DISCUSSION PAPER SERIES

Discussion paper No. 253

The Impact of Compatibility on Incentives to Innovate in a Network Goods Market: A Duopoly Case

Tsuyoshi Toshimitsu

(School of Economics, Kwansei Gakuin University)

July 2023

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho

Nishinomiya 662-8501, Japan

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Tsuyoshi TOSHIMITSU*

School of Economics, Kwansei Gakuin University

Abstract

Based on a horizontal product differentiation model associated with network externalities, we consider the impact of compatibility (interconnectivity) on incentives to innovate in a network goods industry in the cases of Cournot quantity and Bertrand price duopoly. We demonstrate that the effect of compatibility on incentives to innovate depends on network externalities and product substitutability. In particular, an increase in the degree of compatibility increases the incentives to innovate if the degree of network externalities is relatively large and if the degree of product differentiation is sufficiently large, irrespective of the mode of competition. Then, we then examine the same problem in a Hotelling-type unit-linear market and show that an increase in the degree of compatibility reduces the incentives to innovate.

Keywords: innovation; network externality; compatibility; a fulfilled expectation; cost-reducing R&D; Cournot duopoly; Bertrand duopoly

JEL Classification: D43, L13, L15, O31

* Corresponding author: School of Economics, Kwansei Gakuin University, 1-155, Nishinomiya, Japan, 662-8501, Tel: +81 798 54 6440, Email: ttsutomu@kwansei.ac.jp

1. Introduction

The interconnectivity (and compatibility) of goods and services and network structures, such as the magnitude and scope of network systems in network industries, is very important in the current digital economy and network society where people spend increasing amounts of time and money on Internet services (e.g., e-commerce, mobile games, and search engine sites). Furthermore, we can observe that there are various degrees of compatibility (ranging from incompatible to perfectly compatible) and of network effects among the network industries. In such an environment, competition in the network industries is undertaken at various levels that involve a mix of strategic investments, such as process and product R&D and price and quantity competition (e.g., telecommunications and Internet services). For example, as Heywood et al. (2022, p. 356) discuss, "the extent to which one firm's R&D may allow it to lower costs and capture customers can be limited by the lack of compatibility. In addition, it is recognized that the extent of compatibility can influence the introduction of new technology."[1] Furthermore, regarding compatibility, they comment that, "reflecting this interconnection, firm compatibility decisions by network firms raise public policy issues regarding both anti-competitive behavior and reduced technological progress."[2]

The main research question of this paper is how such compatibility affects the incentives to innovate and undertake R&D activities designed to reduce costs. That is, does an increase in the degree of compatibility improve or reduce the incentives to innovate? If an increasing degree of compatibility increase such incentives, compatibility standardization may be not necessarily anticompetitive. Conversely, we are interested in the question of under what conditions does an increase in the degree of compatibility reduce the incentives to innovate?

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[1] Regarding this point, Heywood et al. (2022, p. 356) cited Farrell & Saloner (1986) and Kristiansen (1996)

[2] Regarding this point, Heywood et al. (2022, p. 356) cited Gandal (2002).

Since the seminal research by Katz and Shapiro (1985), there has been much research analyzing process (that is, cost-reducing) and product (that is, quality-improving) R&D competition in the presence of network externalities. In particular, many studies have investigated how network externalities and compatibility affect R&D activities and outcomes. In this paper, focusing on the impact of compatibility on innovation, we review the recent papers by Bond-Smith (2019), Buccella et al. (2022), Heywood et al. (2022), Kim (2000), Knauff and Karbowski (2021), and Sääskilahti (2006).[3]

Bond-Smith (2019) develops a model of the relationship between innovative entry and compatibility decisions by incumbents. In particular, he examines two regimes: (1) a regime involving compatibility, where all firms are compatible with all rivals; and (2) a regime involving autarky, where one firm (incumbent monopoly) is an autarky, but all rivals are compatible. As we examine duopolistic competition in this paper, the case of full compatibility (incompatibility) corresponds to regime (1) and (2), respectively, in Bond-Smith (2019).

Heywood et al. (2022) consider noncooperative R&D competition and cooperative R&D, given an endogenous choice of compatibility. In particular, they assume a three-stage game where firms simultaneously choose the degree of compatibility in the first stage, choose R&D investments in the second stage, and then engage in Cournot competition in the third stage. Furthermore, they consider an "incumbent–entrant" asymmetric competition model, where an incumbent firm has an installed base consumers, whereas the entrant does not. In this case, they show that under incompatibility, the incumbent invests more in R&D than does the entrant, and that incompatibility is more likely to occur with cooperative R&D.

Knauff and Karbowski (2021) consider process R&D competition in markets with network effects and imperfect compatibility. They show that network effects increase the noncooperative

[3] With the exceptions of Bond-Smith (2019) and Kim (2000), the studies that we review introduce technological spillover effects into the cost function of their models. In this paper, however, we review their models by omitting these effects.

R&D investments, whereas they reduce the cooperative R&D investments. They assume technological spillover effects, installed base consumers, and consumers' responsive (active) expectations. If the assumptions were removed, the model would become basically the same as Remark 1 of this paper, where we show that an increase in the degree of compatibility reduces incentives to innovate.

The studies reviewed above use the model of a backbone market following Crémer et al. (2000), which is an extension of Katz and Shapiro (1986). Regarding the model, Roson (2002) reviews the two papers, Crémer et al. (2000) and Foros and Hansen (2001), which consider the issues of competition and quality determination in the market for Internet access services. As Robson (2002) summarizes, the two papers reach opposite conclusions, with the differences between them depending on their alternative hypotheses on the overall market sizes. In particular, Crémer et al. (2000) adopt the well-known model by Katz and Shapiro (1985), whereas Foros and Hansen (2001) adopt a unit-linear market following a conventional Hotelling framework. As Robson (2002) points out, the market expansion effect is present in the former, whereas it is absent in the latter. In other words, the total number of consumers is given as constant in the latter framework, although the total market sizes, that is, the aggregate demand, change as a result of a quality enhancement.

For example, using a unit-linear market in a Hotelling framework, Kim (2000) considers quality-improving technological innovation and Sääskilahti (2006) considers cost-reducing innovation. Kim (2000, Theorem 5) shows that the effect of an increase in the degree of compatibility on the profit of the innovative firm is ambiguous, whereas the profit of the noninnovative rival firm is increased. This is because an increase in the degree of compatibility raises the price of the innovative firm, leading to it losing the market share, which implies that the effect of compatibility on innovation can be negative.[4] Sääskilahti (2006) first shows that

[4] In this case, the innovative firm corresponds to a high-quality firm, whereas the noninnovative

network compatibility is neutralized in the decision regarding cost-reducing investment in the case of symmetric qualities between firms. However, Sääskilahti (2006, Proposition 3) demonstrates that in the case of asymmetric qualities, with sufficiently high price sensitivity, the effect of an increase in the degree of compatibility on the investment of a high (low) quality firm is negative (positive). When price sensitivity is sufficiently low, the opposite signs hold.[5]

Finally, in a study that is closely related to ours, Buccella et al. (2022), assume a homogeneous product with network externalities based on the utility function presented by Shrivastav (2021).[6] They compare the investments, quantities, and profits in the equilibrium in the case of full compatibility with those in the case of incompatibility. Furthermore, the relationships between the outcomes in the each case depend on the degree of network externalities and technological spillover effects. In particular, based on their model, we can show that the investment level of the innovating firm in the case of incompatibility is larger than that in the case of full compatibility. This implies that an increase in the degree of compatibility reduces the incentives of the innovating firm to invest.[7]

In this paper, we consider the impact of compatibility on the incentives to innovate and undertake R&D activities designed to reduce costs in a network goods market. That is, we demonstrate the effects of an increase in the degree of compatibility on the marginal profit resulting from the cost-reducing activity of an innovating firm. In particular, we focus on the cases of perfect compatibility and incompatibility. As discussed in more detail below, the former

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firm a low-quality firm. In Remark 3 of this paper, we address the effect of compatibility on innovation in the case of asymmetric firms.

[5] In Section 3, we examine the symmetric quality case and show that the effect of an increase in the degree of compatibility reduces (increases) the incentives to innovate if the innovating firm is initially efficient (inefficient) compared with the rival firm. Even without the initial cost difference, the effect is negative. This result differs from Sääskilahti (2006), who assume responsive (active) expectations of consumers (see Appendix 4).

[6] Following Sääskilahti (2006), Buccella et al. (2022) introduce technological spillover effects into the cost function. We omit these effects in discussing their model.

[7] See Remark 1 of this paper.

(latter) corresponds to a single-industry-wide (firm-specific) network system. In other words, the former implies a network industry established with compatibility standards. In this paper, we explore the incentives to innovate under perfect compatibility (that is, compatibility standard) compared with those under imperfect compatibility. We demonstrate the conditions under which the incentives to innovate are larger under perfect compatibility are larger than under incompatibility. We explore the problem based on a horizontally differentiated product models of Cournot and Bertrand duopoly and then on the model in a unit-linear market à la Hotelling.

2. The Model

2.1 Set up

Following Shrivastav (2021), we use the following linear inverse demand function.[8]

$$p_i = a - q_i - \gamma q_j + n\left(q_i^e + \phi q_j^e\right), \quad i, j = 1, 2, \quad i \neq j, \tag{1}$$

where $a\left(> c_i\right)$ implies the intrinsic size of a network product market and $\gamma \in \left[0,1\right]$ denotes product substitutability. When $\gamma \to 1\,(0)$, *product i* becomes perfectly substitutable (independent). If $\gamma = 1$, the products are homogeneous. Furthermore, $n \in \left[0,1\right)$ denotes a network effect and $\phi \in \left[0,1\right]$ denotes the degree of compatibility (interconnectivity). If $\phi = 1\left(0\right)$, products (and services) among firms are perfectly compatible (incompatible). As discussed below, we refer to the case of a perfectly compatible (an incompatible) product as a

[8] Regarding the inverse and direct demand functions, that is, Equations (1) and (2), see Hoernig (2012) and Naskar and Pal (2020), in which product substitutability is assumed to be equal to compatibility. Relaxing the assumption, Shrivastav (2021) demonstrates the ranking of equilibrium R&D investments under Bertrand and Cournot competition.

single industry-wide (a firm-specific) network system. $q_i^e\left(q_j^e\right)$ is the expected output of *firm i* (*j*). Thus, $n\left(q_i^e+\phi q_j^e\right)$ expresses the expected network sizes for the product of *firm i*.

Using Equation (1), the corresponding direct demand function of firm *i* is given by:

$$q_i=\frac{(1-\gamma)a-p_i+\gamma p_j+N_i-\gamma N_j}{1-\gamma^2},\quad i,j=0,1,\quad i\neq j, \tag{2}$$

where $N_i\equiv n\left(q_i^e+\phi q_j^e\right)$ and $N_j\equiv n\left(q_j^e+\phi q_i^e\right)$.

