DISCUSSION PAPER SERIES

Discussion paper No.293

Network Connectivity and R&D Competition in a Hotelling Model: The Role of Market Coverage and Consumer Expectations

Tsuyoshi TOSHIMITSU
(School of Economics, Kwansei Gakuin University)

May 2025

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

Network Connectivity and R&D Competition in a Hotelling Model: The Role of Market Coverage and Consumer Expectations[†]

Tsuyoshi TOSHIMITSU[*]
School of Economics, Kwansei Gakuin University

Abstract

Network connectivity, compatibility, and horizontal interoperability are important functions in network industries. Based on the framework of a Hotelling model, we consider the impact of connectivity between network goods on incentives to innovate and profits. We focus on the role of market coverage (i.e., full and partial coverage) and consumer expectations (i.e., rational and active expectations). We demonstrate that in the case of full market coverage, as the degree of connectivity increases, research and development (R&D) activities decrease, but profits increase. Then, relaxing the assumption of market coverage, we demonstrate that in the case of partial market coverage, as the degree of connectivity increases, R&D activities and profits increase. Furthermore, regarding the full market coverage case, we examine the case that the formation of consumer expectations is active and demonstrate that an improvement in connectivity does not affect R&D activities, but increases profits.

Keywords
Network externality, Connectivity, Compatibility, Horizontal interoperability, R&D competition, Market coverage, Consumer expectations

JEL Classifications
L13, L15, L31, L32, D43

[†] Acknowledgment: A part of the first draft of this paper was presented at the Brown Bag Seminars organized by the Department of Economics at Simon Fraser University. Many participants provided useful comments, for which I would like to express my gratitude. In particular, I owe much to Nicolas Schmitt for his advice. Any errors or omissions are the sole responsibility of the author.

[*] Corresponding author: School of Economics, Kwansei Gakuin University; 1-155, Nishinomiya, Japan, 662-8501; Email. ttsutomu@kwansei.ac.jp

1. Introduction

Episode 1: An exchange of emails with a colleague who recently went on a family trip to see the aurora:

"I took some pictures of the aurora with the A-phone, which has the latest camera, and I'd like to transfer them directly to you, but yours didn't work with the A-phone."

"I have a G-phone, so that might not be possible, but I have an A-pad, which is a little old."

"The image quality might be a little poor, but next time we meet, I'll transfer the pictures from my A-phone to your A-pad."

Episode 2: A conversation with a colleague who commutes between university and downtown:

"It seems like next week; it'll be easier to transfer between the A and B lines at station C on the S-train."

"It'll be more convenient, since I had to walk a bit to transfer every time."

"I used to take the bus to university, too, but from now on I'll start taking the S-train."

1.1 Background and research questions

In a modern digital economy, networking is not only spreading to all economic activities, but also to every aspect of our daily lives. Network connectivity, compatibility, and "horizontal interoperability" are important functions in network industries.[1] However,

---

[1] "Horizontal interoperability" is the form of interconnection between users (e.g., consumers), following the terminology of Çavuş (2024).

these functions are not limited to current information and communication technology industries, and play an important role in transportation such as railways and airlines (e.g., mutual access, alliances, and seamless operation) and banking systems such as automatic teller machines, and so on. Thus, an improvement in the quality of connectivity (compatibility and horizontal interoperability) can be beneficial to consumers who use goods and services with such functions.

However, these trends will surely intensify competition among firms providing network goods and services. Consequently, intense competition occurs at various levels and stages of the production and product sales process, including product (service) and process research and development (R&D) investments, prices and sales (quantity) competition, among many others.

In this regard, Heywood et al. (2022, pp. 355–356, emphasis added) views compatibility as:

> "The extent to which one firm's R&D may allow it to lower costs and capture customers can be limited by the lack of compatibility. In addition, it is recognized that the extent of compatibility can influence the introduction of new technology [and that] reflecting this interconnection, firm compatibility decisions by network firms raise public policy issues regarding both *anti-competitive behavior* and *reduced technological progress*."

The first problem addressed in this paper is how network connectivity affects incentives to undertake R&D activities and profits.[2] That is, does an increase in the

---

[2] In this paper, we will not consider investments in connectivity itself among internet services (e.g., Crémer et al., 2000; Foros and Hansen, 2001; Ji and Daito, 2008), that is,

degree of network connectivity improve or reduce incentives to innovate? We are most interested in the conditions under which it is possible for network connectivity to reduce incentives for firms to innovate. For example, about the two previous episodes, one might consider the following: regarding episode 1, will the G-phone conduct R&D activities to equip its mobile telephones with high-quality cameras to compete with the A-phone? If the degree of compatibility (or connectivity) between the A- and G-phones were to increase, would the G-phone dare to undertake such R&D activities? Conversely, in that case, will the A-phone develop mobile telephones equipped with more high-quality cameras in the future? Second, regarding episode 2, if the number of passengers using the S-train increases because of convenient transfers, that is, by the improvement of connectivity, will the S-train introduce new trains or improve passenger service?

In addition to the first problem, the second is whether an increase in the degree of connectivity induces anticompetitive behavior when firms compete on R&D activities. For example, Economides and White (1994) discuss the economic and legal implications of compatibility and networks. They argue that compatibility is equivalent to the more general concept of complementarity and conclude that network arrangements bring benefits to firms, whereas compatibility may lead to *anti-competitive consequences*. Relating to this point, Shy (2001) also argues that compatibility is anti-competitive.

In considering these problems, as will be addressed in detail below, we focus on two key concepts—market coverage and consumer expectations. Before doing so, we briefly review the related literature.

---

an endogenous decision of network connectivity. We assume that the degree of connectivity is exogenously given as a parameter.

1.2 Literature review

Since the seminal studies by Farrell and Saloner (1985) and Katz and Shapiro (1985), there has been an increasing number of studies analyzing R&D investment competition in the presence of network externalities and compatibility (connectivity). So, in this paper, by focusing on the characteristics of demand functions assumed in related models, we review the related literature.[3]

Based on the model of the backbone market as in Crémer et al. (2000), which is an extension of Katz and Shapiro (1985), Knauff and Karbowski (2021) and Heywood, et al. (2022) consider cost-reducing R&D investment competition under network effects. Both introduce technological knowledge spillover a là d'Aspremont and Jacquemin (1988) and compare noncooperative R&D with cooperative R&D investments.

Second, some models apply the utility function of Hoernig (2012), in which a representative (homogeneous) consumer has a quasi-linear (e.g., quadratic) utility function including network effects, and purchases all the network goods provided in the market. For example, see Naskar and Pal (2020), Shrivastav (2021), and Buccella et al. (2023). Using a horizontally differentiated duopoly model with network effects, Shrivastav (2021) demonstrates the ranking of cost-reducing R&D investments for Bertrand and Cournot duopolistic competition.[4] Furthermore, Shrivastav (2021, Appendix B) also finds the effects of compatibility on R&D investments, and argues that

---

[3] We follow Roson's (2002) review of Crémer et al. (2000) and Foros and Hansen (2001). In particular, Crémer et al. (2000) adopt the well-known model in Katz and Shapiro (1985), whereas Foros and Hansen (2001) adopt a unit-linear market following a conventional Hotelling location model.

[4] Naskar and Pal (2020) assume that the degree of compatibility is equal to that of product differentiation. However, relaxing this assumption, Shrivastav (2021) examines more general cases.

the following results hold in both Bertrand and Cournot competition: (i) if R&D investments are strategic complements, as compatibility increases, R&D investments increase; and (ii) if R&D investments are strategic substitutes, as compatibility increases, R&D investments first decrease, and then increase. Buccella et al. (2023) assume a homogeneous product with network externalities and technological spillover effects. They then compare the investments, quantities, and profits in the full compatibility case with those in the incompatibility case, declaring that if there are no technological spillover effects, the level of investment in the incompatibility case is higher than in the full compatibility case.

As Roson (2002) points out, what the demand functions in the first and second models have in common is that an increase in compatibility (connectivity) leads to an increase in the overall market size. However, in the models in the following studies, an increase in connectivity does not necessarily lead to market expansion.

