DISCUSSION PAPER SERIES

Discussion paper No.29

The Effects of the Asymmetry of Information Intrafirms on Oligopolistic Market Outcomes

Tetsuya Shinkai
Kwansei Gakuin University

Makoto Okamura
Hiroshima University

April 2006

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

The Effects of the Asymmetry of Information Intrafirms on Oligopolistic Market Outcomes*

Tetsuya Shinkai†
School of Economics, Kwansei Gakuin University

Makoto Okamura
Economics Department, Hiroshima University

April 2, 2006

Abstract

We consider an oligopoly with a principal-agent relationship, in which a firm's marginal cost is decreasing in a manager's managerial effort and is subject to an additive uncertainty. Two types of firms operate: one displays symmetric information between the owner and the manager, another presents asymmetric information. We show that if the marginal cost's derivative of the manager is sufficiently small, then the expected effort level in an asymmetric information firm exceeds that in a symmetric one. We also show that the expected total output and consumer surplus may reduce at equilibrium, as the number of symmetric information firms increases.

Keywords: asymmetric information, incentive scheme, competition effort, oligopoly . JEL Classification: D82, L13

* The authors would like to express their gratitude to Hideshi Itoh, Shinsuke Kambe, Toshihiro Matsumura, Tadashi Sekiguchi and participants at the Contract Theory Workshop, the Kyoto Institute of Economic Research and Industrial Organization Workshop and the Institute of Social Science at the University of Tokyo for their helpful comments and suggestions.

†School of Economics, Kwansei Gakuin Univsersity, 1-155, Uegahara ichiban-cho, Nishinomiya, Hyogo 662-8501, Japan. e-mail: shinkai@kwansei.ac.jp, Phone 81-798-54-6967, Fax 81-798-51-0944

1 Introduction

A basic result illustrated in the economics of information is that the asymmetry of information between principals and agents reduces agents' effort levels in an implemented incentive contract in a monopolistic firm. In the oligopolistic context, however, since the external competition to the firm affects internal incentives within the firm, the more difficult the external competition, the higher the effort level agents may choose. In their seminar[OLE3] paper, Fershtman and Judd[1] present different insights on why managerial compensation contracts may not depend solely on realized profits. They also examine how the structure of internal incentives within a firm interacts with the market structure external to the firm. They construct an oligopolistic model where the manager of each firm has an incentive to maximize a linear combination of profits and sales. We also investigate how the structure of internal incentives within a firm interacts with the market structure external to the firm, by directing attention to the asymmetry of information within oligopolistic firms.

We explore the implication of the Laffont–Tirole type agency problems on oligopolistic market outcomes[1]. In other words, we analyze market competition in our model by incorporating the principal–agency relation into each firm in the oligopoly. In particular, we examine how variation in the mixture of firms with different types of information structure affects the level of managers and welfare in oligopolistic market outcomes. The main difference between this study and earlier work is that we consider an oligopolistic market where some firms exhibit asymmetric rather than symmetric information between owners and managers.

Thus, in this paper, we consider an oligopolistic competition in which k ($\leqq n$) firms exist with a symmetric information structure and $n - k$ firms with asymmetric information. We also examine the effect of the number of firms with symmetric information on the oligopolistic market outcomes and welfare by conducting comparative statics on k, the number of the symmetric information firms in the oligopoly. In Section 2, we present a model. We

[1]See Laffont and Tirole[4].

derive the equilibrium strategies of our perfect subgame in Section 3. In Section 4, we conduct a comparative static analysis of equilibrium behavior with respect to k, as derived in Section 3. Section 5 presents some brief concluding remarks.

2 The Model

We consider a Cournot oligopoly model with n firms in which each firm produces a homogeneous good. The owner of a firm hires a manager and instructs him or her to improve efficiency by reducing marginal production costs. Assume that each firm has constant returns to technology: $C_i = c_i q_i$, where q_i is the quantity of output of firm i $(i = 1, \cdots, n)$. A realized marginal cost c_i is expressed as

$$c_i = c_0 - e_i + u_i, \tag{1}$$

where e_i is the cost-reduction effort level of the firm i's manager, and u_i denotes the cost uncertainty of firm i. For simplicity, assume that $u_i's$ $(i = 1, \cdots, n)$ are independent identically uniformly distributed random variables on the support $[\underline{u}, \overline{u}]$ with $0 < \underline{u} < \overline{u}$. The probability density function $f(u)$ of u_i is given by

$$f(u) = \frac{1}{\overline{u} - \underline{u}}, \quad \forall u \in [\underline{u}, \overline{u}] \quad \textit{and} \quad f(u) = 0, \quad \textit{otherwise}. \tag{2}$$

These distributions are common knowledge. The constant c_0 is an intrinsic cost and is normalized to zero. We assume that $0 \leqq e_i \leqq u_i$. That is, the cost reduction due to the manager's effort is at most u_i, since cost never takes a negative value. The manager can observe marginal cost. We assume that there are two types of firm with respect to the available information system. The first type of firm has a symmetric information system in which the owner can observe not only the marginal cost but also the realization of cost uncertainty u_i. In this case, both the manager and the owner of the firm know the realization of cost uncertainty u_i. We refer to this type of firm as a *symmetric information firm* hereafter[2]. Another type of firm with an asymmetric information system also exists, where the owner can observe the marginal cost c_i but not the realization of cost uncertainty u_i. In this case, the realization of marginal cost comprises private information for the manager, and the owner cannot observe the effort level exerted by the manager. We refer to this type of firm as an *asymmetric information firm* hereafter[3]. We also assume that the number of symmetric information firms *is exogenously given* by k, $(k = 0, 1, \cdots, n)$.

When the manager chooses the level of e_i, he or she incurs a disutility of $\psi(e_i)$. For simplicity, we assume that the disutility function $\psi(e_i)$ is assumed to be

$$\psi(e_i) = \frac{1}{2}se_i^2, (s > 0). \tag{3}$$

Both owners and the managers are risk neutral. The utility of firm i's manager is expressed as

$$U_i = w_i - \psi(e_i) = w_i - \frac{1}{2}se_i^2,$$

where w_i is the manager's pay. The outside opportunity utility is normalized to zero. The inverse market demand function is assumed to be linear and is given by $p = a - Q$, where $Q = \sum_{i=1}^{n} q_i$ is the total output, and we assume that $a > \overline{u} > 0$. The owner's utility is then given by

$$V_i = E[\pi_i - w_i] = E[(a - \sum_{j=1}^{n} q_j - (u_i - e_i))q_i - w_i], \tag{4}$$

where E is an expectation operator w.r.t. u_i. For the owners of symmetric information firms, there is no agency problem in the first instance. We assume that c_i is observable and contractible for asymmetric information firms. Since the quantity at the Cournot equilibrium is a function of the value of c_i, if c_i is observable for firm i's owner, he or she can write

