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Credit constraints, firm selection, and endogenous growth: The international transmission of a credit crunch via trade

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Credit constraints, firm selection, and endogenous growth: The international transmission of a credit crunch via trade*

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Abstract

This study develops a two-country, heterogeneous-firm model of trade and growth with country-specific credit constraints to examine the role of firm selection as a channel for the international transmission of a credit crunch in one country. For a permanent credit crunch, we derive the following results analytically. First, the long-run growth rate of the global economy inevitably declines. Second, even in a country not directly affected by the credit crunch, its growth rate declines in both the short run and the long run. Third, this decline occurs because export profitability declines in that country's tradable goods sector, prompting firms to shift from exporting to selling domestically. In addition, we conduct a numerical analysis to compare the effects of a credit crunch under two scenarios: financial autarky (i.e., trade in goods only) and financial integration. We find that, irrespective of the credit crunch being permanent or temporary, the effects on trade and growth are similar in both scenarios, suggesting that regulating international financial transactions may fail to mitigate the cross-border transmission of the shock.

JEL classification: F12; F43; O16; O41
Keywords: Endogenous growth; Heterogeneous firms; Asymmetric countries; Financial frictions; Country-specific credit crunch

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1 Introduction

Since the 2007–2009 financial crisis, many developed countries have experienced prolonged recessions.[1] Against this background, several studies have integrated mechanisms of endogenous growth into business cycle models to explore how adverse financial shocks lead to secular stagnation. Although these findings are insightful, most studies have focused on closed economies. Therefore, to theoretically elucidate the slow recovery of an economy, models must rely on the occurrence of adverse financial shocks within that economy.

It has been recognized, however, that not all depressed countries encountered significant issues in their domestic financial markets following the financial crisis. For instance, the IMF (2009) estimated the potential valuation loss for the U.S. financial sector from 2007 to 2010 to be \$2.712 trillion. In contrast, although Japan experienced a substantial decline in its GDP after the financial crisis, the potential valuation loss for the Japanese financial sector was estimated at \$149 billion, approximately only 5% of the U.S. loss.[2] Based on this observation, another line of research has examined how financial shocks originating in one country spread to others through international trade or financial transactions. Nonetheless, most of these studies assume the long-run growth rate to be exogenous. Consequently, while it is known that the financial crisis impacted global economic activity in both the short run and long run, economic growth models capable of analyzing this effect have not yet been adequately developed.

This study investigates how a country-specific credit crunch influences global economic growth through trade and firm selection dynamics. To this end, we develop a two-country endogenous growth model that integrates heterogeneous firms, international trade, and financial frictions. In contrast to existing models that assume closed economies or exogenous growth, our framework captures the long-run consequences of financial shocks via trade reallocation and firm dynamics. In our model, the long-run growth rate of the world economy is determined by the expansion of intermediate good varieties as in River-Batiz and Romer's (1991) canonical open-economy endogenous growth models. To incorporate endogenous firm exits in response to shocks into the model, following Melitz (2003), we introduce firm heterogeneity into the intermediate goods sector. In addition, each country has a sector comprising heterogeneous entrepreneurs who produce a non-tradable input used for domestic intermediate goods firms' fixed inputs to enter and serve the domestic and export markets. Following Baldwin and Robert-Nicoud (2008), Ourens (2016), and Naito (2017b, 2019, 2021), we refer to such input as the "knowledge good" for convenience. A feature of our model is

[1]The average growth rate in the U.S. for the decade 1997–2007 was 2.06%, while that for the following decade was only 0.66%. Even when we exclude the two years of the Great Recession (2007 and 2008) and change the sample decade to 2009–2019, the average growth rate was only 1.51%. In the 20 countries in the Euro area, the average growth rates were 1.95% and 1.15% for 1997–2007 and 2009–2019, respectively. The data are obtained from OECD Statistics (https://stats.oecd.org/).

[2]The IMF also estimated total financial sector write-downs of \$1.193 trillion for the 21 countries comprising the Euro area and the U.K. Taking the number of countries into account, the devaluation is relatively small compared to that of the U.S. See Table 1.3 in the IMF (2009, p.35) for more details.

that entrepreneurs face credit constraints. Most static Melitz models assume that labor is the only fundamental factor of production and that firms need labor as a fixed input to enter and serve each market. However, firms often require specialized knowledge, such as market research consulting and supply chain improvement, to enter markets. The creation of such an intangible input in our model can be broadly interpreted as research and development (R&D) activities in the real economy. Several empirical studies, as discussed below, have demonstrated that small and medium-sized firms tend to face credit constraints in their R&D activities because of credit market imperfections. This tendency is incorporated into the model.

Within this framework, we derive the following analytical results for permanent credit crunches. First, a country-specific credit crunch reduces long-run growth not only in the crisis-hit country but also in the unaffected country. Second, while the short-run growth effects differ between the two economies, the unaffected country's growth rate remains persistently below its pre-crisis balanced growth path (BGP) level. Depending on the severity of the shock, the unaffected country may undergo a prolonged period of slower growth during the transition to the new BGP, potentially even longer than that experienced by the crisis-hit country. Third, these growth dynamics stem from the credit crunch's asymmetric impact on the productivity thresholds of intermediate goods firms. In the affected country, the shock accelerates firms' exit, whereas in the unaffected country, it diminishes export profitability, prompting firms to reorient their production toward the domestic market. Thus, both economies converge to lower growth rates along the new BGP despite taking different short-run adjustment paths.

We also compare the effects of a credit crunch under financial autarky and financial integration. After analytically deriving the BGP under the assumption of initial symmetry between the two countries, we extend our analysis to a general case using numerical methods. In both scenarios, we find that the impact of a country-specific credit crunch on the BGP growth rate and the productivity cutoffs of intermediate goods firms does not differ significantly from the case without international lending and borrowing. This result holds even in the case of a temporary credit crunch, as confirmed by our numerical simulations.

Our findings highlight the significant role of trade in goods as a transmission channel for financial shocks. Specifically, country-specific financial disturbances propagate internationally through shifts in the supply and demand of goods, substantially influencing global economic activity. These results align with the empirical evidence from historical financial crises. For instance, Forbes (2004) demonstrates that trade linkages, rather than financial linkages, serve as the primary mechanism for the cross-border transmission of crises. This conclusion is further supported by Claessens et al. (2012), who find that trade connections played a more substantial role than financial ties in crisis spillovers during the 2007–2009 global financial crisis. Similarly, Hosono et al. (2016) show that Japanese firms were primarily affected by the 2007–2009 crisis through trade-related channels rather than financial linkages.

The intuition underlying these results can be explained as follows. For convenience, let us refer to the two countries as "country 1" and "country 2" and assume that a credit crunch occurs in country

1. A credit crunch reduces entrepreneurs' borrowing capacity, which in turn lowers the productivity of aggregate capital in producing the knowledge good. This productivity decline increases the price of the knowledge good in country 1. Since the knowledge good serves as a fixed input for intermediate goods firms, a higher price accelerates the exit of firms after productivity realization, ultimately reducing country 1's growth rate. From the perspective of country 2's intermediate goods firms, the credit crunch in country 1 reduces export demand, making foreign sales less profitable than domestic sales. Consequently, the productivity cutoff for domestic activities in country 2 decreases while the export cutoff rises. This shift reduces the demand for the knowledge good in country 2, leading to a fall in its real price. As a result, entrepreneurs' income flows diminish, slowing net worth accumulation and, consequently, economic growth as well in country 2.

1.1 Related literature

This study is situated within three distinct lines of literature. First, it aligns with the literature on the impacts of adverse financial shocks on real economic activity. This body of literature can be further divided into two categories. The first includes studies that integrate endogenous growth and business cycle dynamics, such as Bianchi et al. (2019), Guerron-Quintana and Jinnai (2019), Ikeda and Kurozumi (2019), and Queralto (2020). A common feature of these studies is that they conduct quantitative analyses by introducing the mechanisms of R&D-driven endogenous growth and financial frictions into DSGE models to assess the extent to which temporary financial shocks contribute to economic downturns in a single country. The first three studies assume closed economies, whereas the last study assumes a small open economy.[3] In contrast, we conduct a specific qualitative analysis to analytically clarify the spillover effects of a financial shock from one country to the other and its impacts on both short and long-run growth rates. Another stream of research uses two-country models to elucidate the mechanism of international transmission of financial shocks. Examples of such studies include Devereux and Yetman (2010), Devereux and Sutherland (2011), Kollmann et al. (2011), Dedola and Lombardo (2012), Perri and Quadrini (2018), and Yao (2019). Devereux and Yetman (2010) deal with the international transmission of productivity shocks in one country, while the others use financial shocks in one country as the trigger event for a financial crisis. However, their open-economy models assume the long-run growth rate to be exogenous. In addition, several studies have analyzed the collapse of international trade during the 2007–2009 global financial crisis.[4] For instance, Eaton et al. (2016) incorporated the trade structure of Eaton and Kortum (2002)

[3] In these studies, the sources of long-run growth also differed. Bianchi et al. (2019) employed a growth model with vertical innovations, whereas the studies of Guerron-Quintana and Jinnai (2019), Ikeda and Kurozumi (2019), and Queralto (2020) used an expanding variety framework, that is, horizontal innovations. Recently, Ohdoi (2024) developed a simple model featuring the banking sector, financial frictions, and endogenous growth with quality ladders, and shows that endogenous R&D investment and a shock hindering banks' financial intermediary function can be key to generating both a prolonged recession and a drop in firms' stock prices.

[4] During the financial crisis, especially in 2008Q4 and 2009Q1, the volume of international trade declined more significantly than the decline in GDP. See, for example, Figure 1 in Bems et al. (2013, p.377).

into a dynamic general equilibrium model to quantify the impacts of various shocks on the world economy. Moreover, in this study, the long-run growth rate is assumed to be zero. Therefore, the implications for the long-run growth rate are outside the scope of their analysis.

Second, our study is closely related to the studies building on dynamic trade models with endogenous growth and heterogeneous firms. Baldwin and Rebert-Nicoud (2008) were among the first to posit an endogenous growth version of Melitz's (2003) model. Specifically, they introduced firm heterogeneity into the endogenous growth model with expanding variety and two symmetric countries and demonstrated that whether trade opening enhances higher growth depends on the degree of international spillovers of R&D. Ourens (2016) re-examined the welfare effect of trade opening in Baldwin and Rebert-Nicoud's symmetric two-country model. Sampson (2016) incorporated technology diffusion into the multi-country model, where the productivity of new entrants is drawn from the distribution of the productivity of incumbent firms. However, although the author showed that trade opening always raises welfare through faster growth and static welfare gains, they still assumed symmetric countries. Haruyama and Zhao (2017) embedded firm heterogeneity into a two-country Schumpeterian growth model characterized by creative destruction while assuming that the two countries are symmetric.[5] By contrast, Naito (2017a) introduced Melitz-type firm heterogeneity and international asymmetry in several aspects into an AK growth model. Using this framework, he demonstrated that a unilateral decline in iceberg trade costs in one country raises the growth rates of all countries at all points in time and always improves their welfare. In addition, Naito (2017b) introduced the international asymmetry in trade costs into Baldwin and Rebert-Nicoud's (2008) model and examined the growth and welfare effects of trade liberalization in the form of its unilateral decline.[6] Thus, the source of asymmetry in these studies is the international difference in trade costs, and credit market imperfections are not considered.

Finally, our study is related to the growing body of literature on the effect of financial market imperfections on international trade. Here, we focus on studies on trade combining financial frictions and Melitz-type firm heterogeneity.[7] A typical study is Chaney (2016), who constructed a static two-country model in which firms are subject to financial constraints on fixed costs to enter the export markets. Similarly, Feenstra et al. (2014) analyzed a static two-country model, whereas Manova (2013) considered a multi-country model with financial frictions. All of these studies constructed static models. By contrast, Kohn et al. (2016) explored a dynamic, small open economy model with international lending and borrowing. The authors built a model without capital in which a part

[5]Several studies have relaxed the assumption of symmetric countries in dynamic Melitz models while abstracting the mechanism of endogenous growth. Examples of such studies include Bonfiglioli et al. (2019) and Brooks and Dovis (2020).

[6]While Naito (2017b) used the knowledge-driven specification of R&D activities, Naito (2019) employed the lab-equipment specification and showed that under the lab-equipment specification, unilateral trade liberalization always enhances the long-run growth and welfare of both countries. Replacing the iceberg trade costs with tariffs in Naito (2019), Naito (2021) examined whether the optimal tariff can be zero.

[7]If we include the literature using other trade models, many studies have introduced financial frictions. See, for example, Matsuyama (2005) and Antrás and Caballero (2009).

of working capital (wage payments) is subject to collateral constraints. Given that firms use their capital stocks as collateral so that their borrowings are limited by the levels of capital holdings, Kohn et al. (2020) introduced capital into their earlier study. The authors conducted a numerical analysis and found that a stronger financial constraint reduces exports.[8]

1.2 Facts motivating this study

As previously mentioned, in our model, entrepreneurs produce an intangible good called the "knowledge good." This type of good serves as a fixed input for firms entering the tradable goods sector and for supplying goods to both domestic and export markets. In our model, the knowledge creation can be interpreted as an R&D activity in the real economy. Numerous scholars have pointed out the possibility that an R&D investment is limited by credit constraints (sometimes more than the usual physical investment). To the best of our knowledge, Himmelberg and Petersen (1994) were the first to empirically clarify this possibility. Using data from 179 small firms in the U.S. high-tech industry, they found that a significant positive relationship exists between R&D investment and cash flow; that is, internal funds. Based on this result, the authors argued that external financing may not be sufficiently available for R&D investments. Empirical studies that employ an approach similar to that of Himmelberg and Petersen include Harhoff (1998) and Bond et al. (2006).

However, Kaplan and Zingales (2000) argued that the cash flow sensitivity of R&D investment is not an appropriate measure for credit constraints. Consequently, recent studies have more directly investigated firms' access to external funds. Bakhtiari et al. (2020) reviewed the literature on financial constraints and the performance of small and medium-sized firms. Among others, Czarnitzki and Hottenrott (2011) created a credit rating index representing the degree of access to external funding ranging from 1 to 6 (6=the best value), and used it as an explanatory variable for credit constraints to demonstrate that easing credit constraints encourages R&D investment for German firms. They also divided firms into four categories based on their size and created interaction terms for the credit rating index with firm size. Their results indicate that smaller firms tend to face more severe credit constraints, suggesting that R&D-oriented small firms are more likely to face difficulties in acquiring external financing. Hottenrott and Peters (2012) conducted a survey of German firms in which they asked them to imagine receiving additional funds equal to 10% of their previous year's turnover, and then specify how they would allocate such hypothetical additional funds. If firms respond that they would use the additional funds for R&D projects, we can conclude that their R&D activities are financially constrained. Specifically, they revealed that financial constraints are driven by innovative capabilities through increasing resource requirements.

Thus, it has been empirically proven that firms engaged in R&D activities (i.e., the firms creating the knowledge good) face credit constraints. Nevertheless, the empirical studies do not reveal the reasons for credit constraints. Accordingly, we assume that entrepreneurs producing the knowledge

[8]See also Leibovici (2021). Kohn et al. (2022) present a useful overview of the studies on financial frictions and trade.

good face credit constraints through the same mechanism as Moll (2014) and Itskhoki and Moll (2019). In this setting, the external funds available to entrepreneurs are proportional to their net worth and, accordingly, their knowledge good creation is proportional to their net worth. Thus, entrepreneurs' size and access to external funds are positively correlated, which is consistent with Czarnitzki and Hottenrott's empirical results to some extent.

1.3 Organization of the paper

The remainder of this paper is organized as follows. Sections 2 and 3 set up the model and characterize the two-country equilibrium under financial autarky (i.e., balanced trade), respectively. Section 4 examines the effects of the credit crunch occurring in one country on both countries' firm selections and growth rates. Section 5 alters the model to include international lending and borrowing and investigates the effects of the credit crunch in this case. Section 6 performs numerical analysis to compare the effects of the credit crunch under financial autarky and financial integration to assess whether regulating international financial transactions can mitigate the cross-border transmission of the shock. Section 7 discusses how the results change when each of the underlying assumptions and restrictions is removed. Finally, Section 8 concludes the paper.

2 Model

Time is continuous and indexed by $t \in [0, \infty)$. The world economy comprises two countries: country 1 and country 2. We attach the asterisk "$*$" to variables and parameters in country 2, except in cases where they take the same values as those in country 1. Each country has a non-tradable final good sector, a tradable intermediate good sector, and a non-tradable knowledge creation sector. There are two primary factors, labor and capital, both of which can not move internationally. Thus, the two countries are interrelated through trade in intermediate goods. To focus on this type of trade, we assume for the moment that no other international transactions exist. In Section 5, we consider a world economy with integrated financial markets. In Section 7.2, we consider a situation in which the final good is also tradable.

