

We investigate the positioning of $\kappa^{d*}(\theta)$ relative to $\kappa^{d}(\theta)$ for $\theta \in [0,\ 1]$ to clarify the configuration of the two types of equilibria.

(Insert Figure 4 around here.)

As the following lemma states, we identify three distinct cases of equilibrium configuration based on the magnitudes of firms' profit flows. One case is illustrated in Figure 4. In the figure, $\kappa^{d}(\theta)$ is the relative magnitude of duopoly profits when taking a_h and a_l as given, whereas $\kappa^{d*}(\theta)$ represents the threshold level of the relative magnitude of duopoly profits between the preemptive and semi-preemptive equilibria for any $\theta \in [0,\ 1]$. Specifically, Figure 4 illustrates a case in which there exists a threshold of $\widehat{\theta} \in (0,\ 1)$ that separates preemptive equilibria from semi-preemptive equilibria. We also have two other cases: one is the case in which only the preemptive equilibria occur for all $\theta \in [0,\ 1]$, while the other is the case in which only the semi-preemptive equilibria occur for all $\theta \in [0,\ 1]$. These results are summarized in the following proposition.

Lemma 2 *There are two types of equilibria: preemptive equilibria and semi-preemptive equilibria, with firm h as the leader and firm l as the follower. The following three cases of equilibrium configuration arise depending on the magnitude of profit flows.*
(i) The preemptive equilibrium (resp. the semi-preemptive equilibrium) occurs for $\theta < \widehat{\theta}$ *(resp.* $\theta > \widehat{\theta}$*) when*

$$\beta\left(\Pi_h^{d*}(1)\right)^{\beta-1}\left(\Pi_l^{m*}-\Pi_l^{d*}(1)\right) > \left(\Pi_l^{m*}\right)^{\beta}-\left(\Pi_l^{d*}(1)\right)^{\beta} \ \text{and} \tag{20}$$

$$\beta\left(\Pi_h^{d*}(0)\right)^{\beta-1}\left(\Pi_l^{m*}-\Pi_l^{d*}(0)\right) < \left(\Pi_l^{m*}\right)^{\beta}-\left(\Pi_l^{d*}(0)\right)^{\beta}. \tag{21}$$

(ii) The preemptive equilibrium occurs for all $\theta \in [0,\ 1]$ when

$$\beta\left(\Pi_h^{d*}(1)\right)^{\beta-1}\left(\Pi_l^{m*}-\Pi_l^{d*}(1)\right) \leq \left(\Pi_l^{m*}\right)^{\beta}-\left(\Pi_l^{d*}(1)\right)^{\beta}. \tag{22}$$

(iii) The semi-preemptive equilibrium occurs for all $\theta \in [0,\ 1]$ when

$$\beta\left(\Pi_h^{d*}(0)\right)^{\beta-1}\left(\Pi_l^{m*}-\Pi_l^{d*}(0)\right) \geq \left(\Pi_l^{m*}\right)^{\beta}-\left(\Pi_l^{d*}(0)\right)^{\beta}. \tag{23}$$

Proof. See Appendix B. ■

Lemma 2 indicates that the equilibrium configuration depends on the proportion of savvy consumers θ. Specifically, in case (i) where two types of equilibria coexist within the range of $\theta \in [0,\ 1]$, a preemptive equilibrium occurs when θ is small, while a semi-preemptive equilibrium occurs when θ is large.

Indeed, the equilibrium configuration depends on other factors beyond the proportion of savvy consumers. Notably, as stated in the following proposition, we can re-expound Lemma 2 by focusing on the degree of quality difference in add-on services, which is crucial for the welfare analysis in the next section. Additionally, we verify that an increase in the proportion of savvy consumers θ causes the equilibrium to shift from a preemptive equilibrium to a semi-preemptive equilibrium.

Proposition 2 *Define $\delta \equiv a_h - a_l\ (> 0)$. There exist two thresholds, δ_1 and δ_0, such that:*

(i) When $\delta \in (\delta_1,\ \delta_0)$, the preemptive equilibrium occurs for $\theta < \widehat{\theta}$, while the semi-preemptive equilibrium occurs for $\theta > \widehat{\theta}$.

(ii) When $\delta \leq \delta_1$, the preemptive equilibrium occurs for all $\theta \in [0,\ 1]$.

(iii) When $\delta \geq \delta_0$, the semi-preemptive equilibrium occurs for all $\theta \in [0,\ 1]$.

In addition, the preemptive (resp. the semi-preemptive) equilibrium is less (resp. more) likely to occur as the proportion of savvy consumers θ increases.

Proof. See Appendix C. ■

(Figure 5 around here.)

Figure 5 summarizes the statements of Proposition 2. In the figure, the curve represents the locus of κ^{d*}, i.e., the threshold between a preemptive equilibrium and a semi-preemptive equilibrium, in $(\theta,\ \delta)$ space. Proposition 2 states that when the quality difference in add-on services is small (resp. large), a preemptive (resp. semi-preemptive) equilibrium occurs, regardless of the proportion of savvy consumers. For intermediate values of the quality difference, a preemptive (resp. semi-preemptive) equilibrium occurs when the proportion of savvy consumers is low (resp. high). Furthermore, in terms of the proportion of savvy consumers, a preemptive (resp. semi-preemptive) equilibrium is less (resp. more) likely to occur as the proportion of savvy consumers θ increases.

In general, when the proportion of non-savvy consumers is high or the quality difference in add-on services is small, a preemptive equilibrium is likely to occur, as illustrated in Figure 5. In contrast, when the proportion of non-savvy consumers is low or the quality difference is large, a semi-preemptive equilibrium is likely to occur.

6 Welfare Analysis

In this section, we examine how the proportion of savvy consumers influences the expected social welfare in our model.

6.1 Benchmark: Homogeneous Add-On Services

Before conducting welfare analysis for the equilibria derived in Section 5, we examine the case where add-on services are homogeneous between the two firms, referred to as the case of *homogeneous add-on services*, as a benchmark. By comparing the equilibrium in this case with the equilibria derived in Section 5, we can clarify why an increase in the proportion of savvy consumers may reduce social welfare in those equilibria.

In the case of homogeneous add-on services, we represent the demand for add-on services by $q(p) = a - p$. We then obtain the following proposition.

Proposition 3 *When add-on services are homogeneous, the expected social welfare increases with the proportion of savvy consumers.*

Proof. See Appendix D. ■

As shown in Appendix D, the social surplus flow in a duopoly increases with the proportion of savvy consumers θ, while in a monopoly, the social surplus flow is independent of θ. In addition, the investment timings for both the leader and the follower do not depend on θ. The independence of investment timings from θ is reflected in the stochastic discount factors, which do not vary with θ either.[13] Combining these factors, we confirm that when add-on services are homogeneous, the expected social welfare increases as the proportion of savvy consumers increases.

6.2 Vertically Differentiated Add-On Services

We turn to the case of vertically differentiated add-on services.

6.2.1 Social surplus flow

First, we derive the social surplus flow in a duopoly. Two firms' profit flows are already presented by (5) and (6) in Section 4. We then derive the consumer surplus in a duopoly. Substituting the equilibrium prices into savvy and non-savvy consumers' indirect utility

[13] In the case of homogeneous add-on services, the stochastic discount factors are represented by $\left(\frac{Y}{Y^P}\right)^\beta$ and $\left(\frac{Y}{Y^{F\#}}\right)^\beta$ (see Appendix D). For the definition of stochastic discount factors, refer to footnote 6 of Huisman and Kort (2015).

and rearranging them, we obtain the consumer surplus in a duopoly as follows:

$$\begin{aligned}
U^{d*}(\theta) &\equiv \theta U^{S*}(\theta) + (1-\theta) U^{N*}(\theta) \\
&= \theta \left\{ \int_0^{\widehat{x}^*} \left[R - P_l^* + s_l(p_l^*) - tx\right] dx + \int_{\widehat{x}^*}^1 \left[R - P_h^* + s_h(p_h^*) - t(1-x)\right] dx \right\} \\
&\quad + (1-\theta) \left\{ \int_0^{\widehat{\widehat{x}}^*} \left[R - P_l^* + s_l(p_l^*) - tx\right] dx + \int_{\widehat{\widehat{x}}^*}^1 \left[R - P_h^* + s_h(p_h^*) - t(1-x)\right] dx \right\} \\
&= R - \frac{5}{4}t + \frac{3-2\theta}{4}\left(A_h(\theta) + A_l(\theta)\right) \\
&\quad + \frac{1}{144t}\left(\theta(5-4\theta)^2 - 3(1-\theta)^2(2\theta-1)\right)\left(\Lambda(\theta)\right)^2, \qquad (24)
\end{aligned}$$

where $\Lambda(\theta) \equiv A_h(\theta) - A_l(\theta)$. Thus, the social surplus flow in a duopoly, $SS^{d*}(\theta)$, is described as follows:

$$\begin{aligned}
SS^{d*}(\theta) &\equiv U^{d*}(\theta) + \Pi_l^{d*}(\theta) + \Pi_h^{d*}(\theta) \\
&= R - \frac{1}{4}t + \frac{3-2\theta}{4}\left(A_h(\theta) + A_l(\theta)\right) \\
&\quad + \frac{1}{144t}\left(10\theta^3 - 21\theta^2 - 3\theta + 19\right)\left(\Lambda(\theta)\right)^2. \qquad (25)
\end{aligned}$$

Then, we have $\frac{dSS^{d*}(\theta)}{d\theta} > 0$, since

$$\begin{aligned}
\frac{dSS^{d*}(\theta)}{d\theta} &= \frac{1}{4}\left\{-2\left(A_h(\theta) + A_l(\theta)\right) + (3-2\theta)\left(A_h'(\theta) + A_l'(\theta)\right)\right\} \\
&\quad + \frac{1}{144t}\left\{\left(30\theta^2 - 42\theta - 3\right)\left(\Lambda(\theta)\right)^2 + 2\left(10\theta^3 - 21\theta^2 - 3\theta + 19\right)\Lambda(\theta)\Lambda'(\theta)\right\} \\
&= \frac{1-\theta}{2(2-\theta)}\left(A_h(\theta) + A_l(\theta)\right) + \frac{\left(\Lambda(\theta)\right)^2}{2-\theta}\left(10\theta^3 + 18\theta^2 - 93\theta + 70\right), \qquad (26)
\end{aligned}$$

where $10\theta^3 + 18\theta^2 - 93\theta + 70 > 0$ for all $\theta \in [0,\ 1]$. This means that an increase in the proportion of savvy consumers θ contributes to the enhancement of social surplus flow, as shown in the case of homogeneous add-on services.

