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Extension to a Multi-Period Overlapping Generations Model: Based on the Two-Period Overlapping Generations Model

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Extension to a Multi-Period Overlapping Generations Model: Based on the Two-Period Overlapping Generations Model†

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Abstract

In macroeconomics, the Ramsey model and the two-period overlapping generations (OLG) model are regarded as standard frameworks. The latter is widely used due to its simplicity and its ability to explain a variety of economic phenomena. Although three-period models also exist, they are often constrained by analytical limitations, such as the inability to incorporate capital accumulation. As multi-period models, the altruism-based Ramsey model and the continuous-time OLG model have been developed; however, neither is well-suited for analyzing independent decision-making across generations, elderly labor supply, or pay-as-you-go pension schemes. Against this background, this paper constructs a four-period model consisting of youth, middle age, early old age, and late old age. By incorporating Romer-type capital externalities, it analytically derives the complex capital accumulation equations and theoretically demonstrates the existence, stability, and potential multiplicity of steady states.

Keywords：Overlapping Generations Model, Dynamics of Capital Stock

JEL Classifications：E0, H2, J1

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1. Introduction

As standard analytical models in macroeconomics, the Ramsey model and the overlapping generations (OLG) model are often employed. The OLG model is typically presented in a two-period framework. This is because the two-period formulation not only enables the explanation of a wide range of economic phenomena but also offers greater tractability, which explains its widespread use. However, three-period models are also sometimes adopted. In such cases, the first period is often modeled as a choice between education and leisure, and the dynamics of capital are represented in the form of a first-order difference equation. Alternatively, some analyses employ a three-period setting in the context of a small open economy without considering capital accumulation.

Furthermore, when extending to a multi-period overlapping generations (OLG) framework, one could base the analysis on a dynasty model such as Barro and Becker (1989). While the standard Ramsey model assumes infinitely lived households, a model with altruism—where the utility of children directly enters the utility function of parents—implies that each child's utility function also incorporates the utility of their own children. As a result, the household's optimal allocation ultimately takes into account the welfare of all future generations. From the perspective of any given period, such a framework may be represented as one in which many generations overlap. However, in practice, all intertemporal allocations are decided by a single representative individual. Thus, the distinctive feature of the OLG framework—namely, the coexistence of heterogeneous agents making independent decisions in any given period, which can generate dynamic inefficiency—is not adequately captured by this approach.

Another approach is the continuous-time overlapping generations (OLG) model of the Blanchard–Yaari type (Blanchard, 1985). In this framework, agents face a constant probability of death, and correspondingly, new cohorts are continuously born into the model, thereby representing a multi-period OLG structure. However, this model is relatively difficult to use, and it is not well-suited for recent analyses concerning elderly labor supply or pay-as-you-go pension systems. Although multi-period models are desirable in that they allow economic analysis to better reflect reality, they often involve difficult issues related to the stability of the dynamic system. This may explain why multi-period OLG models have seldom been developed for analytical purposes.

This paper extends the conventional two-period overlapping generations (OLG) model by constructing a four-period framework consisting of youth, middle age, early old age, and late old age, and analyzes capital dynamics and the steady-state growth rate. While the derivation of the capital accumulation equation becomes more complex in the four-period model, it is shown that incorporating Romer-type capital externalities enables an analytical solution.[1] In a simplified setting that excludes early-old-age labor and pay-as-you-go pensions, the existence, stability, and possible multiplicity of steady states are theoretically confirmed. Furthermore, numerical simulations incorporating the increase in early-old-age labor participation and the pay-as-you-go pension system in Japan reveal that both factors reduce capital accumulation and the growth rate, a result consistent with the conventional two-period model. Overall, this study demonstrates the usefulness of multi-period OLG models for future economic analysis.

The structure of this paper is as follows. Section 2 describes the model setup, and Section 3 derives the equilibrium solution. Section 4 presents numerical simulations incorporating elderly labor supply and the pay-as-you-go pension system, and Section 5 concludes.

