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Network Connectivity, Strategic R&D Competition, and Market Structure: A Hotelling Linear Market Model

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Title

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Keyword
Network externality, Connectivity, interoperability, R&D competition, Hotelling linear market, Fulfilled expectations, Lerner index

JEL Classifications
L13, L15, L31, L32, D43

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Using the framework of a Hotelling linear market, we consider the impact of network connectivity (horizontal interoperability) between network goods on strategic R&D competition and profits. We first demonstrate that in the case of a fully covered (mature) market, as network connectivity increases, R&D activities decrease, but profits increase. Then, relaxing the assumption of market coverage, we demonstrate that in the case of a partially covered and uncovered (immature) market, as network connectivity increases, R&D activities at first decrease, and then increase given strong network externalities. Otherwise, the R&D activities monotonically increase. However, regardless of the strength of the network externalities, profits increase. Regarding quantity competition in the immature market, we obtain the same results in the case of price competition. We also consider the implication of network connectivity for market competitiveness.

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1. Introduction

In a modern digital economy, networking is not only spreading to all economic activities, but also every aspect of our daily lives, ubiquitously, as it were, “No online, No life.” This trend will surely intensify competition among firms providing network goods and services in digital markets. Consequently, severe competition takes place at various levels and stages of the production and product sales process, including product (service) and process research and development (R&D) investments, prices and sales (quantity) competition, among many others.

However, we often observe that digital markets also tend to become monopolistic or more highly concentrated, which leads to the situations of winner-takes-all or a winner-takes-most, is the case for Google, Apple, Facebook, and Amazon. This undesirable characteristic of digital markets may limit the potential for competition.

In the current environment, where people spend increasing amounts of their time and money on Internet services (e.g., e-commerce, mobile games, and search engine sites), network connectivity (compatibility) and the ‘horizontal interoperability’ between goods and services or between platforms in digital markets are critical factors for both providers (firms) and users (consumers).[1]

In this regard, Economides and White (1994) discuss the economic and legal (i.e., antitrust policy) implications of compatibility and networks. They argue that compatibility is equivalent to the more general concept of complementarity and conclude

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[1] Horizontal interoperability denotes the form of the interconnection between users (Çavuş, 2024). For the analysis of compatibility promotion between platforms, see Spaeth and Niederhöfer (2022).

that it and network arrangements bring benefits to firms, whereas compatibility may lead to anti-competitive consequences, at least in some cases. More recently, Heywood et al. (2022, pp. 355–356) think about compatibility as:

> The extent to which one firm's R&D may allow it to lower costs and capture customers can be limited by the lack of compatibility. In addition, it is recognized that the extent of compatibility can influence the introduction of new technology [and that] reflecting this interconnection, firm compatibility decisions by network firms raise public policy issues regarding both anti-competitive behavior and reduced technological progress.

The main research question posed by this paper is how network connectivity (compatibility and horizontal interoperability) affects the incentives to innovate and undertake R&D activity. That is, does an increase in the degree of network connectivity improve or reduce the incentives to innovate? Consequently, will the resulting digital markets be competitive? Although policy analysis is beyond the scope of this paper, if network connectivity promotes the incentives to innovation, interoperability obligation policy and open technology standards are not necessarily anti-competitive. For this reason, we are most interested in the question of under what conditions is it possible for network connectivity to reduce the incentives for firms to innovate.

There has been an increasing volume of studies analyzing process (cost-reducing) and product (quality-improving) R&D competition in the presence of network externalities since the seminal work by Katz and Shapiro (1985). In this paper, by focusing on the differences in demand systems, we follow Roson's (2002) review of Crémer et al. (2000) and Foros and Hansen (2001), which consider the issues of competition and quality determination in the market for Internet access services. Roson (2002) argues these two studies come to opposing conclusions, with the differences depending on their alternative hypotheses concerning overall market sizes, whereby Crémer et al. (2000) adopt the well-known model in Katz and Shapiro (1985), whereas Foros and Hansen (2001) adopt a unit-linear market following a conventional Hotelling location model.

More concretely, the former assumes that a representative (homogeneous) consumer has a quasi-linear utility function with network effects, and purchases all of the network goods provided in the market (hereinafter, the linear demand model). However, in the latter, the assumption is made that each consumer in a unit-linear market has an individual preference for the goods (i.e., heterogeneity), and then purchases either one or none of the goods (hereinafter, the location demand model).[2]

For example, if exogenous variables change in the market, it is possible for the total market size to expand in the linear demand model, because new consumers enter the market while incumbent consumers purchase more goods than before. However, an increase in the total market size is not possible in the location demand model because the number of consumers and the number of purchases per consumer are exogenously given (thus limited). As discussed below, we relax this assumption of market coverage in this paper and examine the case of an uncovered market.

We now review the closely related literature. First, regarding the linear demand model, Buccella et al. (2023) and Shrivastav (2021) consider competition in R&D investment. Using a horizontally differentiated duopoly model with network externalities, Shrivastav (2021) demonstrates the ranking of R&D investments for Bertrand and Cournot

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[2] From the perspective of platform economics, we interpret the two types of consumers as follows. The homogenous consumer implies a multi-homing user that can connect various platforms; the heterogeneous consumer implies a single-homing user that can use only a specific platform. See Belleflamme and Peitz (2021).

duopolistic competition. Shrivastav (2021, Appendix B) also demonstrates the effects of compatibility on R&D investment, and argues that the following results hold in both Bertrand and Cournot competition: (i) if R&D investments are strategic complements, as compatibility increases, R&D investments increase; and (ii) if R&D investments are strategic substitutes, as compatibility increases, R&D investments first decrease, and then increase.

In their analysis, Buccella et al. (2023) assume a homogeneous product with network externalities and technological spillover effects. They then compare the investments, quantities, and profits in the full compatibility case with those in the incompatibility case declaring that if there are no technological spillover effects, the level of the investment in the incompatibility case is higher than in the full compatibility case. As will be shown below, we provide results (i) and (ii) from Shrivastav (2021) and that of Buccella et al. (2023), even though our model differs from both.

Regarding the location demand model, Kim (2000) conducts quality-improving technological innovation and considers the effect of compatibility on the incentives to innovate. Kim (2000, Theorem 5) shows that the effect of an increase in compatibility on the profit of the innovative firm is ambiguous, whereas the profit of the non-innovative rival firm is increased. In this case, the assumption is made that the innovative firm corresponds to a high-quality firm, whereas the non-innovative firm is a low-quality firm. This is because an increase in compatibility raises the prices of the innovative firm, leading it to lose the market share, which implies that the effect of compatibility on innovation can be negative.

In other work, Sääskilahti (2006) considers cost-reducing innovation and shows that network compatibility is neutralized in the decision regarding cost-reducing investment given symmetric qualities between firms. However, Sääskilahti (2006, Proposition 3) demonstrates that in the case of asymmetric qualities, the effect of an increase in compatibility on the investment of the high (low) quality firm is negative (positive).

In this paper, using the framework of a Hotelling linear market model, we consider the impact of network connectivity between network goods on strategic R&D activities (i.e., quality-improving and/or cost-reducing investments) in a digital market.

When considering the effect of market pressure on the innovation incentives, we exploit network connectivity as the measure of market competitiveness and focus on two cases of market structure. The first case is full market coverage, as assumed in the existing models, in which all heterogeneous consumers purchase either one of two network goods, that is, the case of a fully covered (or mature) market. However, it is natural to assume that there are enough potential consumers (users) in digital markets and therefore that the size of these markets still has room to expand. Thus, as the second case, we conduct the analysis of a partially covered and uncovered (or immature) market and obtain the results like those in the linear demand model (e.g., Shrivastav, 2021) because we allow for market expansion effects.

For example, as discussed in Economides and White (1994), if an improvement in network connectivity and horizontal interoperability in digital markets may be anti-competitive, we must examine how network connectivity changes the market demand function through strategic R&D competition. Vives (2008), for instance, examines the relationship between strategic commitment effects and the Lerner index (e.g., the absolute value of the elasticity of demand) in the case of cost-reducing R&D competition. To consider the implications of network connectivity for the incentives to invest in R&D activity, whether anti-competitive or not, we examine how the differences in market structure affects the Lerner index and market demand.