Next, we derive the social surplus flow in a monopoly, $SS^{m*}(\theta)$. As mentioned in Section 5, firm h always becomes a leader when add-on services are vertically differentiated, irrespective of the level of θ. Hence, the monopoly's profit is represented by

$$\Pi_h^{m*} = R - t + \pi_h\left(p^M\right),$$

where $\pi_h\left(p^M\right) = \frac{(a_h)^2}{4}$. The consumer surplus flow in a monopoly $U^{m*}(\theta)$ is

$$\begin{aligned} U^{m*}(\theta) &\equiv \theta U^{mS*}(\theta) + (1-\theta)\, U^{mN*}(\theta) \\ &= \int_0^1 \left[R - P^{m*} + s_h\left(p^M\right) - tx\right] dx \\ &= \int_0^1 \left[t + s_h\left(p^M\right) - tx\right] dx \\ &= \frac{1}{2}t + s_h\left(p^M\right) \equiv U^{m*}, \text{ where } s_h\left(p^M\right) = \frac{(a_h)^2}{8}. \end{aligned}$$

Then, the social surplus flow in a monopoly $SS^{m*}(\theta)$ is obtained as follows:

$$\begin{aligned} SS^{m*}(\theta) &\equiv U^{m*} + \Pi_h^{m*} \\ &= R - \frac{1}{2}t + \frac{3(a_h)^2}{8} \equiv SS^{m*}. \end{aligned} \quad (27)$$

Here, we should note that in contrast to the case of homogeneous add-on services, the social surplus flow in a monopoly may be larger than that in a duopoly; $SS^{m*}(\theta) > SS^{d*}(\theta)$. This is because only firm h provides goods to consumers in a monopoly, while not only firm h but also firm l provides goods to consumers in a duopoly. In fact, this situation occurs when the proportion of savvy consumers is small, as shown in the numerical examples presented in the next subsection.

6.2.2 Expected social welfare

As analyzed in Section 5, there are two types of equilibrium: preemptive equilibria and semi-preemptive equilibria. The follower's (firm l's) investment timing is represented by eq.(10) in both types of equilibria. By contrast, the leader's (firm h's) investment timing differs between the two types when $Y_h^{L*} = Y_l^P(\theta)$ in the preemptive equilibrium. Otherwise, it is the same between the two types of equlibria: $Y_h^{L*} = \widetilde{Y}_h^{L*}$.

These observations require us to consider two distinct cases of expected social welfare. One case is the expected social welfare in the semi-preemptive equilibrium and the preemptive equilibrium with $Y_h^{L*} = \widetilde{Y}_h^{L*}$, where the investment timing of each firm is identically represented between the two types of equilibrium. The other case is the expected social welfare in the preemptive equilibrium with $Y_h^{L*} = Y_l^P$, where the leader's investment timing differs between the preemptive and semi-preemptive equilibria. We denote the expected social welfare for the former and latter cases by $\widetilde{W}^*(Y;\theta)$ and $W^*(Y;\theta)$, respectively. These are represented as follows:

$$\begin{aligned}\widetilde{W}^*(Y;\theta) &= \left(\frac{\widetilde{Y}_h^{L*} SS^{m*}}{r-\alpha} - I\right)\left(\frac{Y}{\widetilde{Y}_h^{L*}}\right)^{\beta} \\ &+ \left(\frac{Y_l^{F*}(\theta)\left(SS^{d*}(\theta) - SS^{m*}\right)}{r-\alpha} - I\right)\left(\frac{Y}{Y_l^{F*}(\theta)}\right)^{\beta}.\end{aligned}$$

$$\begin{aligned}W^*(Y;\theta) &= \left(\frac{Y_l^P(\theta)\, SS^{m*}}{r-\alpha} - I\right)\left(\frac{Y}{Y_l^P(\theta)}\right)^{\beta} \\ &+ \left(\frac{Y_l^{F*}(\theta)\left(SS^{d*}(\theta) - SS^{m*}\right)}{r-\alpha} - I\right)\left(\frac{Y}{Y_l^{F*}(\theta)}\right)^{\beta}.\end{aligned}$$

The difference between $\widetilde{W}^*(Y;\theta)$ and $W^*(Y;\theta)$ stems solely from the leader's investment timing Y_h^{L*}. As mentioned in Section 5.2, $Y_l^P(\theta)$ in $W^*(Y;\theta)$ is affected by the proportion of savvy consumers θ, whereas $\widetilde{Y}_h^{L*}$ in $\widetilde{W}^*(Y;\theta)$ is not. This distinction is reflected in the first term of $\widetilde{W}^*(Y;\theta)$ and $W^*(Y;\theta)$.

As a final remark, the stochastic discount factors, $\left(\frac{Y}{Y_l^{F*}(\theta)}\right)^\beta$ and $\left(\frac{Y}{Y_l^{P}(\theta)}\right)^\beta$, are influenced by θ. This complicates the welfare evaluation of θ considerably. Therefore, to analyze the effect of θ on expected social welfare, we conduct a numerical simulation. The main result from the simulation is summarized in the following proposition.

Proposition 4 *When add-on services are vertically differentiated between the two firms, the expected social welfare can decrease as the proportion of savvy consumers increases.*

(Insert Figures 6-1, 6-2, and 6-3 around here.)

To understand the reasoning behind this result, we present three numerical examples with three different levels of $a_h \in \{3,\ 3.8,\ 4\}$ and the following set of other parameters: $\alpha = 0.015$, $r = 0.05$, $\sigma = 0.1$, $I = 100$, $R = 8$, $t = 3$, $Y_0 = 0.2$, and $a_l = 2$.[14]

Figure 6-1 illustrates the case where $a_h = 4$. In this case, there exist the preemptive equilibria with $Y_h^{L*} = \widetilde{Y}_h^{L*}$ for $\theta \in [0, 0.64)$ and the semi-preemptive equilibria for $\theta \in [0.64, 1]$. The expected social welfare in this case is $\widetilde{W}^*(Y;\theta)$ for all $\theta \in [0,1]$. Differentiating $\widetilde{W}^*(Y;\theta)$ with respect to θ, we have

$$\begin{aligned}\frac{\partial \widetilde{W}^*(Y;\theta)}{\partial\theta} &= \frac{1}{r-\alpha}\left(\frac{Y}{Y_l^{F*}(\theta)}\right)^\beta \left\{Y_l^{F*\prime}(\theta)\left(SS^{d*}(\theta)-SS^{m*}\right)+Y_l^{F*}(\theta)\,SS^{d*\prime}(\theta)\right\}\\ &\quad -\beta Y\left(\frac{Y_l^{F*\prime}(\theta)\left(SS^{d*}(\theta)-SS^{m*}\right)}{r-\alpha}-I\right)\left(\frac{Y}{Y_l^{F*}(\theta)}\right)^{\beta-1}\left(Y_l^{F*}(\theta)\right)^{-2}Y_l^{F*\prime}(\theta)\\ &= \left(\frac{Y}{Y_l^{F*}(\theta)}\right)^\beta \times\left[\frac{Y_l^{F*}(\theta)}{r-\alpha}SS^{d*\prime}(\theta)\right.\\ &\quad\left.+\frac{\beta Y_l^{F*\prime}(\theta)}{Y_l^{F*}(\theta)}I-\frac{(\beta-1)\,Y_l^{F*\prime}(\theta)}{r-\alpha}\left(SS^{d*}(\theta)-SS^{m*}\right)\right]. \qquad (28)\end{aligned}$$

[14] These parameter values satisfy the assumption made in Section 4 and the market full-coverage condition.

We identify three distinct channels through which an increase in θ affects $\widetilde{W}^*(Y;\theta)$. First, an increase in θ raises the social surplus flow in a duopoly $SS^{d*}(\theta)$, which enhances the expected social welfare (the first term in the square bracket in (28)). Second, an increase in θ delays the follower's investment timing $Y_l^{F*}(\theta)$, which extends the duration of the leader's monopoly. This delay reduces the present value of the follower's investment cost, resulting in a cost-saving effect (the second term in the square bracket in (28)). This channel also positively contributes to the enhancement of $\widetilde{W}^*(Y;\theta)$.

Third, and most importantly, as the follower delays investment, the transition from monopoly to duopoly happens later as θ increases (the third term of the square bracket in (28)). As long as the social surplus flow in a duopoly $SS^{d*}(\theta)$ exceeds that in a monopoly SS^{m*},[15] the delay in achieving $SS^{d*}(\theta)$ can lead to a decrease in expected social welfare. Because the difference in social surplus $SS^{d*}(\theta) - SS^{m*}$ becomes larger as θ increases, this negative impact can outweigh the positive impacts from the first two channels, thereby reducing $\widetilde{W}^*(Y;\theta)$ at high value of θ. Consequently, a reduction in $\widetilde{W}^*(Y;\theta)$ is observed for $\theta \in (0.85, 1]$ in Figure 6-1.

