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Redistributive Policy and R&D-based Growth

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Abstract

This study examines how redistributive policy attempting to reduce inequality by taxing the bequests of the rich and redistributing the revenue to the poor affects economic growth in an overlapping generations model of R&D-based growth with both product development and process innovation. We show that such a policy simultaneously increases growth and reduces inequality in the long run. When the market structure adjusts, partially reducing inequality in the short run, the effect of redistributive policy on economic growth depends on the values of the social return to variety parameter. However, when the market structure adjusts fully in the long run, the redistributive policy decreases the entry of new firms but raises economic growth and reduces inequality. These favorable predictions of redistributive bequest taxation on growth and inequality are partly consistent with the empirical findings that redistribution is generally benign in terms of economic growth and that lower post-transfer inequality is correlated with faster and more durable growth.

Keywords: R&D, Product Development, Process Innovation
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1 Introduction

In recent years, there has been a growing interest in the links between rising inequality and economic growth. While the empirical literature seems to have converged toward a tentative consensus that inequality is generally harmful to growth, at least over the medium-term (e.g., Persson and Tabellini 1994; Easterly 2007; Hatler et al.2014; Berg et al. 2018), the policy implications are far from clear. The traditional argument that redistribution in the form of higher taxes and subsidies reduces incentives to work and invest has been emphasized in Alesina and Rodrick's (1994) and Persson and Taballini's (1994) models. Even if inequality is bad for growth, considering such a tradeoff between redistribution and growth, taxes and transfer may be worse for growth than the inequality itself.

However, several authors have recognized that redistribution need not be inherently bad for growth. For example, in the presence of imperfect credit and insurance markets, redistribution can be growth-enhancing through an "opportunity creation effect" that allows more individuals to invest in education, as in Galor and Zeira (1993), or through an "insurance effect" that provides a certain degree of insurance to entrepreneurs and stimulates research and growth, as in García-Peñelosa and Wen (2008). Moreover, the progressive taxation that is used to finance public education and public health expenditures may increase the return on the private educational investments of the poor, increase the average level of education in an economy, and promote growth (e.g., Saint Paul and Verdier, 1993, 1997; Benabou 2000).

The evidence of the relationship between redistribution and growth is not clear-cut. Studies that look at presumptive indicators of redistribution, such as taxes or government spending, tend to suggest that more redistribution is detrimental to growth. However, surprisingly little evidence shows that increases in tax rates impede medium to long-run economic growth (e.g., Tanzi and Zee 1997; Jaimovich and Rebelo 2012). With respect to spending, many papers find that some categories of public spending that are retributive (e.g., public spending on health, education, and infrastructure) have no apparent adverse impact on growth or are positively related to growth (e.g., Benabou 2000; Bleaney et al. 2001; Lindert 2004).

Using a recently complied data set that clearly distinguishes between market (pre-tax and transfer) and net (post-tax and transfer) inequality, Ostry et al. (2014) and Berg et al. (2018) examine the role of both redistribution and inequality in growth using a common empirical framework. Across various estimation methods, data samples, and robustness checks, they find that redistribution is generally benign in terms of economic growth, and the lower post-tax and transfer inequality is correlated with faster and more durable growth. Their results indicate that fiscal redistribution, unless it is extreme, may be both pro-growth and pro-equality via equity-inducing effects.

Motivated by these recent developments in theoretical analyses and empirical findings, we examine, in a theoretical model, how redistributive policy that attempts to reduce inequality by taxing the bequests of the rich and redistributing the revenue to the poor affects economic growth in an overlapping generations model of R&D-based growth with both product development and process innovation. Technological progress via R&D innovation has been identified as the primary driving force of modern economic growth (e.g., Romer 1990). However, to the best of my knowledge, only a few existing studies examine the effect of redistributive policy on economic growth in an R&D-based growth framework. In line with the literature of the second-generation R&D-based growth model pioneered by Peretto (1998), Segestrom (1998), and Howitt (1999), the model developed in this paper features two dimensions of technological progress. In the vertical dimension, incumbent production firms invest in process innovation to improve the quality of their product. In the horizontal dimension, the product development sector creates new product designs for firms entering the production sector.

In this Schumpeterian growth model with an endogenous market structure (EMS) measured by the equilibrium number of firms, we examine the effect of the redistributive policy on growth and inequality. Then, we show that such a redistributive policy simultaneously raises growth and reduces inequality in the long run. When the market structure adjusts partially in the short run, the effect of redistributive policy on economic growth depends on the values of the social return to variety parameter, while it reduces inequality. However, when the market structure adjusts fully in the long run, the redistributive policy decreases the number of firms but raises economic growth and reduces inequality. Thus, redistributive bequest taxation is perhaps appropriate for stimulating long-run economic growth and reducing inequality. By employing the second-generation R&D-based growth model, this paper sheds light on the new mechanism that the redistributive policy simultaneously raises growth and reduces inequality.

The intuition behind our theoretical results is explained as follows. The redistributive bequest taxation increases the income of the poor but decreases the income of the rich, which lowers inequality. Since the rich save a larger proportion of their income than the poor do, such a redistributive policy decreases the economy's aggregate savings, decreasing the demand for a share of each production firm. These factors decrease the market value of blueprints of a new variety and the equilibrium entry of new firms, negatively affecting the per capita output growth rate in the short run. In the long run, however, the decline in the entry of new firms increases the market size of each production firm. Given that the market size of a production firm determines its incentives for process innovation, the higher redistributive bequest tax rate increases long-run economic growth.[1] These favorable predictions for redistributive bequest taxation on growth and inequality are partly consistent with the empirical findings that redistribution is generally benign in terms of economic growth and that lower post-tax and transfer inequality is correlated with faster and more durable growth.

The counterintuitive long-run positive effect of redistributive bequest taxation on economic growth is based on the feature of the second-generation R&D-based growth model. In the conventional second-generation R&D-based growth model, the research productivity of variety R&D is specified to preclude the counterfactual scale effect prediction of economic growth. This specification links the steady-state number of firms to the market size (i.e., the labor size) of the economy in an intuitive manner, making it so only the vertical dimension of technological progress (i.e., process innovation) works as a plausible engine of economic growth in the long run. [2] This feature plays a key role in deriving our counterintuitive positive effect of redistributive bequest taxation on economic growth.

This paper is closely related to the following two branches of the literature. First, this paper is related to a few pioneering theoretical studies that analyze the effects of redistributive policy on growth and inequality in an R&D-based growth model (e.g., Chou and Talmain., 1996; Caucutt et al. 2003, 2006; García-Peñelosa and Wen, 2008). Chou and Talmain (1996) incorporate heterogeneous households with different initial assets into the variety expanding growth model and show that when the labor Engel curve is concave, the more equal wealth distribution raises the aggregate labor supply and the growth rate. Caucutt et al. (2003, 2006) develop a numerical simulation model of endogenous technical change where the stock of skilled labor matters for growth and shows that an increase in tax progressivity may increase the long-run skill premium, decrease the upward mobility of the poor, and decrease the human capital investment of both skilled and unskilled individuals. García-Peñelosa and Wen (2008) examine the effects of redistributive taxation on growth and inequality in a Schumpeterian model with risk-averse agents and show that redistribution provides insurance to entrepreneurs and increases the growth rate. In contrast to these studies, we employ a Schumpeterian growth model with EMS (i.e., the second-generation R&D-based growth model). These differences in the modeling strategy of technical change enable us to reveal

[1]Laincz and Peretto (2006) provide empirical evidence for a positive relationship between the average firm size and economic growth.

[2]The advantage of the second-generation R&D-based growth model is that we can eliminate the well-known undesirable scale effect (e.g., Jones 1995) while keeping the policy effect property supported by the recent growing empirical literature (e.g., Laincz and Peretto, 2006; Ha and Howitt, 2007; Madsen, 2008; Madsen et al., 2010; Ang and Madsen, 2011). The model is also consistent with empirical evidence concerning industrial organization (e.g., Adames and Jaffe, 1996; Pagano and Schivardi, 2003).

different aspects of the relationship between the redistributive policy and economic growth that have yet to be examined in the literature. In this sense, our research complements these existing theoretical analyses.