2. Model

2.1 Households

Households are assumed to live for four periods. The overlapping generations (OLG) model is typically formulated with two periods. In the two-period setting, individuals are young in the first period, during which they work and earn labor income, and old in the second period, when they consume the savings accumulated during youth. By extending the model to four periods, the working-age population can be divided into a young and a middle-aged generation, while the old can be divided into an early-old-age generation and a late-old-age generation. The following sections set out the model required to derive the equilibrium of this four-period framework. In what follows, the first, second, third, and fourth periods are

[1] A three-period overlapping generations (OLG) model incorporating youth, middle age, and old age was examined in Yasuoka (2025). In that study, population dynamics were also analyzed in the case where fertility decisions are made during both youth and middle age.

referred to as youth, middle age, early old age, and late old age, respectively.

The utility function u_t is assumed to take the following logarithmic form.

$$u_t = \alpha lnc_{1t} + \beta lnc_{2t+1} + \gamma lnc_{3t+2} + (1-\alpha-\beta-\gamma) lnc_{4t+3}$$
$$0 < \alpha, 0 < \beta, 0 < \gamma, \alpha + \beta + \gamma < 1. \tag{1}$$

Let c_{1t} denote consumption in the first period, c_{2t+1} consumption in the second period, c_{3t+2} consumption in the third period, and c_{4t+3} consumption in the fourth period. Here, t represents time.

The budget constraint in the first period is given by:

$$c_{1t} + s_{1t} = (1-\tau)w_t \tag{2}$$

In this formulation, s_{1t} represents savings in the first period, τ the contribution rate for the pension system, and w_t the wage rate. Pension contributions are assumed to be collected from the young, middle-aged, and early-old-age cohorts.

The budget constraint in the second period is given by:

$$c_{2t+1} + s_{2t+1} = (1-\tau)w_{t+1} + (1+r_{t+1})s_{1t} \tag{3}$$

Here, s_{2t+1} denotes savings in the second period, and r_{t+1} denotes the interest rate.

The budget constraint in the third period is given by:

$$c_{3t+2} + s_{3t+2} = (1+r_{t+2})s_{2t+1} + (1-\tau)w_{t+2}l \tag{4}$$

Here, s_{3t+2} denotes savings in the third period, and l denotes the labor supply in the third period. In the case of the third period, corresponding to early old age, labor supply is not a full unit but rather l units, where $0 < l < 1$.

The budget constraint in the fourth period is given by:

$$c_{4t+3} = (1+r_{t+3})s_{3t+2} + P_{t+3} \tag{5}$$

Here, s_{3t+2} denotes savings in the third period, and P_{t+3} denotes the pension benefits received in the fourth period. These pension benefits are those provided to the late-old-age generation.

	t period	t+1 period	t+2 period	t+3 period	t+4 period
	Young	Adult	Early old age	Late old age	
t generation	c_1t, s_1t	c_2t+1, s_2t+1	c_3t+2, s_3t+2	c_4t+3	
	w_t	w_t+1	w_t+2	P_t+3	
		Young	Adult	Early old age	Late old age
t+1 generation		c_1t+1, s_1t+1	c_2t+2, s_2t+2	c_3t+3, s_3t+3	c_4t+4
		w_t+1	w_t+2	w_t+3	P_t+4
			Young	Adult	Early old age
t+2 generation			c_1t+2, s_1t+2	c_2t+3, s_2t+3	c_3t+4, s_3t+4
			w_t+2	w_t+3	w_t+4
				Young	Adult
t+3 generation				c_1t+3, s_1t+3	c_2t+4, s_2t+4
				w_t+3	w_t+4

Figure 1. Structure of the Four-Period Overlapping Generations Model

Figure 1 illustrates the structure of the four-period overlapping generations (OLG) model. An individual who belongs to the young generation in period t is referred to as the t-generation. In any given period, four generations coexist simultaneously. While the conventional two-period OLG model involves the coexistence of two generations within each period, the present model incorporates four overlapping generations. This feature complicates the analysis of the capital accumulation dynamics, which makes both the formulation and examination of multi-generation OLG models particularly challenging.