The rest of the paper is organized as follows. In Section 2, using the framework of a Hotelling linear market, we derive the equilibrium in R&D activity competition in the case of full market coverage, and consider how network connectivity affects R&D activity and profits. We also investigate the effect of network connectivity on the Lerner index in the case of cost-reducing R&D activity. In Section 3, by relaxing the assumption of market coverage, we assess the impact of network connectivity on R&D activity and profits in the case of a partial market coverage and examine the effect of network connectivity on the Lerner index. In Section 4, we develop quantity competition in the case of a partial market coverage and investigate the same issues as in the previous sections, demonstrating that the main results do not depend on the mode of competition. Finally, in Section 5, we summarize our findings and discuss some remaining problems.

2. A Mature Network Product Market: The Full Market Coverage Case

2.1 Setup

As a benchmark, we consider a mature market where firms providing network products and services compete on prices and innovative activities (e.g., product and/or process R&D investments) and all the consumers in the market purchase either of two products.[3] Let us refer to this network product market as a “battlefield”, which corresponds to the

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[3] From the perspective of the economics of platforms, we can interpret consumers in the model as single-homing users in the digital market.

full market coverage case assumed in the extant literature[4].

Following Kim (2000), Foros and Hansen (2001), and Sääskilahti (2006), we introduce network effects associated with connectivity (compatibility and horizontal interoperability) into a standard Hotelling linear market model. *Firm i*, $i, j = 0,1,$ which is located at both ends of the unit-linear market, provides network goods, *Product i*. Consumers are indexed by l and uniformly distributed with density equal to one on the interval $\left[0,1\right]$. That is, the total size of the market (number of consumers) is limited to unity. We assume that consumer $l \in \left[0,1\right]$ has the following surplus (net utility) function:

$$U = \begin{cases} v_0 - tl - p_0 + N_0 \quad if\ buying \quad product \quad 0 \\ \\ v_1 - t\left(1-l\right) - p_1 + N_1\ if\ buying \quad product \quad 1 \end{cases}, \tag{1}$$

where v_i is the intrinsic value (quality level) of *product i* and t is a transportation cost, implying a product differentiation parameter, p_i, being the price of *product i*, N_i denotes network effects, which are explicitly specified below.

Based on equation (1), the consumer indexed l^*, whose surplus is indifferent between *products 0* and *1*, is given by $l^* = \frac{1}{2} + \frac{v_0 - v_1 - p_0 + p_1 + N_0 - N_1}{2t}$. Thus, the demand function of *product* (*firm*) *0* is expressed as:

$$x_0 = l^* = \frac{t + v_0 - v_1 - p_0 + p_1 + N_0 - N_1}{2t}, \tag{2}$$

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[4] Jahn and Prüfer (2008) refer to it as a “battle zone.”

where $N_i = n\left(x_i^e + \phi x_j^e\right)$, $i, j = 0,1$, $i \neq j$. Parameter $n\left(>0\right)$ expresses network effects, $\phi \in \left[0,1\right]$ denotes the degree of connectivity, and x_i^e the expected network size of *product i*. Regarding the demand function of *product 1*, based on equation (2), we have $x_1 = 1 - x_0$. See *Figure 1*. The (gross) profit function of *firm i* is given by $\pi_i = \left(p_i - c_i\right)x_i$, where c_i is the marginal cost of production of *firm i*, $i = 0,1$.

[Insert Figure 1]

The structure of the game is in two stages. At the first stage, the firms decide the level of R&D activity (e.g., product and/or process R&D investments), and at the second stage, the firms compete on prices. We assume that consumers have rational (passive) expectations for the network sizes of the products and derive a subgame perfect Nash equilibrium by backward induction.

2.2 The impacts of network connectivity

In the second stage of price competition, the first-order condition (FOC) of profit maximization of *firm i* is given by $\dfrac{\partial \pi_i}{\partial p_i} = x_i - \dfrac{p_i - c_i}{2t} = 0$, $i = 0,1$. At the fulfilled expectation equilibrium, $x_i^e = x_i = \dfrac{p_i - c_i}{2t}$, we derive the following outcomes.

$$p_i - c_i = \frac{\left\{3t - n(1-\phi) + a_i - a_j\right\}t}{3t - n(1-\phi)}, \tag{3}$$

$$x_i = \frac{p_i - c_i}{2t} = \frac{3t - n(1-\phi) + a_i - a_j}{2\{3t - n(1-\phi)\}}, \quad i, j = 0,1, \quad i \neq j, \tag{4}$$

where $a_i \equiv v_i - c_i (> 0), i = 0,1.$ Hereinafter, a_i denotes the R&D activities of *firm i*. That is, in assuming that a_i is a variable expressing product (quality-improving) and/or process (cost-reducing) innovation, it holds that $da_i > 0 \Leftrightarrow dv_i - dc_i > 0,$ where $dv_i > 0$ (and/or $-dc_i > 0$) implies an increase in the level of quality-improving (and/or cost-reducing) R&D activities.

In the first stage of competition for R&D activities, we assume the following R&D activities (investments) cost function, $F(a_i) = \frac{k}{2}(a_i)^2, \quad k > 0.$ The net profit function of *firm i* is expressed as:

$$\Pi_i = (p_i - c_i)x_i - F(a_i) = \frac{(p_i - c_i)^2}{2t} - \frac{k}{2}(a_i)^2, \quad i = 0,1.$$

The FOC is given by:

$$\frac{\partial \Pi_i}{\partial a_i} = \frac{p_i - c_i}{3t - n(1-\phi)} - ka_i = \frac{\{3t - n(1-\phi) + a_i - a_j\}t}{\{3t - n(1-\phi)\}^2} - ka_i = 0. \tag{5}$$

Additionally, we derive the following second-order condition (SOC) and cross effect:

$$\frac{\partial^2 \Pi_i}{\partial a_i^2} = \frac{t}{\{3t - n(1-\phi)\}^2} - k < 0 \text{ and } \frac{\partial^2 \Pi_i}{\partial a_i \partial a_j} = \frac{-t}{\{3t - n(1-\phi)\}^2} < 0.$$

Because the cross effect is negative, the relationship between each firms' R&D activities is as strategic substitutes.

Based on equation (5), it holds at the equilibrium that

$$a_0 = a_1 = \frac{t}{k\left\{3t - n(1-\phi)\right\}} \equiv a^*. \tag{6}$$

In view of equation (6), the effect of an increase in connectivity on the R&D activity is given by $\frac{da^*}{d\phi} < 0$. Thus, we summarize the result as follows.

Proposition 1.

An increase in connectivity reduces R&D activities in the mature market.

Equation (6) implies that the R&D activity function of connectivity is monotonically decreasing. Using equation (5), the effect of an increase in connectivity on the marginal net profit is given by $\frac{\partial^2 \Pi_i}{\partial a_i \partial \phi} = -\frac{n\left\{3t - n(1-\phi) + 2(a_i - a_j)\right\}t}{\left\{3t - n(1-\phi)\right\}^3}, \quad i, j = 0,1, \quad i \neq j.$

At the symmetric equilibrium, it holds that $\left.\frac{\partial^2 \Pi_i}{\partial a_i \partial \phi}\right|_{a_i = a_j = a^*} = -\frac{nt}{\left\{3t - n(1-\phi)\right\}^2} < 0.$ That is, because an increase in connectivity reduces the marginal net profit, it decreases the incentive to R&D activity.