Third, Foros and Hansen (2001) assume that each consumer in a unit-linear market of a Hotelling model has an individual preference for the goods (i.e., heterogeneity) and then purchases either one or none of the goods. Regarding this location demand model, Kim (2000) assumes quality-improving innovation and considers the effect of compatibility on incentives to innovate. Kim (2000, Theorem 5) shows that the effect of an increase in compatibility on the profit of the innovative firm is ambiguous, whereas the profit of the non-innovative rival firm is increased. In this case, the assumption is made that the innovative firm is a high-quality firm, whereas the non-innovative firm is a low-quality firm. This is because an increase in compatibility raises the prices of the innovative firm, leading it to lose market share, which implies that the effect of compatibility on innovation can be negative.

Applying a linear Hotelling market model, the approach of Sääskilahti (2006), which is very close to ours, considers cost-reducing innovation and shows that network compatibility is neutralized in the decision regarding cost-reducing investment given "symmetric qualities" (i.e., identical strength of network effects between firms).[5] However, Sääskilahti (2006, Proposition 3) demonstrates that in the case of "asymmetric qualities" (i.e., different strengths of network effects between firms), the effect of an increase in compatibility on the investment of the high (low) "quality" firm is negative (positive).

1.3 The key concepts: Market coverage and consumer expectations

We assume that there are heterogeneous consumers with individual preferences for network products, as in Kim (2000), Foros and Hansen (2001), and Sääskilahti (2006). Based on the framework of a Hotelling linear market model, we consider the impact of connectivity between network goods on strategic R&D activities (i.e., quality-improving and/or cost-reducing investments) and on profits, noticing the following two key concepts: structure of market coverage and formation of consumer expectations for network sizes.

First, regarding market coverage, we address the cases of full and partial coverage. In the partial market coverage case, competing firms can expand their market share. Thus, as mentioned in the literature review, while the market expansion effect does not occur in the full market coverage case, there is room for this effect to occur in the partial market

---

[5] Sääskilahti (2006) assumes a technological knowledge spillover a là d'Aspremont and Jacquemin (1988). We do not assume technological spillover effects, which are supply-side externalities, but assume symmetric network (consumption) externalities, which implies demand-side externalities.

coverage case. In this case, the impact of connectivity on incentives to innovate depends on the difference in market coverage.[6]

Second, regarding the formation of consumer expectations for network sizes, we consider two types of expectations, following the approach of Katz and Shapiro (1985). That is, we deal with "rational" and "active" expectations.[7] "Rational" expectations follow the concept of a fulfilled expected equilibrium. That is, consumers can "rationally" form their expectations of network sizes at the equilibrium. Accordingly, under the expectations, consumers do not believe that the announcements of network sizes in advance by firms are credible, in other words, firms cannot commit to their announcements of network sizes. On the other hand, under "active" expectations, consumers believe the announcements of actual sizes (outputs or number of consumers) are equal to expected sizes and thus the firms can commit to their actual output.[8, 9]

The rest of the paper is organized as follows. In Section 2, using the framework of a Hotelling linear market, we derive demand functions in the cases of full and partial market coverage and present cost and profit functions of R&D activities. In Sections 3, we derive the equilibrium in R&D competition in the cases of full and partial market coverage, and consider how connectivity affects R&D activities and profits. In Section 4, by relaxing

---

[6] For example, Kim (2000), Foros and Hansen (2001), and Sääskilahti (2006) assume the full market coverage case.

[7] In the terminology of Suleymanova and Wey (2012), "rational" corresponds to "strong" and "actual" to "weak". Furthermore, following Hurkens and López (2014), "rational" corresponds to "passive" and "active" to "responsive".

[8] Regarding consumer expectations, Kim (2000) and Foros and Hansen (2001) assume "rational" expectations, whereas Sääskilahti (2006) assumes "active" expectations.

[9] Following the definition of Shy (2001, Definition 2.4, p. 20), "actual" expectations imply that consumers have *perfect foresight* if, at the time of purchase, they can correctly anticipate how many consumers will be buying each brand.

the assumption of consumer expectations, as for the full market coverage case, we examine the impact of connectivity on R&D activities and profits under active expectations and compare the outcomes to those under rational expectations. We find that the impact of connectivity on competitiveness depends on the difference in the formation of consumer expectations. Finally, in Section 5, we summarize our findings and discuss some remaining problems.

2. Model

2.1 Demand functions and market coverage

We introduce network externalities associated with connectivity (compatibility and horizontal interoperability) into a Hotelling linear market model. *Firm i*, which is located at both ends of a unit linear market, provides *product i*, $i = 0,1$. Consumers are uniformly distributed over a unit line of the closed interval $[0,1]$ according to their subjective taste preferences. That is, consumer $l \in [0,1]$ has the following surplus (net utility) function:

$$U_l = \begin{cases} v_0 - tl - p_0 + N_0 & \textit{if buying} \quad \textit{product} \quad 0 \\ v_1 - t(1-l) - p_1 + N_1 & \textit{if buying} \quad \textit{product} \quad 1 \\ 0 & \textit{if buying} \quad \textit{nothing} \end{cases}, \tag{1}$$

where v_i denotes the intrinsic (stand-alone) value of *product i*, implying the level of quality of *product i*, t is a transportation cost, implying product substitutability,[10] p_i

---

[10] The smaller (larger) the transformation cost, the higher (lower) the production

is price of *product i*, and N_i denotes network effects of *product i*, which are explicitly specified below.

Using Equation (1), we derive demand functions in the cases of full and partial market coverage. Consumer indexed l^*, whose surplus is indifferent between *products 0* and *1*, is given by $l^* = \frac{1}{2} + \frac{v_0 - v_1 - p_0 + p_1 + N_0 - N_1}{2t}$.

First, for the case of full market coverage to hold, the following conditions must be met: $U_{l=l^*} > 0$ and $U_{l=0}\left(U_{l=1}\right) > 0$. Taking Equation (1), we obtain the following conditions.

FMC: $t \leq v_0 - p_0 + N_0 + v_1 - p_1 + N_1 \equiv T$ and $v_i - p_i + N_i > 0, \quad i = 0,1.$ See Appendix A (1), in which we demonstrate the conditions under which the *FMC* holds in equilibrium.

Given the conditions, all consumers in the unit-linear market purchase either of two network products. Thus, the demand function of *firm 0* is given by:

$$x_0 = l^* = \frac{t + v_0 - v_1 - p_0 + p_1 + N_0 - N_1}{2t}, \tag{2}$$

where $N_i \equiv n\left(x_i^e + \phi x_j^e\right), \quad i, j = 0,1, \quad i \neq j.$ Parameter $n\left(>0\right)$ expresses the strength of network externalities,[11] $\phi \in \left[0,1\right]$ denotes the degree of network

---

substitutability. Thus, as will be discussed below, the structure of market coverage depends on the level of a transformation cost. Although the intrinsic values also affect the structure of market coverage, we do not discuss that case in this paper.

[11] Following the terminology of Sääskilahti (2006), regarding the strength of network

connectivity (hereinafter, connectivity), and x_i^e denotes the expected network size of *product i*, which also expresses the expected number of consumers. Thus, $x_i^e + \phi x_j^e$ is the total expected network size of network products in the market. In this case, nx_i^e expresses the "within-group" (direct) network effects for consumers purchasing *product i* from themselves, and $n\phi x_j^e$ expresses the "cross-group" (indirect) network effects for consumers purchasing *product i* from consumers purchasing *product j*.[12] Regarding the demand function of *product 1*, based on Equation (2), we have $x_1 = 1 - x_0$.

Second, using the same procedure as in the full market coverage case, for the case of partial market coverage to hold, the following conditions must be met: $U_{l=l^*} < 0$ and $U_{l=0}\left(U_{l=1}\right) > 0$. Hence, we obtain the following conditions.

PMC: $t > v_0 - p_0 + N_0 + v_1 - p_1 + N_1 \equiv T$ and $v_i - p_i + N_i > 0, \quad i = 0,1$. See Appendix A (2), in which we demonstrate the conditions under which the *PMC* holds in equilibrium.

