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Effects of Exchange Rate Volatility on Behaviors of Affliate Firms in a Foreign Oligopoly under the Revision of Supply Chains

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Abstract

This paper investigates the changes in exchange rate volatility on an international oligopolistic market in a foreign country that accepts n affiliate firms through foreign direct investment (FDI) from a home country. Under the revision of the supply chains of essential products such as high-tech products, the affiliate firms are forced to procure their *essential* intermediate products from firms in their home country, even though they are expensive. We derive a Cournot equilibrium of the oligopolistic foreign market, in which affiliate firms compete with foreign firms under foreign exchange rate uncertainty when the number of affiliates, n, is exogenously given. In the equilibrium, we show the affiliate firms/the foreign firms aggressively expand their outputs when the relative risk aversion coefficient is large /small at equilibrium. Affiliate firms may earn *ex-post* expected profits less than the expected profits of the foreign firms even when the relative risk aversion coefficient is small at equilibrium. However, whether the change in the foreign exchange rate may be profitable for the ex-post profits of the affiliate and parent firms is indeterminate.

Keywords: risk aversion, exchange rate volatility, affiliate firms, foreign oligopolistic market, revision of supply chains

JEL classification: G32, L13, L12

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1 Introduction

Recently, events such natural disasters, the COVID-19 pandemic and so on, have frequently disrupted global supply chains. Faced with a substantial increase in China's military capacity and the Russian invasion of Ukraine, governments in the US and other countries in the Western Bloc, including Japan, Australia and South Korea, have reduced their import of parts and intermediate products from China to revise the global supply chains of high-tech products such as semiconductors in light of national security concerns (Todo,(2022), Inoue and Todo,(2023)).

Under such circumstances, many affiliate companies of foreign countries that manufacture in these Western Bloc countries founded by direct investments may increase their procurement of essential parts or intermediate products from firms in their home countries. In such cases, the affiliate firms are forced to procure such essential parts or intermediate products from firms in their home country, even though they are expensive.

A survey on the business conditions of Japanese-affiliated companies overseas made public by the Japan External Trade Organization (JETRO) in November 2022 reports that approximately 65% of Japanese-affiliated companies are expected to achieve profitability in 2022.

Home country parent firms also reseive part of the profits from their affiliates. Based on the above, firms in the home country have to choose their outputs in the foreign country's market by using an *ex-ante expectation* of the exchange rate, since their affiliate firms procure essential parts or intermediate products from firms in the home country and would thus remit a part of their profit as dividends. Hence, they are forced to face foreign exchange rate uncertainty.

Under such rapid changes in the worldwide environment, businesses and economies throughout the world may suffer some losses. Foreign exchange markets may become confused and uncertain. Even in such an environment, parent firms have to compete through their affiliates with foreign rival firms in a foreign oligopolistic market.

For parent manufacturers, it is important to earn the allotment of the profits of their affiliates in the foreign country if the scale of the domestic market they face shows a trend toward shrinking. Many governments often offer manufacturers total or partial exemption from taxation for the remitted allotments or dividends from their affiliates in foreign countries[1]. Currently, Japan is one of the largest creditor countries in the world. Therefore, recently, it has become more important for the Japanese government to induce affiliates in foreign countries to remit the dividends or part of the profits of such affiliates to their parent multinational firms. Hasegawa and Kiyota (2017) positively explore the effect of the dividend exemption system on profit repatriation by Japanese multinational firms. They positively explore the effect of recently moving the Japanese government away from a worldwide income tax system toward a territorial tax system (with dividend exemption) based on profit repatriation by Japanese multinational firms. By using unique confidential survey data for Japanese multinational corporations, the authors find that dividend payments by foreign affiliates to parent companies in the home

[1]Indeed, the Japanese government introduced the Foreign Dividend Exclusion system in 2009, which exempts dividends remitted by Japanese-owned foreign affiliates to their parent firms from home-country taxation.

country also became more responsive to withholding tax rates on dividends levied by the government of the host country after the tax reform.

In such a case, the extent of the relative risk aversion for the volatility of an exchange rate, which a parent firm in a home country faces, is crucial with regard to its decision on the production strategy of its affiliate firm, which procures parts or intermediate goods from firms that produce them and remits part of its profit to the parent firm in the home country.

In their seminal work, Sung and Lapan (2000) investigate how exchange-rate uncertainty affects the foreign investment decision of *risk-neutral* multinational firms (MNFs) *in a monopoly setting.* They show that a MNF would open only one plant in the home country or a foreign country under the assumption that the firm can open plants, each with decreasing average costs in the two different countries. However, under uncertainty (under a mean-preserving spread exchange rate distribution or a uniform exchange rate distribution), the authors demonstrate that under sufficient large exchange rate volatility, the firm can increase expected profit by opening several plants and show that if the MNF faces a competitor in a foreign market, then the exchange rate risk induces the MNF to open plants in both markets, consequently preventing entry by the local competitor.

Lahiri and Mesa (2006) explore the effects of exchange rate volatility of both the host country and the parent country on host-government policy related to the local content requirement (LCR) on export-oriented foreign direct investment (FDI) in the context of an oligopolistic market in a third country in a type of Brander and Spencer (1987) trade model. Namely, the authors assume that there are domestic identical risk-neutral firms and *foreign* identical *risk-averse* firms in the domestic (host) country, and these firms compete in a Cournot oligopolistic market of a homogeneous good in a consuming third country where there are no producers for the good. Hence, they do not examine how changes in the volatility of the exchange risk impact the behavior of affiliate firms and host foreign country firms or the equilibrium outcomes in the host country's oligopolistic market as the two types of firms compete with one another.

Under the assumption that the exchange rates follow log-normal distributions, the authors show that an increase in the volatility of the foreign exchange rate decreases the optimal LCR level both under free entry and exit of foreign firms and when the number of foreign firms is fixed. They also find that the government uses a less strict LCR policy when the number of foreign firms is endogenous than when it is exogenous.

There are few studies on international oligopolistic competition in a foreign market through affiliate firms engaging in FDI in which these affiliates not only procure their important intermediate goods or parts from firms that produce but also repatriate their part of their profit to parent firms in the home country under the exchange rate risk .

Therefore, in this paper, we consider an international oligopoly model with the oligopolistic market in a foreign country. The international home firms compete with host foreign firms in a host foreign country's oligopolistic market through their affiliate firms by way of FDI. In particular, we explore how the volatility of the exchange rate risk affects the behavior of affiliates through the extent of risk aversion for the exchange risk and that of foreign firms in a foreign oligopolistic market in the absence of both free entry and free exit of affiliate firms.

We assume that the affiliates procure all intermediate goods or parts from firms that produce them in the home country for the production of their final goods. We also suppose that they have to repatriate a portion of the profit they earn in the foreign market to their parent firms. We do not consider any policies by the government such as LCR, tariff or production subsidies for firms in their own host foreign country, or the economic welfare of the equilibrium outcome.

