DISCUSSION PAPER SERIES

Discussion paper No. 283

Price and Quantity Competition in a Hotelling Linear Market Model with Network Connectivity

Tsuyoshi Toshimitsu
(School of Economics, Kwansei Gakuin University)

December 2024

MASTERY FOR SERVICE

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

Price and Quantity Competition in a Hotelling Linear Market Model with Network Connectivity

Tsuyoshi TOSHIMITSU*

School of Economics, Kwansei Gakuin University

Abstract

Introducing network externalities into a Hotelling linear market model, we consider the profit ranking of Bertrand and Cournot equilibria, the problem of endogenous choice of strategic variables, and welfare efficiency. In particular, focusing on network connectivity (horizontal interoperability) between network products, we demonstrate the following results: (i) firms earn higher (lower) profits under Bertrand competition rather than under Cournot competition if network connectivity is sufficiently large (small); (ii) firms choose price (quantity) contracts if network connectivity is sufficiently large (small); (iii) social efficiency is achieved under Bertrand competition if network connectivity is sufficiently large.

Keywords: Hotelling linear market model, Bertrand competition, Cournot competition, network connectivity, fulfilled expectations, rational expectations

JEL Classification: D43, L13, L15, L22

* Corresponding author: Tsuyoshi TOSHIMITSU, School of Economics, Kwansei Gakuin University, 1-155, Nishinomiya, Japan, 662-8501, Email: ttsutomu@kwansei.ac.jp

1. Introduction

Forty years ago, Singh and Vives (1984) established a horizontally differentiated duopoly model and showed that Cournot competition yields higher profits than Bertrand competition and that choosing a quantity contract is the dominant strategy for each firm, given that their products are substitutes. This is one of the important theoretical problems in the field of industrial organization. Since their seminal paper, many researchers have considered the profit ranking and the endogenous choice of strategic variables in various models; for example, introducing asymmetric costs, vertical structures, mixed duopoly, R&D investment competition, and other variables.

We contribute to this literature by considering the problem posed by Singh and Vives (1984), when network externalities and connectivity (compatibility and horizontal interoperability) are introduced. In particular, we focus on the role of network connectivity in a network (digital) market in the model, in which we consider a demand system that differs from the related papers.

We review three papers that are very closely related to ours, namely Pal (2014), Toshimitsu (2016), and Shrivastav (2021). Based on the well-known model in Katz and Shapiro (1985), they introduce network externalities into a horizontally differentiated product model and derive a linear demand function (e.g., Hoernig, 2012).[1] The

[1] Pal (2014) and Shrivastav (2021) assume that consumers have rational expectations, whereas Toshimitsu (2016) assumes that they have active expectations. In addition, Shrivastav (2023) examines how network compatibility affects strategic R&D competition in the cases of Bertrand and Cournot competition. Although Shrivastav (2023) does not directly investigate the problems posed by Singh and Vives (1985), based on his model, we can derive the results mentioned in the text.

relationship between product substitutability and network compatibility (connectivity) in the demand function significantly affects the ranking of profits in equilibrium and the endogenous determination of strategic variables.

Pal (2014) assumes that product substitutability is equal to compatibility and shows that, given strong network externalities, firms earn higher profits under Bertrand competition than under Cournot competition and that choosing the quantity contract is the dominant strategy for each firm. He concludes that, unless network externalities are weak, firms face the prisoner's dilemma, and *Pareto* inferior outcomes result.

Toshimitsu (2016) assumes that product substitutability is not necessarily equal to compatibility and that the degree of compatibility is asymmetrical between firms. However, following Shrivastav (2021), symmetric compatibility is assumed regarding the endogenous choice of strategic variables, we find that firms choose a price (quantity) contract if the degree of network externalities is larger (smaller) than that of product substitutability.[2] Thus, regarding the ranking of profits in equilibrium, firms earn higher profits under Bertrand (Cournot) competition than under Cournot (Bertrand) competition if the degree of network externalities is larger (smaller) than that of product substitutability. This implies that firms' outcomes are *Pareto* superior if the degree of network externalities is larger (smaller) than that of product substitutability.

The authors of the related papers assume a homogeneous (representative) consumer who has a standard quadratic utility function that includes network externalities. This implies that the representative consumer (and thus all consumers) necessarily purchases every network product provided in the market. For example, the representative consumer

[2] Shrivastav (2023) assumes that product substitutability is not necessarily equal to compatibility, but that compatibility is symmetric.

(user) purchases two network products in the case of a duopolistic competition. The two network products may be two types of software that have the same functionality but different brand names (horizontally differentiated products) or the two network products may have two different operating systems. It may be convenient for the user to have both operating systems to connect with both types of software.