Given the conditions, in the case of partial market coverage, there are some potential consumers not purchasing network products. Using Equation (1) (i.e., $U_l = 0$), the

---

externalities, we assume symmetric "qualities" of network products (i.e., $n_0 = n_1 = n$).

[12] Following the terminology of Sääskilahti (2006), the "within group" network effect corresponds to a "home" network and the utility is labelled as an "intra-network utility". Similarly, the "cross-group" network effect corresponds to a "rival" network and the utility as an "inter-network utility".

marginal consumer purchasing *product 0* is given by $l_0 = \frac{v_0 - p_0 + N_0}{t} = z_0$. Similarly, for *product 1*. Thus, we obtain the following demand function:

$$z_i = \frac{v_i - p_i + N_i}{t}, \quad i = 0,1, \tag{3}$$

where $N_i = n\left(z_i^e + \phi z_j^e\right), \quad i, j = 0,1, \quad i \neq j.$ Using Equation (3), it also holds that $z_0 + z_1 < 1 \Leftrightarrow t > T.$

[Insert Figures 1 and 2]

2.2 Profit function, R&D activities, and game structure

Using Equations (1) and (2), in the cases of full (partial) market coverage, the gross profit function of *firm i* is expressed as $\pi_i^f = \left(p_i - c_i\right)x_i \quad \left(\pi_i^p = \left(p_i - c_i\right)z_i\right)$, where c_i is the marginal cost of production of *firm i*, $i = 0,1.$ Superscript *f* (*p*) denotes the full (partial) market coverage case under rational expectations.

We consider product (quality-improving) and/or process (cost-reducing) R&D activities, in which the quality level corresponds to the intrinsic value of network products. For the analysis below, regarding the variables expressing the quality level and marginal cost, we assume $v_i = v + \beta_i$ and $c_i = c - \varepsilon_i\left(\geq 0\right), \quad i = 0,1.$ $\beta_i\left(\varepsilon_i\right)$ expresses the degree of quality-improving (cost-reducing) R&D activity and $v\ \left(c\right)$ expresses the initial level of quality (marginal cost) before implementing R&D activities. Furthermore, we define the variables as follows: $a_i \equiv a + \alpha_i$, where $a \equiv v - c > 0, \quad \alpha_i \equiv \beta_i + \varepsilon_i,$

$i = 0,1.$ The variable a denotes the initial level of quality net of marginal cost, and thus $\alpha_i (\geq 0)$ is a combined variable expressing the quality-improving and cost-reducing R&D activities. Thus, $d\alpha_i > 0 \Leftrightarrow d\beta_i + d\varepsilon_i > 0,$ where $d\beta_i > 0$ (and/or $d\varepsilon_i > 0$) shows an increase in the level of quality-improving (and/or cost-reducing) R&D activities.

The firms incur fixed costs for their R&D activities. In particular, we assume the following R&D activities (investments) cost function: $F(\alpha_i) = \frac{k}{2}(\alpha_i)^2, \ k > 0.$ The net profit function of *firm i* is expressed as $\Pi_i^m = \pi_i^m - F(\alpha_i), \ m = f, p, \ i = 0,1.$

The structure of the game consists of two stages. At the first stage, the firms decide the level of R&D activities, and at the second stage, the firms compete on prices. We assume that consumers have rational expectations for the network sizes of the products and form the expectations before the second stage (or after the first stage). We exploit the concept of a fulfilled expectation equilibrium presented by Katz and Shapiro (1985) and derive a subgame perfect Nash equilibrium in the game by backward induction.

3. Analysis: Market Coverage and R&D Competition

3.1 Equilibrium and impact of connectivity under full market coverage case[13]

In the second stage of price competition, the firm decides the price to maximize profit, given the expected network sizes. Taking Equation (2), the first-order condition (FOC)

---

[13] This subsection is based on Toshimitsu (2024).

of profit maximization of *firm i* is given by $\frac{\partial \pi_i^f}{\partial p_i} = x_i - \frac{p_i - c_i}{2t} = 0,\ \ i = 0{,}1.$ At the fulfilled expectation equilibrium, $x_i^e = x_i = \frac{p_i - c_i}{2t}$, we obtain the following price-cost margin and output:

$$p_i^f - c_i = \frac{\left\{3t - n(1-\phi) + \alpha_i - \alpha_j\right\}t}{3t - n(1-\phi)}, \tag{4}$$

$$x_i^f = \frac{p_i^f - c_i}{2t} = \frac{3t - n(1-\phi) + \alpha_i - \alpha_j}{2\left\{3t - n(1-\phi)\right\}},\ \ i, j = 0{,}1,\ \ i \neq j, \tag{5}$$

where $t > \frac{n}{3}\left(\geq \frac{n(1-\phi)}{3}\right)$.

In the first stage of competition for R&D activities, the net profit function of *firm i* is expressed as $\Pi_i^f = \left(p_i^f - c_i\right)x_i^f - F\left(\alpha_i\right) = \frac{\left(p_i^f - c_i\right)^2}{2t} - \frac{k}{2}\left(\alpha_i\right)^2,\ \ i = 0{,}1.$ The FOC with respect to R&D activity is given by:

$$\frac{\partial \Pi_i^f}{\partial \alpha_i} = \frac{p_i^f - c_i}{3t - n(1-\phi)} - k\alpha_i = \frac{\left\{3t - n(1-\phi) + \alpha_i - \alpha_j\right\}t}{\left\{3t - n(1-\phi)\right\}^2} - k\alpha_i = 0. \tag{6}$$

Additionally, we derive the following second-order condition (SOC) and cross effect:[14]

$$\frac{\partial^2 \Pi_i^f}{\partial {\alpha_i}^2} = \frac{t}{\left\{3t - n(1-\phi)\right\}^2} - k < 0 \ \text{ and } \ \frac{\partial^2 \Pi_i^f}{\partial \alpha_i \partial \alpha_j} = \frac{-t}{\left\{3t - n(1-\phi)\right\}^2} < 0.$$

Because the cross effect is negative, the R&D activities are strategic substitutes.

---

[14] Because the sign of the determinant of the Hessian matrix is positive, the local maximum value condition is $k\left\{3t - n(1-\phi)\right\}^2 - 2t > 0.$

Based on Equation (6), it holds at the equilibrium that

$$\alpha_i^f = \frac{t}{k\left\{3t - n(1-\phi)\right\}} \equiv \alpha^{f*}(\phi), \quad i = 0,1. \tag{7}$$

In view of Equation (7), the effect of an increase in connectivity on the R&D activity is given by $\frac{d\alpha^{f*}(\phi)}{d\phi} = -\frac{nt}{k\left\{3t - n(1-\phi)\right\}^2} < 0$. Thus, we summarize the result as follows.

Proposition 1.

An increase in connectivity reduces R&D activity in the full market coverage case.

Why does an improvement in connectivity between network products suppress incentives to innovate? Using Equation (6), the effect of an increase in connectivity on the marginal net profit is given by $\frac{\partial^2 \Pi_i^f}{\partial \alpha_i \partial \phi} = -\frac{n\left\{3t - n(1-\phi) + 2(\alpha_i - \alpha_j)\right\}t}{\left\{3t - n(1-\phi)\right\}^3}$, $i, j = 0,1, \quad i \neq j$. In particular, at the symmetric equilibrium, it holds that $\left.\frac{\partial^2 \Pi_i^f}{\partial \alpha_i \partial \phi}\right|_{\alpha_0^f = \alpha_1^f = \alpha^{f*}} = -\frac{nt}{\left\{3t - n(1-\phi)\right\}^2} < 0$. Because an increase in connectivity reduces the marginal net profit, it decreases incentives to innovate. This is because an increase in connectivity leads to demand spillovers to the rival firm through the "cross-group" network effects.

