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Policy Analysis in Endogenous Fertility Model with Human Capital Accumulation

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Policy Analysis in Endogenous Fertility Model with Human Capital Accumulation[†]

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Abstract

This paper sets up an endogenous fertility model with human capital accumulation and uses simulation analysis to evaluate how four child-care support policies ((i) child allowances, (ii) policies to subsidize childcare services, (iii) childcare leave benefits, and (iv) education subsidy policies) affect the fertility rate and the amount of human capital accumulation. This paper presents consideration of the fertility function of a constant elasticity of substitution (CES) function for child-care time and child-care services. The analysis results show that policies (i)–(iii) increase fertility in the short term, but in the long term, households' disposable income declines because of lower human capital accumulation, leading to a lower fertility rate. A policy of subsidy for education investment can raise fertility in the long run by virtue of an increase in human capital accumulation.

Keywords: Child Allowance, Child-Care Service, Child-Care Time, Education Investment, Endogenous Fertility

JEL: J13, H20

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1. Introduction

This paper sets up an endogenous fertility model with human capital accumulation and uses simulation analysis to determine how four child-care support policies affect the fertility rate and the level of human capital accumulation: (i) child allowances, (ii) policies to subsidize childcare services, (iii) childcare leave benefits, and (iv) education subsidy policies. A number of related reports of the literature describe endogenous fertility models. Based on factors considered in the determination of fertility, there are first of all models by which fertility is determined using child-care time, Galor and Weil (1996) being a representative example. A representative model in which fertility is determined using child-care services is that of van Groezen, Leers and Meijdam (2003). Apps and Rees (2004), Ferrero and Iza (2004) and Hirazawa and Yakita (2009) have all presented models in which fertility is determined using both child-care time and child-care services. Ferrero and Iza (2004) assume a perfect substitution fertility function between child-care time and child-care service and demonstrate that economic growth increases the wage level, leading to a shift from child-care through child-care time to child-care through the use of child-care services. Hirazawa and Yakita (2009) assume a Cobb–Douglas type fertility function and demonstrate that a reduction in pension contributions leads to Pareto improvement.

This paper presents consideration of the fertility function of a constant elasticity of substitution (CES) function for child-care time and child-care services. Also, we assess a model of the quantity and quality of children. The model includes not only determination of the number of children but also investment in education for children. Zhang (1997) assumes an endogenous fertility model that incorporates human capital accumulation and shows that child allowances increase fertility but reduce human capital accumulation. They also show that education subsidy policies increase human capital accumulation, but decrease fertility. These results demonstrate the tradeoff between quality and quantity of children. However, although various child-care support policies exist, few reports of the related literatures have described comparison of these policies. This paper presents consideration of four child-care support policies with the aim of comparing effects of each policy on fertility and human capital accumulation. In terms of the model setting, one characteristic of this paper is that, unlike related reports of the literature, it assumes a CES-type fertility function, which allows for more realistic policy analysis.

The results of the analysis are the following. Results show that (i) a child allowance, (ii) a subsidy for child-care services, and (iii) child-care time all raise fertility rates in the short term. In the long term, results show that the fertility rate declines. Instead of making the choice to increase the number of children, these three policies reduce investment in education for children, which engenders reduction in future human capital accumulation, and consequently lower incomes, which in turn engenders a reduction in child-care expenditures because of lower disposable income, i.e. lower fertility rates. Results also show that the degree of substitution between child-care services and child-care time has no intrinsic effect on the fertility-raising effects of these three policies. However, when the relation between child-care services and child-care time is complementary, subsidies to child-care time were found to increase not only the child-care time but also demand for child-care services. Subsidies for investment in education reduce fertility in the short term. However, in the long run, human capital accumulation increases household disposable income, which in turn increases child-care expenditure, i.e. fertility. The paper is presented with the following structure. Section 2 presents a description of the model setup. Section 3 includes four policy analyses: (i) child allowances, (ii) subsidy for child-care services, (iii) subsidy for child-care time, and (iv) subsidy for educational investment. In addition, section 3 provides a simulation analysis of how these policies affect fertility rates and human capital accumulation, not only in the case of the Cobb–Douglas function but also for a CES-type function. Section 4 provides a summary.