[2] Hart[2] calls this type of firm an *entrepreneur firm.*

[3] Hart[2] call this type of firm a *managerial firm.*

down the contract, forcing the manager to produce, if the quantities are verifiable, the exact equilibrium quantity. In the first stage, the owner offers a contract to resolve the agency problem. Through the revelation principle, for an equilibrium of any mechanism, there exists a truth-telling equilibrium of a direct mechanism that is equivalent. Therefore, we focus on the direct mechanism defined by $\{w_i(u_i), c_i(u_i)\}$, where u_i denotes the report of firm i's manager. The contract offered by the owner takes the following form: "if the manager announces u_i, the owner will pay him or her $w_i(u_i)$, when he or she realizes the cost target $c_i(u_i)$, and will give no reward otherwise". We also assume the following information structure. In the first stage, an executed contract within firm i is unobservable to the owner and firm $j(\neq i)'$s manager, that is, it is private information. Note that this is true for symmetric information firms. At the beginning of the second stage, the contracts executed in the first stage become common knowledge, so each firm can observe the level of its rivals' marginal costs. In the second stage, and knowing all firms' marginal costs, each manager chooses output to maximize profits.

The timing of the game is as follows.

Stage 0: The manager of firm i observes the realization of u_i. In the symmetric information firm i, its owner also learns the realization of u_i.

Stage1: Each owner of the asymmetric information firms offers a contract $\{w_i(u_i), c_i(u_i)\}$ that specifies the wage and target level of marginal cost contingent on the managers reported value of u_i. The manager decides whether or not to accept the contract. If he or she accepts, he or she reports u_i, the contract is executed and the manager chooses his or her effort level to reduce the marginal cost to the level $c_i(u_i)$. If he or she rejects the offer, the game ends. At this point, contracts executed in other firms are not observable.

Stage 2: At the beginning of this stage, the contracts executed in the first stage and the marginal costs become public information, and each manager chooses the level of output and the profits are realized.

In the above specification, there are no agency problems in the stage of market competition. However, the effort level of the owner of the asymmetric firms made in the contracting stage influences the marginal costs of firms and the outputs in the market game equilibrium.

3 Derivation of the Equilibrium

In this section, and from the timing of the game specified in the previous section, we solve the subgame perfect equilibrium. We solve this game through backward induction. First, let us derive the equilibrium strategies of the symmetric information firms.

3.1 Derivation of the equilibrium strategies

In the second stage, since the marginal costs $(c_1, c_2, \cdots, c_n)$ are common knowledge, the Cournot–Nash equilibrium quantities are derived as

$$q_i^* = \frac{1}{n+1}(a - nc_i + \sum_{j \neq i} c_j) \qquad (i \neq j, \quad i, j = 1, 2, \cdots, n) \tag{5}$$

if $a - nc_i + \sum_{j \neq i} c_j \geq 0, q_i^* = 0$ otherwise. We also have an equilibrium of gross profits

$$\pi_i^* = (q_i^*)^2 = \frac{1}{(n+1)^2}(a - nc_i + \sum_{j \neq i} c_j)^2 \tag{6}$$

if $a - nc_i + \sum_{j \neq i} c_j \geq$ and $\pi_i^* = 0$, otherwise.

Denote the set of firms in the oligopoly by $N = \{1, 2, \cdots, n\}$. By assumption, the number of symmetric information firms k $(k = 0, 1, 2, \cdots, n)$ is exogenously given. We assume that firm 1, firm 2, $\cdots$,firm k are symmetric information firms. Let $K = \{1, 2, \cdots, k\} \subseteq N$, the set of symmetric information firms. The set of asymmetric information firms can be expressed by $N \cap K^C = \{k+1, k+2, \cdots n\}$,where K^C is the complement of K. Note that the owner of the symmetric information firm learns the realization of u_i. The decision problem facing the owner of the symmetric information firm $j \in K$ is

$$\max_{w_j(u_j),c_j(u_j)} \frac{1}{(n+1)^2} E_{u_k}[(a - nc_j(u_j) + \sum_{k\neq j} c_k(u_k))^2 - w_j(u_j)]$$
$$s.t. \quad w_j(u_j) - \psi(e_j) = \ w_j(u_j) - \frac{1}{2}s(u_j - c_j(u_j))^2 \geq 0, \forall u_j,$$

where the constraint above describes the individual rationality condition. The equality in the constraint holds, since the marginal cost and effort have a one-to-one correspondence by $e_j = u_j - c_i$. We can rewrite this problem into the expected net profit maximization problem w.r.t. e_i as follows.

$$\max_{w_j(u_j),e_j(u_j)} \frac{1}{(n+1)^2} E_{u_k}[(a-n(u_j-e_j(u_j))+\sum_{k\neq j}\{u_{k-}e_k(u_k)\})^2-w_j(u_j)] \quad (7)$$

$$s.t. \quad w_j(u_j) - \psi(e_j) = \ w_j(u_j) - \frac{1}{2}s(e_j(u_j))^2 \geq 0, \forall u_j \in [\underline{u}, \overline{u}]$$

Since the owner prefers to offer a reward as low as possible, the constraint must be binding; the net surplus of the manager is reduced to zero. That is, we have

$$w_j(u_j) = \frac{1}{2}s(e_j(u_j))^2, \forall u_j \in [\underline{u}, \overline{u}]\,. \quad (8)$$

Substituting (8) into the maximand of (7), we have

$$\frac{1}{(n+1)^2}[(a - n(u_j - e_j(u_j)) + \sum_{t\neq j, t\in K} \{E_{u_t}[u_t] - E_{u_t}[e_t(u_t)]\})^2$$
$$+ \sum_{t'\in N\cap K^C} \{E_{u_{t'}}[u_{t'}] - E_{u_{t'}}[e_{t'}(u_{t'})]\})^2 - \frac{1}{2}s(e_j(u_j))^2], \forall u_j.$$

The partial derivative of the above expression w.r.t. $e_j(u_j)$ is given by

$$\frac{2n}{(n+1)^2}[(a-n(u_j-e_j(u_j))+\sum_{t\neq j,t\in K}\{\widehat{u}-E^{SI}_{u_t}[e_t(u_t)]\} \\ +\sum_{t'\in N\cap K^C}\{\widehat{u}-E^{AI}_{u_{t'}}[e_{t'}(u_{t'})]\}]-s\cdot e_j(u_j)=0, \tag{9}$$

where $\widehat{u}=E[u_t]=\frac{\underline{c}+\overline{c}}{2}$,for all $t\in N$, since u_t are independent identically uniformly distributed random variables.