2.1 Final good sector

We omit the time subscript unless doing so would cause confusion. The final good is produced from differentiated intermediate goods and labor under constant returns to scale and perfect competition.

$$Y = \frac{1}{\alpha} L^{1-\alpha} \left(\int_{\omega \in \Omega} x(\omega)^{\alpha} d\omega \right),$$

where Y is the output, L is the demand for labor, Ω is the set of available varieties of intermediate goods, $x(\omega)$ is the demand for variety $\omega \in \Omega$, and $\alpha \in (0, 1)$ is the cost share of intermediate goods. The elasticity of substitution between any two pairs of varieties is $1/(1-\alpha) > 0$. We include the term $1/\alpha$ for notational simplicity.[9]

Let P denote the final good's price, w denote the wage rate, and $p(\omega)$ denote the demand price of variety ω. Profit maximization under perfect competition yields the following demand functions:

$$L = (1-\alpha)PY/w,$$
$$x(\omega) = p(\omega)^{-1/(1-\alpha)}LP^{1/(1-\alpha)}, \omega \in \Omega.$$

In country 2,

$$L^* = (1-\alpha)P^*Y^*/w^*,$$
$$x^*(\omega) = p^*(\omega)^{-1/(1-\alpha)}L^*P^{*1/(1-\alpha)}, \omega \in \Omega^*.$$

2.2 Intermediate goods sector

Under monopolistic competition, each firm produces an intermediate good using the knowledge and final goods as the fixed and variable inputs, respectively. We follow Naito (2017a) and assume that at every point in time, firms must pay a fixed entry cost and then draw their productivity, denoted by φ. The fixed entry cost at time t is given by $P_{K,t}f_E$, where P_K is the price of the knowledge good and $f_E > 0$ is the fixed amount of the knowledge good required for entry.

Let $G(\varphi)$ denote the cumulative distribution function of φ. Given the realization of φ, firms decide their options among exiting markets, serving only the domestic market, and serving both the domestic and export markets. Let $j(= D, X)$ denote the market index. The fixed cost for market j is given by $P_{K,t}f_j$, where $f_j > 0$ is the fixed amount of the knowledge good to serve market j. As in much of the trade literature, exports are subject to trade costs; if the final good firm in country 2 wants to use one unit of an intermediate good produced in country 1, it must import $\tau \geq 1$ units of this good.

Hereafter, we drop the notation ω because it is sufficient to identify each firm with its location and productivity level φ. Let $y_D(\varphi)$ and $p_D(\varphi)$ denote the output and supply prices for the domestic market, respectively. Similarly, let $y_X(\varphi)$ and $p_X(\varphi)$ denote the output and supply prices for the export market, respectively. That is, $x^* = y_X/\tau$ by definition, and the demand and supply prices are related such that $p^* = \tau p_X$. In this stage, the firm maximizes its profit in market $j(= D, X)$:

$$\pi_j(\varphi) \equiv \left(p_j(\varphi) - \frac{P}{\varphi}\right) y_j(\varphi) - P_K f_j,$$

subject to the following demand conditions:

$$y_D(\varphi) = p_D(\varphi)^{-1/(1-\alpha)}LP^{1/(1-\alpha)}, \tag{1}$$
$$\frac{y_X(\varphi)}{\tau} = (\tau p_X(\varphi))^{-1/(1-\alpha)}L^*P^{*1/(1-\alpha)}. \tag{2}$$

Profit maximization results in the supply price in market j as

$$p_j(\varphi) = \frac{P}{\alpha\varphi}. \tag{3}$$

[9]See, for example, Acemoglu (2009, Ch.13).

Let φ_j denote the cutoff level of productivity in market j, defined in the same manner as in Melitz (2003): $\pi_j(\varphi_j) = 0$. The output of a firm whose productivity is just the cutoff productivity is expressed as

$$y_j(\varphi_j) = \frac{\alpha\varphi_j}{1-\alpha} f_j q, \tag{4}$$

where $q \equiv P_K/P$ denotes the real price of the knowledge good, that is, the price of this good in terms of the domestic final good.

From (1)–(3), we can express the output $y_j(\varphi)$ for any φ as $y_j(\varphi) = (\varphi/\varphi_j)^{1/(1-\alpha)} y_j(\varphi_j)$, which is a well-known result of the Melitz model. Substituting this result and (3)–(4) into $\pi_j(\varphi)$ yields

$$\pi_j(\varphi) = P_K f_j \left[(\varphi/\varphi_j)^{\alpha/(1-\alpha)} - 1 \right].$$

In this equation, the term $(\varphi/\varphi_j)^{\alpha/(1-\alpha)}$ can be interpreted as the firm's productivity relative to the exit cutoff in market j. We let $H(\varphi_j)$ denote its aggregate over the firms surviving in market j:

$$H(\varphi_j) \equiv \int_{\varphi \geq \varphi_j} (\varphi/\varphi_j)^{\alpha/(1-\alpha)} \, dG(\varphi), \quad H'(\varphi_j) = -\frac{\alpha}{1-\alpha} \frac{H(\varphi_j)}{\varphi_j} - G'(\varphi_j) < 0.$$

The free entry condition requires that the fixed entry cost equals the sum of the expected net profits across all markets. Because firms pay a fixed entry cost and draw their productivity at each point in time, the free entry condition is

$$P_K f_E = \int_{\varphi \geq \varphi_D} \pi_D(\varphi) dG(\varphi) + \int_{\varphi \geq \varphi_X} \pi_X(\varphi) dG(\varphi).$$

Following the literature, we let

$$\Pi(\varphi_j) \equiv \int_{\varphi \geq \varphi_j} \left[(\varphi/\varphi_j)^{\alpha/(1-\alpha)} - 1 \right] dG(\varphi) = H(\varphi_j) - (1 - G(\varphi_j)),$$
$$\Pi'(\varphi_j) = -\frac{\alpha}{1-\alpha} \frac{H(\varphi_j)}{\varphi_j} < 0.$$

From the definition of the cutoff φ_j, the term $\int_{\varphi_j}^{\infty} \pi_j(\varphi) dG(\varphi)$ can be rewritten as $\Pi(\varphi_j) f_j P_K$. Therefore, the free entry condition is rewritten as

$$f_E = \sum_{j=D,X} f_j \Pi(\varphi_j). \tag{5}$$

Let N^e denote the number of entrants, which is endogenously determined as discussed below. Then, the number of surviving firms in market j is given by $(1 - G(\varphi_j))N^e$. Accordingly, the measure of Ω is given by the sum of $(1 - G(\varphi_d))N^e$ and $(1 - G(\varphi_x^*))N^{e*}$. Because entry and exit occur at each point in time, the total profit of intermediate goods firms becomes zero.[10]

We assume that the fixed inputs (f_E, f_D, f_X) and the distribution function G are the same between the two countries. Let $q^* \equiv P_K^*/P^*$ denote the real price of the knowledge good in country

[10] The ex-post profits of productive firms are given by $\left[\int_{\varphi \geq \varphi_D}^{\infty} \pi_D(\varphi) dG(\varphi) + \int_{\varphi \geq \varphi_X}^{\infty} \pi_X(\varphi) dG(\varphi) \right] N^e - P_K f_E (1 - G(\varphi_D))N^e$ on the aggregate. Under the free entry condition, this is rewritten as $P_K f_E G(\varphi_D) N^e$, which is equal to the sum of the ex-post negative profits of the exiting firms.

2. For the intermediate goods firms in country 2, the equations corresponding to (1)–(5) are given as follows:

$$p_D^*(\varphi) = P^* L^{*1-\alpha} y_D^*(\varphi)^{\alpha-1},$$
$$\tau^* p_X^*(\varphi) = P L^{1-\alpha} (y_X^*(\varphi)/\tau^*)^{\alpha-1},$$
$$p_j^*(\varphi) = \frac{P^*}{\alpha\varphi}, \ j = D, X,$$
$$y_j^*(\varphi_j^*) = \frac{\alpha\varphi_j^*}{1-\alpha} f_j q^*, \ j = D, X,$$
$$f_E = \sum_{j=D,X} f_j \Pi(\varphi_j^*).$$

2.3 Entrepreneurs (the knowledge good sector)

Preferences, technologies, and constraints Entrepreneurs comprise a continuum of heterogeneous agents with unit mass, indexed by $i \in [0,1]$. An entrepreneur has the following expected utility:

$$\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \ln c_{i,t} dt \right],$$

where c is consumption and $\rho > 0$ is the subjective discount rate. Each entrepreneur produces the knowledge good by using capital under constant returns to scale technology and perfect competition:

$$y_{Ki,t} = z_{i,t} k_{i,t},$$

where y_K is the output of the knowledge good, k is the capital, and z is the marginal productivity of capital. We assume that productivity z is independent and identically distributed (iid) across agents and over time. At each point in time, z is drawn from a stationary distribution, where the cumulative distribution function is expressed as $F(z)$.

The budget constraint evaluated in the domestic final good is given by

$$(q_t z_{i,t} k_{i,t} - r_t^b b_{i,t})dt + db_{i,t} = (c_{i,t} + \iota_{i,t})dt,$$

where b is the debt in terms of the domestic final good, r^b is the real interest rate, and ι is the investment for capital. The level of capital changes according to $dk_{i,t} = (\iota_{i,t} - \delta k_{i,t})dt$.[11]

Each entrepreneur's borrowing is subject to the following credit constraint:

$$b_{i,t} \leq \left(1 - \frac{1}{\theta}\right) k_{i,t}, \quad \theta \geq 1.$$

This inequality implies that, at most, a fraction $1 - 1/\theta \in [0, 1)$ of capital can be financed by external funds. By varying θ, we can trace all degrees of financial frictions. The case of $\theta = 1$ results in $1 - 1/\theta = 0$, implying that the entrepreneur's investment must be self-financed. The case of $\theta \to \infty$ results in $1 - 1/\theta \to 1$, which implies that entrepreneurs can finance all their capital

[11] Since debts and capital at the individual level can be discontinuous over time as shown below, we do not differentiate them with respect to time here.

investment through external financing; that is, there is no financial friction. Let $a_i \equiv k_i - b_i$ denote an entrepreneur's net worth. The credit constraint can be rewritten as

$$k_{i,t} \leq \theta a_{i,t}. \tag{6}$$

Thus, θ corresponds to the maximum leverage ratio. Using net worth, we can arrange the budget constraint as

$$da_{i,t} = \left[r_t^b a_{i,t} + \left(q_t z_{i,t} - \delta - r_t^b\right) k_{i,t} - c_{i,t}\right] dt. \tag{7}$$

Optimization Entrepreneurs maximize their utility subject to (6) and (7). Following Moll (2014) and Itskhoki and Moll (2019), we assume that at each point in time, the entrepreneur can determine the amount of investment after observing its productivity, z. Consequently, the optimization problem is essentially the same as that of their model. Because differences in entrepreneurs' behaviors are attributed to heterogeneities in a and z, we drop the subscript i for the moment and use (a, z) to identify individuals.

We can solve the optimization problem in two stages. First, at each point in time, the entrepreneur determines k to maximize the net profit, $(qz - \delta - r^b)k$, subject to the credit constraint (6) while taking $(a,\, z)$ as given. Therefore, the optimal choice is given by

$$k_t = \begin{cases} 0 & \text{if } z_t < \widetilde{z}_t, \\ \theta a_t & \text{if } z_t \geq \widetilde{z}_t, \end{cases}$$

where $\widetilde{z}$ denotes the productivity cutoff for entrepreneurs to survive and produce the knowledge good:

$$\widetilde{z}_t \equiv \frac{r_t^b + \delta}{q_t}.$$

We can rewrite the budget constraint as $da_t = (R_t(z_t)a_t - c_t)dt$, where

$$R_t(z_t) = \begin{cases} r_t^b & \text{if } z_t < \widetilde{z}_t, \\ r_t^b + \theta q_t \left(z_t - \widetilde{z}_t\right) & \text{if } z_t \geq \widetilde{z}_t. \end{cases}$$

Second, the entrepreneur chooses the time paths of consumption and net worth to maximize the lifetime utility subject to the budget constraint. Since instantaneous utility is a logarithmic function, this problem yields the following results:

$$c_t = \rho a_t, \quad da_t = (R_t(z_t) - \rho)a_t dt.$$

We provide the derivations of these equations in Appendix A.1.

Aggregation Let us define $A \equiv \int_0^1 a_i di$, $K_t \equiv \int_0^1 k_i di$, $B \equiv \int_0^1 b_i di$, and $Y_K \equiv \int_0^1 y_{Ki} di$. In this model, each entrepreneur's net worth at date t, $a_{i,t}$, has already been determined. Then the productivity z_t^i is determined as an iid shock. Therefore, they are independent of each other. Using this fact, we obtain

$$K_t = \theta(1 - F(\widetilde{z}_t))A_t,$$
$$B_t = K_t - A_t = [\theta(1 - F(\widetilde{z}_t)) - 1]A_t.$$

The aggregate output of the knowledge good is given by

$$Y_{K,t} = Z(\widetilde{z}_t)K_t,$$

where $Z(\widetilde{z}) \equiv \int_{z \geq \widetilde{z}} z dF(z)/(1 - F(\widetilde{z}))$ represents the average productivity of capital conditional on active entrepreneurs. Thus, the knowledge good is produced by AK technology. However, in this study, the average productivity is endogenous and affected by financial frictions through its effect on the entrepreneur's cutoff, as first pointed out by Moll (2014).

We define dA_t as $dA_t \equiv \int da_{i,t} di$. Using $da_{i,t} = (R_t(z_{i,t}) - \rho)a_{i,t}dt$ and the fact that a_i and z_i are independent from each other, we obtain

$$dA_t = A_t \left[r_t^b + \theta(1 - F(\widetilde{z}_t)) \left(q_t Z(\widetilde{z}_t) - \delta - r_t^b \right) - \rho \right] dt.$$

The right-hand side of the equation does not contain any stochastic components. Hereafter, let a dot over a variable denote the time derivative of this variable; for example, $\dot{A}_t \equiv dA_t/dt$. Using $r_t^b + \delta = q_t \widetilde{z}$, we can obtain the dynamic equation of A as follows:

$$\dot{A}_t/A_t = q_t \widetilde{z}_t \left(1 + \theta \Psi(\widetilde{z}_t)\right) - \delta - \rho, \tag{8}$$

where $\Psi(\widetilde{z})$ is defined as

$$\Psi(\widetilde{z}) \equiv (1 - F(\widetilde{z}_t)) \left(\frac{Z(\widetilde{z})}{\widetilde{z}} - 1 \right) = \int_{z \geq \widetilde{z}} (z/\widetilde{z} - 1) dF(z),$$
$$\Psi'(\widetilde{z}) = \int_{z \geq \widetilde{z}} -z/(\widetilde{z})^2 dF(z) = -\frac{\Psi(\widetilde{z}) + 1 - F(\widetilde{z})}{\widetilde{z}} < 0.$$

In (8), $q\widetilde{z}\left(1 + \theta\Psi(\widetilde{z})\right) - \delta$ corresponds to the entrepreneurs' rate of return on their aggregate net worth. As shown by the definition of $R(z)$, any entrepreneur has a guaranteed rate of return of $r^b (= q\widetilde{z} - \delta)$. Thus, $\theta q \widetilde{z} \Psi(\widetilde{z})$ represents the excess from the guaranteed rate of return.

In this study, we focus on the international differences in the degree of financial frictions. For this purpose, we assume that the distribution functions of z are identical in both countries. Thus, the entrepreneur's productivity cutoff in country 2 $\widetilde{z}^*$ is given by $\widetilde{z}^* = (r^{b*} + \delta)/q^*$, where r^{b*} is the real interest rate for lending and borrowing in country 2. The behavior of entrepreneurs in country 2 is summarized as follows:

$$K^* = \theta^*(1 - F(\widetilde{z}^*))A^*,$$
$$B^* = [\theta^*(1 - F(\widetilde{z}^*)) - 1]A^*,$$
$$Y_K^* = Z(\widetilde{z}_t^*)K^*,$$
$$\dot{A}^*/A^* = q^* \widetilde{z}^* \left(1 + \theta^* \Psi(\widetilde{z}^*)\right) - \delta - \rho.$$

2.4 Workers

There is a continuum of homogeneous workers with the constant mass of $L > 0$. They are myopic and do not save. Each worker chooses consumption c_w and labor supply l to solve the following optimization problem at each point in time:

$$\max_{c_{w,t}, l_t} \ln c_{w,t} - \frac{l_t^{1+\nu}}{1+\nu},$$

subject to the budget constraint $w_t l_t = P_t c_{w,t}$. The optimal choice yields $l_t = 1$ and $c_{w,t} = w_t/P_t$ for all t. Then, the aggregate supply of labor always equals the population L and the aggregate consumption of workers is given by $w_t L/P_t$.[12]

2.5 Autarky equilibrium

In Appendix A.3, we characterize the autarky equilibrium. In this appendix, we demonstrate that under autarky, the economy is on the BGP from the initial time, and the output of the final good always increases through the expansion of the variety of intermediate goods.

3 Equilibrium with trade

Throughout this study, we choose country 1's final good as the numeraire: $P_t = 1$ for all t. As we have already stated, for the moment, we assume no international lending and borrowing. Hereafter, we refer to such a situation as "financial autarky."[13] Because we focus on the cross-country asymmetry in the degree of financial frictions, we assume that the labor size and trade costs are symmetric across the two countries:

$$L = L^*, \quad \tau = \tau^*.$$

3.1 The intermediate goods firms' cutoffs given P^*

Following Melitz (2003) and other studies, we assume the following inequality,

$$T \equiv \tau \left(\frac{f_X}{f_D}\right)^{(1-\alpha)/\alpha} > 1,$$

[12] If the workers can save, each worker's optimization problem is given by

$$\max \int_0^\infty e^{-\rho_w t} \left(\ln c_{w,t} - \frac{l_t^{1+\nu}}{1+\nu} \right) dt,$$

subject to $\dot{a}_{w,t} = r_t^b a_{w,t} + (w_t/P_t) l_t - c_{w,t}$, where $\rho_w > 0$ is the discount rate applied to workers and a_w denotes their asset holding. As pointed out by Moll (2014), if the workers' discount rate is sufficiently high such that $\rho_w > r_t$, and if they face a tight borrowing constraint such that they cannot borrow (i.e., $a_{w,t} > 0$), their optimal choice yields $c_{w,t} = (w_t/P_t) l_t$ in the long run.

