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Public Education, Pension and Debt Policy

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Public Education, Pension and Debt Policy[†]

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Abstract

In some OECD countries, public debt is increasing to the point that fiscal reforms should be considered. Our paper sets a government budget constraint with the deficit of primary balance and examines how such a policy affects public debt in the long run. In the model, we consider policies of three types to reduce the deficit of primary balance: decreases in pension benefits and public education investment, and an increase in income tax. A decrease in pension benefit or an increase in tax revenues can inevitably raise the capital stock per unit of effective labor. Depending on the parametric conditions, they can also reduce the public debt per unit of effective labor and the ratio of public debt to gross domestic product (GDP).

JEL Classification: H63, H20, E61, I28

Keywords: Education Investment, Fiscal Sustainability, Pension, Public Debt

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1. Introduction

In some OECD countries, public debt is increasing to the point that fiscal reforms should be considered. Especially in Japan, the public debt per gross domestic product (GDP) has reached a level high enough that it raises doubts about fiscal sustainability, as shown by Fig. 1.

[Insert Fig. 1 around here.]

Some fiscal rule is necessary to constrain current generations and prevent from passing a tax burden to future generations. In European countries, the Maastricht Treaty sets such a fiscal rule.¹ However, in Japan, no such fiscal rule exists. Because of the large deficit of the primary balance, the public debt in Japan continues to increase. As shown by Fig. 2, because of Japan's aging society with fewer children, social security costs continue to increase. However, because of budget constraints, investment in public education can not be pulled up. These policies raise the fiscal deficit. Therefore, the government in Japan should consider measures to reduce public debt or the deficit of primary balance.

[Insert Fig. 2 around here.]

This paper sets a model with public debt for examination of how a policy affects public debt. Concretely, our paper presents an examination of what policy should be provided to decrease the deficit of primary balance. We consider policies of three types to reduce the deficit of primary balance: decreases of pension benefits or public education investment, and an increase in income tax. The results demonstrate that a decrease in pension benefit and an increase in the tax rate can reduce the public debt stock depending on the parametric condition.

Many papers describe related work conducted in the field of public debt finance. Greiner (2007, 2008) described the condition of the sustainability of the fiscal deficit finance. Barro (1990), Futagami, Morita and Shibata (1993), Yakita (2008), and Maebayashi (2013) set endogenous growth models with public capital. Especially, Yakita (2008), Arai (2011), and Teles and Mussolini (2014) respectively consider fiscal deficit models with public investment. Results obtained by Yakita (2008) point to increased public capital financed by a fiscal deficit, which raises public debt in the long run.

Futagami, Iwaisako and Ohdoi (2008) set a model with productive government spending as public capital in a public debt finance model with a fiscal rule of a constant ratio of public debt to physical capital stock. Using that model, they derive the two steady-state equilibrium: one for a low income level and the other for a high income level. Minea and Villieu (2013) consider the fiscal rule

¹ The Maastricht Treaty was enacted at 1992. The treaty levies the rule that the ratio of public debt stock to gross domestic product (GDP) be less than 60%. The ratio of fiscal deficit to GDP must be less than 3%.

of debt policy. Ono (2003) and Kunze (2014) examine how an aging society affects the level of the public debt stock and the income level.

Chalk (2000) and Moraga and Vidal (2008) consider fiscal sustainability in an overlapping generations model. In these papers, if the ratio of public debt to physical capital converges to a constant level in the long run, then fiscal sustainability holds. Therefore, even if a deficit of the primary balance exists, one can achieve fiscal sustainability. Needless to say, the upper bound of the primary balance deficit exists to maintain fiscal sustainability. Oguro and Sato (2014) report that fiscal sustainability can not always be brought about by an increase in income taxation because an increase in the tax burden reduces household saving; then interest rates are pulled up.

The remainder of this paper comprises the following sections. Section 2 sets the model. Section 3 presents derivation of the equilibrium. Section 4 explains derivation of the steady state. Section 5 presents an examination of whether a policy to decrease the deficit of primary balance can reduce public debt in the long run or not. Section 6 concludes our analyses.

2. Model

In this economic model, agents of three types exist: household, firm, and government. This section explains the model settings.

2.1 Households

Individuals live in two periods: young and old. Our paper sets the overlapping generations model, i.e., a young generation and an old generation exist in each period. No population growth occurs. This model sets the population size as unity. The household's utility function is assumed by the following log utility function as

$$u_t = \alpha \ln c_{1t} + (1 - \alpha) \ln c_{2t+1}, 0 < \alpha < 1, \quad (1)$$

where c_{1t} and c_{2t+1} respectively denote consumption during young and old periods. t denotes the period.