Figure 6-2 illustrates the case where $a_h = 3.8$. We explain this case by classifying two regions of θ: $\theta \in [0.16,\ 1]$ and $\theta \in [0, 0.16)$. First, as in the previous case with $a_h = 4$, there exist the preemptive equilibria with $Y_h^{L*} = \widetilde{Y}_h^{L*}$ for $\theta \in [0.16, 0.8167)$ and the semi-preemptive equilibria for $\theta \in [0.8167, 1]$. Hence, the expected social welfare is represented by $\widetilde{W}^*(Y;\theta)$ for $\theta \in [0.16,\ 1]$. However, unlike the previous case, $\widetilde{W}^*(Y;\theta)$ does not decline for any θ. This is because the quality difference in add-on services becomes smaller than that in the previous case, which results in a smaller difference of $SS^{d*}(\theta) - SS^{m*}$, mitigating the negative impact of the third channel generated by an increase in θ on $\widetilde{W}^*(Y;\theta)$.

Second, for $\theta \in [0, 0.16)$, the preemptive equilibrium with $Y_h^{L*} = Y_l^P(\theta)$ emerges and the expected social welfare is $W^*(Y;\theta)$. Since the second term in $W^*(Y;\theta)$ is identical

[15] As mentioned in Section 6.2.1, there is a case in which $SS^{d*}(\theta) < SS^{m*}$ when add-on services are vertically differentiated. In the case of Figure 6-1, we obtain $SS^{d*}(\theta) < SS^{m*}$ for $\theta < 0.47$.

to that of $\widetilde{W}^*(Y;\theta)$, the three channels explained for $\widetilde{W}^*(Y;\theta)$ also apply to $W^*(Y;\theta)$. By contrast, the first term in $W^*(Y;\theta)$, which differs from that in $\widetilde{W}^*(Y;\theta)$, significantly impacts on $W^*(Y;\theta)$ through the change in $Y_l^P(\theta)$. Denoting the first term in $W^*(Y;\theta)$ as $\psi(\theta)$, we identify the effects of θ by differentiating it with respect to θ:

$$\begin{aligned}\psi'(\theta) &= \frac{SS^{m*}}{r-\alpha}Y_l^{P\prime}(\theta)\left(\frac{Y}{Y_l^P(\theta)}\right)^{\beta} - \beta Y\left(\frac{Y_l^P(\theta)\,SS^{m*}}{r-\alpha} - I\right)\left(\frac{Y}{Y_l^P(\theta)}\right)^{\beta-1}\left(Y_l^P(\theta)\right)^{-2}Y_l^{P\prime}(\theta) \\ &= -\left(\frac{Y}{Y_l^P(\theta)}\right)^{\beta}\frac{(\beta-1)}{Y_l^P(\theta)}Y_l^{P\prime}(\theta)\left(\frac{Y_l^P(\theta)\,SS^{m*}}{r-\alpha} - \frac{\beta}{\beta-1}I\right),\end{aligned}$$

where $Y_l^{P\prime}(\theta) > 0$ due to the delay of the follower's investment timing $Y_l^{F*}(\theta)$. This expression indicates that the shortened monopoly duration has not only a positive impact by saving costs due to the delayed investment, but it also has a negative impact by forgoing the social surplus in a monopoly that would otherwise have been obtained. Whether $\psi(\theta)$ decreases with θ or not depends on the relative strength of these effects. When $a_h = 3.8$, the latter negative impact dominates the former positive impact as θ increases, which results in a reduction in $W^*(Y;\theta)$ for $\theta \in [0, 0.16)$.

Finally, Figure 6-3 illustrates the case where $a_h = 3$. In this case, the preemptive equilibrium with $Y_h^{L*} = Y_l^P(\theta)$ exists for all $\theta \in [0,1]$, and the expected social welfare is $W^*(Y;\theta)$ for all $\theta \in [0,1]$. Due to the lower a_h, the social surplus flow in a monopoly, SS^{m*}, becomes smaller than that in Figure 6-2. Consequently, the negative impact of θ on $\psi(\theta)$ diminishes. By contrast, as θ increases, the difference in social surplus flow between a duopoly and a monopoly, $SS^{d*}(\theta) - SS^{m*}$, increases, indicating that the third channel in the second term of $W^*(Y;\theta)$ dominates the other effects. Therefore, $W^*(Y;\theta)$ decreases as θ increases for $\theta \in (0.665, 1]$.

Proposition 4 and its associated results provide crucial welfare implications for consumer protection policies such as information disclosure and consumer education. Our analysis shows that in all types of equilibria, the social surplus flow increases as the proportion of non-savvy consumers decreases. Nevertheless, we find that an increase in the proportion of savvy consumers negatively affects expected social welfare due to the delay in a follower's (i.e., a low-quality firm's) entry and the prolonged duration of a leader's monopoly. Moreover, a leader's entry may also be delayed by the delay of a follower's entry timing. Based on this reasoning, we recommend a policy mix of consumer protection and market-entry promotion. While consumer protection policies can enhance social welfare by making consumers fully understand the costs and benefits of add-on services, they may also discourage firms from entering a market. Therefore, consumer protection policies should be implemented alongside entry-promotion policies to enhance social welfare. The importance of the policy mix can be understood by inspecting real-world policies in mobile telecommunications markets worldwide.

7 Conclusion

We have studied how heterogeneity in consumer savviness regarding vertically differentiated add-on services affects pricing, investment timing, and social welfare in a duopolistic market within a real-options framework. Our analysis yields three main findings.

First, the high-quality firm sets a lower core-good price than the low-quality firm when the proportion of non-savvy consumers is high and a higher core-good price when it is low. Additionally, add-on prices increase with the proportion of non-savvy consumers. Second, regarding investment timing, the high-quality firm always enters the market first; however, the type of equilibrium depends on consumer composition. A preemptive equilibrium arises when the proportion of non-savvy consumers is high or the quality difference in add-on services is small, while a semi-preemptive equilibrium occurs when the proportion of non-savvy consumers is low or the quality difference is large. Third, our welfare analysis reveals a counterintuitive result: when add-on services are vertically differentiated, the expected social welfare may decrease as the proportion of savvy consumers rises.

This reduction is attributed to the delayed investment of the follower and the prolonged monopoly of the leader, despite an increase in social surplus flow.

These results suggest that consumer protection policies can enhance transparency but may inadvertently discourage market entry. Therefore, a policy mix that combines consumer protection with measures to promote entry is essential for improving social welfare. This insight is particularly relevant to markets such as mobile telecommunications, where such policy mixes are actively implemented.

Our analysis abstracts from firms' strategic quality disclosure and their endogenous choice of add-on service quality. Incorporating these factors, along with learning about savviness over time or empirical calibration, represents promising directions for future research.

Appendix

Appendix A: The proof of Lemma 1

From (17) in the text, we obtain

$$\xi_l\left(Y^*;\kappa^{d*}\right)=\frac{Y^*\Pi_l^{m*}}{r-\alpha}-I-\frac{Y_h^{F*}\left(\Pi_l^{m*}-\Pi_l^{d*}\right)}{r-\alpha}\left(\frac{Y^*}{Y_h^{F*}}\right)^{\beta}-\left[\frac{Y_l^{F*}\Pi_l^{d*}}{r-\alpha}-I\right]\left(\frac{Y^*}{Y_l^{F*}}\right)^{\beta}=0. \tag{29}$$

From (18), we obtain

$$\begin{aligned}
&\left.\frac{\partial\xi_l\left(Y;\kappa^{d*}\right)}{\partial Y}\right|_{Y=Y^*}\\
=\;&\frac{\Pi_l^{m*}}{r-\alpha}-\beta\frac{\left(\Pi_l^{m*}-\Pi_l^{d*}\right)}{r-\alpha}\left(\frac{Y^*}{Y_h^{F*}}\right)^{\beta-1}-\beta\left[\frac{Y_l^{F*}\Pi_l^{d*}}{r-\alpha}-I\right]\left(\frac{Y^*}{Y_l^{F*}}\right)^{\beta-1}\frac{1}{Y_l^{F*}}\\
=\;&0.
\end{aligned} \tag{30}$$

By (30)$\times$ (Y^*/β),

$$\frac{Y^*}{\beta}\frac{\Pi_l^{m*}}{r-\alpha}-\frac{Y_h^{F*}\left(\Pi_l^{m*}-\Pi_l^{d*}\right)}{r-\alpha}\left(\frac{Y^*}{Y_h^{F*}}\right)^{\beta}-\left[\frac{Y_l^{F*}\Pi_l^{d*}}{r-\alpha}-I\right]\left(\frac{Y^*}{Y_l^{F*}}\right)^{\beta}=0. \tag{31}$$

Then, subtracting (31) from (29), we derive

$$Y^*=\frac{\beta}{\beta-1}\frac{r-\alpha}{\Pi_l^{m*}}I. \tag{32}$$

Substituting (32) into (29) and using the definition of $\kappa^d \equiv \Pi_l^{d*}/\Pi_h^{d*}$, we obtain the following equation for κ^{d*}.

$$\left(\kappa^{d*}\right)^{\beta}-\beta\kappa^{d*}+\beta\chi\left(\theta\right)-\left(\chi\left(\theta\right)\right)^{\beta}=0, \tag{33}$$

where $\chi\left(\theta\right)\equiv\frac{\Pi_l^{m*}}{\Pi_h^{d*}(\theta)}$.