From equations (2)–(5), the lifetime budget constraint is given as follows:

$$c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} + \frac{c_{3t+2}}{(1+r_{t+1})(1+r_{t+2})} + \frac{c_{4t+3}}{(1+r_{t+1})(1+r_{t+2})(1+r_{t+3})} = (1-\tau)w_t + \frac{(1-\tau)w_{t+1}}{1+r_{t+1}} + \frac{(1-\tau)w_{t+2}l}{(1+r_{t+1})(1+r_{t+2})} + \frac{P_{t+3}}{(1+r_{t+1})(1+r_{t+2})(1+r_{t+3})} \tag{6}$$

The allocation that maximizes the utility function (1) subject to the lifetime budget constraint (6) can be derived as follows.

$$c_{1t} = \alpha\left((1-\tau)w_t + \frac{(1-\tau)w_{t+1}}{1+r_{t+1}} + \frac{(1-\tau)w_{t+2}l}{(1+r_{t+1})(1+r_{t+2})} + \frac{P_{t+3}}{(1+r_{t+1})(1+r_{t+2})(1+r_{t+3})}\right) \tag{7}$$

$$\frac{c_{2t+1}}{1+r_{t+1}} = \beta\left((1-\tau)w_t + \frac{(1-\tau)w_{t+1}}{1+r_{t+1}} + \frac{(1-\tau)w_{t+2}l}{(1+r_{t+1})(1+r_{t+2})} + \frac{P_{t+3}}{(1+r_{t+1})(1+r_{t+2})(1+r_{t+3})}\right) \tag{8}$$

$$\frac{c_{3t+2}}{(1+r_{t+1})(1+r_{t+2})} = \gamma\left((1-\tau)w_t + \frac{(1-\tau)w_{t+1}}{1+r_{t+1}} + \frac{(1-\tau)w_{t+2}l}{(1+r_{t+1})(1+r_{t+2})} + \frac{P_{t+3}}{(1+r_{t+1})(1+r_{t+2})(1+r_{t+3})}\right) \tag{9}$$

$$\frac{c_{4t+3}}{(1+r_{t+1})(1+r_{t+2})(1+r_{t+3})} = (1-\alpha-\beta-\gamma)\left((1-\tau)w_t + \frac{(1-\tau)w_{t+1}}{1+r_{t+1}} + \frac{(1-\tau)w_{t+2}l}{(1+r_{t+1})(1+r_{t+2})} + \frac{P_{t+3}}{(1+r_{t+1})(1+r_{t+2})(1+r_{t+3})}\right) \tag{10}$$

2.2 Firms

Firms produce the final good by employing capital stock and labor. Here, we assume the following production function with Romer (1986)-type externalities, as in Grossman and Yanagawa (1993).

$$Y_t = AK_t^{\theta}(B_tL_t)^{1-\theta}, 0 < A, 0 < \theta < 1, B_t = \frac{K_t}{L_t} \tag{11}$$

Here, K_t denotes the capital stock, L_t labor input, A total factor productivity, and θ the capital share. Moreover, B_t represents productivity, measured as the capital stock per worker. In considering the firm's profit-maximizing behavior, B_t is taken as given. Under the assumption of a perfectly competitive market, factor prices are equal to the marginal products of the corresponding inputs, which yields the following equations.

$$w_t = \frac{(1-\theta)A}{2+l}K_t \tag{12}$$

$$1 + r_t = \theta A \tag{13}$$

The population size of each generation is assumed to be one. In this case, both the young and middle-aged generations supply one unit of labor inelastically, while the early-old-age generation supplies l units of labor. Hence, note that aggregate labor supply is given by $L_t = 2 + l$.

2.3 Government

The government levies a proportional tax at rate τ on the wage incomes of the young, middle-aged, and early-old-age generations, and provides pension benefits to the late-old-age generation. Under the assumptions of a pay-as-you-go pension system and a balanced budget, pension benefits are given by the following equation.

$$P_{t+3} = \tau(w_{t+3} + w_{t+3} + w_{t+3}l) = \tau(2 + l)w_{t+3} \tag{14}$$

Pension benefits in period $t + 3$ are financed by contributions collected from the young, middle-aged, and early-old-age generations in the same period. In the parentheses, the first term represents tax revenues from the young generation, the second term from the middle-aged generation, and the third term from the early-old-age generation.