During the young period, younger people work inelastically to gain wage income, which is allocated for consumption during the young period and for savings to fund consumption during the old period. Moreover, the government levies a labor income tax to provide the pensions and public education investment. Then, the household's lifetime budget constraint is

$$c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} = (1 - \tau)w_t h_t + \frac{p_{t+1}}{1 + r_{t+1}}. \quad (2)$$

Therein, $1 + r_{t+1}$ and w_t respectively denote an interest rate and a wage rate. h_t represents the human capital stock. p_{t+1} stands for the pension benefit. τ expresses the tax rate to provide public education investment and a pension benefit ($0 < \tau < 1$). Maximizing utility function (1) subject to budget constraint (2) is reduced to the following consumption allocations.

$$c_{1t} = \alpha \left((1 - \tau)w_t h_t + \frac{p_{t+1}}{1 + r_{t+1}} \right), \quad (3)$$

$$c_{2t+1} = (1 - \alpha) \left((1 - \tau)w_t h_t + \frac{p_{t+1}}{1 + r_{t+1}} \right). \quad (4)$$

It is assumed that the human capital is formed by only public education investment e_t as shown below:²

$$h_{t+1} = H e_t^\delta h_t^{1-\delta}, 0 < \delta < 1, 0 < H. \quad (5)$$

2.2 Firms

Firms produce final goods by inputting the capital stock and labor in a perfectly competitive market. For the analyses presented in this paper, it is assumed that the production function is

$$Y_t = A K_t^\gamma H_t^{1-\gamma}, 0 < \gamma < 1, 0 < A. \quad (6)$$

Therein, Y_t signifies final goods; K_t denotes the physical capital stock in t period. H_t stands for the aggregate human capital. Defining the population size of younger people as N_t , then aggregate human capital is given as $H_t = N_t h_t$. Considering profit maximization in perfectly competitive market, demand for the physical capital stock and labor input are shown as

$$w_t = A(1 - \gamma)k_t^\gamma, \text{ and} \quad (7)$$

$$1 + r_t = A\gamma k_t^{\gamma-1}, \quad (8)$$

where $k_t = \frac{K_t}{H_t}$. Physical capital stock is assumed to be fully depreciated in one period.

2.3 Government

The government provides a pension benefit for older people and public education investment for younger people. The analyses presented in this paper rely on the assumption that the government can issue bonds to collect revenues: debt finance or a fiscal deficit is allowed in addition to the taxation for labor income. Then, the government budget constraint is shown as

$$B_{t+1} = N_t e_t + N_{t-1} p_t - \tau w_t H_t + (1 + r_t) B_t. \quad (9)$$

In that equation, B_t denotes the public debt stock. Also, $N_t e_t$ stands for aggregate expenditure for public education investment. $N_{t-1} p_t$ signifies the aggregate expenditure for pension. N_{t-1} expresses the elderly population size. With $N_t e_t + N_{t-1} p_t - \tau w_t H_t > 0$, this budget shows the primary deficit. We present specific consideration of this situation, which resembles that in Japan.

We assume that public education investment e_t and pension benefit p_t are proportionally provided, respectively, as $e_t = \phi w_t h_t$ and $p_t = \varepsilon w_t h_t$ ($0 < \phi < 1$, $0 < \varepsilon < 1$). Then, with (5),

² Glomm and Ravikumar (1992) assume that the growth rate of human capital is represented by the schooling time, public education investment, and parental human capital. In Greiner (2008), the growth rate of human capital is determined by public education investment and human capital stock.

(7), (8) and (9), the government budget constraint becomes the following.

$$b_{t+1} = \frac{1}{H\phi^\delta A^\delta (1-\gamma)^\delta k_t^{\gamma\delta}} \left((\phi + \varepsilon - \tau)A(1-\gamma)k_t^\gamma + A\gamma k_t^{\gamma-1} b_t \right), \quad (10)$$

In that equation, $b_t = \frac{B_t}{H_t}$. The growth rate of human capital is given as

$$1 + g \equiv \frac{h_{t+1}}{h_t} = H\phi^\delta A^\delta (1-\gamma)^\delta k_t^{\gamma\delta}. \quad (11)$$

3. Equilibrium

This section presents derivation of the equilibrium. Considering human capital accumulation, the growth rate of the human capital is given as (11). Furthermore, the dynamics of public debt b_t is given by (10). Then, defining $\Delta b_t = b_{t+1} - b_t$, the loci of $\Delta b_t = 0$ are given as the following equation. The loci are depicted as shown in Fig. 3.

$$b_t = \frac{(\phi + \varepsilon - \tau)A(1-\gamma)}{H\phi^\delta A^\delta (1-\gamma)^\delta - A\gamma k_t^{-(1-\gamma(1-\delta))}} k_t^{\gamma(1-\delta)}. \quad (12)$$

[Insert Fig. 3 around here.]

Our paper presents specific consideration of the deficit of primary balance $\phi + \varepsilon > \tau$ and positive public debt $b_t > 0$. For them, $H\phi^\delta A^\delta (1-\gamma)^\delta - A\gamma k_t^{-(1-\gamma(1-\delta))} > 0$ should be assumed to consider the setting of the model economy. From differentiation of (12) with respect to b_t and k_t ,

we obtain both positive and negative slope. With $k_t < \left[\frac{A^{1-\delta}}{(1-\delta)H\phi^\delta (1-\gamma)^\delta} \right]^{\frac{1}{1-\gamma(1-\delta)}}$, the slope of (12) is negative. Alternatively, the slope of (12) is positive.