We should note that the threshold κ^{d*} becomes a function of θ, i.e., $\kappa^{d*}\left(\theta\right)$, by solving (33). Indeed, denoting (33) by an implicit function of $F\left(\kappa^{d*},\ \theta\right)=0$ and totally differentiating it with respect to κ^{d*} and θ, we obtain

$$\frac{d\kappa^{d*}}{d\theta}=\frac{\chi'\left(\theta\right)\left[\left(\chi\left(\theta\right)\right)^{\beta-1}-1\right]}{\left(\kappa^{d*}\right)^{\beta-1}-1},$$

where $\chi'\left(\theta\right)=-\Pi_l^{m*}\left(\Pi_h^{d*}\left(\theta\right)\right)^{-2}\Pi_h^{d*\prime}\left(\theta\right)<0$. Then, because $\beta>1$, $\kappa^{d*}<1$, and $\chi\left(\theta\right)>1$, we conclude that

$$\frac{d\kappa^{d*}}{d\theta}>0.$$

To prove the uniqueness of κ^{d*} at a given θ, we define $\Omega\left(\kappa^d\right)$ such that

$$\Omega\left(\kappa^d\right)\equiv\left(\kappa^d\right)^{\beta}-\beta\kappa^d+\beta\chi\left(\theta\right)-\left(\chi\left(\theta\right)\right)^{\beta}.$$

It is apparent that when $\Omega\left(\kappa^{d}\right)=0$, the solution is κ^{d*} by (33). Because

$$\Omega'\left(\kappa^{d}\right)=\beta\left(\left(\kappa^{d}\right)^{\beta-1}-1\right)<0,$$

we verify that given θ, $\Omega\left(\kappa^{d}\left(\theta\right)\right)\gtreqless 0$ if and only if $\kappa^{d}\left(\theta\right)\lesseqgtr\kappa^{d*}\left(\theta\right)$. This implies that at a given θ, κ^{d} monotonically decreases, so that $\kappa^{d*}\left(\theta\right)$ is uniquely determined for any $\theta\in[0,\ 1]$. ■

Appendix B: The proof of Lemma 2

Because $\kappa^{d}\left(\theta\right)$ is decreasing in θ and $\kappa^{d*}\left(\theta\right)$ is increasing in θ, we have the following three cases. See Figure 4 as a reference.

Case 1: $\kappa^{d*}\left(0\right)<\kappa^{d}\left(0\right)$ and $\kappa^{d}\left(1\right)<\kappa^{d*}\left(1\right)$.

Case 2: $\kappa^{d*}\left(1\right)<\kappa^{d}\left(1\right)$.

Case 3: $\kappa^{d}\left(0\right)<\kappa^{d*}\left(0\right)$.

We check each of these cases.

Case 1: $\kappa^{d*}\left(0\right)<\kappa^{d}\left(0\right)$ and $\kappa^{d}\left(1\right)<\kappa^{d*}\left(1\right)$.

This case corresponds to (i) in the proposition. That is, there exists a unique threshold $\widehat{\theta}$ such that the preemptive equilibrium (resp. the semi-preemptive equilibrium) occurs for $\theta<\widehat{\theta}$ (resp. $\theta>\widehat{\theta}$).

When $\kappa^{d*}\left(0\right)<\kappa^{d}\left(0\right)$, $\Omega\left(\kappa^{d*}\left(0\right)\right)=0$ and $\Omega\left(\kappa^{d}\left(0\right)\right)<0$ because $\Omega'\left(\kappa^{d}\right)<0$. Substituting $\kappa^{d}\left(0\right)=\left(\Pi_{l}^{d*}\left(0\right)/\Pi_{h}^{d*}\left(0\right)\right)$ into $\Omega\left(\kappa^{d}\left(0\right)\right)<0$ and rearranging it, we obtain (21).

Similarly, when $\kappa^{d}\left(1\right)<\kappa^{d*}\left(1\right)$, $\Omega\left(\kappa^{d*}\left(1\right)\right)=0$ and $\Omega\left(\kappa^{d}\left(1\right)\right)>0$ because $\Omega'\left(\kappa^{d}\right)<0$. Substituting $\kappa^{d}\left(1\right)=\left(\Pi_{l}^{d*}\left(1\right)/\Pi_{h}^{d*}\left(1\right)\right)$ into $\Omega\left(\kappa^{d}\left(1\right)\right)>0$ and rearranging it, we obtain (20).

Case 2: $\kappa^{d}\left(1\right)<\kappa^{d*}\left(1\right)$.

This is the opposite case of the second inequality of Case 1. Then, we obtain the opposite case of (20), i.e., (22). This case corresponds to (ii) in the proposition. That is, the preemptive equilibrium occurs for all $\theta \in [0,\ 1]$ in this case.

Case 3: $\kappa^{d}(0) < \kappa^{d*}(0)$.

This is the opposite case of the first inequality of Case 1. Then, we obtain the opposite case of (21), i.e., (23). This case corresponds to (iii) in the proposition.

Summarizing the above results gives the proposition. ■

Appendix C: The proof of Proposition 2

Let us define $f(x) = x^{\beta}$ where $\beta > 1$. Since $\beta > 1$, $f(x)$ is increasing and convex in x.

First, we prove (ii) in Proposition 2. We rewrite (22) in case (ii) of Lemma 2.

$$\beta \left(\Pi_h^{d*}(1)\right)^{\beta-1} \left(\Pi_l^{m*} - \Pi_l^{d*}(1)\right) \leq f\left(\Pi_l^{m*}\right) - f\left(\Pi_l^{d*}(1)\right), \tag{22'}$$

where

$$\begin{aligned}
\Pi_l^{m*} &= R - t + \frac{(a_l)^2}{4}, \\
\Pi_h^{d*}(1) &= \frac{1}{2t}\left(t + \frac{1}{6}\left((a_h)^2 - (a_l)^2\right)\right), \\
\Pi_l^{d*}(1) &= \frac{1}{2t}\left(t - \frac{1}{6}\left((a_h)^2 - (a_l)^2\right)\right).
\end{aligned}$$

According to the mean value theorem, the right-hand side of (22') can be rewritten as

$$f\left(\Pi_l^{m*}\right) - f\left(\Pi_l^{d*}(1)\right) = \beta \left(\Pi_1\right)^{\beta-1}\left(\Pi_l^{m*} - \Pi_l^{d*}(1)\right), \tag{34}$$

where $\Pi_1 \in \left(\Pi_l^{d*}(1),\ \Pi_l^{m*}\right)$. Substituting this into (22'), we have

$$\begin{aligned}
\beta \left(\Pi_h^{d*}(1)\right)^{\beta-1} \left(\Pi_l^{m*} - \Pi_l^{d*}(1)\right) &\leq \beta \left(\Pi_1\right)^{\beta-1}\left(\Pi_l^{m*} - \Pi_l^{d*}(1)\right) \\
\Leftrightarrow \Pi_h^{d*}(1) &\leq \Pi_1.
\end{aligned} \tag{35}$$

We define $\delta \equiv a_h - a_l$. In this proof, we consider a case in which a change in δ is generated by a change in a_h by following the numerical examples in Section 6.[16] Then, the left-hand side of (35), $\Pi_h^{d*}(1)$, increases with δ. Furthremore, we can show that the right-hand side of (35), Π_1, decreases with δ. In fact, rearranging (34), we have

$$\frac{f\left(\Pi_l^{m*}\right) - f\left(\Pi_l^{d*}(1)\right)}{\Pi_l^{m*} - \Pi_l^{d*}(1)} = \beta\left(\Pi_1\right)^{\beta-1}. \tag{34'}$$

Since $f(x)$ is increasing and convex in x and $\Pi_l^{d*}(1)$ is decreasing in δ,[17] the left-hand side of (34') decreases with δ. This implies that Π_1 on the right-hand side of (34') decreases with δ when $\beta > 1$.

Since $\Pi_h^{d*}(1)$ increases and Π_1 decreases as δ increases, there exists an upper bound δ_1 such that (35) holds for $\delta \leq \delta_1$. This means that the preemptive equilibrium occurs for all $\theta \in [0,\ 1]$ when $\delta \leq \delta_1$.

Next, we prove Proposition 2 (iii). The proof is similar to that of (ii). We rewrite (23) in case (iii) of Lemma 2.

$$\beta\left(\Pi_h^{d*}(0)\right)^{\beta-1}\left(\Pi_l^{m*} - \Pi_l^{d*}(0)\right) \geq f\left(\Pi_l^{m*}\right) - f\left(\Pi_l^{d*}(0)\right), \tag{23'}$$

where

$$\begin{aligned}
\Pi_h^{d*}(0) &= \frac{1}{2t}\left(t + \frac{1}{12}\left((a_h)^2 - (a_l)^2\right)\right), \\
\Pi_l^{d*}(0) &= \frac{1}{2t}\left(t - \frac{1}{12}\left((a_h)^2 - (a_l)^2\right)\right).
\end{aligned}$$

By the mean value theorem, the right-hand side of (23') can be represented as

[16] Instead, a change in a_l also produces a change in δ, which generates the same analytical result as a change in a_h.

[17] Also, we should remember that $\Pi_l^{m*} > \Pi_l^{d*}(1)$ and Π_l^{m*} is constant when only a_h changes.

$$f\left(\Pi_l^{m*}\right) - f\left(\Pi_l^{d*}(0)\right) = \beta\left(\Pi_0\right)^{\beta-1}\left(\Pi_l^{m*} - \Pi_l^{d*}(0)\right), \tag{36}$$

where $\Pi_0 \in \left(\Pi_l^{d*}(0),\ \Pi_l^{m*}\right)$. Substituting this into (23'), we have

$$\begin{aligned} \beta\left(\Pi_h^{d*}(0)\right)^{\beta-1}\left(\Pi_l^{m*} - \Pi_l^{d*}(0)\right) &\geq \beta\left(\Pi_0\right)^{\beta-1}\left(\Pi_l^{m*} - \Pi_l^{d*}(0)\right) \\ \Leftrightarrow\ \Pi_h^{d*}(0) &\geq \Pi_0. \end{aligned} \tag{37}$$

The left-hand side of (37), $\Pi_h^{d*}(0)$, increases with δ. We also show that the right-hand side of (37), Π_0, decreases with δ. Rearranging (22'), we have

$$\frac{f\left(\Pi_l^{m*}\right) - f\left(\Pi_0^{d*}(0)\right)}{\Pi_l^{m*} - \Pi_l^{d*}(0)} = \beta\left(\Pi_0\right)^{\beta-1}. \tag{36'}$$

Since $f(x)$ is increasing and convex in x, $\Pi_l^{d*}(0)$ is decreasing in δ, and Π_l^{m*} is constant and greater than $\Pi_l^{d*}(1)$, the left-hand side of (36') decreases with δ. Thus, Π_0 on the right-hand side of (36') decreases with δ when $\beta > 1$.

As δ decreases, $\Pi_h^{d*}(1)$ decreases and Π_0 increases. Therefore, there exists a lower bound δ_1 such that (37) holds for $\delta \geq \delta_0$. This means that the semi-preemptive equilibrium occurs for all $\theta \in [0,\ 1]$ when $\delta \geq \delta_0$.