2. Model

This model economy consists of three types of agents: households, firms, and a government.

2.1 Households

We assume the following fertility function as

$$n_t = \left(Ae_t^{\rho} + Bl_t^{\rho}\right)^{\frac{1}{\rho}}, \rho < 1, 0 < A, 0 < B. \tag{1}$$

In that equation, n_t, e_t, and l_t respectively denote the fertility (number of children), demand for child-care services, and child-care time. Also, t denotes the period. Regarding the definition of the fertility function, Apps and Rees (2004), for example, assume a first-order homogeneous function. By contrast, Hirazawa and Yakita (2009) assume a Cobb–Douglas type fertility function. As described in this paper, a relaxed form of the Cobb–Douglas type fertility function is assumed in the functional form. The reason for specifying the functional form is to perform a simulation analysis, as described below.

We assume the household utility function as shown below:

$$u_t = \alpha ln c_{1t} + \beta ln c_{2t+1} + (1 - \alpha - \beta) ln n_t h_{t+1}, 0 < \alpha, 0 < \beta, \alpha + \beta < 1. \tag{2}$$

Therein, u_t, c_{1t}, c_{2t+1}, and h_{t+1} respectively represent the level of utility, consumption in young period, consumption in old period, and human capital stock of children. The individuals in households exist in two periods: a young period and an old period. During the young period, they care for children. The same utility function includes fertility and the human capital stock of children, as assumed by de la Croix and Doepke (2003).

The equation of human capital accumulation is assumed as

$$h_{t+1} = Cg_t^{\delta},\ 0 < C,\ 0 < \delta < 1, \tag{3}$$

In that equation, h_t and g_t respectively denote the human capital stock and education investment.

Generally, although a one-degree homogeneous human capital function is assumed, we do not consider functional forms in which the human capital stock of the parents directly affects the human capital formation of their children. In fact, this is not a necessary effect. This assumption is necessary to have a steady state.

The budget constraint in the young period is shown as

$$s_t + c_{1t} + (1-\gamma_t)ze_t + (1-\sigma_t)g_t n_t = (1-l_t)wh_t + \varepsilon_t l_t wh_t + q_t n_t - T_t, \tag{4}$$

where s_t, z, w, q_t, γ_t, ε_t, T_t, and σ_t respectively stand for saving, the price of child-care services, the wage rate, child allowance, subsidy for child-care service, subsidy rate for child-care time, lump-sum taxation, and the subsidy rate for education investment.

The budget constraint for old period is

$$c_{2t+1} = (1+r)s_t, \tag{5}$$

where r represents the interest rate.

We derive the optimal household allocations to maximize utility (1) subject to constraints (2)–(5). The optimal allocations are derived as presented below.

$$c_{1t} = \alpha(wh_t - T_t), \tag{6}$$

$$c_{2t+1} = (1+r)\beta(wh_t - T_t), \tag{7}$$

$$(1-\gamma_t)z_t e_t = \frac{Ae_t^{\rho}}{Ae_t^{\rho} + Bl_t^{\rho}}\left((1-\alpha-\beta)(wh_t - T_t) + (q_t - (1-\sigma_t)g_t)n_t\right) \tag{8}$$

$$(1-\varepsilon_t)wh_t l_t = \frac{Bl_t^{\rho}}{Ae_t^{\rho} + Bl_t^{\rho}}\left((1-\alpha-\beta)(wh_t - T_t) + (q_t - (1-\sigma_t)g_t)n_t\right), \tag{9}$$

$$(1-\sigma_t)g_t n_t = (1-\alpha-\beta)\delta(wh_t - T_t). \tag{10}$$

2.2 Firms

We assume the following production function:

$$Y_t = K_t^{\mu} L_t^{1-\mu}, 0 < \mu < 1. \tag{11}$$

Therein, Y_t, K_t and L_t respectively denote the output, physical capital stock, and effective labor input. Considering profit maximization, we can obtain the following equations:

$$w_t = (1-\mu)k_t^{\mu}, 0 < \mu < 1, \tag{12}$$

$$1 + r_t = \mu k_t^{\mu-1}. \tag{13}$$

In those equations, $k_t = \frac{K_t}{L_t}$ represents the capital–labor ratio. We assume that the physical capital stock depreciates during a period. For these analyses, we assume a small open economy, with the interest rate given by the foreign interest rate. Therefore, the capital–labor ratio k_t is also given. The wage rate is also given from equation (12). Child-care services are also produced from final goods. However, it is assumed that payment of z is necessary to purchase one unit of childcare services.

2.3 Government

The government will implement four policies to support childcare: (i) child allowances, which provide benefits in relation to the number of children; (ii) child-care time, which provides benefits in relation to child-care time; (iii) subsidies for the use of child-care services; and (iv) subsidies for investment in education for children. Funding will be provided by a lump-sum tax. No public bonds will be issued. The policies will be financed on a balanced budget. The government budget constraint is

$$T_t = q_t n_t + \gamma_t z e_t + \varepsilon_t w l_t + \sigma_t g_t n_t. \tag{14}$$

3. Policy analysis

This section presents examination of the following policy effects on fertility and others with simulation.

(i) child allowances, which provide benefits in relation to the number of children

(ii) subsidies for child-care time, which provide benefits in relation to child-care time

(iii) subsidies for the use of child-care services

(iv) subsidies for investment in education for children

We consider fertility functions of three types

i) Cobb–Douglas fertility function ($\rho = 0$)

ii) Substitutive fertility function ($\rho = 0.5$)

iii) Complementary fertility function ($\rho = -0.5$)

We set the parameters such that fertility is unity. The parameter settings are shown in Table 1.

Table 1: Parameter settings and endogenous variables in the steady state

Case	z	ρ	A	B	e	l	n	g	h
I	0.014706	0	0.75	0.25	2.942837	0.039238	1	0.10039	1
II	0.002264	0	0.5	0.5	12.74291	0.078475	1	0.10039	1
III	0.032459	0.5	0.75	0.25	1.777778	0	1	0.10039	1
IV	0.014426	0.5	0.5	0.5	3.999999	0	1	0.10039	1
V	0.001894	-0.5	0.75	0.25	8.05303	0.11547	1	0.10039	1

Child-care service cost z is given such that fertility is unity in the steady state. Cases I and II show the Cobb–Douglas function case; Cases III and IV show the substitutive fertility function case. Case V shows the complementary fertility function case. Others parameter are shown in Table 2.

Table 2: Parameter settings and endogenous variables in the steady state

α	0.43
β	0.14
$1-\alpha-\beta$	0.43
δ	0.635
C	4.304549
$1+r$	1.347849
k	0.116904
w	0.36766
h	1