Solving this w.r.t.$e_j(u_j)$, we obtain

$$e^{SI}_j(u_j)=\frac{2n}{(n+1)^2s-2n^2}[a-nu_j+(n-1)\widehat{u}-\sum_{t\neq j,t\in K}E^{SI}_{u_t}[e_t(u_t)] \\ -\sum_{t'\in N\cap K^C}E^{AI}_{u_{t'}}[e_{t'}(u_{t'})]], \tag{10}$$

where the superscripts 'SI' ('AI') of the expectation operator imply that the firm is a 'symmetric (asymmetric) information firm.' We present two assumptions.

Assumption 1

$$s>\frac{2n^2}{(n+1)^2}$$

Assumption 2

$$\frac{4n^2+(n+1)^2s}{2n}\geq a\geq n\underline{u}$$

Remember that $0\leqq e_i\leqq u_i$. Therefore, it is plausible to hold the following inequality.

$$0 \leqq E[e_i^{SI}(u_i)] \leqq E[u_i] = \widehat{u}, \text{and } 0 \leqq E[e_i^{AI}(u_i)] \leqq E[u_i] = \widehat{u}$$

Assumption 2 guarantees that these inequalities hold.

Solving (10) w.r.t. $E_{u_j}^{SI}[e_j(u_j)]$, we get

$$E[e_j^{SI}(u_j)] = \frac{2n}{(n+1)^2 s - 2n(n-k+1)}[a - \widehat{u} - (n-k)E_{u_{t'}}^{AI}[e_{t'}(u_{t'})]]. \quad (11)$$

Noting that u_t are independent identically distributed random variables and taking expectation on both sides of equation (10), we obtain

$$E[e_j^{SI}(u_j)] = \frac{2n}{(n+1)^2 s - 2n^2}[a - \widehat{u} - (k-1)E_{u_t}^{SI}[e_t(u_t)] - (n-k)E_{u_{t'}}^{AI}[e_{t'}(u_{t'})]].$$

Note that $E_{u_t}^{SI}[e_t(u_t)] = E[e_j^{SI}(u_j)]$ for any $t \in K$ and $E_{u_{t'}}^{AI}[e_{t'}(u_{t'})] = E^{AI}[e_m(u_m)]$ for any $t' \in N \cap K^C$ because of the symmetry of the firms that have the same type of information system.

Substituting (11) into (10) and rearranging terms yields

$$\begin{aligned} e_j^{SI}(u_j) \quad = \quad & \frac{2n}{(n+1)^2 s - 2n^2}[\frac{(n+1)^2 s - 2n^2}{(n+1)^2 s - 2n(n-k+1)} a - n u_j \\ & + \frac{(n-1)((n+1)^2 s - 2n(n-k+1)) + 2n(k-1)}{(n+1)^2 s - 2n(n-k+1)} \widehat{u} \\ & - (n-k) E_{u_{t'}}^{AI}[e_{t'}(u_{t'})]]. \end{aligned} \quad (12)$$

Now, we are ready to derive the equilibrium strategies of the asymmetric information firms.

Since the owner of the asymmetric information firm $m(\in N \cap K^C)$ can observe neither u_m nor e_m, the manager can earn a strictly positive information rent, $U_m(u_m)$, because of the asymmetry of information. The owner has to impose the following incentive compatibility constraints when he or she determines the offer in the first stage:

$$w_m(u_m) - \psi(u_m - v_m(u_m)) \geq w_m(u_m^{'}) - \psi(u_m - v_m(u_m^{'})) \qquad (13)$$

for any u_m and $u_m^{'}$ in $[\underline{u}, \overline{u}]$. It is well known that we can rewrite the incentive compatibility constraints (13) into the following equivalent conditions:

$$\begin{aligned} U_m^{'}(u_m) &= -\psi^{'}(u_m - c_m(u_m)), \\ c_m^{'}(u_m) &\geq 0, \end{aligned} \qquad (14)$$

for piecewise differentiable functions $U_m(\cdot)$ and $c(\cdot)$.[4] It is clear that the functions $U_m(u_m) = w_m(u_m) - \psi(e_m(u_m)) = w_m(u_m) - \frac{1}{2}s(e_m(u_m))^2 = > 0$ and $c_m = u_m - e_m(u_m)$ are piecewise differentiable. We can easily rewrite the inequality (14) as $e_m^{'}(u_m) \leqq 1$. Replacing $w_m(u_m)$ by $U_m(u_m) + \psi(e_m(u_m))$, the problem facing the owner of firm $m \in N \cap K^C$ to solve is

$$\max_{U_m(u_m), e_m(u_m)} \frac{1}{(n+1)^2} \int_{\underline{u}}^{\overline{u}} \{ \ [a - n(u_m - e_m(u_m)) + \sum_{t \in K} \{E_{u_k}^{SI}[u_t] - E_{u_k}^{SI}[e_t(u_t)]\}$$
$$+ \sum_{t' \neq m, \quad t' \in N \cap K^C} \{E_{u_{t'}}^{AI}[u_{t'}] - E_{u_{t'}}^{AI}[e_{t'}(u_{t'})]\}]^2 - U_m(u_m) - \psi(e_m(u_m)) \ \} f(u_m) du_m,$$

subject to

[4] See, for example, chapter 7 in Fudenberg and Tirole (1991).

$$\begin{aligned}
U_m^{'}(u_m) &= -\psi^{'}(u_m - c_m(u_m)), & (15)\\
e_m^{'}(u_m) &\leqq 1, & (16)\\
U_m(u_m) &\geq 0, \forall u_m \in [\underline{u}, \overline{u}]\,. & (17)
\end{aligned}$$

By integrating (15) to yield

$$U_m(u_m) = \int_{u_m}^{\overline{u}} \psi^{'}(e_m(u))du + U_m(\overline{u})$$

and

$$U_m^{'}(u_m) = -\psi^{'}(u_m - c_m(u_m)) = -se_m(u_m) \leqq 0,$$

we can see that the individual rationality constraint (17) is equivalent to $U_m(\overline{u}) \geq 0$. The owner wishes to pay the lowest possible wage to the manager, and the constraint is binding; that is, $U_m(\overline{u}) = 0$. The expected rent to the manager is represented as

$$\begin{aligned}
\int_{\underline{u}}^{\overline{u}} U_m(u_m)f(u_m)du_m &= \int_{\underline{u}}^{\overline{u}} \int_{u_m}^{\overline{u}} \psi^{'}(e_m(u))duf(u_m)du_m\\
&= \int_{\underline{u}}^{\overline{u}} h(u_m)\psi^{'}(e_m(u_m))f(u_m)du_m, \quad (18)
\end{aligned}$$

where $h(u_m) = \frac{F(u_m)}{f(u_m)}$. Using (18), the problem facing the owner of firm m can be reduced to