The total effect of an increase in connectivity on the net profit of *firm i* is then expressed as:

$$\frac{d\Pi_i^*}{d\phi} = \frac{\partial \Pi_i}{\partial a_i}\frac{da_i}{d\phi} + \frac{\partial \Pi_i}{\partial a_j}\frac{da_j}{d\phi} + \frac{\partial \Pi_i}{\partial \phi} = \frac{\partial \Pi_i}{\partial a_j}\frac{da_j}{d\phi} + \frac{\partial \Pi_i}{\partial \phi}, \quad i, j = 0,1, \quad i \neq j,$$

where $\frac{\partial \Pi_i}{\partial a_i} = 0$ is by the FOC. Using equation (3), the direct effect of connectivity on net profit is $\frac{\partial \Pi_i}{\partial \phi} = \frac{p_i - c_i}{t} \frac{-n\left(a_i - a_j\right)t}{\left\{3t - n(1-\phi)\right\}^2}$.[5] The direct effect of the rival firm's R&D activity on the net profit is $\frac{\partial \Pi_i}{\partial a_j} = -\frac{p_i - c_i}{3t - n(1-\phi)} < 0$. Thus, at the symmetric equilibrium, $a_i = a_j = a^*$, the total effect of an increase in connectivity on net profit is given by:

$$\left.\frac{d\Pi_i^*}{d\phi}\right|_{a_i = a_j} = \underset{-}{\frac{\partial \Pi_i}{\partial a_j}} \underset{-}{\frac{da_j}{d\phi}} > 0, \text{ where } \left.\frac{\partial \Pi_i}{\partial \phi}\right|_{a_i = a_j} = 0. \tag{7}$$

Given equation (7), the direct effect of connectivity on net profit is cancelled out at the symmetric equilibrium. However, while an increase in connectivity reduces the rival firm's R&D activity, the reduction leads to an increase in net profit because there is strategic substitution between the firms' R&D competition. Thus, we summarize the result as follows.

Proposition 2.

An increase in connectivity increases net profit in the mature market.

Propositions 1 and 2 imply that while the perfect network connectivity case is

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[5] Based on equation (4), we have $a_i > (<) a_j \Leftrightarrow x_i > (<) x_j$. That is, the higher the level of R&D activity, the greater the market share. Assuming that *firm i* is a large company, an increase in connectivity reduces market share, i.e., $\frac{dx_i}{d\phi} = \frac{-n\left(a_i - a_j\right)t}{\left\{3t - n(1-\phi)\right\}^2} < 0$, and thus, the net profit of the large company. Conversely, for a small company.

preferable for the firms, the level of R&D activities is at its lowest, i.e., $\Pi_i^*(\phi=0)<\Pi_i^*(\phi=1)$ and $a^*(\phi=0)>a^*(\phi=1)$. In addition, as in equation (6), the network effects do not affect the R&D activities in the mature network product market. If standardization and open network policies weaken a firm's R&D activities, they are desirable for the firms. Although a policy analysis is beyond the scope of this paper, we need to discuss the role of standardization and open network policies in mature digital markets.

Remark 1. Network connectivity, the Lerner index, and strategic commitment effect

Focusing on the strategic commitment effects; as in Proposition 1, an increase in connectivity reduces R&D activities. Drawing on Vives (2008), we investigate the relationship between the Lerner index (i.e., the inverse of the absolute value of the elasticity of demand) and network connectivity. We deal with the cost-reducing R&D activities and assume $a_i = v - c_i,\ \ i = 0,1.$

We define the Lerner index as follows: $L_i = \dfrac{p_i - c_i}{p_i} = \dfrac{1}{\eta_i},\ \ i = 0,1,$ where η_i denotes the (absolute value of) elasticity of demand (hereinafter, just elasticity).

Using equations (3) and (6), the R&D activity in the symmetric subgame perfect Nash equilibrium is expressed as $a^*(\phi) = v - c^*(\phi)$. Thus, it holds that $\dfrac{da^*}{d\phi} = -\dfrac{dc^*}{d\phi}$. The elasticity is given by $\eta_i = \eta^* = 1 + \dfrac{c^*(\phi)}{t}$. Based on Proposition 1, it holds that

$\frac{d\eta^*}{d\phi} = \frac{-1}{t}\left(-\frac{dc^*}{d\phi}\right) > 0$. As network connectivity increases, the elasticity increases, and thus, the Lerner index decreases. The demand curve becomes more elastic. This implies that increasing network connectivity weakens the power of price control of firms.

Therefore, when the degree of network connectivity is large, firms seek to obtain higher profits by refraining from R&D activities. Conversely, in the case of non-connectivity between network goods (e.g., the firm-specific network system), firms can control their prices. Accordingly, an increase in connectivity in a fully covered market is not necessarily anti-competitive.

3. An Uncovered Network Product Market and Market Expansion Effect

3.1 Equilibrium in the case of an uncovered market with 'hinterland'

By relaxing the assumption of market coverage, we consider a partially covered and uncovered (immature) market where there exists some potential consumers purchasing the network products.[6] In particular, consumers indexed by l are uniformly distributed with density equal to one on the open interval $l \in (-\infty, \infty)$. By a similar way to the mature market case, we assume that the two firms are located at *0* and *1*. In this case, there are three types of consumers, as follows (see *Figure 2*).

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[6] We extend the location model in the previous section, following Jahn and Prüfer (2008), and Tolotti and Yepez (2020). Dyskeland and Foros (2023) point out that allowing market expansion by relaxing the assumption provides results in line with those from the (homogeneous) representative consumer model (i.e., the linear demand model).

[Insert Figure 2]

Type (i): A consumer $l_{0\square 1} \in [0,1]$ has a net utility function given by equation (1).

Type (ii): A consumer $l_{-\infty\square 0} \in (-\infty, 0)$ has the following net utility function:

$$U_{-\infty\square 0} = \begin{cases} v_0 - tl_{-\infty\square 0} - p_0 + N_0 \, if \; buying \quad product \quad 0 \\ \\ 0 \qquad\qquad\qquad if \; not \quad buying \end{cases} . \tag{8.1}$$

Type (iii): A consumer $l_{1\square +\infty} \in (1, \infty)$ has the following net utility function:

$$U_{1\square +\infty} = \begin{cases} v_1 - t(1 + l_{1\square +\infty}) - p_1 + N_1 \, if \; buying \quad product \quad 1 \\ \\ 0 \qquad\qquad\qquad if \; not \quad buying \end{cases} \tag{8.2}$$

The demand function of consumers in the ‘battlefield’ of Type (i) is the same as equation (2). The markets, in which consumers of Types (ii) and (iii) exist, correspond to the ‘hinterlands’ of *products 0* and *1*, where some consumers do not initially purchase the network products. Using equation (8.1), the marginal consumer purchasing *product 0* is given by $l^{**}_{-\infty\square 0} = \dfrac{v_0 - p_0 + N_0}{t}$. Thus, we derive the following demand function for consumers of Type (ii).

$$y_0 = \frac{v_0 - p_0 + N_0}{t}. \tag{9.1}$$

Regarding the demand function of consumers of Type (iii), using the same way as for Type (ii), we have

$$y_1 = \frac{v_1 - p_1 + N_1}{t}. \tag{9.2}$$

Therefore, based on equations (2) and (9.1) (or (9.2)), we obtain the following demand function for *firm (product) i*:

$$q_i \equiv x_i + y_i = \frac{t + 3v_i - v_j - 3p_i + p_j + 3N_i - N_j}{2t}, \quad i, j = 0,1, \quad i \neq j, \tag{10}$$

where $N_i = n\left(q_i^e + \phi q_j^e\right)$ and $q_i^e \equiv x_i^e + y_i^e$.