We consider a Cournot oligopolistic market game in a host foreign country that accepts n affiliate firms through FDI from a home country. First, we derive equilibria in the case where the number of affiliates, n, is exogenously given under the assumption that the exchange rates follow log-normal distributions, as Lahiri and Mesa (2006) assume. We investigate how changes in the exchange rate volatility of the international oligopolistic market equilibria impact outcomes.

The rest of this paper is organized as follows. In Section 2, we present our model, and we examine an equilibrium in the absence of the free entry and exit of affiliate firms. We also examine the properties of equilibrium outcomes in the equilibria. In Section 3, we explore how changes in the volatility of the exchange rate have an effect on the equilibrium outcomes in the absence of the free entry and exit of affiliate firms. Finally, Section 5 concludes this paper.

2 Model

Here, we consider an international oligopoly model within the oligopolistic market of a foreign country. The international home firms (*IH* firms hereafter) compete in a host foreign country's (country 2) oligopolistic market through their affiliate firms (*A* firms hereafter) by foreign direct investment (*FDI*). Furthermore, we assume that *A* firm i internally reserves its profit at the foreign market equilibrium at *the retained earnings rate* of s $(0 < s < 1)$ and remits its profit at *the repatriation rate* of $1-s$ from the foreign market to the head office of its *IH* firm in the home country. We thus derive a Cournot equilibrium under foreign exchange rate uncertainty when the number of *IH* firms, n, is either exogenous or endogenous. Then, we explore the effects of foreign exchange rate volatility on equilibrium outcomes.

Assume there are two countries: country 1 (home country) and country 2 (foreign country). In country 2's oligopoly, the n international firms of home country 1 compete in the oligopolistic market of host foreign country 2 through the FDI of their affiliate companies (*A* firms) with m foreign firms (*F* firms).

Each *IH* firm in home country 1 has constant returns to scale technology according to $c_i^H, i = 1, \cdots, n$, as indicated by the home currency.

We assume that an international firm supplies its product in both a domestic market and a foreign market. Although each *IH* firm procures its *essential parts or intermediate goods* from the home country to assemble its final products, the affiliate firms in foreign country 2 procure all parts or intermediate goods from home country 1 by importation. Each foreign firm supplies its product in the foreign country market and procures all essential parts or intermediate goods from foreign country 2. We only focus on *the foreign country's market competition* between A and F firms since affiliate A firms choose their outputs for the home country market independently of the outputs for the foreign country's market.

The competition among A and F firms in an oligopolistic foreign market under exchange rate uncertainty and the relationship between each parent firm in home country and its affiliate firm that we consider in our study are depicted in Figure 1.

Foreign firms have constant returns to scale technology, and their marginal and the average common cost is given by $c^F \equiv c_j^F, j = 1, \cdots m$, as indicated by the foreign currency. Each affiliate A firm of the foreign country incurs marginal cost $\widetilde{c}^A$ to produce its product; this is a random variable because it depends on the exchange rate

between the home and the foreign country's currencies $\widetilde{\epsilon}$, and is thus exogenous to the model . $\widetilde{\epsilon}$ is assumed to be a log-normally distributed random variable, that is, $\widetilde{\epsilon} = \exp(\widetilde{X}), \widetilde{X} \sim N(\mu, \sigma^2)$; we also assume that $\ln 2 \approx 0.693\,15 > \sigma^2 > 0.0095$[2]. We assume that a foreign country government never imposes an import tariff for inputs imported from the home country.

[2]We estimate μ and σ^2 from the exchange rate of Japanese yen per unit of some foreign currencies, i.e., the Chinese yuan, the Indian rupee, the Thai baht, the Malaysia ringgit, the Korean 100 won, etc., as $\widetilde{\epsilon} = \exp(\widetilde{X}), \widetilde{X} \sim N(\mu, \sigma^2)$ by using Monthly Foreign Exchange Quartation Data published from the Mizuho Bank Corporation in Japan. All of the estimated sample variances $\widehat{\sigma^2}$ of these Currencies are included in the open interval $(0.01, 0.03)$. Namely, under our assumption that $\widetilde{\epsilon}$ is a log-normally distributed, we can estimate that the variance of $\ln \widetilde{\epsilon}$ is very small. Hence, we assume that $0.0095 < \sigma^2 < \ln 2$ to satisfy the Assumption that $\mathbf{a}_\epsilon > 0$, which we place thereafter on the essential parameter $\mathbf{a}_\epsilon$. Monthly Foreign Exchange Quartation Data published from Mizuho Bank Corporation in Japan are available from the next website: https://www.mizuhobank.co.jp/market/csv/m_quote.csv

$$\widetilde{c}^A = c^H / \widetilde{\epsilon} \tag{1}$$

Then, it is well known that the mean and the variance of $\widetilde{\epsilon}$ are given by the following:

$$\mu_{\widetilde{\epsilon}} = \exp(\mu + \sigma^2/2) \tag{2}$$

and

$$0 < \sigma^2_{\widetilde{\epsilon}} = \mu^2_{\widetilde{\epsilon}}(e^{\sigma^2} - 1) < (e^{0.0095} - 1) \times \mu^2_{\widetilde{\epsilon}} \approx 9.5453 \times 10^{-3} \times \mu^2_{\widetilde{\epsilon}} < \mu^2_{\widetilde{\epsilon}}$$
$$\text{for } \ln 2 \approx 0.69315 > \sigma^2 > 0.0095. \tag{3}$$

We assume that the mean of exchange rate $\widetilde{\epsilon}$[3] is as follows:

$$\mu_{\widetilde{\epsilon}} > 2. \tag{4}$$

We can easily derive the following:

$$E_{\widetilde{\epsilon}}[1/\widetilde{\epsilon}] = \exp(-\mu + \sigma^2/2) = \frac{\mu_{\widetilde{\epsilon}}}{e^{2\mu}}, \tag{5}$$

[3]We also estimate $\widehat{\mu_\epsilon}$, which is the sample mean of the exchange rate of Japanese yen per unit of some foreign currencies, i.e., the Chinese yuan, the Indian rupee, the Thai baht, the Malaysia ringgit, the Korean 100 won, etc., as $\widetilde{\epsilon} = \exp(\widetilde{X}), \widetilde{X} \sim N(\mu, \sigma^2)$ by using Monthly Foreign Exchange Quartation Data published from Mizuho Bank Corporation in Japan. All of the estimated sample means of the exchange rate $\widehat{\mu_\epsilon}$ of these currencies are included in the open interval $(2, 29)$. Therefore, we reject the assumption.

$$Q^{FM} \equiv \sum_{i=1}^{n} q_i^A + \sum_{j=n+1}^{n+m} q_j^F = Q^A + Q^F, \tag{6}$$

$$p^F = a^F - Q^{FM} = a^F - \sum_{i=1}^{n} q_i^A - \sum_{j=1}^{m} q_j^F, \tag{7}$$