Second, this paper is related to recent studies of a Schumpeterian growth model with EMS. For example, using the Schumpeterian growth model with EMS, Chu et al. (2016) explore the effect of patent breadth and R&D policy, Chu and Ji (2016) explore the effect of unionization, Ji et al. (2016) explore the effect of monetary policy, and Morimoto and Tabata (2020) explore the effect of higher education subsidy policy. However, to the best of my knowledge, the relationship between redistributive policy and economic growth has yet to be examined rigorously in the literature. Therefore, our research complements these existing studies and tackles an issue that has gone unexplored in the literature.

This paper is organized as follows. Section 2 presents the basic model. Section 3 investigates the dynamic and the steady-state equilibrium properties of the economy. Section 4 employs a numerical analysis to examine the effects of redistributive bequest taxation on economic growth, inequality, and welfare. Finally, section 5 briefly discusses the limitations of our analysis and concludes the paper.

2 Model

This section introduces a two-period OLG model of R&D-based growth with redistributive policy, endogenous productivity growth, and variety expansion. The economy consists of three sectors (i.e., a final goods sector, an intermediate goods sector, and a product development sector). The final goods sector produces homogeneous goods for sales in a perfectly competitive market, with both a variety of imperfectly substitutable intermediate goods and labor as inputs. On the other hand, the intermediate goods sector consists of monopolistically competitive firms that produce differentiated product varieties for firms in the final goods sector. Productivity growth arises as a result of process innovation undertaken by the intermediate-goods firms to improve the quality of their product. Finally, the product development sector creates new product designs for firms entering the intermediate goods sector.

2.1 Individuals

Individuals in this economy live for two periods, young and old, and the cohort born in period t is called generation t.[3] In each period, the size of newly born cohorts is given by L. There are two groups of families, "rich" and "poor," denoted

[3]Some basic settings in our household side of the model are inherited from Bossmann et al. (2007) and Maebayashi and Konishi (2021).

by R and P, respectively. We assume that a constant fraction $\phi \in (0, 1)$ of individuals are the rich and a constant fraction $1 - \phi \in (0, 1)$ of individuals are the poor. Each individual supplies one unit of labor inelastically and earns labor income in their young period. In the old period, they are retired and consume their savings and leave bequests to their children.

The lifetime utility of individual $i \in \{R, P\}$ born in period t is expressed as follows:

$$U_t^i = (1 - \alpha^i) \ln c_{1,t}^i + \alpha^i \left[(1 - \beta) \ln c_{2,t+1}^i + \beta \ln b_{t+1}^i \right], \quad (1)$$

where $c_{1,t}^i$ and $c_{2,t+1}^i$ represent their consumption during their youth and old age, respectively, and b_{t+1}^i represents the bequest passed onto the child, reflecting a joy of giving motives for savings. $\alpha^i \in (0, 1)$ is the intertemporal preference parameter and $\beta \in (0, 1)$ is the relative importance parameter of consumption when old and the bequest is made. We assume that β does not differ between the rich and poor following Bossmann et al. (2007) and Maebayashi and Konishi (2021). In addition, we assume that $\alpha^R > \alpha^P$ based on Becker (1980) and several empirical evidences (e.g., Lawrance, 1991; Harrison et al., 2002). Under this assumption, the rich save a larger proportion of their income than the poor do, which is also supported by recent empirical studies (e.g., Dynan et al, 2004; Bozio et al, 2013; Alan et al, 2014; Néstor, 2017). Moreover, we assume that the wealth endowment of the rich old generation in the initial period is larger than that of the poor old generations (i.e., $s_{-1}^R > s_{-1}^P$). The budget constraints of these individuals are expressed as follows:

$$c_{1,t}^i + s_t^i = w_t + b_t^i + T_t, \quad (2)$$

$$c_{2,t+1}^i + (1 + \tau) b_{t+1}^i = (1 + r_{t+1}) s_t^i, \quad (3)$$

where w_t is wage rate, r_{t+1} is interest rate, $\tau \in (0, 1)$ is the tax on the bequests, and T_t is the uniform lump-sum transfer received from the government. By maximizing (1) subject to (2) and (3), we can derive the following optimal conditions:

$$s_t^i = \alpha^i \left(w_t + b_t^i + T_t \right), \quad (4)$$

$$b_{t+1}^i = \frac{\beta(1 + r_{t+1})}{1 + \tau} s_t^i. \quad (5)$$

From (4) and (5), saving are determined as follows:

$$s_t^i = \alpha^i \left[w_t + T_t + \frac{\beta(1 + r_t)}{1 + \tau} s_{t-1}^i \right]. \quad (6)$$

Equation (6) indicates that the savings of the current generation s_t^i are linked to the savings of their parents s_{t-1}^i because the bequests from their parents depend on their wealth income from (5). Therefore, assumption $\alpha^R > \alpha^P$ shows that the rich save more than the poor do and leave more wealth to their offspring. This means that the rich tend to accumulate more wealth than the poor do.

2.2 Final Good

Following Aghion and Howitt (2005) and Peretto (2011, 2015), we assume that final goods Y_t are produced by competitive firms using the production function

$$Y_t = \int_0^{N_t} X_t(j)^\theta \left[Z_t(j)^\gamma Z_t^{1-\gamma} \left(\frac{L_t}{N_t^\eta} \right) \right]^{1-\theta} dj, \quad (7)$$

where $\{\theta, \gamma, \eta\} \in (0, 1)$. $X_t(j)$ is the quantity of intermediate good $j \in [0, N_t]$, N_t is the mass of available intermediate goods in period t and L_t is the service of labor purchased from the household. In equilibrium, the relationship $L_t = L$ holds. The productivity of intermediate good $X_t(j)$ depends on its own quality $Z_t(j)$ and on the average quality $Z_t \equiv \frac{1}{N_t}(\int_0^{N_t} Z_t(j)dj)$ of all intermediate goods capturing technology spillovers. The quality is the good's ability to raise the productivity of L_t. The private return to quality is determined by γ, and the degree of technology spillovers is determined by $1 - \gamma$. The parameter η captures the degree of congestion of the services of labor across intermediate goods. For $\eta = 0$, there is no congestion, meaning that services of labor can be shared by all intermediate goods with no loss of productivity. This is a case of extreme economies of scope in the use of labor services. The reduced-form representations of the production function in equilibrium (shown later) manifest themselves as strong social increasing returns to product variety. In contrast, $\eta = 1$ yields full congestion, where there are no economies of scope and therefore no social returns to variety.

Profit maximization yields the following optimal conditions:

$$P_t(j) = \theta X_t(j)^{\theta-1} \left[Z_t(j)^\gamma Z_t^{1-\gamma} \left(\frac{L}{N_t^\eta} \right) \right]^{1-\theta}, \quad (8)$$

$$w_t = (1 - \theta)\frac{Y_t}{L}, \quad (9)$$

where $P_t(j)$ is the price of intermediate good j. Competitive producers of final goods pay $\theta Y_t = \int_0^{N_t} P_t(j)X_t(j)dj$ for intermediate goods and $(1 - \theta)Y_t = w_t L$ for labor.

2.3 Intermediate goods and in-house R&D

Monopolistic firms produce differentiated intermediate goods with a linear technology that requires one unit of final good to produce one unit of intermediate good $j \in [0, N_t]$. In addition, to improve the quality of its product, the firm in industry j also devotes $Z_t I_t(j)$ units of final good to R&D. In line with Aghion and Howitt (2005), the number of final inputs required to improve the product quality increases proportionally with the average quality of products Z_t.

Based on Grossmann (2007, 2009), the firm can increase product quality according to the technology

$$Z_t(j) = kZ_{t-1}I_t(j), \tag{10}$$

where k is the efficiency parameter of in-house R&D and $I_t(j)$ is R&D in effective units of the final output. The productivity of in-house R&D, $I_t(j)$, is given by the exogenous parameter k times the average quality of product in period $t-1$, Z_{t-1}. We adopt the level of the average quality of the product in period $t-1$ as a proxy for the stock of public technological knowledge in period t, which accumulates within firms as a byproduct of process innovation. Following the process innovation framework employed by Peretto (2011, 2015) and others, we model knowledge spillovers into process innovations among firms as a function of the average quality of products observable by the R&D departments of firms.