Regarding case (i) of Proposition 2, it is straightforward to show that the preemptive and semi-preemptive equilibria coexist for $\delta \in (\delta_1,\ \delta_0)$. Then, from Lemma 2 (i), the preemptive equilibrium occurs for $\theta < \widehat{\theta}$, while the semi-preemptive equilibrium occurs for $\theta > \widehat{\theta}$.[18]

Finally, we derive the trace of the threshold between the preemptive and the semi-preemptive equilibria, κ^{d*}, in terms of δ and θ. Note that the threshold κ^{d*} is thoroughly represented in terms of δ and θ in (19) (or (33)); $\kappa^{d*} = \kappa^{d*}(\theta,\ \delta)$. Then, as in the proof of Lemma 1, denoting (19) by an implicit function of $F\left(\kappa^{d*},\ \theta,\ \delta\right) = 0$ and totally

[18] In the case of $\delta_0 < \delta_1$, case (i) of Proposition 2 and Lemma 2 do not exist.

differentiating it with respect to θ and δ with $d\kappa^{d*} = 0$, we obtain

$$\left(1 - (\chi(\theta))^{\beta-1}\right) \left[\chi'(\theta)\, d\theta + \frac{\partial \chi(\theta)}{\partial \delta} d\delta\right] = 0,$$

where $1 < (\chi(\theta))^{\beta-1}$ and $\chi'(\theta) < 0$. In addition, we have

$$\frac{\partial \chi(\theta)}{\partial \delta} = -\frac{\Pi_l^{m*}}{\left(\Pi_h^{d*}(\theta)\right)^2} \frac{\partial \Pi_h^{d*}(\theta)}{\partial \delta} < 0.$$

Therefore, we obtain

$$\frac{d\delta}{d\theta} < 0,$$

which states that the proportion of savvy consumers θ negatively relates to the quality difference δ on the threshold κ^{d*}. This implies that the preemptive (resp. the semi-preemptive) equilibrium is less (resp. more) likely to occur as the proportion of savvy consumers θ increases. ■

Appendix D: The proof of Proposition 3

At first, we derive the social surplus flow when add-on services are homogeneous.

Social surplus flow

When add-on services are homogeneous, the equilibrium prices and each firm's equilibrium demand are derived as follows:

$$p_l^{\#} = p_h^{\#} = \frac{(1-\theta)\, a}{2-\theta} \equiv p^{\#},$$

$$P_l^{\#} = P_h^{\#} = t - \pi\left(p^{\#}\right) \equiv P^{\#},$$

$$\widehat{x}^{\#} = \widehat{\widehat{x}}^{\#} = \frac{1}{2},$$

where $\pi\left(p^{\#}\right)=p^{\#}\left(a-p^{\#}\right)=(1-\theta)\left(\frac{a}{2-\theta}\right)^{2}$.

Substituting the equilibrium prices into a firm's profit, we obtain the equilibrium profit of firm $i\ (=l,\ h)$, $\Pi_i^{d\#}$, as follows:

$$\Pi_l^{d\#}=\Pi_h^{d\#}=\frac{1}{2}t.$$

Note that the proportion of savvy consumers, θ, does not affect each firm's profit in equilibrium.

Next, we derive the consumer surplus flow in a duopoly, $U^{d\#}(\theta)$. We obtain $U^{d\#}(\theta)$ by using $\widehat{x}^{\#}=\widehat{\widehat{x}}^{\#}=1/2$, as follows:

$$\begin{aligned}
U^{d\#}(\theta) &\equiv \theta U^{S\#}(\theta)+(1-\theta)U^{N\#}(\theta)\\
&= \theta\left\{\int_0^{\frac{1}{2}}\left[R-P_l^{\#}+s\left(p_l^{\#}\right)-tx\right]dx+\int_{\frac{1}{2}}^{1}\left[R-P_h^{\#}+s\left(p_h^{\#}\right)-t(1-x)\right]dx\right\}\\
&\quad +(1-\theta)\left\{\int_0^{\frac{1}{2}}\left[R-P_l^{\#}+s\left(p_l^{\#}\right)-tx\right]dx+\int_{\frac{1}{2}}^{1}\left[R-P_h^{\#}+s\left(p_h^{\#}\right)-t(1-x)\right]dx\right\}\\
&= R-\frac{5}{4}t+\frac{3-2\theta}{2}\left(\frac{a}{2-\theta}\right)^{2}.
\end{aligned}$$

In contrast to a firm's profit $\Pi_i^{d\#}$, the consumer surplus $U^{d\#}(\theta)$ depends on the proportion of savvy consumers θ. Furthermore, we verify that $U^{d\#}(\theta)$ is increasing in θ, as follows:

$$\frac{dU^{d\#}(\theta)}{d\theta}=\frac{a^2(1-\theta)}{(2-\theta)^3}\geq 0 \text{ for all } \theta\in[0,\ 1].$$

This result indicates the existence of "search externalities" caused by an increase in savvy consumers: as the proportion of savvy consumers θ increases, total consumer surplus increases.[19]

We then obtain the social surplus flow (or the instantaneous social surplus) in a

[19] See Section 3.1 of Armstrong (2015) for the classification of externalities caused by an increase in savvy consumers.

duopoly, $SS^{d\#}(\theta)$, as follows:

$$\begin{aligned} SS^{d\#}(\theta) &\equiv U^{d\#}(\theta) + \Pi_l^{d\#} + \Pi_h^{d\#} \\ &= R - \frac{1}{4}t + \frac{3-2\theta}{2}\left(\frac{a}{2-\theta}\right)^2. \end{aligned} \tag{38}$$

Since $\frac{dSS^{d\#}(\theta)}{d\theta} \geq 0$ holds, the social surplus flow in a duopoly also increases with the proportion of savvy consumers.

Next, we derive the social surplus flow in a monopoly, $SS^{m\#}(\theta)$. A monopoly's profit is

$$\Pi^{m\#} = R - t + \pi\left(p^M\right),$$

where $\pi\left(p^M\right) = \frac{a^2}{4}$. The consumer surplus flow in a monopoly, $U^{m\#}(\theta)$, is

$$\begin{aligned} U^{m\#}(\theta) &\equiv \theta U^{mS\#}(\theta) + (1-\theta)\, U^{mN\#}(\theta) \\ &= \theta\left\{\int_0^1 \left[R - P^{m\#} + s\left(p^M\right) - tx\right] dx\right\} + (1-\theta)\left\{\int_0^1 \left[R - P^{m\#} + s\left(p^M\right) - tx\right] dx\right\} \\ &= \frac{1}{2}t + s\left(p^M\right) \equiv U^{m\#}, \end{aligned}$$

where $s\left(p^M\right) = \frac{a^2}{8}$. Then, the social surplus flow in a monopoly, $SS^{m\#}(\theta)$, is obtained as follows.

$$\begin{aligned} SS^{m\#}(\theta) &\equiv U^{m\#} + \Pi^{m\#} \\ &= R - \frac{1}{2}t + \frac{3a^2}{8} \equiv SS^{m\#}. \end{aligned} \tag{39}$$

(39) implies that the social surplus flow in a monopoly is not affected by the proportion of savvy consumers θ. In addition, comparing (38) and (39), we verify that the social surplus flow in a duopoly is larger than that in a monopoly, as follows:

$$SS^{d\#}(\theta) - SS^{m\#} = \frac{1}{4}t + \frac{\theta(4-3\theta)a^2}{8(2-\theta)^2} \geq 0 \text{ for all } \theta \in [0,\ 1].$$

Expected social welfare

We now derive the expected social welfare in the case of homogeneous add-on services. To do so, we first derive a value function when a firm becomes a follower or a leader.

When a firm becomes a follower, its value function is represented as follows:

$$V^F(Y) = \begin{cases} \Psi Y^\beta \text{ if } Y < Y^{F\#} \\ \frac{Y\Pi^{d\#}}{r-\alpha} - I \text{ if } Y \geq Y^{F\#} \end{cases} \tag{40}$$

where $\Psi \equiv \left(Y^{F\#}\right)^{-\beta}\left[\frac{Y^{F\#}\Pi^{d\#}}{r-\alpha} - I\right]$ and $\Pi^{d\#} = \frac{1}{2}t$. The follower's investment timing $Y^{F\#}$ is characterized by

$$Y^{F\#} = \frac{\beta}{\beta-1}\frac{r-\alpha}{\Pi^{d\#}}I \tag{41}$$

We should note that $Y^{F\#}$ does not depend on θ.

It is obvious that in the case of homogeneous add-on services, only the preemptive equilibrium occurs.[20] Then, each firm's value function as a leader is the following.

$$V^L(Y) = \begin{cases} \frac{Y\Pi^{m\#}}{r-\alpha} - I + \frac{Y^{F\#}\left(\Pi^{d\#}-\Pi^{m\#}\right)}{r-\alpha}\left(\frac{Y}{Y^{F\#}}\right)^\beta \text{ if } Y < Y^{F\#} \\ \frac{Y\Pi^{d\#}}{r-\alpha} - I \text{ if } Y \geq Y^{F\#}. \end{cases} \tag{42}$$

Moreover, the leader's investment timing $Y^P(\theta)$ is characterized by

$$\frac{Y^P(\theta)\Pi^{m\#}}{r-\alpha} - I + \frac{Y^{F\#}\left(\Pi^{d\#}-\Pi^{m\#}\right)}{r-\alpha}\left(\frac{Y^P(\theta)}{Y^{F\#}}\right)^\beta = \Psi\left(Y^P(\theta)\right)^\beta. \tag{43}$$

Note that since all variables do not depend on θ in (43), the leader's investment timing is not affected by θ, either; $Y^P(\theta) = Y^P$.

[20] A mixed-strategy equilibrium can occur in the case of homogeneous add-on services. However, to compare the expected social welfare in this case with that in the case of vertically differentiated add-on services, we restrict our attention to pure strategy equilibria.