[13] Since balanced trade is always achieved in this situation, we use "financial autarky" and "balanced trade" interchangeably.

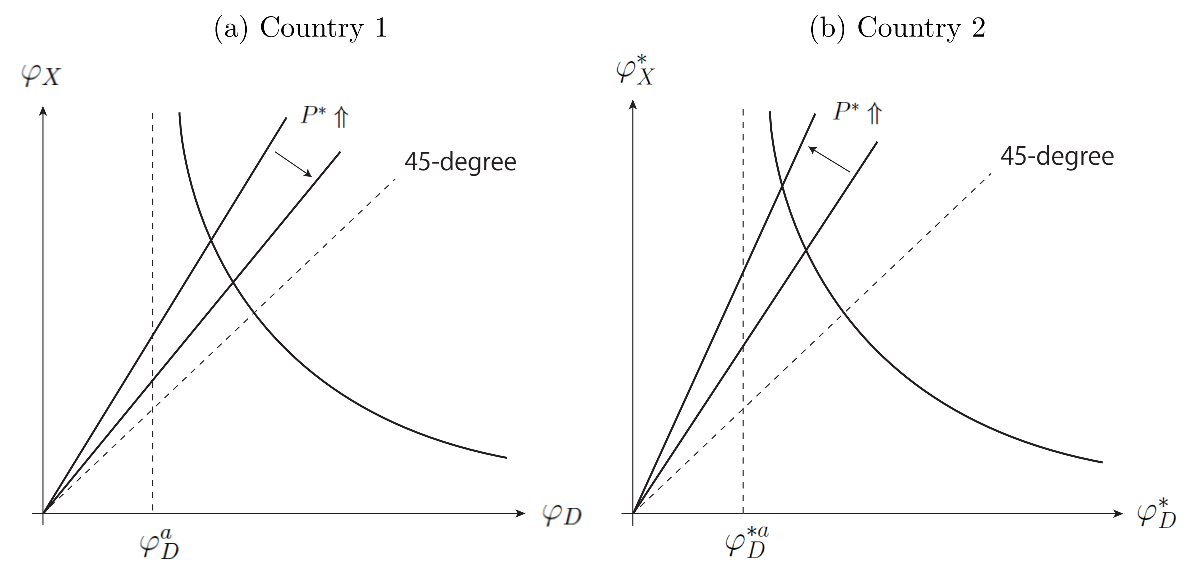

Figure 1: The intermediate goods firms' cutoffs given P^*

which is useful to obtain the result that the productivity cutoff for exporting is higher than that for domestic operations. From (1)–(4), we obtain the following equation showing how φ_D and φ_X are related:

$$\frac{\varphi_{X,t}}{\varphi_{D,t}} = TP_t^{*-1/\alpha}. \tag{9}$$

This intuition can be explained as follows: As evident from (1) and (2), the higher the price of the final good in a country, the greater the demand for intermediate goods in that country. For intermediate goods firms in country 1, a larger value of P^* implies a greater demand for their products in country 2. This means that exporting to country 2 becomes more profitable when P^* is larger than when it is smaller, leading to a decrease in φ_X/φ_D.

Given P^*, the two cutoffs φ_D and φ_X are determined from (5) and (9). Panel (a) of Figure 1 depicts how they are determined, where the downward-sloping curve represents (5) and the upward-sloping line represents (9). The free entry condition (5) provides the negative relationship between φ_D and φ_X. The intuition is straightforward. If φ_X decreases, it will positively affect the aggregate profit in the export market. Under the free entry condition, such positive effects must be offset by negative impacts on the aggregate profit in the domestic market. Moreover, φ_D approaches φ_D^a if φ_X is sufficiently large. Thus, as this figure shows, (5) and (9) uniquely determine φ_D and φ_X.

We can obtain the cutoffs for country 2's intermediate goods firms by following the same procedure as for those of country 1. We obtain

$$\frac{\varphi^*_{X,t}}{\varphi^*_{D,t}} = TP_t^{*1/\alpha}. \tag{10}$$

The right-hand side is an increasing function with respect to P^*. This is simply because, for intermediate goods firms in country 2, a larger value of P^* means that the profit from supplying goods to the domestic market increases relative to exporting them. Panel (b) of Figure 1 depicts how φ_D^* and φ_X^* are determined and how a change in P^* affects them. We can summarize these results as the following lemma:

Lemma 1. *Given P_t^*, the productivity cutoffs for the intermediate goods firms are uniquely determined, and $\varphi_D'(P_t^*) > 0$, $\varphi_X'(P_t^*) < 0$, $\varphi_D^{*\prime}(P_t^*) < 0$, and $\varphi_X^{*\prime}(P_t^*) > 0$.*

Following the literature building on heterogeneous-firm trade models, we focus on the situation where not all operating firms can export: $\varphi_X > \varphi_D$ and $\varphi_X^* > \varphi_D^*$. From (9) and (10), these inequalities are expressed as the following inequalities that P^* must satisfy in the equilibrium:

$$P_{\min}^* \leq P^* \leq P_{\max}^*,$$

where $P_{\min}^* \equiv T^{-\alpha} < 1$ and $P_{\max}^* \equiv T^{\alpha} > 1$.[14]

Substituting (3) and (4) for $j = D$ into (1), we obtain the following relationship between the real price of the knowledge good $q(= P_K)$ and productivity cutoff φ_D:

$$q_t = \chi L(\varphi_{D,t})^{\alpha/(1-\alpha)}, \tag{11}$$

where $\chi \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)/f_D > 0$. If the price of the knowledge good is high relative to that of the final good, the fixed cost of serving the domestic market is high. This lowers the profit of the intermediate goods firms; thus, the cutoff φ_D increases. Therefore, (11) represents the positive relationship between q and φ_D.

Applying Lemma 1 to (11), we obtain the real price of the knowledge good in each country as a function of P^*:

$$q_t = q(P_t^*) \equiv \chi L\left(\varphi_D(P_t^*)\right)^{\alpha/(1-\alpha)}, \quad q'(P^*) > 0, \tag{12}$$

$$q_t^* = q^*(P_t^*) \equiv \chi L\left(\varphi_D^*(P_t^*)\right)^{\alpha/(1-\alpha)}, \quad q^{*\prime}(P^*) < 0. \tag{13}$$

Note that q and q^* coincide if $\varphi_D = \varphi_D^*$.

3.2 Market-clearing conditions

The market-clearing condition for the final good in country 1 is

$$Y_t = C_t + \dot{K}_t + \delta K_t + N_t^e \sum_{j=D,X} \int_{\varphi \geq \varphi_{j,t}} y_{j,t}(\varphi)/\varphi dG(\varphi).$$

where C is the aggregate consumption in country 1. Since $P_t = 1$ for all t, C_t is given by $\rho A_t + w_t L_t$. The market equilibrium condition for the knowledge good is given by

$$Y_{K,t} = N_t^e \left(f_E + \sum_{j=D,X} f_j (1 - G(\varphi_{j,t})) \right). \tag{14}$$

[14] Because $T > 1$, $P_{\max}^* > P_{\min}^*$ is satisfied automatically.

Since $P_t = 1$, we can obtain the equation representing the balance of payment in country 1 as:[15]

$$\dot{B}_t = r_t^b B_t - (EX_t - IM_t), \tag{15}$$

where EX and IM are country 1's exports and imports, respectively:

$$EX \equiv N^e \int_{\varphi \geq \varphi_X} p_X(\varphi) y_X(\varphi) dG(\varphi),$$
$$IM = EX^* \equiv N^{e*} \int_{\varphi \geq \varphi_X^*} p_X^*(\varphi) y_X^*(\varphi) dG(\varphi).$$

Since $B_t = 0$ for all t, (15) implies that trade is always balanced:

$$EX_t = IM_t. \tag{16}$$

The market-clearing conditions in country 2 are analogous. According to Walras' law, the equation for balanced trade in country 2, $EX_t^* = IM_t^*$, is redundant.

3.3 Equilibrium characterization

When global financial linkages are absent, equilibrium conditions imply $B_t = 0$ and $B_t^* = 0$. From the definitions of B and B^*, we obtain $\theta(1 - F(\widetilde{z}_t)) - 1 = 0$ and $\theta^*(1 - F(\widetilde{z}^*)) - 1 = 0$. Therefore, the entrepreneurs' cutoffs $\widetilde{z}_t$ and $\widetilde{z}_t^*$ are, respectively, given by

$$\widetilde{z}_t = \overline{z}(\theta) \equiv F^{-1}(1 - 1/\theta), \quad \widetilde{z}_t^* = \overline{z}(\theta^*) \equiv F^{-1}(1 - 1/\theta^*). \tag{17}$$

Thus, without international financial transactions, the international difference in the cutoffs comes solely from the difference in the degree of financial friction. We obtain

$$\overline{z}'(\theta) = \frac{1 - F(\overline{z}(\theta))}{\theta F'(\overline{z}(\theta))} > 0,$$

which implies that even less productive entrepreneurs begin to invest and produce the knowledge good with a decrease in θ. This is simply because the decrease in θ corresponds to the decrease in the entrepreneurs' borrowing capacity in country 1.

We derive a dynamical system for the economy. Let m^* denote the real wealth of country 2 relative to that in country 1: $m^* \equiv A^*/A$. From (8) and its counterpart in country 2, the dynamic equation of m^* is given by

$$\begin{aligned}
\frac{\dot{m}_t^*}{m_t^*} &\equiv \frac{\dot{A}_t^*}{A_t^*} - \frac{\dot{A}_t}{A_t} \\
&= q^*(P_t^*)\overline{z}(\theta)(1 + \theta\Psi(\overline{z}(\theta))) - q(P_t^*)\overline{z}(\theta^*)(1 + \theta^*\Psi(\overline{z}(\theta^*))).
\end{aligned} \tag{18}$$

Thus, given θ and θ^*, the level of final good price P^* governs the dynamics of the relative wealth.

[15] Recall that B_t represents the aggregate real debts in country 1.

The other equation constituting the dynamical system is derived from (16), which is the condition for a balanced trade. Let μ denote the share of the export sales in the total sales of country 1's intermediate goods firms. We can express μ as a function of P^*:

$$\mu_t = \mu(P_t^*) \equiv \frac{f_X H(\varphi_X(P_t^*))}{\sum_{j=D,X} f_j H(\varphi_j(P_t^*))}, \quad \mu'(P^*) > 0. \tag{19}$$

In Appendix A.2, we provide the derivation of this equation and the formal proof of $\mu'(P^*) > 0$. Briefly, the sign of $\mu'(P^*)$ comes from Lemma 1 and $H'(\varphi_j) < 0$. This result is intuitive. As P^* increases, the profitability of exporting increases for intermediate goods firms in country 1. Consequently, the share of export sales rises. Similarly, let μ^* denote the share of the export sales in the total sales of country 2's intermediate goods firms, which can be obtained by simply adding an asterisk to φ_D and φ_X in (19). From Lemma 1 and $H'(\varphi_j^*) < 0$, it follows that $\mu^{*\prime}(P^*) < 0$.

Using $\mu(P^*)$ and $\mu^*(P^*)$, we can express EX and IM, respectively.

$$EX_t = \frac{q(P_t^*)\theta A_t \int_{z \geq \tilde{z}_t} z dF(z)}{1-\alpha} \mu(P_t^*), \tag{20}$$

$$IM_t = \frac{q^*(P_t^*) P_t^* \theta^* A_t^* \int_{z \geq \tilde{z}_t^*} z dF(z)}{1-\alpha} \mu^*(P_t^*). \tag{21}$$

The derivation of this equation is provided in Appendix A.4. These equations hold true regardless of whether the financial market is autarkic or completely integrated. Substituting (20) and (21) into (16) and using the fact that $\tilde{z}_t = \overline{z}(\theta)$ and $\tilde{z}_t^* = \overline{z}(\theta^*)$ under financial autarky, we obtain the following relationship between m^* and P^* :

$$m_t^* = m^*(P_t^*) \equiv \frac{\mu(P_t^*) q\left(P_t^*\right)}{\mu^*(P_t^*) q^*(P_t^*) P_t^*} \frac{\theta \int_{z \geq \overline{z}(\theta)} z dF(z)}{\theta^* \int_{z \geq \overline{z}(\theta^*)} z dF(z)}. \tag{22}$$

Equation (18) indicates that the dynamics of m_t^* depend on P_t^*. Equation (22) shows that the change in P_t^* is determined solely by m_t^*. Thus, (18) and (22) constitute the autonomous dynamical system of the economy with international trade.

From (18), we obtain the following equation when $\dot{m}^* = 0$:

$$q(P^*)\overline{z}(\theta)(1 + \theta\Psi(\overline{z}(\theta))) = q^*(P^*)\overline{z}(\theta^*)(1 + \theta^*\Psi(\overline{z}(\theta^*))). \tag{23}$$

If this equation holds, both countries' assets grow at the same rate. As we have already shown $q'(P^*) > 0$ and $q^{*\prime}(P^*) < 0$, we can state the following proposition:

Proposition 1. *The BGP of the two-country economy uniquely exists if* $q(P_{\min}^*)\overline{z}(\theta)(1+\theta\Psi(\overline{z}(\theta))) < q^*(P_{\min}^*)\overline{z}(\theta^*)(1 + \theta^*\Psi(\overline{z}(\theta^*)))$ *and* $q(P_{\max}^*)\overline{z}(\theta)(1 + \theta\Psi(\overline{z}(\theta))) > q^*(P_{\max}^*)\overline{z}(\theta^*)(1 + \theta^*\Psi(\overline{z}(\theta^*)))$.

Henceforth, let P^{*tr} denote the final good price that solves (23), where "*tr*" indicates the BGP with international trade. Once P^{*tr} is obtained, we can obtain the BGP value of m^* from (22):

$$m^* = m^{*tr} \equiv \frac{\mu(P^{*tr}) q\left(P^{*tr}\right)}{\mu^*(P^{*tr}) q^*(P^{*tr}) P^{tr*}} \frac{\theta \int_{z \geq \overline{z}(\theta)} z dF(z)}{\theta^* \int_{z \geq \overline{z}(\theta^*)} z dF(z)}.$$

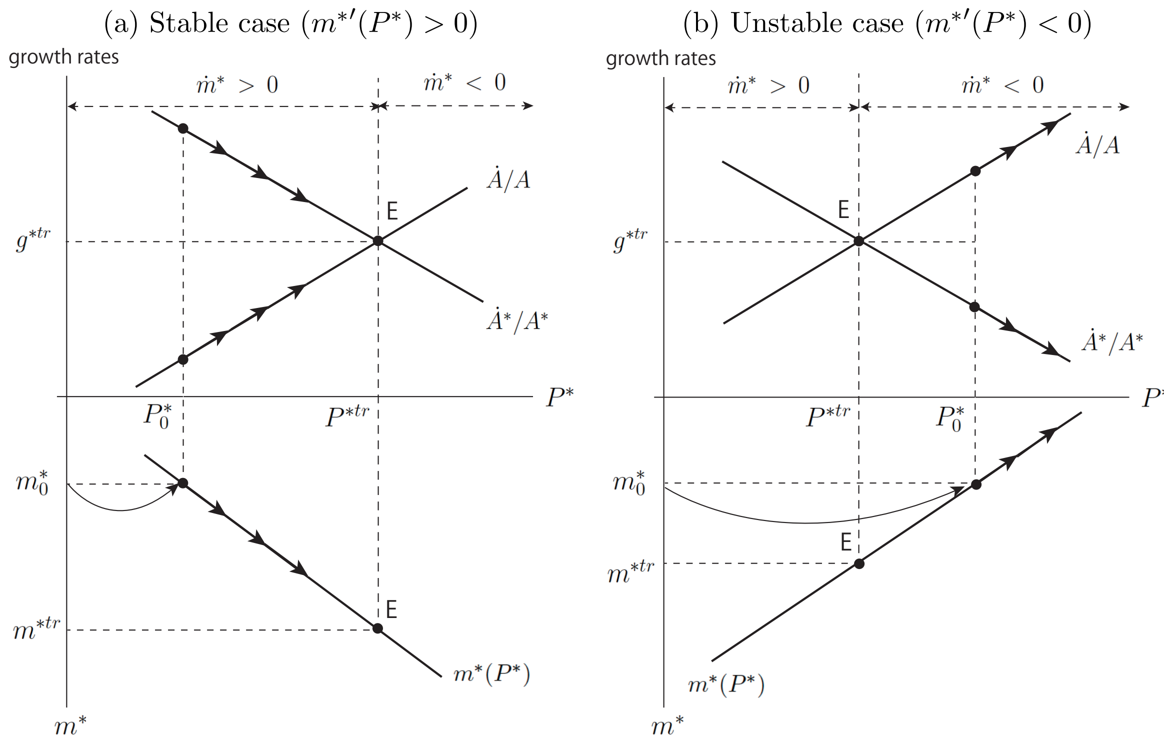

Figure 2: Stability of the BGP

Next, we examine the stability of the BGP. The relative wealth m^* is a predetermined variable whose initial value is historically given, while the final good price P^* is a forward-looking variable. More specifically, under financial autarky, P_0^* is determined from (22) given m_0^*. Since $q'(P^*) > 0$ and $q^{*\prime}(P^*) < 0$, Equation (18) implies the following relationship:

$$\begin{aligned}\dot{m}_t^* \gtreqless 0 &\Leftrightarrow \dot{A}_t/A_t \lesseqgtr \dot{A}_t^*/A_t^* \\ &\Leftrightarrow q(P^*)\bar{z}(\theta)(1+\theta\Psi(\bar{z}(\theta))) \lesseqgtr q^*(P^*)\bar{z}(\theta^*)(1+\theta^*\Psi(\bar{z}(\theta^*))) \\ &\Leftrightarrow P_t^* \lesseqgtr P^{*tr}.\end{aligned}$$

This relationship is depicted in the top quadrant of Figure 2, where the horizontal axis represents P^* and the vertical axis represents $\dot{A}/A$ and $\dot{A}^*/A^*$. The bottom quadrant of Figure 2 depicts Equation (22), which shows how P^* is determined given m^* under financial autarky. Panels (a) and (b) illustrate the cases where $m^{*\prime}(P^*) > 0$ and $m^{*\prime}(P^*) < 0$, respectively. We can analyze the dynamic behavior of the economy by using Figure 2. Without any loss of generality, we assume $m_0^* < m^{*tr}$. As shown in panel (a), if $m^{*\prime}(P^*) > 0$, then P_0^* is smaller than P^{*tr}. In this case, m_t^* increases over time until it reaches the steady-state value, m^{*tr}. Since $m^{*\prime}(P^*) > 0$, P_t^* also increases, and eventually, the economy converges to the BGP, represented by point E in each quadrant of the figure. By contrast, as shown in panel (b), if $m^{*\prime}(P^*) < 0$, then P_0^* is larger than P^{*tr}. Under this condition, m_t^* decreases over time, while P_t^* continues to increase. As a result, both m_t^* and P_t^* move away from point E, indicating that the BGP is unstable and the economy never converges to it.