$$\begin{aligned}
&\max_{e_m(u_m)} \frac{1}{(n+1)^2} \int_{\underline{u}}^{\overline{u}} \{\ [a - n(u_m - e_m(u_m)) + \sum_{t \in K} \{E_{u_t}^{SI}[u_t] - E_{u_t}^{SI}[e_t(u_t)]\}\\
&+ \sum_{t' \neq m,\quad t' \in N \cap K^C} \{E_{u_{t'}}^{AI}[u_{t'}] - E_{u_{t'}}^{AI}[e_{t'}(u_{t'})]\}]^2 - \psi(e_m(u_m))\\
&-h(u_m)\psi^{'}(e_m(u_m))\ \}f(u_m)du_m,
\end{aligned}$$

subject to

$$e_m^{'}(u_m) \leqq 1. \tag{19}$$

Let us solve the above problem by ignoring the constraint (19). We observe that $f(u_m) = \frac{1}{\overline{u}-\underline{u}}$ and $h(u_m) = u_m - \underline{u}$. Using these and (3), differentiating partially w.r.t. $e_m(u_m)$ the above maximand, we obtain the first-order condition of this optimization *without constraint*:

$$\begin{aligned}&\frac{2n}{(n+1)^2}\,[a - n(u_m - e_m(u_m)) + k(\widehat{u} - E^{SI}_{u_k}[e^{SI}_t(u_t)]) \\ &+(n-k-1)(\widehat{u} - E^{AI}_{u_{t'}}[e_{t'}(u_{t'})])] - s \cdot e_m(u_m) - s(u_m - \underline{u}) = 0.\end{aligned} \tag{20}$$

Solving (20) w.r.t. $e_m(u_m)$, we obtain

$$\begin{aligned}e^{AI}_m(u_m) = {}& \frac{2n}{(n+1)^2 s - 2n^2}[a - nu_m + (n-1)\widehat{u} - (n-k-1)E^{AI}_{u_{t'}}[e_{t'}(u_{t'})] \\ & -kE^{SI}_{u_k}[e^{SI}_t(u_t)]] - \frac{(n+1)^2 s}{(n+1)^2 s - 2n^2}(u_m - \underline{u}).\end{aligned} \tag{21}$$

Substituting (11) into (21) and taking the expectation and rearranging the terms yields

$$\begin{aligned}E^{AI}_{u_m}[e_m(u_m)] \;=\; & \frac{2n}{(n+1)^2 s - 2n^2}[\frac{(n+1)\{(n+1)^2 s - 2n\}}{(n+1)^2 s - 2n(n-k+1)}a - nu_m \\ & + \frac{(n+1)\{(n+1)s - 2n\}}{(n+1)^2 s - 2n(n-k+1)}\widehat{u} \\ & - \frac{-2n(n-1) + \{(n+1)^2 s - 2n^2\}(n-k-1)}{(n+1)^2 s - 2n(n-k+1)}E^{AI}_{u_m}[e_m(u_m)]] \\ & - \frac{(n+1)^2 s}{(n+1)^2 s - 2n^2}(\widehat{u} - \underline{u}).\end{aligned}$$

Solving the above w.r.t. $E^{AI}_{u_m}[e_m(u_m)]$, we get

$$\begin{aligned}E_{u_m}^{AI}[e_m(u_m)] &= \frac{1}{\{(n+1)^2s-2n\}\{(n+1)s-2n\}}[\frac{2n\{(n+1)s-2n\}}{(n+1)^2s-2n(n-k+1)}a \\ &-\{(n+1)^3s^2-2n(n+1)(n-k)s-4n^2\}\widehat{u} \\ &+\{(n+1)^2s-2n(n-k+1)\}(n+1)s\underline{u}]. \qquad (22)\end{aligned}$$

Substituting (22) into (11) and rearranging yields

$$E_{u_j}^{SI}[e_j^{SI}(u_j)] = \frac{2n}{(n+1)^2s-2n}[a-\widehat{u}-\frac{(n+1)(n-k)}{(n+1)s-2n}s(\widehat{u}-\underline{u})]. \qquad (23)$$

By substituting (22) and (23) into (21), we obtain

$$\begin{aligned}e_m^{AI}(u_m) &= \frac{2n}{(n+1)^2s-2n^2}[\frac{(n+1)^2s-2n^2}{(n+1)^2s-2n}(a-\widehat{u})-n(u_m-\widehat{u}) \\ &+\frac{(n+1)\{(n-k-1)(n+1)^2s-2n(n(n-k)-1)\}}{\{(n+1)s-2n\}\{(n+1)^2s-2n\}}s(\widehat{u}-\underline{u})] \\ &-\frac{(n+1)^2s}{(n+1)^2s-2n^2}(u_m-\widehat{u}). \qquad (24)\end{aligned}$$

From (24) and Assumption 1, we have $\frac{de_m^{AI}(u_m)}{du_m} = -\frac{2n^2+(n+1)^2s}{(n+1)^2s-2n^2} < 0$, so the solution (24) of the unconstrained problem satisfies the constraint (19). Hence, we can see that (24) is the equilibrium effort level of the manager of the *asymmetric information* firm.

By substituting (23) into (11), we obtain the equilibrium effort level of the manager of the *symmetric information* firm:

$$\begin{aligned}e_j^{SI}(u_j) &= \frac{2n}{(n+1)^2s-2n^2}[\frac{(n+1)^2s-2n^2}{(n+1)^2s-2n}(a-\widehat{u})-n(u_j-\widehat{u}) \\ &+\frac{(n+1)(n-k)\{(n+1)^2s-2n^2\}}{\{(n+1)s-2n\}\{(n+1)^2s-2n\}}s(\widehat{u}-\underline{u})]. \qquad (25)\end{aligned}$$

Combining (22), (23), (24) and (25), we easily have

$$e_j^{SI}(u_j) = E[e_j^{SI}(u_j)] - \frac{2n^2}{(n+1)^2 s - 2n^2}(u_j - \widehat{u}) \tag{26}$$

and

$$e_m^{AI}(u_m) = E[e_m^{AI}(u_m)] - \frac{(n+1)^2 s + 2n^2}{(n+1)^2 s - 2n^2}(u_m - \widehat{u}). \tag{27}$$

From (5), (22), (23), (25) and the fact that u_j is private information of firm j in the first stage, a somewhat tedious calculation yields the *ex ante equilibrium quantity of the symmetric information firm* $j(\in K)$:

$$\begin{aligned} q_j^{SI}(u_j) &= \frac{1}{n+1}[a - n(u_j - e_j^{SI}(u_j)) + (n-1)\widehat{u} - \sum_{t \neq j, t \in K} E_{u_t}^{SI}[e_t(u_t)] \\ &\quad - \sum_{t' \in N \cap K^C} E_{u_{t'}}^{AI}[e_{t'}(u_{t'})]] \\ &= \frac{(n+1)}{(n+1)^2 s - 2n}(a - \widehat{u}) - \frac{n(n+1)s}{(n+1)^2 s - 2n^2}(u_j - \widehat{u}) \\ &\quad + \frac{(n+1)^2 (n-k) s}{\{(n+1)^2 s - 2n\}\{(n+1)s - 2n\}}(\widehat{u} - \underline{u}). \end{aligned} \tag{28}$$