In this paper, we exploit the linear market model à la Hotelling in which each consumer (user) has an individual preference for network products (i.e., heterogeneous preferences) and purchases either one or none of the products (e.g., Shy, 2001). In the demand system, an individual user has a specific software (and operating system), and their satisfaction increases if they can connect with the users who have a different software and operating system. This situation may be a more natural assumption for the current digital economy than the assumption of homogeneous consumers in the related papers.[3]

2. A Hotelling Linear Market Model

2.1 Setup: demand and inverse demand functions

We consider the partial market coverage with some potential consumers in the network goods market. Following Tolotti and Yepez (2020) and Dyskeland and Foros (2023), we conduct a Hotelling linear market where two firms (providers), $i = 0,1,$ are located at

[3] From the perspective of the economics of platforms (e.g., Belleflamme and Peitz, 2021), we interpret the two types of market systems as follows. The homogeneous consumer corresponds to a multihoming user who can connect with various platforms, whereas the heterogeneous consumer corresponds to a singlehoming user who can use only a specific platform.

positions *0* and *1*, in the linear market.[4] Consumers indexed by l are uniformly distributed with a density equal to one on an open interval $l \in (-\infty, \infty)$. We can derive the following demand function for *product* (*firm*) *i*. For the detailed derivation of the demand function, see Appendix A and Figure 1.

$$q_i = \frac{t + 3v_i - v_j - 3p_i + p_j + 3N_i - N_j}{2t}, \quad i, j = 0,1, \quad i \neq j, \tag{1}$$

where $N_i = n\left(q_i^e + \phi q_j^e\right)$. v_i is an intrinsic value (the quality level) of *product i*, t is the transportation cost and implies product substitutability, p_i is the price of *product i*. Furthermore, N_i denotes network effects, $n(>0)$ is a parameter of network externalities, $\phi \in [0,1]$ is the degree of connectivity (horizontal interoperability), and q_i^e is the expected network size of *product i*.

For the following analysis, using equation (1), we derive the following indirect demand function:

$$p_i = \frac{2t + 4v_i - 3tq_i - tq_j + 4N_i}{4}, \quad i, j = 0,1, \quad i \neq j. \tag{2}$$

The profit function of *firm i* is given by $\pi_i = (p_i - c_i)q_i$, where c_i is the marginal cost of production of *firm i*, $i = 0,1$. We assume that consumers form rational (passive) expectations regarding the network sizes of the network products, and we exploit the concept of a fulfilled expectation equilibrium presented by Katz and Shapiro

[4] The location of firms is exogenously given because the difference in location implies differences in patents based on science and technological knowledge (and therefore different operating systems, in our context). In that sense, network connectivity in our model may perform a role as a converter (connector) between operating systems.

(1985).

[Insert Figure 1.]

2.2 Bertrand price competition

Using equation (1), we derive the following first-order condition (FOC) to maximize the profit function with respect to the price of *firm i*: $\frac{\partial \pi_i}{\partial p_i} = q_i - \frac{3}{2t}\left(p_i - c_i\right) = 0$. At the fulfilled expectation equilibrium, that is, $q_i^e = q_i = \frac{3}{2t}\left(p_i - c_i\right)$ and $q_j^e = q_j = \frac{3}{2t}\left(p_j - c_j\right)$, we obtain the following for *firm i*:

$$2t\left(2t + 3a_i - a_j\right) - \left\{12t - 3n(3-\phi)\right\}(p_i - c_i) + \left\{2t - 3n(1-3\phi)\right\}(p_j - c_j) = 0.$$

Similarly, for *firm j*, we have:

$$2t\left(2t + 3a_j - a_i\right) - \left\{12t - 3n(3-\phi)\right\}(p_j - c_j) + \left\{2t - 3n(1-3\phi)\right\}(p_i - c_i) = 0.$$

Thus, we obtain the following price net of marginal costs in equilibrium:

$$p_i^B - c_i = \frac{t\left[\left\{7t - 6n(1-\phi)\right\}t + (17t - 12n)a_i - 3(t - 4n\phi)a_j\right]}{D^B}, \quad (3)$$

where $D^B \equiv \left\{7t - 6n(1-\phi)\right\}\left\{5t - 3n(1+\phi)\right\} > 0$, $a_i \equiv v_i - c_i\left(>0\right)$, $i, j = 0,1$, and $i \neq j$. Superscript *B* denotes Bertrand price competition. We assume that $17t - 12n > 0$.

Using the FOC, the output and profit in equilibrium are expressed by

$$q_i^B = \frac{3}{2t}\left(p_i^B - c_i\right) \text{ and } \pi_i^B = \frac{3}{2t}\left(p_i^B - c_i\right)^2.$$

For the following analysis, we assume symmetric firms, $v_i = v_j = v$ and $c_i = c_j = c$. Thus, it holds that $a_i = a_j = a = v - c > 0$. In this case, we have the following outcomes:

$$p^B - c = \frac{(t+2a)t}{5t - 3n(1+\phi)}, \tag{4}$$

$$q^B = \frac{3(t+2a)}{2\{5t - 3n(1+\phi)\}}, \tag{5}$$

and

$$\pi^B = \frac{3t(t+2a)^2}{2\{5t - 3n(1+\phi)\}^2}, \tag{6}$$

where $5t - 3n(1+\phi) > 0$.[5]

---

[5] Regarding D^B, the following relationship holds:

$$7t - 6n(1-\phi) > (<) 5t - 3n(1+\phi) \Leftrightarrow \phi > (<) \frac{3n - 2t}{9n} \equiv \phi^*,$$

where $\phi^* > (<) 0 \Leftrightarrow n > (<) \frac{2t}{3}$ and $\frac{t}{4n} > \phi^*$. Then, the following two conditions need to hold for $D^B > 0$:

(i) If $\phi > \max\{0, \phi^*\}$, then $\frac{5t}{3n} - 1 \equiv \overline{\phi} > \phi$.