Assuming that the marginal cost of production (operation) is constant, the profit function of *firm i* is expressed as $\pi_i=\left(p_i-c_i\right)q_i,\quad i=0,1.$ In this paper, following Yi (1999), we assume that *firm 0* is the only firm with the capability to invest in R&D to reduce the marginal cost.[9] Furthermore, consumers' expectations of the network sizes, we assume passive (rational) expectations and adopt the concept of a fulfilled expectation equilibrium (Katz and Shapiro, 1985).[10]

2.2 Compatibility and incentives to innovate in the case of Cournot duopoly

The first-order condition (FOC) for the profit maximization by *firm i* is given by: $\frac{\partial\pi_i}{\partial q_i}=p_i-c_i-q_i=0,\quad i=0,1.$ Taking Equation (1), at the fulfilled expectation Cournot equilibrium, that is, $q_i^e=q_i$, we derive the following output of *firm i*:

$$q_i\left[c_i,c_j;\phi\right]=\frac{(2-n-\Gamma)a-(2-n)c_i+\Gamma c_j}{D},\quad i,j=0,1,\quad i\neq j, \tag{3}$$

[9] Yi (1999) examines how the strength of competition (an increase in the number of firms) affects incentives to innovate in a Cournot oligopoly in a homogeneous product market.

[10] In the case of responsive (active) expectation, our main results do not change. However, see Remark 3, where we explore the effects of compatibility in a model with a Hotelling structure. Furthermore, in Appendix 4, we show that the effect vanishes if there are symmetric marginal costs during the initial situation under responsive expectations.

where $\Gamma \equiv \gamma - n\phi$, $\Gamma > (<) 0 \Leftrightarrow \gamma > (<) n\phi$, and $D \equiv (2-n-\Gamma)(2-n+\Gamma) > 0.$

We refer to a network effect multiplied by compatibility, that is, $n\phi(<1)$, as network compatibility, which is an increasing function of the degree of compatibility.

In what follows, using Equation (3), we show the effects of an infinitesimal cost reduction by *firm 0* on the outputs of the firms, as follows:

$$-\frac{dq_0}{dc_0} = \frac{2-n}{D} > 0, \tag{4.1}$$

$$-\frac{dq_1}{dc_0} = -\frac{\Gamma}{D} > (<) 0 \Leftrightarrow \gamma < (>) n\phi. \tag{4.2}$$

It is clear from Equation (4.1) that the cost reduction achieved by innovation increases the output of *firm 0* (hereinafter, we refer to this as the cost-reduction effect). However, as in Equation (4.2), the effect on the output of rival *firm 1* depends on the degree of product substitutability and network compatibility. In particular, if $\gamma < n\phi$, the cost reduction of *firm 0* increases the output of *firm 1*. This is counterintuitive. However, because the degree of network compatibility is larger than that of product substitutability, the strategic relationship between the firms becomes complementary. That is, an increase in the output of *firm 0* as a result of the cost reduction increases the output of *firm 1*. This implies spillovers on the demand side as a result of a network effect. Conversely, if $\gamma > n\phi$, to be intuitively plausible, a relationship of strategic substitution holds between the firms, so that an increase in the output of *firm 0* resulting from the cost reduction decreases the output of *firm 1*.

Using the FOC, the equilibrium profit is expressed as $\pi_i = (p_i - c_i)q_i = \left(q_i\left[c_i, c_j; \phi\right]\right)^2$, $i, j = 0,1,\ i \neq j.$ Following the method by Yi (1999), we explore the impact of compatibility on incentives to innovate. In particular, the marginal profit of an infinitesimal cost reduction to *firm 0* is given by:

$$
\begin{aligned}
-\frac{d\pi_0}{dc_0} &= -\frac{d(p_0-c_0)}{dc_0}q_0+(p_0-c_0)\left(-\frac{dq_0}{dc_0}\right) \\
&= 2(p_0-c_0)\left(-\frac{dq_0}{dc_0}\right)=2q_0\left(-\frac{dq_0}{dc_0}\right).
\end{aligned} \tag{5}
$$

Taking Equations (3) and (4.1), Equation (5) is rewritten as:

$$
-\frac{d\pi_0}{dc_0}=\frac{2(2-n)}{D}q_0\left[c_0,c_1;\phi\right]=2(2-n)X_0\left[c_0,c_1;\phi\right], \tag{6}
$$

where $X_0\left[c_0,c_1;\phi\right]\equiv\dfrac{(2-n)(a-c_0)-\Gamma(a-c_1)}{D^2}$. Hereinafter, for the analysis, we define $X_0\left[c_0,c_1;\phi\right]$ as the benefit function of an infinitesimal cost reduction to *firm 0*, and explore the effects of an increase in the degree of compatibility on the benefit function.

Based on Equation (6), we obtain the following:

$$
\frac{dX_0}{d\phi}=\frac{n\left\langle\left\{(2-n)^2+3\Gamma^2\right\}(a-c_1)-4(2-n)\Gamma(a-c_0)\right\rangle}{D^3}. \tag{7}
$$

Assuming that $c_0=c_1=c(>0)$ initially holds, given Equation (7), we derive the following relationship:

$$
\left.\frac{dX_0}{d\phi}\right|_{c_0=c_1=c}=\frac{nA(2-n-\Gamma)(2-n-3\Gamma)}{D^3}>(<)0\Leftrightarrow 2-n-3\Gamma>(<)0, \tag{8}
$$

where $2-n-\Gamma>0$ and $A\equiv a-c>0$. The above Equation is rewritten as:

$$
\left.\frac{dX_0}{d\phi}\right|_{c_0=c_1=c}>(<)0\Leftrightarrow \phi>(<)\phi^C\left[n,\gamma\right]\equiv\frac{1}{3}-\frac{2-3\gamma}{3n}\left(<\frac{\gamma}{n}\right), \tag{9}
$$

where $\phi\in\left[0,1\right]$, $\gamma\in\left[0,1\right]$, and $n\in\left(0,1\right)$.[11] Superscript C denotes Cournot duopoly. The magnitude of $\phi^C\left[n,\gamma\right]$ depends on parameters n and γ. In particular, we derive the

[11] Given Equations (7) or (8), if $n=0$, the effect of an increase in the degree of compatibility on the benefit is zero. Thus, hereinafter, we assume that $n\neq 0$.

following relationships:

$$\phi^C[n,\gamma] > (<) 0 \Leftrightarrow \gamma > (<) \frac{2-n}{3}, \tag{10.1}$$

$$\phi^C[n,\gamma] > (<) 1 \Leftrightarrow \gamma > (<) \frac{2(1+n)}{3}. \tag{10.2}$$

Based on Equations (10.1) and (10.2), we can draw Figure 1, where the following properties of $\phi^C[n,\gamma]$ hold in each area.[12]

(1) $\phi^C[n,\gamma] < 0$ in area *I*, where $\gamma < \frac{2-n}{3}$ for $0 < n < 1$.

(2) $\phi^C[n,\gamma] > 1$ in area *II*, where $\gamma > \frac{2(1+n)}{3}$ for $0 < n < \frac{1}{2}$.

(3) $0 < \phi^C[n,\gamma] < 1$ in area *III*, where $\frac{2(1+n)}{3} > \gamma > \frac{2-n}{3}$ for $0 < n < \frac{1}{2}$ and $1 \geq \gamma > \frac{2-n}{3}$ for $\frac{1}{2} < n < 1$.

With respect to the effects of an increase in the degree of compatibility on incentives to innovate, we summarize the following results:

Proposition 1

(i) If $\gamma < \frac{2-n}{3}$ for $0 < n < 1$, an increase in the degree of compatibility promotes incentives to innovate.

[12] If $\gamma = \frac{2}{3}$, based on Equation (9), it holds that $\left.\frac{dX_0}{d\phi}\right|_{c_0=c_1=c} > (<) 0 \Leftrightarrow \phi > (<) \frac{1}{3}$. This relationship arises in area *III*.

(ii) If $\gamma > \dfrac{2(1+n)}{3}$ for $0 < n < \dfrac{1}{2}$, an increase in the degree of compatibility reduces incentives to innovate.

(iii) If $\dfrac{2(1+n)}{3} > \gamma > \dfrac{2-n}{3}$ for $0 < n < \dfrac{1}{2}$ and $1 \geq \gamma > \dfrac{2-n}{3}$ for $\dfrac{1}{2} < n < 1$, the effect of an increase in the degree of compatibility on incentives to innovate is not unidirectional (see Equation (9)).

Proof. See Appendix 1.

How an increase in the degree of compatibility affects the incentives to innovate depends on the properties of the network products, that is, the degree of product substitutability and the degree of the network effect.[13] In the case where product substitutability is high (low) and the network effect is small (large), the increase in the degree of compatibility affects incentives to innovate negatively (positively). This is because the direct output–expansion effect caused by an increase in the degree of compatibility through the network effect is smaller (larger) than the indirect cost-reduction effect, which may possibly be negative, resulting from an increase in the degree of compatibility through strategic relationships between the firms.[14] Therefore, as will be addressed below, when the products are sufficiently close to being homogenous, the incentives to innovate in the incompatible case (that is, a firm-specific network system) are larger than that in the perfectly compatible case (that is, a single industry-wide network system).

[13] The higher the degree of product substitutability, the more intense the competitiveness. The stronger the network effect, the weaker the competitiveness.
[14] If the degree of network compatibility is larger than that of product substitutability, the effect of an increase in compatibility on the cost-reduction effect is positive. In this case, it holds that $\left.\dfrac{dX_0}{d\phi}\right|_{c_0=c_1=c} > 0.$

Conversely, if the products are differentiated sufficiently, the opposite result arises because of a strategic complementary relationship, which is caused by a high degree of network compatibility.

Next, we examine the relationship between the benefit (marginal profit) level in the case of incompatibility and that in the case of perfect compatibility. In particular, the benefit function is rewritten as:

$$X_0\left[c_0 = c_1 = c;\phi\right] = \frac{A}{\left\{2-\gamma-n(1-\phi)\right\}\left\{2+\gamma-n(1+\phi)\right\}^2}. \tag{11}$$

In the cases of perfect compatibility and incompatibility, Equation (11) is given, respectively, by:

$$X_0\left[\phi = 1\right] = \frac{A}{(2-\gamma)(2+\gamma-2n)^2}, \tag{12.1}$$

$$X_0\left[\phi = 0\right] = \frac{A}{(2-\gamma-n)(2+\gamma-n)^2}. \tag{12.2}$$

Based on Equation (9), we directly derive the following results. In area *I*, because $\phi \geq 0 > \phi^C\left[n,\gamma\right]$, we have $\left.\frac{dX_0}{d\phi}\right|_{c_0=c_1=c} > 0$; thus, it holds that $X_0\left[\phi=1\right] > X_0\left[\phi=0\right]$. Similarly, in area *II*, because $\phi \leq 1 < \phi^C\left[n,\gamma\right]$, we have $\left.\frac{dX_0}{d\phi}\right|_{c_0=c_1=c} < 0$; thus, it holds that $X_0\left[\phi=0\right] > X_0\left[\phi=1\right]$.

However, the relationships of the benefit levels in the cases of incompatibility and perfect compatibility in area *III* are ambiguous. In particular, in view of Equation (9), the benefit function of compatibility, that is, $X_0\left[\phi;n,\gamma\right]$, is *U*-shaped and reaches a minimum at $\phi^C\left[n,\gamma\right]$. Furthermore, the relationship between $X_0\left[\phi=0\right]$ and $X_0\left[\phi=1\right]$ is not unidirectional, as it depends on parameters n and γ.