As mentioned in the introduction, affiliate firm A remits a $(1-s)$ portion of its profit to its head office in home country 1. Therefore, the head office is interested in the amount of the expected remittance from its affiliate. Hence, we can define the amount of the remittance to the IH firm head office from its affiliate in foreign country 2 as follows:

$$\begin{aligned} \pi_i^{IH} &\equiv (1-s)\widetilde{\epsilon}(p^F - \widetilde{c}^A)q_i^A \\ &= (1-s)\widetilde{\epsilon}(a^F - \sum_{i=1}^{n} q_i^A - \sum_{j=1}^{m} q_j^F - \widetilde{c}^A)q_i^A \\ &= (1-s)\widetilde{\epsilon}(a^F - Q^A - Q^F - c^H/\widetilde{\epsilon})q_i^A, i = 1, ..., n. \end{aligned} \tag{8}$$

From (8), (7), and (6), the certainty equivalence of the expected profit of A firm i is given by the following:

$$\begin{aligned} E_{\widetilde{\epsilon}}\left[CE\pi_i^{IH}\right] &= (1-s)E_{\widetilde{\epsilon}}\left[CE\pi_i^A\right] \\ &= (1-s)\{E_{\widetilde{\epsilon}}\left[\widetilde{\epsilon}(p^F - \widetilde{c}^A)q_i^A\right] - \gamma SD_{\widetilde{\epsilon}}\left[\widetilde{\epsilon}(p^F - \widetilde{c}^A)q_i^A\right]\} \\ &= (1-s)[\mathbf{a}_\epsilon\{a^F - Q^A - Q^F\} - (1-\gamma)c^H]q_i^A, \end{aligned} \tag{9}$$

where $E(\cdot)$ and $SD(\cdot)$ stand for expectation and standard deviation operators, respectively, and

$$\mathbf{a}_\epsilon = \mu_{\varepsilon} - \gamma\sigma_\epsilon,$$

where γ is a relative risk averse coefficient. Throughout this paper, we assume that $0 < \gamma < 1$[4]. We can interpret $\mathbf{a}_\epsilon$ as *the home currency compensation coefficient against the exchange rate risk* since it devaluates the mean of the exchange rate of one unit of foreign currency corrected in terms of the relative risk aversion coefficient of the head office of the affiliate firm.

We assume the following:

$$\mathbf{a}_\epsilon = \mu_{\tilde{\epsilon}} - \gamma\sigma_{\tilde{\epsilon}} \ > 0. \tag{10}$$

Hereafter, we assume the following:

$$c^H > \mathbf{a}_\epsilon c^F. \tag{11}$$

[5]

[4]This is a representative CRRA (constant relative risk-averse) utility function of wealth: w is $u(w) = w^{1-\gamma}$ for $0 < \gamma < 1$. We can ascertain that $u(w)$ is a concave increasing function in w for $0 < \gamma < 1$, and the relative risk averse measure RRA=-$\frac{u''(w)w}{u'(w)} = \gamma$. If $\gamma > 1$, then a representative CRRA utility function of w is $u(w) = w_0 - w^{-(\gamma-1)}$ for $\gamma > 1$, where w_0 is the initial wealth. We also ascertain that $u(w)$ is a concave increasing function in $w^{-(\gamma-1)}$ for $\gamma > 1$, and the relative risk averse measure RRA=-$\frac{u''(w)w}{u'(w)} = \gamma$. However, an organization or individual with this type of utility function is a public utility foundation or a person living on unearned income. Here, we assume that a parent private IH firm and its affiliate firm are constantly relative risk averse; thus, it seems to be natural that they have the following CRRA utility function: $u(w) = w^{1-\gamma}$ for $0 < \gamma < 1$.

[5]This condition (11) seems to be curious for readers. However, we would like to remind the reader that all parts or intermediates imported from home country are essential for affiliate firms in the foreign country to assemble their final products in the foreign country under the exchange rate risk. Hence, we assume that the affiliates have to procure these parts from the home country, although their procurement costs through importation are

From (3), this assumption is equivalent to the following:

$$0 < \gamma < \min\{(e^{\sigma^2} - 1)^{-1/2} \equiv \overline{\gamma}(\sigma^2), 1\}, \tag{12}$$

since $\mathbf{a}_\epsilon = \mu_{\tilde{\epsilon}} - \gamma\sigma_{\tilde{\epsilon}} = \mu_{\tilde{\epsilon}}(1 - \gamma(e^{\sigma^2} - 1)^{1/2})$[6]. Then, (11) can be written by the following:

$$c^H > \mathbf{a}_\epsilon c^F = \mu_{\tilde{\epsilon}}(1 - \gamma(e^{\sigma^2} - 1)^{1/2})c^F > 0. \tag{13}$$

Note that the assumption given by inequality (11) implies that the marginal cost (or unit cost of inputs) of home country firms is higher than the marginal cost (or unit cost of inputs) of foreign country firms compensated by *the home currency compensation coefficient against the exchange rate risk.*

3 Derivation of an Equilibrium

In this subsection, at first, we derive a Cournot game equilibrium in a foreign market where the free entry or exit of affiliate firms in not available according to some regulation such as the foreign direct investment control of a foreign country government, and the number of affiliates in the foreign market is exogenously given. We also examine the properties of the equilibrium outcome.

more expensive than the costs of those parts made in the foreign country.

[6]Note that the upper bound of γ is obtained by the assumption (**??**) that guarantees $\mathbf{a}_\epsilon > 0$ for $0.0095 < \sigma^2 < \ln 2$. $0 < \overline{\gamma}(\sigma^2) < 10.23$, since $\overline{\gamma}(\sigma^2)$ is decreasing in σ^2 and $\sigma^2 > 0.0095$, as we will show in the preceding section. However, we assume that $0 < \gamma < 1$ as the parent IH firm and its affiliate firm have a constant relative risk-averse utility in footnote 6.

Then, A firm i chooses q_i^A to maximize the certainty equivalent of *ex-ante* expected profit $E_{\widetilde{\epsilon}}\left[CE\pi_i^A\right]$. The first-order condition for q_i^A of A firm i is given by the following:

$$\frac{\partial E_{\widetilde{\epsilon}}\left[CE\pi_i^A\right]}{\partial q_i^A} = \mathbf{a}_\epsilon\{a^F - Q^A - Q^F - q_i^A - (1-\gamma)c^H\} = 0,\ i = 1, ..., n. \quad (14)$$

The profit of foreign firm j is defined by the following:

$$\pi_j^F = (p^F - c^F)q_i^F. \quad (15)$$

From (6) and (15), the first-order condition for q_j^F of foreign firm j is given by the following:

$$\frac{\partial \pi_j^F}{\partial q_j^F} = a^F - Q^A - Q^F - c^F - q_j^F = 0,\ j = 1, ..., m. \quad (16)$$