From (5), (23), (24), (25) and the fact that u_m is private information of firm j in the first stage, we can derive the *ex ante equilibrium quantity of the asymmetric information firm* $m(\in N \cap K^C)$:

$$\begin{aligned} q_m^{AI}(u_m) &= \frac{1}{n+1}[a - n(u_m - e_m^{AI}(u_m)) + (n-1)\widehat{u} - \sum_{t \in K} E_{u_t}^{SI}[e_t(u_t)] \\ &\quad - \sum_{t' \neq m, t' \in N \cap K^C} E_{u_{t'}}^{AI}[e_{t'}(u_{t'})]] \\ &= \frac{(n+1)}{(n+1)^2 s - 2n}(a - \widehat{u}) - \frac{n(n+1)s}{(n+1)^2 s - 2n^2}(2u_m - \widehat{u} - \underline{u}) \\ &\quad + \frac{(n+1)^2 s^2 \{(n-k-1)(n+1)^2 s - 2n(n(n-k)-1)\}}{\{(n+1)^2 s - 2n\}\{(n+1)s - 2n\}}(\widehat{u} - \underline{u}). \end{aligned} \tag{29}$$

Taking the expectation of (28) and (29), we obtain the expected equilibrium quantities of firm $j(\in K)$ and firm $m(\in N \cap K^C)$:

$$\begin{aligned} E[q_j^{SI}(u_j)] &= E[\frac{(n+1)}{(n+1)^2 s - 2n}(a - \widehat{u}) - \frac{n(n+1)s}{(n+1)^2 s - 2n^2}(u_j - \widehat{u}) \\ &+ \frac{(n+1)^2(n-k)s}{\{(n+1)^2 s - 2n\}\{(n+1)s - 2n\}}(\widehat{u} - \underline{u})] \\ &= \frac{(n+1)}{(n+1)^2 s - 2n}(a - \widehat{u}) + \frac{(n+1)^2(n-k)s}{\{(n+1)^2 s - 2n\}\{(n+1)s - 2n\}}(\widehat{u} - \underline{u}) \end{aligned} \tag{30}$$

and

$$\begin{aligned} E[q_m^{AI}(u_m)] &= \frac{(n+1)s}{(n+1)^2 s - 2n}(a - \widehat{u}) + \frac{(n+1)s}{(n+1)^2 s - 2n^2} \cdot \\ &\{\frac{-(k+1)^2(n+1)^3 s^2 + 2n(n+1)(n(k+2)+1)s - 4n^3}{\{(n+1)^2 s - 2n\}\{(n+1)s - 2n\}}\}(\widehat{u} - \underline{u}). \end{aligned} \tag{31}$$

From (28), (29), (30) and (31), we see that

$$q_j^{SI}(u_j) = E_{u_j}^{SI}[q_j^{SI}(u_j)] - \frac{n(n+1)s}{(n+1)^2 s - 2n^2}(u_j - \widehat{u}) \tag{32}$$

and

$$q_m^{AI}(u_m) = E_{u_m}^{AI}[q_m(u_m)] - \frac{2n(n+1)s}{(n+1)^2 s - 2n^2}(u_m - \widehat{u}). \tag{33}$$

Remarks

Note that the cost reduction due to the manager's effort is at most u_i. If $0 \leqq E^{AI}[e_m(u_m)] \leqq \widehat{u}$ and $0 \leqq E[e_i^{SI}(u_i)] \leqq E[u_i] = \widehat{u}$, then we have from (11)

$$\begin{aligned} E[e_j^{SI}(u_j)] &= \frac{2n}{(n+1)^2 s - 2n(n-k+1)} [a - \widehat{u} - (n-k) E_{u_{t'}}^{AI}[e_{t'}(u_{t'})]] \\ &\leqq \frac{2n}{(n+1)^2 s - 2n(n-k+1)} [a - \widehat{u}] \leqq \widehat{u}. \end{aligned}$$

We can show that the final inequality holds under Assumption 2, i.e.

$$a \leqq \frac{(n+1)^2 s - 2n(n-k)}{2n} \widehat{u} < \frac{4n^2 + (n+1)^2 s}{2n} \widehat{u}. \tag{34}$$

Rearranging (20) and taking the expectation of both sides w.r.t. u_m yields

$$E^{AI}[e_m(u_m)] = \frac{2n}{(n+1)^2 s - 2n(k+1)} [a - \widehat{u} - k E_{u_k}^{SI}[e_t^{SI}(u_t)]] - \frac{(n+1)^2 s}{(n+1)^2 s - 2n(k+1)} (\widehat{u} - \underline{u}).$$

If $0 \leqq E^{AI}[e_m(u_m)] \leqq \widehat{u}$ and $0 \leqq E[e_i^{SI}(u_i)] \leqq E[u_i] = \widehat{u}$, then the right-hand side of the above equation has to satisfy

$$\begin{aligned} & \frac{2n}{(n+1)^2 s - 2n(k+1)} [a - \widehat{u} - k E_{u_k}^{SI}[e_t^{SI}(u_t)]] - \frac{(n+1)^2 s}{(n+1)^2 s - 2n(k+1)} (\widehat{u} - \underline{u}) \\ \leqq & \frac{2n}{(n+1)^2 s - 2n(k+1)} [a - \widehat{u}] - \frac{(n+1)^2 s}{(n+1)^2 s - 2n(k+1)} (\widehat{u} - \underline{u}) \\ < & \frac{2n}{(n+1)^2 s - 2n(k+1)} [a - \widehat{u}] \leqq \widehat{u}, \end{aligned}$$

where the first inequality follows from inequalities $0 \leqq E^{SI}[e_m(u_m)] \leqq \widehat{u}$, the second follows from Assumption 1 and the fact that $\widehat{u} - \underline{u} > 0$. The third inequality implies that $a \leqq \frac{(n+1)^2 s - 2nk}{2n} \widehat{u}$, but this follows from Assumption 2, since $a \leqq \frac{(n+1)^2 s - 2nk}{2n} \widehat{u} < \frac{(n+1)^2 s + 2n^2}{2n} \widehat{u}$.

We are now ready to present the main results concerning the equilibrium outcome.

3.2 Properties in the equilibrium outcome of each firm

At first, we present the results of both the expected effort level of managers of symmetric information firm $j(\in K)$ and the asymmetric information firm $m(\in N \cap K^C)$.