In industry j, the monopolistic firm's net profit in period t is

$$\pi_t(j) = [P_t(j) - 1]\, X_t(j) - Z_t I_t(j). \tag{11}$$

The monopolistic firm in industry j maximize (11) subject to (8) and (10). Profit maximization yields the following optimal conditions:

$$P_t = \frac{1}{\theta}, \tag{12}$$

$$I_t = (1-\theta)\theta\gamma \frac{Y_t}{Z_t N_t}. \tag{13}$$

Using equations (11) to (13) and $\theta Y_t = N_t P_t X_t$, we obtain the following maximum net profits for each intermediate firm in period t:

$$\pi_t = (1-\theta)\theta(1-\gamma)\frac{Y_t}{N_t}. \tag{14}$$

Because of the ex-ante homogeneity of the individuals, all intermediate goods firms behave in the same way. Thus, we omit index j whenever this does not lead to confusion.

2.4 Product Development Sector

Between periods t and $t+1$, competitive R&D firms in the product development sector devote I_t^N units of final outputs, develop $N_{t+1} - N_t$ new blueprints, and sell these blueprints to intermediate goods firms at their market values of V_t. Thus, given a research productivity of $1/\delta_t$, output is expressed as follows:

$$N_{t+1} - N_t = (1/\delta_t)I_t^N, \tag{15}$$

Following Peretto (2011), research productivity $1/\delta_t$ is a given for each firm but depends on the aggregate level, negatively on the average quality of product, as follows:

$$\delta_t = \delta Z_t, \tag{16}$$

where δ is the inefficiency parameter of product variety R&D. This specification implies that the cost of the creation of a newly designed product adds to the existing stock of product design and increases proportionally with the average quality of product Z_t.

Under the assumption of free entry in the product development sector, the expected gain of $(V_t/\delta_t)I_t^N$ from R&D must not exceed the cost of I_t^N for a finite size of R&D activities at equilibrium. Thus, we have the following condition:

$$V_t = \delta_t = \delta Z_t. \tag{17}$$

We next consider no-arbitrage conditions. The market value of intermediate goods firms V_t (i.e., the market value of blueprints) is related to the risk-free gross interest rate $1 + r_{t+1}$. Shareholders of intermediate goods firms that purchased these shares during period t obtain dividends of π_{t+1} during period $t + 1$ and can sell these shares to the subsequent generation at a value of V_{t+1}. In the financial market, the total returns from holding the stock of a particular intermediate firm must be equal to the returns on the risk-free asset $(1 + r_{t+1})V_t$, which implies the following no-arbitrage condition:

$$1 + r_{t+1} = \frac{\pi_{t+1} + V_{t+1}}{V_t}. \tag{18}$$

2.5 Redistributive policy

The only tax instrument of the government is the tax on bequests. Following Bossman et al. (2007), the government taxes the bequests of all old individuals at rate τ and redistributes its revenue among the all young individuals in a lump sum. Every young individual receives the same transfer T_t that corresponds to the average tax revenue per tax case,

$$T_t = \tau \left[\phi b_t^R + (1 - \phi)\, b_t^P\right]. \tag{19}$$

Since the rich leave more wealth than the poor, these policies indicate a net transfer of income from the rich to the poor.

2.6 Market-clearing conditions

The market equilibrium condition of final goods is given by

$$Y_t = \left(c_{1,t}^R + c_{2,t}^R\right)\phi L + \left(c_{1,t}^P + c_{2,t}^P\right)(1 - \phi)L + \delta_t\,(N_{t+1} - N_t) + N_t\,(X_t + Z_t I_t). \tag{20}$$

In addition, as shown in Appendix A, we can obtain the following asset market equilibrium condition:

$$V_t N_{t+1} = A_t \equiv s_t^R \phi L + s_t^P (1-\phi) L \tag{21}$$

where A_t expresses the total assets (savings) held by young agents in period t. This condition states that the total savings of young individuals in period t must be used for investing in new inventions ($V_t(N_{t+1} - N_t)$) or purchasing existing stocks that were owned by preceding generations ($V_t N_t$).

2.7 Aggregation

Substituting (8) and (12) into the production function (7) and imposing symmetry yield

$$Y_t = \theta^{\frac{2\theta}{1-\theta}} Z_t N_t^{\sigma} L, \tag{22}$$

where $\sigma \equiv 1-\eta$. Thus the reduced-form representation of the production function of the economy features a social return to variety equal to σ and a social return to quality equal to 1. Therefore, the growth rate of per capita output $y_t \equiv \frac{Y_t}{L}$ is

$$G_t^y \equiv \frac{y_{t+1}}{y_t} = \frac{Z_{t+1}}{Z_t}\left(\frac{N_{t+1}}{N_t}\right)^{\sigma} = G_t^Z \left(G_t^N\right)^{\sigma}, \tag{23}$$

which is determined by quality growth rate $G_t^Z \equiv \frac{Z_{t+1}}{Z_t}$ and the variety growth rate $G_t^N \equiv \frac{N_{t+1}}{N_t}$. The degree of social return to variety parameter σ determines the relative importance of variety expansion on per capita output growth.

Using (10), (13) and (22), the product quality growth rate G_t^Z is given by the following expression:

$$G_t^Z \equiv \frac{Z_{t+1}}{Z_t} = k(1-\theta)\theta^{\frac{1+\theta}{1-\theta}} \gamma \frac{L}{N_{t+1}^{1-\sigma}} \equiv G^Z\left(N_{t+1}\right), \tag{24}$$

where $G_N^Z\left(N_{t+1}\right) < 0$. As depicted in Figure 1, the quality growth rate G_t^Z in period t is negatively related to the number of intermediate-good firms N_{t+1} in period $t+1$. As the number of firms N_{t+1} increases, each firm's market size decreases, which motivates firms to invest less in process innovation and thereby lowers the product quality growth rate G_t^Z.

3 Dynamics of inequality and product variety

In this section, we derive the global transition dynamics of the economy and investigate how the expansion of variety and inequality relate to each other.

3.1 Dynamic system

In this subsection, we derive the dynamic system of the market equilibrium based on the model in the preceding section. By substituting (5), (6), (9), (14), (17), (18), and (19) into $A_t = s_t^R \phi L + s_t^P (1-\phi) L$, we obtain

$$A_t = \bar{\alpha}(1-\theta)Y_t + \beta\left[\frac{(1-\theta)\theta(1-\gamma)}{\delta}\frac{Y_t}{Z_{t-1}N_t} + \frac{Z_t}{Z_{t-1}}\right]\frac{\left(\alpha^R + \bar{\alpha}\tau\right)s_{t-1}^R \phi L + \left(\alpha^P + \bar{\alpha}\tau\right)s_{t-1}^P(1-\phi)L}{1+\tau}, \tag{25}$$

where $\bar{\alpha} \equiv \alpha^R\phi + \alpha^p(1-\phi)$.

By dividing (25) by A_{t-1}, we obtain the growth in aggregate savings A_t as follows:

$$\frac{A_t}{A_{t-1}} = \bar{\alpha}(1-\theta)\frac{Y_t}{A_{t-1}} + \beta\left[\frac{(1-\theta)\theta(1-\gamma)}{\delta}\frac{Y_t}{Z_{t-1}N_t} + \frac{Z_t}{Z_{t-1}}\right]\frac{\left(\alpha^R + \bar{\alpha}\tau\right)\varphi_{t-1} + \left(\alpha^P + \bar{\alpha}\tau\right)(1-\varphi_{t-1})}{1+\tau}, \tag{26}$$

where $\varphi_t \equiv \frac{s_t^R \phi L}{A_t}$. Note that $1 - \varphi_t \equiv \frac{s_t^P(1-\phi)L}{A_t}$ holds from the definition of φ_t and A_t. As stressed by Maebayashi and Konishi (2021), since φ_t represents the ratio of total savings of the rich to aggregate savings, φ_t serves as a convenient measure of inequality. If φ_t is close to the population ratio of the rich, ϕ, the economy expresses weak inequality (i.e., equality). Conversely, if φ_t is close to 0 or 1, the economy expresses strong inequality. In the following analyses, we assume that $\varphi_{-1} > \phi$. It means that the wealth endowment of the rich old generation in the initial period is larger than that of the poor old generations (i.e., $s_{-1}^R > s_{-1}^P$). Under this initial condition and the assumption $\alpha^R > \alpha^P$, in equilibrium, the relationship $\varphi_t > \phi$ holds for all period t. The rich tend to accumulate more wealth than the poor do for all period t.