Summing (14) and (16) on i and j, respectively, we obtain the following:

$$(1-s)\mathbf{a}_\epsilon\{na^F - (n+1)Q^A - nQ^F\} - (1-s)n(1-\gamma)c^H = 0$$

and

$$ma^F - mQ^A - (m+1)Q^F - mc^F = 0.$$

Solving the above two equations with respect to Q^A and Q^F, we obtain the following:

$$Q^{*A} = \frac{n}{m+n+1}\{a^F + mc^F - (m+1)(1-\gamma)c^H/\mathbf{a}_\epsilon\} \tag{17}$$

and

$$Q^{*F} = \frac{m}{m+n+1}\{a^F - (n+1)c^F + n(1-\gamma)c^H/\mathbf{a}_\epsilon\}. \tag{18}$$

Since q_i^A and q_j^F are symmetrical in $i = 1, ..., n$ and $j = 1, ..., m$, respectively, from (17) and (18), we obtain the following:

$$q^{*A} \equiv q_i^{*A} = \frac{1}{m+n+1}\left\{a^F + mc^F - (m+1)(1-\gamma)c^H/\mathbf{a}_\epsilon\right\} \tag{19}$$

and

$$q^{*F} \equiv q_j^{*F} = \frac{1}{m+n+1}\{a^F - (n+1)c^F + n(1-\gamma)c^H/\mathbf{a}_\epsilon\}. \tag{20}$$

From (19) and (20), we have the following:

$$q^{*A} + (1-\gamma)c^H/\mathbf{a}_\epsilon - c^F = q^{*F}. \tag{21}$$

Substituting (17) and (18) into (7), we obtain equilibrium prices on the foreign market as follows:

$$p^{*F} = \frac{1}{m+n+1}\{a^F + mc^F + n(1-\gamma)c^H/\mathbf{a}_\epsilon\}. \tag{22}$$

From (19) (22) and (9), the certainty equivalent of the expected profit of affiliate firm i in the foreign market and IH firm i in the home country at equilibrium are given by the following:

$$
\begin{aligned}
E_{\widetilde{\epsilon}}\left[CE\pi_i^{*A}\right] &= E_{\widetilde{\epsilon}}\left[\widetilde{\varepsilon}(p^{*F}-\widetilde{c}^{A})q_i^{*A}\right]-\gamma SD_{\widetilde{\epsilon}}\left[\widetilde{\varepsilon}(p^{*F}-\widetilde{c}^{A})q_i^{*A}\right] \\
&= \mathbf{a}_{\epsilon}\left(q^{*A}\right)^2, \qquad (23)
\end{aligned}
$$

and

$$
E_{\widetilde{\epsilon}}\left[CE\pi_i^{*IH}\right]=(1-s)E_{\widetilde{\epsilon}}\left[CE\pi_i^{*A}\right], \qquad (24)
$$

where $\mathbf{a}_{\epsilon}=\mu_{\widetilde{\epsilon}}-\gamma\sigma_{\widetilde{\epsilon}}$, $\sigma_{\widetilde{\epsilon}}$ and γ stand for the standard deviation of $\widetilde{\epsilon}$ and the relative risk aversion coefficient, respectively.

The *ex-post* expected profit *evaluated home currency* of affiliate firm i in the foreign market and *IH* firm i in the home country market at the short-run equilibrium are as follows:

$$
\begin{aligned}
E_{\widetilde{\epsilon}}\left[\pi_{iHC}^{*A}\right] &= E_{\widetilde{\epsilon}}\left[\widetilde{\varepsilon}(p^{*F}-\widetilde{c}^{A})q_i^{*A}\right] \\
&= E_{\widetilde{\epsilon}}\left[\widetilde{\varepsilon}(a^F-Q^{*A}-Q^{*F}-c^H/\widetilde{\epsilon})q_i^{*A}\right] \\
&= [\frac{\mu_{\epsilon}}{m+n+1}(a^F+mc^F+n(1-\gamma)c^H/\mathbf{a}_{\epsilon})-c^H]\times q^{*A}, \qquad (25)
\end{aligned}
$$

and

$$
E_{\widetilde{\epsilon}}\left[\pi_{iHC}^{*IH}\right]=(1-s)E_{\widetilde{\epsilon}}\left[\pi_{iHC}^{*A}\right]. \qquad (26)
$$

To compare the *ex-post* expected profit of foreign firm j at the short-run equilibrium, we have to derive the *ex-post* expected profit *evaluated by foreign currency* of the affiliate firm i in the foreign market. This is given by the following:

$$
\begin{aligned}
E_{\widetilde{\epsilon}}\left[\pi^{*A}_{iFC}\right] &= E_{\widetilde{\epsilon}}\left[(p^{*F} - \widetilde{c}^{A})q^{*A}_{i}\right] \\
&= (p^{*F} - c^{H}E_{\widetilde{\epsilon}}[1/\widetilde{\epsilon}])q^{*A}_{i} \\
&= (a^{F} - Q^{*A} - Q^{*F} - c^{H}\cdot\exp(-\mu + \sigma^{2}/2))q^{*A}_{i} \qquad (27)
\end{aligned}
$$

where the third line holds from the (5).

The *ex-post* expected profit of foreign firm j in the foreign country market at the short-run equilibrium is given by the following:

$$\pi^{*F}_{j} = (p^{*F} - c^{F})q^{*F}_{j} = (q^{*F}_{j})^{2}. \qquad (28)$$

$$
\begin{aligned}
E_{\widetilde{\epsilon}}\left[\pi^{*F}_{jHC}\right] &= E_{\widetilde{\epsilon}}\left[(p^{*F}-c^{F})q^{*F}_{j}\right] \\
&\quad = E_{\widetilde{\epsilon}}\left[(a^{F}-Q^{*A}-Q^{*F}-c^{F})q^{*F}_{j}\right] \\
&= E_{\widetilde{\epsilon}}\left[(q^{*F}_{j})^{2}\right] \\
&= E_{\widetilde{\epsilon}}[\frac{1}{(m+n+1)^{2}}\{a^{F}-(n+1)\widetilde{\varepsilon}c^{F}+n(1-\gamma)c^{H}/\mathbf{a}_{\epsilon}\}^{2}] \\
&= \frac{1}{(m+n+1)^{2}}E_{\widetilde{\epsilon}}[\{a^{F}+n(1-\gamma)c^{H}/\mathbf{a}_{\epsilon}\}^{2}-2(n+1)c^{F}\{a^{F}+n(1-\gamma)c^{H}/\mathbf{a}_{\epsilon}\}\widetilde{\varepsilon} \\
&\quad +(n+1)^{2}\widetilde{\varepsilon}^{2}(c^{F})^{2}] \\
&= \frac{1}{(m+n+1)^{2}}[\{a^{F}+n(1-\gamma)c^{H}/\mathbf{a}_{\epsilon}\}^{2}-2(n+1)c^{F}\{a^{F}+n(1-\gamma)c^{H}/\mathbf{a}_{\epsilon}\}\mu_{\widetilde{\epsilon}} \\
&\quad +(n+1)^{2}(c^{F})^{2}\mu^{2}_{\widetilde{\epsilon}}\cdot e^{\sigma^{2}}]]
\end{aligned} \tag{29}
$$

Proof of these propositions is provided in the Appendix.