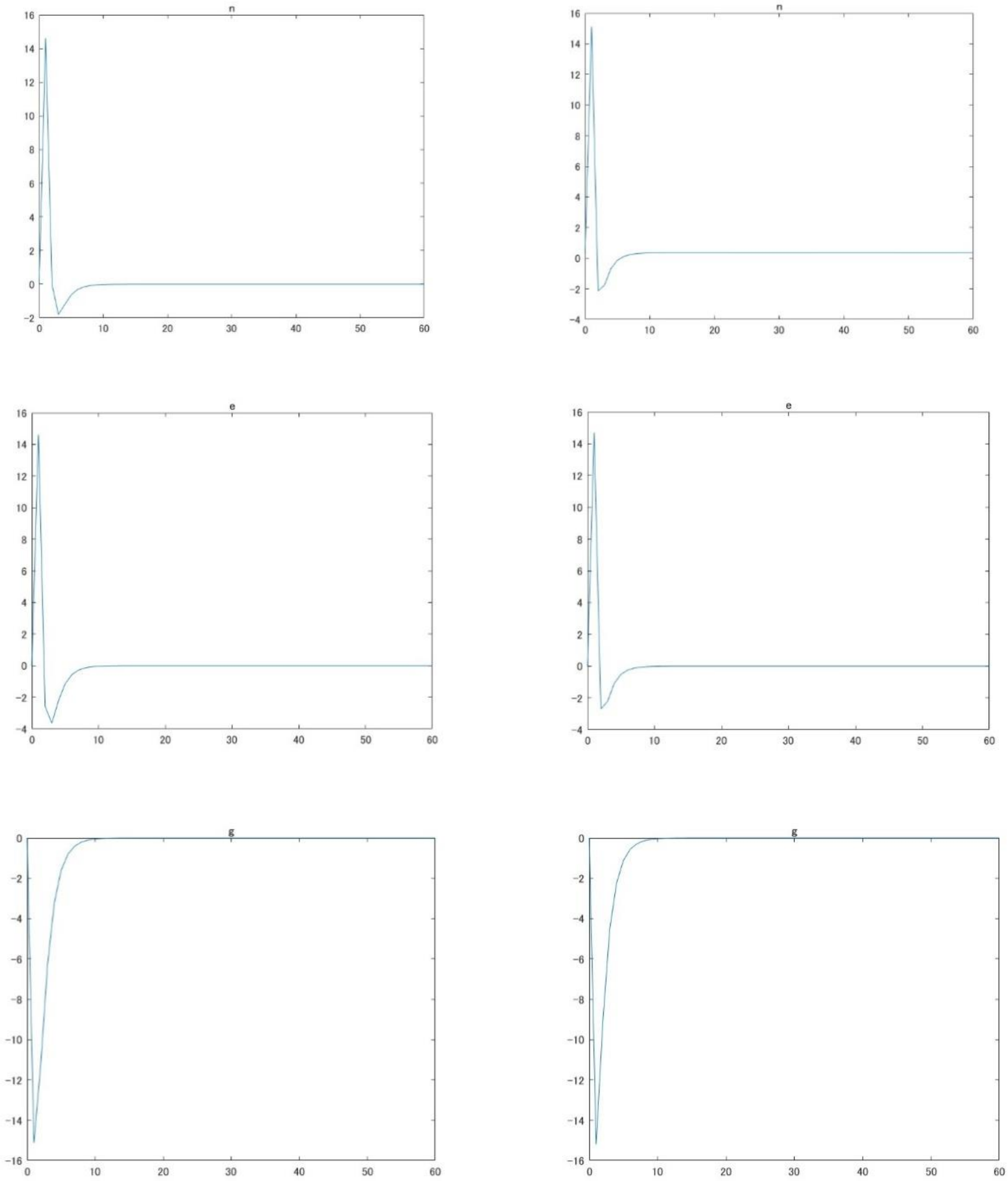
Oshio (2001) examines a case in which the consumption and fertility preference parameters are equal for the younger individuals. The present analyses are set up similarly. In addition, β is a discount factor. de la Croix and Doepke (2003) calculated the discount factor for one quarter as 0.99^{120} for a preference parameter of consumption at a young age of 1, because the discount factor for one quarter is 0.99 and one period of the overlapping generations model is 30 years. Another parameter set in de la Croix and Doepke (2003) is δ. Also, C is set so that the steady state level of human capital is 1. Because the interest rate in Japan in recent years has been 1% per annum and because the overlapping generations model has one period of 30 years, we calculated 1.01^{30}. Then, the capital–labor ratio k and the wage rate were derived from (12) and (13) and from the fact that the capital–labor ratio in recent years is $\mu = 0.3$.

Finally, we consider the following function of shocks as

$$T_t = \phi T_{t-1} + f. \tag{15}$$

Here, we consider a 1 percentage point tax shock at some point in time, which is assumed to be used to fund spending on each policy. Then, because $\phi = 0.5$, half of the policy shock will remain in the next period and beyond. For example, in the case of a child allowance, a policy expenditure other than the child allowance is assumed to be zero. The size of the child allowance collected through the lump-sum tax is expected to decrease over time.