Second, we examine the impact of connectivity on net profit, which is expressed as $\Pi_i^f = \Pi_i^f\left[\alpha_i^f(\phi), \alpha_j^f(\phi), \phi\right]$, $i, j = 0,1, \quad i \neq j$. The total effect of an increase in connectivity on the net profit of *firm i* is given by:

$$\frac{d\Pi_i^f}{d\phi}=\frac{\partial\Pi_i^f}{\partial\alpha_i^f}\frac{d\alpha_i^f}{d\phi}+\frac{\partial\Pi_i^f}{\partial\alpha_j^f}\frac{d\alpha_j^f}{d\phi}+\frac{\partial\Pi_i^f}{\partial\phi}=\frac{\partial\Pi_i^f}{\partial\alpha_j}\frac{d\alpha_j^f}{d\phi}+\frac{\partial\Pi_i^f}{\partial\phi},\ \ i,j=0,1,\ \ i\neq j,$$

where $\frac{\partial\Pi_i}{\partial\alpha_i}=0$ by the FOC. Using Equation (4), the direct effect of connectivity on net profit is $\frac{\partial\Pi_i^f}{\partial\phi}=\frac{p_i^f-c_i}{t}\frac{-n\left(\alpha_i-\alpha_j\right)t}{\left\{3t-n(1-\phi)\right\}^2}$.[15] At the symmetric equilibrium, it holds that $\left.\frac{\partial\Pi_i^{f*}}{\partial\phi}\right|_{\alpha_0^f=\alpha_1^f=\alpha^{f*}}=0$. In addition, the direct effect of the rival firm's R&D activity on the net profit is $\frac{\partial\Pi_i^f}{\partial\alpha_j^f}=-\frac{p_i^f-c_i}{3t-n(1-\phi)}<0$. Thus, the total effect of an increase in connectivity on the net profit is given by:

$$\left.\frac{d\Pi_i^{f*}}{d\phi}\right|_{\alpha_0^f=\alpha_1^f=\alpha^{f*}}=\underset{-}{\frac{\partial\Pi_i^f}{\partial\alpha_j^f}}\underset{-}{\frac{d\alpha_j^f}{d\phi}}>0. \tag{8}$$

Given Equation (8), the direct effect of connectivity on net profit is cancelled out at the symmetric equilibrium. However, although an increase in connectivity reduces the rival firm's R&D activity, the reduction leads to an increase in net profit because of strategic

---

[15] Based on Equation (5), we have the following relationship: $\alpha_i>(<)\alpha_j \Leftrightarrow x_i>(<)x_j$. That is, the higher the level of R&D activity, the larger the market share. Assuming that *firm i* is a firm with a larger market share (i.e., a large firm), an increase in connectivity reduces its market share as follows: $\frac{dx_i}{d\phi}=\frac{-n\left(\alpha_i-\alpha_j\right)t}{\left\{3t-n(1-\phi)\right\}^2}<0$. Thus, the direct effect on the net profit of the large firm is negative. This is because of demand spillover to the small firm through the "cross-group" network effects.

substitutes in R&D competition. We summarize the result as follows.

Proposition 2.

An increase in connectivity increases net profit in the full market coverage case.

Based on Propositions 1 and 2, it holds that $\alpha^{f*}(\phi=0)>\alpha^{f*}(\phi=1)$ and $\Pi_i^{f*}(\phi=0)<\Pi_i^{f*}(\phi=1)$. This implies that providing the network products with perfect connectivity is desirable for the firms, although the level of R&D activities is the lowest. In addition, in view of Equation (4), the equilibrium price is expressed as $p_i^f(\phi)=t+c_i(\phi),$ so that we obtain $p_i^f(\phi=1)>p_i^f(\phi=0)$. That is, the firms provide their network products with lower quality and higher price (i.e., degraded network products), compared with the case of imperfect connectivity.

3.2 Equilibrium and impact of connectivity under partial market coverage case

By relaxing the assumption of market coverage, we consider the case of partial market coverage, in which there are some potential consumers not purchasing network products before changing connectivity.

Following the approach in the previous subsection, and using Equation (3), we derive the following outcomes at the equilibrium in price competition:

$$p_i^p-c_i=\frac{\left[\{2t-n(1-\phi)\}a+(2t-n)\alpha_i+n\phi\alpha_j\right]t}{D^p}, \tag{9}$$

$$z_i^p=\frac{p_i^p-c_i}{t}=\frac{\{2t-n(1-\phi)\}a+(2t-n)\alpha_i+n\phi\alpha_j}{D^p}, \tag{10}$$

where $D^p \equiv \{2t - n(1+\phi)\}\{2t - n(1-\phi)\} > 0$ for $t > n \geq \frac{n(1+\phi)}{2}$, $i, j = 0,1$, $i \neq j.$

The net profit function is $\Pi_i^p = \left(p_i^p - c_i\right) z_i^p - F(\alpha_i) = \frac{\left(p_i^p - c_i\right)^2}{t} - \frac{k}{2}\left(\alpha_i\right)^2.$ Thus, the FOC in R&D competition is given by:

$$\frac{\partial \Pi_i^p}{\partial \alpha_i} = \frac{2\left(2t-n\right)}{D^p}\left(p_i^p - c_i\right) - k\alpha_i = 0, \quad i = 0,1. \tag{11}$$

The SOC, the cross effect, and the effect of an increase in the rival firm's R&D activity on the net profit are respectively given by $\frac{\partial^2 \Pi_i^p}{\partial \alpha_i^2} = 2t\left(\frac{2t-n}{D^p}\right)^2 - k < 0,$ $\frac{\partial^2 \Pi_i^p}{\partial \alpha_i \partial \alpha_j} = \frac{2t\left(2t-n\right)n\phi}{(D^p)^2} \geq 0 \Leftrightarrow \phi \geq 0,$ and $\frac{\partial \Pi_i^p}{\partial \alpha_j} = \frac{2n\phi}{D^p}\left(p_i^p - c_i\right) \geq 0 \Leftrightarrow \phi \geq 0.$

The second equation implies strategic complements in R&D competition. Here, we should consider the outcomes shown in the second and third equations, which differ from those in the full market coverage case. In view of Equation (5), when the market is fully covered, the rival firm's R&D activity takes away the firm's market share. Conversely, in view of Equation (10), the firm's market share increases through the "cross-group" network effects by the rival firm's R&D activity (i.e., market expansion effects) unless there exists null connectivity. In addition, the direct effect of an increase in connectivity on net profit is positive, even in the symmetric equilibrium.[16] That is, an increase in

---

[16] Regarding the direct effect, we derive: $\frac{\partial \Pi_i^p}{\partial \phi} = \frac{2}{t}\left(p_i^p - c_i\right)\frac{ntQ}{(D^p)^2} > 0,$ where

connectivity can expand the market share of firms in the partial market coverage case, which differs from the full market coverage case.

Exploiting Equations (9) and (11), we obtain the following R&D activities in equilibrium:

$$\alpha_i^p = \frac{2t(2t-n)a}{R^p} \equiv \alpha^{p*}(\phi), \quad i = 0,1, \tag{12}$$

where $R^p \equiv k\left\{2t-n(1-\phi)\right\}\left\{2t-n(1+\phi)\right\}^2 - 2t(2t-n) > 0.$ Based on Equation (12), we have $\frac{d\alpha^{p*}(\phi)}{d\phi} > 0,$ because $\frac{dR^p}{d\phi} < 0.$ The R&D activity function of connectivity is monotonically increasing.

In addition, regarding the impact on the net profit, we can directly obtain $\left.\frac{d\Pi_i^{p*}}{d\phi}\right|_{\alpha_0^p=\alpha_1^p=\alpha^{p*}} > 0.$[17] Therefore, we summarize the results in the partial market coverage case as follows.

Proposition 3.

An increase in connectivity increases R&D activity in the partial market coverage case.