Proposition 1 *Suppose that* $c^H > \mu_{\widetilde{\epsilon}}c^F$ *and* $0.0095 < \sigma^2 < \ln 2 = 0.693\,15$. *If* $0 < \gamma \leq \frac{c^H-\mu_{\widetilde{\epsilon}}c^F}{c^H-(e^{\sigma^2}-1)^{1/2}\mu_{\widetilde{\epsilon}}c^F}$ $(\frac{c^H-\mu_{\widetilde{\epsilon}}c^F}{c^H-(e^{\sigma^2}-1)^{1/2}\mu_{\widetilde{\epsilon}}c^F} < \gamma < 1)$. *Then the equilibrium output of the affiliate firm is less than or equal* (*greater than*) *the equilibrium output of the foreign firm, i.e.,* $q^{*A} \leq (>)\ q^{*F}$.

As the extent of the relative risk aversion for the volatility of an exchange rate γ is small, the numerator of $(1-\gamma)c^H/\mathbf{a}_\epsilon$ is large when the exchange risk σ^2 ($\sigma^2 < \ln 2 \approx 0.693\,15$) is so small since $\mathbf{a}_\epsilon = \mu_{\widetilde{\epsilon}}(1-\gamma(e^{\sigma^2}-1)^{1/2})$. The denominator of $(1-\gamma)c^H/\mathbf{a}_\epsilon$ can approximate $c^H/\mu_{\widetilde{\epsilon}}$; accordingly, the output of the foreign firm *F*, q^{*F} is large when c^H is large under the assumption (11). Then, the output of affiliate firm *A*, q^{*F} is small according to the strategic substitute property results shown in $q^{*A} \leq q^{*F}$.

Proposition 2 *Suppose that* $c^H > \mu_{\tilde{\epsilon}} c^F$ *and* $0.0095 < \sigma^2 < \ln 2 = 0.693\,15$, $a^F + mc^F - 9.235(m+n+1)c^H/\mathbf{a}_\epsilon$ *and* $0 < \gamma < 1$. *Then, the equilibrium expected certainty equivalence of the affiliate firm is less than the equilibrium expected profit of the affiliate firm evaluated by the home currency, i.e.,* $E_{\tilde{\epsilon}}\left[CE\pi_i^{*A}\right] < E_{\tilde{\epsilon}}\left[\pi_{iHC}^{*A}\right]$.

From (23), the equilibrium expected certain equivalence of the affiliate firm $E_{\tilde{\epsilon}}\left[CE\pi_i^{*A}\right]$ is the expected profit of the affiliate $E_{\tilde{\epsilon}}\left[\pi_{iHC}^{*A}\right]$ devaluated by $\mathbf{a}_\epsilon$, which is the mean of the exchange rate of one unit of foreign currency corrected in terms of the relative risk aversion coefficient of the parent firm in the home country. Therefore, the results of this proposition hold.

Proposition 3 *Suppose that* $c^H - \mu_{\tilde{\epsilon}} c^F > c^H - 2\mu_{\tilde{\epsilon}} c^F \geq 0$. If $0 < \gamma < 1$, *then the equilibrium expected profit of the foreign firm is greater than the equilibrium ex-post expected profit of the affiliate firm evaluated by foreign currency* , i.e., $\pi^{*F} > E_{\tilde{\epsilon}}\left[\pi_{iFC}^{*A}\right]$.

Based on Proposition 1, at equilibrium, the foreign firm can be expected ex post to earn more than the affiliate firm being evaluated by foreign currency. Combining Propositions 1, 2 and 3, we obtain the following proposition:

Proposition 4 *Suppose that* $c^H - \mu_{\tilde{\epsilon}} c^F > c^H - 2\mu_{\tilde{\epsilon}} c^F \geq 0$. If $0 < \gamma < 1$, *then the equilibrium expected profit of the foreign firm evaluated by home currency is also greater than the equilibrium ex-post expected profit of the affiliate firm evaluated by home currency* , *and the equilibrium expected* *certainty equivalence of the affiliate firm is the smallest, i.e., ,* $E_{\widetilde{\epsilon}}\left[CE\pi_i^{*A}\right] < E_{\widetilde{\epsilon}}\left[\pi_{iHC}^{*A}\right] < E_{\widetilde{\epsilon}}\left[\pi_{jHC}^{*F}\right]$.

4 Effect of the Change in the Exchange Rate Risk on Equilibrium Outcomes

In the following, we explore how changes in the volatility of the exchange rate have an effect on the equilibrium outcome.

We posit a lemma before presenting the results. From the (2) and (3), we have the following:

$\frac{\partial \mu_{\widetilde{\epsilon}}}{\partial \sigma^2} = \exp(\mu + \sigma^2/2)/2 = \mu_{\widetilde{\epsilon}}/2 > 0$

$\frac{\partial \sigma^2_{\widetilde{\epsilon}}}{\partial \sigma^2} = e^{\sigma^2} \exp(2\mu+\sigma^2) + (e^{\sigma^2}-1)\exp(2\mu+\sigma^2) = (2e^{\sigma^2}-1)\exp(2\mu+\sigma^2) > 0.$

Hence, when $\widetilde{\epsilon}$ is a log-normally distributed random variable, that is, $\widetilde{\epsilon} = \exp(\widetilde{X})$, $\widetilde{X} \sim N(\mu, \sigma^2)$,

then we see that $\frac{\partial \mu_{\widetilde{\epsilon}}}{\partial \sigma^2} > 0$, $\frac{\partial \sigma^2_{\widetilde{\epsilon}}}{\partial \sigma^2} > 0$. In other words, an increase in the exchange rate risk increases the mean and variance of the exchange rate.

We also have the following:

$\frac{\partial \mathbf{a}_{\epsilon}}{\partial \sigma^2} = \mu_{\widetilde{\epsilon}}(1 - \gamma(2e^{\sigma^2} - 1)(e^{\sigma^2} - 1)^{-1/2})/2 \gtreqqless 0 \Longleftrightarrow \gamma \lesseqqgtr (e^{\sigma^2} - 1)^{1/2}/(2e^{\sigma^2} - 1) \equiv \gamma^*(\sigma^2)$.

From (10), we can easily derive the next lemma without proof.

Lemma 1 *For any* $0.0095 < \sigma^2 < \ln 2$, *if* $1 > \gamma \geq \gamma^*(\sigma^2)$, *then* $\frac{\partial}{\partial \sigma^2}\mathbf{a}_\epsilon \leq 0$. *If* $0 < \gamma < \gamma^*(\sigma^2)$, *then* $\frac{\partial}{\partial \sigma^2}\mathbf{a}_\epsilon > 0$.