In the second stage of price competition, based on equation (10), we have the following FOC: $\dfrac{\partial \pi_i}{\partial p_i} = q_i - \dfrac{3}{2t}\left(p_i - c_i\right) = 0$. At the fulfilled expectations equilibrium, $q_i^e = q_i = \dfrac{3}{2t}\left(p_i - c_i\right)$, it holds that

$$p_i - c_i = \frac{\left[\{7t - 6n(1-\phi)\}t + (17t - 12n)a_i - 3(t - 4n\phi)a_j\right]t}{D} \tag{11}$$

and

$$q_i = \frac{3\left[\{7t - 6n(1-\phi)\}t + (17t - 12n)a_i - 3(t - 4n\phi)a_j\right]}{2D}, \tag{12}$$

where $D \equiv \{7t - 6n(1-\phi)\}\{5t - 3n(1+\phi)\} > 0$. Hereinafter, we assume the following upper level of network externality: $\dfrac{5t}{6} > n.$

In the first stage, where the firms compete on R&D activities by incurring R&D investment costs, the firms simultaneously decide the level of R&D activities to maximize the net profit: $\Pi_i = \left(p_i - c_i\right)q_i - F(a_i) = \dfrac{3}{2t}\left(p_i - c_i\right)^2 - \dfrac{k}{2}(a_i)^2$. The following

FOC and SOC are respectively given by

$$\frac{\partial \Pi_i}{\partial a_i} = \frac{3t\left(17t - 12n\right)}{D}\left(p_i - c_i\right) - ka_i = 0 \tag{13}$$

and

$$\frac{\partial^2 \Pi_i}{\partial a_i^{\ 2}} = 3\left\{\frac{\left(17t - 12n\right)t}{D}\right\}^2 - k < 0. \tag{14}$$

The effect of an increase in the rival firm's R&D activity on net profit and the cross effect are respectively given by

$$\frac{\partial \Pi_i}{\partial a_j} = -\frac{3t\left(t - 4n\phi\right)}{D}\left(p_i - c_i\right) > (<) 0 \Leftrightarrow \phi > (<) \Phi \tag{15}$$

and

$$\frac{\partial^2 \Pi_i}{\partial a_i \partial a_j} = -\frac{9t\left(17t - 12n\right)\left(t - 4n\phi\right)}{D^2} > (<) 0 \Leftrightarrow \phi > (<) \Phi, \tag{16}$$

where $\Phi \equiv \frac{t}{4n}\left(< \frac{5t}{6n}\right)$. In view of equations (15) and (16), we summarize the results as Lemma 1.

Lemma 1.

(i) When $\Phi < 1,$ *if* $\phi > (<) \Phi,$ *the relationship between the firms' R&D activities is as strategic complements (substitutes). In this case, an increase in the rival firm's R&D activity increases (decreases) the net profit.*

(ii) When $\Phi > 1,$ *the relationship between the firms' R&D activities is as strategic substitutes. In this case, an increase in the rival firm's R&D activity decreases the net*

profit.

Regarding equations (15) and (16), we have the following relationship: $\phi > (<) \Phi \Leftrightarrow n\phi > (<) \frac{t}{4}$. That is, $n\phi(<1)$ denotes network connectivity and $\frac{t}{4}$ denotes product substitutability. In other words, as discussed in Section 1, the former implies (product) complementarity. Thus, if the former is larger (smaller) than the latter, we understand that the network products are complements (substitutes). In particular, as in Lemma 1 (i), if network connectivity is larger than product substitutability, i.e., $n\phi > \frac{t}{4}$, the relationship between the R&D activities is strategic complements and the effect of the rival firm's R&D activity on the net profit is positive. Otherwise, the opposite results hold. However, as in Lemma 1 (ii), when the network externality is sufficiently small, i.e., $\frac{t}{4} > n(\geq n\phi)$, the network products are substitutes. Thus, the firms' R&D activities are strategic substitutes and the effect of the rival firm's R&D activity on net profit is negative.

3.2 The impact of network connectivity on R&D activities

Using equations (11) and (13), we obtain the following R&D activity in equilibrium.

$$a_0 = a_1 = \frac{3t^2(17t-12n)\{7t-6n(1-\phi)\}}{kD^2 - 6t(17t-12n)\{7t-6n(1-\phi)\}} \equiv a^{**}. \tag{17}$$

Give equation (17), the effect of an increase in connectivity on the R&D activity is given by:

$$\frac{da^{**}}{d\phi} > (<) 0 \Leftrightarrow \phi > (<) \frac{1}{3} - \frac{2t}{9n} \equiv \phi^{**}, \tag{18}$$

where $\min\{\Phi, 1\} > \phi^{**}$. Furthermore, it holds that $\phi^{**} > (<) 0 \Leftrightarrow n > (<) \frac{2t}{3}$. Given the assumption, we have the following two cases: (i) if $\frac{5t}{6} > n > \frac{2t}{3}$, then $\phi^{**} > 0$, and (ii) if $\frac{2t}{3} > n$, then $\phi^{**} < 0$.

Using equation (18), and based on the above two cases, we derive the following Proposition 3.

Proposition 3.

(i) If $\frac{5t}{6} > n > \frac{2t}{3}$, *then the following relationship holds:*

$$\frac{da^{**}}{d\phi} > (<) 0 \Leftrightarrow \phi > (<) \phi^{**}.$$

(ii) If $\frac{2t}{3} > n$, *then it holds that* $\frac{da^{**}}{d\phi} > 0$.

First, regarding Proposition 3 (i), it holds that $1 > \Phi > \phi^{**} > 0$ (see *Figure 3A*).[7] If $\Phi > (<) \phi$, the relationship between the firms' R&D activities is as strategic substitutes

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[7] Based on $D > 0$ and the assumption, we have the following lower and upper bounds of connectivity: $\underline{\phi} \equiv 1 - \frac{7t}{6n} < \min\{0, \phi^{**}\}$ and $\overline{\phi} \equiv \frac{5t}{3n} - 1 > \max\{1, \Phi\}$.

(complements). Given $\Phi > \phi \geq 0,$ it holds that $\phi^{**} > (<)\phi,$ and the R&D activities first decrease, then increase. That is, the R&D activity function of connectivity is a *U*-shaped curve. This result like that of Shrivastav (2021, p. 159). Using equation (13), we derive the following relationship: $\left.\frac{\partial^2 \Pi_i}{\partial a_i \partial \phi}\right|_{a_i = a_j = a^{**}} > (<)0 \Leftrightarrow n\phi > (<)n\phi^{**}.$ That is, if the degree of network connectivity is larger (smaller) than the threshold, $n\phi^{**},$ then an increase in connectivity increases (decreases) the marginal net profit with respect to the R&D activities. This result differs from that in the full market coverage case, in which the effect on the marginal net profit is always negative. This is because market expansion effects through an increase in connectivity positively affect the marginal net profit. If $\Phi > \phi > \phi^{**},$ even though the relationship between the R&D activities is as strategic substitutes, an increase in connectivity improves the R&D activities via the market expansion effects

If $1 \geq \phi > \Phi,$ the relationship between the R&D activities is as strategic complements. Thus, an increase in connectivity increases the R&D activities.

Second, regarding Proposition 3 (ii), where $\phi \geq 0 > \phi^{**},$ an increase in connectivity always increases the R&D activities, irrespective of the strategic relationships (see *Figures 3B* and *3C*). In other words, the R&D activity function of connectivity is monotonically increasing.

[Insert Figures 3A, 3B and 3C]

As shown in Proposition 1, the R&D activity is a monotonically decreasing function of connectivity in the mature market case. However, in the immature market, where market expansion effects are allowed, the R&D activity function of connectivity is either a *U*-shaped curve or a monotonically increasing curve. The results then differ from those in the mature market case. This is because of the market expansion effects through an increase in connectivity.

In addition, regarding the level of the R&D activities, we obtain $a^{**}(\phi=1) > a^{**}(\phi=0)$. That is, the level of the R&D activities under an open network system (i.e., horizontal interoperability between the network products) is higher than that under a firm-specific network system.

Therefore, if the digital market is not fully covered (immature), there is potential demand (customers) in the market, such that an increase in connectivity and interoperability promotes the incentives to innovate R&D activities. The results imply that standardization and interoperability policies in digital markets are preferable from the perspective of R&D promotion.

3.3 The impact of network connectivity on net profit

By a similar way as for the mature market, we investigate how an increase in connectivity affects the net profit. We can express the net profit function of connectivity as follows: $\Pi_i^{**} = \Pi_i\left[a_i[\phi], a_j[\phi], \phi\right]$. Thus, the impact of network connectivity on the net profit is given by:

$$\left.\frac{d\Pi_i^{**}}{d\phi}\right|_{a_i=a_j=a^{**}} = \frac{\partial\Pi_i}{\partial a_i}\frac{da_i}{d\phi} + \frac{\partial\Pi_i}{\partial a_j}\frac{da_j}{d\phi} + \frac{\partial\Pi_i}{\partial\phi} = \frac{\partial\Pi_i}{\partial a_j}\frac{da_j}{d\phi} + \frac{\partial\Pi_i}{\partial\phi}, \qquad (19)$$

where $\frac{\partial \Pi_i}{\partial a_i} = 0$ is from the FOC. Given the second equation (19), the first term denotes the indirect (strategic) effect through network connectivity and the second term denotes the direct effect. We summarize the properties of the two terms at the symmetric equilibrium as the following Lemma 2.