If the distribution function is given in general form, it is difficult to determine whether $m^{*\prime}(P^*) >$

0 is satisfied. However, if we assume that φ follows the Pareto distribution, we can show that $m^{*\prime}(P^*) > 0$ always holds and hence the BGP is globally stable. For example, suppose that $G(\phi)$ is given by

$$G(\varphi) = 1 - \varphi^{-\kappa}. \tag{24}$$

To make $\Pi(\varphi_j)$ well defined, we assume $\kappa > \alpha/(1-\alpha)$.

Lemma 2. *If $G(\varphi) = 1 - \varphi^{-\kappa}$, $m^{*\prime}(P^*) > 0$ always holds and hence the BGP is globally stable.*

Proof. See Appendix A.5. □

Therefore, we proceed with our analysis assuming that $m^{*\prime}(P^*) > 0$ holds.

4 Long and short-run effects of the country-specific credit crunch

We examine how a credit crunch in one country affects both countries' firm selections and growth rates. Following Buera and Moll (2015), we formulate the credit crunch as a decrease in the value of θ or θ^*. Without any loss of generality, we focus on a decrease in θ. In addition, we focus on the case of a permanent crunch, where the value of θ decreases permanently. In Section 6.3, we examine how transitory shocks to θ affects the world economy using numerical analysis.

4.1 Long-run effects

First, we examine the long-run effects of a permanent credit crunch in country 1. By totally differentiating (23) and with θ^* remaining unchanged, we obtain

$$\begin{aligned}&\left[q'(P^{*tr})\overline{z}(\theta)(1+\theta\Psi(\overline{z}(\theta))) - q^{*\prime}(P^{*tr})\overline{z}(\theta^*)(1+\theta^*\Psi(\overline{z}(\theta^*)))\right] dP^{*tr}\\ &\quad = -q(P^{*tr})\left[\overline{z}(\theta)\Psi(\overline{z}(\theta)) + \left(1+\theta\Psi(\overline{z}(\theta)) + \overline{z}(\theta)\theta\Psi'(\overline{z})\right)\overline{z}'(\theta)\right] d\theta.\end{aligned}$$

Since $q' > 0$ and $q^{*\prime} < 0$, the sign of the terms in the left-hand side parentheses is positive. In the right-hand side parentheses, the first term $\overline{z}\Psi(\overline{z}) > 0$ is the direct effect of a decrease in θ, which makes active entrepreneurs earn less. The second term is the indirect effect induced by selecting active entrepreneurs. Within the parentheses of the second term, $1 + \theta\Psi > 0$, whereas $\overline{z}\theta\Psi' < 0$. However, based on the definition of Ψ, we can demonstrate the following result:

$$1 + \theta\Psi(\overline{z}) + \theta\overline{z}\Psi'(\overline{z}) = 1 + \theta\Psi(\overline{z}) - \theta(\Psi(\overline{z}) + 1 - F(\overline{z})) = 1 - \theta(1 - F(\overline{z})) = 0,$$

where the last equality holds because $B = 0$. Thus, only the direct effect exists, resulting in

$$\frac{dP^{*tr}}{d\theta} = -\frac{q(P^{*tr})\overline{z}(\theta)\Psi(\overline{z}(\theta))}{q'(P^{*tr})\overline{z}(\theta)(1+\theta\Psi(\overline{z}(\theta))) - q^{*\prime}(P^{*tr})\overline{z}(\theta^*)(1+\theta^*\Psi(\overline{z}(\theta^*)))} < 0.$$

Therefore, with a decrease in θ, the final good price rises in country 2.

On the BGP, assets in countries 1 and 2 grow at the same rate, which is expressed as

$$g^{tr} \equiv q^*(P^{*tr})\overline{z}(\theta^*)(1+\theta^*\Psi(\overline{z}(\theta^*))) - \delta - \rho,$$

which shows that g^{tr} decreases due to a decrease in θ because $q^{*\prime} < 0$. Thus, we can state the following proposition:

Proposition 2. *The growth rate of the BGP decreases with a permanent credit crunch in a country.*

The credit crunch in country 1 directly leads to slower growth in that country. So, what lies behind the decline in country 2's growth rate? The primary factor is the firm selection in country 2's intermediate goods sector. As discussed in Section 3.1, a higher price for the final good in a country corresponds to greater demand for intermediate goods within that country. A decrease in θ leads to a reduction in $1/P^*$, causing a decrease in the demand for intermediate goods in country 1. As depicted in panel (b) of Figure 1, this makes exporting less profitable for intermediate goods firms in country 2. Therefore, the productivity cutoff φ_X^* rises while φ_D^* falls. With $q^* = \chi L(\varphi_D^*)^{\alpha/(1-\alpha)}$, the real price of the knowledge good declines, contributing to a decrease in country 2's growth rate. Therefore, we can state the following proposition:

Proposition 3. *Suppose that a permanent credit crunch occurs in one country.*

1. *In this country, the intermediate goods firms' productivity cutoff for survival ($\varphi_{D,t}$) increases, while that for exporting ($\varphi_{X,t}$) decreases.*

2. *In the partner country, that for survival ($\varphi_{D,t}^*$) decreases, while that for exporting ($\varphi_{X,t}^*$) increases.*

4.2 Short-run effects

Using figures similar to Figure 2, we examine the short-run effects of a permanent credit crunch. Owing to the decrease in θ, the curves of both $\dot{A}_t/A_t$ in the top quadrant and $m^*(P^*)$ in the bottom quadrant shift downward.[16]

[16]Note that A^*/A^* is not affected by θ. From the equation of $\dot{A}/A$, we obtain

$$\frac{\partial \dot{A}/A}{\partial \theta} = q(P^{*tr}) \left[\overline{z}(\theta)\Psi(\overline{z}(\theta)) + \left(1 + \theta\Psi(\overline{z}(\theta)) + \overline{z}(\theta)\theta\Psi'(\overline{z})\right) \overline{z}'(\theta)\right] = q(P^{*tr})\overline{z}(\theta)\Psi(\overline{z}(\theta)) > 0.$$

In addition, from (22), we obtain

$$\frac{\partial m^*(P^*)}{\partial \theta} = \frac{\mu(P_t^*)q\left(P_t^*\right)}{\mu^*(P_t^*)q^*(P_t^*)P_t^*} \frac{1}{\theta^* \int_{z \geq \overline{z}(\theta^*)} zdF(z)} \frac{\partial \theta \int_{z \geq \overline{z}(\theta)} zdF(z)}{\partial \theta},$$

where

$$\begin{aligned}\frac{\partial \theta \int_{z \geq \overline{z}(\theta)} zdF(z)}{\partial \theta} &= \int_{z \geq \overline{z}(\theta)} zdF(z) - \theta\overline{z}(\theta)F'(\overline{z}(\theta))\overline{z}'(\theta) \\ &= \int_{z \geq \overline{z}(\theta)} zdF(z) - \overline{z}(\theta)(1 - F(\overline{z}(\theta))) \\ &= (1 - F(\overline{z}(\theta))) \left[\frac{\int_{z \geq \overline{z}(\theta)} zdF(z)}{(1 - F(\overline{z}(\theta)))} - \overline{z}(\theta)\right] > 0.\end{aligned}$$

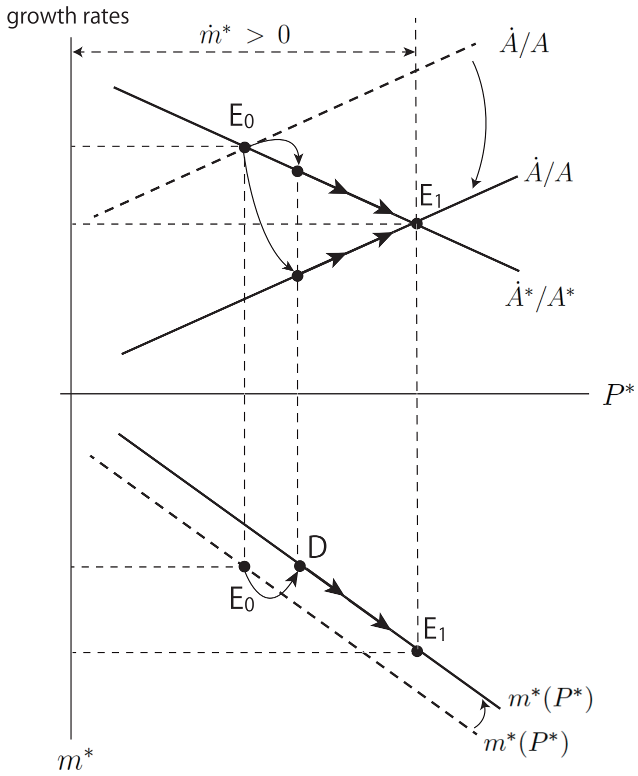

Figure 3: Short-run effects (1): Case of $\dot{A}/A < \dot{A}^*/A^*$

Possible cases for the dynamics are illustrated in Figures 3 and 4. In both figures, the old and new BGPs are represented by points E_0 and E_1, respectively. Suppose that the world economy is on the old BGP at the initial date and θ falls permanently. Immediately after the credit crunch, the economy first moves to point D from E_0 as P^* jumps and increases given m^*. Thus far, these results are common to both figures. Briefly, such a jump in P^* occurs because a decrease in θ puts downward pressure on country 1's exports. However, since trade must be balanced by (22), country 2's exports also should decline. As discussed in the analysis of long-run effects, this occurs as $1/P^*$ falls, and the final good firms in country 1 reduce their demand for intermediate goods relative to the firms in country 2.

Figure 3 illustrates the case where the $m^*(P^*)$ curve does not shift considerably, and hence the initial jump of P^* is not very large. In this case, point D is located northwest of point E_1. As indicated by the arrows in the bottom quadrant, m^* and P^* increase over time until they reach point E_1. The upper quadrant illustrates how the growth rates of the two countries' assets change during this process. Immediately after θ falls, the two growth rates decline discretely from point E_0, with the decline being larger in country 1. Therefore, during the transition process, country 1's growth rate is lower than that of country 2 (i.e., $\dot{A}/A < \dot{A}^*/A^*$). However, as already mentioned, an increase in P^* leads to a slowdown in exports for intermediate goods firms in country 2, which in turn lowers country 2's growth rate. Therefore, the two growth rates gradually approach each other, and eventually, the world economy arrives at point E_1, where the new BGP growth rate is lower than that at point E_0. Note that both countries' growth rates decrease at all points in time

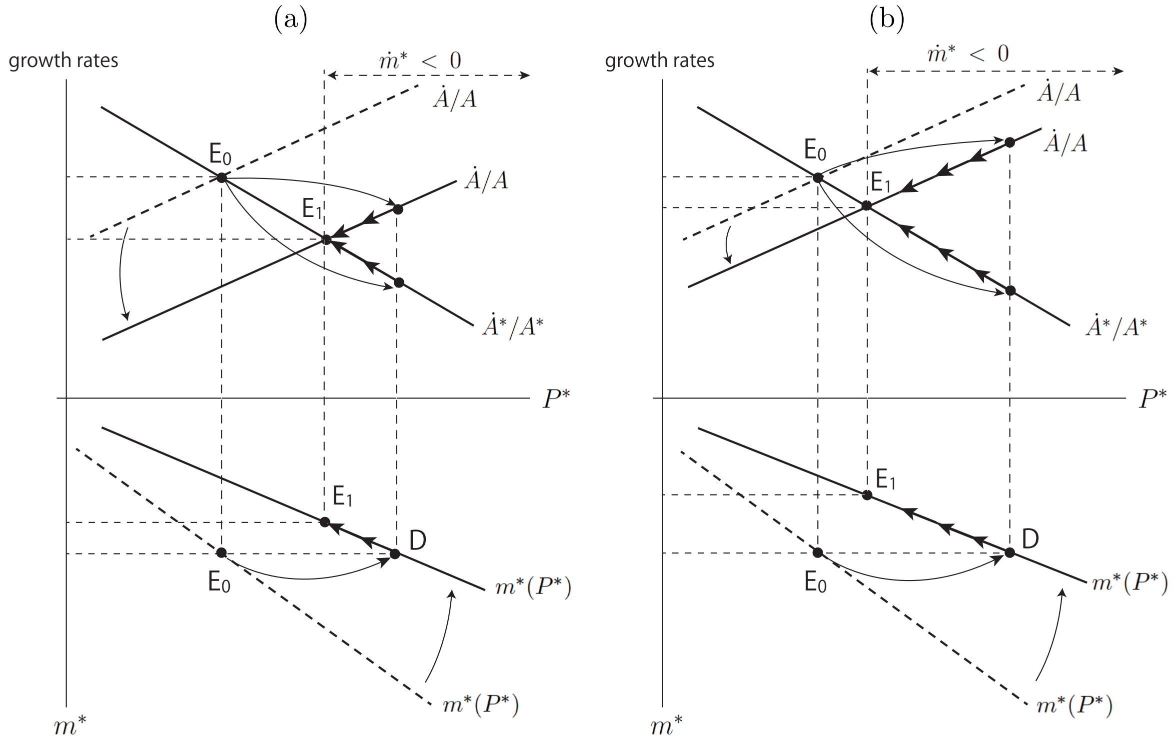

Figure 4: Short-run effects (2): Case of $\dot{A}/A > \dot{A}^*/A^*$

compared with the old BGP.

The two panels in Figure 4 depict the case where the shift in the $m^*(P^*)$ curve is large enough such that the initial jump of P^* overshoots point E_1. In this case, m^* and P^* decrease over time after the initial jump, as depicted in the bottom quadrants of the two panels. The most distinguishing feature of Figure 4 compared to Figure 3 is that country 2 has a lower growth rate than country 1 during the transition process in Figure 4 (i.e., $\dot{A}/A > \dot{A}^*/A^*$). This is due to the large initial jump in P^*, which significantly slows down exports in country 2. In this case, P^* decreases through time and country 2's exports gradually revive and its growth rate recovers. The two countries then reach a new BGP where, again, the common growth rate is lower than before the credit crunch. The difference between panel (a) and panel (b) lies in the magnitude of the downward shift in the $\dot{A}/A$ curve. In panel (a), the shift is considerable so that the initial decline in $\dot{A}/A$ occurs. By contrast, in panel (b), the downward shift is not as considerable, which leads to the initial increase in $\dot{A}/A$ from E_0.

It is difficult to identify which of the three possible cases actually occurs theoretically. However, when the decrease in θ requires only a relatively small jump in P^* to restore a new trade balance, the transition dynamics follow those in Figure 3 or Figure 4-(a). In these cases, the growth rates of both countries decline at all time points compared to those in the old BGP. In addition, it should be noted that in all three possible cases, the growth rate is lower at all time points in the country that does not directly experience the credit crunch. Therefore, we state the following proposition, which is the most important result of this study.

Proposition 4. *Suppose that a permanent credit crunch occurs in country 1. Then, compared to the old BGP, the growth rate decreases at all points in time in country 2, which does not experience a credit crunch. In addition, if the credit crunch requires only a relatively small jump in P^* to restore a new trade balance, the growth rates of both countries decrease at all points in time.*

As already noted, country 2 experiences lower growth because exporting becomes less profitable due to an increase in the cutoff φ_X^*. This decline in export profitability is triggered by lower demand for intermediate goods in country 1. This result is partly consistent with Japan's experience during the 2007–2009 financial crisis. For instance, Kawai and Takagi (2009) pointed out that a major factor underlying the collapse of Japanese exports was a worldwide shrinkage of demand and trade following the Lehman shock.