(ii) If $\phi < \phi^*$, then $\phi > \underline{\phi} \equiv 1 - \frac{7t}{6n}$.

In addition, it holds that $\overline{\phi} > \underline{\phi}$, because we assume that $17t - 12n > 0$. Therefore, for $D^B > 0$ to hold, it is necessary that $\overline{\phi} > \phi > \underline{\phi}$.

2.3 Cournot quantity competition

Using equation (2), the FOC for *firm i* is given by $\frac{\partial \pi_i}{\partial q_i} = p_i - c_i - \frac{3t}{4} q_i = 0.$ Thus, at the fulfilled expectations, that is, $q_i^e = q_i$ and $q_j^e = q_j$, we have:

$$2t + 4a_i - 2(3t - 2n)q_i - (t - 4n\phi)q_j = 0.$$

Similarly, for *firm j*, we have:

$$2t + 4a_j - 2(3t - 2n)q_j - (t - 4n\phi)q_i = 0.$$

Thus, we obtain the following output in equilibrium.

$$q_i^C = \frac{\{5t - 4n(1-\phi)\}t + 4(3t - 2n)a_i - 2(t - 4n\phi)a_j}{D^C}, \tag{7}$$

where $D^C \equiv \{7t - 4n(1+\phi)\}\{5t - 4n(1-\phi)\} > 0,$ $i, j = 0,1,$ and $i \neq j.$ Superscript *C* denotes Cournot quantity competition.[6] Furthermore, using the FOC, the price net of marginal costs and the profit in equilibrium are expressed by:

[6] Regarding D^C, the following relationship holds:

$$7t - 4n(1+\phi) > (<) 5t - 4n(1-\phi) \Leftrightarrow \frac{t}{4n} > (<) \phi.$$

Thus, it is necessary for $D^C > 0$ that the following conditions need to hold.

(i) If $\phi > \frac{t}{4n}$, then $\frac{7t}{4n} - 1 > \phi.$ That is, it holds that $\frac{7t}{4n} - 1 > \phi > \frac{t}{4n}$ because $3t - 2n > 0.$

(ii) If $\phi < \frac{t}{4n}$, then $\phi > 1 - \frac{5t}{4n}.$ That is, it holds that $\frac{t}{4n} > \phi > 1 - \frac{5t}{4n}$ because $3t - 2n > 0.$

Here, if $17t - 12n > 0,$ it holds that $3t - 2n > 0.$ Therefore, for $D^C > 0$ to hold, it is necessary that $\frac{7t}{4n} - 1 > \phi > 1 - \frac{5t}{4n}.$

$p_i^C - c_i = \frac{3t}{4} q_i^C$ and $\pi_i^C = \frac{3t}{4}\left(q_i^C\right)^2$, respectively.

In a similar manner to the Bertrand price competition case, we have the following outcomes for Cournot quantity competition, given symmetric firms:[7]

$$p^C - c = \frac{3\left(t+2a\right)t}{2\left\{7t - 4n(1+\phi)\right\}}, \tag{8}$$

$$q^C = \frac{2\left(t+2a\right)}{7t - 4n(1+\phi)}, \tag{9}$$

and

$$\pi^C = \frac{3t\left(t+2a\right)^2}{\left\{7t - 4n(1+\phi)\right\}^2}. \tag{10}$$

2.4 Comparison[8]

Exploiting the results derived in the previous subsections, that is, equations (4)–(6) and equations (8)– (10), we obtain the following lemma.

Lemma 1.

(1) $q^B > q^C$.

(2) $p^B - c > (<) p^C - c \Leftrightarrow z \equiv n(1+\phi) > (<) t.$

---

[7] It holds that $\frac{7t}{4n} - 1 > \frac{5t}{3n} - 1 > \phi.$

[8] See Appendix B for the active (responsive) expectations case. We compare equilibrium outcomes between Bertrand price and Cournot quantity competition in Lemma B.

(3) $\pi^B > (<) \pi^C \Leftrightarrow F[z,t] < (>) 0, \qquad F[z,t] = 2z^2 - 4tz + t^2, \quad \text{where} \quad z < \frac{5t}{3}.$

Thus, the following relationship holds: $z > (<) \left(\frac{2-\sqrt{2}}{2} \right) t \Leftrightarrow \pi^B > (<) \pi^C.$

In Lemma 1, parameter $z \equiv n(1+\phi)$, where $n \le z \le 2n$, expresses the total effects of network connectivity. These effects are divided into two parts: first, the "within-group network effects," expressed as $n \times 1$, and second, the "cross-group network effects," expressed as $n \times \phi$. Regarding the cross-group network effects, if $\phi = 0(1)$, consumers purchasing one network product cannot (can perfectly) operate the other network product because the products are not connected (perfectly connected) and thus incompatible (completely compatible).