With respect to the benefit levels in the cases of incompatibility and perfect compatibility, we derive the following results (see also Figure 2).

Corollary 1

(i) If $\frac{2}{3} \geq \gamma \geq 0$ and $1 > n > 0$, it holds that $X_0[\phi = 1] > X_0[\phi = 0]$.

(ii) If $1 \geq \gamma > \frac{10}{13}$ and $1 > n > 0$, or if $\gamma > \frac{2(1+n)}{3}$ and $0 < n < \frac{2}{13}$, it holds that $X_0[\phi = 0] > X_0[\phi = 1]$.

(iii) If $\frac{10}{13} > \gamma > \frac{2}{3}$ and $1 > n > \frac{2}{13}$, or if $\gamma < \frac{2(1+n)}{3}$ and $\frac{2}{13} > n > 0$, the relationships between the benefits of incompatibility and those of perfect compatibility are ambiguous.

Proof. See Appendix 2.

Corollary 1 (i) (Corollary 1 (ii)) implies that if the products are differentiated (substitutionary) sufficiently, irrespective of the degree of the network effect, the effect of perfect compatibility on the benefits is larger (smaller) than the effect of incompatibility.

To confirm Proposition 1 and Corollary 1, we examine the cases of a homogeneous product market and a significant innovation.

Remark 1. Cournot duopoly in a homogeneous product market (*CH*)

Substituting $\gamma = 1$ into Equation (9), we have the following relationship.

$$\left.\frac{dX_0}{d\phi}\right|_{c_0=c_1=c} > (<)0 \Leftrightarrow \phi > (<)\phi^{CH}[n] = \frac{1}{3} + \frac{1}{3n}(>0). \tag{13}$$

Given that homogeneous products are perfect substitutes, the degree of product substitutability is necessarily larger than the degree of network compatibility, that is, $\gamma = 1 > n\phi$. In this case, although the *firm 0*'s cost reduction increases its output, this reduces the output of *firm 1* because of the strategic substitutionary relationship. Thus, an increase in the degree of compatibility has a negative influence on the cost-reduction effect.

Given Equation (13), if $\frac{1}{2} > n,$ it holds that $\phi^{CH}[n] > 1.$ Because the following relationship holds $\phi < 1 < \phi^{CH}[n] \Leftrightarrow \left.\frac{dX_0}{d\phi}\right|_{c_0=c_1=c} < 0,$ we have $X_0[\phi = 0] > X_0[\phi = 1].$ That is, if $\frac{1}{2} > n,$ an increase in the degree of compatibility reduces the benefits and decreases incentives to innovate, which implies that the negative cost-reduction effect outweighs the positive output-expansion effect. Conversely, if $\frac{1}{2} < n,$ the benefit function of compatibility is *U*-shaped and the value of $X_0[\phi]$ reaches its minimum at $\phi^{CH} = \frac{1}{3} + \frac{1}{3n}(<1).$ In this case, using Equations (12.1) and (12.2), we derive the following relationship: $X_0[\phi = 0] > X_0[\phi = 1].$

Therefore, in the case of a homogeneous product market, an increase in the degree of compatibility reduces incentives to innovate and, despite the degree of the network effect being sufficiently large, the benefit level under perfect compatibility is lower than that under incompatibility. In other words, incentives to innovate are stronger under a firm-specific network system than under a single industry-wide network system when the network market involves products and services that are almost identical. This yields the policy implication that compatibility standardization may reduce incentives to innovate in an identical product and service network industry.

Remark 2. A significant innovation

Similar to Yi (1999), we consider the effects of an increase in the degree of compatibility on benefits in the case of a significant but not drastic innovation and reconfirm Proposition 1.

We assume that $c_1 = c_{0(before)} = \overline{c}$ before the innovation and that $c_{0(after)} = \underline{c}\left(< \overline{c}\right)$ after the innovation by *firm 0*. Using the definition of the profit in the equilibrium before and after the innovation, we derive the following increase in *firm 0*'s profit.

$$\Delta\pi_0 \equiv \pi_{0(after)} - \pi_{0(before)} = \left(q_0\left[c_{0(after)} = \underline{c}, c_1 = \overline{c};\phi\right]\right)^2 - \left(q_0\left[c_{0(before)} = c_1 = \overline{c};\phi\right]\right)^2$$

$$= \left\langle q_0\left[\underline{c},\overline{c};\phi\right] - q_0\left[\overline{c};\phi\right]\right\rangle \left\langle q_0\left[\underline{c},\overline{c};\phi\right] + q_0\left[\overline{c};\phi\right]\right\rangle.$$

Based on Equation (3), the above Equation is revised as:

$$\Delta\pi_0 = \left\langle \frac{(2-n)(\overline{c}-\underline{c})}{D} \right\rangle \left\langle \frac{2(2-n-\Gamma)(a-\overline{c}) + (2-n)(\overline{c}-\underline{c})}{D} \right\rangle. \tag{14}$$

Thus, the effect of an increase in the degree of compatibility on the benefit is given by

$$\frac{d\Delta\pi_0}{d\phi} = H\left\langle (a-\overline{c})D - 2\Gamma\left\{2(2-n-\Gamma)(a-\overline{c}) + (2-n)(\overline{c}-\underline{c})\right\}\right\rangle, \tag{15}$$

where $H \equiv \dfrac{2n(2-n)(\overline{c}-\underline{c})}{D^3} > 0.$ Based on Equation (15), we obtain the following relationship:

$$\frac{d\Delta\pi_0}{d\phi} > (<) 0 \Leftrightarrow \overline{A}(2-n-\Gamma)(2-n-3\Gamma) - 2\Gamma(2-n)(\overline{c}-\underline{c}) > (<) 0, \tag{16}$$

where $2-n-\Gamma > 0$ and $\overline{A} \equiv a - \overline{c} > 0.$[15] In view of Equation (16), and using Equation

[15] Regarding Equation (16), if $\overline{c} - \underline{c} \to 0,$ we have Equation (8).

(8), we directly obtain the following results.

Result 1: $\Gamma \leq 0,$ that is, $n\phi \geq \gamma,$ it holds that $\dfrac{d\Delta\pi_0}{d\phi} > 0.$

Result 2: If $\Gamma > 0,$ that is, $n\phi < \gamma,$ and $2 - n - 3\Gamma > 0,$ that is, $\dfrac{\gamma}{n} > \phi > \phi^C[n,\gamma],$ the following relationship holds

$$\frac{d\Delta\pi_0}{d\phi} > (<) 0 \Leftrightarrow \frac{\bar{A}(2-n-\Gamma)(2-n-3\Gamma)}{2\Gamma(2-n)} > (<) \bar{c} - \underline{c}.$$

Result 3: If $\Gamma > 0,$ that is, $n\phi < \gamma,$ and $2 - n - 3\Gamma \leq 0,$ that is, $\phi \leq \phi^C[n,\gamma] < \dfrac{\gamma}{n},$ it holds that $\dfrac{d\Delta\pi_0}{d\phi} < 0.$

Result 1 corresponds to Proposition 1 (i), where the degree of network compatibility is larger than the degree of product substitutability and there is a relationship of strategic complements between the firms in turn. In addition, an increase in the degree of compatibility improves the incentives to innovate. Conversely, in Results 2 and 3, an increase in the degree of compatibility affects the cost-reduction effect negatively. Result 3 corresponds to Proposition 1 (ii), where the negative cost-reduction effect outweighs the positive output-expansion effect. Furthermore, although Result 2 appears to correspond to Proposition 1 (iii), the outcomes depend on the magnitude of cost reduction. For example, when the magnitude of the cost reduction is sufficiently large, an increase in the degree of compatibility reduces the incentives to innovate. This case corresponds to Result 3. Therefore, we have confirmed that Proposition 1 holds in the case of a significant innovation.

2.3 The case of Bertrand duopoly competition

In this section, we consider the case of Bertrand duopoly competition and confirm whether the mode of competition affects the role of compatibility on incentives to innovate. Following a similar method to that in the previous section, and using Equation (2), that is, $q_i = \dfrac{(1-\gamma)a - p_i + \gamma p_j + N_i - \gamma N_j}{1-\gamma^2}$, $i, j = 0,1,\ \ i \neq j$, we obtain the following FOC of profit maximization with respect to *firm i*:

$$(1-\gamma)a - 2p_i + \gamma p_j + N_i - \gamma N_j + c_i = 0, \quad i, j = 0,1, \quad i \neq j.$$

Similarly for *firm j*, we have the FOC. Thus, at the fulfilled expectation Bertrand equilibrium, we derive the following equilibrium price for firm i:

$$p_i\left[c_i, c_j; \phi\right] = \frac{(1-\gamma^2)\left(2-\gamma^2-n-\Gamma\right)a + \left\langle (1-n)(2-\gamma^2-n) - \Gamma^2 \right\rangle c_i + (1-\gamma^2)\Gamma c_j}{D^B}, \quad (17)$$

where $D^B \equiv \left(2-\gamma^2-n-\Gamma\right)\left(2-\gamma^2-n+\Gamma\right) > 0$ and superscript B denotes Bertrand competition. The profit per unit of output is given by:

$$p_i\left[c_i, c_j; \phi\right] - c_i = (1-\gamma^2)\frac{\left(2-\gamma^2-n-\Gamma\right)a - (2-\gamma^2-n)c_i + \Gamma c_j}{D^B}, \quad (18)$$

where it holds that $q_i\left[c_i, c_j; \phi\right] = \dfrac{p_i\left[c_i, c_j; \phi\right] - c_i}{1-\gamma^2}$, based on the FOC. Furthermore, Equation (18) can be rewritten as:

$$p_i\left[c_i, c_j; \phi\right] - c_i = (1-\gamma^2)\frac{\left(2-\gamma^2-n\right)(a-c_i) - \Gamma(a-c_j)}{D^B}, \quad i, j = 0,1, \quad i \neq j. \quad (19)$$

Given Equation (19), we derive the following results for the effects of a cost reduction by *firm 0* on the profit per unit of output (or on the output) of the firms:

$$-\frac{d(p_0 - c_0)}{(1-\gamma^2)dc_0} = -\frac{dq_0}{dc_0} = \frac{2-\gamma^2-n}{D^B} > 0, \quad (20.1)$$

$$-\frac{d(p_1-c_1)}{(1-\gamma^2)dc_0}=-\frac{dq_1}{dc_0}=-\frac{\Gamma}{D^B}>(<)0 \Leftrightarrow \gamma<(>)n\phi. \tag{20.2}$$

Equations (20.1) and (20.2) remind us of Equations (4.1) and (4.2), respectively. In particular, the cost reduction by *firm 0* increases the output of rival *firm 1* when the degree of network compatibility is larger than the degree of product substitutability. This implies that the cost-reduction effect is not affected by the mode of competition.