3. Equilibrium

We now derives the law of motion for capital. The capital stock fully depreciates within one period, and the savings of the young, middle-aged, and early-old-age generations are invested and become the capital stock in the next period.

$$K_{t+1} = s_{1t} + s_{2t+1} + s_{3t+2} \tag{15}$$

$$K_{t+1} = X_1K_t - X_2K_{t+1} - X_3K_{t+2} - X_4K_{t+3} + X_5K_{t-1} + X_6K_{t-2} \tag{16}$$

The terms X_1 through X_6 are defined as follows.

$$X_1 = \Big(1 - \alpha + 1 - (\alpha + \beta) + l\big(1 - (\alpha + \beta + \gamma)\big)\Big)(1 - \tau)\frac{A(1 - \theta)}{2 + l} \tag{17}$$

$$X_2 = \left(\frac{\alpha}{1 + r} + \frac{(\alpha + \beta)l}{1 + r} + \frac{\alpha + \beta + \gamma}{1 + r}\frac{\tau(2 + l)}{1 - \tau}\right)(1 - \tau)\frac{A(1 - \theta)}{2 + l} \tag{18}$$

$$X_3 = \left(\frac{\alpha l}{(1 + r)^2} + \frac{\alpha + \beta}{(1 + r)^2}\frac{\tau(2 + l)}{1 - \tau}\right)(1 - \tau)\frac{A(1 - \theta)}{2 + l} \tag{19}$$

$$X_4 = \frac{\alpha\tau(2 + l)}{(1 + r)^3}\frac{A(1 - \theta)}{2 + l} \tag{20}$$

$$X_5 = \Big((1 + r)\big(1 - (\alpha + \beta)\big) + (1 + r)\big(1 - (\alpha + \beta + \gamma)\big)\Big)(1 - \tau)\frac{A(1 - \theta)}{2 + l} \tag{21}$$

$$X_6 = (1+r)^2\big(1-(\alpha+\beta+\gamma)\big)(1-\tau)\frac{A(1-\theta)}{2+l} \tag{22}$$

Let $g_t = \frac{K_{t+1}}{K_t}$. Then we obtain the following equation.

$$(1+X_2)g_t = X_1 - X_3 g_{t+1}g_t - X_4 g_{t+2}g_{t+1}g_t + \frac{X_5}{g_{t-1}} + \frac{X_6}{g_{t-1}g_{t-2}} \tag{23}$$

For analytical tractability, we consider a simplified economy in which only the young and middle-aged generations supply labor and earn labor income, the early-old-age generation does not work ($l = 0$), and no pay-as-you-go pension system is in place ($\tau = 0$). In this case, equation (23) can be written as follows.

$$(1+X_2)g_t = X_1 + \frac{X_5}{g_{t-1}} + \frac{X_6}{g_{t-1}g_{t-2}} \tag{24}$$

Let g denote the steady-state growth rate. In the steady state, the condition $g = g_t = g_{t-1} = g_{t-2}$ holds, so equation (24) can be written as follows.

$$(1+X_2)g^3 - X_1 g^2 - X_5 g - X_6 = 0 \tag{25}$$

Since $0 < X_2$ and $0 < X_6$, it follows that the cubic function in g necessarily has one solution. This establishes the existence of a solution. A graphical illustration is shown below.

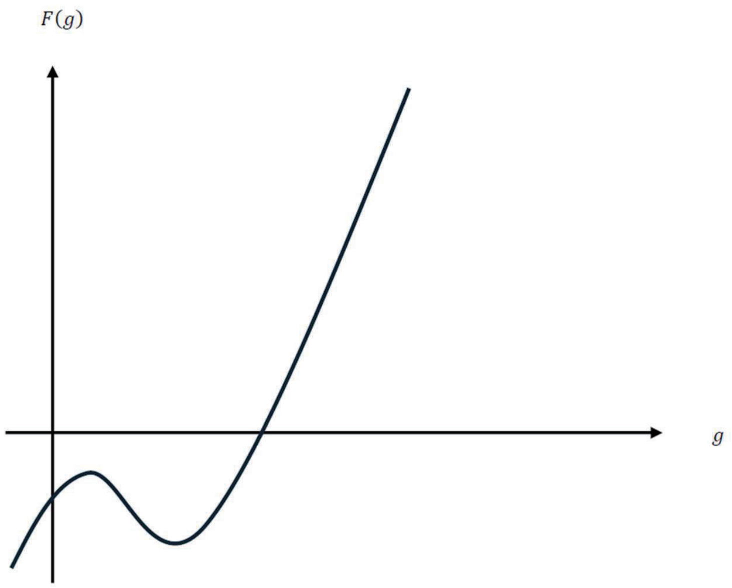

Figure 2 . Growth Rate at the Steady State

Let $F(g)$ denote the left-hand side of equation (25). Since $0 < X_2$ and $0 < X_6$, it follows that there always exists at least one solution for g. Needless to say, there may also be multiple solutions. The proof for such a case is as follows. Because equation (25) is a cubic function, the existence of at least one solution for g implies that the function must have at least one factor. To determine whether additional factors exist, one needs to check whether the quadratic equation obtained by factoring (25) admits real solutions. If this quadratic has real roots, then multiple steady-state growth rates g exist, implying the possibility of multiple steady-state equilibria. If the gross growth rate is not at least one, income will converge to zero in the long run; therefore, the condition $F(1) < 0$ is required to avoid such an outcome.