Regarding Lemma 1 (1) and (2), we have the following effects of an increase in network externalities on the outputs and prices: $\frac{dq^B}{dn} > \frac{dq^C}{dn} > 0$ and $\frac{d(p^B - c)}{dn} > (<) \frac{d(p^C - c)}{dn} \Leftrightarrow F[z,t] < (>) 0.$[9] The effect of output in the Bertrand price competition case is larger than that in the Cournot quantity competition case. However, the effect on price (or price net of marginal costs) depends on the degree of network connectivity. If the total effect of network connectivity is sufficiently small (large), the effect of an increase in price is larger (smaller) in the Bertrand price competition case than in the Cournot quantity competition case. This price effect affects

[9] We can obtain the same results for the effects of an increase in the degree of connectivity as those obtained for an increase in the degree of network externalities.

the relationship of the profits between Bertrand price and Cournot quantity competition. In particular, regarding Lemma 1 (2) and (3), if the total effects are sufficiently large, the price and profit in the Bertrand price competition case are larger than those in the Cournot quantity competition case.

3. Endogenous Determination of Strategic Variables and Efficiency

3.1 Mixed strategy competition: price and quantity contract[10]

We assume that *firm i* (*j*) chooses a quantity, *Q* (price, *P*) contract which is expressed as [*firm i*'s strategy, *firm j*'s strategy] $=\left[Q,P\right]$. Using equations (1) and (2), the inverse demand function for *firm i* and the direct demand function for *firm j* are given respectively by:

$$p_i = \frac{t + 3v_i - v_j - 2tq_i + p_j + 3N_i - N_j}{3} \tag{11}$$

and

$$q_j = \frac{2t + 4v_j - 4p_j - tq_i + 4N_j}{3t}, \quad i, j = 0,1, \quad i \neq j. \tag{12}$$

By the similar maneuver to the previous section, we derive the following outcomes in equilibrium:

$$q_i^M[Q,P] = \frac{6\left\langle \{5t - 4n(1-\phi)\}t + 4(3t - 4n)a_i - 2(t - 4n\phi)a_j \right\rangle}{D^M} \tag{13}$$

---

[10] For a comparison of the equilibrium outcomes under the mixed strategy competition, see Appendix C, in which we summarize the results as Lemma C.

and

$$p_j{}^M[Q,P]-c_j=\frac{3t\left\langle\{7t-6n(1-\phi)\}t+(17t-12n)a_j-3(t-4n\phi)a_i\right\rangle}{D^M}, \tag{14}$$

where $D^M \equiv 8(3t-2n)\{4t-n(3-\phi)\}+(t-4n\phi)\{3t-4n(1-3\phi)\}>0,$ and superscript *M* denotes the mixed strategy competition. We assume that $3t-4n>0.$[11]

Assuming symmetry, $a_i=a_j=a=v-c,$ we have the following for *firm i*:

$$q_i{}^M[Q,P]=\frac{6\left\langle\{5t-4n(1-\phi)\}\right\rangle(t+2a)}{D^M}, \tag{15}$$

$$p_i{}^M[Q,P]-c=\frac{2t}{3}q_i{}^M[Q,P]=\frac{4t\left\langle\{5t-4n(1-\phi)\}\right\rangle(t+2a)}{D^M}, \tag{16}$$

and

$$\pi_i{}^M[Q,P]=\frac{24t\left\langle\{5t-4n(1-\phi)\}\right\rangle^2(t+2a)^2}{(D^M)^2}. \tag{17}$$

Similarly, for *firm j*, we have:

$$q_j{}^M[Q,P]=\frac{4}{3t}\left(p_j{}^M[Q,P]-c\right)=\frac{4\left\langle\{7t-6n(1-\phi)\}\right\rangle(t+2a)}{D^M}, \tag{18}$$

$$p_j{}^M[Q,P]-c=\frac{3t\left\langle\{7t-6n(1-\phi)\}\right\rangle(t+2a)}{D^M}, \tag{19}$$

and

$$\pi_j{}^M[Q,P]=\frac{12t\left\langle\{7t-6n(1-\phi)\}\right\rangle^2(t+2a)^2}{(D^M)^2}. \tag{20}$$

---

[11] If this condition holds, we have that $17t-12n>0.$

3.2 Endogenous choice of strategic variables

We can express the notation of the profits in the equilibrium derived earlier, that is, in equations (6), (10), (17), and (20), as follows: $\pi^B = \pi_i^B[P,P] = \pi_j^B[P,P]$, $\pi^C = \pi_i^C[Q,Q] = \pi_j^C[Q,Q]$, $\pi_i^M[Q,P] = \pi_j^M[P,Q]$, and $\pi_j^M[Q,P] = \pi_i^M[P,Q]$. Exploiting these equations, we derive the following lemma (which is similarly, derived for *firm j*).

Lemma 2.

(1) $\pi_i^B[P,P] > (<) \pi_i^M[Q,P] \Leftrightarrow \phi > (<) \dfrac{t}{4n}.$

(2) $\pi_i^C[Q,Q] > (<) \pi_i^M[P,Q] \Leftrightarrow \dfrac{t}{4n} > (<) \phi.$

With respect to the relationship between network externalities and product substitutability, we make the following assumption.