The profit at the equilibrium is expressed as: $\pi_i=\left(p_i-c_i\right)q_i=\dfrac{\left(p_i\left[c_i,c_j;\phi\right]-c_i\right)^2}{1-\gamma^2}$, $i,j=0,1,\quad i\neq j.$ Thus, the benefit of an infinitesimal cost reduction to *firm 0* is given by:

$$-\frac{d\pi_0^B}{dc_0}=2(1-\gamma^2)(2-\gamma^2-n)X_0^B\left[c_0,c_1;\phi\right], \tag{21}$$

where $X_0^B\left[c_0,c_1;\phi\right]\equiv\dfrac{(2-\gamma^2-n)(a-c_0)-\Gamma(a-c_1)}{(D^B)^2}$ is the benefit of an infinitesimal cost reduction to *firm 0* in the case of Bertrand duopoly. Based on Equation (21), we obtain the following equation:

$$\frac{dX_0^B}{d\phi}=\frac{nD^B(a-c_1)-4n\left\langle(2-\gamma^2-n)(a-c_0)-\Gamma(a-c_1)\right\rangle\Gamma}{(D^B)^3}. \tag{22}$$

Assuming initially that $c_0=c_1=c(>0)$ holds, we derive the following relationship:

$$\left.\frac{dX_0^B}{d\phi}\right|_{c_0=c_1=c}=\frac{nA\left(2-\gamma^2-n-\Gamma\right)\left(2-\gamma^2-n-3\Gamma\right)}{(D^B)^3}>(<)0$$

$$\Leftrightarrow 2-\gamma^2-n-3\Gamma>(<)0, \tag{23}$$

where $2-\gamma^2-n-\Gamma>0.$ Equation (23) can be rewritten as:

$$\left.\frac{dX_0^B}{d\phi}\right|_{c_0=c_1=c}>(<)0 \Leftrightarrow \phi>(<)\phi^B\left[n,\gamma\right]\equiv\frac{1}{3}-\frac{2-3\gamma-\gamma^2}{3n}\left(<\frac{\gamma}{n}\right), \tag{24}$$

where $\phi \in [0,1]$, $\gamma \in [0,1)$, and $n \in (0,1)$. Equation (24) is qualitatively similar to Equation (9) in the case of Cournot duopoly.

With respect to $\phi^B[n,\gamma]$, we derive the following relationships:

$$\phi^B[n,\gamma] > (<) 0 \Leftrightarrow n > (<) 2 - 3\gamma - \gamma^2. \tag{25.1}$$

$$\phi^B[n,\gamma] > (<) 1 \Leftrightarrow n < (>) \frac{\gamma^2 + 3\gamma - 2}{2}, \tag{25.2}$$

Using Equations (25.1) and (25.2), we draw Figure 3, where the following properties of $\phi^B[n,\gamma]$ hold in each area.

(1) $\phi^B[n,\gamma] < 0$ in area *I*, where $n < 2 - 3\gamma - \gamma^2$ for $0 < n < 1$.[16]

(2) $\phi^B[n,\gamma] > 1$ in area *II*, where $n < \dfrac{\gamma^2 + 3\gamma - 2}{2}$, for $0 < n < 1$.[17]

(3) $0 < \phi^B[n,\gamma] < 1$ in area *III*, where $1 > n > Max\left\{2 - 3\gamma - \gamma^2, \dfrac{\gamma^2 + 3\gamma - 2}{2}\right\}$.[18]

With respect to areas *I* and *II*, based on Equation (24), we derive the results the following results:

$$\text{If } n < 2 - 3\gamma - \gamma^2, \text{ then } \left.\frac{dX_0^B}{d\phi}\right|_{c_0 = c_1 = c} > 0 \text{ and } X_0^B[\phi = 1] > X_0^B[\phi = 0].$$

$$\text{If } n < \frac{\gamma^2 + 3\gamma - 2}{2}, \text{ then } \left.\frac{dX_0^B}{d\phi}\right|_{c_0 = c_1 = c} < 0 \text{ and } X_0^B[\phi = 0] > X_0^B[\phi = 1].$$

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16 $0 < \gamma < \dfrac{\sqrt{17} - 3}{2} \approx 0.562$ for $n = 0$, and $0 < \gamma < \dfrac{\sqrt{13} - 3}{2} \approx 0.303$ for $n = 1$.

17 $\dfrac{\sqrt{17} - 3}{2} < \gamma < 1$ for $n = 0$, and $\gamma = 1$ for $n = 1$.

18 $2 - 3\gamma - \gamma^2 > (<) \dfrac{\gamma^2 + 3\gamma - 2}{2} \Leftrightarrow \gamma < (>) \dfrac{\sqrt{17} - 3}{2}$.

However, because it holds that $0<\phi^B[n,\gamma]<1$ in area *III*, the benefit function of compatibility is *U*-shaped and it reaches its minimum at $\phi^B[n,\gamma]$. Furthermore, the relationship between the benefit of incompatibility and the benefit of perfect compatibility is not unidirectional. These results are qualitatively similar to those in the case of Cournot duopoly. This is because we can easily derive the same results for the effects of an increase in the degree of compatibility on the output and the cost-reduction effects as in the case of Cournot duopoly (see Equations (A.1) and (A.2) in Appendix 1).

At $c_0=c_1=c(>0)$, $X_0^B[c_0,c_1;\phi]$ is expressed as:

$$X_0^B[\phi;n,\gamma]=\frac{A}{\{(2+\gamma)(1-\gamma)-n(1-\phi)\}\{(2-\gamma)(1+\gamma)-n(1+\phi)\}^2}. \tag{26}$$

Based on Equation (26), we obtain the following benefits regarding the perfectly compatible and incompatible cases, respectively:

$$X_0^B[\phi=1]=\frac{A}{\{(2-\gamma)(1+\gamma)-2n\}^2(2+\gamma)(1-\gamma)}, \tag{27.1}$$

$$X_0^B[\phi=0]=\frac{A}{\{(2-\gamma)(1+\gamma)-n\}^2\{(2+\gamma)(1-\gamma)-n\}}. \tag{27.2}$$

Therefore, we summarize the results as Proposition 2 and Corollary 2, which are qualitatively similar to Proposition 1 and Corollary 1 in the case of Cournot duopoly.

Proposition 2

(i) If $n<2-3\gamma-\gamma^2$ for $0<n<1$, an increase in the degree of compatibility promotes the incentives to innovate.

(ii) If $n<\dfrac{\gamma^2+3\gamma-2}{2}$ for $0<n<1$, an increase in the degree of compatibility reduces the incentives to innovate.

(iii) If $1 > n > Max\left\{2-3\gamma-\gamma^2, \frac{\gamma^2+3\gamma-2}{2}\right\} > 0,$ the effect of an increase in the degree of compatibility on the incentives to innovate is not unidirectional (see Equation (24)).

Corollary 2

(i) If $0 < \gamma \le \frac{\sqrt{17}-3}{2}$ and $0 < n < 1,$ it holds that $X_0[\phi = 1] > X_0[\phi = 0]$.

(ii) If $n < \frac{\gamma^2+3\gamma-2}{2}$ and $0 < n < \frac{3\sqrt{41}-13}{50}$, or if $\gamma > \frac{3\sqrt{41}-13}{10}$ and $\frac{3\sqrt{41}-13}{50} < n < 1,$ it holds that $X_0[\phi = 0] > X_0[\phi = 1]$.

(iii) If $\frac{\sqrt{17}-3}{2} < \gamma < \frac{3\sqrt{41}-13}{10}$ and $\frac{3\sqrt{41}-13}{50} < n < 1,$ or if $n > \frac{\gamma^2+3\gamma-2}{2}$ and $0 < n < \frac{3\sqrt{41}-13}{50}$, the relationship between the benefit of incompatibility and that of perfect compatibility is ambiguous.

Proof. See Appendix 3 and Figure 4.

In view of Propositions 1 and 2 and Corollaries 1 and 2, the impact of compatibility on the marginal profit in relation to the cost reduction does not depend on the mode of competition. In particular, when the products are sufficiently differentiated and/or network effects are relatively strong, the incentives to innovate and the marginal gross profit under a single industry-wide network system, that is, the full compatibility (compatibility standardization) case, are larger than those under a firm-specific network system, that is, the incompatible case.

3. Discussion: The Case of a Hotelling Market

In this section, based on the unit-linear market of the Hotelling type and assuming a full coverage market, we consider the effect of an increase in the degree of compatibility on the incentives to innovate, and reconfirm the results of Foros and Hansen (2001). In particular, focusing on the marginal costs of the firms in the initial situation, we demonstrate that an increase in the degree of compatibility increases the incentives to innovate if *firm 0* is inefficient compared with *firm 1*.

We consider a unit-linear market where there is a continuum of consumers, indexed by $l \in [0,1]$. For simplicity, consumers are uniformly distributed with a density of one in the market where two firms exist at both ends of the market, that is, *firm 0* (*1*) locates at 0 (1). Given the prices, each consumer purchases at most one unit of either *product 0* or *product 1*.[19] The marginal consumer has the same surplus from purchasing one unit of either *product 0* or *product 1*, that is, l^*. In this case, the following relationship holds with respect to the surplus of the marginal consumer:

$$v - p_0 - tl^* + N_0 > (<) v - p_1 - t\left(1 - l^*\right) + N_1, \tag{28}$$

where $N_i = n\left(q_i^e + \phi q_j^e\right)$, $i, j = 0,1,\ i \neq j,$ t denotes a transportation cost, and v denotes an intrinsic quality value, which is assumed to be identical between the firms.[20] Thus, we obtain the following demand functions for *firms 0* and *1*, respectively.

[19] *Firm i* provides *product i*, $i = 0,1$. Because we assume a full coverage market, all consumers purchase either of the products.