Furthermore, letting $x_t = \frac{dg_{t+1}}{dg_t}$, we obtain the following equation. This expression is derived by totally differentiating with respect to g_t, g_{t-1} and g_{t-2} in the neighborhood of the steady state.

$$(1 + X_2)x_{t+1} = -\left(\frac{X_5}{g^2} + \frac{X_6}{g^3}\right) - \frac{X_6}{g^3}\frac{1}{x_t} = 0 \tag{26}$$

If the stability condition is satisfied and, in the steady state, $x_{t+1} = x_t = x < 1$, then it follows that $dg_{t+1} < dg_t$, implying that g_t converges to a certain value. The conditions under which x_t first converges to x are as follows.

$$\frac{dx_{t+1}}{dx_t} = \frac{X_6}{1 + X_2}\frac{1}{g^3}\frac{1}{x^2} < 1 \tag{27}$$

In this case, $\frac{dg_{t+1}}{dg_t}$ converges to a constant value. However, even if $\frac{dg_{t+1}}{dg_t}$ converges to a constant, the growth rate does not converge to the steady-state growth rate g unless the condition $-1 < \frac{dg_{t+1}}{dg_t} < 1$ holds. To examine this condition, it is necessary that the solutions to the quadratic equation derived from (26) in the steady state satisfy $-1 < x < 1$.

$$(1 + X_2)x^2 + \left(\frac{X_5}{g^2} + \frac{X_6}{g^3}\right)x + \frac{X_6}{g^3} = 0 \tag{28}$$

The solutions to the quadratic equation are given as follows.

$$x = \frac{-\left(\frac{X_5}{g^2} + \frac{X_6}{g^3}\right) \pm \sqrt{\left(\frac{X_5}{g^2} + \frac{X_6}{g^3}\right)^2 - 4\frac{X_6(1 + X_2)}{g^3}}}{2(1 + X_2)} \tag{29}$$

4. Numerical Examples

In the previous section, we analytically derived the capital accumulation equation and the steady-state growth rate in the four-period overlapping generations model. However, this derivation was obtained under the assumption that neither labor supply in early old age nor a pay-as-you-go pension system is considered, which somewhat limits its realism. In this section, we incorporate the recent increase in labor supply by the early-old-age generation as well as the pay-as-you-go pension scheme. While these extensions render the analytical derivation of the equilibrium intractable, we instead examine the properties of the equilibrium through numerical simulations. The parameter settings for the numerical analysis are specified as follows.

A	4.16
α	0.5475
β	0.245
γ	0.134
θ	0.3

Table 1 . Parameter Settings

In De la Croix and Doepke (2003), the quarterly discount factor is set at 0.99. In the present model, one period corresponds to 20 years in young and middle age period, so the discount factor over 20 years is obtained by computing 0.99^{80}. On the other hand, in early old age, and late old age, one period corresponds to 15 years and the discount factor over 15 years is obtained by computing 0.99^{60}. Based on this, the parameters α , β , and γ are determined. The capital share θ is set at 0.3, reflecting the capital share observed in recent advanced economies. Furthermore, since the real economic growth rate in Japan has been close to zero in recent years, the parameter A is adjusted such that $g = 1$. The numerical results based on these parameter settings are as follows.

	Case1	Case2	Case3	Case4
l	0	0.2	0	0.1
τ	0	0	0.2	0.2
g	0.999	0.881	0.722	0.692

Table 2 . Results of Numerical Examples

Table 2 presents the results of the numerical simulations, which are consistent with intuitive expectations. The introduction of a pay-as-you-go pension system reduces capital accumulation, and in this study as well, it is shown that economic growth declines through the reduction in capital accumulation.