Assumption 1. Strong network externalities exist such that $\dfrac{t}{4} < n.$

This assumption shows that the degree of network externalities is large compared with the degree of product substitutability. Given this assumption, we derive the following proposition.

Proposition 1.

If $\phi > (<) \dfrac{t}{4n}(<1)$, then choosing price (quantity) contract is the dominant strategy for each firm.

Taking Assumption 1 and Lemma 1 (3), the following equation holds: $\dfrac{t}{4n} > \left(\dfrac{2-\sqrt{2}}{2}\right)\dfrac{t}{n} - 1$. Considering Proposition 1, we derive the following proposition regarding the equilibrium profits in the case of the endogenous choice of strategy game.

Proposition 2.

(1) Choosing the price contract results in a *Pareto* superior outcome if $\phi > \dfrac{t}{4n}$.

(2) Choosing the quantity contract results in a *Pareto* inferior (superior) outcome if

$$\frac{t}{4n} > \phi > \left(\frac{2-\sqrt{2}}{2}\right)\frac{t}{n} - 1 \quad \left(\left(\frac{2-\sqrt{2}}{2}\right)\frac{t}{n} - 1 > \phi \geq 0\right).$$

Regarding Proposition 2 (2), if the degree of total network connectivity is an intermediate value, that is, $\dfrac{t+4n}{4} > n(1+\phi) = z > \left(\dfrac{2-\sqrt{2}}{2}\right)t$, each firm will choose a quantity contract. However, each firm's profit in equilibrium is lower than the profit achieved when they choose a price contract: the firms face a prisoner's dilemma. In other words, if the total effect of network connectivity is either sufficiently large or small, firms can choose *Pareto* superior strategies.

Assumption 1 is necessary for Propositions 1 and 2 to hold. Now, we investigate the case of weak network externalities, that is, $\frac{t}{4} > n.$ Based on Lemma 1 and 4, with respect to choosing optimal strategies and the profits in equilibrium, we derive the following corollaries.

Corollary 1.

Given $\frac{t}{4} > n\left(\Leftrightarrow \frac{t}{4n} > 1 \geq \phi\right),$ choosing the quantity contract is the dominant strategy for each firm.

Corollary 2.

(1) Choosing the quantity contract results in a *Pareto* superior if $\frac{\left(2-\sqrt{2}\right)t}{4} > n.$

(2) Choosing the quantity contract results in a *Pareto* inferior (superior) outcome if

$$1 > \phi > \left(\frac{2-\sqrt{2}}{2}\right)\frac{t}{n} - 1 \quad \left(\left(\frac{2-\sqrt{2}}{2}\right)\frac{t}{n} - 1 > \phi \geq 0\right).$$

In view of Proposition 1 and Corollary 1, we summarize the following results regarding the endogenous choice of strategies in a network product market. In a market characterized by either weak network externalities or lower network connectivity (e.g., nonconnectivity), firms choose quantity contracts. However, in a market characterized by strong network externalities and higher network connectivity (e.g., perfect horizontal interoperability), firms choose price contracts. Furthermore, regarding *Pareto* optimality, based on Proposition 2 and Corollary 2, whatever strategies firms choose, if the degree of network connectivity is an intermediate value, the firms face a prisoner's dilemma.

3.3 Consumer welfare and social efficiency

We consider the consumer surplus from purchasing *product 0*.[12] Using equations (A.1) and (A.2), we can express the following consumer surplus in equilibrium, that is, $CS_0^{\ m}$, $m = B, C.$

$$CS_0^{\ m} = \int_0^{l^{*}} \left\{ v_0 - tl - p_0^{\ m} + N_0^{\ m} \right\} dl + \int_{-l^{-*}}^{0} \left\{ v_0 + tl^{-} - p_0^{\ m} + N_0^{\ m} \right\} dl^{-}$$
$$= \int_0^{l^{*}} \left\{ a_0 - tl - (p_0^{\ m} - c) + n(1+\phi) q_0^{\ m} \right\} dl + \int_{-l^{-*}}^{0} \left\{ a_0 + tl^{-} - (p_0^{\ m} - c) + n(1+\phi) q_0^{\ m} \right\} dl^{-},$$

where $l^{*} = \frac{1}{2},\ \ -l^{-*} = y_0^{\ m} = q_0^{\ m} - \frac{1}{2}.$

Thus, we can derive the following:

$$\tilde{CS}_0^{\ m} \equiv CS_0^{\ m} + \frac{t}{2} = \left\{ \frac{t+2a}{2} - \frac{t - 2n(1+\phi)}{2} q_0^{\ m} - (p_0^{\ m} - c) \right\} q_0^{\ m}. \quad (21)$$

Using equations (4), (5), (8), (9), and (21), the consumer surpluses in equilibrium in the cases of Bertrand price and Cournot quantity competition are given respectively by:

$$\tilde{CS}_0^{\ B} = \left(\frac{t}{2} \right) \left\{ \frac{3(t+2a)}{2\left[5t - 3n(1+\phi)\right]} \right\}^2 \text{ and } \tilde{CS}_0^{\ C} = \left(\frac{t}{2} \right) \left\{ \frac{2(t+2a)}{7t - 4n(1+\phi)} \right\}^2.$$

Therefore, we can derive the following result: $\tilde{CS}_0^{\ B} > \tilde{CS}_0^{\ C}$. Consumer surplus under Bertrand price competition is larger than that under Cournot quantity competition,

---

[12] Given the assumption of symmetry, we have the same results for consumer surplus from purchasing *product 1* as those for the *product 0* case.

regardless of the degree of network connectivity (see Lemma 1 (1)). The price contract is better for consumers. However, which contract is better for firms depends on the relationship between network connectivity and product substitutability. In this case, the next question is which contract is socially efficient? In view of Proposition 2 and Corollary 2, we derive the following proposition.