The decrease in global growth is largely attributed to Lemma 1, which shows that $\varphi_{D,t}$ and $\varphi_{X,t}^*$ respond in the same direction to shocks. To the best of our knowledge, this co-movement was first shown by Demidova (2008), who employed a static model to examine the impacts of a unilateral increase in labor productivity in one country on the productivity cutoffs in two countries. Our results indicate that Demidova's mechanism is applicable in dynamic models.

5 Equilibrium with financial integration

This section introduces international lending and borrowing into our model. Now bonds issued by active entrepreneurs can be bought by the other country's lenders (i.e., inactive entrepreneurs) as well as by domestic ones, although capital is not directly mobile between the two countries.

Note that the analysis in Section 3.1 does not include variables or parameters related to financial markets. Therefore, the determination of cutoffs for intermediate goods firms (given P^*) discussed there holds even under financial integration.

5.1 The BGP conditions

Since B and B^* are respectively evaluated in terms of the domestic final good, the market-clearing condition of bonds is given by

$$B_t + P_t^* B_t^* = 0. \tag{25}$$

Recall that the real interest rate of bonds in a country is defined in terms of its country's final good. Therefore, the interest parity is given by $r^b = r^{b*} + \dot{P}^*/P^*$. This is rewritten as

$$q_t \widetilde{z}_t = q_t^* \widetilde{z}_t^* + \dot{P}_t^*/P_t^*. \tag{26}$$

The equation for a balanced trade (16) is no longer necessarily valid. Rather, (15) becomes one of the equilibrium conditions.[17]

[17] According to Walras' law, the balance of payment equation in country 2, $\dot{B}^* = r^{b*}B^* - (EX^* - IM^*)/P^*$, is redundant.

Here we provide the conditions that the economy satisfies on the BGP. The equation of interest parity (26) with $\dot{P}^* = 0$ implies

$$q(P^*)\widetilde{z} = q^*(P^*)\widetilde{z}^*. \tag{27}$$

On the BGP, the growth rate of net worth is equal between the two countries: $\dot{A}/A = \dot{A}^*/A^*$. From (8), its counterpart in country 2, and (27), we obtain the following condition:

$$\theta\Psi(\widetilde{z}) = \theta^*\Psi(\widetilde{z}^*). \tag{28}$$

Let $v \equiv B/A$ and $v^* = B^*/A^*$. Using the definitions of B and B^*, we can rewrite the market-clearing condition for debts (25) as

$$\underbrace{\theta(1 - F(\widetilde{z})) - 1}_{=v} + m^*P^* \underbrace{[\theta^*(1 - F(\widetilde{z}^*)) - 1]}_{=v^*} = 0. \tag{29}$$

In Appendix A.6, we show that the dynamic equation of v is given by

$$\begin{aligned} \dot{v}_t &= (\rho - q(P_t^*)\widetilde{z}_t\theta\Psi(\widetilde{z}_t))\, v_t \\ &\quad - \frac{1}{1-\alpha}\left[\mu(P_t^*)q(P_t^*)\theta \int_{z \geq \widetilde{z}_t} zdF(z) - m_t^*P_t^*\mu^*(P_t^*)q^*(P_t^*)\theta^* \int_{z \geq \widetilde{z}_t^*} zdF(z)\right]. \end{aligned} \tag{30}$$

On the BGP, B grows at the same rate as A or becomes zero. In either situation, $\dot{v} = 0$. Imposing $\dot{v} = 0$ in (30) and using $v = \theta(1 - F(\widetilde{z})) - 1$, we obtain

$$\begin{aligned} &(\rho - q(P^*)\widetilde{z}\theta\Psi(\widetilde{z}))\, [\theta(1 - F(\widetilde{z})) - 1] \\ &\quad = \frac{1}{1-\alpha}\left[\mu(P^*)q(P^*)\theta \int_{z \geq \widetilde{z}} zdF(z) - m^*P^*\mu^*(P^*)q^*(P^*)\theta^* \int_{z \geq \widetilde{z}^*} zdF(z)\right]. \end{aligned} \tag{31}$$

The BGP is characterized as the variables $\widetilde{z}$, $\widetilde{z}^*$, P^*, and m^* satisfying (27), (28), (29), and (31). Once these variables are determined, the other variables, including the BGP growth rate, can be determined accordingly.

5.2 The symmetric BGP

To analytically characterize the BGP under financial integration in this asymmetric two-country model, the following difficulty arises: trade is not necessarily balanced at zero. In particular, the left-hand side of (31) may not be zero. Therefore, it is difficult to analytically solve (27), (28), (29), and (31) in general situations. At first, we consider the BGP when $\theta = \theta^*$; the two countries are perfectly symmetric. In this case, (28) yields

$$\Psi(\widetilde{z}) = \Psi(\widetilde{z}^*),$$

which implies $\widetilde{z} = \widetilde{z}^*$. Substituting this result into (29) yields

$$(1 + P^*m^*)[\theta(1 - F(\widetilde{z})) - 1] = 0.$$

Because we focus on the equilibrium in which $m^* \equiv A^*/A$ and P^* are positive and finite, this equation implies

$$v = \theta(1 - F(\widetilde{z})) - 1 = 0.$$

Then, $\widetilde{z} = \widetilde{z}^* = \overline{z}(\theta)$. Thus, the net foreign assets are zero on the BGP when the degree of financial frictions is the same between the two countries. From (27) with $\widetilde{z} = \widetilde{z}^* = \overline{z}(\theta)$, we find that the real prices of the knowledge good are equal. From (12) and (13), $\varphi_D = \varphi_D^*$ holds. Hence,

$$P^* = 1,$$

which results in $\mu = \mu^*$. Since $\theta(1 - F(\widetilde{z})) - 1 = 0$, (31) implies that trade is balanced in this case. Therefore, we obtain the relative wealth m^* on the BGP from (22) as

$$m^* = 1.$$

In the case of financial integration, the autonomous dynamical system no longer only contains the dynamics of m_t^*; it also contains those of P_t^* and v_t. In Appendix A.7, we provide the autonomous dynamical system under financial integration. Furthermore, in Appendix A.8, we provide the condition for the local stability of the BGP where the two countries are symmetric.

Lemma 3. *The symmetric BGP under $\theta = \theta^*$ is locally saddle-point stable if and only if $\rho < q(1)\overline{z}(\theta)\theta\Psi(\overline{z}(\theta))$.*

Proof. See Appendix A.8. □

Let g^{fi} denote the BGP growth rate under financial integration. When the two countries are symmetric,

$$g^{fi} = q(1)\overline{z}(\theta)(1 + \theta\Psi(\overline{z}(\theta))) - \delta - \rho.$$

Since $q(1)\overline{z}(\theta) - \delta$ is equal to the real interest rate, the condition for the saddle-point stability in Lemma 3 is rewritten as $g^{fi} > r^b$.[18]

5.3 Long-run effects of the country-specific credit crunch

Suppose that the economy is on the BGP with $\theta = \theta^*$ at the initial date and θ unilaterally decreases with θ^* remaining unchanged. We investigate the characteristics of the new BGP.

Equation (28) represents the international equalization of the rate of return on the entrepreneur's aggregate net worth. Thus, this equation represents a positive relationship between the two cutoffs $\widetilde{z}$ and $\widetilde{z}^*$. By differentiating (28) and evaluating the differential coefficient on the old BGP (i.e., $\widetilde{z} = \widetilde{z}^* = \overline{z}(\theta)$), we obtain

$$-\theta\Psi'(\overline{z}(\theta))(d\widetilde{z} - d\widetilde{z}^*) = \Psi(\overline{z}(\theta))d\theta. \tag{32}$$

[18]Note that in our heterogeneous-agent framework, the transversality condition for each entrepreneur is satisfied, even though the growth rate exceeds the interest rate. We have already verified that each entrepreneur's consumption is given by $c_t = \rho a_t$. Therefore, the transversality condition for them is satisfied as $\lim_{t\to\infty} \mathbb{E}_t[e^{-\rho t}(a_t/c_t)] = \lim_{t\to\infty} \rho^{-1}e^{-\rho t} = 0$.

Equation (32) shows that the BGP condition (28) implies an asymmetric effect of θ on $d\widetilde{z}$ and $d\widetilde{z}^*$. When θ becomes smaller, it negatively affects the growth rate in country 1. With country 2's borrowing capacity θ^* held constant, equalizing the growth rates requires a higher (lower) entry of entrepreneurs in country 1 (2).

By contrast, (29) represents the negative relationship between $\widetilde{z}$ and $\widetilde{z}^*$ given the other variables. By differentiating this equation and evaluating the differential coefficient on the old BGP, we obtain the following equation: [19]

$$\varepsilon(\theta)(d\widetilde{z} + d\widetilde{z}^*) = \frac{d\theta}{\theta}, \tag{33}$$

where $\varepsilon(\theta)$ is defined as

$$\varepsilon(\theta) \equiv \frac{F'(\overline{z}(\theta))}{1 - F(\overline{z}(\theta))} > 0.$$

Equation (33) shows that the international asset market equilibrium (29) provides the symmetric effect of θ between $d\widetilde{z}$ and $d\widetilde{z}^*$. As a direct effect, a credit crunch in a country decreases the demand for funds by active entrepreneurs in that country. Under financial market integration, this credit crunch increases the global supply of funds relative to the global demand. However, the global asset market must remain in equilibrium. In both countries, less productive entrepreneurs, who would be inactive without a credit crunch, become active and borrow funds to produce the knowledge good. Note that $\varepsilon(\theta)$ is the ratio of the active entrepreneurs with their productivity given by $\overline{z}(\theta)$ to their total mass. When $\varepsilon(\theta)$ is small, a credit crunch in a country has a significant impact on the entrepreneurs' productivity cutoffs.

Let $\eta(\theta) \equiv -\Psi'(\overline{z}(\theta))/\Psi(\overline{z}(\theta)) > 0$. From (32) and (33), we obtain

$$\frac{d\widetilde{z}}{d\theta} = \frac{1}{2\theta}\left(\frac{1}{\varepsilon(\theta)} + \frac{1}{\eta(\theta)}\right) > 0,$$
$$\frac{d\widetilde{z}^*}{d\theta} = \frac{1}{2\theta}\left(\frac{1}{\varepsilon(\theta)} - \frac{1}{\eta(\theta)}\right) \gtreqless 0 \Leftrightarrow \eta(\theta) \gtreqless \varepsilon(\theta).$$

From these equations, we state the following proposition.

Proposition 5. *Consider the financially integrated two-country BGP with $\theta = \theta^*$ initially. With a permanent decrease only in θ, $\widetilde{z}$ necessarily decreases. By contrast, $\widetilde{z}^*$ increases (decreases) if $\eta(\theta) < (>)\varepsilon(\theta)$.*

Thus, a credit crunch in one country affects the supply and demand of funds in the other country through a change in the number of active entrepreneurs. This implies that a credit crunch can change the position of foreign assets. Because B is defined as debts, the net foreign debts relative to the total wealth in country 1 is given by $B/A \equiv v = \theta(1 - F(\widetilde{z})) - 1$. We obtain

$$\begin{aligned}\frac{dv}{d\theta} &= (1 - F(\overline{z})) - \theta F'(\overline{z})\frac{d\widetilde{z}}{d\theta} \\ &= \frac{1}{\theta} - \varepsilon(\theta)\frac{d\widetilde{z}}{d\theta} \\ &= \varepsilon(\theta)\frac{d\widetilde{z}^*}{d\theta} \gtreqless 0 \Leftrightarrow \eta(\theta) \gtreqless \varepsilon(\theta).\end{aligned}$$

[19] To obtain (33), we use the facts that $m^* = 1$, $P^* = 1$, and $(1 - F(\overline{z}(\theta))) = 1/\theta$ hold on the old BGP.

Because $v = 0$ on the old BGP, this result shows the following proposition:

Proposition 6. *Consider the financially integrated two-country BGP with $\theta = \theta^*$ initially. With a permanent decrease only in θ, v becomes positive (negative) if $\eta(\theta) < (>)\varepsilon(\theta)$.*

As a direct effect, the credit crunch in country 1 decreases the demand for funds in this country. Under financial market integration, this is expected to make that country a creditor nation. However, this proposition shows that such an expectation is not necessarily correct. If $\eta(\theta) < \varepsilon(\theta)$, country 1 becomes a debtor country due to its credit crunch.

Totally differentiating (27) and using the facts that $\widetilde{z} = \widetilde{z}^* = \overline{z}(\theta)$, $P^* = 1$, and $q(1) = q^*(1)$ hold on the old BGP, we obtain

$$\left(q'(1) - q^{*\prime}(1)\right) \overline{z}(\theta) dP^* = -q(1)(d\widetilde{z} - d\widetilde{z}^*).$$

On the left-hand side, $q'(1) - q^{*\prime}(1)$ is positive. Using (32), we obtain

$$\frac{dP^*}{d\theta} = \frac{-q(1)}{\theta\eta(\theta)\overline{z}(\theta)(q'(1) - q^{*\prime}(1))} < 0. \tag{34}$$

Thus, for the change in P^*, we obtain the same qualitative results as in the financial autarky case. This also means that the impact of credit tightness in a country on the intermediate goods firms' cutoffs is qualitatively the same as in the financial autarky case.

Finally, the effect of a unilateral change in θ on the BGP growth rate, $g^{fi} = q(P^*)\widetilde{z}(1+\theta\Psi(\widetilde{z})) - \delta - \rho$, is given by

$$\frac{dg^{fi}}{d\theta} = q(1)\overline{z}(\theta)\Psi(\overline{z}(\theta)) + q(1)\left[1 + \theta\Psi(\overline{z}(\theta)) + \overline{z}(\theta)\theta\Psi'(\overline{z}(\theta))\right]\frac{d\widetilde{z}}{d\theta} + \overline{z}(\theta)(1 + \theta\Psi(\overline{z}(\theta)))q'(1)\frac{dP^*}{d\theta}. \tag{35}$$

We have already shown $1 + \theta\Psi(\overline{z}) + \overline{z}\theta\Psi'(\overline{z}) = 0$. Therefore, the second term on the right-hand side vanishes. Using (34), the definition of η, and $1 + \theta\Psi(\overline{z}) = -\overline{z}\theta\Psi'(\overline{z})$, we can obtain

$$\frac{dg^{fi}}{d\theta} = -\frac{q^{*\prime}(1)}{q'(1) - q^{*\prime}(1)}q(1)\overline{z}(\theta)\Psi(\overline{z}(\theta)) > 0.$$

Thus, starting from the old symmetric BGP, the long-run growth effect of the credit crunch is the same as that in the case of financial autarky, even though the international financial transaction occurs due to this shock.

6 Numerical analysis

6.1 Setting of parameters

Suppose that $G(\varphi)$ is specified by (24). In addition, we assume that the entrepreneurs' productivity z satisfies $z \in [1, \beta]$ and follows the following upper-truncated Pareto distribution:

$$F(z) = \frac{1 - z^{-\gamma}}{1 - \beta^{-\gamma}},$$

where $\gamma > 1$ is the shape parameter. Then, we can write (28) and (29) as

$$\frac{\theta}{1-\beta^{-\gamma}}\left(\frac{\widetilde{z}^{-\gamma}}{\gamma-1}+\beta^{-\gamma}\right)=\frac{\theta^*}{1-\beta^{-\gamma}}\left(\frac{\widetilde{z}^{*-\gamma}}{\gamma-1}+\beta^{-\gamma}\right), \tag{28'}$$

$$\theta\left(\frac{\widetilde{z}^{-\gamma}-\beta^{-\gamma}}{1-\beta^{-\gamma}}\right)-1+m^*P^*\left[\theta^*\left(\frac{\widetilde{z}^{*-\gamma}-\beta^{-\gamma}}{1-\beta^{-\gamma}}\right)-1\right]=0, \tag{29'}$$

respectively. If $\beta \to \infty$, z follows a standard Pareto distribution.[20]

We normalize the population to unity: $L = 1$. Then, this model has 12 parameters still to be determined: $\rho, \delta, \alpha, f_D, f_X, f_E, \kappa, \tau, \gamma, \beta, \theta$, and θ^*.[21] We set $\rho = 0.04$ and $\delta = 0.025$, both of which are standard in the literature. We borrow the value of the elasticity of substitution between varieties, given by $1/(1-\alpha)$ here, from Ghironi and Melitz (2005) and set it at 3.4. This results in $\alpha \simeq 0.71$. For the shape parameter of the distribution of φ, we set $\kappa = 5$, which is consistent with the estimation results in Balistreri et al. (2011), who estimated that κ ranges from 3.924 to 5.171.

When setting the baseline degree of financial frictions, we assume that they are symmetric between the two countries; that is, $\theta = \theta^*$. Buera and Nicolini (2017) reported that the average ratio of debts to non-financial assets for the U.S. non-financial business sector between 1997Q3 and 2007Q3 was 0.69. In our model, the debts of the active entrepreneurs can be represented as $(\theta-1)(1-F(\widetilde{z}_t))A_t$. Then, the ratio of the active entrepreneurs' debts to capital (non-financial assets) in country 1 is given by

$$\frac{(\theta-1)(1-F(\widetilde{z}_t))A_t}{K_t}=\frac{\theta-1}{\theta}.$$

We set the ratio at 0.69, which results in

$$\theta=\theta^*=\overline{\theta}\equiv 3.226.$$

In the next section, we examine the effects of a unilateral deviation of θ from $\overline{\theta}$ with θ^* unchanged.