$Q \equiv (a+\alpha_j)D^p + 2(n\phi)\left[\{2t-n(1-\phi)\}a + (2t-n)\alpha_i + n\phi\alpha_j\right] > 0.$

[17] We can express net profit as follows: $\Pi_i^p = \Pi_i^p\left[\alpha_i^p(\phi), \alpha_j^p(\phi), \phi\right], \quad i, j = 0,1,$ $i \neq j.$ Hence, taking the FOC, the total effects of an increase in interoperability on the net profit of *firm i* is given by $\frac{d\Pi_i^p}{d\phi} = \frac{\partial\Pi_i^p}{\partial\alpha_j}\frac{d\alpha_j^p}{d\phi} + \frac{\partial\Pi_i^p}{\partial\phi}, \quad i, j = 0,1, \quad i \neq j.$ As mentioned in the text, the first and second terms of the right-hand side of the equation are positive, unless null connectivity exists.

Proposition 4.

An increase in connectivity increases net profit in the partial market coverage case.

Based on Propositions 3 and 4, we have $\alpha^{p^*}(\phi=0)<\alpha^{p^*}(\phi=1)$ and $\Pi_i^{p^*}(\phi=0)<\Pi_i^{p^*}(\phi=1)$. The results imply that providing network products with perfect connectivity is preferable for the firms, and the level of R&D activities is the highest. For example, under a common network environment, e.g., identical operating system and perfect compatibility, the firms will provide the most upgraded network products and services for consumers in the immature digital markets.

3.3 Market coverage matters: Market expansion effects and technological progress

Considering the difference in market coverage, we can interpret the results derived above as follows. Proposition 3 is opposite to Proposition 1. This depends on the assumption of market coverage. As mentioned above, under full market coverage, an increase in connectivity induces demand spillovers to rival firms, and thus, reduces incentives to innovate. However, under partial market coverage, an increase in connectivity expands the market share (i.e., market expansion effects); as a result, it improves incentives to innovate. Therefore, the impact of connectivity on incentives to innovate (i.e., technological progress) depends on market structure, that is, whether the market is fully covered or not. If the market is not completely covered, in other words, if there is an opportunity for firms to expand their market shares, an improvement in connectivity promotes incentives to innovate by market expansion effects. This is the answer to the first problem.

Furthermore, looking at the results alone, Propositions 2 and 4 are formally the same, but the effects on net profits are completely different. In the full market coverage case, the firms earn more profits by degrading the network products, while under partial market coverage, they can earn more profits by upgrading the network products. This is because an increase in connectivity intensifies competition for a limited market share in the full market coverage case, while it has the effect of expanding the market size of each firm in the partial market coverage case.

4. Discussion: Active Expectations and R&D Competition

So far, we have assumed rational expectations, such that consumers form their expectations of the network sizes of the products before the firms decide the prices. Relaxing this assumption, we examine the R&D incentive problem in the case of active (responsive) expectations. That is, as discussed in the introduction, firms can commit to the announcement of the network sizes of their products and consumers believe that the announcements are credible. Accordingly, consumers form expectations of network sizes using these announcements.

In this section, we consider the full market coverage case. Regarding the partial market coverage case, see Appendix B, in which we demonstrate that the results are not significantly different from those under rational expectations.

4.1 Equilibrium and impact of connectivity under active expectations[18]

Under the assumption of active expectations, consumers believe the announcement of outputs (i.e., number of consumers) in advance by the firms, and form expectations of network sizes of the products, based on these announcements and prices. Thus, it holds that $x_i^e = x_i[p_i, p_j, x_i^e, x_j^e], \ \ i, j = 0,1, \ \ i \neq j.$ Taking Equation (2), we derive the following direct demand function, which the firms face:

$$x_i = \frac{t - n(1-\phi) + v_i - v_j - p_i + p_j}{2\{t - n(1-\phi)\}}, \quad i, j = 0,1, \ \ i \neq j. \tag{13}$$

Regarding price competition in the second stage, based on Equation (13), the FOC is given by $\dfrac{\partial \pi_i^{fA}}{\partial p_i} = x_i - \dfrac{p_i - c_i}{2\{t - n(1-\phi)\}} = 0, \ \ i = 0,1,$ where superscript *fA* denotes the case of full market coverage under active expectations. Similarly, for firm *j*, we obtain the following equations at this stage:

$$p_i^{fA} - c_i = \{t - n(1-\phi)\} + \frac{\alpha_i - \alpha_j}{3}, \tag{14}$$

$$x_i^{fA} = \frac{p_i^{fA} - c_i}{2\{t - n(1-\phi)\}} = \frac{1}{2} + \frac{\alpha_i - \alpha_j}{6\{t - n(1-\phi)\}}, \tag{15}$$

where $i, j = 0,1, \ \ i \neq j.$

In the first stage of R&D competition, the net profit function of *firm i* is expressed as: $\Pi_i^{fA} = \left(p_i^{fA} - c_i\right)x_i^{fA} - F\left(\alpha_i\right) = \dfrac{\left(p_i^{fA} - c_i\right)^2}{2\{t - n(1-\phi)\}} - \dfrac{k}{2}\left(\alpha_i\right)^2, \ \ i = 0,1.$ The FOC is

---

[18] The model in this subsection is basically like Sääskilahti (2006). However, we do not assume technological spillover and assume symmetric "qualities".

given by the following equation:[19]

$$\frac{\partial \Pi_i^{fA}}{\partial \alpha_i} = \frac{p_i^{fA} - c_i}{3\{t - n(1-\phi)\}} - k\alpha_i = \frac{1}{3} + \frac{\alpha_i - \alpha_j}{9\{t - n(1-\phi)\}} - k\alpha_i = 0. \qquad (16)$$

Based on Equation (16), it holds in the equilibrium that

$$\alpha_i^{fA} = \frac{1}{3k} \equiv \alpha^{fA*}(\phi), \quad i = 0,1. \qquad (17)$$

In view of Equation (17), the effect of an increase in connectivity on the R&D activity is given by $\frac{d\alpha^{fA*}(\phi)}{d\phi} = 0$. Thus, we summarize the result as follows.

Proposition 1A.

An increase in connectivity does not affect R&D activity under active expectations.

As we will explain below, under active expectations, the firms can decide the prices taking consumer expectations of the network sizes into account, that is, by internalizing network (consumption) externalities. Thus, changing connectivity does not affect the decision of R&D activities. This implies network neutrality with respect to connectivity (compatibility), under which consumers have *perfect foresight*, as mentioned by Shy (2001) and Sääskilahti (2006).

---

[19] We derive the following SOC and cross effect:

$$\frac{\partial^2 \Pi_i^{fA}}{\partial \alpha_i^2} = \frac{1}{9(t - n + n\phi)} - k < 0 \text{ and } \frac{\partial^2 \Pi_i^{fA}}{\partial \alpha_i \partial \alpha_j} = \frac{-1}{9(t - n + n\phi)} < 0.$$

Because the cross effect is negative, the R&D activities are strategic substitutes. Additionally, the condition for the local maximum value is $9k(t - n + n\phi) - 2 > 0$, because the sign of the determinant of the Hessian matrix is positive.

Second, using the FOC, i.e., $\frac{\partial \Pi_i^{fA}}{\partial \alpha_i} = 0,$ the total effect on net profit is expressed as: $\frac{d\Pi_i^{fA}}{d\phi} = \frac{\partial \Pi_i^{fA}}{\partial \alpha_j}\frac{d\alpha_j^{fA}}{d\phi} + \frac{\partial \Pi_i^{fA}}{\partial \phi},\quad i, j = 0,1,\quad i \neq j.$ In this case, the direct effect of connectivity on net profit is $\frac{\partial \Pi_i^{fA}}{\partial \phi} = \frac{n\left(p_i^{fA} - c_i\right)\left(p_j^{fA} - c_j\right)}{2\left(t - n + n\phi\right)^2} > 0.$ In addition, the effect of the rival firm's R&D activity on net profit is $\frac{\partial \Pi_i^{fA}}{\partial \alpha_j^{fA}} = -\frac{p_i^{fA} - c_i}{3\left(t - n + n\phi\right)} < 0.$ Thus, in view of Proposition 1A, the total effect of an increase in connectivity on net profit is:

$$\left.\frac{d\Pi_i^{fA*}}{d\phi}\right|_{\alpha_0^{fA}=\alpha_1^{fA}=\alpha^{fA*}} = \underset{-}{\frac{\partial \Pi_i^{fA}}{\partial \alpha_j^{fA}}}\underset{0}{\frac{\partial \alpha_j^{fA}}{\partial \phi}} + \underset{+}{\frac{\partial \Pi_i^{fA}}{\partial \phi}} = \underset{+}{\frac{\partial \Pi_i^{fA}}{\partial \phi}} > 0. \tag{18}$$

Thus, we have the following result.