Substituting (17), (22), and $V_t N_{t+1} = A_t$ into (26) and rearranging it, we can express the dynamics of variety N_t as follows:

$$N_{t+1} = \Gamma^R\left(N_t, \varphi_{t-1}; \tau\right) + \Gamma^P\left(N_t, \varphi_{t-1}; \tau\right) \equiv \mu\left(N_t, \varphi_{t-1}; \tau\right), \tag{27}$$

where $\Gamma^R\left(N_t, \varphi_{t-1}; \tau\right) \equiv \alpha^R\left\{\phi f\left(N_t\right) + \beta\left[\theta(1-\gamma)f\left(N_t\right) + N_t\right]\frac{\varphi_{t-1}+\phi\tau}{1+\tau}\right\}$, $\Gamma^P\left(N_t, \varphi_{t-1}; \tau\right) \equiv \alpha^P\left\{(1-\phi)f\left(N_t\right) + \beta\left[\theta(1-\gamma)f\left(N_t\right) + N_t\right]\frac{1-\varphi_{t-1}+(1-\phi)\tau}{1+\tau}\right\}$ and $f\left(N_t\right) \equiv \frac{1-\theta}{\delta}\theta^{\frac{2\theta}{(1-\theta)}}LN_t^{\sigma}$. From (27), $\mu\left(N_t, \varphi_{t-1}; \tau\right)$ satisfies (a) $\mu_N\left(N_t, \varphi_{t-1}; \tau\right) > 0$, (b) $\mu_\varphi\left(N_t, \varphi_{t-1}; \tau\right) > 0$, and (c) $\mu_\tau\left(N_t, \varphi_{t-1}; \tau\right) < 0$ for $\varphi_{t-1} > \phi$ and $\mu_\tau\left(N_t, \varphi_{t-1}; \tau\right) > 0$ for $\varphi_{t-1} < \phi$. See Appendix B for details. The property (a) indicates that an increase in the number of specialized intermediate goods N_t increases the wage rate, which positively affects the aggregate savings of the economy, and thereby increases the demand for shares of each intermediate-goods firm. These factors increase the market value of blueprints of a new variety, which positively affects the equilibrium number of product varieties. The property (b) indicates that the greater inequality φ_{t-1} positively affects the aggregate savings of the economy, and thereby increases the equilibrium number of product varieties. Since $\alpha^R > \alpha^P$, the rich save more than the poor do and leave more wealth to their offspring. Therefore, the inequality φ_{t-1} driven by the rich contributes to the growth in aggregate savings as well as the equilibrium number of product varieties. The property (c) indicates that in the region where the relationship $\varphi_{t-1} > \phi$ holds, the redistributive policy negatively affects the aggregate savings of the economy, and thereby decreases the equilibrium number of product varieties. The redistributive policy increases the income of poor, whereas it decreases the income of rich. Since the rich save a larger proportion of income than the poor, the redistributive policy decreases the aggregate savings of the economy and the equilibrium number of product varieties.

Moreover, using (5), (6), (9), (14), (17), (18), (19), (21), (22), (27) and the definition of φ_t, we can express the dynamics of the share of aggregate savings held by the rich φ_t as follows:

$$\varphi_t = \frac{\Gamma^R (N_t, \varphi_{t-1}; \tau)}{\mu (N_t, \varphi_{t-1}; \tau)} \equiv \lambda (N_t, \varphi_{t-1}; \tau) . \tag{28}$$

From (28), $\lambda (N_t, \varphi_{t-1}; \tau)$ satisfies (a) $\lambda_N (N_t, \varphi_{t-1}; \tau) > 0$ for $\varphi_{t-1} > \phi$ and $\lambda_N (N_t, \varphi_{t-1}; \tau) < 0$ for $\varphi_{t-1} < \phi$, (b) $\lambda_\varphi (N_t, \varphi_{t-1}; \tau) > 0$, and (c) $\lambda_\tau (N_t, \varphi_{t-1}; \tau) < 0$ for $\varphi_{t-1} > \phi$ and $\lambda_\tau (N_t, \varphi_{t-1}; \tau) > 0$ for $\varphi_{t-1} < \phi$. See Appendix C for details. Property (a) indicates that in the region where the relationship $\varphi_{t-1} > \phi$ holds, an increase in the number of specialized intermediate goods N_t increases the share of aggregate savings held by the rich and thereby it increases the level of inequality. An increase in the number of specialized intermediate goods N_t increases the wage rate, which motivates the both poor and rich individuals to save more. However, since $\alpha^R > \alpha^P$, the rich save more than the poor do and leave more wealth to their offspring. Therefore, an increase in the number of specialized intermediate goods N_t contributes to an increase in the level of inequality in period t. Property (b) indicates that greater inequality φ_{t-1} in period $t-1$ leads to an increase in inequality in period t. In our model, greater inequality φ_{t-1} in period $t-1$ increases the equilibrium number of specialized intermediate goods N_t and the wage rate in period t, which motivates both poor and rich individuals to save more. However, since $\alpha^R > \alpha^P$, the rich save more than the poor do and leave more wealth to their offspring. Therefore, the inequality φ_{t-1} in period $t-1$ contributes to an increase in the level of inequality in period t. Property (c) indicates that in the region where the relationship $\varphi_{t-1} > \phi$ holds, the redistributive policy increases the savings of the poor, whereas it decreases the savings of the rich and thereby decreases the level of inequality.

The above two difference equations (27) and (28) together with the initial values N_0 and φ_{-1} characterize the dynamics of the economy. Note that both N_t and φ_{t-1} in period t are predetermined variables.

3.2 Phase diagram

The characterization of the global dynamics of the economy requires the depiction of the phase diagram of the dynamic system defined by (27) and (28). We begin with the derivation of the $N_{t+1} = N_t$ locus on the (N_t, φ_{t-1}) plane. Let NN be the geometric place of all pairs (N_t, φ_{t-1}) such that the variety N_t is at steady-state:

$$NN = \{(N_t, \varphi_{t-1}) \mid \Delta N_t \equiv N_{t+1} - N_t = \mu(N_t, \varphi_{t-1}; \tau) - N_t = 0\} \tag{29}$$

where

$$\Delta N_t \begin{cases} > 0, \\ = 0, \\ < 0, \end{cases} \quad \text{if and only if} \quad \varphi_{t-1} \begin{cases} > \Gamma(N_t; \tau), \\ = \Gamma(N_t; \tau), \\ < \Gamma(N_t; \tau), \end{cases} \tag{30}$$

where $\Gamma(N_t; \tau) \equiv \left\{ \frac{[\xi(N_t) - \bar{\alpha}](1+\tau)}{\beta[\theta(1-\gamma)+\xi(N_t)]} - (\alpha^P + \bar{\alpha}\tau) \right\} \frac{1}{(\alpha^R - \alpha^P)}$, $\xi(N_t) \equiv \frac{N_t}{f(N_t)}$, $\xi_N(N_t) > 0$, $\xi_{NN}(N_t) < 0$, $\lim_{N_t \to 0} \xi(N_t) = 0$, $\lim_{N_t \to \infty} \xi(N_t) = \infty$. The properties of the NN locus, as depicted in Figure 2, are:

- $(\hat{N}, 0) \in NN$ for some $\hat{N} > 0$,
- $\frac{d\varphi_{t-1}}{dN_t} |_{NN} = \frac{(1+\tau)[\theta(1-\gamma)+\bar{\alpha}]\xi_N(N_t)}{\beta(\alpha^R - \alpha^P)[\theta(1-\gamma)+\xi(N_t)]^2} > 0$,
- $\frac{d^2\varphi_{t-1}}{dN_t^2} |_{NN} = \frac{(1+\tau)[\theta(1-\gamma)+\bar{\alpha}]\left\{\xi_{NN}(N_t))[\theta(1-\gamma)+\xi(N_t)]^2 - 2[\theta(1-\gamma)+\xi(N_t)]\xi_N(N_t)^2\right\}}{\beta(\alpha^R - \alpha^P)[\theta(1-\gamma)+\xi(N_t)]^4} < 0$.