[20] Even if the intrinsic values of the firms are not symmetric, our main results do not change. See also Remark 3.

$$l^* = \frac{t - (p_0 - N_0) + (p_1 - N_1)}{2t} = q_0, \quad (29.1)$$

$$1 - l^* = \frac{t - (p_1 - N_1) + (p_0 - N_0)}{2t} = q_1. \quad (29.2)$$

The FOC of *firm i* under price competition is given by $\frac{\partial \pi_i}{\partial p_i} = q_i - \frac{p_i - c_i}{2t} = 0,\ i = 0,1.$

Thus, using Equation (29.1), we have

$$t - 2p_i + p_j + N_i - N_j + c_i = 0,\ i, j = 0,1,\quad i \neq j. \quad (30)$$

Using the FOC, it holds that $q_i^e = q_i = \frac{p_i - c_i}{2t},\ i = 0,1,$ at the fulfilled expectation equilibrium. Thus, Equation (30) can be rewrtten as:

$$\begin{aligned} &2t^2 - \{4t - n(1-\phi)\} p_i + \{2t - n(1-\phi)\} p_j \\ &+ \{2t - n(1-\phi)\} c_i + n(1-\phi) c_j = 0, \end{aligned} \qquad i, j = 0,1,\quad i \neq j. \quad (31)$$

Based on Equation (31), we derive the following price at the equilibrium.

$$p_i^H = t + \frac{\{2t - n(1-\phi)\} c_i + t c_j}{3t - n(1-\phi)},\quad i, j = 0,1,\quad i \neq j, \quad (32)$$

where we assume that $3t - n(1-\phi) > 0$ and superscript H denotes a Hotelling model.[21] Furthermore, the profit per output and the output are given by:

$$p_i^H - c_i = t - \frac{t(c_i - c_j)}{3t - n(1-\phi)}, \quad (33)$$

[21] Given Equation (32), we have the following effect of compatibility on the price: $\frac{dp_i^H}{d\phi} = \frac{tn(c_i - c_j)}{\{3t - n(1-\phi)\}^2} > (<) 0 \Leftrightarrow c_i > (<) c_j$. That is, if *firm i* is less (more) efficient compared with *firm j* at the time of the initial situation, an increase in the degree of compatibility increases (decreases) the price. This implies that an increase in the degree of compatibility is anticompetitive (procompetitive).

$$q_i^H = \frac{1}{2} - \frac{c_i - c_j}{2\{3t - n(1-\phi)\}}, \quad i, j = 0,1, \quad i \neq j. \tag{34}$$

In a similar manner to the previous sections, we explore the effect of compatibility on the benefits by reducing the marginal cost. Given Equations (33) and (34), the profit of *firm 0* is expressed as: $\pi_0^H = \left(p_0^H - c_0\right)q_0^H = \frac{1}{2t}\left(p_0^H - c_0\right)^2 = \frac{t}{2}\left\{1 - \frac{c_0 - c_1}{3t - n(1-\phi)}\right\}^2$. Thus, the benefit (marginal profit) of an infinitesimal cost reduction to *firm 0* is given by:

$$-\frac{d\pi_0^H}{dc_0} = \frac{t}{3t - n(1-\phi)}\left\{1 - \frac{c_0 - c_1}{3t - n(1-\phi)}\right\} \equiv X_0^H\left[c_0, c_1; \phi\right], \tag{35}$$

where $X_0^H\left[c_0, c_1; \phi\right]$ is the benefit function of the degree of compatibility in the case of a unit-linear market. We obtain the following effect on the benefit:

$$\frac{dX_0^H}{d\phi} = -\frac{nt}{\{3t - n(1-\phi)\}^2}\left\{1 - \frac{2(c_0 - c_1)}{3t - n(1-\phi)}\right\}. \tag{36}$$

Assuming that initially $c_0 = c_1$ holds, given Equation (35), we derive the following equation:

$$\left.\frac{dX_0^H}{d\phi}\right|_{c_0 = c_1} = -\frac{nt}{\{3t - n(1-\phi)\}^2} < 0. \tag{37}$$

Equation (37) shows that the benefit is a decreasing function of compatibility. Therefore, in the case of price competition in a unit-linear market, the effect of incompatibility on the benefit is larger than that that of perfect compatibility, that is, $X_0^H\left[\phi = 0\right] > X_0^H\left[\phi = 1\right]$.

Considering Equation (34), we derive the following output–expansion effect: $\frac{dq_0^H}{d\phi} = \frac{(c_0 - c_1)n}{2\{3t - n(1-\phi)\}^2}$. Provided that initially symmetric marginal costs hold, that is., $c_0 = c_1$, the output expansion effect becomes 0 because of the full coverage market. Furthermore, regarding the effect of an increase in the degree of compatibility on the cost-reduction effect, we derive $\dfrac{d\left(-\dfrac{dq_0^H}{dc_0}\right)}{d\phi} = \dfrac{-n}{2\{3t - n(1-\phi)\}^2} < 0$. That is, the higher the degree of compatibility the smaller the cost reduction effect. Thus, if we assume symmetric marginal costs, as shown in Equation (37), an increase in the degree of compatibility reduces the incentives to innovate.

We assume that there are asymmetric marginal costs of the firms at an initial situation. In this case, based on Equation (36), we directly obtain the following results.

Result 1: If $c_0 \leq c_1$, the same results arise as in the case of symmetric marginal costs.

Result 2: If $c_0 > c_1$, it holds that $\dfrac{dX_0^H}{d\phi} > (<) 0 \Leftrightarrow c_0 - c_1 > (<) \dfrac{3t - n(1-\phi)}{2}$.[22]

We are interested in Result 2. Given that *firm 0* is sufficiently inefficient compared with rival *firm 1* at the initial situation, a larger degree of compatibility improves *firm 0*'s incentives to innovate.[23] That is, the output–expansion effect depends on the initial cost differences in the firms, that is., $\dfrac{dq_0^H}{d\phi} > (<) 0 \Leftrightarrow c_0 > (<) c_1$. In particular, if *firm 0* is less (more) efficient than *firm 1* before the innovation, an increase in the degree of compatibility increases (decreases) *firm 0*'s output. Result 2 implies that the magnitude of a positive output–expansion effect is larger than that of a negative cost-reduction effect.[24]

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[22] It is a necessary and sufficient condition for nonnegative production of *firm 0*, that is, $q_0^H > 0$ that the following inequality holds; $3t - n(1-\phi) > c_0 - c_1$.

[23] Because Foros and Hansen (2001) assume symmetric marginal costs, they do not demonstrate the result.

[24] See Appendix 4, where we address the case of responsive expectations and show that the effect of an increase in the degree of compatibility vanishes if there are symmetric marginal costs at the initial situation.

Remark 3. The effect on product (quality-improving) R&D under passive expectations[25]

We have addressed the effect of compatibility on incentives in the case of R&D designed to reduce costs. Following Kim (2000), who assumes responsive (active) expectations, we examine how an increase in the degree of compatibility affect an incentive to improve the level of quality, that is., the usage value of services, $v_i, \ \ i = 0,1.$[26]

In this case, Equation (27) can be revised as follows:

$$v_0 - p_0 - tl^* + N_0 > (<) v_1 - p_1 - t\left(1 - l^*\right) + N_1.$$

Following the same procedure as in the previous section, the demand function of *firm 0* is given by:

$$l^* = \frac{t + \left(v_0 - p_0 + N_0\right) - \left(v_1 - p_1 + N_1\right)}{2t} = q_0,$$

Assuming that $c_0 = c_1 = c \geq 0,$ we derive the following outcomes of *firm 0* in the equilibrium.

$$p_0^{Hv} - c = t\left\{1 + \frac{v_0 - v_1}{3t - n(1-\phi)}\right\},$$

$$q_0^{Hv} = \frac{1}{2}\left\{1 + \frac{v_0 - v_1}{3t - n(1-\phi)}\right\},$$

where superscript *Hv* denotes the case of product R&D. Thus, because the profit is expressed as $\pi_0^{Hv} = \frac{t}{2}\left\{1 + \frac{v_0 - v_1}{3t - n(1-\phi)}\right\}^2$, we have the effect of an increase in the quality-improving innovation on profit:

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[25] Assuming that $v_i = -c_i, \ i = 0,1,$ we obtain the same results as in the case of process R&D.

[26] Kim (2000) does not examine how increasing compatibility affects the incentives to improve the quality level. However, we can demonstrate that if the firms have symmetric quality levels at the initial situation, the increasing compatibility does not work. Furthermore, Sääskilahti (2006) shows that network compatibility is neutralized in the investment decisions in the symmetric quality case, assuming that consumers have responsive (active) expectations. For the case of responsive expectations, see Appendix 4.

$$\frac{d\pi_0^{Hv}}{dv_0} = \frac{t}{3t - n(1-\phi)} \left\{ 1 + \frac{v_0 - v_1}{3t - n(1-\phi)} \right\} \equiv X_0^{Hv} \left[v_0, v_1; \phi \right].$$

We obtain the following effect of an increase in the degree of compatibility on the benefit:

$$\frac{dX_0^{Hv}}{d\phi} = -\frac{nt}{\{3t - n(1-\phi)\}^2} \left\{ 1 + \frac{2(v_0 - v_1)}{3t - n(1-\phi)} \right\}.$$

Therefore, we derive the following results.

Result 1: If $v_0 \geq v_1$, it holds that $\dfrac{dX_0^{Hv}}{d\phi} < 0$.

Result 2: If $v_0 < v_1$, it holds that $\dfrac{dX_0^{Hv}}{d\phi} > (<) 0 \Leftrightarrow v_1 - v_0 > (<) \dfrac{3t - n(1-\phi)}{2}$.

Result 1 implies that if the quality level of the product or service of *firm 0* is higher than that of *firm 1* before the innovation, an increase in the degree of compatibility weakens the incentives to innovate. However, as shown in Result 2, if the quality level of the product (service) of *firm 0* is lower than that of *firm 1* and the degree of the quality difference between the firms is sufficiently large before the innovation, an increase in the degree of compatibility strengthens that incentives to innovate.

Recalling the results in the case of process R&D, we can declare that the incentives to innovate for a firm with less efficient and/or a lower quality production operating under a single industry-wide network system are larger than that under a firm-specific network system. In other words, this result may suggest regulation of a network system and compatibility standardization regarding the R&D investments of small-sized firms in network industries.

4. Conclusion

We have considered how the impact of compatibility (interconnectivity) between products and services providing by firms affects the incentives to innovate, that is, in relation to process R&D activities, in network industries, and demonstrated that if the products of the firms are sufficiently substitutable, such as if they are homogeneous products, an increase in the degree of compatibility decreases the incentives to innovate. Conversely, if the products are differentiated to some extent and the degree of network effects is relatively large, then the opposite results arise. That is, an increase in the degree of compatibility improves the incentives to innovate. In particular, an increase in the degree of compatibility directly expands the output (market share) of an innovating firm and influences the cost-reduction effect, which depends on product substitutability and network compatibility. If the degree of network compatibility is larger than that of product substitutability, the strategic relationship between the firms becomes complementary. In this case, an increase in the degree of compatibility increases the cost-reduction effect. Otherwise, it decreases the cost-reduction effect because the relationship between the firms is one of strategic substitutes. In the latter case, the impact of compatibility on innovation depends on the relationship between the positive output–expansion effect and the negative cost-reduction effect. Thus, for example, it would be preferable for the network system to be imperfectly (perfectly) compatible to advance process R&D activities in network industries with homogeneous (heterogeneous) products and services. In other words, compatibility standardization may be appropriate to improve process R&D activities in network industries where the products and services are differentiated sufficiently.

We have examined the incentive problem using Cournot and Bertrand duopoly models based on standard linear demand functions and shown that the effects of compatibility on incentives to innovate do not depend on the mode of competition. Furthermore, we have reconfirmed these main results using a unit-linear market based on a Hotelling framework and demonstrated that in the case of symmetric costs between the firms at the initial situation, an increase in the degree of compatibility reduces the incentives to innovate. That is, market expansion effects do not exist in the model based on the Hotelling framework, where we assume a full coverage market. This result is similar to the case of a homogeneous product market. However, in the case of asymmetric costs, especially if the innovating firm is less efficient than the rival firm, an increase in the degree of compatibility increases innovation.