3.1 Child allowance



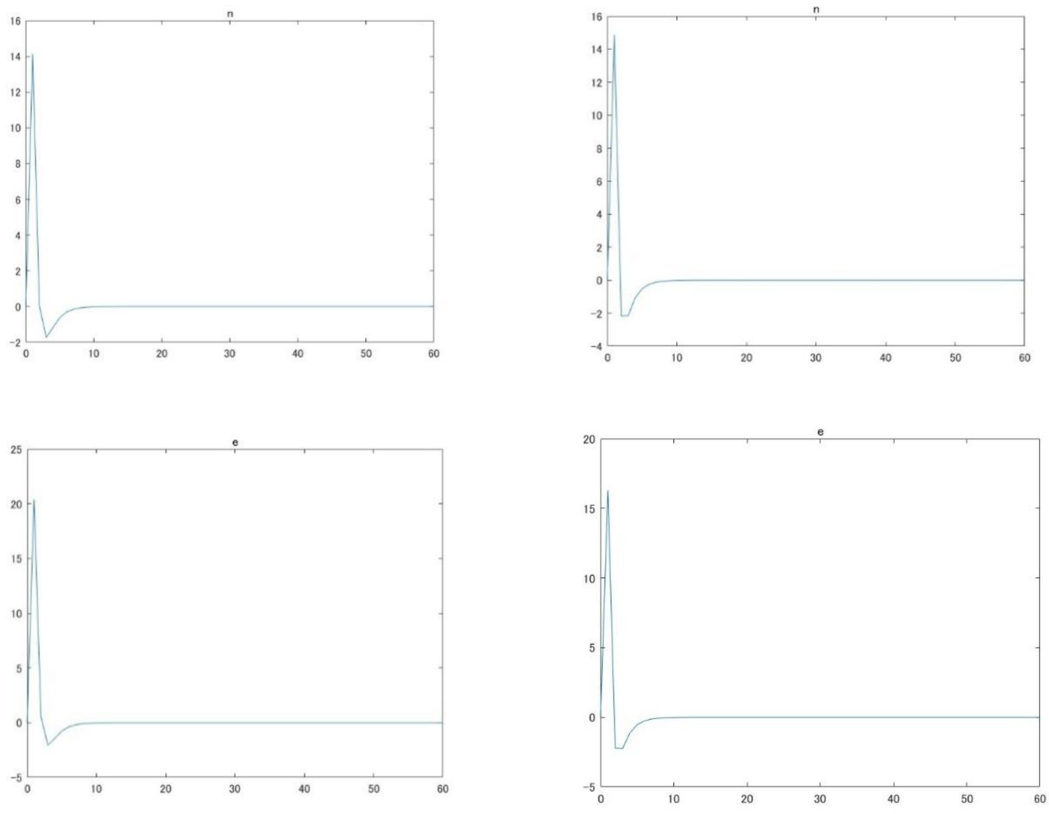
Case I   Case III

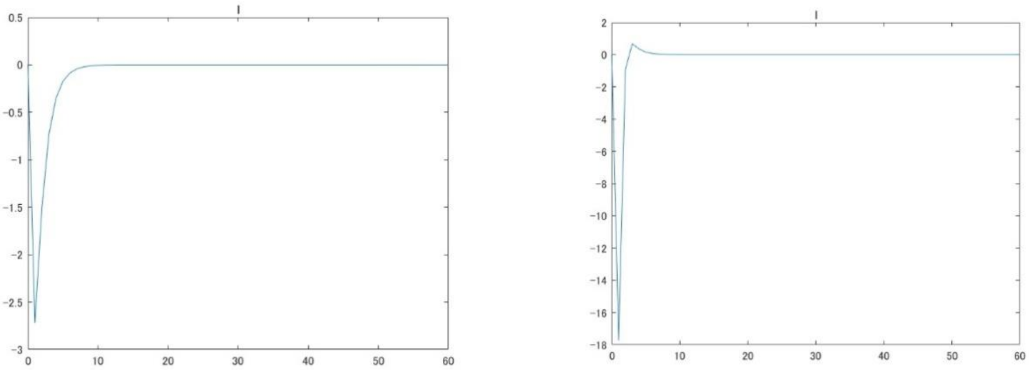
Fig. 1 Child allowance.