Lemma 2

(i) Regarding the direct effect on the net profit, it holds that

$$\left.\frac{\partial \Pi_i}{\partial \phi}\right|_{a_i=a_j=a^{**}} = \frac{9\left(p_i - c_i\right)n\left[7t - 6n(1-\phi)\right]^2 (t + 2a^{**})}{D^2} > 0.$$

(ii) Regarding the indirect effect on the net profit, it holds that

$$\underset{\operatorname{sgn} A}{\frac{\partial \Pi_i}{\partial a_j}} \underset{\operatorname{sgn} B}{\frac{da_j}{d\phi}} > (<) 0 \Leftrightarrow \left(4n\phi - t\right)\left[2t - 3n(1-3\phi)\right] > (<) 0.$$

We first address Lemma 2 (i). In the mature market, the direct effect cancels out because the level of the R&D activities is equal at the symmetric equilibrium. However, in the immature market, the direct effect is positive because of market expansion effects. Regarding Lemma 2 (ii), we derive the following relationships:

$$\operatorname{sgn} A \Leftrightarrow \frac{\partial \Pi_i}{\partial a_j} > (<) 0 \Leftrightarrow \phi > (<) \Phi,$$

$$\operatorname{sgn} B \Leftrightarrow \left.\frac{da_j}{d\phi}\right|_{a_i=a_j=a^{**}} > (<) 0 \Leftrightarrow \phi > (<) \phi^{**}.$$

Thus, we obtain $\operatorname{sgn}\left\{\frac{\partial\Pi_i}{\partial a_j}\frac{da_j}{d\phi}\right\} = \operatorname{sgn}\left\{(\phi-\Phi)\left(\phi-\phi^{**}\right)\right\}$, where $\Phi > \phi^{**}$. In particular, if $\min\{\Phi,1\} > \phi > \max\left\{\phi^{**},0\right\}$, the indirect effect is negative, i.e., $\operatorname{sgn}\left\{\frac{\partial\Pi_i}{\partial a_j}\frac{da_j}{d\phi}\right\} < 0.$ Otherwise, the indirect effect is positive. Thus, we must only examine the negative indirect case because the direct effect is always positive. Therefore, with respect to the impact of network connectivity on the net profit, we derive the following Proposition 4.

Proposition 4.

An increase in the degree of network connectivity increases the net profit in the immature market.

Proposition 4 is formally the same as Proposition 2 in the case of the mature market. However, Proposition 4 is more general because it includes the result of Proposition 2. Considering strategic relationships between the R&D activities, we consider the economic implications of Proposition 4. In the mature market, there is one case, in which the R&D activities are strategic substitutes and the effect of an increase in connectivity on the R&D activities is negative, i.e., $\frac{\partial\Pi_i}{\partial a_j} < 0$ and $\left.\frac{da_j}{d\phi}\right|_{a_i=a_j=a^*} < 0.$ As a result, because the indirect effect is positive, i.e., $\frac{\partial\Pi_i}{\partial a_j}\frac{da_j}{d\phi} > 0,$ the impact of network connectivity on the net profit is positive, given that the direct effect is zero at the symmetric equilibrium.

In the immature market, there are the following three cases. The first is like that in the mature market, although the indirect effect is positive. However, as mentioned earlier, in the second case, in which the indirect effect is negative but the direct effect is positive, the R&D activities are strategic substitutes and an increase in connectivity improves the R&D activities, i.e., $\frac{\partial \Pi_i}{\partial a_j} < 0$ and $\left.\frac{da_j}{d\phi}\right|_{a_i = a_j = a^{**}} > 0$. Thus, the indirect effect is negative, i.e., $\frac{\partial \Pi_i}{\partial a_j}\frac{da_j}{d\phi} < 0$. However, because the positive direct effect on the net profit by market expansion effects over weights the negative indirect effect, the impact on the net profit is positive (see Appendix 1). The third case is that the R&D activities are strategic complements and an increase in connectivity improves the R&D activities, i.e., $\frac{\partial \Pi_i}{\partial a_j} > 0$ and $\left.\frac{da_j}{d\phi}\right|_{a_i = a_j = a^{**}} > 0$. Because the direct and indirect effects are positive, the impact of network connectivity on the net profit is positive.

Remark 2. Network connectivity, the Lerner index, and strategic commitment effects

In the same manner as Remark 1, we consider the role of connectivity for the R&D activities with market competition. Using equations (11) and (17), in the symmetric subgame perfect Nash equilibrium, the Lerner index and the absolute value of the elasticity of demand are expressed as: $L^{**} = \frac{1}{\eta^{**}}$ and

$\eta^{**} = 1 + \left\{ \frac{c^{**}(\phi)}{t} \right\} \left\{ \frac{5t - 3n(1+\phi)}{t + 2a^{**}(\phi)} \right\}$, where $a^{**}(\phi) = v - c^{**}(\phi)$. Thus, we derive

$\frac{d\eta^{**}}{d\phi} = \frac{G}{t\left(t+2a^{**}\right)^2}$, where

$$G \equiv \left\{ \left(\frac{dc^{**}}{d\phi} \right) \left[5t - 3n(1+\phi) \right] - 3nc^{**} \right\} (t + 2a^{**}) - 2c^{**} \left[5t - 3n(1+\phi) \right] \left(\frac{da^{**}}{d\phi} \right)$$
$$= -\left\{ \left(-\frac{dc^{**}}{d\phi} \right) \left[5t - 3n(1+\phi) \right] + 3nc^{**} \right\} (t + 2a^{**}) - 2c^{**} \left[5t - 3n(1+\phi) \right] \left(-\frac{dc^{**}}{d\phi} \right).$$

Based on Proposition 3, if $\phi > \max\left\{0, \phi^{**}\right\}$, that is, $n\phi > \frac{3n-2t}{9n}$, it holds that $-\frac{dc^{**}}{d\phi} > 0$. Given significantly strong network connectivity, an increase in connectivity facilitates the cost-reducing R&D activities. Because it holds that $G < 0$, we obtain $\frac{d\eta^{**}}{d\phi} < 0$, and thus, $\frac{dL^{**}}{d\phi} > 0$.[8] An increase in connectivity decreases the elasticity. In the case of an immature market, potential consumers in the 'hinterland' (i.e., previously non-purchasers) are encouraged to become customers of the firm because the other firm's product is also available via an increase in network connectivity. In other words, if the network connectivity is large, the firms can control their prices in the market, and thus

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[8] If $\phi^{**} > \phi \geq 0$, it holds that $-\frac{dc^{**}}{d\phi} < 0$. In this case, we have as follows:

$$G = -\left\{ \left(-\frac{dc^{**}}{d\phi} \right) \left[5t - 3n(1+\phi) \right] \right\} (t + 2v) - 3nc^{**}(t + 2a^{**}).$$

Because the first term is positive, but the second is negative, the sign of G is ambiguous.

have greater incentives to promote the R&D activities. This result differs from that of the mature market case (see Remark 1). For example, suppose perfectly horizontal interoperability between the network products. The firms can then obtain more profits by raising prices and reducing marginal costs. If so, standardization policy can induce R&D investments competition, although it may also increase the burden on consumers.

In view of Remarks 1 and 2, we can interpret the implications of network connectivity in the indicator of market competition as follows. In the case of a fully covered market, an increase in network connectivity reduces the Lerner index, that is, the firm's power of price control, and thus weakens the incentives to innovate. In this case, the degree of connectivity expresses an indicator of competitive pressure. However, in the case of an imperfectly covered and uncovered market, an increase in network connectivity increases the number of customers who choose only one specific firm's network product, because the other network product from the competing firm is also available. This implies that an increase in connectivity increases the Lerner index, and thus heightens the incentives to innovate. In this case, the degree of connectivity is an opposing indicator of competitive pressure.

4. Quantity Competition Case

In this section, we consider the case of quantity competition in an immature (partially covered and uncovered) market and demonstrate the same results as in the case of price competition. This implies that the impact on network connectivity on R&D activity and net profits do not depend on the mode of competition.