Then, the expected social welfare in the case of homogeneous add-on services is represented by

$$\begin{aligned}
W^{\#}(Y;\theta) &= \left(\frac{Y^P SS^{m\#}}{r-\alpha} - I\right)\left(\frac{Y}{Y^P}\right)^{\beta} \\
&\quad + \left(\frac{Y^{F\#}\left(SS^{d\#}(\theta) - SS^{m\#}\right)}{r-\alpha} - I\right)\left(\frac{Y}{Y^{F\#}}\right)^{\beta} \\
&= \left(\frac{Y}{Y^P}\right)^{\beta}\left\{\frac{Y^P SS^{m\#}}{r-\alpha} - I + \left(\frac{Y^P}{Y^{F\#}}\right)^{\beta}\left(\frac{Y^{F\#}\left(SS^{d\#}(\theta) - SS^{m\#}\right)}{r-\alpha} - I\right)\right\},
\end{aligned} \tag{44}$$

where $Y\ (= Y_0)$ is an initial value that is assumed to be lower than Y^P. In (44), both of $\left(\frac{Y}{Y^P}\right)^{\beta}$ and $\left(\frac{Y}{Y^{F\#}}\right)^{\beta}$ are interpreted as a "stochastic discount factor" when Y follows the geometric Brownian motion. We should note that these stochastic discount factors are independent of θ in this benchmark case. Therefore, the expected social welfare $W^{\#}(Y;\theta)$ increases in θ as $SS^{d\#}(\theta)$ increases in θ. ■

Appendix E: On the multiplicative form of $\varphi_{it}(\mathbf{P},\mathbf{p}) \equiv Y_t\Pi_i(\mathbf{P},\mathbf{p})$ $(i = l,\ h)$

In this appendix, we provide an example in which a firm's profit flow $\varphi_{it}(\mathbf{P},\mathbf{p})$ becomes the multiplicative function of Y_t; $\varphi_{it}(\mathbf{P},\mathbf{p}) \equiv Y_t\Pi_i(\mathbf{P},\mathbf{p})$ $(i = l,\ h)$. When a consumer's indirect utility and a linear demand function of add-on services are formulated in the following way, we obtain the multiplicative form of $Y_t\Pi_i(\mathbf{P},\mathbf{p})$.

$$\begin{aligned}
\overleftrightarrow{U}_l(x) &= Y_t(R - tx) + P_l + s_l(p_l), \\
\overleftrightarrow{U}_h(x) &= Y_t(R - (1-t)x) + P_h + s_h(ph), \\
\overleftrightarrow{q}_i(p) &= \sqrt{Y_t}a_i - p, \quad i = l,\ h.
\end{aligned}$$

Using the above formulation, we derive the equilibrium prices and profits in a duopoly. From each firm's profit-maximization problem, we obtain the equilibrium prices and profits as follows:

$$\begin{aligned}
\overleftrightarrow{p}_l^* &= \frac{(1-\theta)\sqrt{Y_t}a_l}{2-\theta} \text{ and } \overleftrightarrow{p}_h^* = \frac{(1-\theta)\sqrt{Y_t}a_h}{2-\theta}, \\
\overleftrightarrow{P}_l^* &= Y_t\left\{t - \frac{1}{6}\left((4-5\theta)A_l(\theta) + (2-\theta)A_h(\theta)\right)\right\}, \\
\overleftrightarrow{P}_h^* &= Y_t\left\{t - \frac{1}{6}\left((2-\theta)A_l(\theta) + (4-5\theta)A_h(\theta)\right)\right\}, \\
\overleftrightarrow{\Pi}_l^{d*} &= \frac{Y_t}{2t}\left(t - \frac{1}{6}(2-\theta)\left(A_h(\theta) - A_l(\theta)\right)\right)^2 = Y_t\Pi_l^{d*}, \\
\overleftrightarrow{\Pi}_h^{d*} &= \frac{Y_t}{2t}\left(t + \frac{1}{6}(2-\theta)\left(A_h(\theta) - A_l(\theta)\right)\right)^2 = Y_t\Pi_h^{d*}.
\end{aligned}$$

Next, we derive the equilibrium prices and profits in a monopoly. Because firm h becomes a leader in our model, the equilibrium prices and profits in a monopoly are represented as follows:

$$\begin{aligned}
\overleftrightarrow{P}_h^{m*} &= R - t, \\
\overleftrightarrow{p}_h^{m*} &= \frac{a_h}{2}, \\
\overleftrightarrow{\Pi}_h^{m*} &= Y_t\left(R - t + \pi_h\left(p_h^M\right)\right) = Y_t\Pi_h^{m*}.
\end{aligned}$$

Therefore, we verify that a firm's equilibrium profit flow takes the multiplicative form of Y_t; $\overleftrightarrow{\Pi}_l^{d*} = Y_t\Pi_l^{d*}$, $\overleftrightarrow{\Pi}_l^{d*} = Y_t\Pi_l^{d*}$, and $\overleftrightarrow{\Pi}_h^{m*} = Y_t\Pi_h^{m*}$.

In addition, we can ensure that consumer surplus flow also becomes the multiplicative function of Y_t by using the above formulation. For example, in the case of homogeneous add-on services, we obtain

$$\begin{aligned}
\overleftrightarrow{U}^{d\#}(\theta) &= Y_t\left\{R - \frac{5}{4}t + \frac{3-2\theta}{2}\left(\frac{a}{2-\theta}\right)^2\right\} = Y_t U^{d\#}(\theta) \quad \text{in a duopoly}, \\
\overleftrightarrow{U}^{m\#}(\theta) &= Y_t\left\{\frac{1}{2}t + s_h\left(p^M\right)\right\} = Y_t U^{m\#} \quad \text{in a monopoly}.
\end{aligned}$$

Therefore, social surplus flow also becomes the multiplicative function of Y_t in the case of homogeneous add-on services. Similarly, we can show that social surplus flow takes the multiplicative form of Y_t when add-on services are vertically differentiated.[21]

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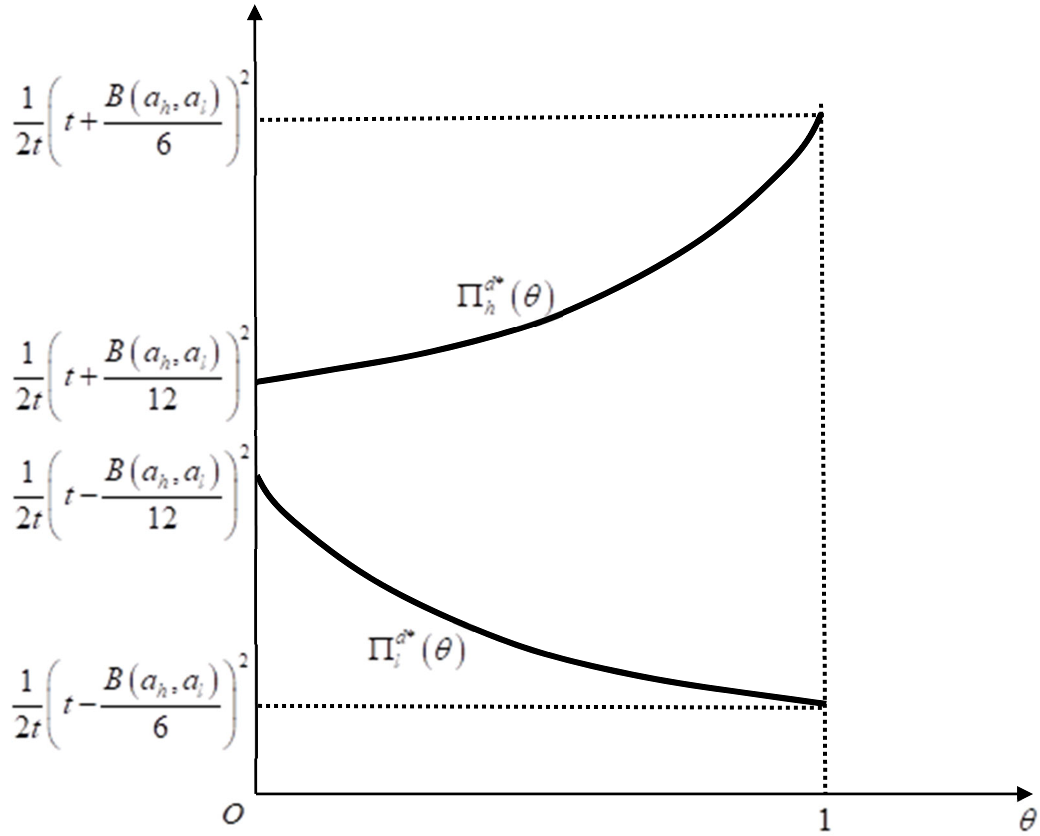


Figure 1 The Effect of the Change in Θ on Firms' Profit Flows in a Duopoly

Note: $B(a_h,a_l) \equiv (a_h)^2 - (a_l)^2$.