Proposition 1 $E^{AI}_{u_m}[e_m(u_m)] > E^{SI}_{u_j}[e^{SI}_j(u_j)]$, *for* $\frac{2n^2}{(n+1)^2} < s < \frac{2n}{n+1}$,

$$E^{AI}_{u_m}[e_m(u_m)] \leqq E^{SI}_{u_j}[e^{SI}_j(u_j)], \text{ for } \quad \frac{2n}{n+1} \leqq s.$$

Proof. From (22) and (23), we have

$$E^{AI}_{u_m}[e_m(u_m)] - E^{SI}_{u_j}[e^{SI}_j(u_j)] = -\frac{(n+1)s}{(n+1)s - 2n}(\widehat{u} - \underline{u}). \tag{35}$$

The right-hand side of (34) has a positive (negative) sign, if $\frac{2n^2}{(n+1)^2} < s < \frac{2n}{n+1}$($\frac{2n}{n+1} \leqq s$), since Assumption 1 and $\widehat{u} = \frac{\underline{u}+\overline{u}}{2} > \underline{u}$. ■

It is well known that the expected effort level of the manager of the asymmetric information firm is always higher than that of an symmetric information firm in a monopoly setting, since the owner facing asymmetric information has to pay more through information rent. The result obtained in Proposition 1 states that if we extend the model to Cournot oligopoly with a principal–agent relationship in each intrafirm, the existence of market competition can reverse this result if some conditions hold. That is, when s,the extent of the derivative of the marginal disutility of the manager, is sufficiently large, the expected effort level of the manager of the symmetric information firm is higher than that of the asymmetric information firm in an oligopoly setting. This is the same result as in a monopoly setting. When s is sufficiently small, however, the reverse holds. The expected effort level of the manager of the symmetric information firm is lower than that of the asymmetric information firm. Note that this result is robust with respect to a change in the number of symmetric information firms, since the difference of the expected effort levels of the two firms given does not depend upon k. (See (34).)

The expected utility of the manager of the asymmetric information firm exceeds that of the symmetric information firm. We can easily show the following corollary.

Corollary 2

For, $m \in N \cap K^C$, and $j \in K$,
$E^{AI}[U_m(u_m)] > E^{SI}U_j(u_j) = 0$.

Using the result shown in Proposition 1, we can provide a stronger result concerning the equilibrium managers' effort levels in both types of firm. Under the symmetry within the same type of firms, we define the left hand-side of the first-order condition for the owner of a *symmetric firm* (removed subscript j of u_j) (9) as a manager's *Marginal Net Value of effort in a Symmetric Information firm*, and denoting this by $MNV^{SI}(u)$, then (9) is expressed as

$$\begin{aligned} MNV^{SI}(e) \equiv & \frac{2n}{(n+1)^2}[(a - n(u-e) + (k-1)\{\widehat{u} - E^{SI}_{u_t}[e_t(u_t)]\} \\ & +(n-k)(\widehat{u} - E^{AI}_{u_{t'}}[e_{t'}(u_{t'})])] - s \cdot e = 0. \end{aligned}$$

Similarly, denoting the left-hand side of the first-order condition for the owner of an *asymmetric firm* (removed subscript m of u_m), (20) by $MNV^{AI}(e)$, then we can rewrite (20) into

$$\begin{aligned} MNV^{AI}(e) \equiv & \frac{2n}{(n+1)^2}[(a - n(u-e) + k\{\widehat{u} - E^{SI}_{u_t}[e_t(u_t)]\} \\ & +(n-k-1)(\widehat{u} - E^{AI}_{u_{t'}}[e_{t'}(u_{t'})])] - s \cdot e - s(u - \underline{u}) = 0. \end{aligned}$$

Then, subtracting both sides of the former from those of the latter:

$$MNV^{AI}(e) - MNV^{SI}(e) = \frac{2n}{(n+1)^2}\{E^{AI}[e] - E^{SI}[e]\} - s(u - \underline{u}). \quad (36)$$

Then we can express (20) as

$$MNV^{AI}(e) = MNV^{SI}(e) + \frac{2n}{(n+1)^2}\{E^{AI}[e] - E^{SI}[e]\} - s(u - \underline{u}) = 0,$$

or equivalently

$$MNV^{SI}(e) = \frac{2n}{(n+1)^2}\{E^{SI}[e] - E^{AI}[e]\} + s(u - \underline{u}), \tag{37}$$

and note that $e^{AI}(u)$ becomes the solution of this equation, $MNV^{SI}(e) = \frac{2n}{(n+1)^2}\{E^{SI}[e] - E^{AI}[e]\} + s(u - \underline{u})$. On the other hand, the first-order condition (9) is simply written as

$$MNV^{SI}(e) = 0, \tag{38}$$

and note that $e^{SI}(u)$ becomes the solution of this equation, $MNV^{SI}(e) = 0$. From (35) and (36), we can show that the following result holds.

Proposition 3 *$e^{AI}(u) > e^{SI}(u)$, for $\frac{2n^2}{(n+1)^2} < s < \frac{2n}{n+1}$ and u such that $u < \widehat{u}$ and $e^{SI}(u) > e^{AI}(u)$ for $\frac{2n}{n+1} < s$ for $\forall u \in [\underline{u}, \overline{u}]$.*

Proof. From the definition of $MNV^{SI}(e)$ and Assumption 1, $\frac{dMNV^{SI}(e)}{de} = \frac{2n^2}{(n+1)^2} - s < 0$. From (34) in the proof of Proposition 1, $\frac{2n}{(n+1)^2}\{E^{SI}[e] - E^{AI}[e]\} + s(u - \underline{u}) = \frac{2n}{(n+1)^2}\{\frac{(n+1)s}{(n+1)s-2n}(\widehat{u} - \underline{u})\} + s(u - \underline{u})$. For u such that $u < \widehat{u}$, $\frac{2n}{(n+1)^2}\{E^{SI}[e] - E^{AI}[e]\} + s(u - \underline{u}) = \frac{2n}{(n+1)^2}\{\frac{(n+1)s}{(n+1)s-2n}(\widehat{u} - \underline{u})\} + s(u - \underline{u}) < \frac{2n}{(n+1)^2}\{\frac{(n+1)s}{(n+1)s-2n}(\widehat{u} - \underline{u})\} + s(\widehat{u} - \underline{u})$
$= \frac{(n+1)^2 s - 2n^2}{(n+1)\{(n+1)s-2n\}} s(\widehat{u} - \underline{u}) < 0$ for $\frac{2n^2}{(n+1)^2} < s < \frac{2n}{n+1}$. For $\frac{2n}{n+1} < s$, $\frac{2n}{(n+1)^2}\{E^{SI}[e] - E^{AI}[e]\} + s(u - \underline{u}) = \frac{2n}{(n+1)^2}\{\frac{(n+1)s}{(n+1)s-2n}(\widehat{u} - \underline{u})\} + s(u - \underline{u}) > 0$, for $\forall u \in [\underline{u}, \overline{u}]$. Hence we show that the right-hand side of equation (35), $\frac{2n}{(n+1)^2}\{E^{SI}[e] - E^{AI}[e]\} + s(u - \underline{u}) < 0$,for $\frac{2n^2}{(n+1)^2} < s < \frac{2n}{n+1}$ and u such that $u < \widehat{u}$. While for $\frac{2n}{n+1} \leqq s$, $\frac{2n}{(n+1)^2}\{E^{SI}[e] - E^{AI}[e]\} + s(u - \underline{u}) > 0$, for