The condition for $\Delta b_t > 0$ to hold is given as

$$b_t < \frac{(\phi + \varepsilon - \tau)A(1-\gamma)}{H\phi^\delta A^\delta (1-\gamma)^\delta - A\gamma k_t^{-(1-\gamma(1-\delta))}} k_t^{\gamma(1-\delta)}. \quad (13)$$

The capital market equilibrium condition is given as $K_{t+1} + B_{t+1} = N_t s_t$. Also, s_t denotes household saving shown by $s_t = (1-\tau)w_t h_t - c_{1t}$. Then, we can obtain the following equation.

$$k_{t+1} = \frac{\frac{(1-\tau)(1-\alpha)A^{1-\delta}(1-\gamma)^{1-\delta}}{H\phi^\delta} k_t^{\gamma(1-\delta)} - b_{t+1}}{1 + \frac{\alpha\varepsilon(1-\gamma)}{\gamma}}. \quad (14)$$

Therein, b_{t+1} is given by (10). Defining $\Delta k_t = k_{t+1} - k_t$, the loci of $\Delta k_t = 0$ is shown as

$$b_t = \frac{(1-\gamma)((1-\alpha)(1-\tau) - (\phi + \varepsilon - \tau))}{\gamma} k_t - \frac{\left(1 + \frac{\alpha\varepsilon(1-\gamma)}{\gamma}\right) H\phi^\delta A^{\delta-1} (1-\gamma)^\delta}{\gamma} k_t^{2-\gamma(1-\delta)}. \quad (15)$$

The condition to be $\Delta k_t > 0$ is given as the following.

$$b_t < \frac{(1-\gamma)((1-\alpha)(1-\tau) - (\phi + \varepsilon - \tau))}{\gamma} k_t - \frac{\left(1 + \frac{\alpha\varepsilon(1-\gamma)}{\gamma}\right) H\phi^\delta A^{\delta-1} (1-\gamma)^\delta}{\gamma} k_t^{2-\gamma(1-\delta)}. \quad (16)$$

Then the dynamics of k_t can be presented as the following figure.

[Insert Fig. 4 around here.]

For given k_t, b_t, h_t , the human capital stock in $t+1$ h_{t+1} or the growth rate of human capital $1+g$ is given as (11). The physical capital stock per unit of effective labor in $t+1$ k_{t+1} is given as (10) and (14). The public debt per unit of effective labor in $t+1$ b_{t+1} is given as (10). Then, the wage rate w_t and interest rate $1+r_t$ are given respectively as (7) and (8). Consumption c_{1t} and c_{2t+1} are obtainable. Consequently, we can obtain all endogenous variables in the equilibrium.

However, the steady state does not always exist. The case of Fig. 5(a) shows that the two steady state equilibrium exist. As shown by Fig. 5(b), there exists the case of no steady state equilibrium.

[Insert Fig. 5(a) around here.]

[Insert Fig. 5(b) around here.]

In the case of Fig. 5(b), the public debt per unit of effective labor b continues increasing. However, in the case of Fig. 5(a), if an initial b exists at the neighborhood of the steady state equilibrium P, b converges to the steady state equilibrium. Our paper presents consideration that this case holds fiscal sustainability.

4. Steady State

In this section, we present derivation of the steady state equilibrium. Defining k, b as the physical capital stock per unit of effective labor and public debt per unit of effective labor, respectively, k is given as (12) and (14) as

$$\left(1 + \frac{\alpha\varepsilon(1-\gamma)}{\gamma}\right)k = \frac{(1-\tau)(1-\alpha)A^{1-\delta}(1-\gamma)^{1-\delta}}{H\phi^\delta}k^{\gamma(1-\delta)} - \frac{(\phi + \varepsilon - \tau)A(1-\gamma)}{H\phi^\delta A^\delta(1-\gamma)^\delta - A\gamma k^{-(1-\gamma(1-\delta))}}k^{\gamma(1-\delta)}. \quad (17)$$

Calculating (17), we can obtain

$$\left(1 + \frac{\alpha\varepsilon(1-\gamma)}{\gamma}\right)k^{1-\gamma(1-\delta)} = \frac{(1-\tau)(1-\alpha)A^{1-\delta}(1-\gamma)^{1-\delta}}{H\phi^\delta} - \frac{(\phi + \varepsilon - \tau)A(1-\gamma)}{H\phi^\delta A^\delta(1-\gamma)^\delta - A\gamma k^{-(1-\gamma(1-\delta))}}. \quad (18)$$

Defining L as the left-hand side of (18) and R as the right-hand side of (18), the steady state equilibrium is obtainable as the two intersect points shown in Fig. 6.

[Insert Fig. 6 around here.]

5. Policy Effects

This section presents examination of the policy of a decrease in the primary deficit by decreases in public education ϕ and pension benefit ε , and an increase in tax rate τ at the stable steady state equilibrium.

5.1 Decrease in pension benefits

A decrease in pension benefit ε pulls down the loci of $\Delta b_t = 0$, as shown by (12) and Fig. 7. As shown by (15) and Fig. 7, a decrease in ε pulls up the loci of $\Delta k_t = 0$ because of an increase in incentives for saving.

[Insert Fig. 7(a) around here.]

[Insert Fig. 7(b) around here.]