Fig. 1 presents effects of child allowances. This figure compares Case I and Case III. They are almost identical. The child allowance increases the purchase of childcare services, which in turn increases fertility. Although not shown in the figure, child-care time has also increased. However, investment in education has decreased. This outcome is consistent with reports from the related literature. Once the fertility rate has increased, it will instead decrease. The reason for this is attributable to a decrease in income associated with a decrease in human capital caused by a decrease in investment in education. Because of the decrease in income, fewer child-care services are purchased, leading to a decrease in fertility compared to the pre-policy level.

3.2 Subsidy for child care services

As shown by Fig.2, subsidies for childcare services increase the demand for childcare services but decrease the time spent on childcare. However, the former effect is larger, leading to an increase in the fertility rate. The reason for the decrease in hours of childcare is that the increased tax burden forces people to work more hours. However, in the case of the substitutive fertility function, substitution occurs, whereby the demand for inexpensive childcare services is increased and the childcare time, which is expensive, is reduced, leading to a larger decrease in childcare time than in the case of the Cobb–Douglas type fertility function.



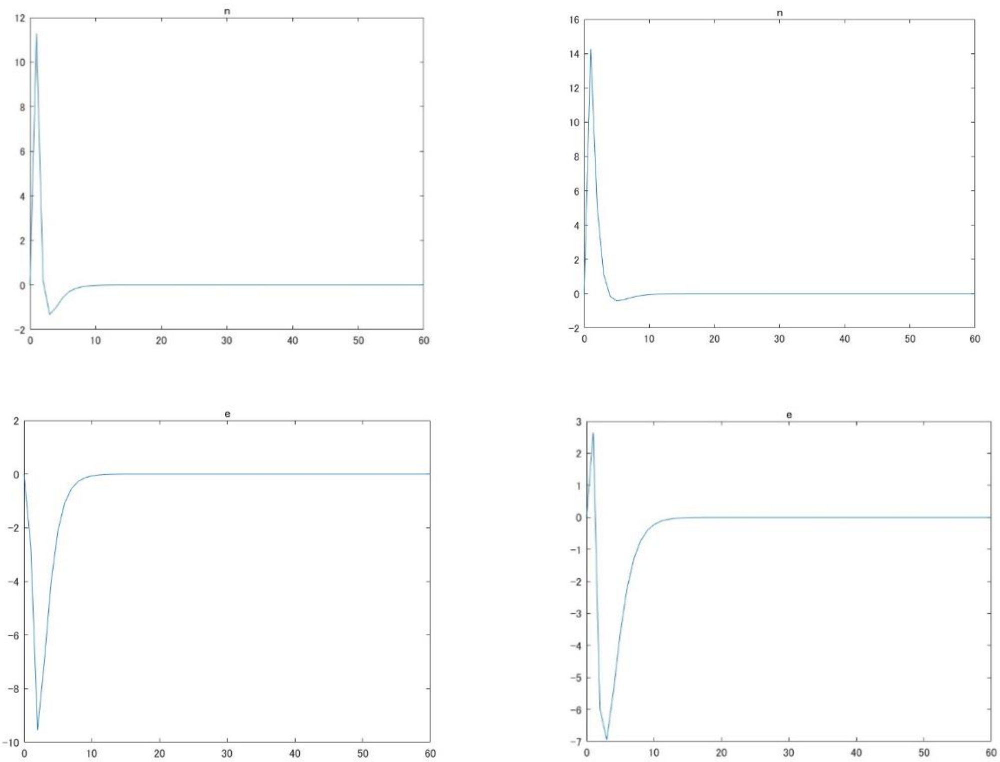


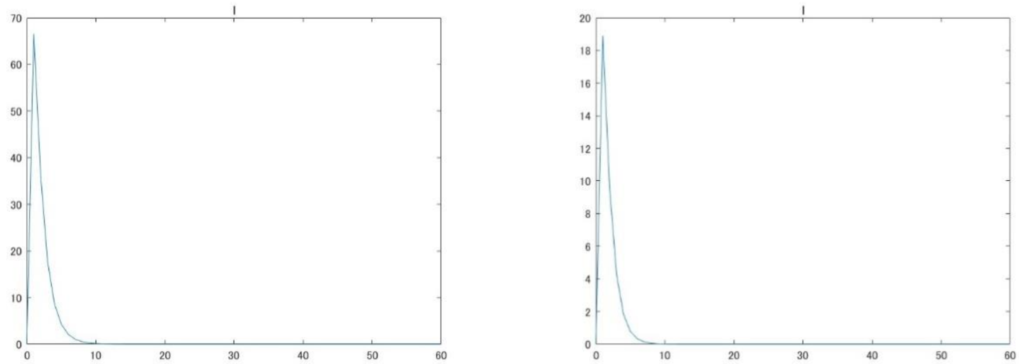
Case I Case IV

Fig. 2 Subsidy for child-care services.

3.3 Subsidy for child-care time

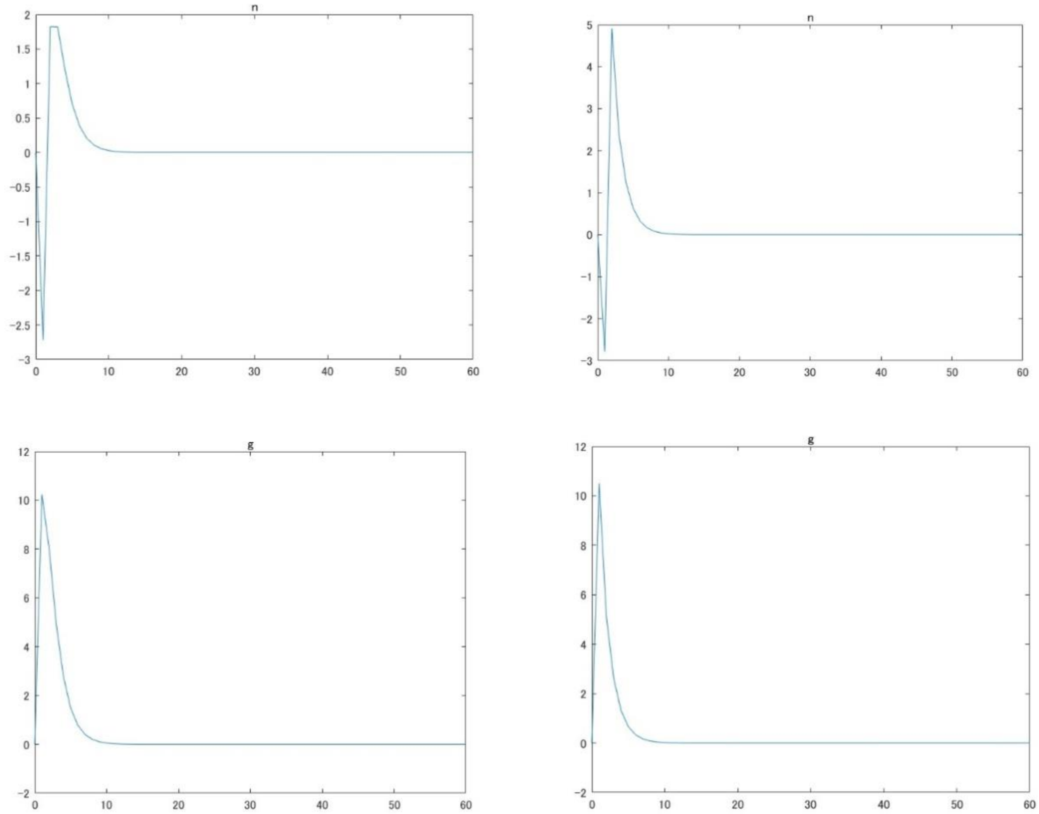
A subsidy for child-care time increases the time spent on child care but reduces the demand for childcare services. Fig.3 shows the results of Case I and Case V. Case I shows the case of Cobb-Douglas type fertility function. Then, even if the demand for childcare services reduces, the fertility can be pulled up because of an increase in the time spent on child care. Case V shows the case of complementary fertility function. In this case, the fertility increases.