Proposition 2A

An increase in connectivity increases net profit under active expectations.

The indirect effect on profit is null because R&D activities do not change by increasing connectivity. Thus, the marginal cost (and/or intrinsic value) is constant. However, the direct effect on profit is positive because an increase in the degree of connectivity increases the price, that is, the price-cost margin. As a result, an increase in connectivity increases net profit. This result differs from that under rational expectations, in which the direct effect is null in the symmetric equilibrium.

4.2 Expectations matter: internalization of network externalities and anti-competitiveness

With respect to the full market coverage case, we consider the economic implications of difference in the formation of consumer expectations.[20] Taking Equations (4), (7), (14), and (17), and considering the outcomes in each symmetric equilibrium, we derive the following equations:[21]

$$\alpha^{f*} = \frac{t}{k\{3t - n(1-\phi)\}} \geq \alpha^{fA*} = \frac{1}{3k}, \tag{19}$$

$$p^f - c\left(\alpha^{f*}\right) = t \geq p^{fA} - c\left(\alpha^{fA*}\right) = t - n + n\phi, \tag{20}$$

and

$$\Pi^{f*} = \frac{t}{2} - \frac{1}{2k}\left\{\frac{t}{3t - n(1-\phi)}\right\}^2 \geq \Pi^{fA*} = \frac{t - n + n\phi}{2} - \frac{1}{18k}. \tag{21}$$

In view of Equations (19), (20), and (21), first, with perfect connectivity, these outcomes are the same. In other words, unless there is perfect connectivity, the outcomes under rational expectations are larger than those under active expectations.

Based on Equation (19), we obtain the following results. Under active expectations, firms reflect network effects in their pricing, and thus, there is no need to consider network effects when determining the level of R&D activity (i.e., *internalizing network externalities*), so it remains constant regardless of connectivity, and is kept at the level it would be without network effects. Conversely, under rational expectations, the level of

---

[20] Here, we assume that the firms invest in cost-reducing R&D activities, but not in quality-improving R&D activities. Thus, it holds that $\varepsilon_i = \alpha_i,\ i = 0,1.$

[21] Regarding the outputs, it holds that $x^{f*} = x^{fA*} = \frac{1}{2}.$

R&D activity is determined by considering network effects. In doing so, the greater the connectivity, the more one's market share will be lost to the other firm through the "cross-group" network effects, so the level of R&D activity will be kept low. Under perfect connectivity, network effects cancel each other out, so the level of R&D activity is kept to the level it would be in the case of no network effects (same as in the case of active expectations).

Equation (20) implies that the price-cost margin under rational expectations is larger than that under active expectations. In addition, as connectivity increases, these prices increase: the price under rational expectations increases because of an increase in the marginal cost, whereas the price under active expectations increases because of improving network effects.

Because the price-cost margin under rational expectations is larger than that under active expectations and the market share of each firm in equilibrium is a half, the gross profit under rational expectations is larger than that under active expectations. However, the fixed investment cost in the former is larger than that in the latter. In this case, the difference in the gross profits (the price-cost margin) is larger than that in the fixed investment cost. As a result, Equation (21) holds. It is preferable for firms that consumers *rationally* form expectations of network sizes and *never believe* any ex-ante announcement of network sizes by firms.

As mentioned in the introduction, we should address Shy's (2001) result "compatibility is anti-competitive" as the second problem. Following Vives (2008), who employs the Lerner index as an indicator of market competitiveness, we focus on the difference in formation of consumer expectations and investigate the relationship between the Lerner index (i.e., the inverse of the absolute value of the elasticity of demand) and connectivity in the case of full market coverage.

We define the Lerner index at the equilibrium as: $L^m = \frac{p^m - c^m}{p^m} = \frac{1}{\eta^m}$, $m = f*, fA*,$ where η^m denotes the (absolute value of) elasticity of demand (hereinafter, simply elasticity). Furthermore, we deal with cost-reducing R&D activities, given a symmetric quality level (i.e., $v_0 = v_1 = v$) and thus, we have $a_i = v - c_i$, $i = 0{,}1.$

First, we examine the case of rational expectations. Using Equations (3) and (6), the R&D activity in the symmetric subgame perfect Nash equilibrium is expressed as $a^{f*}(\phi) = v - c^{f*}(\phi).$ Thus, it holds that $\frac{da^{f*}}{d\phi} = -\frac{dc^{f*}}{d\phi}.$ The elasticity is given by $\eta^{f*} = 1 + \frac{c^{f*}(\phi)}{t}.$ Based on Proposition 1, it holds that $\frac{d\eta^{f*}}{d\phi} = \frac{-1}{t}\left(-\frac{dc^{f*}}{d\phi} \right) > 0.$ As connectivity increases, the Lerner index decreases, and thus, the elasticity increases. The demand curve becomes more elastic. This implies that increasing network connectivity weakens the price controlling power of firms.

Second, in a similar way to the first case, we examine the case of active expectations. In this case, we derive the elasticity $\eta^{fA*} = 1 + \frac{c^{fA*}(\phi)}{t - n(1-\phi)}.$ Taking Proposition 1A, the impact of connectivity on the elasticity is given by $\frac{d\eta^{fA*}}{d\phi} = \frac{-nc^{fA*}(\phi)}{\{t - n(1-\phi)\}^2} < 0.$ That is, the demand curve becomes more inelastic, and thus, the Lerner index increases. This implies that increasing network connectivity strengths the price controlling power of firms.

Therefore, the answer to the second problem depends on the formation of consumer expectations. If firms can commit to the announcement of network sizes of their network products and consumers believe it, that is, under active expectations, Shy's (2001) result holds. This is because, as mentioned above, the firms can internalize consumption externalities and reflect the internalization in their pricing. However, if consumers rationally form expectations of network sizes, firms are forced to set their prices, given those expectations. In this case, an increase in the degree of connectivity reduces the mark-up ratio; however, this does not necessarily imply that the market becomes competitive. Because the firms reduce the amount of R&D activity as connectivity increases. As a result, marginal costs, and thus prices, increase compared with the case of imperfect connectivity.

5. Concluding Remarks

Introducing network externalities into a standard Hotelling linear market model, we consider the impact of connectivity (compatibility and horizontal interoperability) between network products on R&D activities and profits. From two perspectives, we demonstrated their effects in each case: one is market coverage (i.e., full and partial) and the other is consumer expectations (i.e., rational and active). Regarding the effects of an increase in connectivity on R&D activity, we summarize the results in Table 1.

[Insert Table 1]

First, in the partial market coverage case, irrespective of how consumer expectations are formed, improving connectivity promotes R&D activity. This is because an improvement in connectivity between network products expands the market share of each firm, and thus, induces incentives to innovate. These are market expansion effects. However, in the full market coverage case, an improvement does not promote incentives to innovate. Under rational expectations, an increase in connectivity induces demand spillover to the rival firm, and thus, reduces incentives to innovate. In addition, under active expectations, because firms set their prices considering consumers' expectations of network seizes (i.e., internalizing consumer externalities), the change in connectivity does not affect R&D activity (i.e. network neutrality).

Second, regarding the impact on net profits, irrespective of market coverage and consumer expectations, the same results hold in all cases. That is, an increase in connectivity increases net profits. In this case, as for the full market coverage, we notice the effects on net profits depend on the formation of consumer expectations. Under rational expectations, in the symmetric equilibrium, the direct effect of connectivity on net profit is null. However, the indirect effect is positive because strategic relationships between R&D activities are substitutionary, although the effect on net profit from the rival firms' R&D activity is negative. As a result, the impact is positive. On the other hand, under active expectations, an increase in connectivity does not affect R&D activity, but it does raise the price. Thus, it increases net profit.