Thus, as depicted in Figure 2, the NN locus is monotonically increasing and a strictly concave curve with a positive horizontal intercept $\hat{N}$. Moreover, since $\Delta N_t > 0$ for $\varphi_{t-1} > \Gamma(N_t; \tau)$, for a given pair (N_t, φ_{t-1}) along NN locus, an increase in φ_{t-1} holding N_t constant, causes $\varphi_{t-1} > \Gamma(N_t; \tau)$, and therefore $\Delta N_t > 0$. Similarly, a decrease in φ_{t-1} holding N_t constant, causes $\varphi_{t-1} < \Gamma(N_t; \tau)$, and therefore $\Delta N_t < 0$. In addition, as shown in Appendix D, we can see that the relationships $\Gamma_\tau(N_t; \tau) > 0$ for $\varphi_{t-1} > \phi$ and $\Gamma_\tau(N_t; \tau) < 0$ for $\varphi_{t-1} < \phi$ hold. Therefore, as depicted in Figure 3, when the government introduces redistributive policy and increases the tax on bequests τ, the NN locus shifts upward when $\varphi_{t-1} > \phi$, whereas it shifts downward when $\varphi_{t-1} < \phi$.

Next, we derive the $\varphi_t = \varphi_{t-1}$ locus on the (N_t, φ_{t-1}) plane. Let $\varphi\varphi$ be the geometric place of all pairs (N_t, φ_{t-1}) such that the share of aggregate savings held by the rich φ_t is at steady-state:

$$\varphi\varphi = \{(N_t, \varphi_{t-1}) \mid \Delta\varphi_{t-1} \equiv \varphi_t - \varphi_{t-1} = \lambda(N_t, \varphi_{t-1}; \tau) - \varphi_{t-1} = 0\} \tag{31}$$

where

$$\Delta\varphi_{t-1}\begin{cases}>0,\\=0,\\<0,\end{cases}\quad \text{if and only if}\quad \varphi_{t-1}\begin{cases}<Q(N_t;\tau)\\=Q(N_t;\tau)\\>Q(N_t;\tau)\end{cases} \tag{32}$$

where $Q(N_t;\tau) \equiv \{\varphi_{t-1} \mid \varphi_{t-1} = \lambda(N_t,\varphi_{t-1};\tau)\}$. The properties of $\varphi\varphi$ locus, as depicted in Figure 2, are:

- $(0,\varphi_L) \in \varphi\varphi$ for some $\varphi_L > \phi$,
- $(\infty,\varphi_H) \in \varphi\varphi$ for some $\varphi_H < 1$,
- $\frac{d\varphi_{t-1}}{dN_t}\mid_{\varphi\varphi}= \frac{\lambda_N(N_t,\varphi_{t-1};\tau)}{1-\lambda_\varphi(N_t,\varphi_{t-1};\tau)} > 0$.

From (28), since $\lambda_\varphi(N_t,\varphi_{t-1};\tau) > 0$, $\lambda_{\varphi\varphi}(N_t,\varphi_{t-1};\tau) < 0$ and $0 < \lambda(N_t,0;\tau) < \lambda(N_t,1;\tau) < 1$, evaluating the value of $\lambda_\varphi(N_t,\varphi_{t-1};\tau)$ at the $\varphi\varphi$ locus, we can see that the relationship $\lambda_\varphi(N_t,\varphi_{t-1};\tau) \in (0,1)$ holds. Thus, as depicted in Figure 2, the $\varphi\varphi$ locus is a monotonically increasing and strictly concave curve with a positive vertical intercept φ_L and a positive upper bound φ_H. Moreover, since $\Delta\varphi_{t-1} < 0$ for $\varphi_{t-1} > Q(N_t;\tau)$, for a given pair (N_t,φ_{t-1}) along $\varphi\varphi$ locus, an increase in φ_{t-1} holding N_t constant, causes $\varphi_{t-1} > Q(N_t;\tau)$, and therefore $\Delta\varphi_{t-1} < 0$. Similarly, a decrease in φ_{t-1} holding N_t constant, causes $\varphi_{t-1} < Q(N_t;\tau)$, and therefore $\Delta\varphi_{t-1} > 0$. In addition, as shown in Appendix E, we can see that the relationship $Q_\tau(N_t;\tau) < 0$ for $\varphi_{t-1} > \phi$ holds. Therefore, as depicted in Figure 3, when the government introduces redistributive policy and increases the tax on bequests τ, the $\varphi\varphi$ locus shifts downward when $\varphi_{t-1} > \phi$.

3.3 Steady-state equilibrium

Now, we investigate the steady-state of the economy wherein both N_t and φ_{t-1} are constant. Let (N^*,φ^*) be the steady-state equilibrium of the dynamic system defined by (27) and (28). As depicted in Figure 2, there exists a unique steady-state equilibrium E (N^*, φ^*) that is determined by the intersections of the NN locus and the $\varphi\varphi$ locus on the (N_t,φ_{t-1}) plane. The steady-state measure of inequality φ^* lies above the rich population ratio ϕ (i.e., $\varphi^* > \phi$). As N_t and φ_{t-1} in period t are predetermined variables, their initial values, N_0 and φ_{-1}, are historically given. Therefore, as inferred from Figure 2, all economies, irrespective of their initial endowments of (N_0, φ_{-1}), eventually approach this steady-state equilibrium $E(N^*, \varphi^*)$. More precisely, linearizing (27) and (28) around the steady-state equilibrium $E(N^*, \varphi^*)$, we obtain

$$\begin{pmatrix} N_{t+1}-N^* \\ \varphi_t-\varphi^* \end{pmatrix} = J\begin{pmatrix} N_t-N^* \\ \varphi_{t-1}-\varphi^* \end{pmatrix}, \tag{33}$$

where

$$J = \begin{pmatrix} \mu_N^* & \mu_\varphi^* \\ \lambda_N^* & \lambda_\varphi^* \end{pmatrix}, \tag{34}$$

$\mu_N^* \equiv \mu_N(N^*, \varphi^*; \tau) \in (0,1)$, $\mu_\varphi^* \equiv \mu_\varphi(N^*, \varphi^*; \tau) > 0$, $\lambda_N^* \equiv \lambda_N(N^*, \varphi^*; \tau) > 0$ and $\lambda_\varphi^* \equiv \lambda_\varphi(N^*, \varphi^*; \tau) \in (0,1)$. Here, let J be the Jacobian matrix evaluated at the steady-state equilibrium E (N^*, φ^*).

From investigations of (33) and (34), we can obtain the following proposition:

Proposition 1 *The eigenvalues of the Jacobian matrix J, ω_1 and ω_2, are real and have a modulus of less than one (i.e., $\omega_1, \omega_2 \in (-1, 1)$). The steady-state equilibrium E (N^*, φ^*) of the dynamic system defined by (27) and (28) is a sink.*

Proof of Proposition 1 is given in Appendix F. Suppose that both eigenvalues, ω_1 and ω_2, are positive (i.e., $\omega_1, \omega_2 \in (0, 1)$), the orbits in the neighborhood of E (N^*, φ^*) are monotone, and we have stable nodes. However, suppose that the eigenvalues have opposite signs (i.e., $-1 < \omega_1 < 0 < \omega_2 < 1$) and the orbits in the neighborhood of E (N^*, φ^*) are oscillatory, which corresponds to stable spirals. In either case, all economies, irrespective of their initial endowments of (N_0, φ_{-1}), eventually approach to the steady-state equilibrium $E(N^*, \varphi^*)$. Moreover, concerning comparative statics in the steady-state equilibrium E(N^*, φ^*), we obtain the following proposition:

Proposition 2 *The relationships $\frac{dN^*}{d\tau} < 0$ and $\frac{d\varphi^*}{d\tau} < 0$ hold.*

Proof for this proposition is shown in Appendix G. As depicted in Figure 3, the introduction of redistributive policy and an increase in the tax on bequests τ negatively affects the share of aggregate savings held by the rich as well as the aggregate savings of the economy, and thereby decreases the steady-state level of inequality as well as the number of product varieties (i.e., $\varphi^* > \varphi'$ and $N^* > N'$).