Case I Case V

Fig. 3 Subsidy for child-care time.



Case I Case IV

Fig. 4 Subsidy for education investment.

3.4 Subsidy for education investment

Finally, we consider a subsidy to investment in education, while comparing Case I and Case IV. Fig.4 shows the results. Subsidies to investment in education increase demand for investment in education; the fertility rate decreases. This is true because instead of investing in education, child-care services and child-care time are reduced. However, a phase emerges during which increased investment in education increases human capital stock accumulation, and consequently, income, which increases demand for child-care services and which increases fertility.

4. Conclusions

Using an endogenous fertility model with human capital accumulation, we examine four child-care support policies: (i) a child allowance, (ii) a subsidy for childcare service, (iii) child-care time, and (iv) an education subsidy. In addition, we consider how these policies affect fertility and human capital accumulation. In addition to a Cobb–Douglas type function with child-care services and child-care time as input factors for the fertility determination function, a CES function with a constant elasticity of substitution was also assumed. Also, how the elasticity of substitution alters the effects of the childcare support policies was considered. When child-care services and child-care time are substitutive, it is apparent that that child-care time is reduced considerably by subsidizing child-care services compared to the Cobb–Douglas model. In the complementary case, the study revealed that a subsidy for child-care time increases demand for child-care services and child-care time.

Policies such as a child allowance, a subsidy for child-care services and a subsidy for child-care time raise fertility in the short term. However, in the long term, fertility declines as the substitution of quality for number of children occurs, leading to less investment in education for children, and consequently lower incomes because of smaller human capital accumulation. Results of this study demonstrate that fertility has been reduced. Conversely, this paper finds that a subsidy for educational investment temporarily reduces fertility, but that higher household incomes achieved through the accumulation of human capital increase fertility.

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Program Examples (Case I Child allowance)

```
//1. variables
var h g n T q e l;
varexo f;

//2. parameter
parameters alpha beta delta w A B z phi C;

//2.1 parametervalue
alpha = 0.43;
beta =0.14;
delta = 0.635;
w=0.367660208;
A=0.75;
B=0.25;
z=0.0147062867712498;
phi=0.5;
C=4.304549;

//3.equations
model;
h=C*g(-1)^delta;
n=(1-alpha-beta)*delta*(w*h-T)/(g);
n = (e^A*l^B);
e
=A/(A+B)*((1-alpha-beta)*(w*h-T)+(q-g)*n)/z
;
l
=B/(A+B)*((1-alpha-beta)*(w*h-T)+(q-g)*n)/(
w*h);
T = q*n;
T = phi*T(-1)+f;
end;

//4. initial value
initval;
n=1;
e=12;
l=0.07;
g=0.1;
T=0;
q=0;
h=1;
end;

//5. steady state
steady;
check;

//6. simulation
shocks;
var f;
periods 1;
values 0.01;
end;

//7. results
simul(periods=60);

n1=(n/1-1)*100;
e1=(e/2.94283710784528-1)*100;
l1=(l/0.0392375-1)*100;
g1=(g/0.10039-1)*100;
T1=(T/1)*100;

figure(1)
plot(0:60, n1(1:61)); title('n')
figure(2)
plot(0:60, e1(1:61)); title('e')
figure(3)
plot(0:60, l1(1:61)); title('l')
```

```
figure(4)
plot(0:60, g1(1:61)); title('g')
figure(5)
plot(0:60, T1(1:61)); title('T')
```