From Lemma 1, an increase in the exchange rate risk increases (decreases) the home currency compensation coefficient against the exchange rate risk $\mathbf{a}_\epsilon$ when the relative risk averse coefficient γ is small (large).

Denoted by $\overline{\gamma}(\sigma^2) \equiv (e^{\sigma^2} - 1)^{-1/2}$, which is the upper bound of γ given by the assumption (**??**), we can easily show that $\gamma^*(\sigma^2) < 1 < \overline{\gamma}(\sigma^2) \equiv (e^{\sigma^2} - 1)^{-1/2} < \overline{\gamma}(0.0095) \approx 10.23$ for any σ^2 such that $\ln 2 > \sigma^2 > 0.0095$, where the last inequality is from (**??**).

Thus, we see the following:

$$
\begin{aligned}
\frac{\partial}{\partial \sigma^2}\gamma^*(\sigma^2) &= \frac{e^{\sigma^2}(3 - 2e^{\sigma^2})}{2(2e^{\sigma^2} - 1)^2(e^{\sigma^2} - 1)^{1/2}} \gtreqless 0 \Leftrightarrow \\
\sigma^2 &\lesseqgtr \ln 3 - \ln 2 = 0.405\,47 \text{ for } \ln 2 > \sigma^2 > 0.0095,
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial}{\partial \sigma^2}\overline{\gamma}(\sigma^2) &= \frac{\partial}{\partial \sigma^2}((e^{\sigma^2} - 1)^{-1/2}) = -\frac{1}{2}\frac{e^{\sigma^2}}{(e^{\sigma^2} - 1)^{\frac{3}{2}}} < 0, \\
\text{for } \ln 2 &> \sigma^2 > 0.0095.
\end{aligned}
$$

Therefore, the upper bound of γ, $\overline{\gamma}(\sigma^2)$ that guarantees $\mathbf{a}_\epsilon > 0$ does not affect the optimal choice of the output q^{*A}; only $\gamma^*(\sigma^2)$ affects it since we assume that $0 < \gamma < 1$.

Now, we present the next lemma.

Lemma 2 *The upper bound of* γ*,* $\overline{\gamma}(\sigma^2)$ *is decreasing in the exchange risk* σ^2*, but* $\gamma^*(\sigma^2)$*, the threshold for the direction of the change in the home currency compensation coefficient against the exchange rate risk* $\mathbf{a}_\epsilon$ *in Lemma 1, is increasing/decreasing in the exchange risk* σ^2 when $0.0095 < \sigma^2 < \ln 3 - \ln 2 \approx 0.405\,47$ / $\ln 3 - \ln 2 < \sigma^2 < \ln 2$.

For these properties of $\gamma^*(\sigma^2)$ and $\overline{\gamma}(\sigma^2)$ in Lemma 2, see Figure 2.

[Insert here Figure 2]

Note that the difference between $\overline{\gamma}(\sigma^2)$ and $\gamma^*(\sigma^2)$ becomes narrow as the exchange rate risk σ^2 increases, as shown in Figure 2.

The firm with a very small relative risk for the exchange rate risk does not estimate the home currency compensation coefficient against the exchange risk as being large, while the firm with a large relative risk estimate does so when the exchange rate risk becomes large enough. Considering the results of these two lemma together, we can conclude that an increase in the exchange risk causes the extent of the risk aversion of the firm to be strong and then makes the firm's estimate of the home currency compensation coefficient against the exchange risk shrink.

By using Lemma 1, we can conduct a comparative statics analysis of the equilibrium outputs, the expected profits of firms, the expected certain equivalence of profits and the prices based on the volatility of the exchange rate σ^2.

Next, we conduct a comparative analysis on the equilibrium outcome derived above, namely, the volatility exchange rate σ^2. We begin with a comparative statics analysis on exchange rate volatility σ^2. We present the next proposition. For the proof, see the Appendix.

Proposition 5

Suppose that the exchange risk is distributed within a smaller range; Thus, $0.0095 < \sigma^2 < \ln 2 \approx 0.693\,15$.

If the parent IH firm is weakly risk averse, i.e., $0 < \gamma \leq \gamma^*(\sigma^2) < 1 < \overline{\gamma}(\sigma^2)$ */ strongly risk averse, i.e.,* $0 < \gamma^*(\sigma^2) < \gamma < 1 \leq \overline{\gamma}(\sigma^2)$*, then the equilibrium output of affiliate A firm* q^{*A} *and the total output of affiliate firms* Q^{*A} *nondecrease / decrease, as the volatility of the exchange rate* σ^2 *increases.*

If the parent IH firm is weakly risk averse, i.e., $0 < \gamma \leq \gamma^*(\sigma^2) < 1 < \overline{\gamma}(\sigma^2)$ */ strongly risk averse, i.e.,* $0 < \gamma^*(\sigma^2) < \gamma < 1 < \overline{\gamma}(\sigma^2)$*, then the equilibrium output of the foreign F firm* q^{*F}*, the total output of foreign firms* Q^{*F}*, and the equilibrium price in the foreign market* p^{*F} *do not increase/increase, as the volatility of the exchange rate* σ^2 *increases.*

From the above proposition, when the relative risk aversion coefficient is smaller than a threshold $\gamma^*(\sigma^2)$, we can see that if the volatility of the exchange rate increases, then each A firm aggressively expands its output into a foreign market. Therefore, Q^{*A} increases enough that the equilibrium price decreases if foreign F firms decrease their outputs q^{*F} to mitigate the decrease in price, since the number of A firms n is given in this case. Hence, Q^{*F} decreases since they are strategic substitutes in the Cournot competition.

As the increase effect of Q^{*A} surpasses that of the decrease effect of Q^{*F}, the total equilibrium output $Q^{*A} + Q^{*F}$ increases and p^{*F} decreases.

In the following, we examine how the change in the exchange rate risk affects the *ex-post* expected profit of firms and certainty equivalence of the expectations of firms.

We thus present the next proposition. Proof of the proposition is provided in the Appendix.