The remaining parameters to be set are $\gamma, f_D, f_X, f_E, \tau$, and β. We choose the value of γ as $\gamma = 3$. In Section 7.1, we examine the robustness of this inequality for other values of γ. We normalize f_D to 10, which leads to $\chi = 0.013$. We calibrate f_X, f_E, τ, and β against the following target values for the four endogenous variables. First, the BGP growth rate g^{fi} is set to 0.02. Second, the real interest rate in country 1 r^b is set to 0.015.[22] Third, the proportion of exporting firms is set at 0.21, as reported by Bernard et al. (2003, p.1270; Table 1). Finally, the import penetration ratio, which is one minus the share of expenditure on domestic intermediate goods, is set to 0.081, which was borrowed from Sampson (2016, Table 1 on p.351). Using these target values, we obtain $f_E = 1.419$, $f_X = 4.198$, $\tau \simeq 1.962$, and $\beta \simeq 2.06$. The resulting values generate

[20] If $\beta \to \infty$, however, (28') and (29') are rewritten as $\theta z^{-\gamma} = \theta^* z^{*-\gamma}$ and $\theta\widetilde{z}^{-\gamma} - 1 + m^*P^*\left[\theta^*\widetilde{z}^{*-\gamma} - 1\right] = 0$, respectively. From these equations, we obtain $(1 + m^*P^*)[\theta\widetilde{z}^{-\gamma} - 1] = 0$. That is, if z follows a standard Pareto distribution, the net foreign asset is always zero and financial autarky arises although there are international financial transactions. To avoid such a situation, we assume the upper bound for z.

[21] Recall that h and χ are defined as $h \equiv \alpha/[\kappa(1-\alpha)-\alpha]$ and $\chi \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)/f_D$, respectively.

[22] Because $g^{fi} > r^b$, the local saddle-point stability is guaranteed.

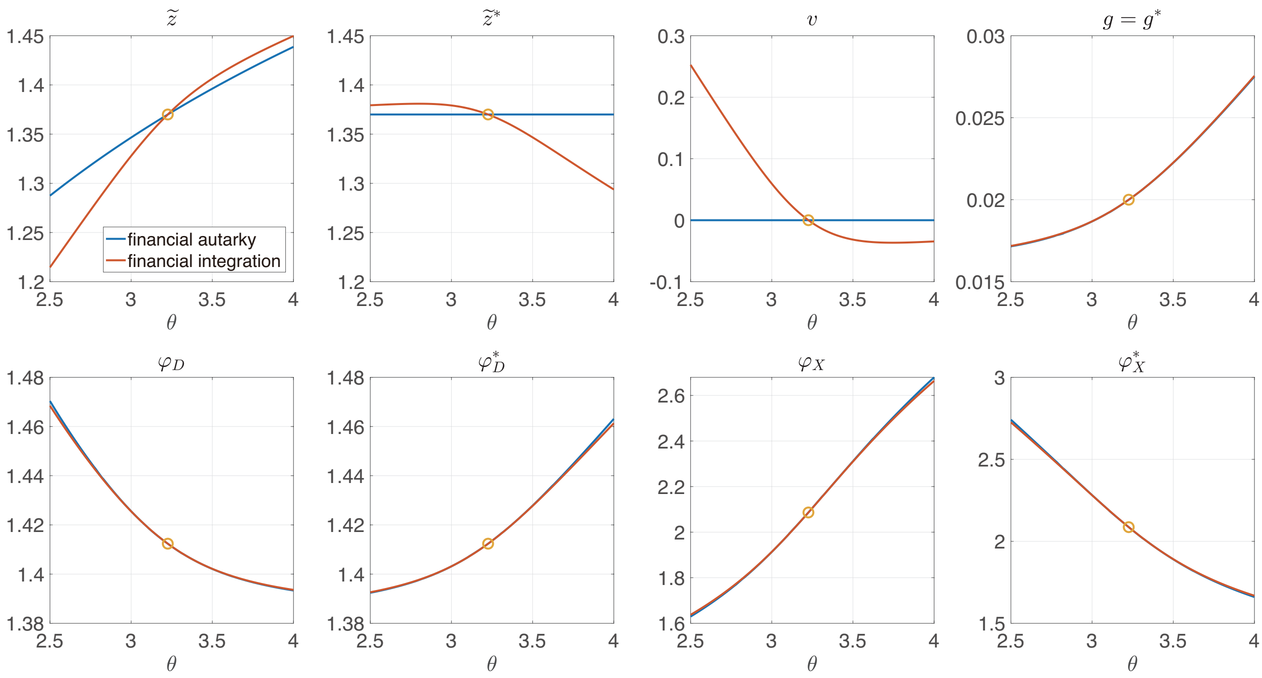

Figure 5: Effects of a permanent change in θ on key variables on the BGP

$T \equiv \tau(f_X/f_D)^{(1-\alpha)/\alpha} \simeq 1.366 > 1$. We provide the derivations of these parameter values in Appendix A.9.

6.2 Comparing long-run effects under financial autarky and integration

In this section, we consider the case in which θ changes permanently from its baseline value while θ^* remains unchanged. Figure 5 illustrates how such a unilateral change in θ affects key variables on the BGP. In each panel, the blue and red lines represent changes in the variable under financial autarky and integration, respectively. A circle marker in each panel indicates the benchmark value. As the first two panels show, the impacts on $\widetilde{z}$ and $\widetilde{z}^*$ differ between financial autarky and integration. This is because, under financial integration, the credit crunch in country 1 changes the cutoff for entrepreneurs in country 2 through the asset market. Under the calibrated parameters, $\eta(\theta) \simeq 1.379 < \varepsilon(\theta) \simeq 3.102$ holds true. According to Proposition 5, the validity of this inequality implies that $d\widetilde{z}/d\theta > 0$ and $d\widetilde{z}^*/d\theta < 0$. As the second panel shows, the decrease in θ indeed raises the productivity cutoff for entrepreneurs in country 2 and, consequently, decreases the mass of active entrepreneurs who borrow funds to produce the knowledge good. Therefore, as shown in the third panel, the relative demand for funds decreases in country 2 such that country 1 (which directly experienced the credit crunch) increases its net foreign debts. This effect does not occur under financial autarky, as indicated by the blue line in this panel.

However, as shown in the fourth panel, such a difference in the impact on the entrepreneurs' cutoffs between these two cases only results in a negligible difference in the growth effects. Furthermore, as shown in the four panels in the bottom row, the effects on the cutoffs for the intermediate goods firms are almost the same for the two cases. International asset transactions are usually considered an international propagation channel for financial shocks. However, the results obtained here imply that even if the two countries quit international financial transactions, trade in goods still facilitates a sufficiently large international propagation of a country-specific financial shock.

6.3 Comparing responses to temporary shocks under financial autarky and integration

Thus far, we have analyzed the effects of a permanent change in θ. In this section, we numerically analyze the responses of key variables in the case of a temporary credit crunch. Many previous studies have considered temporary shocks to the extent of financial frictions to quantitatively examine the impact of financial crises on real economic activity. In contrast, this study aims to qualitatively identify the importance of firm selection in the tradable goods sector as a transmission channel for country-specific shocks. Therefore, in this section, we examine whether the similarity in the effects on growth and trade between financial autarky and financial integration, as shown in Section 6.2, also holds in the case of temporary shocks.

We assume that θ varies with time. More specifically, we assume that the economy is initially on the symmetric BGP with $\theta = \theta^* = \overline{\theta}$, and that θ_0 unanticipatedly decreases by 10% from $\overline{\theta}$. Subsequently, θ_t gradually returns to the baseline value according to the following equation:

$$\dot{\theta}_t = -\varrho(\theta_t - \overline{\theta}),$$

where $\overline{\theta}$ is set at 3.226, as in Section 6.1. In this equation, $\varrho > 0$ is a parameter specifying the persistence of the shock, set at 0.02. We assume that θ^* remains unchanged as before, that is, $\theta_t^* = \overline{\theta}$ for all t.

Figure 6 presents the responses of the major variables. The first panel displays the shock in country 1. In the second and subsequent panels, the blue and red lines represent the responses under financial autarky and integration, respectively. In each case, we calculate the responses by using the linear approximation of the dynamical system.[23] In all the panels, the dashed lines represent the initial symmetric BGP values. As already stated, the relative wealth m^* is a state variable, while the final good price in country 2, P^*, is a forward-looking one. Hence, P^* immediately changes in response to the credit crunch. Foreign debt per assets in country 1, $v \equiv B/A$, is the state variable. Although it never changes from zero under financial autarky, it does so under financial integration. This leads to differences in the responses of $\widetilde{z}$ and $\widetilde{z}^*$, particularly $\widetilde{z}^*$ as shown in the sixth panel.

The seventh to tenth panels illustrate the effects of the credit crunch on the productivity cutoffs of intermediate goods firms. Overall, the magnitude of the initial change immediately after the shock is greater under financial integration. This is because P^* increases more sharply under financial integration. As explained in Section 4, P^* is a key variable for intermediate goods firms, as it affects the relative magnitude of demand for their goods between the two countries. Consequently,

[23]More specifically, we use the linear approximation of (18) and (22) in the case of financial autarky, and use the linear approximation of the system described in Appendix A.7 in the case of financial integration.

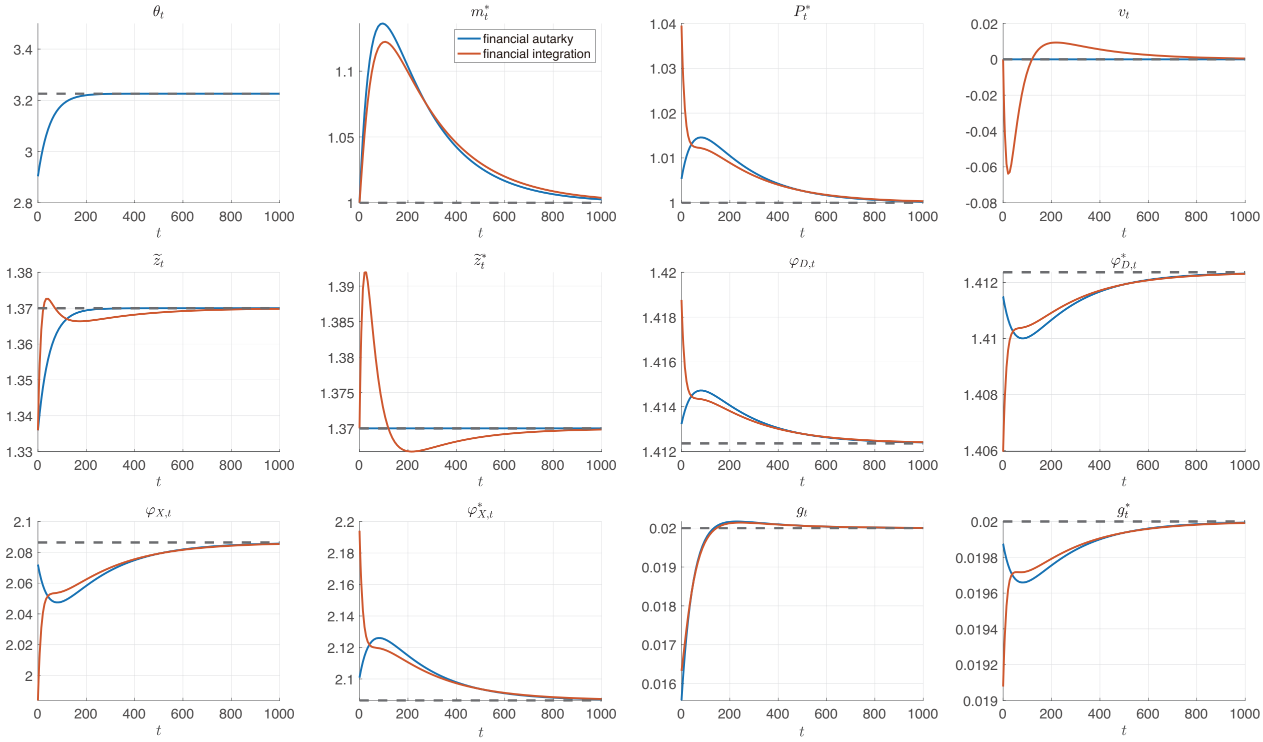

Figure 6: Dynamic responses of major variables to a temporary decrease in θ

the changes in the cutoffs align with the change in P^*. However, these panels also indicate that the persistence of the effects is similar between financial autarky and integration. After a period of time, the movements of the cutoffs are almost identical in both cases.

Focusing on the last two panels, we observe the time variation in the growth rates in the two countries. For country 1, the responses are quite similar across both cases at all time points, although some difference is noted at the initial stage. This indicates that the direct effect of the country's credit crunch (i.e., the decrease in θ) is dominant over other effects arising from whether international financial markets are open. In contrast, for country 2, the responses differ between the two cases, reflecting differences in P^*. More specifically, while the initial decline is larger under financial integration, the effect of the shock is more persistent under financial autarky, implying that it takes longer for the growth rate to return to its initial value. In Section 6.2, which focuses on the effects of a permanent credit crunch, we highlight the strong role of trade as a transmission channel for the shock. The result obtained in this section shows that even if the shock is temporary, trade exerts a persistent recessionary effect. Moreover, a comparison of the last two panels reveals some interesting findings. The decline in growth is indeed larger in country 1, which experiences a credit crunch. However, country 1 also recovers more quickly. Meanwhile, country 2 experiences a smaller decline in its growth, but takes longer to recover than country 1. Notably, this slower recovery is observed under both financial autarky and integration. This result is partly consistent with the fact that countries that did not directly experience the shock during the 2007-09 financial crisis also struggled to recover economically.

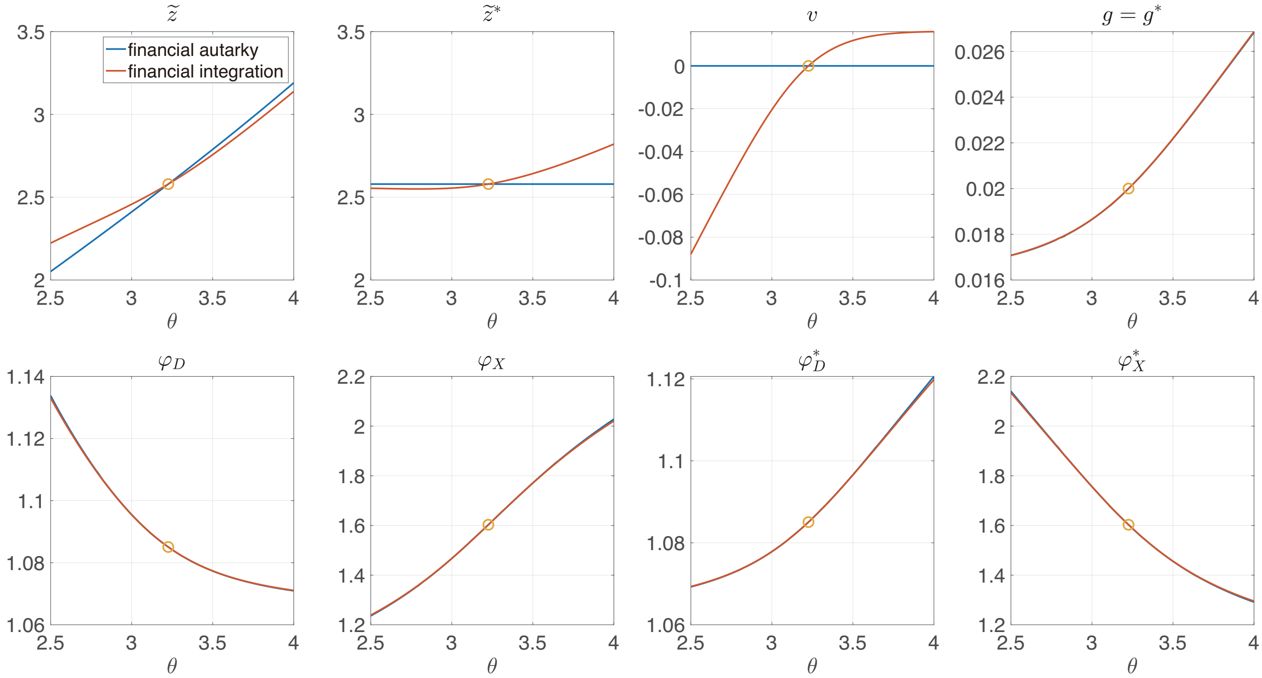

Figure 7: Sensitivity

7 Discussion

7.1 Sensitivity in the long-run effects

We consider the case of $\eta(\theta) < \varepsilon(\theta)$ in the numerical analysis of Section 6.2. If we change the value of γ to 5, it follows that $\eta(\theta) \simeq 1.598 < \varepsilon(\theta) \simeq 7.189$. Then the sign of the inequality does not change. If we change the value of γ to 1.5, it follows that $\eta(\theta) \simeq 0.7323 > \varepsilon(\theta) \simeq 0.4119$. Figure 7 shows the numerical results for the change in θ in this case. A comparison of the second and third panels of Figures 5 and 7 reveals that the results of comparative statics are reversed for $\widetilde{z}^*$ and v, as foreseen in Propositions 5 and 6. However, no significant differences in the two figures are observed for the other key variables.