Proposition 3.

(1) The price contract is socially efficient if $1 > \phi > \dfrac{t}{4n}$.

(2) The quantity contract is socially inefficient if $\min\left\{\dfrac{t}{4n}, 1\right\} > \phi > \left(\dfrac{2-\sqrt{2}}{2}\right)\dfrac{t}{n} - 1$.

As shown in Proposition 3 (1), to achieve social efficiency, it is necessary for network connectivity to be to sufficiently large. In other words, social efficiency is achieved in the digital market with strong network externalities and high network connectivity (horizontal interoperability). However, Proposition 3 (2) implies that, if the degree of network connectivity has an intermediate value associated with weak network externalities, a socially inefficient situation arises in the digital market. Furthermore, taking Proposition 2 (2) and Corollary 2 (1), if the degree of network connectivity is sufficiently small, the quantity contract is preferable for the firms, but not for the consumers. That is, a conflict of benefits arises between firms and consumers regardless of whether the situation is socially efficient.

4. Concluding Remarks

Introducing network externalities into a Hotelling linear market model, we have considered the profit ranking of Bertrand and Cournot equilibria, the problem of endogenous choice of strategic variables, price vs. quantity, and social efficiency. In particular, we have focused on the role of network connectivity (i.e., horizontal interoperability) between network products and demonstrated the following results.

(1) With respect to profit ranking, firms earn higher (lower) profits under Bertrand price competition than under Cournot quantity competition, if the degree of network connectivity is sufficiently large (small).

(2) With respect to the endogenous choice of strategic variables, given strong network externalities, if the degree of network connectivity is sufficiently large (small), the firms choose the price (quantity) contract. As a result, Bertrand price (Cournot quantity) competition results in a *Pareto* superior outcome. However, if the degree of network connectivity is an intermediate value, the Cournot quantity competition yields in *Pareto* inferior outcome.

(3) Given weak network externalities, firms choose the quantity contract, which yields a *Pareto* superior outcome.

(4) Consumer surplus is always larger under Bertrand price competition than under Cournot quantity competition. Thus, given strong network externalities, the degree of network connectivity is sufficiently large, a socially efficient situation arises in the Bertrand price competition market. However, if the degree of network connectivity is an intermediate value, a socially inefficient situation arises in the

Cournot quantity competition market.

Some problems remain in our study that point to future research directions. In our Hotelling-type location model, we have assumed that the degree of connectivity is exogenously given. The connectivity (compatibility) and horizontal interoperability between network products are critical factors in digital markets. In addition, from the perspective of competitive policy, the role of connectivity (i.e., compatibility standardization) is important for users (not only consumers but also firms under horizontal and vertical relationships). That is, we should consider how connectivity (interoperability) is endogenously determined by users and administrators. In that case, the results in this paper suggest that the determination is related to the mode of competition, that is, an optimal contract (strategic variables). We plan to examine the endogenous choice of strategic variables (price and quantity) and connectivity (nonconnection and perfect connection) in future research.

Appendix A. The Derivation of the Demand Function in a Hotelling Linear Market

Consumer indexed by l are uniformly distributed with a density equal to one on an open interval $l \in (-\infty, \infty)$. Two firms are located at *0* and *1*. In this case, there are three types of consumers, as follows:

(i) Consumer $l \in [0,1]$ has the following net utility (surplus) function:

$$U = \begin{cases} v_0 - tl - p_0 + N_0 \quad \text{if purchasing} \quad \text{product} \quad 0 \\ \\ v_1 - t(1-l) - p_1 + N_1 \; \text{if purchasing} \quad \text{product} \quad 1 \end{cases}, \qquad (A.1)$$

where v_i is an intrinsic value (the quality level) of *product I*; t is a transportation cost, which implies product substitutability; and p_i is the price of *product i*. Furthermore, N_i denotes network effects, which we will explicitly specify below.

(ii) Consumer $l^- \in (-\infty, 0)$ has the following net utility function:

$$U^- = \begin{cases} v_0 - t(0-l^-) - p_0 + N_0 \; \text{if purchasing} \quad \text{product} \quad 0 \\ \\ 0 \qquad\qquad \text{if not} \quad \text{purchasing} \end{cases}. \qquad (A.2)$$

(iii) Consumer $l^+ \in (1, \infty)$ has the following net utility function:

$$U^+ = \begin{cases} v_1 - t(l^+ - 1) - p_1 + N_1 \; \text{if purchasing} \quad \text{product} \quad 1 \\ \\ 0 \qquad\qquad \text{if not} \quad \text{purchasing} \end{cases}. \qquad (A.3)$$

Consumers belonging to type (i) purchase one of the two network products (in a fully covered market). However, type (ii) (type (iii)) consumers either purchase *product 0* (*1*) or do not purchase any products. That is, this open network products market is a partially covered and partially uncovered market where there are some potential consumers who may purchase the network products.