At the stable steady state equilibrium, a decrease in ε reduces public debt per unit of effective labor b and increases the physical capital stock per unit of effective labor k in the case of $k <$

$\left[\frac{A^{1-\delta}}{(1-\delta)H\phi^\delta(1-\gamma)^\delta}\right]^{\frac{1}{1-\gamma(1-\delta)}}$, as shown by Fig. 7(a). However, as shown by Fig. 7(b), the public debt per

unit of effective labor b can be pulled up in the case of $k > \left[\frac{A^{1-\delta}}{(1-\delta)H\phi^\delta(1-\gamma)^\delta}\right]^{\frac{1}{1-\gamma(1-\delta)}}$. By virtue of an increase in k , the human capital growth rate given by (11) increases. Then, the following proposition can be established.

Proposition 1

A decrease in pension benefit ε raises the capital stock per unit of effective labor at the stable steady state. Then, with $k < \left[\frac{A^{1-\delta}}{(1-\delta)H\phi^\delta(1-\gamma)^\delta} \right]^{\frac{1}{1-\gamma(1-\delta)}}$, the public debt stock per unit of effective labor decreases. Alternatively, the public debt stock per unit of effective labor can be pulled up. The income growth rate $1 + g$ always rises.

A decrease in pension benefit ε increases the capital stock per unit of effective labor k directly. However, an increase in k has the effect of an increase in the net government expenditure $(\phi + \varepsilon - \tau)w$. Then the public debt per unit of effective labor b can be pulled up. However, as long as $k < \left[\frac{A^{1-\delta}}{(1-\delta)H\phi^\delta(1-\gamma)^\delta} \right]^{\frac{1}{1-\gamma(1-\delta)}}$, b is always pulled down.

Because of $\frac{B_t}{Y_t} = \frac{b_t}{Ak_t}$, one can check the ratio of public debt to GDP. As shown by Proposition 1,

b decreases and k increases. As a result, the ratio of public debt to GDP decreases.

5.2 Decrease in public education

A decrease in public education ϕ pulls up the loci of $\Delta k_t = 0$, as shown by (15). However, the effect on the loci of $\Delta b_t = 0$ is ambiguous because a decrease in ϕ reduces the deficit of primary balance, but the public debt per unit of effective labor increases because a decrease in ϕ reduces the human capital accumulation or effective labor.³ Therefore, the effects on k and b are ambiguous.

5.3 Increase in the income tax rate

An increase in income tax rate τ decreases the deficit of primary balance. The loci of $\Delta b_t = 0$ is pulled down. However, an increase in income tax rate τ pulls up the loci of $\Delta k_t = 0$ because an increase in τ increases the physical capital stock thanks to a decrease in the crowding out effect on k . Shifts of $\Delta b_t = 0$ and $\Delta k_t = 0$ are the same with Fig. 7(a) and Fig. 7(b). Then, the following proposition can be established.

Proposition 2

An increase in income tax rate τ raises the capital stock per unit of effective labor at the stable steady

³ Defining $m = \frac{(\phi + \varepsilon - \tau)A(1-\gamma)}{H\phi^\delta A^\delta (1-\gamma)^\delta - A\gamma k_t^{-(1-\gamma(1-\delta))}}$, one can obtain $\frac{\partial m}{\partial \phi} = \frac{(1-\delta)\phi^\delta - \frac{A\gamma k_t^{-(1-\gamma(1-\delta))}}{HA^\delta(1-\gamma)^\delta} - \delta\phi^{\delta-1}(\varepsilon-\tau)A(1-\gamma)}{(\phi + \varepsilon - \tau)(\phi^\delta - \frac{A\gamma k_t^{-(1-\gamma(1-\delta))}}{HA^\delta(1-\gamma)^\delta})}$ as the

differentiation form. The sign of $\frac{\partial m}{\partial \phi}$ is ambiguous.

state. Then, with $k < \left[\frac{A^{1-\delta}}{(1-\delta)H\phi^\delta(1-\gamma)^\delta} \right]^{\frac{1}{1-\gamma(1-\delta)}}$, the public debt stock per unit of effective labor decreases. Alternatively, the public debt stock per unit of effective labor can be pulled up. The income growth rate $1 + g$ always rises.

An increase in τ reduces private saving. However, a direct decrease in the primary deficit facilitates physical capital accumulation. Finally, k increases. Because of an increase in k , the net government expenditure $(\phi + \varepsilon - \tau)w$ can be raised. Then, the ratio of public debt to GDP decreases.

We devote attention to whether welfare can be improved or not by a decrease in public debt. A decrease in pension benefit reduces the welfare of older people during the reform of a decrease in the public debt. Welfare in the future period can be pulled up by virtue of an increase in the output per unit of effective labor. However, this is not a Pareto-improving policy.

An increase in the income tax rate can raise the welfare of all generations. An increase in the tax rate reduces the welfare of the younger people at the reform. However, in the old period, the pension benefit can be pulled up by virtue of an increase in the output per unit of effective labor. For subsequent generations, an increase in the output per unit of effective labor has a positive effect on the welfare. Therefore, if the effect of the output per unit of effective labor is strong, then this can be a Pareto-improving policy.