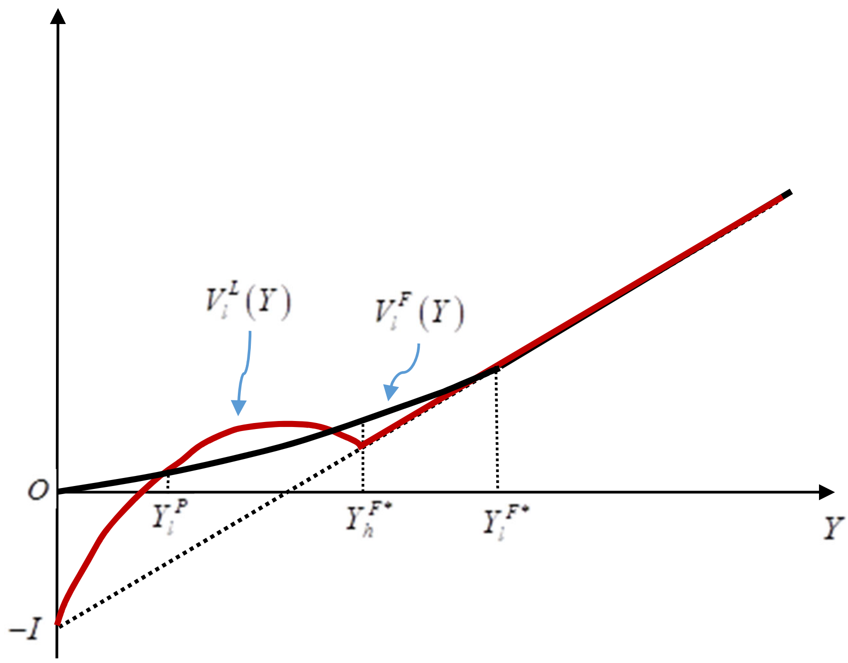


Figure 2-1 Firm *l*'s Value Functions in Preemptive Equilibrium

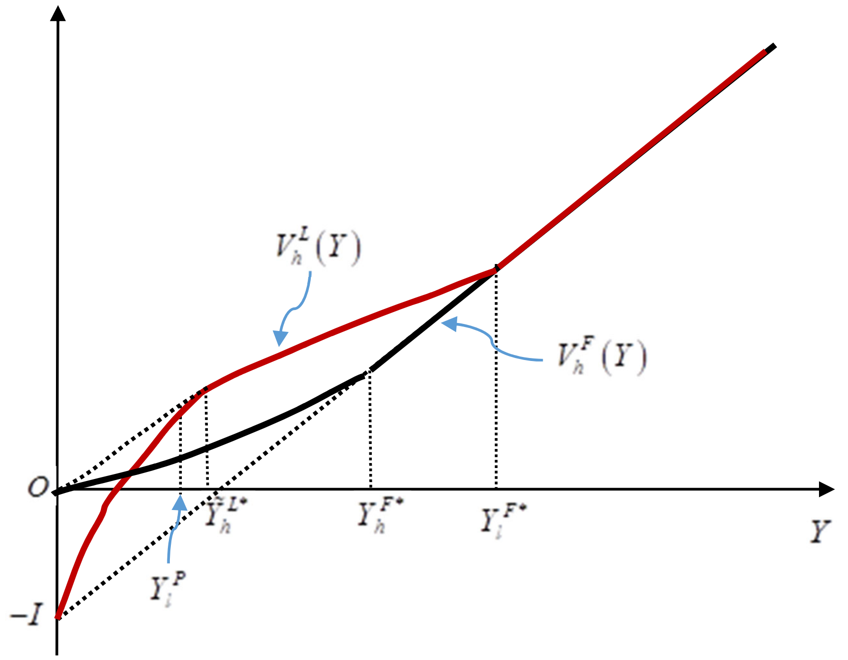


Figure 2-2 Firm *h*'s Value Functions in Preemptive Equilibrium

Figure 2 Preemptive Equilibrium

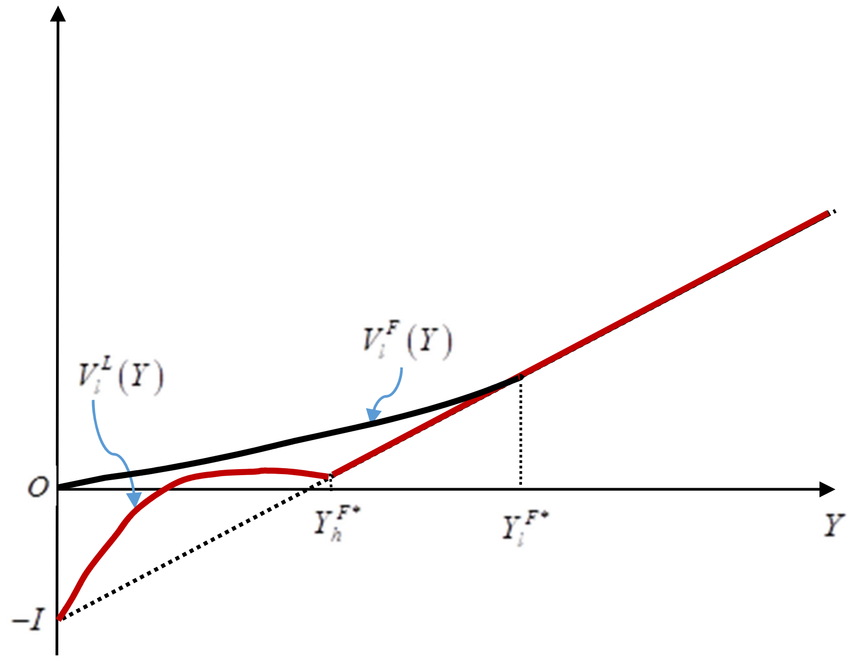


Figure 3-1 Firm *l*'s Value Functions in Semi-Preemptive Equilibrium

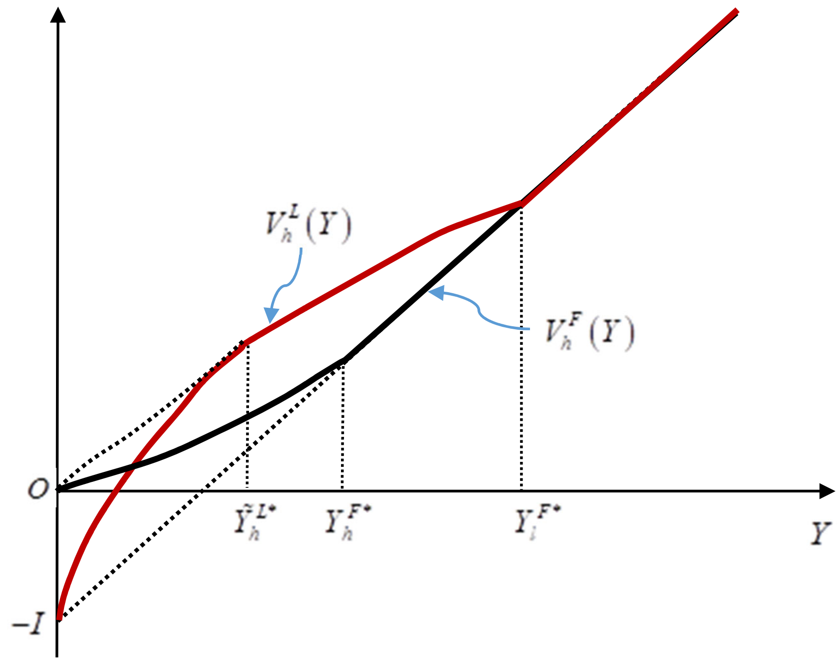


Figure 3-2 Firm *h*'s Value Functions in Semi-Preemptive Equilibrium

Figure 3 Semi-Preemptive Equilibrium

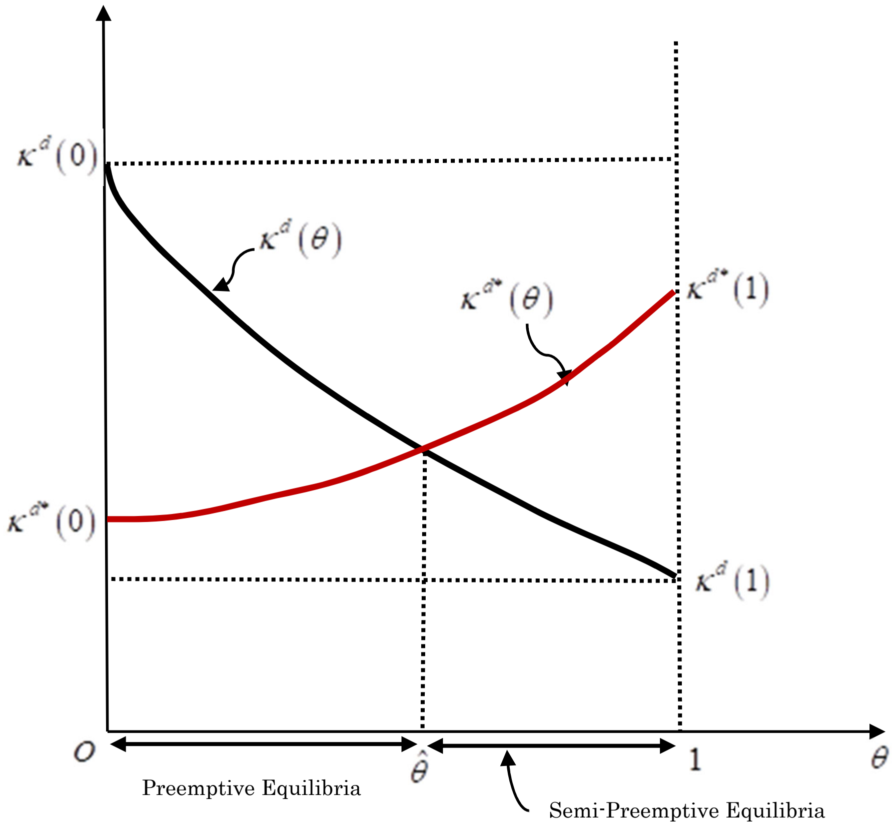


Figure 4 Equilibrium Configuration

Note:

(i) $\kappa^{d}(\theta) \equiv \dfrac{\Pi_l^{d*}(\theta)}{\Pi_h^{d*}(\theta)}$.

(ii) $\kappa^{d*}(\theta)$ is defined by the solution of $\left(\kappa^{d*}\right)^{\beta} - \beta\kappa^{d*} + \beta\chi(\theta) - \left(\chi(\theta)\right)^{\beta} = 0$,

where $\chi(\theta) \equiv \dfrac{\Pi_l^{m*}}{\Pi_h^{d*}(\theta)}$.