7.2 On the non-tradability of the final good

Throughout this study, we assume that the final good produced in each country cannot be traded internationally. If we allow international trade in the final good, $P_t^* = 1$ for all t. This in turn leads to $\varphi_D = \varphi_D^*$. From (12) and (13), $q(1) = q^*(1)$ holds in this case. First, consider the case of financial autarky as in Section 3. The condition for balanced growth under financial autarky (23) is now given by $\overline{z}(\theta)(1 + \theta\Psi(\overline{z}(\theta))) = \overline{z}(\theta^*)(1 + \theta^*\Psi(\overline{z}(\theta^*)))$, which is further rewritten as $\theta = \theta^*$, namely, the condition for balanced growth fails to hold unless the two countries are perfectly symmetric. Next, we consider the case of financial integration as in Section 5. Since $q(1) = q^*(1)$ holds, the interest parity (27) is now given by $\widetilde{z} = \widetilde{z}^*$. The condition for balanced growth under financial integration (28) is rewritten as $\theta = \theta^*$, which still does not hold, except for the knife-edge case of perfectly symmetric countries. Therefore, in our framework of international asymmetry in the degree of financial frictions, the non-tradability of the final good is required to obtain the BGP equilibrium.

8 Concluding remarks

After the financial crisis, some countries experienced prolonged economic downturns despite suffering relatively less damage to their domestic financial sectors. In this study, we provide a novel mechanism through which domestic financial shocks are transmitted to foreign economies via trade and firm selection dynamics, with long-run implications for aggregate productivity.

One advantage of this study over previous ones is the tractability of the proposed model. This enabled us to analytically elucidate both short-run and long-run effects. Specifically, we demonstrated that a credit crunch in one country lowers the growth rate of its trading partner country at all points in time. The economic slowdown in the partner country occurs due to the decreased profitability of exports for trading partner firms, leading some exporters to transition their focus solely to domestic sales. We also compared the effects of a credit crunch under financial autarky and integration and found that the impacts on trade and growth are remarkably similar in both scenarios. The key takeaway from our results is that international trade in goods facilitates significant international propagation of country-specific financial shocks. Our results highlight a novel channel through which credit shocks propagate internationally, even in the absence of cross-border finance: via firm selection and trade reallocation. In addition, these findings, indicating economic downturns resulting from sluggish export growth, are consistent with Japan's experience following the financial crisis.

In this study, we relied on certain assumptions to derive results analytically while introducing heterogeneities in two types of agents: intermediate goods firms and entrepreneurs. Therefore, our model is open to extensions. First, it would be beneficial to assume that the productivity of each agent has some degree of persistence rather than being determined by iid shocks. Second, it is important to investigate the effects of the credit crunch by assuming that not only entrepreneurs but also other agents face credit constraints. Both extensions are crucial for quantitatively evaluating the international propagation of financial shocks through international trade and financial transactions. Nevertheless, the results obtained in this study provide a useful benchmark.

Appendix

A.1 Entrepreneurs' intertemporal optimization

This maximization problem in Section 2.3 is rewritten as the following Hamilton–Jacobi–Bellman (HJB) equation:

$$\rho U_t(a,z)dt = \max \Big\{ (\ln c)dt + \mathbb{E}_t\left[dU_t(a,z)\right] \ : \ da = \left(R_t(z)a - c\right)dt \Big\},$$

where $U_t(a,z)$ is the value function. We can solve this problem using "guess and verify." We guess that the value function takes the form $U_t(a,z) = \zeta(\phi_t(z) + \ln a)$ with ζ and $\phi_t(z)$ being unknown. Using this guess leads to $\mathbb{E}_t[dU_t(a,z)] = \zeta\left(\mathbb{E}_t[d\phi_t(z)] + da/a\right)$. The right-hand side of the HJB equation is rewritten as

$$\max_c \; \ln c + \frac{\zeta(R_t(z)a - c)}{a} + \zeta \frac{1}{dt}\mathbb{E}_t\left[d\phi_t(z)\right].$$

Given ζ, the first-order condition is given by $1/c = \zeta/a$. This result must be consistent with the HJB equation. Substituting this result back into the HJB equation, we obtain

$$\rho\zeta(\phi_t(z) + \ln a) = \ln a - \ln\zeta + \zeta R_t(z) - 1 + \zeta\frac{1}{dt}\mathbb{E}_t\left[d\phi_t(z)\right],$$

the equality of which must be satisfied for all a. This implies $\zeta = 1/\rho$, which, in turn, yields $c_t = \rho a_t$. The dynamics of the net worth are given by $da_t = (R_t(z_t) - \rho)\, a_t dt$.

A.2 Derivation of (19)

In the main text, we define μ_t as the share of the export sales in the total sales of country 1's intermediate goods firms.

$$\mu_t \equiv \frac{\int_{\varphi \geq \varphi_{X,t}} p_X(\varphi) y_X(\varphi) dG(\varphi)}{\sum_{j=D,X}\int_{\varphi\geq\varphi_{j,t}} p_j(\varphi) y_j(\varphi) dG(\varphi)}.$$

We omit the time subscript hereafter. Using $p_j(\varphi) = 1/(\alpha\varphi)$, $y_j(\varphi) = (\varphi/\varphi_j)^{1/(1-\alpha)} y_j(\varphi_j)$, and $y_j(\varphi_j) = \alpha f_j \varphi_j q/(1-\alpha)$,

$$p_j(\varphi)y_j(\varphi) = \frac{f_j q}{1-\alpha}\left(\frac{\varphi}{\varphi_j}\right)^{\alpha/(1-\alpha)}, \quad j = D, X. \tag{A.1}$$

Substituting (A.1) into the definition of μ,

$$\mu = \frac{f_X \int_{\varphi\geq\varphi_X}\left(\frac{\varphi}{\varphi_j}\right)^{\alpha/(1-\alpha)} dG(\varphi)}{\sum_{j=D,X} f_j \int_{\varphi\geq\varphi_j}\left(\frac{\varphi}{\varphi_j}\right)^{\alpha/(1-\alpha)} dG(\varphi)}.$$

Applying the definition of $H(\varphi_j)$ to this equation yields $\mu(P^*)$ in (19). Its derivative with respect to P^* and $\mu'(P^*)$ is given by

$$\mu'(P^*) = \frac{f_X\left[H'(\varphi_x)\varphi_X'(P^*) f_D H(\varphi_D) - H(\varphi_X) f_D H'(\varphi_D)\varphi_D'(P^*)\right]}{\left(\sum_{j=D,X} f_j H(\varphi_j(P^*))\right)^2}.$$

Since $\varphi_D' > 0$ and $\varphi_X' < 0$ from Lemma 1 and $H'(\varphi_j) < 0$, we can show that $\mu'(P^*) > 0$.

We obtain $\mu^*(P^*)$ in a similar way. The share of the export sales in the total sales of country 2's intermediate goods firms, μ^*, is defined as

$$\mu^* \equiv \frac{\int_{\varphi\geq\varphi_X^*} p_X^*(\varphi) y_X^*(\varphi) dG(\varphi)}{\sum_{j=D,X}\int_{\varphi\geq\varphi_j^*} p_j^*(\varphi) y_j^*(\varphi) dG(\varphi)}.$$

Using $p_j^*(\varphi) = P^*/(\alpha\varphi)$, $y_j^*(\varphi) = (\varphi/\varphi_j^*)^{1/(1-\alpha)} y_j^*(\varphi_j)$, and $y_j^*(\varphi_j^*) = \alpha f_j \varphi_j^* q^*/(1-\alpha)$,

$$p_j^*(\varphi) y_j^*(\varphi) = \frac{f_j P^* q^*}{1-\alpha} \left(\frac{\varphi}{\varphi_j^*} \right)^{\alpha/(1-\alpha)}, \quad j = D, X. \tag{A.2}$$

Substituting (A.2) into the definition of μ^* and applying the definition of H into the resulting equation, we obtain

$$\mu^* = \mu^*(P^*) \equiv \frac{f_X H(\varphi_X^*(P^*))}{\sum_{j=D,X} f_j H(\varphi_j^*(P^*))}.$$

Since $\varphi_D^{*\prime} < 0$, and $\varphi_X^{*\prime} > 0$ from Lemma 1 and $H'(\varphi_j) < 0$, we can show that $\mu^{*\prime}(P^*) < 0$.

A.3 Autarky equilibrium

In this appendix, we characterize the autarky equilibrium as the benchmark for the main analysis. We focus on country 1 and choose the final good as the numeraire: $P_t = 1$ for all t. Without international trade, the free entry condition (5) is replaced by

$$f_E = f_D \Pi(\varphi_{D,t}).$$

Thus, under autarky, the intermediate goods firms' cutoff is always constant, denoted by φ_D^a.

Equation (11) in Section 3 is derived by substituting (3) and (4) for $j = D$ into (1). Therefore, this equation holds not only in the two-country economy but also in the autarky economy. Equation (11) then determines the price of the knowledge good as $q^a \equiv \chi L(\varphi_D^a)^{\alpha/(1-\alpha)}$. In addition, in the absence of international financial transactions, $A = K$ holds, implying that the entrepreneurs' cutoff productivity is given by $\overline{z}(\theta)$, which is defined in the main text. Substituting this result into (8), we can show that the growth rate of net worth is always constant:

$$\frac{\dot{A}_t}{A_t} = g^a \equiv q^a \overline{z}(\theta)(1 + \theta\Psi(\overline{z}(\theta))) - \delta - \rho, \tag{A.3}$$

Under autarky, the market-clearing condition of the knowledge good is given by

$$Y_{K,t} = N_t^e \left(f_E + (1 - G(\varphi_D^a) f_D \right).$$

Since $B_t = 0$ for all t, K_t always equals A_t. Therefore, the mass of entrants grows at the same rate as the net worth from the initial date:

$$N_t^e = \frac{Y_{K,t}}{f_E + f_D(1 - G(\varphi_D^a))} = \frac{Z(\overline{z}(\theta)) A_t}{f_E + f_D(1 - G(\varphi_D^a))}.$$

The output of the final good under autarky is given by

$$Y_t = \frac{L^{1-\alpha} N_t^e}{\alpha} \int_{\varphi \geq \varphi_{D,t}} y_{D,t}(\varphi)^\alpha dG(\varphi) = \frac{L^{1-\alpha} N_t^e}{\alpha} H(\varphi_D^a) \left(y_D(\varphi_D^a) \right)^\alpha.$$

Thus, under autarky, the economy is on the BGP from the initial time and the output of the final good always continues to grow through the expansion of the variety of intermediate goods.

A.4 Derivations of (20) and (21)

In the main text, we respectively define EX and IM as

$$EX \equiv N^e \int_{\varphi \geq \varphi_X} p_X(\varphi) y_X(\varphi) dG(\varphi), \tag{A.4}$$

$$IM (= EX^*) \equiv N^{e*} \int_{\varphi \geq \varphi_X^*} p_X^*(\varphi) y_X^*(\varphi) dG(\varphi). \tag{A.5}$$

Substituting (A.1) and (A.2) into (A.4) and (A.5), and using the definition of $H(\cdot)$, we obtain

$$EX = N^e \frac{q f_X H(\varphi_X)}{1-\alpha}, \tag{A.6}$$

$$IM = N^{e*} \frac{q^* P^* f_X H(\varphi_X^*)}{1-\alpha}. \tag{A.7}$$

Next, we derive N^e and N^{e*}. From (14) and $Y_K = \theta A \int_{z \geq \tilde{z}} z dF(z)$, we can obtain N^e as

$$N^e = \frac{\theta A \int_{z \geq \tilde{z}} z dF(z)}{f_E + \sum_{j=D,X} f_j (1 - G(\varphi_j))}.$$

Here note that $\Pi(\varphi_j) = H(\varphi_j) - (1 - G(\varphi_j))$. Then the free entry condition (5) is rewritten as

$$f_E + \sum_{j=D,X} f_j (1 - G(\varphi_j)) = \sum_{j=D,X} f_j H(\varphi_j),$$

which implies

$$N^e = \frac{\theta A \int_{z \geq \tilde{z}} z dF(z)}{\sum_{j=D,X} f_j H(\varphi_j)}.$$

Analogously, N^{e*} is given by

$$N^{e*} = \frac{\theta^* A^* \int_{z \geq \tilde{z}^*} z dF(z)}{\sum_{j=D,X} f_j H(\varphi_j^*)}.$$

Substituting the resulting N^e and N^{e*} into (A.6) and (A.7), we obtain

$$EX = \frac{q(P^*) \theta A \int_{z \geq \tilde{z}} z dF(z)}{1-\alpha} \mu(P^*),$$

$$IM = \frac{q^*(P^*) P^* \theta^* A^* \int_{z \geq \tilde{z}^*} z dF(z)}{1-\alpha} \mu^*(P^*).$$

A.5 Proof of Lemma 2

First, we derive the following two equations useful for the proof. Totally differentiating (5), (9), and (10), and arranging the results, we obtain

$$\frac{P^* \varphi_D'(P^*)}{\varphi_D(P^*)} = \frac{1}{\alpha} \frac{f_X H(\varphi_X)}{\sum_{j=D,X} f_j H(\varphi_j)} = \frac{\mu}{\alpha}, \tag{A.8}$$

$$\frac{P^* \varphi_D^{*\prime}(P^*)}{\varphi_D^*(P^*)} = -\frac{\mu^*}{\alpha}. \tag{A.9}$$

Note that these equations hold without the Pareto specification of G.

Then we show Lemma 2. Under financial autarky, the BGP is globally stable if and only if $dm^*/dP^* > 0$ holds in (22). As stated in Lemma 2, we specify G as $G(\varphi) = 1 - \varphi^{-\kappa}$. In this case, we can calculate μ as follows:[24]

$$\mu = \frac{f_X \varphi_X^{-\kappa}}{f_D \varphi_D^{-\kappa} + f_X \varphi_X^{-\kappa}} = \frac{h f_X}{f_E} \varphi_X^{-\kappa}, \tag{A.10}$$

and we obtain μ^* as $h f_X \varphi_X^{*-\kappa} / f_E$. Using (9), (10), and (A.10), we can rewrite (22) as

$$\begin{aligned} m^* &= \left(\frac{\varphi_X}{\varphi_X^*}\right)^{-\kappa} \left(\frac{\varphi_D}{\varphi_D^*}\right)^{\alpha/(1-\alpha)} \frac{1}{P^*} \frac{\theta \int_{z \geq \overline{z}(\theta)} z dF(z)}{\theta^* \int_{z \geq \overline{z}(\theta^*)} z dF(z)} \\ &= \left(\frac{P^{*-1/\alpha} \varphi_D}{P^{*1/\alpha} \varphi_D^*}\right)^{-\kappa} \left(\frac{\varphi_D}{\varphi_D^*}\right)^{\alpha/(1-\alpha)} \frac{1}{P^*} \frac{\theta \int_{z \geq \overline{z}(\theta)} z dF(z)}{\theta^* \int_{z \geq \overline{z}(\theta^*)} z dF(z)} \\ &= \left(\frac{\varphi_D}{\varphi_D^*}\right)^{\alpha/(1-\alpha)-\kappa} P^{*2\kappa/\alpha - 1} \frac{\theta \int_{z \geq \overline{z}(\theta)} z dF(z)}{\theta^* \int_{z \geq \overline{z}(\theta^*)} z dF(z)}. \end{aligned} \tag{A.11}$$

Let a hat over a variable denote its rate of change (e.g., $\widehat{x} = dx/x$). From (A.11), we obtain

$$\frac{\widehat{m}^*}{\widehat{P}^*} = \left(\frac{\alpha}{1-\alpha} - \kappa\right) \left(\frac{\widehat{\varphi}_D}{\widehat{P}^*} - \frac{\widehat{\varphi}_D^*}{\widehat{P}^*}\right) + \frac{2\kappa}{\alpha} - 1.$$

Using (A.8) and (A.9), we obtain

$$\frac{\widehat{m}^*}{\widehat{P}^*} = \frac{\kappa[2 - (\mu + \mu^*)]}{\alpha} + \frac{\mu + \mu^*}{1-\alpha} - 1.$$

Recall that we assume $h > 0$; that is, $\kappa > \alpha/(1-\alpha)$ for $\Pi(\varphi_j) = h\varphi_j^{-\kappa}$ to be well defined. Then the following inequality holds:

$$\frac{\widehat{m}^*}{\widehat{P}^*} > \frac{2}{1-\alpha} - 1 > 0.$$

Namely, when $G(\varphi) = 1 - \varphi^{-\kappa}$, $dm^*/dP^* > 0$ is always satisfied. This, in turn, shows that the BGP under financial autarky is globally stable when $G(\varphi) = 1 - \varphi^{-\kappa}$.

A.6 Derivation of (30)

Differentiating the definition of v with respect to time yields

$$\dot{v} = \dot{B}/A - v\dot{A}/A.$$

Using (8) and (15), we can arrange this equation as follows:

$$\begin{aligned} \dot{v} &= r^b v - \frac{EX - IM}{A} - v\left[r^b + \theta \int_{z \geq \widetilde{z}} \left(qz - \delta - r^b\right) dF(z) - \rho\right] \\ &= \left[\rho - \theta \int_{z \geq \widetilde{z}} (qz - q\widetilde{z})\, dF(z)\right] v - \frac{EX - IM}{A} \\ &= \left(\rho - q(P^*)\widetilde{z}\theta\Psi(\widetilde{z})\right) v - \frac{EX - IM}{A}. \end{aligned} \tag{A.12}$$

[24]When $G(\varphi) = 1 - \varphi^{-\kappa}$, we can calculate $H(\varphi_j) = (h+1)\varphi_j^{-\kappa}$, where $h > 0$ is shown in the main text. Substituting this into (19) and using the fact that $f_D \varphi_D^{-\kappa} + f_X \varphi_X^{-\kappa} = f_E/h$ holds from the free entry condition, we obtain μ.