Yakita (2008) sets the productive government expenditure model with public debt. This setting resembles the setting of education investment in this paper. Maebayashi (2013) sets the model with pension benefit and productive government expenditure without public debt. Compared with these studies, we can derive that cutting of pension benefits reduces the public debt per unit of effective labor. This result is not obtained in related reports of the relevant literature. In addition, an increase in the income tax rate can raise the capital stock per unit of effective labor because of a decrease in the public debt per unit of effective labor. This result can be achieved using the public debt model.

6. Conclusions

Our paper presents an examination of how a decrease in the primary balance deficit affects the income growth rate and the level of public debt stock. A decrease in the pension benefit for the older period can invariably increase the capital stock per unit of effective labor. Depending on the level of the capital stock per unit of effective labor, a decrease in the pension benefit reduces the public debt per unit of effective labor directly and the ratio of the public debt to GDP. This result is obtained using the case of an increase in the income tax rate.

This result is interesting. Even if the government cuts the pension benefit and levies income taxation on households to decrease the public debt stock as one pillar of fiscal reform, the public debt increases conversely, depending on the level of the capital stock per unit of effective labor.

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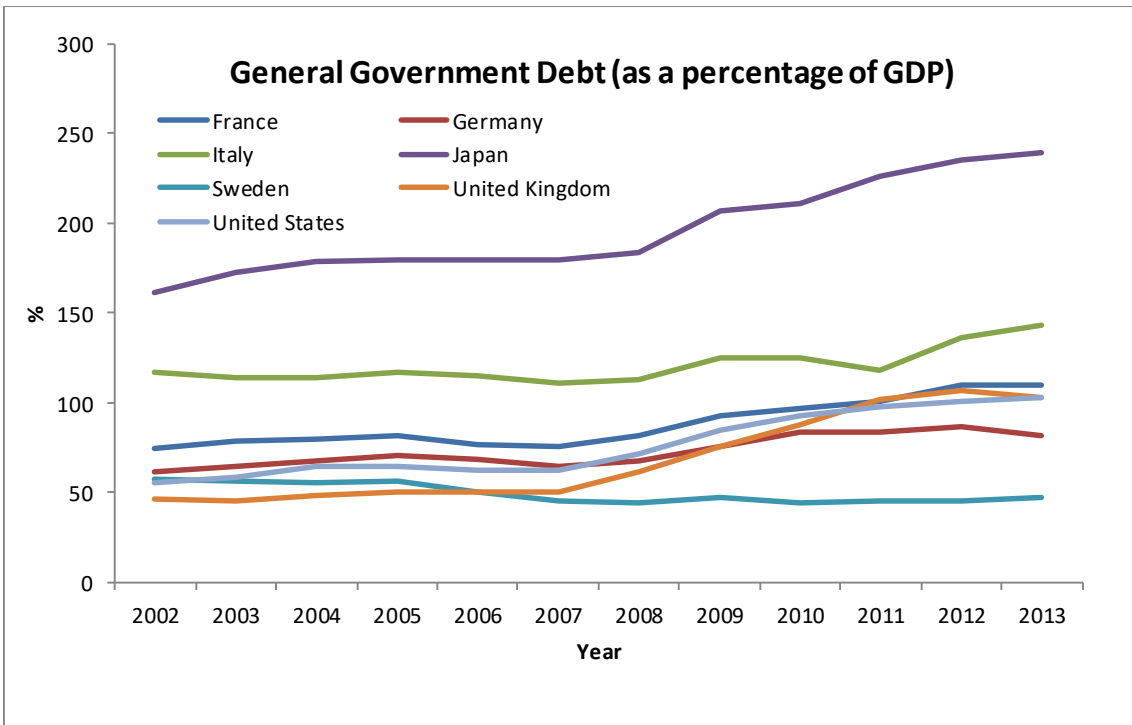


Fig. 1: General Government Debt (as a percentage of GDP).
 (Data: OECD Factbook Economic, Environmental and Social Statistics)