We consider whether pay-as-you-go pension can increase the social welfare. We assume the following social welfare function SW_t.

$$\begin{aligned} SW_t = &(1-\alpha-\beta-\gamma)lnc_{4t} + \rho(\gamma lnc_{3t+1} + (1-\alpha-\beta-\gamma)lnc_{4t+1}) \\ &+ \rho^2(\beta lnc_{2t+2} + \gamma lnc_{3t+2} + (1-\alpha-\beta-\gamma)lnc_{4t+2})u_t \\ &+ \sum_{s=t+3}^{\infty} \rho^s u_s, 0 < \rho < 1. \end{aligned}$$

ρ denotes the discount factor of generation's utility. A welfare analysis can be carried out by determining the contribution rate τ that maximizes this social welfare function.

Moreover, labor supply in early old age intuitively reduces economic growth by lowering capital accumulation. Since individuals in early old age are able to earn income from working, the need for precautionary savings is reduced. The behavior of the model is thus consistent with that of the two-period overlapping generations model, which, in turn, underscores the usefulness of the two-period framework in a wide range of analytical contexts.

5. Conclusion

This paper extends the conventional two-period overlapping generations (OLG) model—comprising youth and old age—by constructing a four-period OLG model in which individuals live through youth, middle age, early old age, and late old age, and by analyzing capital dynamics and the steady-state growth rate. The main difficulty in the four-period model lies in deriving the capital accumulation equation. By employing a production function with Romer-type capital externalities, this paper demonstrates that such a derivation can be achieved analytically. In a simplified framework without early-old-age labor and a pay-as-you-go pension system, the existence, stability, and potential multiplicity of steady states are established. The contribution of this study is to show that the analysis of multi-period OLG models—whether three- or four-period—can indeed be conducted in a tractable manner.

Furthermore, reflecting recent economic circumstances in Japan, such as the increase in labor supply among the early-old-age generation and the presence of a pay-as-you-go pension system, a numerical simulation is conducted. The results indicate that both early-old-age labor supply and the pension system reduce capital accumulation and lower economic growth, which is consistent with the findings of the conventional two-period OLG framework.

While the stability of steady-state equilibria in multi-period OLG models is usually analyzed using Schur's theorem (the Schur–Cohn stability criterion), this paper provides an alternative approach and demonstrates the stability of equilibria without relying on that method.

Appendix

We show stability condition as the other analysis. Now, we derive the Taylor approximation (28) as follows,

$$dg_{t+2} + X_7 dg_{t+1} + X_8 dg_t = 0 \tag{30}$$

where $dg_t = g_t - g$ and

$$X_7 = \frac{1}{1+X_2}\frac{1}{g^2}\left(X_5 + \frac{X_6}{g}\right) \tag{31}$$

$$X_8 = \frac{X_6}{1+X_2}\frac{1}{g^3} \tag{32}$$

Then, dg_t is derived as follows.

$$dg_t = C_1 X_9^t + C_1 X_{10}^t \tag{33}$$

where

$$X_9 = -\frac{X_7 + \sqrt{X_7^2 - 4X_8}}{2} \tag{34}$$

$$X_{10} = -\frac{X_7 - \sqrt{X_7^2 - 4X_8}}{2} \tag{35}$$

The condition to have the balanced growth path is $-1 < \frac{-X_7 \pm \sqrt{X_7^2 - 4X_8}}{2} < 1$. C_1 and C_2 are constant value and are given by the initial value.

We can the other analysis about stability condition. Considering (28), we can consider the following quadratic equations.

$$g^2 + X_7 g + X_8 = 0 \tag{36}$$

Then, we can obtain the following equations.

$$dg_{t+2} - X_9 dg_{t+1} = X_{10}(dg_{t+2} - X_9 dg_{t+1}) \tag{37}$$

$$dg_{t+2} - X_{10} dg_{t+1} = X_9(dg_{t+2} - X_{10} dg_{t+1}) \tag{38}$$

We obtain the following equations.

$$dg_{t+1} - X_9 dg_t = (dg_2 - X_9 dg_1) X_{10}^{t-1} \tag{39}$$

$$dg_{t+1} - X_{10} dg_t = (dg_2 - X_{10} dg_1) X_9^{t-1} \tag{40}$$

Then, we obtain

$$dg_{t+1} = \frac{(dg_2 - X_9 dg_1) X_{10}^{t-1} - (dg_2 - X_{10} dg_1) X_9^{t-1}}{X_{10} - X_9} \tag{41}$$

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