Proposition 6

Suppose that the exchange risk is distributed within smaller range, thus $0.0095 < \sigma^2 < \ln 2 \approx 0.693\,15$, *and the number of foreign firms is relatively less than that of the affiliate firms,* $m + 1 < n$ *and* $a^F + mc^H > (m + 1)(1 - \gamma)c^H/\mathbf{a}_\epsilon$. *If the relative risk averse coefficient* γ *of the parent IH firm is large, i.e.,* $\gamma^*(\sigma^2) < \gamma < 1$, *then the equilibrium ex-post expected profits of affiliate firm A and parent firm IH* $E_{\widetilde{\epsilon}}\left[\pi_i^{*IH}\right]$ *and* $E_{\widetilde{\epsilon}}\left[\pi_i^{*A}\right]$ *increase as the volatility of the exchange rate* σ^2 *increases. However, if the relative risk averse coefficient* γ *of the parent IH firm is small, i.e.,* $0 < \gamma \leq \gamma^*(\sigma^2) < 1$, *then how the equilibrium ex-post expected profits of affiliate firm A and parent firm IH* $E_{\widetilde{\epsilon}}\left[\pi_i^{*IH}\right]$ *and* $E_{\widetilde{\epsilon}}\left[\pi_i^{*A}\right]$ *change may be indeterminate.*

If the relative risk aversion coefficient γ *of the parent IH firm is small, i.e.,* $0 < \gamma < \gamma^*(\sigma^2) < 1$ / *large,* $\gamma^*(\sigma^2) < \gamma < 1$, *then the equilibrium certainty equivalence of their expected profits* $E_{\widetilde{\epsilon}}\left[CE\pi_i^{*IH}\right]$ and $E_{\widetilde{\epsilon}}\left[CE\pi_i^{*A}\right]$ *increase / decrease as the volatility of the exchange rate* σ^2 *increases. If the relative risk aversion coefficient* γ *of the parent IH firm is small,* $0 < \gamma \leq \gamma^*(\sigma^2) < 1$, *large,* $\gamma^*(\sigma^2) < \gamma < 1$, *then the equilibrium ex-post expected profit* *of the foreign F firm π^{*F} does not increase / increases as the volatility of the exchange rate σ^2 increases.*

As the volatility of the exchange rate σ^2 increases, then both $E_{\tilde{\epsilon}}\left[\pi_i^{*IH}\right]$ *and* $E_{\tilde{\epsilon}}\left[\pi_i^{*A}\right]$ increase when the number of foreign firms is relatively less than that of the affiliate firms; i.e., $m+1 < n$ and γ is large, that is, $\gamma^*(\sigma^2) < \gamma < 1$. However, when $m+1 < n$ and γ is small ($0 < \gamma \leq \gamma^*(\sigma^2)$) or when $m+1 \geq n$, whether they increase is indeterminate as σ^2 increases.

Note that the signs for $E_{\tilde{\epsilon}}[CE\pi_i^{*IH}]$ and $E_{\tilde{\epsilon}}\left[CE\pi_i^{*A}\right]$ increase / decrease when the relative risk aversion coefficient γ of the parent IH firm is small $(0 < \gamma \leq \gamma^*(\sigma^2))$/large ($\gamma^*(\sigma^2) < \gamma < 1$) in the equilibrium. The ex-post expected profit of foreign firms does not increase/increases when the relative risk-averse coefficient γ of the parent IH firm is small, i.e., $0 < \gamma \leq \gamma^*(\sigma^2) < 1$/ large, i.e., $\gamma^*(\sigma^2) < \gamma < 1 < \overline{\gamma}(\sigma^2)$, as the volatility of the exchange rate σ^2 increases.

5 Conclusion

We consider an international oligopoly model given the oligopolistic market in a foreign country. The international home parent firms (*IH* firms) compete in a host foreign country oligopolistic market through their affiliates by *FDI*. Furthermore, we assume that an A firm (affiliate firm) i internally reserves its profit at the foreign market equilibrium for the ratio of s $(0 < s < 1)$ and remits its profit for the ratio of $1 - s$ from the foreign market to the head office of its parent firm *IH* in the home country. We derive a Cournot equilibrium under foreign exchange rate uncertainty for when the number of

IH firms, n, is exogenously given. Then, we explore the effects of foreign exchange rate volatility on the equilibrium outcome.

In the equilibrium, we show that if the measure of the relative risk aversion of *IH* firms is small, then affiliate firms aggressively expand their outputs, while foreign firms defensively decrease their outputs given the equilibrium price in the foreign market. However, if γ is small, then *IH* firms are severely reluctant to expand in response to the increase in the exchange rate risk.

The *ex ante* certainty equivalent of the expected profits of the affiliate firm and the parent firm increase/decrease, but the *ex-post* expected profits of the foreign firm do not increase/increase with the volatility of the exchange rate when the relative risk aversion coefficient is small/large at equilibrium. However, the ex-post expected profits of the affiliate and parent firms increase when the number of the affiliate firms is relatively larger than that of the foreign firms, and γ is large as the volatility of the exchange rate increases. However, the effect of the volatility of the exchange rate on them in other cases is indeterminate.

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6 References

Brander, J. and B. Spencer (1987),"Foreign direct investment with unemployment and endogenous taxes," *Journal of International Economics*, vol. 22, issue 3-4, 257-279.

Hasegawa, M. and K. Kiyota (2017), "The Effect of Moving to a Territorial Tax Systems on Profit Repatriation: Evidence from Japan," *Journal of Public Economics*, Vol. 153, pp.92-110.

Japanese External Trade Organization (JETRO), FY2022 'Survey on Business Conditions of Japanese Affiliated Companies Overseas: Global Edition,' https://www.jetro.go.jp/ext_images/en/reports/survey/pdf/2022/rp_global2022.pdf

Lahiri, S. and F. Mesa "Local content requirement on foreign direct investment under exchange rate volatility," *International Review of Economics and Finance*, Vol. 15 (2006), pp346-363.

Sung, H. and H. E. Lapan, "Strategic Foreign Direct Investment and Exchange-rate Uncertainty," *International Economic Review*, Vol. 41, (2000) (2), pp.411-423.

Todo, Y. (2022), 'Resilient and Innovative Supply Chains: Evidence-based policy

and managerial implications,' RIETI Policy Discussion Paper Series 22-P-024, https://www.rieti.go.jp/jp/publications/pdp/22p024.pdf

Inoue H. and Y. Todo (2023), " Propagation of Overseas Economic Shocks through Global Supply Chains: Firm-level Evidence (Revised),' RIETI Discussion Paper Series 22-E-062, https://www.rieti.go.jp/jp/publications/dp/22e062.pdf