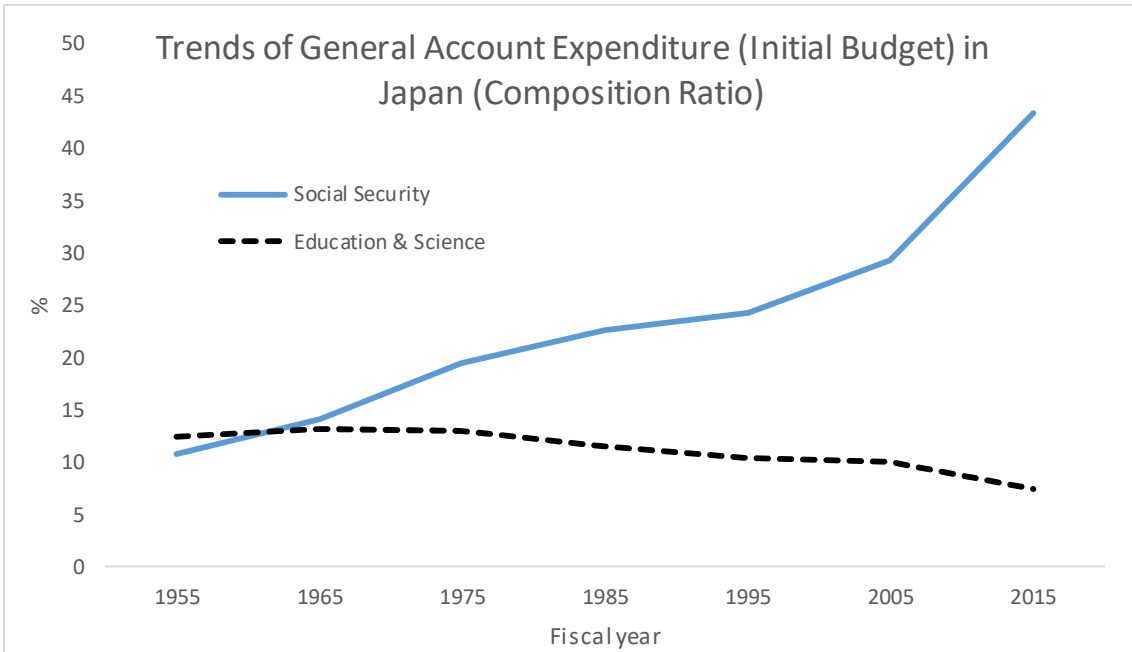


Fig. 2: Trends of General Account Expenditure (Initial Budget in Japan) (Composition ratio).
 (Data: Ministry of Finance Japan 'Trends of General Account Expenditure')

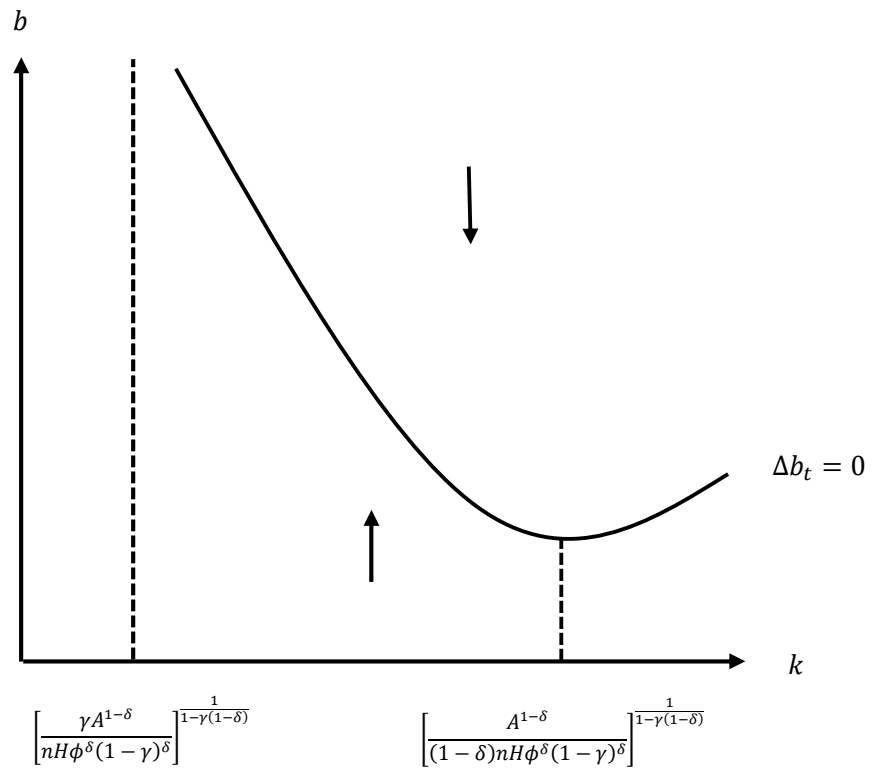


Fig. 3: Phase of b .

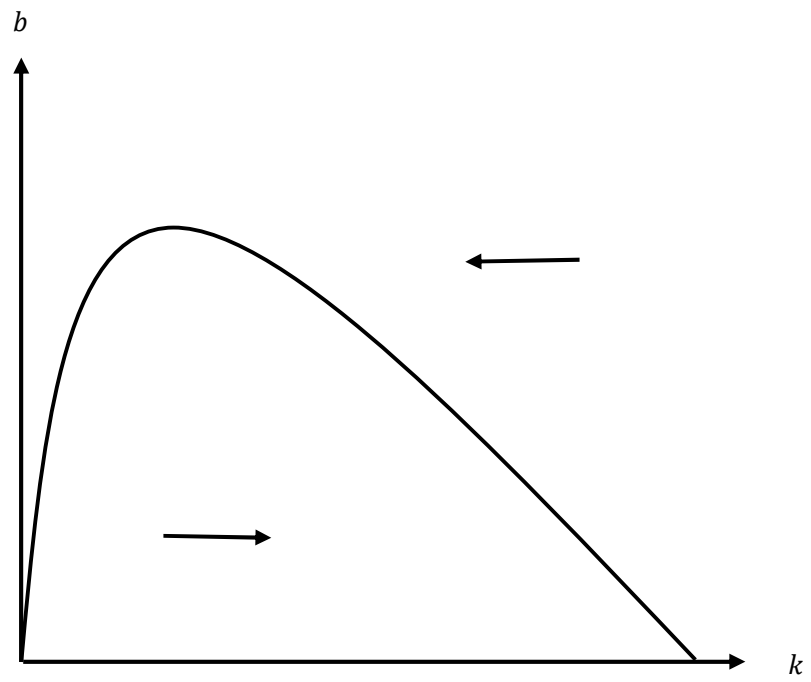


Fig. 4: Phase of k .