In the steady-state equilibrium where the relationships $N_t = N_{t-1} = N^*$ and $\varphi_t = \varphi_{t-1} = \varphi^*$ hold, from (23) and (24), the per capita output growth rate becomes equivalent to the product quality growth rate as follows:

$$G^{y*} \equiv G_t^y \mid_{N_t = N^*} = G^{Z*} \equiv G^Z(N^*). \tag{35}$$

From (24), since the quality growth rate G_t^Z in period t is negatively related to the amount of product variety N_{t+1} in period $t+1$, as depicted in Figure 1, the decline in the level of product variety due to an increase in the tax on bequests τ leads to the higher steady-state growth rate of product quality (i.e., $G^Z(N^*) < G^Z(N')$). Intuitively, the introduction of redistributive policy and an increase in the tax on bequests τ discourages the entry of new firms, which in turn expands each firm's market size and increases incentives for process innovation. Summarizing these results, we obtain the following proposition:

Proposition 3 *The relationships* $\frac{dG^{Z*}}{d\tau} > 0$ *and* $\frac{dG^{y*}}{d\tau} > 0$ *hold.*

The introduction of redistributive policy and an increase in the tax on bequests τ increases the steady-state growth rate of product quality and per capita output by reducing the steady-state number of firms.

In summary, concerning the long-run (i.e., steady-state) effects of redistributive policy on per capita output growth and inequality, we obtain the following results.

Result 1 *The introduction of redistributive policy decreases the steady-state level of inequality, reduces the entry of new firms, and expands the market size of each firm. The larger market size increases incentives for process innovation and increases the steady-state growth rate of product quality and per capita output.*

3.4 Welfare

In this subsection, we solve for the welfare levels of both the rich and poor individuals in generation t. Using equations (1) to (5), (9), (14), (16) to (18), (21), (22) and $\varphi_t \equiv \frac{s_t^R \phi L}{A_t}$, the lifetime utility level of individuals $i \in \{R, P\}$ born in period t (i.e., generation t) is given by

$$U_t^i = \ln\left[\frac{\Omega_i}{(1+\tau_{t+1})^{\beta\alpha^i}}(1+r_{t+1})^{\alpha^i}(w_t + b_t^i + T_t)\right], \tag{36}$$

where $\Omega_i \equiv (1-\alpha^i)^{1-\alpha^i}(\alpha^i)^{\alpha^i}\left[(1-\beta)^{1-\beta}\beta^\beta\right]^{\alpha^i}$,

$$b_t^i = \frac{\beta(1+r_t)}{1+\tau_t} s_{t-1}^i = \begin{cases} \frac{\beta(1+r_t)}{1+\tau_t}\frac{\varphi_{t-1}}{\phi}\delta\frac{Z_{t-1}N_t}{L}, & \text{for} \quad i = R, \\ \frac{\beta(1+r_t)}{1+\tau_t}\frac{1-\varphi_{t-1}}{1-\phi}\delta\frac{Z_{t-1}N_t}{L}, & \text{for} \quad i = P, \end{cases}$$

$$T_t = \frac{\beta(1+r_t)}{1+\tau_t}\tau_t\left[\phi s_{t-1}^R + (1-\phi)\, s_{t-1}^P\right] = \frac{\beta(1+r_t)}{1+\tau_t}\tau_t\delta\frac{Z_{t-1}N_t}{L},$$

$$w_t = (1-\theta)\frac{Y_t}{L} = (1-\theta)\theta^{\frac{2\theta}{1-\theta}} Z_t N_t^\sigma,$$

$$1 + r_{t+1} = \frac{(1-\theta)\theta(1-\gamma)}{\delta}\frac{Y_{t+1}}{Z_t N_{t+1}} + \frac{Z_{t+1}}{Z_t} = G_t^Z\left[\frac{(1-\theta)\theta(1-\gamma)}{\delta}\frac{\theta^{\frac{2\theta}{1-\theta}} L}{N_{t+1}^{1-\sigma}} + 1\right].$$

Here, in order to clarify the explanation of the following numerical simulation analyses, we introduce the time index of the tax rate on bequests τ_t explicitly. From the above expressions, we can summarize the effects of redistributive policy on the lifetime utility of individuals in generation t as follows.

Let us first explain the direct effects of redistributive policy on welfare. Given the value of w_t and r_{t+1}, the introduction of redistributive policy and an increase in the tax on bequests affects the lifetime utility of individuals in generation t directly through the following two channels. First, since $\frac{\partial b^i_{t+1}}{\partial \tau_{t+1}} < 0$, an increase in the tax on bequests τ_{t+1} in period $t+1$ negatively affects the lifetime utility of both poor and rich individuals in generation t through its distortionary effects on bequests for their children. Second, since $\frac{\partial (b^R_t + T_t)}{\partial \tau_t} < 0$ and $\frac{\partial (b^P_t + T_t)}{\partial \tau_t} > 0$, an increase in the tax on bequests τ_t in period t and the resulting income transfer from the rich to the poor negatively affects the lifetime utility of the rich individuals in generation t, whereas it positively affects the lifetime utility of the poor individuals in generation t.

Let us next explain the indirect effects of redistributive policy on welfare. The redistributive policy also indirectly affects the lifetime utility of individuals in generation t through its negative impacts on the level of product variety N_t and the level of inequality φ_t and its positive impacts on the level of product quality Z_t. The lower level of product variety N_t negatively affects the lifetime utility of individuals in generation t through declines in the wage rate w_t, bequests from their parents b^i_t, and lump-sum transfers from the government T_t, whereas it positively affects the lifetime utility of individuals in generation t through an increase in the interest rate r_t. The lower level of inequality φ_{t-1} negatively affects the lifetime utility of rich individuals in generation t through a decline in bequests from their parents b^R_t. In contrast, it positively affects the lifetime utility of poor individuals in generation t through an increase in bequests from their parents b^P_t. The higher level of product quality Z_t positively affects the lifetime utility of individuals in generation t through increases in the wage rate w_t, bequests from their parents b^i_t and lump-sum transfers from the government T_t. The higher product quality growth rate G^Z_t also positively affects the lifetime utility of individuals in generation t through an increase in the interest rate r_t.

Unfortunately, even focusing on the steady-state equilibrium, it is difficult for us to analytically clarify the welfare implications of redistributive policy. Therefore, to obtain further insights, we resort to numerical simulation analysis in the following section.

4 Numerical Analysis

In this section, to obtain further insights with respect to the effects of redistributive policy on the per capita output growth rate, inequality, and welfare in the transition process, we resort to the numerical simulations of our model. We consider the case where the economy is initially in the steady-state equilibrium where the tax rate on bequests τ is given by zero (i.e., $\tau_k = 0$ for all period $k < 3$). Then, our steady-state economy introduces the redistributive policy from period 3. More concretely, in our benchmark analysis, the tax rate on bequests in period 3 and subsequent periods is increased from 0 to 0.2 (i.e., $\tau_k = 0.2$ for all periods $k \geq 3$).

As inferred from (23), in the transition process, the per capita output growth rate is determined not only by the quality growth rate but also the variety growth rate. Therefore, in contrast to the steady-state equilibrium analyzed in the previous section, the effects of redistribution policy on variety growth play a significant role in determining the overall effects of redistribution policy on per capita output growth. The main objective of the following numerical exercises is not to calibrate our simple model to actual data but to supplement the qualitative results of our theoretical model. Therefore, although we chose the parameter values carefully, the quantitative results obtained in our paper should be interpreted with caution.

4.1 The Model Parameterization

The model features the following set of parameters $\{\phi, \beta, \alpha^R, \alpha^P, \gamma, \sigma, \delta, k\}$. Table 1 lists our preset parameters, the values of which reflect available empirical estimates or existing numerical studies. We follow Maebayashi and Konishi (2021) to set the population share of the rich ϕ to 0.5, the relative importance parameter of consumption when old and leaving a bequest β to 0.3, the intertemporal preference parameter of the rich α^R to 0.45, and the poor α^P to 0.25. Then, we set the degree of technology spillover $1 - \gamma$ to 0.833, which corresponds to the baseline simulation parameter value of Iacopetta et al. (2019).