Before discussing some remaining problems, we should comment on our approach to the model. In this paper, we have analyzed how the impact of compatibility affects the incentives to undertake cost-reducing R&D activity; that is, we have examined the effect of an increase in the degree of compatibility on the marginal gross profit owing to a reduction in marginal costs, that is, the benefit. We have not investigated R&D investment competition between firms, that is, strategic investment games. However, by assuming that a cost function includes an investment cost function (a fixed cost), e.g., $C\left(q_i, k_i\right) = c(k_i)q_i + \frac{f}{2}k_i^{\,2}$, $c(k_i) = \overline{c} - \delta k_i$, where k_i denotes an investment, and introducing the stage of R&D investment competition into the present model, we demonstrate the effects of compatibility on the R&D investments, outputs, and profits. Assuming symmetry, for example, we may yield the same results as the related papers, such as Shrivastav (2021).[27]

We note some remaining issues. First, we should confirm our main results by extending our duopolistic model to an oligopolistic one. Second, we have dealt with process innovation, that is, cost-reducing R&D activity. In Remark 3, where we assume that the usage value of services in a Hotelling-type model represents the level of quality, we have addressed the effects of

[27] Using Equations (5) and (21), we can show that strategic complements (substitutes) arise in the stage of investment competition if the degree of network compatibility is larger (smaller) than that of product substitutability irrespective of the mode of competition. However, in view of Equation (35), strategic substitutes always hold in a model à la Hotelling.

compatibility on the quality level. However, as examined by Kristiansen and Thum (1997), we should explore product R&D to improve the level of quality, in particular, in a vertically differentiated products market with network externalities.[28] Third, by following the definition of Amir and Lazzati (2011), we have treated mixed network goods which implies that the intrinsic utility from consumption is independent of network effects, such as a personal computer connected with network systems. We should explore the case of pure network goods, which are multiplicatively added network effects, such as telecommunications and Internet access services. Fourth, assuming exogenous compatibility, we have analyzed the effects. However, as Heywood et al. (2022) examine endogenous choice of compatibility, we should extend the model by introducing the stage of endogenous compatibility decision. Finally, we have compared the incentive under the perfect compatibility with that under the incompatibility. In this case, if products are sufficiently substitutable, for example, homogenous products, or if the market is full coverage in the Hotelling framework, then the perfect compatibility weakens the incentive to innovate compared with the incompatibility. If we interpret that the perfect compatibility corresponds to compatibility standardization, this result implies that standardization of network systems may reduce firms' innovative activities in network industries. However, we have not explicitly discussed the policy perspectives and implications of the model. We should examine optimal R&D policies in network industries, such as R&D investment subsidy/tax, and compatibility standardization and/or connectivity between various products and services of firms.

[28] Lambertini and Orsini (2005) investigate the existence of equilibrium in a vertically differentiated product market with network externalities. But they do not examine the innovation in a (vertically differentiated) network product market.

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Appendix 1. Proof of Proposition 1

Before proving Proposition 1, we should consider the implications of the benefit (marginal gross profit) function. Using Equation (5), the effects of an increase in the degree of compatibility on the benefits are decomposed into the following two parts:

$$\frac{d\left(-\frac{d\pi_0}{dc_0}\right)}{2d\phi}=\left(\frac{dq_0}{d\phi}\right)\left(-\frac{dq_0}{dc_0}\right)+q_0\frac{d\left(-\frac{dq_0}{dc_0}\right)}{d\phi}. \tag{A.1}$$

The first term on the right-hand side of Equation (A.1), that is, $\frac{dq_0}{d\phi}=\frac{nA}{(2-n+\Gamma)^2}(>0)$ implies that the effect of an increase in the degree of compatibility on output is positive because of direct network effects (hereinafter, we refer to this as the output–expansion effect). However, regarding the second term, the effect of an increase in the degree of compatibility on the cost-reduction effect is not unidirectional:

$$\frac{d\left(-\frac{dq_0}{dc_0}\right)}{d\phi}=-\frac{2n(2-n)\Gamma}{D^2}<(>)0\Leftrightarrow\gamma>(<)n\phi. \tag{A.2}$$

The effect depends on the degrees of product substitutability and network compatibility, which recalls Equation (4.2). If the degree of production substitutability is sufficiently large (small), and/or if the degree of network compatibility is sufficiently small (large), the effect is negative (positive). As a result, an increase in the degree of compatibility decreases (increases) the magnitude of the cost-reduction effect.

In particular, if $\gamma<n\phi,$ the sign of Equation (A.2) is positive.[29] This implies that the

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[29] For the sign of (A.2) to be positive, it is necessary that $1>\frac{\gamma}{n}$. If $1\geq\phi>\frac{\gamma}{n}$ for the degree of compatibility, it holds that $\gamma<n\phi$. However, if the degree of compatibility is low, that is,

relationship of strategic complements between the firms holds. The higher the degree of compatibility, the higher is the degree of network compatibility; thus, the magnitude of the cost-reduction effect increases.

Conversely, if $\gamma > n\phi,$ the sign of Equation (A.2) is negative. That is, an increase in the degree of compatibility reduces the magnitude of the cost-reduction effect. In this case, the higher is the degree of compatibility, the lower is the absolute value, that is, $|\Gamma| = |\gamma - n\phi|.$ In particular, an increase in the degree of compatibility reduces the magnitude of a decrease in the output of *firm 1*. In turn, the degree of the decrease in the price of *firm 0* becomes large. This affects profit negatively. Conversely, if the absolute value is sufficiently large, for example, $|\Gamma| \approx |\gamma|$ if $\phi \to 0,$ the degree of the decrease in the price becomes small, such that the effect on profit can be positive.

Equation (A.1) can be rewritten as:

$$\frac{d\left(-\frac{d\pi_0}{dc_0}\right)}{2d\phi} = \left(\frac{q_0}{\phi}\right)\left(-\frac{dq_0}{dc_0}\right)\left\langle \left(\frac{dq_0}{d\phi}\right)\left(\frac{\phi}{q_0}\right) + \frac{d\left(-\frac{dq_0}{dc_0}\right)}{d\phi}\frac{\phi}{\left(-\frac{dq_0}{dc_0}\right)} \right\rangle, \quad \text{(A.3)}$$

where $\left(\frac{dq_0}{d\phi}\right)\left(\frac{\phi}{q_0}\right)$ denotes the elasticity of the output–expansion effect in relation to the degree of compatibility and $\frac{d\left(-\frac{dq_0}{dc_0}\right)}{d\phi}\frac{\phi}{\left(-\frac{dq_0}{dc_0}\right)}$ denotes the elasticity of the cost-reduction effect in relation to the degree of compatibility. Although the sign of the former is always

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$1 \geq \frac{\gamma}{n} > \phi,$ the sign of (A.2) is negative. Furthermore, if $\frac{\gamma}{n} > 1(\geq \phi),$ the sign of (A.2) is necessarily negative.

positive, that of the latter is not unidirectional, as noted in Equation (A.2). In particular, the term in parentheses in Equation (A.3) can be rewritten as:

$$\langle \bullet \rangle = \frac{n\phi}{2-n+\Gamma} - \frac{2n\phi\Gamma}{(2-n+\Gamma)(2-n-\Gamma)} = \frac{n\phi}{2-n+\Gamma}\left\{1-\frac{2\Gamma}{2-n-\Gamma}\right\}. \quad \text{(A.4)}$$

Equation (A.4) implies the relationship between the elasticities of the output–expansion and cost-reduction effects. Using Equations (A.3) and (A.4), we derive the following Equation, which is equal to Equation (8) multiplied by $2(2-n)$.

$$\frac{d\left(-\frac{d\pi_0}{dc_0}\right)}{2d\phi} = \frac{nA(2-n)(2-n-\Gamma)(2-n-3\Gamma)}{D^3} > (<)0 \Leftrightarrow 2-n-3\Gamma > (<)0$$

$$\Leftrightarrow \phi > (<)\phi^C[n,\gamma] = \frac{1}{3} - \frac{2-3\gamma}{3n} < \frac{\gamma}{n} \quad \text{(A.5)}$$

Based on Equation (A.5), if $n\phi > \gamma,$ it holds that $1 \geq \phi > \frac{\gamma}{n} > \phi^C[n,\gamma].$ Thus, we have $\frac{d\left(-\frac{d\pi_0}{dc_0}\right)}{d\phi} > 0.$ This result implies that if the degree of network compatibility is larger than that of product substitutability, the signs of the elasticities of both the cost-reduction and the output–expansion effects are positive because the firms are strategic complementary. Thus, an increase in the degree of compatibility always improves the incentives to innovate.

Second, if $n\phi < \gamma,$ this implies that the sign of the elasticity of the cost-reduction effect is negative, whereas that of the output–expansion effect is positive. Thus, there are two cases: (a) $\frac{\gamma}{n} > \phi^C[n,\gamma] > \phi$ and (b) $\frac{\gamma}{n} > \phi > \phi^C[n,\gamma].$ In Case of (a) (Case (b)), an increase in the degree of compatibility reduces (promotes) the incentives to innovate. This is because the elasticity of the output–expansion effect is smaller (larger) than that of the cost-reduction effect.

We proceed to prove Proposition 1. First, if $\gamma < \frac{2-n}{3}$ for $0 < n < 1,$ that is, the degree of product substitutability is low (in other words, the products are sufficiently differentiated), then it holds that $\phi \geq 0 > \phi^C[n,\gamma].$ This implies that the elasticity of the cost-reduction effect is either positive or, if it is negative, the magnitude of the elasticity is smaller than that of the positive output–expansion effect. As a result, an increase in the degree of compatibility increases the firm's benefit.

Conversely, if $\gamma > \frac{2(1+n)}{3}$ for $0 < n < \frac{1}{2},$ that is, if the firms' products are close to homogenous and the degree of the network effect is small, then it holds that $\phi \leq 1 < \phi^C[n,\gamma].$ This implies that the elasticity of the negative cost-reduction effect is larger than that of the positive output–expansion effect. As a result, an increase in the degree of compatibility decreases the firm's benefit.

Third, if $\frac{2(1+n)}{3} > \gamma > \frac{2-n}{3}$ for $0 < n < \frac{1}{2},$ that is, if the degree of product substitutability is moderate under weak network effects (that is, smaller than a half) and if $1 \geq \gamma > \frac{2-n}{3}$ for $\frac{1}{2} < n < 1,$ that is, if the degree of product substitutability is high under strong network effects (that is, larger than a half), then it holds that $0 < \phi^C[n,\gamma] < 1.$ In this case, where the benefit function is *U*-shaped, we have the following two cases. First, if $0 \leq \phi < \phi^C[n,\gamma],$ then the magnitude of the elasticity of the negative cost-reduction effect is larger than that of the elasticity of the positive output–expansion effect, so that an increase in the degree of compatibility decreases the firm's benefit. This corresponds to Case (a) mentioned above. Second, if $\phi^C[n,\gamma] < \phi \leq 1,$ the magnitude of the elasticity of the negative cost-reduction effect is smaller than that of the positive output–expansion effect. Accordingly, an increase in the degree of compatibility increases the firm's benefit. This corresponds to Case (b) mentioned above.

Appendix 2. Proof of Corollary 1

Using Equations (9.1) and (9.2), we derive the following Equation:

$$\operatorname{sgn}\left\{X_0[\phi=0]-X_0[\phi=1]\right\}=\operatorname{sgn}F(n,\gamma), \tag{A.7}$$

where $F(n,\gamma)\equiv n^2+(2-5\gamma)n+(2+\gamma)(3\gamma-2)$.