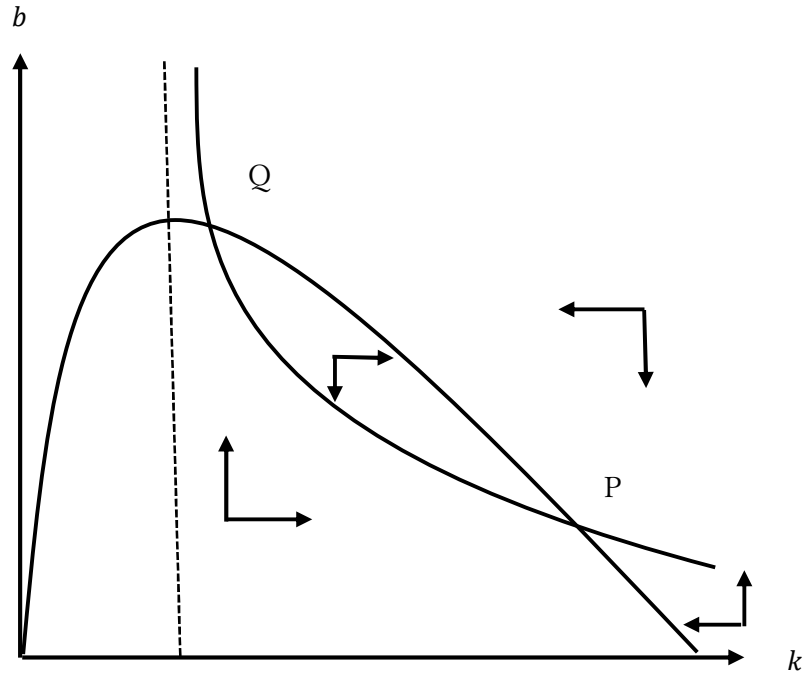


Fig. 5a: Two Steady State Equilibrium (P is sink; Q is the saddle path).

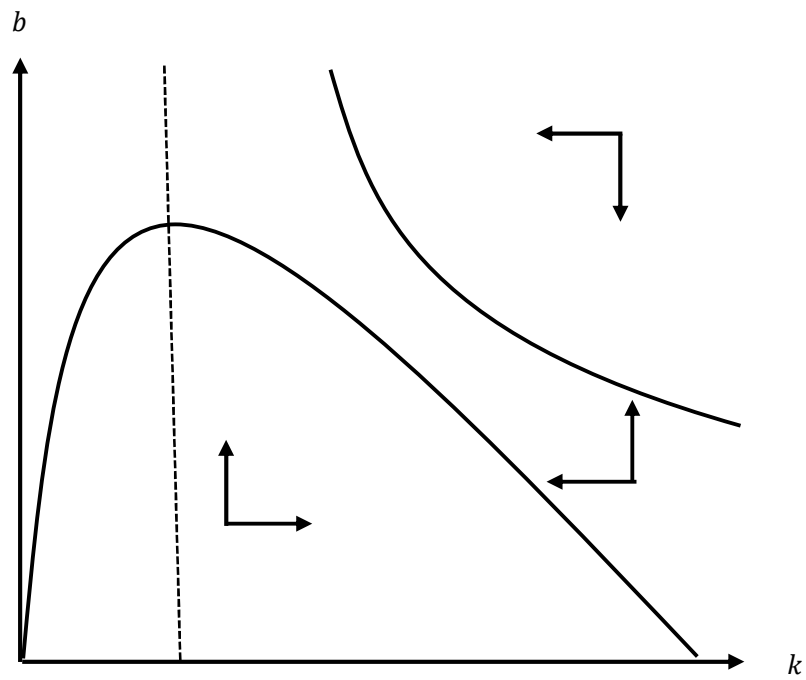


Fig. 5b: No steady state equilibrium.