4.1 The impact of network connectivity on the R&D activities

Using the direct demand function, equation (10), we derive the following indirect demand function:

$$p_i = \frac{2t + 4v_i - 3tq_i - tq_j + 4N_i}{4}, \quad i, j = 0,1, \quad i \neq j. \tag{20}$$

First, regarding quantity competition in the second stage, the FOC of *firm i* is given by $\frac{\partial \pi_i}{\partial q_i} = p_i - c_i - \frac{3t}{4} q_i = 0.$ At the fulfilled expectations, we have

$$2t + 4a_i - 2\left(3t - 2n\right)q_i - \left(t - 4n\phi\right)q_j = 0, \quad i, j = 0,1, \quad i \neq j.$$

By a similar way to the previous section for *firm j*, we obtain the following output at the equilibrium.

$$q_i^C = \frac{\left\{5t - 4n(1-\phi)\right\}t + 4(3t - 2n)a_i - 2(t - 4n\phi)a_j}{D^C}, \tag{21}$$

where $D^C \equiv \left\{7t - 4n(1+\phi)\right\}\left\{5t - 4n(1-\phi)\right\} > 0$ and superscript C denotes quantity competition. Hereinafter, we assume $\frac{7t}{8} > n.$

Second, regarding R&D competition in the first stage, using the FOC derived above, the net profit function of *firm i* is expressed as:

$$\Pi_i^C = \left(p_i - c_i\right)q_i^C - F(a_i) = \frac{3t}{4}\left(q_i^C\right)^2 - \frac{k}{2}\left(a_i\right)^2.$$

The FOC and SOC are respectively given by:

$$\frac{\partial \Pi_i^C}{\partial a_i} = \frac{6t\left(3t - 2n\right)}{D^C} q_i^C - ka_i = 0 \tag{22}$$

and

$$\begin{aligned}&\frac{\partial^2 \Pi_i^C}{\partial a_i^2} = \frac{3t}{2}\left(\frac{4(3t-2n)}{D^C}\right)^2 - k < 0 \\ &\Leftrightarrow k\left(D^C\right)^2 - 24t(3t-2n)^2 > 0.\end{aligned} \tag{23}$$

Furthermore, we have the following cross effect:

$$\frac{\partial^2 \Pi_i^C}{\partial a_i \partial a_j} = -\frac{12t\left(3t-2n\right)\left(t-4n\phi\right)}{\left(D^C\right)^2} > (<) 0 \Leftrightarrow \frac{t}{4n} < (>) \phi. \tag{24}$$

In this case, with respect to the strategic relationships between the firms' R&D activity, we have as follows: $\frac{t}{4n} = \Phi \begin{cases} < \phi \Rightarrow SC \\ > \phi \Rightarrow SS \end{cases}$, where *SC* (*SS*) denotes strategic complements (substitutes). This result is the same as Lemma 1.

Using equations (21) and (22), we derive the following R&D activity at the equilibrium.

$$a_i^C = a_j^C = \frac{6t^2(3t-2n)\left\{5t-4n(1-\phi)\right\}}{k(D^C)^2 - 12t(3t-2n)\left\{5t-4n(1-\phi)\right\}} \equiv a^{C**}. \tag{25}$$

Given equation (25), the following relationship holds.

$$\frac{da^{C**}}{d\phi} > (<) 0 \Leftrightarrow \phi > (<) \frac{1}{3} - \frac{t}{4n} \equiv \phi^{C**}, \tag{26}$$

where $\phi^{C**} < \min\left\{1, \Phi\right\}$. Furthermore, it holds that $\phi^{C**} > (<) 0 \Leftrightarrow n > (<) \frac{3t}{4}$.

Therefore, in view of equation (26), we derive the same result as Proposition 3.

4.2 The impact of network connectivity on the net profit

The effect of the rival firm's R&D activity and the direct impact of network connectivity on the net profit at the symmetric equilibrium are respectively given by:

$$\frac{\partial \Pi_i^C}{\partial a_j} = -\frac{3t\left(t-4n\phi\right)}{D^C} q_i^C > (<) 0 \Leftrightarrow \Phi < (>) \phi \tag{27}$$

and

$$\left.\frac{\partial \Pi_i^C}{\partial \phi}\right|_{a_i=a_j=a_i^{C**}} = \frac{6tq_i^C n\left[5t-4n(1-\phi)\right]^2 (t+2a^{C**})}{\left(D^C\right)^2} > 0. \tag{28}$$

Because the direct effect is positive, as in equation (28), we should focus on the case of a negative indirect effect on the net profit, i.e., $\frac{\partial \Pi_i^C}{\partial a_j}\frac{da_j^C}{d\phi} < 0$.[9] In other words, we should consider the case that the degree of network connectivity is within the following range: $\min\{1, \Phi\} > \phi > \max\left\{0, \phi^{C**}\right\}$.[10] As shown in equations (A.2), (A.3.1), (A.3.2), and (A.5) in Appendix 1, because the conditions regarding parameter k hold in the case of quantity competition, that is, the positive direct effect is over the negative indirect effect under the condition for parameter k, we can prove that

$$\left.\frac{d\Pi_i^{C**}}{d\phi}\right|_{a_i=a_j=a^{C**}} = \frac{\partial \Pi_i^C}{\partial a_j}\frac{da_j^C}{d\phi} + \frac{\partial \Pi_i^C}{\partial \phi} > 0.$$

Therefore, we derive the same result as Proposition 4.

5. Concluding Remarks

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[9] Using equations (24), (26), and (27), we can easily derive the same results as Lemma 2 in the case of price competition.

[10] In the other ranges, the indirect effect is always positive.

Since at least the pioneering works of Schumpeter (1943) and Arrow (1962), research into the relationship between technological innovation and market competition has become one of the most important issues in economics, especially in today's digitalized world. The key research questions are whether competitive pressure promotes the incentives to innovate and how innovation outcomes affect firms, consumers, and the market.

Introducing network externalities into a standard Hotelling linear market model, we examine these questions in this paper. Employing the degree of network connectivity as an indicator of market competitiveness, we focus on two market structure cases, i.e., a fully covered (mature) market, and a partially covered and uncovered (immature) market. We consider the impact of network connectivity (compatibility and horizontal interoperability) between the network goods on the R&D activity of firms, i.e., quality-improving and/or cost-reducing R&D, and on profits. In addition, to examine the implications of network connectivity and market competition, we discuss the relationship between the Lerner index and strategic commitment effects through cost-reducing R&D activity

In the mature market, an increase in network connectivity weakens the incentives to innovate, and thus reduces R&D activity. However, it does increase net profits. This is because strategic substitutes hold in the competition for R&D activity, and the market expansion effects through network effects cancel out at the symmetric equilibrium. Furthermore, as network connectivity increases, the elasticity of demand (Lerner index) increases (decreases).

In the immature market, the effects of an increase in network connectivity on the incentives to innovate and on the level of R&D activity depends on the relationship between network connectivity and product substitutability (conversely, differentiation).

If the degree of network connectivity is larger than a certain value of product substitutability, strategic complementarity between the R&D activity of the firms holds. Otherwise, strategic substitutes hold. The impact on R&D activity is then either monotonically increasing or *U*-shaped (i.e., first decreasing and then increasing). Even as strategic substitutes, R&D activity can increase by market expansion effects through increasing network connectivity. These results differ from those in the mature market case.

Regarding the net profits effect, the same results as in the mature market case hold: however, their economic implications differ. We obtain the following two cases. In the first case where increasing network compatibility increases R&D activity under strategic substitution, the positive direct effect on profit by market expansion outweighs the negative indirect effect. As a result, an increase in connectivity increases net profit. In the second case where increasing network compatibility increases R&D activity under strategic complementarity and the direct effect is positive, the increase in connectivity increases net profit. Further, as network connectivity increases, the elasticity of demand (Lerner index) decreases (increases) given strong network externalities. This result is opposite to that in the mature market case.