There are some remaining problems in this paper. First, we should relax the strict assumption of symmetric firms. Under asymmetric firms, for example, suppose that a firm with a large (small) intrinsic value or high (low) quality is a large (small) company.

In this case, we should examine whether there is a difference in the effects of an improvement in connectivity between the large and small companies. As mentioned in a footnote, because of the asymmetry in “group-cross” network effects, small companies may have greater incentives for R&D activity than large companies.

Second, we have mainly examined how network connectivity affects incentives to innovate and profits. However, we should consider the effect on consumer welfare. For example, as in an information and telecommunications industry, improving quality of connectivity and expanding network connectivity provide direct benefits for the users (consumers). However, the improvements may result in increased usage fees. Thus, we will investigate the impact of improving connectivity on consumer surplus.

Finally, while policy analysis in a network industry is not within the scope of this paper, we do note some implications of the model. The problems of network connectivity, compatibility, and horizontal interoperability are closely related to standardization and compatibility policies facilitating market competition and promoting R&D activity. In this regard, as mentioned above, it is necessary to consider consumer welfare rather than social (total) welfare. From this perspective, we should consider an optimal connectivity (compatibility) policy.

Appendix A.

(1) On condition *FMC*: lower transportation costs (i.e., higher product substitutability)

We should demonstrate the conditions under which the full market coverage case holds in equilibrium.

In the equilibrium, we obtain the following outcomes: $\alpha^{f*} = \dfrac{t}{k\{3t - n(1-\phi)\}}$, $p_i^f - c_i = t,$ and $x_i^f = \dfrac{1}{2}.$ Condition *FMC* is rewritten as

$$t < v_0 - p_0 + N_0 + v_1 - p_1 + N_1 \Leftrightarrow \frac{t}{2} < a + \alpha^{f*} - t + \frac{n(1+\phi)}{2}.$$[22]

Thus, regarding the equation of the second part, we derive $a > H^f(t),$

where $H^f(t) \equiv \dfrac{3}{2}\left\{t - \dfrac{n(1+\phi)}{3}\right\} - \dfrac{t}{3k\left\{t - \dfrac{n(1-\phi)}{3}\right\}}$ for $t > \underline{t}^f \equiv \dfrac{n(1-\phi)}{3}.$ (A.1)

With respect to function $H^f(t),$ it holds that $H^f(t) > (<) 0 \Leftrightarrow t > (<) \hat{t}^f$ for $\hat{t}^f \equiv \left\{ t \geq \dfrac{n(1+\phi)}{3} \middle| H^f(t) = 0 \right\} > \underline{t}^f.$ Furthermore, we have $\dfrac{dH_f(t)}{dt} > 0$ for $t > \underline{t}^f.$ Because $H^f(t)$ is a monotonically increasing function of a transportation cost, $t,$ there exists the following critical level of the transportation cost:

$$\bar{t}^f \equiv \left\{ t > \hat{t}^f \middle| H^f(t) = a > 0 \right\}. \qquad \text{(A.2)}$$

Therefore, the condition under which the full market coverage case in the closed interval holds is given by:

---

[22] If the condition holds, it is plausible that $v_i - p_i + N_i > 0,$ $i = 0,1.$

$$\bar{t}^{f} > t > \underline{t}^{f}. \tag{A.3}$$

(2) On condition *PMC*: higher transportation costs (i.e., lower product substitutability) Using the same approach as above, we demonstrate the conditions under which the partial market coverage case in the closed interval holds in the equilibrium. Namely, the following conditions are necessary for the transportation cost: $v_i - p_i + N_i > 0$, $i = 0,1$, and $t > v_0 - p_0 + N_0 + v_1 - p_1 + N_1$. That is, we derive the following equation:

$$\frac{t}{2} > a + \alpha^{p*} - \left(p_i^p - c_i\right) + n\left(1+\phi\right) z_i^p > 0,$$

where $p_i^p - c_i = \dfrac{\left(a + \alpha^{p*}\right)t}{2t - n(1+\phi)}$ and $z_i^p = \dfrac{a + \alpha^{p*}}{2t - n(1+\phi)}$.[23] Thus, the above condition is revised as $t - \dfrac{n(1+\phi)}{2} > a + \alpha^{p*}$. Substituting Equation (12) into the revised condition and rearranging it, we derive the following equation: $H^p(t) > a$,

where $$H^p(t) \equiv 2\left\{t - \frac{n(1+\phi)}{2}\right\} - \frac{t\left(t - \dfrac{n}{2}\right)}{2k\left\{t - \dfrac{n(1+\phi)}{2}\right\}\left\{t - \dfrac{n(1-\phi)}{2}\right\}}$$ for

$$t > \underline{t}^{p} \equiv \frac{n(1+\phi)}{2}. \tag{A.4}$$

---

[23] It is plausible that $a + \alpha^{p*} - \left(p_i^p - c_i\right) + n\left(1+\phi\right) z_i^p = \dfrac{\left(a + \alpha^{p*}\right)t}{2t - n(1+\phi)} > 0.$

With respect to function $H^p(t)$, it holds that $H^p(t) > (<) 0 \Leftrightarrow t > (<) \hat{t}^p$ for $\hat{t}^p \equiv \left\{ t > \underline{t}^p \middle| H^p(t) = 0 \right\}$. Furthermore, we have $\frac{dH^p(t)}{dt} > 0$ and $H^p(t) > 0$ for $t > \hat{t}^p \left(> \underline{t}^p \right)$. Because $H^p(t)$ is a monotonically increasing function of the transportation cost, t, there exists the following critical level of transportation cost:

$$\bar{t}^p \equiv \left\{ t > \hat{t}^p \middle| H^p(t) = a > 0 \right\}. \tag{A.5}$$

Therefore, the condition that the partial market coverage case in the closed interval holds is given by:

$$t > \bar{t}^p \left(> \hat{t}^p > \underline{t}^p \right). \tag{A.6}$$

Appendix B. Active Expectations in the Partial Market Coverage Case

Under active expectations (i.e., $z_i^e = z_i$), using Equation (3), the following direct demand function is derived.

$$z_i = \frac{(t-n)v_i + n\phi v_j - (t-n)p_i - n\phi p_j}{\{t - n(1-\phi)\}\{t - n(1+\phi)\}}, \quad i, j = 0,1, \ i \neq j. \tag{B.1}$$

Equation (B.1) implies that the network products in the market are complements through network connectivity unless connectivity is not null. For example, we can imagine that the local telecommunications companies connect using long-distance cable. Unless the cable breaks, people can talk on the telephone not only with people living in the same area, but also with people who live far away.

In price competition, the FOC of profit maximization of *firm i* is given by

$\dfrac{\partial \pi_i^{pA}}{\partial p_i} = z_i - \dfrac{p_i - c_i}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}} = 0, \quad i = 0,1.$ In this case, because it holds that $\dfrac{\partial^2 \pi_i^{pA}}{\partial p_i \partial p_j} = \dfrac{\partial z_i}{\partial p_j} = \dfrac{-n\phi}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}} < 0,$ as mentioned above, strategic complements arise in price competition. Using the FOC, we derive the following outcomes in the equilibrium in price competition.

$$p_i^{pA} - c_i = \frac{\delta^{pA} a + \{2(t-n)^2 - (n\phi)^2\}\alpha_i + (t-n)n\phi\alpha_j}{D^{pA}}, \qquad \text{(B.2)}$$

$$z_i^{pA} = \frac{(t-n)(p_i^{pA} - c_i)}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}}, \qquad \text{(B.3)}$$

where $\delta^{pA} \equiv \{2(t-n) - n\phi\}\{t-n(1-\phi)\} > 0$ and $D^{pA} \equiv \{2(t-n)+n\phi\}\{2(t-n)-n\phi\} > 0, \quad i, j = 0,1, \quad i \neq j.$ Thus, based on Equations (B.2) and (B.3), the net profit function is expressed as: $\Pi_i^{pA} = \dfrac{t-n}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}}\left(p_i^{pA} - c_i\right)^2 - \dfrac{k}{2}(\alpha_i)^2.$