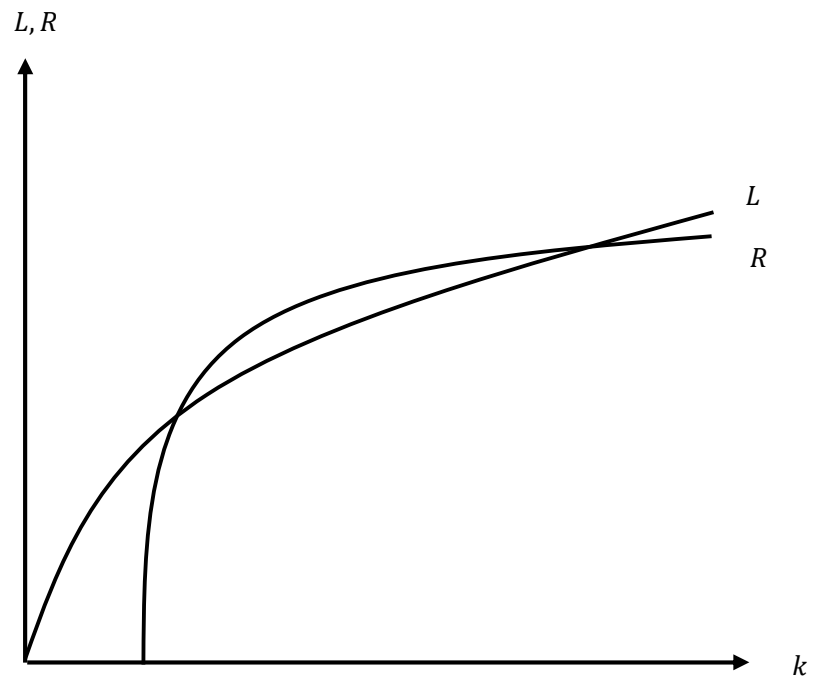


Fig. 6: Existence of steady state equilibrium.

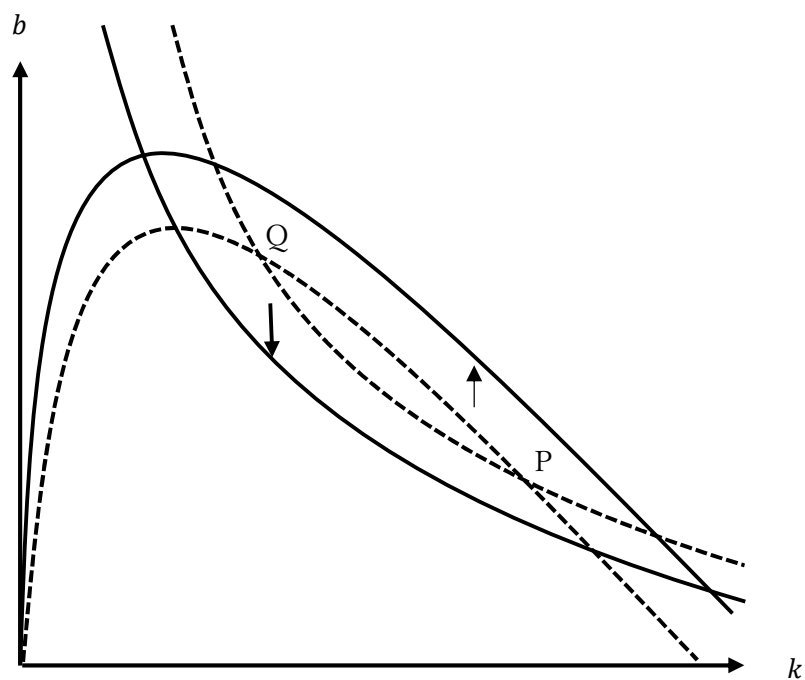


Fig. 7(a): Effects of a decrease in pension benefit on the public debt in the case of $k <$

$$\left[\frac{A^{1-\delta}}{(1-\delta)H\phi^\delta(1-\gamma)^\delta} \right]^{\frac{1}{1-\gamma(1-\delta)}}.$$

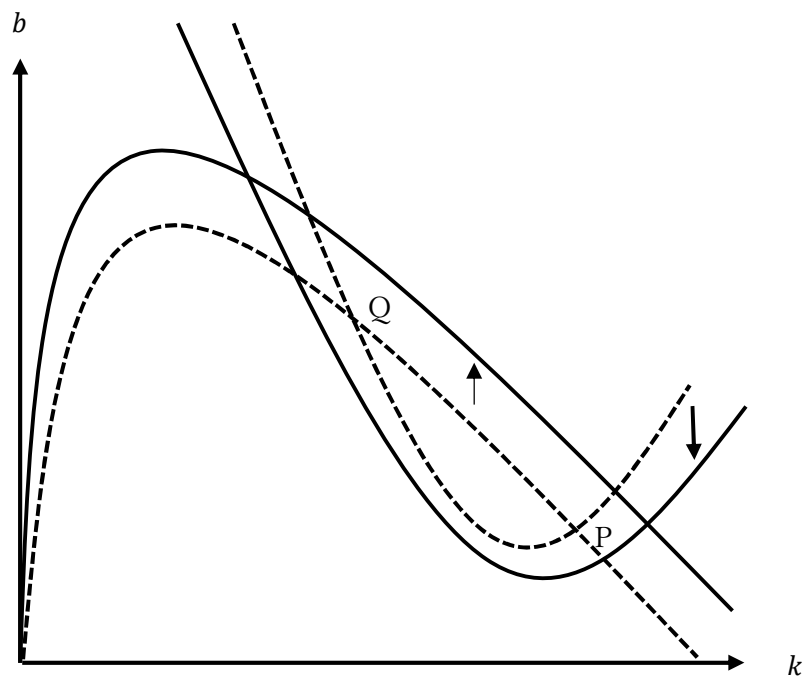


Fig. 7(b): Effects of a decrease in pension benefit on the public debt in the case of $k >$

$$\left[\frac{A^{1-\delta}}{(1-\delta)H\phi^\delta(1-\gamma)^\delta} \right]^{\frac{1}{1-\gamma(1-\delta)}}.$$