$\forall u \in [\underline{u}, \overline{u}]$. Note that e^{SI}, e^{AI} satisfy (35), (36), respectively. Combining these facts and the decreasing property of $MNV^{SI}(e)$,the result follows. ■

We can illustrate this result intuitively. In the definition of $MNV^{SI}(e)$, the first term enclosed in square brackets is associated with the marginal benefit of effort, while the second term is associated with the marginal cost of effort of the *symmetric information firm.* We refer to the former as *strategic effect*, since this shows interaction among oligopolistic firms. This strategic effect does not exist in a monopoly setting. The first term in (20), which is enclosed in square brackets, corresponds to marginal benefit, the second term is the marginal wage and the third term expresses the marginal cost due to information rent enjoyed by the manager (agent) of the *asymmetric information firm.* We denote the third term as *asymmetry of information effect.* There exists *asymmetry of information effect* in the monopoly setting. The *asymmetry of information effect* works to discourage manager effort in both settings. As we will consider, however, in the mixed oligopoly where both symmetric information firms and asymmetric information firms compete à la Cournot, the strategic effect differs between asymmetric information firms and symmetric information firms. This difference is expressed as the first term enclosed in brackets in (36). It takes a positive (nonpositive) value if s is sufficiently small (large) from Proposition 1. The *asymmetry of information effect* expressed by the second term in (36), however always takes a negative value. If this positive difference of strategic effect dominates the negative asymmetry of information effect, the manager of each asymmetric information firm has an incentive to select a higher effort level than that of each symmetric information firm's manager. Proposition 3 asserts that this is the case when the realized marginal cost is lower than its mean and s is sufficiently small.

Next, we establish the expected quantity levels of the managers of the symmetric information firm and the asymmetric information firm.

Proposition 4

$E[q_m^{AI}(u_m)] > E[q_j^{SI}(u_j)]$, for $\frac{2n^2}{(n+1)^2} < s < \frac{2n}{n+1}$,

$E[q_m^{AI}(u_m)] < E_{u_j}^{SI}[q_j^{SI}(u_j)]$, for $\frac{2n}{n+1} < s$.

Proof. From (30) and (31), some manipulation shows that

$$\begin{aligned} & E[q_m^{AI}(u_m)] - E[q_j^{SI}(u_j)] \\ = & -\frac{[(k+1)(n+1)s + (n-k)(n+1)^2 - 2n]}{\{(n+1)^2 s - 2n\}\{(n+1)s - 2n\}}(n+1)s(\widehat{u} - \underline{u}). \end{aligned} \quad (39)$$

For $k = 0$, the expression in the bracket of the numerator of the last expression of (37) becomes $(n+1)s + n(n+1)^2 - 2n$

$= (n+1)s + n(n^2+2n-1) > 0$, for $n \geq 2$. For $k \geq 1$ and $n \geq 2$, since $(n-k)(n+1)^2 \geq 0$, so

$(n-k)(n+1)^2 - 2n \geq -2n$. Hence, the same expression, for $k \geq 1$ and $n \geq 2$

$$\begin{aligned} (k+1)(n+1)s + (n-k)(n+1)^2 - 2n &\geq (k+1)(n+1)s - 2n \\ &> \frac{2n^2(k+1)}{n+1} - 2n \\ &= \frac{2n(kn-1)}{n+1} > 0, \end{aligned} \quad (40)$$

where the second inequality follows from Assumption 1. Since we show that $(k+1)(n+1)s+(n-k)(n+1)^2-2n > 0$, for $n \geq 2$ and $k = 0, 1, \cdots, n$, the last expression of (38) has a positive(negative) sign, if $\frac{2n^2}{(n+1)^2} < s < \frac{2n}{n+1}(\frac{2n}{n+1} < s)$, given Assumption 1 and $\widehat{u} = \frac{\underline{u}+\overline{u}}{2} > \underline{u}$. ■

An economic intuition of the result obtained in Proposition 4 is obvious from that in Proposition 1. In our Cournot model, the smaller the marginal cost of the firm, the larger the equilibrium of output. However, from (1), the higher the level of manager effort, the smaller the marginal cost. Therefore, the property of the expected equilibrium output obtained in the proposition inherits that of the equilibrium expected effort level of the manager presented in Proposition 1.

4 Effect of the Increase of Symmetric Information Firms on the Oligopoly

From the results obtained in the preceding section, we know that the properties of the expected level of manager's effort and the expected quantities of output depend on the initial number of firms with symmetric information k,the total number of firms in the market n, indicating the extent of the competition of the market, and the derivative of the marginal disutility of the manager s.

In the this section, we examine how variation in the mixture of firms with different types of information structure affects the effort level of managers and the expected consumer's surplus at the oligopolistic market equilibrium by conducting comparative statics on k, the number of symmetric information firms.

From (30) and (31), the equilibrium total output of the market is given by

$$E[Q(k)] = kE[q_j^{SI}(u_j)] + (n-k)E[q_m^{AI}(u_m)]. \tag{41}$$

Then, from equations (30),(31) and (39), we obtain the following main result.

Proposition 5 *Assume that* $n \geq 3$.

If $k \leqq \frac{n-1}{2}$, *then* $\frac{\partial E[Q(k)]}{\partial k} < 0$, *for* $\frac{2n^2}{(n+1)^2} < s < \frac{2n}{(n+1)}$, $\frac{\partial E[Q(k)]}{\partial k} > 0$, *for* $\frac{2n}{(n+1)} \leqq s$. *If* $\frac{n-1}{2} < k$, *then* $\frac{\partial E[Q(k)]}{\partial k} \geq 0$, *for* $\frac{2n^2}{(n+1)^2} < s < \frac{2n}{(n+1)}$, $\frac{\partial E[Q(k)]}{\partial k} \leqq 0$, *for* $\frac{2n}{n+1} \leqq s$.