While policy analysis is not within the scope of this paper, we do note some implications of our model. For example, whether (mandated) standardization policy is facilitative for R&D activity in digital markets may depend on the characteristics of market structure. That is, in the case of mature digital markets, policymakers had better accept and retain the environment where firms (providers) freely operate under their own network systems. Conversely, in markets with room for future growth, policymakers should consider implementing policies that promote connectivity (interoperability), as the participation of potential consumers in the market will lead to an expansion of the market, thereby encouraging firms to undertake R&D activity.

Although there are some remaining problems, especially from the perspective of the economics of platforms, we should consider the case of a two-(multi-)sided market.[11] In particular, we assume that the firm (provider) itself is a platform in the case of a one-sided market, in which there are two groups of consumers using network goods, *products 0* and *1*. Thus, an increase in network connectivity (horizontal interoperability) between the firms (i.e., network goods and services) implies an increase in the direct "cross-consumers network effects." In other words, the network connectivity in this paper is the form of interconnection between the two groups of consumers. We examine how the change of such network connectivity affects the firm's incentive to innovate.

However, in the case of a two-sided market, how does an increase in connectivity (compatibility) between two different platforms (e.g., A and B operating systems) affect the behavior of application suppliers and consumers? Alternatively, if network connectivity between various platforms becomes perfectly common, and consequently one huge platform can be built, does the formation of this platform improve social and consumer welfare? Conversely, is it desirable for customers that various (specific) platforms compete in digital markets? These questions are critical issues confronting our future research in this area.

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[11] In addition, we need to extend our model to the following: endogenous choice of connectivity, endogenous choice of location, vertical product (quality) differentiation, and asymmetric (one-way) connectivity.

Appendix 1.

Regarding the effects of an increase in connectivity on the R&D activities, using equation (13), we derive the following Hessian matrix.

$$\begin{pmatrix} \dfrac{\partial^2 \Pi_i}{\partial a_i{}^2} & \dfrac{\partial^2 \Pi_i}{\partial a_i \partial a_j} \\ \dfrac{\partial^2 \Pi_i}{\partial a_j \partial a_i} & \dfrac{\partial^2 \Pi_j}{\partial a_j{}^2} \end{pmatrix} \begin{pmatrix} da_i \\ da_j \end{pmatrix} = \begin{pmatrix} -\dfrac{\partial^2 \Pi_i}{\partial a_i \partial \phi} \\ -\dfrac{\partial^2 \Pi_i}{\partial a_j \partial \phi} \end{pmatrix} d\phi. \qquad \text{(A.1)}$$

Thus, the determinant Δ of the Hessian Matrix is given as:

$$\Delta \equiv \left\{ k - \frac{6t(17t-12n)\left[7t-6n(1-\phi)\right]}{D^2} \right\} \left\{ k - \frac{6t(17t-12n)\left[10t-6n(1+\phi)\right]}{D^2} \right\} > 0.$$

With respect to parameter $k,$ we assume as follows.

$$k > \max\left\{ \frac{6t(17t-12n)\left[7t-6n(1-\phi)\right]}{D^2}, \frac{6t(17t-12n)\left[10t-6n(1+\phi)\right]}{D^2} \right\}. \qquad \text{(A.2)}$$

Furthermore, we derive the following relationship:

$$\Phi > (<)\phi \Leftrightarrow 10t - 6n(1+\phi) > (<) 7t - 6n(1-\phi).$$

Thus, based on the assumption above, we have as follows.

(i) If $\Phi > \phi, \;\; k > \dfrac{6t(17t-12n)\left[10t-6n(1+\phi)\right]}{D^2} \equiv k^{\#}$ (A.3.1)

(ii) If $\Phi < \phi, \quad k > \dfrac{6t(17t-12n)\left[7t-6n(1-\phi)\right]}{D^2} \equiv k^b$ (A.3.2)

Unless $\min\{\Phi,1\} > \phi > \max\{\phi^{**},0\},$ the indirect effect is positive. Because the direct effect is always positive, we need to examine the case that $\min\{\Phi,1\} > \phi > \max\{\phi^{**},0\},$ in which the indirect effect is negative. In this case, we derive as follows.

$$\left.\frac{d\Pi_i^{**}}{d\phi}\right|_{a_i=a_j=a^{**}} = \frac{\partial \Pi_i}{\partial a_j}\frac{da_j}{d\phi} + \frac{\partial \Pi_i}{\partial \phi} > (<) 0 \Leftrightarrow k > (<) \tilde{k}, \tag{A.4}$$

where $\tilde{k} \equiv \dfrac{6t(17t-12n)}{D^2}\left\{7t-6n(1-\phi)+\dfrac{3(t-4n\phi)\left[2t-3n(1-3\phi)\right]}{7t-6n(1-\phi)}\right\}.$

Because it holds that $k^{\#} > \tilde{k},$ based on equation (A.3.1), we have

$$\left.\frac{d\Pi_i^{**}}{d\phi}\right|_{a_i=a_j=a^{**}} > 0 \Leftrightarrow k > k^{\#}\left(>\tilde{k}\right). \tag{A.5}$$

This result implies that, in view of equation (A.4), the positive direct effect outweighs the negative indirect effect. Therefore, an increase in connectivity always improves the net profit.

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***Figure 1*. A Fully Covered Market Case and “Battlefield”**