Hereinafter, based on Equation (A.7) and Proposition 1, we will prove Corollary 1.[30]

(1) Regarding Equation (A.7), if $\frac{10}{13}(\approx 0.769)<\gamma\leq 1$, there are imaginary number solutions. Because it holds that $F(n,\gamma)>0$, we have $X_0[\phi=0]>X_0[\phi=1]$. Furthermore, in view of Proposition 1 (ii), it holds that $\left.\frac{dX_0}{d\phi}\right|_{c_0=c_1=c}<0$ if $\gamma>\frac{2(1+n)}{3}$ for $0<n<\frac{1}{2}$.[31] Thus, we have $X_0[\phi=0]>X_0[\phi=1]$. Therefore, we have proven Corollary 1 (ii).

(2) Given Equation (A.7), if $\frac{1}{3}\geq\gamma\geq 0$, it holds that $F(n,\gamma)<0$ for $0<n<1$. Thus, we have $X_0[\phi=0]<X_0[\phi=1]$.

[30] With respect to Corollary 1 (i) and (ii), we prove the order in reverse.

[31] Substituting $\gamma=\frac{10}{13}$ into $\gamma=\frac{2(1+n)}{3}$, we have $n=\frac{2}{13}$.

Next, we examine the following range for product substitutability: $\frac{10}{13} > \gamma > \frac{1}{3}$, where two real number solutions exist, that is, $n_{+/-}[\gamma] = \frac{5\gamma - 2 \pm \sqrt{(2-\gamma)(10-13\gamma)}}{2}$. Let us divide the range into two subranges: (a) $\gamma > \frac{2-n}{3} (\Leftrightarrow n > 2 - 3\gamma)$ for $\frac{2}{3} \geq \gamma > \frac{1}{3}$, and (b) $\frac{2(1+n)}{3} > \gamma \left(\Leftrightarrow n > \frac{3\gamma - 2}{2} \right)$ for $\frac{10}{13} > \gamma > \frac{2}{3}$.

In Subrange (a), we have the following real number solutions: $n_+[\gamma] > 1 > 2 - 3\gamma > 0 > n_-[\gamma]$. This implies that $F(n, \gamma) < 0$ for $0 < n < 1$. Furthermore, if $\gamma = \frac{2}{3}$, $F(n) = n\left(n - \frac{4}{3}\right) < 0$ for $0 < n < 1$. Therefore, we have proven Corollary 1 (i).

(3) With respect to Subrange (b), we further derive the following two outcomes.

(b.1) If $\frac{2}{3} < \gamma \leq \frac{\sqrt{13}+1}{6} (\approx 0.7676)$, we have $n_+[\gamma] > 1 > n_-[\gamma] > \frac{3\gamma - 2}{2}$. Thus, the following relationship holds:

$$n > (<) n_-[\gamma] \Leftrightarrow F(n, \gamma) < (>) 0 \Leftrightarrow X_0[\phi = 0] < (>) X_0[\phi = 1].$$

(b.2) If $\frac{\sqrt{13}+1}{6} < \gamma < \frac{10}{13}$, we have $1 > n_+[\gamma] > n_-[\gamma] > \frac{3\gamma - 2}{2}$. Thus, the following relationships hold:

$$n < n_-[\gamma] \Leftrightarrow F[n, \gamma] > 0 \Leftrightarrow X_0[\phi = 0] > X_0[\phi = 1],$$

$$n_-[\gamma] < n < n_+[\gamma] \Leftrightarrow F[n, \gamma] < 0 \Leftrightarrow X_0[\phi = 0] < X_0[\phi = 1],$$

$$n_+[\gamma] < n \Leftrightarrow F[n, \gamma] > 0 \Leftrightarrow X_0[\phi = 0] > X_0[\phi = 1].$$

In particular, if $\gamma = \dfrac{10}{13}$, we have $n_{-}[\gamma] = n_{+}[\gamma]$. Thus, it holds that $F[n,\gamma] \geq 0 \Leftrightarrow X_0[\phi = 0] \geq X_0[\phi = 1]$. The relationship between the benefits of incompatibility and perfect compatibility in Subrange (b) is ambiguous. Therefore, we have proven Corollary 1 (iii).

Appendix 3. Proof of Corollary 2

Using a similar manner to that used for Corollary 1 to prove Corollary 2, we explore the magnitude of the benefits in the cases of incompatibility and perfect compatibility in area *III*, considering the results in areas *I* and *II* of Proposition 2.

Using Equations (27.1) and (27.2), we derive the following Equation:

$$\operatorname{sgn}\left\{X_0^B[\phi = 0] - X_0^B[\phi = 1]\right\} = \operatorname{sgn} F^B(n,\gamma), \tag{A.8}$$

where $F^B(n,\gamma) \equiv n^2 + \left(2 - 5\gamma - \gamma^2\right)n - \left(2 + \gamma - \gamma^2\right)\left(2 - 3\gamma - \gamma^2\right)$.

Regarding the quadratic function of network effects, if $\dfrac{3\sqrt{41} - 13}{10}(\approx 0.621) < \gamma < 1$, because there are imaginary number solutions, we have $F^B(n,\gamma) > 0$ for $0 < n < 1$. In addition, it holds that $\left.\dfrac{dX_0}{d\phi}\right|_{c_0 = c_1 = c} < 0$ in area *II* of Figure 2, that is, $n < \dfrac{\gamma^2 + 3\gamma - 2}{2}$ for $0 < n < \dfrac{3\sqrt{41} - 13}{50}$. Thus, it holds that $X_0[\phi = 0] > X_0[\phi = 1]$. Thus, we have proven Corollary 2 (ii).

Next, we examine the range of product substitutability: $\frac{3\sqrt{41}-13}{10} > \gamma > 0$, where two real number solutions exist, that is, $n^{B}{}_{+/-}\left[\gamma\right] = \frac{\gamma^2 + 5\gamma - 2 \pm \sqrt{(1-\gamma)(2+\gamma)(10-13\gamma-5\gamma^2)}}{2}$. Let us divide the range into two subranges: (a) $n > \frac{\gamma^2 + 3\gamma - 2}{2}$ for $0 < \gamma < \frac{\sqrt{17}-13}{2}$, and (b) $n > 2 - 3\gamma - \gamma^2$ for $\frac{\sqrt{17}-3}{2} < \gamma < \frac{3\sqrt{41}-13}{10}$.

(1) In Subrange (a), we have the following real number solutions: $n_{+}^{B}\left[\gamma\right] > 1 > 0 > n_{-}^{B}\left[\gamma\right]$. This implies that $F^{B}\left(n, \gamma\right) < 0$ for $0 < n < 1$. Furthermore, it holds that $\left.\frac{dX_0}{d\phi}\right|_{c_0 = c_1 = c} > 0$ in area *I* of Figure 2. Thus, if $0 < \gamma < \frac{\sqrt{17}-3}{2}$ and $0 < n < 1$, it holds that $X_0\left[\phi = 0\right] < X_0\left[\phi = 1\right]$. Thus, we have proven Corollary 2 (i).

(2) Regarding Subrange (b), similar to Subrange (b) in the case of Cournot duopoly, the relationship between the benefits of incompatibility and perfect compatibility depend on the degree of the parameters and, thus, is not unidirectional. In particular, if $\frac{\sqrt{17}-3}{2} < \gamma < \frac{3\sqrt{41}-13}{10}$ and $\frac{3\sqrt{41}-13}{50} < n < 1$, or if $n > \frac{\gamma^2 + 3\gamma - 2}{2}$ and $0 < n < \frac{3\sqrt{41}-13}{50}$, the relationship is ambiguous. Thus, we have proven Corollary 2 (iii).

Appendix 4. The case of responsive (active) expectations

Given the assumptions of passive expectations and symmetric marginal costs, we have shown that the effect of an increase in the degree of compatibility on incentives to innovate is negative. We should reconfirm the results in the case of responsive expectations.

Using Equations (28.1) and (28.2), given responsive expectations, that is, $q_i^e = q_i$, $i = 0,1$, we obtain the following demand function for *firm 0*:

$$q_0 = \frac{\{t - n(1-\phi)\} - p_0 + p_1}{2\{t - n(1-\phi)\}}, \tag{A.9}$$

where we assume that $t - n(1-\phi) > 0$. In this case, we obtain the following outcomes in the equilibrium.

$$p_0 - c_0 = \frac{3\{t - n(1-\phi)\} - c_0 + c_1}{3}, \tag{A.10}$$

$$q_0 = \frac{p_0 - c_0}{2\{t - n(1-\phi)\}} = \frac{3\{t - n(1-\phi)\} - c_0 + c_1}{6\{t - n(1-\phi)\}}. \tag{A.11}$$

Thus, the profit is represented as $\pi_0 = \dfrac{(p_0 - c_0)^2}{2\{t - n(1-\phi)\}} = 2\{t - n(1-\phi)\}(q_0)^2$. The benefit of an infinitesimal cost reduction to *firm 0* is given by:

$$-\frac{d\pi_0}{dc_0} = -\frac{d(p_0 - c_0)}{dc_0} q_0 + (p_0 - c_0)\left(-\frac{dq_0}{dc_0}\right) = \frac{1}{3} q_0 + (p_0 - c_0)\frac{1}{6\{t - n(1-\phi)\}}.$$

Thus, the effect of an increase in the degree of compatibility on the benefit is given by:

$$\frac{d\left(-\dfrac{d\pi_0}{dc_0}\right)}{d\phi} = \frac{1}{3}\left(\frac{dq_0}{d\phi}\right) + \frac{d(p_0 - c_0)}{d\phi}\left(-\frac{dq_0}{dc_0}\right) + (p_0 - c_0)\frac{d\left(-\dfrac{dq_0}{dc_0}\right)}{d\phi}, \tag{A.12}$$

where $\frac{dq_0}{d\phi} = \frac{n(c_0 - c_1)}{6\{t - n(1-\phi)\}^2} > (<) 0 \Leftrightarrow c_0 > (<) c_1,$ $\frac{d(p_0 - c_0)}{d\phi} = n > 0,$

$\frac{d\left(-\frac{dq_0}{dc_0}\right)}{d\phi} = -\frac{n}{6\{t - n(1-\phi)\}} < 0.$ In particular, the output–expansion effect depends on the cost difference in the initial situation. Furthermore, the effect on the profit per output is positive and the effect on the cost-reduction effect is negative. The results are the same as in the case of passive expectations. Given Equation (40), we derive the following relationship.

$$\frac{d\left(-\frac{d\pi_0}{dc_0}\right)}{d\phi} = \frac{c_0 - c_1}{9\{t - n(1-\phi)\}^2} > (<) 0 \Leftrightarrow c_0 > (<) c_1. \tag{A.13}$$

Therefore, under responsive expectations, in the case that the firms have symmetric marginal costs in the initial situation, that is, $c_0 = c_1$, the effect on the benefits is 0, that is, the change in the degree of compatibility does not affect incentives to innovate. However, if *firm 0* is less (more) efficient compared with *firm 1*, an increase in the degree of compatibility improves (reduces) incentives to innovate.