The value of σ represents the social return to variety. The closest empirical counterpart to our σ is the "elasticity of productivity to variety" calculated by Broda et al. (2006), ranging from 0.05 to 0.2. However, as stressed by Brunnschweiler et al. (2017), the empirical estimates of "gains from variety" that come from empirical studies on international trade are not fully consistent with the notion of "gains from differentiation" originally emphasized by Romer (1990). Therefore, we adopt a conservative approach by setting the baseline value of σ on the high end, $\sigma = 0.2$, and then perform a sensitivity analysis with wide values of σ.

Given these preset parameters, we calibrate the two remaining parameters listed in Panel B of Table 2, the inefficiency parameter of variety R&D (δ) and the efficiency parameter of in-house R&D (k), to match the 2 target values of the endogenous variables listed in Panel A of Table 2. The target value of the mass of firms relative to population, $\frac{N^*}{L} = 0.0327$, equals the OECD-average numbers of firms in 2013, $N^* = 1,181,040$, divided by the average population in the same year, $L = 365,525,680$. [4] Since the target value of this variable is already calcu-

[4] According to the United Nations (2015), the target population (i.e., $L = 365,525,680$)

lated by Brunnschweiler et al. (2017: Table 1, p39), we employ their calculated value as our target value. The target value of the per capita output growth rate, $G^{y*} = 1.02$, approximates the average per capita GDP growth rate of developed countries over a century (Barro and Sala-i-Martin, 2004: Table I.1, p13). We identify the values of (δ, k) by imposing the target values of $\frac{N^*}{L}$ and G^{y*}. Intuitively, we adjust the value of δ to match the target value of $\frac{N^*}{L}$, whereas the value of k is adjusted to match the target value of G^{y*}. These procedures yield the calibrated values of (δ, k) reported in Panel B of Table 2. [5]

4.2 The effects of the redistributive policy

Figures 4-a to 4-f show the numerical examples of the transition path of the tax rate on bequests τ_t (Figure 4-a), the share of aggregate savings held by the rich φ_t (Figure 4-b), the number of firms per capita N_t/L (Figure 4-c), the product variety growth rate G_t^N (Figure 4-d), the product quality growth rate G_t^Z (Figure 4-e), and the per capita output growth rate G_t^y (Figure 4-f) under the benchmark parameter value of the social return to variety parameter σ (i.e., $\sigma = 0.2$). Then, we compare four different values of tax rate on bequests (i.e., $\tau = 0, 0.1, 0.2$ and 0.3). Moreover, Figure 4-g and Figure 4-h show the net welfare gain of rich and poor individuals who belong to generations 1-9 (i.e., those born between periods 1-9), which describes how the lifetime utility levels of rich and poor individuals are affected by the redistributive policy implemented in period 3. The $\tau = 0.1$ line shows the difference in the level of lifetime utility between $\tau = 0.1$ and $\tau = 0$; the $\tau = 0.2$ line, $\tau = 0.3$ line shows the differences between $\tau = 0.2, 0.3$ and 0, respectively.

The introduction of the redistributive policy in period 3 (Figure 4-a) decreases the share of savings held by the rich φ_t and thereby decreases the level of inequality in period 3 and onwards (Figure 4-b). As a result, the level of inequality φ_t gradually converges to its new steady-state value, which is smaller than that in the original steady-state equilibrium. The introduction of the redistributive policy in period 3 also decreases the number of firms per capita in period 4 and onwards (Figure 4-c). Therefore, since $G_3^N = N_4/N_3$, variety growth rate G_t^N suddenly drops

matches the average population of OECD countries in 2013. Moreover, by summing the data reported in the OECD (2016: Ch2, Table 2.1) across countries, the aggregate number of enterprises in 2013 in OECD economies is 46,060,568 (all sizes and sectors). Dividing this number by 39 countries, we obtain the OECD-average numbers of firms in 2013, $N^* = 1,181,040$. See Appendix E of Brunnschweiler et al. (2017) for further details.

[5] The initial values of both the average quality of products Z_0 and the number of firms N_0 are normalized to 1 and the initial saving share of the rich φ_{-1} is given by 0.55. Since the starting point of our numerical simulation exercise is the steady-state equilibrium where the tax on bequests τ is given by zero, our numerical simulation results do not change even when we consider the alternative values of Z_0, N_0 and φ_{-1}.

in period 3, and it gradually returns to its steady-state value of 1 (Figure 4-d), as the number of firms per capita gradually approaches to its new steady-state value which is smaller than that in the original steady-state equilibrium.

As inferred from (24), the quality growth rate G_t^Z in period t is negatively related to the number of intermediate-good firms N_{t+1} in period $t + 1$. Therefore, since the introduction of the redistributive policy in period 3 decreases the number of firms per capita in period 4 and onwards (Figure 4-c), the product quality growth rate G_t^Z begins to increase from period 3, and gradually converges to its new steady-state value, which is higher than that in the original steady-state equilibrium (Figure 4-e).

Equation (23) indicates that the per capita output growth rate G_t^y in Figure 4-f depends on the variety growth rate G_t^N in Figure 4-d and the product quality growth rate G_t^Z in Figure 4-e. Moreover, the degree of social return to variety parameter σ determines the relative importance of the variety growth rate G_t^N on the per capita output growth rate G_t^y. Figures 4-d to 4-f show the transition path of G_t^N, G_t^Z and G_t^y when the degree of the social return to variety parameter σ is 0.2, which is the high end of the empirical estimates of σ by Broda et al. (2006). Thus, the evolutions of G_t^N in Figure 4-d and G_t^Z in Figure 4-e indicate that the introduction of the redistributive policy in period 3 provides two competing impacts upon the per capita output growth rate in period 3 (Figure 4-f). On the one hand, as shown in Figure 4-d, the decline in the growth rate of the product variety in period 3 negatively affects the per capita output growth rate in period 3. On the other hand, as shown in Figure 4-e, the rise in the growth rate of the product quality in period 3 positively affects the per capita output growth rate in period 3. In our benchmark simulation (i.e., $\sigma = 0.2$), since the latter positive effect dominates the former negative effect, the per capita output growth rate in period 3 becomes higher than that in the original steady-state equilibrium. Consequently, the per capita output growth rate under the redistributive policy is always beyond that in the original steady-state.

After period 3, the economy gradually converges to its new steady-state equilibrium. During this transition process, the product variety growth rate gradually approaches 1. Therefore, in the steady-state equilibrium, the per capita output growth rate becomes equivalent to the quality growth rate. Consequently, as shown in Figures 4-e and 4-f, the new steady-state equilibrium's per capita output growth rate becomes higher than in the original steady-state equilibrium. Therefore, as stated in Proposition 3, the long-run effect of the redistributive policy on per capita output growth is positive.

Figures 4-g and 4-h show the net welfare gain of rich and poor individuals who belong to generations 1-9 (i.e., those born between periods 1-9) when the redistributive policy is implemented in period 3. However, in period 3 (i.e., generation 2), the old individuals only pay bequest taxes in their old period and cannot receive any government transfer in their young period. Therefore, their value of welfare gain is negative, and the redistributive policy implemented in period 3 lowers the welfare level of both rich and poor individuals in generation 2. However, since young poor individuals in period 3 (i.e., generation 3) receive a net income transfer at the expense of young and old rich individuals, the introduction of the redistributive policy in period 3 raises the welfare level of poor individuals in generation 3 (Figure 4-h). In contrast, it lowers the welfare level of rich individuals in generation 3 (Figure 4-g).