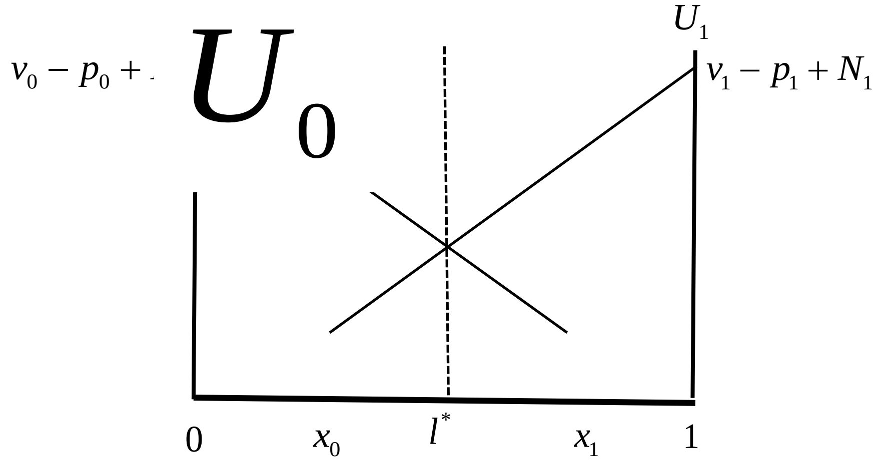


Figure 2. **A Partially Covered Market Case and Market Expansion Effect**

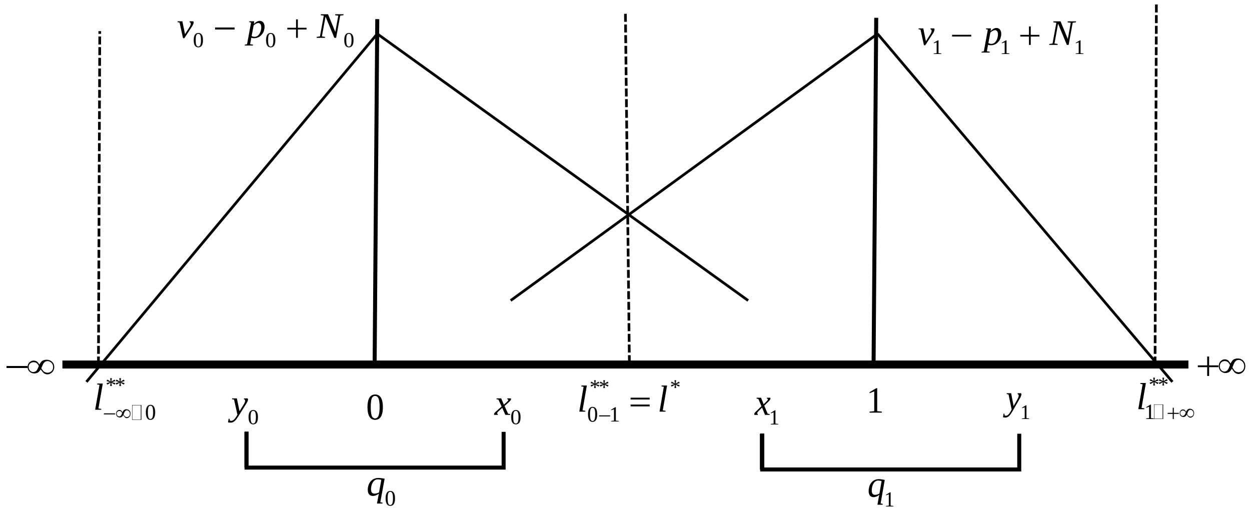
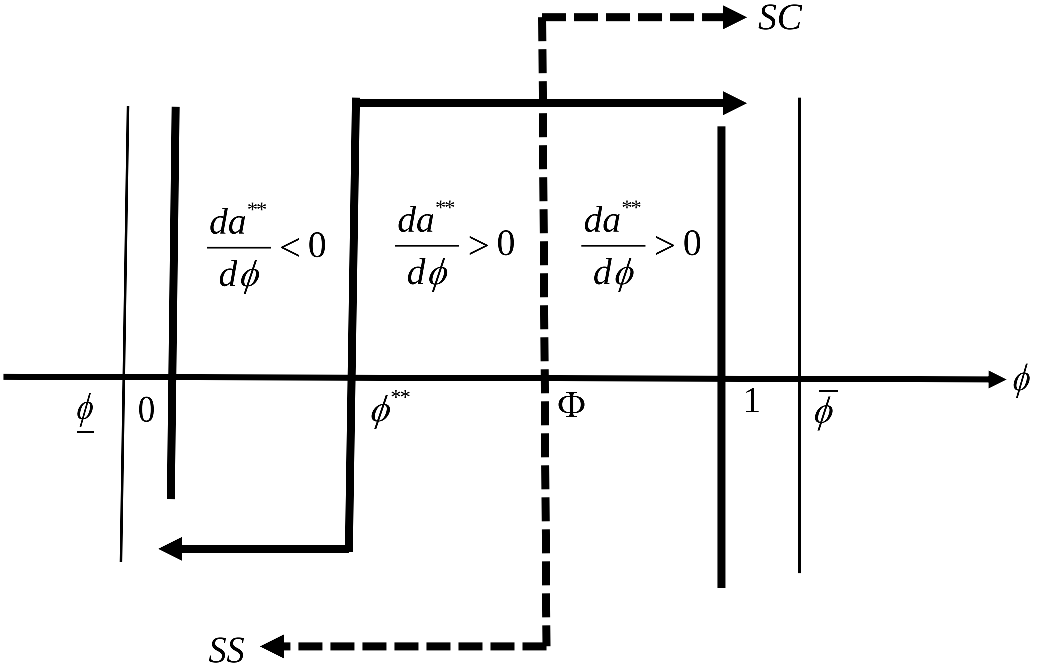
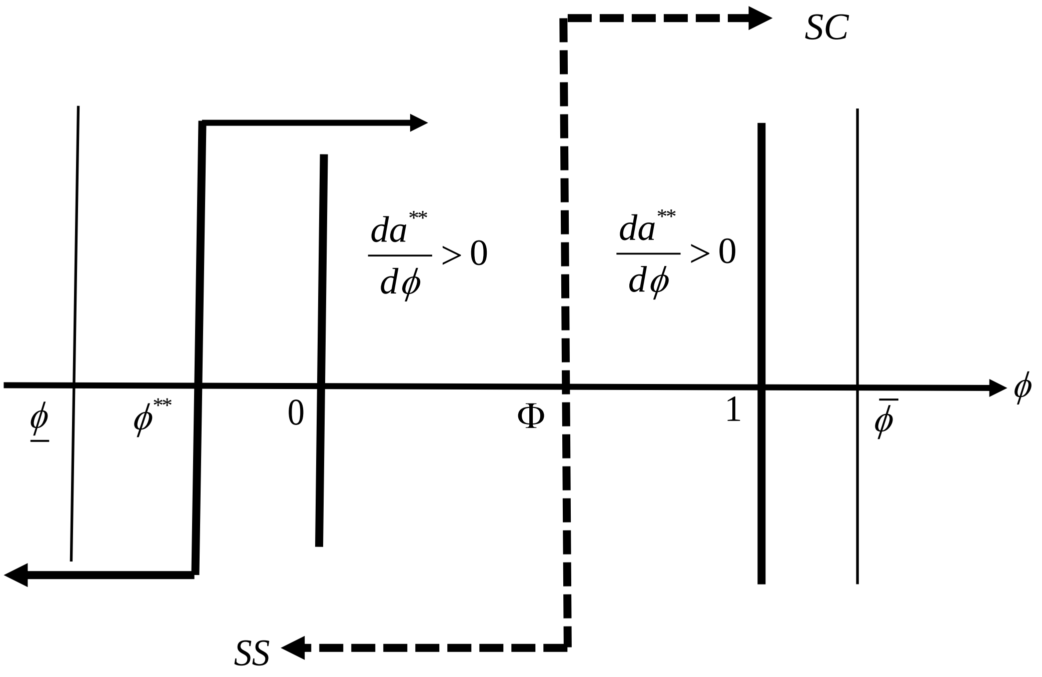


Figure 3A: $\frac{5t}{6} > n > \frac{2t}{3} > \frac{t}{4}$



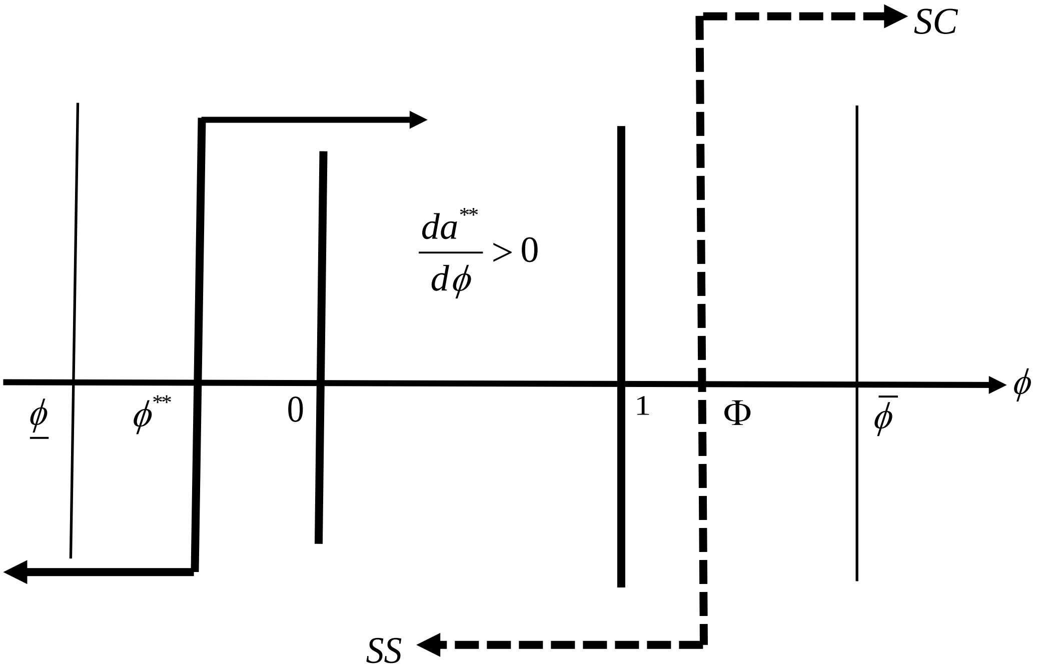
$$\underline{\phi} \equiv 1 - \frac{7t}{6n} < 0 \text{ and } \overline{\phi} \equiv \frac{5t}{3n} - 1 > 1$$

Figure 3B: $\frac{2t}{3} > n > \frac{t}{4}$



$\underline{\phi} \equiv 1 - \frac{7t}{6n} < 0$ and $\overline{\phi} \equiv \frac{5t}{3n} - 1 > 1$

Figure 3C: $\frac{2t}{3} > \frac{t}{4} > n$



$$\underline{\phi} \equiv 1 - \frac{7t}{6n} < 0 \quad \text{and} \quad \overline{\phi} \equiv \frac{5t}{3n} - 1 > 1$$