Figure 1. The effects of an increase in the degree of compatibility on incentives to innovate

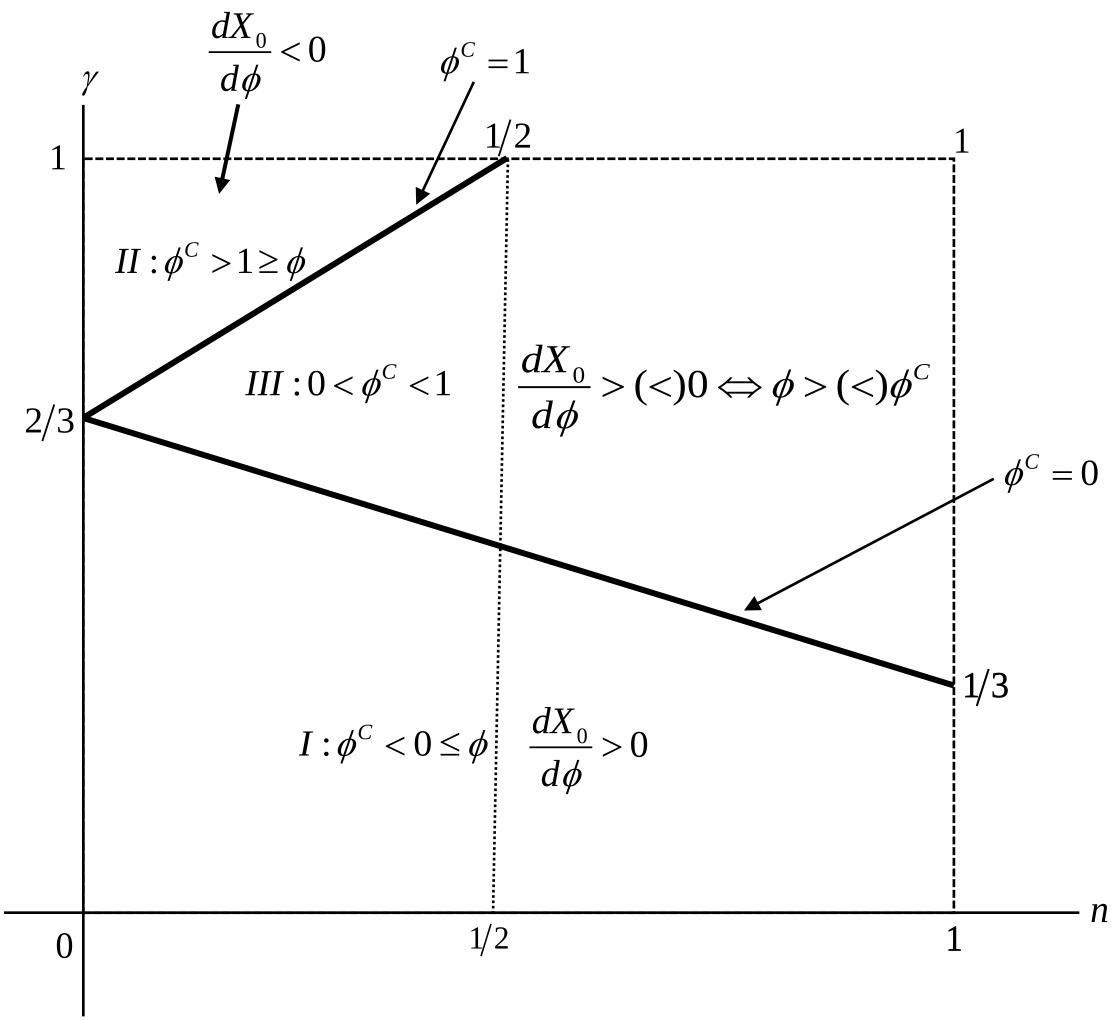

Figure 2. The relationship of the benefits: The incompatibility vs. the perfect compatibility

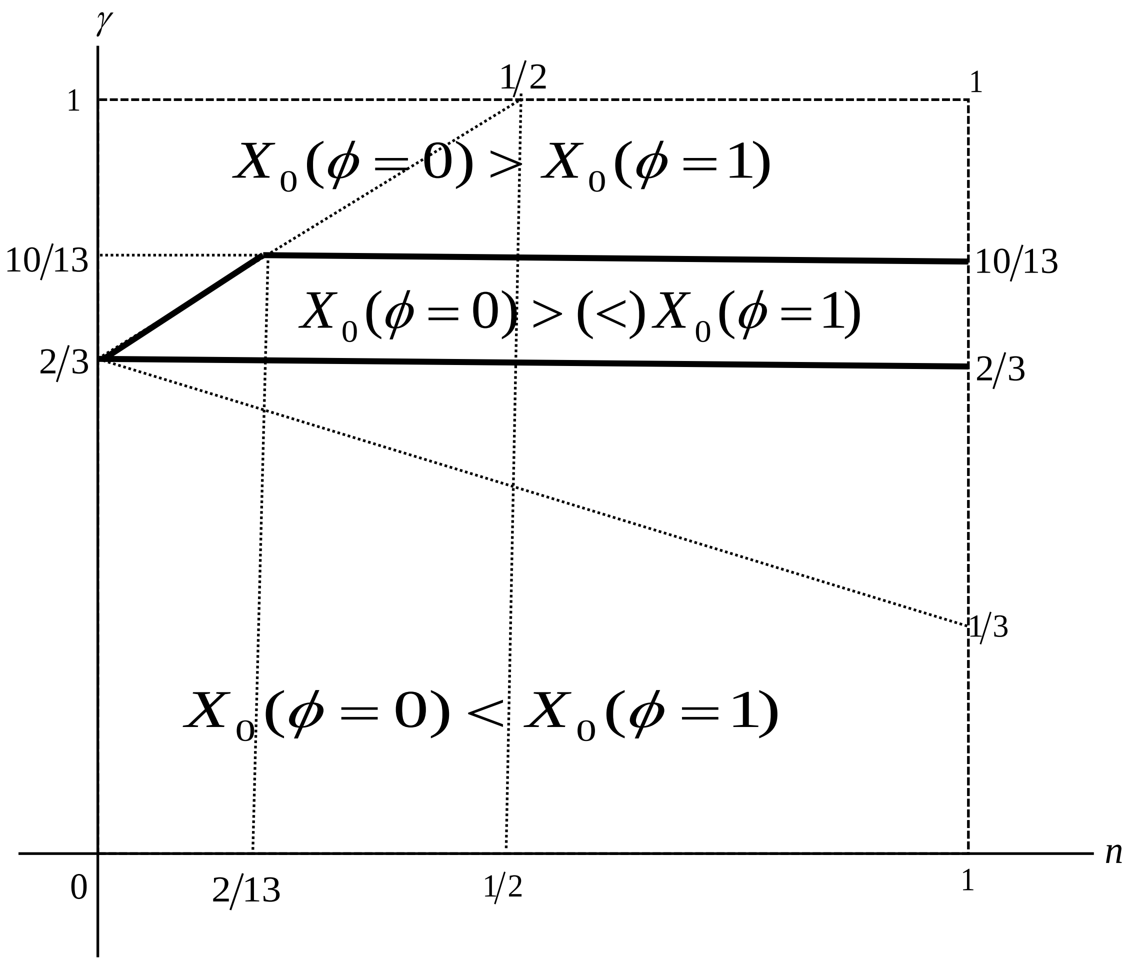

Figure 3. The effects of an increase in the degree of compatibility in the case of Bertrand duopoly

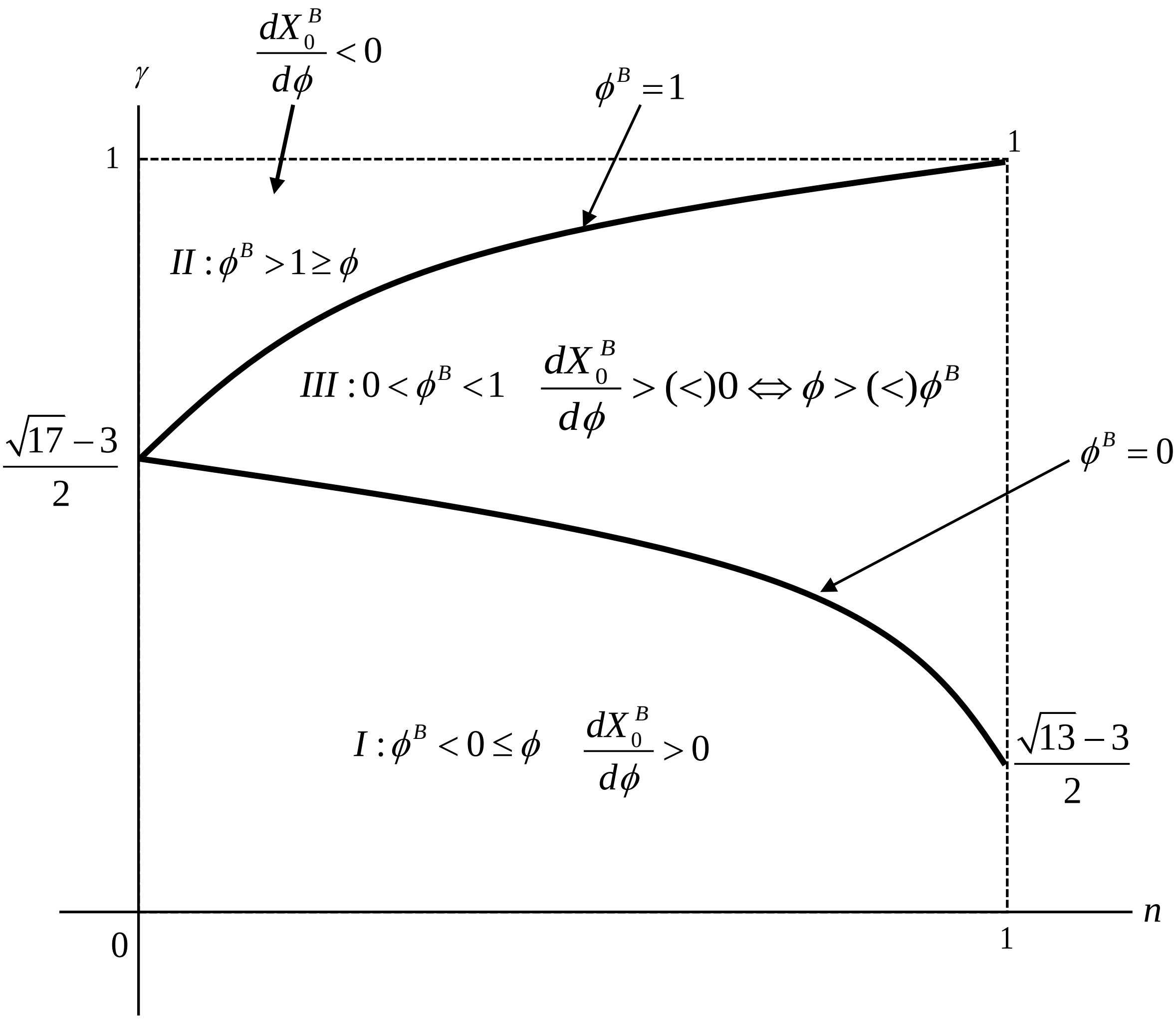

Figure 4. The relationship of the benefits in the case of Bertrand duopoly