In Appendix A.4, we have already shown that EX and IM are expressed as (20) and (21). Therefore, the net export of country 1 relative to its net worth can be expressed as

$$\frac{EX - IM}{A} = \frac{1}{1-\alpha}\left(\mu(P^*)q(P^*)\theta \int_{z\geq\tilde{z}} zdF(z) - m^*P^*\mu^*(P^*)q^*(P^*)\theta^* \int_{z\geq\tilde{z}^*} zdF(z)\right). \quad \text{(A.13)}$$

Substituting (A.13) into (A.12) yields (30).

A.7 The dynamical system under financial integration

Here we derive the autonomous dynamical system. For this, it is convenient to use v instead of $\tilde{z}$ and $\tilde{z}^*$. From $v = \theta(1 - F(\tilde{z})) - 1$, we can express $\tilde{z}$ as a function of v:

$$\tilde{z}_t = \tilde{z}(v_t),$$

where

$$\tilde{z}_v(v) \equiv \frac{\partial \tilde{z}(\cdot)}{\partial v} = \frac{1}{-\theta F'(\tilde{z})} < 0.$$

Using v, (29) can be rewritten as

$$v_t + m_t^* P_t^* [\theta^*(1 - F(\tilde{z}_t^*)) - 1] = 0.$$

Note that this equation holds not only on but also off the BGP, which implies that $\tilde{z}^*$ is a function of v, m^*, and P^*:

$$\tilde{z}_t^* = \tilde{z}^*(v_t, m_t^*, P_t^*),$$

where it follows that as long as $m^* > 0$ and $P^* > 0$,

$$\tilde{z}_v^*(\cdot) \equiv \frac{\partial \tilde{z}^*(\cdot)}{\partial v} = \frac{1}{m^* P^* \theta^* F'(\tilde{z}^*)} > 0,$$
$$\tilde{z}_{m^*}^*(\cdot) \equiv \frac{\partial \tilde{z}^*(\cdot)}{\partial m^*} = \frac{\theta^*(1 - F(\tilde{z}^*)) - 1}{m^* \theta^* F'(\tilde{z}^*)},$$
$$\tilde{z}_{P^*}^*(\cdot) \equiv \frac{\partial \tilde{z}^*(\cdot)}{\partial P^*} = \frac{\theta^*(1 - F(\tilde{z}^*)) - 1}{P^* \theta^* F'(\tilde{z}^*)}.$$

We have already derived the dynamic equation of v as (30). Here we define function Λ as

$$\Lambda(v, m^*, P^*) \equiv \frac{1}{1-\alpha}\left(\mu(P^*)q(P^*)\theta \int_{z\geq\tilde{z}(v)} zdF(z) - m^*P^*\mu^*(P^*)q^*(P^*)\theta^* \int_{z\geq\tilde{z}^*(v,m^*,P^*)} zdF(z)\right).$$

Then, (30) gives the dynamic equation of v, where its right-hand side now contains v, m^*, and P^*:

$$\dot{v}_t = \left(\rho - q(P_t^*)\tilde{z}(v_t)\theta\Psi(\tilde{z}(v_t))\right)v_t - \Lambda(v_t, m_t^*, P_t^*). \quad \text{(A.14)}$$

Using $\tilde{z}_v, \tilde{z}_v^*$, and $\tilde{z}_{m^*}^*$, the properties of Λ are given by

$$\Lambda_v(\cdot) \equiv \frac{\partial \Lambda(\cdot)}{\partial v} = \frac{1}{1-\alpha}\left(-\mu q \theta \tilde{z} F'(\tilde{z})\tilde{z}_v + m^* P^* \mu^* q^* \theta^* \tilde{z}^* F'(\tilde{z}^*)\tilde{z}_v^*\right),$$
$$= \frac{1}{1-\alpha}\left(\mu q \tilde{z} + \mu^* q^* \tilde{z}^*\right),$$
$$\Lambda_{m^*}(\cdot) \equiv \frac{\partial \Lambda(\cdot)}{\partial m^*} = \frac{P^*\mu^*q^*}{1-\alpha}\left(-\theta^* \int_{z\geq\tilde{z}^*} zdF(z) + m^*\theta^*\tilde{z}^*F'(\tilde{z}^*)z_{m^*}^*\right)$$
$$= \frac{P^*\mu^*q^*}{1-\alpha}\left[-\theta^* \int_{z\geq\tilde{z}^*} zdF(z) + \tilde{z}^*[\theta^*(1 - F(\tilde{z}^*)) - 1]\right].$$

The interest parity (26) provides the dynamics of the final good's price in country 2:

$$\frac{\dot{P}_t^*}{P_t^*} = q(P_t^*)\widetilde{z}(v_t) - q^*(P_t^*)\widetilde{z}^*(v_t, m_t^*, P_t^*). \tag{A.15}$$

From (8) and its counterpart in country 2, we can express the dynamic equation of m as follows:

$$\begin{aligned}\frac{\dot{m}_t}{m_t} &\equiv \frac{\dot{A}_t^*}{A_t^*} - \frac{\dot{A}_t}{A_t} \\ &= \Gamma^*(v_t, m_t^*, P_t^*) - \Gamma(v_t, P_t^*). \end{aligned} \tag{A.16}$$

Equations (A.14)–(A.16) jointly constitute the autonomous dynamical system of the model. In (A.16), the functions Γ and Γ^* are defined as

$$\begin{aligned}&\Gamma(v, P^*) \equiv q(P^*)\widetilde{z}(v)\left[1 + \theta\Psi(\widetilde{z}(v))\right] \\ &\Gamma^*(v, m^*, P^*) \equiv q^*(P^*)\widetilde{z}^*(v, m^*, P^*)\left[1 + \theta^*\Psi(\widetilde{z}^*(v, m^*, P^*))\right],\end{aligned}$$

where

$$\begin{aligned}\Gamma_v(\cdot) &\equiv \frac{\partial\Gamma(\cdot)}{\partial v} = q\widetilde{z}_v\left(1 + \theta\Psi(\widetilde{z}) + \widetilde{z}\theta\Psi'(\widetilde{z})\right) = q\widetilde{z}_v[1 - \theta(1 - F(\widetilde{z}))] \\ \Gamma_{P^*}(\cdot) &\equiv \frac{\partial\Gamma(\cdot)}{\partial P^*} = q'\widetilde{z}\left(1 + \theta\Psi(\widetilde{z})\right) > 0, \\ \Gamma_v^*(\cdot) &\equiv \frac{\partial\Gamma^*(\cdot)}{\partial v} = q^*\widetilde{z}_v^*\left(1 + \theta^*\Psi(\widetilde{z}^*) + \widetilde{z}^*\theta^*\Psi'(\widetilde{z}^*)\right) = q^*\widetilde{z}_v^*\left[1 - \theta^*(1 - F(\widetilde{z}^*))\right], \\ \Gamma_{m^*}^*(\cdot) &\equiv \frac{\partial\Gamma^*(\cdot)}{\partial m^*} = q^*\widetilde{z}_{m^*}\left(1 + \theta^*\Psi(\widetilde{z}^*) + \widetilde{z}^*\theta^*\Psi'(\widetilde{z}^*)\right) = q^*\widetilde{z}_{m^*}^*\left[1 - \theta^*(1 - F(\widetilde{z}^*))\right], \\ \Gamma_{P^*}^*(\cdot) &\equiv \frac{\partial\Gamma^*(\cdot)}{\partial P^*} = q^{*\prime}\widetilde{z}^*\left(1 + \theta^*\Psi(\widetilde{z}^*)\right) + q^*\widetilde{z}_{P^*}^*\left(1 + \theta^*\Psi(\widetilde{z}^*) + \widetilde{z}^*\theta^*\Psi(\widetilde{z}^*)\right) \\ &= q^{*\prime}\widetilde{z}^*\left(1 + \theta^*\Psi(\widetilde{z}^*)\right) + q^*\widetilde{z}_{P^*}^*\left[1 - \theta^*(1 - F(\widetilde{z}^*))\right].\end{aligned}$$

A.8 Proof of Lemma 3

To show this lemma, we take the following three steps.

Linearization of the system We examine the local stability of the dynamical system (A.14)–(A.16) with respect to v, m^*, and P^*. Recall the definitions of v and m^*: $v \equiv B/A$ and $m^* \equiv A^*/A$. Since $A^{(*)}$ is the aggregate wealth and B is the net foreign debts, both are predetermined variables, which implies that v_0 and m_0^* are historically given. By contrast, since the final good price P^* is a forward-looking variable, its initial value is endogenously determined. Therefore, the system has one forward-looking and two predetermined variables.

Recall also that on the symmetric BGP, $(v, m^*, P^*) = (0, 1, 1)$. The linearization of the system around the symmetric BGP is given by

$$\begin{pmatrix} \dot{v}_t \\ \dot{m}_t^* \\ \dot{P}_t^* \end{pmatrix} = J \begin{pmatrix} v_t - 0 \\ m_t^* - 1 \\ P_t^* - 1 \end{pmatrix},$$

where the Jacobian matrix J is

$$J = \begin{pmatrix} \rho - q\overline{z}\theta\Psi(\overline{z}) - \Lambda_v(0,1,1) & -\Lambda_{m^*}(0,1,1) & -\Lambda_{P^*}(0,1,1) \\ \Gamma_v^*(0,1,1) - \Gamma_v(0,1,1) & \Gamma_{m^*}(0,1,1) & \Gamma_{P^*}^*(0,1,1) - \Gamma_{P^*}(0,1,1) \\ q\widetilde{z}_v(0) - q^*\widetilde{z}_v^*(0,1,1) & -q^*\widetilde{z}_{m^*}^*(0,1,1) & q'\widetilde{z} - q^{*'}\widetilde{z}^* - q^*\widetilde{z}_{P^*}^*(0,1,1) \end{pmatrix}.$$

When we consider the symmetric BGP, we can obtain

$$\begin{aligned}
&\widetilde{z}_v(0) = -\widetilde{z}_v^*(0,1,1) = -\frac{1}{\theta F'(\overline{z}(\theta))} < 0, \\
&\widetilde{z}_{m^*}^*(0,1,1) = 0, \\
&\widetilde{z}_{P^*}^*(0,1,1) = 0, \\
&\Lambda_v(0,1,1) = \frac{2\mu q\overline{z}(\theta)}{1-\alpha}, \\
&\Lambda_{m^*}(0,1,1) = -\frac{\mu q\theta}{1-\alpha}\int_{z\geq\overline{z}(\theta)} zdF(z), \\
&\Gamma_v(0,1,1) = 0, \\
&\Gamma_{P^*}(0,1,1) = q'(1)\overline{z}(\theta)\left(1+\theta\Psi(\overline{z}(\theta))\right) > 0, \\
&\Gamma_v^*(0,1,1) = 0, \\
&\Gamma_{m^*}^*(0,1,1) = 0, \\
&\Gamma_{P^*}^*(0,1,1) = q^{*'}(1)\overline{z}(\theta)\left(1+\theta\Psi(\overline{z}(\theta))\right) < 0.
\end{aligned}$$

Hence, the Jacobian coefficient matrix J can be simplified as follows:

$$J = \begin{pmatrix} \rho - q\overline{z}\theta\Psi(\overline{z}) - \dfrac{2\mu q\overline{z}}{1-\alpha} & \dfrac{\mu q\theta}{1-\alpha}\int_{z\geq\overline{z}(\theta)} zdF(z) & -\Lambda_{P^*}(0,1,1) \\ 0 & 0 & (q^{*'}(1) - q'(1))\overline{z}\left(1+\theta\Psi(\overline{z})\right) \\ \dfrac{-2q}{\theta F'(\overline{z}(\theta))} & 0 & (q'(1) - q^{*'}(1))\overline{z} \end{pmatrix}.$$

The signs of the determinant and trace of J The determinant of J is calculated as

$$\det J = -\frac{\mu q\theta}{1-\alpha}\int_{z\geq\overline{z}(\theta)} zdF(z)(q^{*'}(1) - q'(1))\overline{z}\left(1+\theta\Psi(\overline{z})\right)\frac{2q}{\theta F'(\overline{z}(\theta))}.$$

Since $q^{*'}(P^*) < 0$ and $q'(P^*) > 0$, we can find $\det J > 0$. As for the trace of J,

$$\text{tr } J = \rho - q\overline{z}\left(\theta\Psi(\overline{z}) + \frac{2\mu}{1-\alpha} - \frac{q'(1) - q^{*'}(1)}{q}\right).$$

From (12) and (13), we obtain

$$\frac{P^*q'(P^*)}{q(P^*)} = \frac{\alpha}{1-\alpha}\frac{P^*\varphi_D'(P^*)}{\varphi_D(P^*)}, \quad \frac{P^*q^{*'}(P^*)}{q^*(P^*)} = \frac{\alpha}{1-\alpha}\frac{P^*\varphi_D^{*'}(P^*)}{\varphi_D^*(P^*)}.$$

Using these results together with (A.8) and (A.9), we obtain

$$\frac{P^*q'(P^*)}{q(P^*)} - \frac{P^*q^{*'}(P^*)}{q^*(P^*)} = \frac{\mu+\mu^*}{1-\alpha}.$$

On the symmetric BGP, $P^* = 1$ and $\mu = \mu^*$ are satisfied, which results in $(q'(1) - q^{*'}(1))/q = 2\mu/(1-\alpha)$. Therefore, the trace is given by

$$\text{tr } J = \rho - q(1)\overline{z}(\theta)\theta\Psi(\overline{z}(\theta)).$$

Proof As the autonomous dynamical system has one forward-looking and two predetermined variables, the uniqueness of the equilibrium path converging to the BGP requires J to have one positive and two negative eigenvalues. Since $\det J > 0$ implies that J has one or three positive eigenvalues, the necessary and sufficient condition for uniqueness is tr $J < 0$, namely, $\rho < q(1)\overline{z}(\theta)\theta\Psi(\overline{z}(\theta))$.

A.9 Determination of β, τ, f_E, and f_X

When we specify the distribution of z as $F(z) = (1 - z^{-\gamma})/(1 - \beta^{-\gamma})$, we can obtain $\Psi(\widetilde{z})$ as

$$\Psi(\widetilde{z}) \equiv \int_{z \geq \widetilde{z}} (z/\widetilde{z} - 1)dF(z) = \frac{1}{1 - \beta^{-\gamma}} \left(\frac{\widetilde{z}^{-\gamma}}{\gamma - 1} + \beta^{-\gamma} \right).$$

From this result and $q\widetilde{z} = r^b + \delta$, we can express the BGP growth rate as

$$g^{fi} = (r^b + \delta) \left[1 + \frac{\theta}{1 - \beta^{-\gamma}} \left(\frac{\widetilde{z}^{-\gamma}}{\gamma - 1} + \beta^{-\gamma} \right) \right] - \delta - \rho. \tag{A.17}$$

Since we assume $\theta^* = \theta$ when we determine the parameters, $m^* = 1$ and $P^* = 1$ hold. Therefore, the asset market equilibrium is rewritten as

$$\theta \frac{\widetilde{z}^{-\gamma} - \beta^{-\gamma}}{1 - \beta^{-\gamma}} = 1. \tag{A.18}$$

We have already determined the values of parameters ρ, δ, θ, and γ. Additionally, we have already determined the target values of g^{fi} and r^b. Then, from (A.17) and (A.18), we obtain the value of β together with the value of $\widetilde{z}$.

In turn, we can obtain the value of q since $q = (r^b + \delta)/\widetilde{z}$. Using this result and $q = \chi L \varphi_D^{\alpha/(1-\alpha)}$, we can determine φ_D.[25] When we specify the distribution of φ as $G(\varphi) = 1 - \varphi^{-\kappa}$, it follows that

$$1 - \mu = \frac{f_D H(\varphi_D)}{\sum_{j=D,X} f_j H(\varphi_j)} = \frac{h f_D}{f_E} \varphi_D^{-\kappa}. \tag{A.19}$$

Under balanced trade, μ is equal to the import penetration ratio.[26] Thus, $\mu = 0.081$. Since we have already set f_D, α, and κ, f_E is determined from (A.19). Since $P^* = 1$, φ_X is given by

$$\varphi_X = T\varphi_D. \tag{A.20}$$

In our model, the fraction of exporting intermediate goods firms is given by

$$\frac{1 - G(\varphi_X)}{1 - G(\varphi_D)} = \left(\frac{\varphi_X}{\varphi_D} \right)^{-\kappa} = T^{-\kappa},$$

[25] Recall that $L = 1$ and $\chi \equiv \alpha^{\alpha/(1-\alpha)}(1-\alpha)/f_D$.

[26] Since we assume a perfect symmetry between the two countries when determining the parameters, the trade balance indeed holds.

which is set at 0.21. Then T is determined as $0.21^{-1/\kappa} \simeq 1.366$. From (A.20), we can determine φ_X. When $G(\varphi) = 1 - \varphi^{-\kappa}$,

$$\mu = \frac{hf_X}{f_E}\varphi_X^{-\kappa},$$

from which f_X is determined. Finally, from the definition of T,

$$T^{-\kappa} = 0.021 = \left[\tau\left(\frac{f_X}{f_D}\right)^{(1-\alpha)/\alpha}\right]^{-\kappa}.$$

From this equation, τ is determined.

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