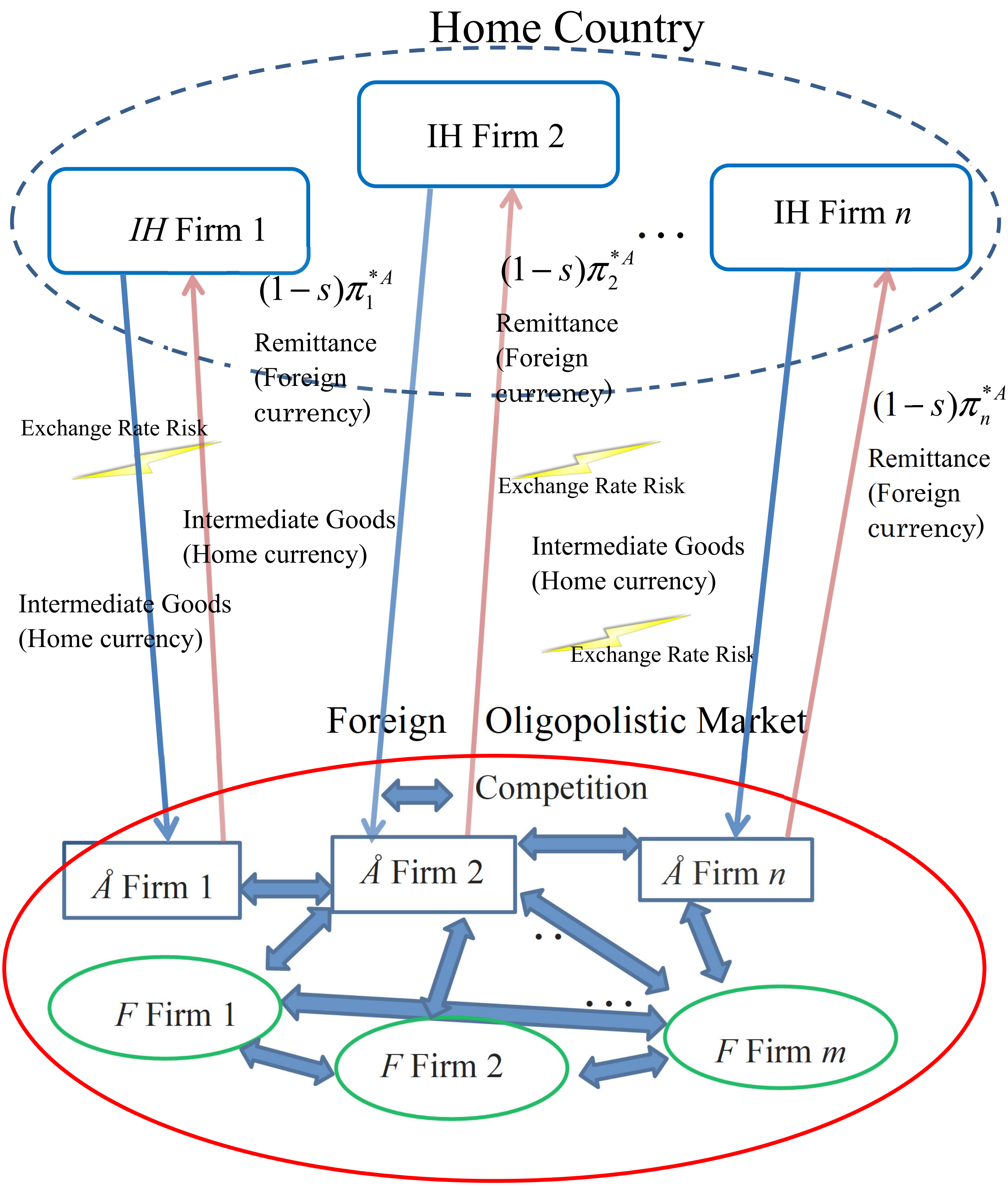
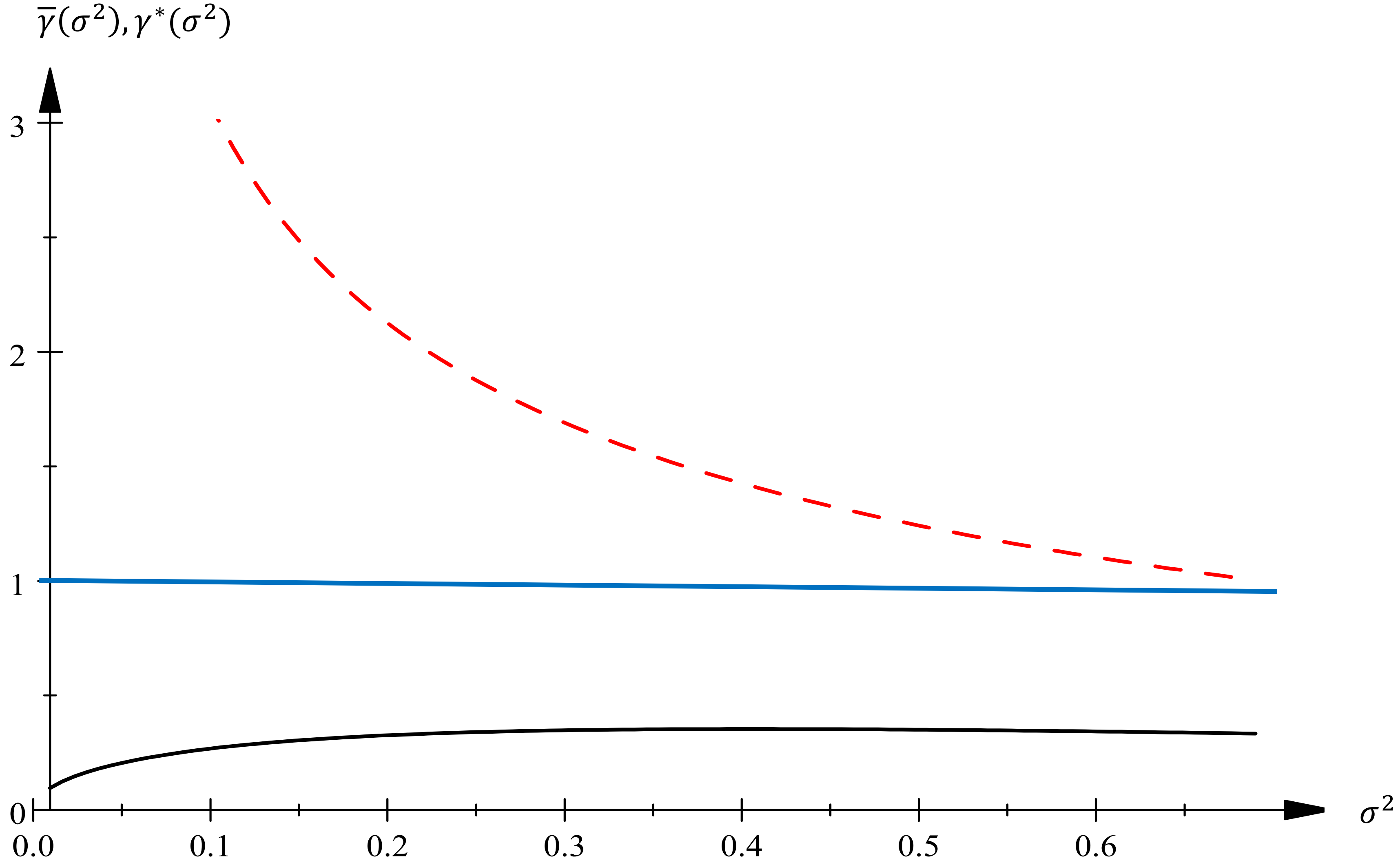


Figure 1 Oligopolistic Competition among Affiliate and Foreign firms in a Foreign Market



$\gamma^*(\sigma)$ Black solid line, $\gamma^*(\sigma) = (exp(\sigma) - 1)^{1/2}/(2\,exp(\sigma) - 1)$, $\overline{\gamma}(\sigma) = (exp(\sigma) - 1)^{-1/2}$ Dotted Light Red line,

Figure 2 $\overline{\gamma}(\sigma^2), \gamma^*(\sigma^2)$