In the first stage of R&D competition, the FOC is given by:

$$\frac{\partial \Pi_i^{pA}}{\partial \alpha_i} = \frac{2(t-n)\{2(t-n)^2 - (n\phi)^2\}}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}D^{pA}}\left(p_i^{pA} - c_i\right) - k\alpha_i = 0, \quad i = 0,1. \qquad \text{(B.4)}$$

The SOC, the cross effect, and the effect of an increase in the rival firm's R&D activity on the net profit are respectively given as follows.

$$\frac{\partial^2 \Pi_i^{pA}}{\partial \alpha_i^2} = \frac{2(t-n)}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}}\left\{\frac{2(t-n)^2 - (n\phi)^2}{D^{pA}}\right\}^2 - k < 0,$$

$$\frac{\partial^2 \Pi_i^{pA}}{\partial \alpha_i \partial \alpha_j} = \frac{2\{2(t-n)^2-(n\phi)^2\}}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}}\left\{\frac{t-n}{D^{pA}}\right\}^2 n\phi \geq 0 \Leftrightarrow \phi \geq 0,$$

and

$$\frac{\partial \Pi_i^{pA}}{\partial \alpha_j} = \frac{2(t-n)^2 n\phi}{\{t-n(1-\phi)\}\{t-n(1+\phi)\}D^{pA}}\left(p_i^{pA}-c_i\right) \geq 0 \Leftrightarrow \phi \geq 0.$$

The outcomes expressed in the second and third equations are the same as those under rational expectations in the partial market coverage case. That is, under partial market coverage, the differences in expectations do not significantly affect the outcomes.

Exploiting Equations (B.2) and (B.4), we obtain the following R&D activities in equilibrium:

$$\alpha_i^{pA} = \frac{2(t-n)\{2(t-n)^2-(n\phi)^2\}a}{R^{pA}} \equiv \alpha^{pA*}(\phi), \quad i=0,1, \tag{B.5}$$

where

$$R^{pA} \equiv k(t-n-n\phi)\{2(t-n)-n\phi\}\{2(t-n)+n\phi\}^2 - 2(t-n)\left\{2(t-n)^2-(n\phi)^2\right\} > 0.$$

Based on Equation (B.5), we can derive $\frac{d\alpha^{pA*}(\phi)}{d\phi} > 0$. In addition, regarding the impact on net profit, we obtain $\left.\frac{d\Pi_i^{pA*}}{d\phi}\right|_{\alpha_0^{pA}=\alpha_1^{pA}=\alpha^{pA*}} > 0$.[24] Therefore, we summarize the

---

[24] The total effects of an increase in interoperability on the net profit of *firm i* is given by $\frac{d\Pi_i^{pA}}{d\phi} = \frac{\partial \Pi_i^{pA}}{\partial \alpha_j}\frac{d\alpha_j^{pA}}{d\phi} + \frac{\partial \Pi_i^{pA}}{\partial \phi}, \quad i,j=0,1, \quad i \neq j.$ The first and second terms imply that the indirect effects are positive. In addition, regarding the third term expressing the direct effect, it holds at the symmetric equilibrium that $\left.\frac{\partial \Pi_i^{pA*}}{\partial \phi}\right|_{\alpha_0^{pA}=\alpha_1^{pA}=\alpha^{pA*}} > 0.$

results as follows.

Proposition 3A.

An increase in connectivity increases R&D activity under active expectations.

Proposition 4A.

An increase in connectivity increases net profit under active expectations.

With respect to the partial market coverage case, the difference in the formation of consumer expectations does not qualitatively change the results regarding R&D activity and profits.

References

Buccella D, Fanti L, Gori L (2023) Network externalities, product compatibility and process innovation. Economics of Innovation and New Technology 32:1156–1189

Çavuş Ç (2024) Does DMA connectivity promote innovation: a comparative study from EU competition law to the DMA. European Competition Journal https://doi.org/10.1080/17441056.2024.2379143

Crémer J, Rey P, Tirole J (2000) Connectivity in the commercial internet. Journal of Industrial Economics 48:433–472

d'Aspremont C, Jacqumin A (1988) Cooperative and noncooperative R&D in duopoly with spillovers. American Economic Review 78:1133–1137

Economides N, White L (1994) Networks and compatibility: implications for antitrust. European Economic Review 38:651–662

Farrell J, Saloner G (1985) Standardization, compatibility, and innovation. Rand Journal of Economics 16:70–83

Foros Ø, Hansen B (2001) Competition and compatibility among internet service providers. Information Economics and Policy 13:411–425

Heywood JS, Wang Z, Ye G (2022) R&D rivalry with endogenous compatibility. Manchester School 90:354–384

Hoernig S (2012) Strategic delegation under price competition and network effects. Economics Letters 117:487–489

Hurkens S, López ÁL (2014) Mobile termination, network externalities and consumer expectations. Economic Journal 124:1005–1039

Ji H-S, Daitoh I (2008) Interconnection agreement between internet service providers and the optimal policy intervention: the case of Cournot-type competition under network externalities. Japanese Economic Review 59: 228–240

Katz M, Shapiro C (1985) Network externalities, competition, and compatibility. American Economic Review 75:424–440

Kim JY (2000) Product compatibility and technological innovation. International Economic Journal 14:87–100

Knauff M, Karbowski A (2021) R&D investments in markets with network effects. Journal of Industry, Trade and Competition 21:225–250

Naskar M, Pal R (2020) Network externalities and process R&D: A Cournot-Bertrand comparison. Mathematical Social Science 103:51–58

Roson R (2002) Two papers on internet connectivity and quality. Review of Network Economics 1:75–80

Sääskilahti P (2006) Strategic R&D and network compatibility. Economics of Innovation and New Technology 15:711–733

Shrivastav S (2021) Network compatibility, intensity of competition and process R&D: a generalization. Mathematical Social Science 109:152–163

Shy O (2001) The Economics of Network Industries. Cambridge University Press, Cambridge, UK

Suleymanova I, Wey C (2012) On the role of consumer expectations in markets with networks. Journal of Economics 105:101–127

Tolotti M, Yepez J (2020) Hotelling-Bertrand duopoly competition under firm-specific network effects. Journal of Economic Behavior and Organization 176:105–128

Toshimitsu, T (2024) Network connectivity, Strategic R&D competition, and Market Structure: A Hotelling linear market model. Discussion Paper Series No. 280, School of Economics, Kwansei Gakuin University.

Vives X (2008) Innovation and competitive pressure. Journal of Industrial Economics 56:419–469

Table 1

Market Coverage / Consumer Expectations	*Full*	*Partial*
Rational	-	+
Active	0	+

Figure 1. Full Market Coverage Case

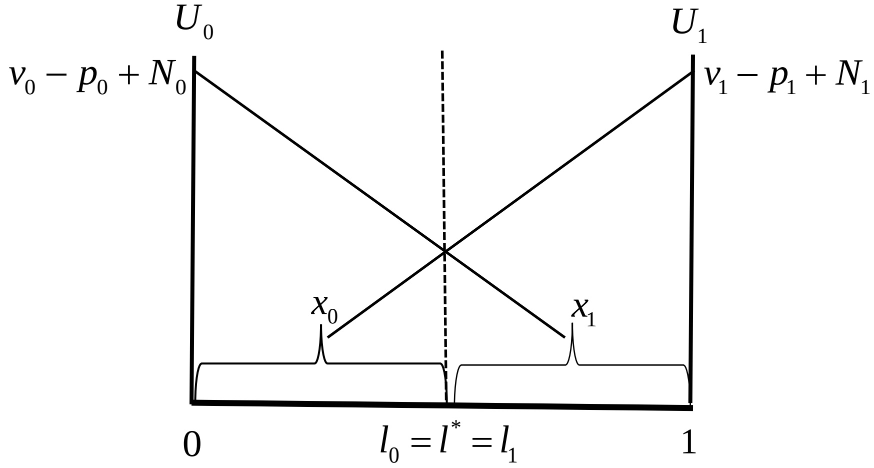

Figure 2. Partial Market Coverage Case