First, regarding type (i) consumers, based on equation (A.1), the consumer-indexed l^*, who is indifferent between *products 0* and *1*, is given by $l^* = \frac{1}{2} + \frac{v_0 - v_1 - p_0 + p_1 + N_0 - N_1}{2t}$. Thus, the demand function for *product* (*firm*) *0* is expressed as:

$$x_0 = l^* = \frac{t + v_0 - v_1 - p_0 + p_1 + N_0 - N_1}{2t}, \tag{A.4}$$

where $N_i = n\left(x_i^e + \phi x_j^e\right)$, $i, j = 0,1$, and $i \neq j$. Parameter $n(>0)$ expresses the network externalities, $\phi \in [0,1]$ denotes the degree of connectivity, and x_i^e denotes the expected network size of *product i*. Regarding the demand function for *product 1*, based on equation (A.4), we have $x_1 = 1 - x_0$.

Second, for type (ii) consumers, using equation (A.2), the marginal consumer purchasing *product 0* is given by $-l^{-*} = \frac{v_0 - p_0 + N_0}{t}$. Thus, we derive the following demand function of the consumers:

$$y_0 = \frac{v_0 - p_0 + N_0}{t}. \tag{A.5}$$

Similarly, for type (iii) consumers, we have the following demand function:

$$y_1 = \frac{v_1 - p_1 + N_1}{t}. \tag{A.6}$$

Therefore, based on equations (A.4) and (A.5) (or (A.6)), we obtain the following demand function of *firm i*:

$$q_i \equiv x_i + y_i = \frac{t + 3v_i - v_j - 3p_i + p_j + 3N_i - N_j}{2t}, \quad i, j = 0,1, \quad i \neq j. \tag{1}$$

See Figure 1 for an illustration of the demand function.

Appendix B. The Case of Active Expectations

1. Demand functions

For convenience of explanation, we first derive the inverse demand function and consider Cournot quantity competition case. Given active (responsive) expectations, that is, $q_i^e = q_i$ and $q_j^e = q_j$, equation (2) can be revised as follows:

$$p_i^A = \frac{2t + 4v_i - (3t - 4n)q_i - (t - 4n\phi)q_j}{4}, \tag{B.1}$$

where superscript *A* denotes active expectations and we assume that $3t - 4n > 0$. Furthermore, using equation (B.1), the direct demand function is given by:

$$q_i^A = \frac{\{t - 2n(1-\phi)\}t + (3t - 4n)(v_i - p_i) - (t - 4n\phi)(v_j - p_j)}{2\{t - 2n(1-\phi)\}\{t - n(1+\phi)\}}. \tag{B.2}$$

2. Cournot quantity competition

Based on equation (B.1), we derive the following first-order condition (FOC), second-order condition (SOC), and cross effect, respectively:

$$\frac{\partial \pi_i^{AC}}{\partial q_i} = p_i^A - c_i - \frac{3t - 4n}{4} q_i = 0, \tag{B.3}$$

$$\frac{\partial^2 \pi_i^{AC}}{\partial q_i^{\,2}} = -\frac{3t-4n}{2} < 0, \tag{B.4}$$

and

$$\frac{\partial^2 \pi_i^{AC}}{\partial q_i \partial q_j} = -\frac{t-4n\phi}{4} > (<) 0 \Leftrightarrow \phi > (<) \frac{t}{4n}, \tag{B.5}$$

where superscript AC denotes Cournot quantity competition under active expectations. In view of equation (B.5), if $\phi > (<) \frac{t}{4n}$, the firms are strategic complements (substitutes). This differs from the case of rational expectations.

Using equations (B.1) and (B.3), we derive the following output in equilibrium.

$$q_i^{AC} = \frac{2\left[\{5t-4n(2-\phi)\}t + 4(3t-2n)a_i - 2(t-4n\phi)a_j\right]}{D^{AC}}, \tag{B.6}$$

where $D^{AC} \equiv \{7t-4n(2+\phi)\}\{5t-4n(2-\phi)\} > 0$. Assuming symmetric firms, $a_i = a_j = a = v - c$, we have the following outcomes:

$$p^{AC} - c = \frac{(3t-4n)(t+2a)}{2\{7t-4n(2+\phi)\}}, \tag{B.7}$$

$$q^{AC} = \frac{2(t+2a)}{7t-4n(2+\phi)}, \tag{B.8}$$

and

$$\pi^{AC} = \frac{(3t-4n)(t+2a)^2}{\{7t-4n(2+\phi)\}^2}, \tag{B.9}$$

where we assume that $7t > 12n \geq 4n(2+\phi)$.