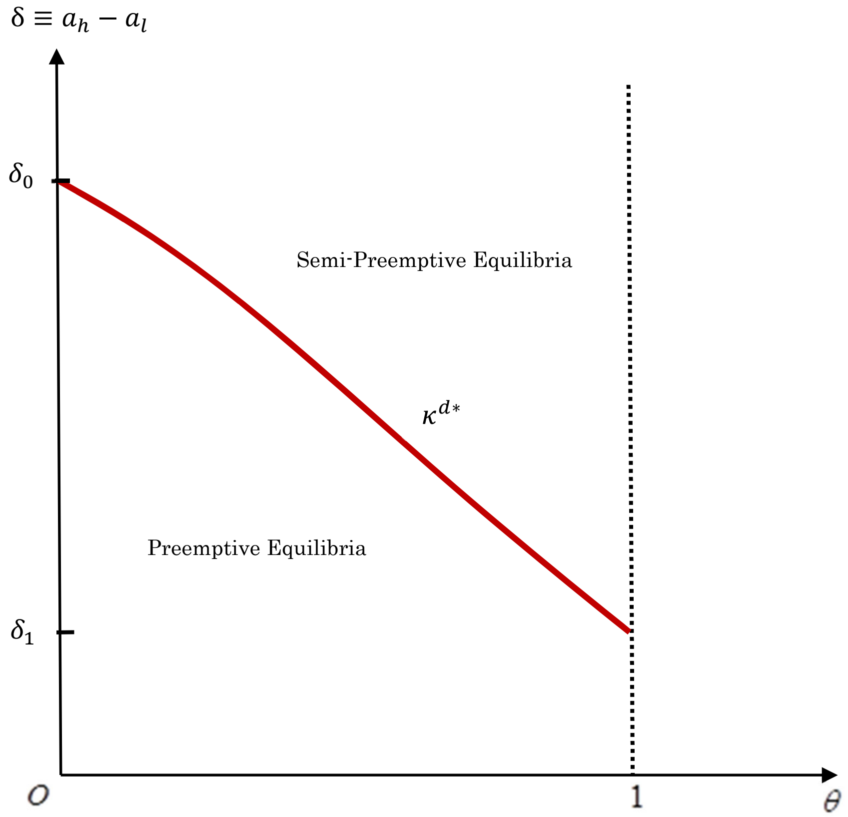


Figure 5 The Relationship between θ and δ on the Threshold κ^{d*}

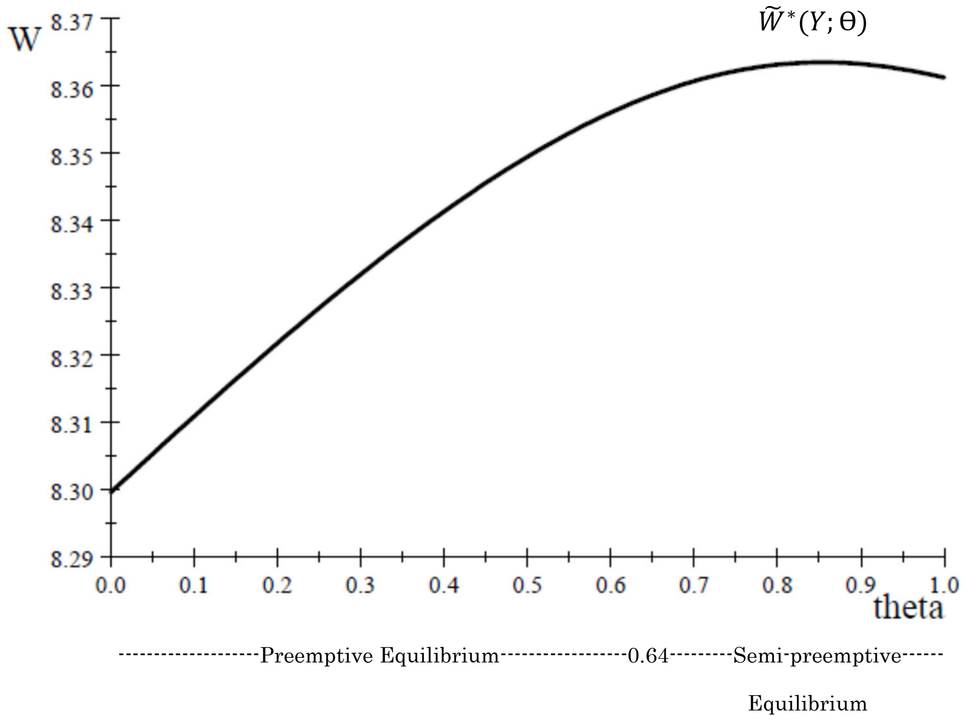
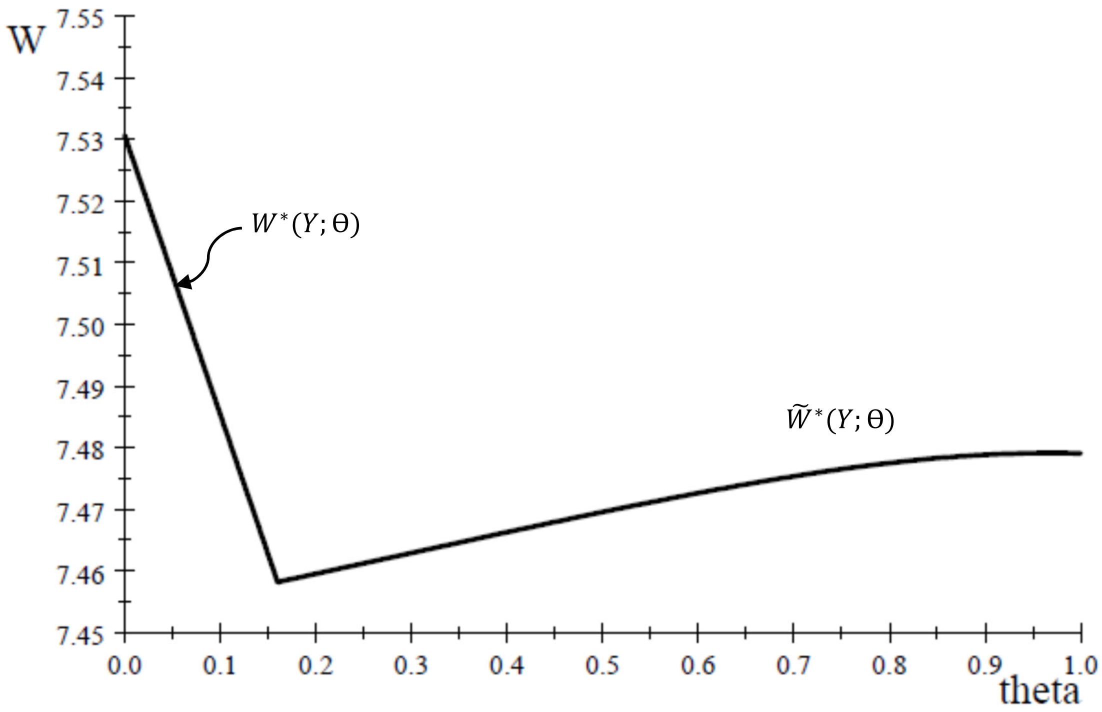


Figure 6-1 Expected Social Welfare (1)

Notes:
$\alpha = 0.015$, $r = 0.05$, $\sigma = 0.1$, $I = 100$, $R = 8$, $t = 3$, $\boldsymbol{a_h} = \mathbf{4}$, $a_l = 2$.



-----------------------------Preemptive Equilibrium--------------------------------------0.8167----Semi-preemptive--

Equilibrium

Figure 6-2 Expected Social Welfare (2)

Notes:
$\alpha = 0.015,\ r = 0.05,\ \sigma = 0.1,\ I = 100,\ R = 8,\ t = 3,\ \boldsymbol{a_h = 3.8},\ a_l = 2.$

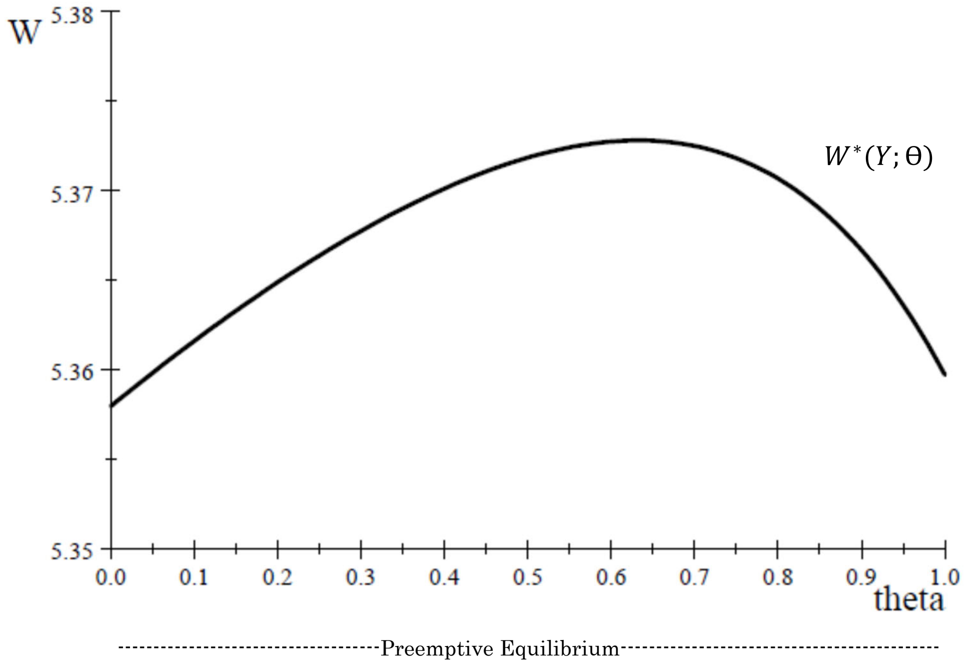


Figure 6-3 Expected Social Welfare (3)

Notes:

$\alpha = 0.015$, $r = 0.05$, $\sigma = 0.1$, $I = 100$, $R = 8$, $t = 3$, $\boldsymbol{a_h} = \mathbf{3}$, $a_l = 2$.