Proof. From equations (30),(31) and (39), we have

$$
\begin{aligned}
\frac{\partial E[Q(k)]}{\partial k} &= E[q_j^{SI}(u_j)] - E[q_m^{AI}(u_m)] \\
&\quad + k\frac{\partial E[q_j^{SI}(u_j)]}{\partial k} + (n-k)\frac{\partial E[q_m^{AI}(u_m)]}{\partial k} \\
&= E[q_j^{SI}(u_j)] - E[q_m^{AI}(u_m)] \\
&\quad + \frac{-ks(n+1)^2}{\{(n+1)^2 s - 2n\}\{(n+1)s - 2n\}}(\widehat{u} - \underline{u}) \\
&\quad + (n-k)\frac{(n+1)s}{(n+1)^2 s - 2n^2}\{\frac{-2(k+1)(n+1)^3 s^2 + 2n(n+1)ns}{\{(n+1)^2 s - 2n\}\{(n+1)s - 2n\}}\}(\widehat{u} - \underline{u}) \\
&= \frac{s(n+1)}{\{(n+1)^2 s - 2n\}\{(n+1)s - 2n\}}(\widehat{u} - \underline{u}) \cdot B, \qquad (42)
\end{aligned}
$$

where $B \equiv (n+1)(2k-n+1)s + (n+1)(n-2k) - 2n$. From Assumption 1, $s > \frac{2n^2}{(n+1)^2}$ and $\frac{2n^2}{(n+1)^2} < \frac{2n}{n+1}$. Set $C = \frac{n^2-(2k+1)n-2k}{(n-2k-1)(n+1)}$. We can easily show that for $\frac{n-1}{2} < k$, $s < C \Leftrightarrow B < 0$ and $s \geq C \Leftrightarrow B \geq 0$, and for $k \leqq \frac{n-1}{2}, s \geq C \Leftrightarrow B < 0$, $s < C \Leftrightarrow B > 0$. Define $f(k) = \frac{n^2-(2k+1)n-2k}{(n-2k-1)(n+1)} - \frac{2n^2}{(n+1)^2}$. Then we see that $f'(k) = -2\frac{n-1}{(n+1)(n-2k-1)^2} < 0$ and $f(0) = \frac{n^2-n}{(-1+n)(n+1)} - 2\frac{n^2}{(n+1)^2} = -\frac{n(n-1)}{(n+1)^2} < 0$ for $n \geq 2$. That is, we have $C = \frac{n^2-(2k+1)n-2k}{(n-2k-1)(n+1)} < \frac{2n^2}{(n+1)^2}$. However, since from Assumption 1 we have $s > \frac{2n^2}{(n+1)^2} > C$, we see that for $\frac{n-1}{2} < k$, $B > 0$, and for $k \leqq \frac{n-1}{2}, s \geq C \Leftrightarrow B \leqq 0$. From all these facts, we can show the result. ∎

This result shows that the equilibrium expected total quantity either increases or decreases, as the number of symmetric information firms increases. As the number of firms with symmetric information, k increases, the equilibrium expected total output may increase and may improve the welfare of consumers. Whether the equilibrium expected total output expands or not depends on the initial number of k and the derivative of the marginal disutility of the manager, s. When the initial value of k is sufficiently small (less than half of the total number of firms) and s is considerably small (large), the equilibrium expected total output decreases (increases). On the other hand, when k is sufficiently large (more than the half of the total number of firms) and s is considerably small (large), the equilibrium expected total output increases (decreases). In our simple Cournot model, the expected consumer surplus is given by a positive quadratic function of the expected total output. That is, the expected consumer surplus is given by

$E[CS(Q(k)] = \frac{1}{2}E[(a - p(Q(k))Q(k))] = \frac{1}{2}E[(Q(k))^2]$.

Hence we present the next proposition without proof.

Proposition 6 *Assume that $n \geq 3$.*

If $k \leqq \frac{n-1}{2}$, then $\frac{\partial E[CS(Q(k)]}{\partial k} < 0$, for $\frac{2n^2}{(n+1)^2} < s < \frac{2n}{(n+1)}$, $\frac{\partial E[CS(Q(k)]}{\partial k} > 0$, for $\frac{2n}{(n+1)} < s$.If $\frac{n-1}{2} < k$, then $\frac{\partial E[CS(Q(k)]}{\partial k} \leqq 0$, for $\frac{2n^2}{(n+1)^2} < s < \frac{2n}{(n+1)}$, $\frac{\partial E[CS(Q(k)]}{\partial k} \leqq 0$, for $\frac{2n}{(n+1)} < s$.

5 Concluding Remarks

In this paper, we examine a Cournot oligopoly model where each firm produces and supplies a homogeneous good. We assume that two types of firm operate with respect to the available information system. In firms with a symmetric information system, the owner can observe not only the marginal cost but also the realization of cost uncertainty u_i. In firms with an asymmetric information system, the owner can observe the marginal cost but cannot observe the realization of cost uncertainty u_i. Since a Laffont–Tirole-type agency problem appears in this case, the realization of marginal cost becomes private information to manager of firm i, and the level of effort chosen by the manager cannot be observed by the owner of the firm. Provided that the number of symmetric types of firm *is exogenously given* by k, $(k = 0, 1, \cdots, n)$, we derive the equilibrium level of the manager of both types of firm and compare one with another. We show that when the extent of the derivative of the marginal disutility of the manager is sufficiently large, the expected effort level of the manager of the symmetric information firm is higher than that of the asymmetric information firm in an oligopoly setting. We also show that when the derivative of the marginal disutility of the manager is sufficiently large, the expected output of the symmetric information firm is higher than that of the asymmetric information firm in an oligopoly setting. We show that the manager of each asymmetric information firm may choose a higher effort level than that of each symmetric information firm's manager, if the realized marginal cost is lower than its mean and s is sufficiently small.

Furthermore, we examine the effects of an increase in the number of firms with symmetric information on the oligopolistic market outcomes, by conducting comparative statics of the equilibrium behaviors with respect to k. That is, we show that the equilibrium expected total output either increases or decreases, as the number of symmetric information firms increases. We find that whether the equilibrium expected total output increases or not depends on the initial number of k and the derivative of the marginal disutility of the manager s. When the initial value of k is relatively small or large (is less than or more than half of the total number of firms), the equilibrium expected total output and the expected consumer surplus decreases (increases) or increases (decreases), if s is considerably small (large). However, we have not provided any properties of the expected net utility of the owner, examination of which is left for our future research.

References

[1] Fershtman, C. and K. L. Judd, (1987), "Equilibrium Incentives in Oligopoly," *Amer. Econ. Rev.*, **77**, No. 5, 927–940.

[2] Fudenberg, D. and J. Tirole, (1991), *Game Theory*, Cambridge, MA: The MIT Press.

[3] Hart, O., (1983), "The Market Mechanism as an Incentive Scheme," *Bell J. Econ.*, **14**, 366–382.

[4] Laffont, J. J. and J. Tirole, (1993), *A Theory of Incentives in Procurement and Regulation*, Cambridge, MA: MIT Press.