3. Bertrand price competition

In a similar manner to the case of Cournot quantity competition, we use equation (B.2) and derive the following FOC, SOC, and cross effect, respectively, for Bertrand price competition:

$$\frac{\partial \pi_i^{AB}}{\partial p_i} = q_i^A - \frac{3t-4n}{2\{t-2n(1-\phi)\}\{t-n(1+\phi)\}}(p_i - c_i) = 0, \quad \text{(B.10)}$$

$$\frac{\partial^2 \pi_i^{AB}}{\partial p_i^2} = -\frac{3t-4n}{2\{t-2n(1-\phi)\}\{t-n(1+\phi)\}} < 0, \quad \text{(B.11)}$$

and

$$\frac{\partial^2 \pi_i^{AB}}{\partial p_i \partial p_j} = \frac{t-4n\phi}{4} > (<) 0 \Leftrightarrow \phi < (>) \frac{t}{4n}, \quad \text{(B.12)}$$

where superscript AB denotes Bertrand price competition under active expectations. Equation (B.12) implies that if $\phi > (<) \frac{t}{4n}$, the firms strategic substitutes (complements).

Considering equations (B.5) and (B.12), if the degree of network connectivity is sufficiently large, the strategic relationships between the firms are opposite to the familiar case without network externalities.

Using equations (B.2) and (B.10), we have the following price (net of marginal cost) in equilibrium.

$$p_i^{AB} - c_i = \frac{T^{AB}t + \left\{2(3t-4n)^2 - (t-4n\phi)^2\right\}a_i - (3t-4n)(t-4n\phi)a_j}{D^{AB}}, \quad \text{(B.13)}$$

where $D^{AB} \equiv \left\{7t-4n(2+\phi)\right\}\left\{5t-4n(2-\phi)\right\} = D^{AC} > 0$ and

$T^{AB} \equiv \{7t - 4n(2+\phi)\}\{t - 2n(1-\phi)\} > 0$. Assuming symmetry, we derive the following outcomes:

$$p^{AB} - c = \frac{\{t - 2n(1-\phi)\}(t + 2a)}{5t - 4n(2-\phi)}, \tag{B.14}$$

$$q^{AB} = \frac{(3t - 4n)(t + 2a)}{2\{t - n(1+\phi)\}\{5t - 4n(2-\phi)\}}, \tag{B.15}$$

and

$$\pi^{AB} = \frac{(3t - 4n)\{t - 2n(1-\phi)\}(t + 2a)^2}{2\{t - n(1+\phi)\}\{5t - 4n(2-\phi)\}^2}, \tag{B.16}$$

where we assume that $t > 2n \geq \max\{2n(1-\phi), n(1+\phi)\}$.[13]

4. Comparison

Using the results derived in the previous sections, we obtain the following lemma.

Lemma B.

(1) $q^{AB} > q^{AC}$.

(2) $p^{AB} - c < p^{AC} - c$.

(3) $\pi^{B} > (<) \pi^{C} \Leftrightarrow \phi > (<) \dfrac{t}{4n}$.

Lemma B is basically similar to Lemma 1 and Proposition 1 in Pal (2014), who

[13] Given this assumption, it holds that $5t - 4n(2-\phi) > 0$.

assumes rational expectations.

Appendix C. Comparison in the Case of Mixed Strategy Competition

Exploiting equations (15) to (20), we can directly derive the following lemma.

Lemma C.

(1) $q_i^M[Q,P] > q_j^M[Q,P]$.

(2) $p_i^M[Q,P] - c > (<) p_j^M[Q,P] - c \Leftrightarrow t - 2n(1-\phi) < (>) 0$.

(3) $\pi_i^M[Q,P] > \pi_j^M[Q,P]$, given $t - n(1-\phi) > 0$.

References

Belleflamme, P. and M. Peitz (2021) *The Economics of Platforms*, Cambridge University Press, Cambridge, UK.

Dyskeland, O. K. and Ø. Foros (2023) Multihoming and market expansion: effects on media platforms' pricing and content creation incentives. *Economics Letters* 111327.

Hoernig, S. (2012) Strategic delegation under price competition and network effects. *Economics Letters* 117 (2): 487–489.

Katz, M. and C. Shapiro (1985) Network externalities, competition, and compatibility. *American Economic Review* 75: 424–440.

Pal, R. (2014) Price and quantity competition in network goods duopoly: a reversal result. *Economics Bulletin* 34 (2): 1019–1027.

Shrivastav, S. (2021) Network compatibility, intensity of competition and process R&D: a generalization. *Mathematical Social Science* 109:152–163.

Shy, O. (2001) *The Economics of Network Industries*, Cambridge University Press, Cambridge, UK.

Singh, N. and X. Vives (1984) Price and quantity competition in a differentiated duopoly. *RAND Journal of Economics* 15: 546–554.

Tolotti, M and J. Yepez (2020) Hotelling–Bertrand duopoly competition under firm-specific network effects. *Journal of Economic Behavior and Organization* 176: 105−128.

Toshimitsu, T. (2016) Price and quantity competition in a differentiated duopoly with network compatibility effects. *Japanese Economic Review* 67 (4): 495–512.

Figure 1. A partially covered (and uncovered) market