As shown in Figure 4-c, the redistributive policy implemented in period 3 decreases the number of firms per capita in period 4 and onwards, which negatively affects the lifetime utility level of individuals in generation $t \geq 4$ through declines in the wage rate, bequests from their parents, and lump-sum transfers from the government. However, as shown in Figure 4-e, the redistributive policy implemented in period 3 also increases the growth rate of the product quality in period 3 and onwards, which positively affects the lifetime utility level of individuals in generation $t \geq 4$ through increases in the wage rate, the interest rate, bequests from their parents, and lump-sum transfers from the government. As the number of firms per capita gradually converges to its new steady-state value, the latter positive welfare effect is more likely to dominate the former negative welfare effect because the latter growth-enhancing effect lasts permanently. Consequently, as shown in Figures 4-g and 4-h, the welfare losses of rich individuals in generation $t \geq 4$ become smaller over time, whereas the welfare gains of rich individuals in generation $t \geq 4$ becomes larger over time. When $\tau = 0.2$, the net welfare gain of rich individuals in generation $t \geq 12$ turns out to be positive. These numerical simulation results indicate that an intergenerational conflict between current and future generations exists concerning the implementation of redistributive policy. As in the pension reform literature (e.g., Breyer and Straub, 1993; Wigger 2001), a well-designed debt financing scheme might be necessary for pursuing a Pareto improving redistributive policy. This issue is beyond the scope of this paper and is left for future research.

4.3 Sensitivity analyses

In our benchmark simulation (i.e., $\sigma = 0.2$), as shown in Figure 4-f, the redistributive policy positively affects the per capita output growth rate in the long run and the short run. However, this result heavily depends on the value of the social return to variety parameter σ used in our benchmark simulation. The empirical estimates of σ by Broda et al. (2006) range from 0.05 to 0.2. Hence, our benchmark parameter value of σ follows the high end of their empirical estimates. Nevertheless, it is difficult to obtain a reliable estimate value of σ. Moreover, the main objective of our numerical exercises is not to calibrate our simple model to actual data but to supplement the qualitative results of our theoretical model. Therefore, to clarify the growth implications of redistributive policy, this subsection performs a sensitivity analysis with wide values of σ in the range of 0.2-0.8.

Figures 5-a to 5-d show the numerical examples of the transition path of the tax rate on bequests τ_t (Figure 5-a), the product variety growth rate G_t^N (Figure 5-b), the product quality growth rate G_t^Z (Figure 5-c), and the per capita output growth rate G_t^y (Figure 5-d) under different values of the social return to variety parameter σ (i.e., $\sigma = 0.2, 0.4, 0.6$ and 0.8), when the tax rate on bequests in period 3 and subsequent periods is increased from 0 to 0.2 (i.e., $\tau_k = 0.2$ for all periods $k \geq 3$). From (23), the redistributive policy implemented in period 3 generates two competing impacts on the per capita output growth rate in period 3 (Figure 5-d). On the one hand, as shown in Figure 5-b, the decline in the growth rate of the product variety in period 3 negatively affects the per capita output growth rate in period 3. On the other hand, as shown in Figure 5-c, the rise in the growth rate of the product quality in period 3 positively affects the per capita output growth rate in period 3. In our simulation, when the degree of social return to variety parameter σ is sufficiently small, as described in the $\sigma = 0.2$ and $\sigma = 0.4$ lines in Figure 5-d, since the latter positive effect dominates the former negative effect, the per capita output growth rate in period 3 becomes higher than that in the original steady-state equilibrium. However, when the degree of social return to variety parameter σ is sufficiently large, as described in the $\sigma = 0.6$ and $\sigma = 0.8$ lines in Figure 5-d, the per capita output growth rate in period 3 becomes lower than that in the original steady-state equilibrium since the former negative effect dominates the latter positive effect. After period 3, since both the growth rate of product variety and product quality rises gradually, the per capita output growth rate also increases gradually. As shown in Figures 5-c and 5-d, the new steady-state equilibrium's per capita output growth rate becomes higher than in the original steady-state equilibrium. Therefore, irrespective of the values of σ, the long-run effect of the redistributive policy on per capita output growth is positive.

These results imply that the short-run effect of the redistributive policy on economic growth is generally ambiguous and depends on the values of the social return to variety parameter σ. Suppose that the degree of social return to variety parameter σ is sufficiently small; then, the per capita output growth rate under the redistributive policy is always beyond that in the original steady-state. Therefore, the redistributive policy positively affects the per capita output growth rate in the long run and the short run. However, suppose that the degree of social return to variety parameter σ is sufficiently large; the length of periods, for which the per capita output growth rate under the redistributive policy is below that in the original steady-state, is relatively long. Therefore, the short-run effect of the redistributive policy on economic growth is negative. Although empirical estimates of σ by Broda et al. (2006) support a sufficiently small value of σ, it is difficult to obtain a reliable estimate value of σ. Therefore, the short-run effect of the redistributive policy on economic growth remains inconclusive.

In summary, concerning the short-run effects of redistributive policy on per capita output growth, we obtain the following results.

Result 2 *Suppose that the degree of social return to variety parameter σ is sufficiently small, then the short-run effect of the redistributive policy on economic growth is positive. However, suppose that the degree of social return to variety parameter σ is sufficiently large, then the short-run effect of the redistributive policy on economic growth is negative.*

5 Concluding Remarks

Employing a two-period overlapping generations model of R&D-based growth with both product development and process innovation, we examined how a redistributive policy for reducing inequality by taxing the bequests of the rich and redistributing the revenue to the poor affects the per capita output growth rate of the economy. We showed that such a policy simultaneously raises growth and reduces inequality in the long run. Moreover, when the market structure adjusts, partially in the short- run, the effect of the redistributive policy on economic growth depends on the values of the social return to variety parameter, while it reduces inequality. However, when the market structure adjusts fully in the long run, the redistributive policy discourages the entry of new firms but enhances economic growth and reduces inequality. These favorable predictions regarding the impact of the redistributive policy on economic growth and inequality are partly consistent with empirical findings that show that the redistribution is generally benign in terms of economic growth and the lower post-tax and transfer inequality is correlated with faster and more durable growth. Thus, the redistributive bequest taxation is perhaps appropriate to stimulate long-run economic growth and reduce inequality.

Before concluding this paper, we note several limitations of our paper and discuss directions for future research. First, we ignore several important elements of the real economy for clarity of our main arguments, such as the risk of R&D activities and the presence of imperfect credit and insurance markets. Although these model simplifications enable us to obtain a clear-cut intuitive prediction regarding the effect of redistribution policy on R&D-based growth, some of them are overly restrictive from an empirical perspective. For example, in the presence of imperfect credit and insurance markets, redistribution can be growth-enhancing through an “opportunity cost effect” that allows more individuals to invest in education, as in Galor and Zeira (1993), or through an "insurance effect" that provides a certain degree of insurance to entrepreneurs and stimulates innovations and growth, as in García-Peñelosa (2008). Since our paper ignores the important roles of the redistribution policy stressed in the existing literature, the application of our simple framework for assessing the quantitative impact of policy reform is limited.

Second, for analytical tractability, this paper assumes that the rich save a larger proportion of their income than the poor and leave more wealth to their offspring. Although this assumption is empirically supported and improves the tractability of the model greatly, the micro foundation of this assumption should be modeled explicitly. As in Galor and Zeira (1993), the presence of imperfect credit markets and the limited investment opportunities of the poor may generate analogous theoretical results. Nevertheless, the micro foundation of this assumption will be a promising direction for future research.

Third, for analytical tractability, this paper assumes that the product development firms that are inventing new varieties have to incur an R&D expenditure one period in advance of production. In contrast, intermediate goods firms can improve their production efficiency instantaneously through their in-house process innovation. Thus, this asymmetric specification of product development and process innovation improves the tractability of the model greatly without altering the main predictions of this paper. Nevertheless, it will be interesting to consider alternative specifications for R&D activities.

Fourth, since this paper uses a two-period OLG framework, if we employ a straightforward interpretation, one period in our model is interpreted as approximately 30 years. The concept of "short-run" in our model does not match the concept of "short-run" in the real world, making comparing our theoretical results with actual data slightly difficult. Therefore, to evaluate the quantitative impact of redistribution policy on economic growth and inequality more precisely, it is necessary to develop a more elaborate numerical version of the large-scale OLG model with various important elements of the real economy, such as the risk of R&D activities and the presence of imperfect credit and insurance markets.

Appendix

Appendix A: The market-clearing condition for assets

Due to perfect competition in the final goods market, the value of the final goods output is expressed as follows:

$$Y_t = w_t L + N_t P_t X_t.$$

Using (11) and (18), the above equation can be rewritten as follows:

$$Y_t = w_t L + [(1 + r_t)V_{t-1} - V_t]\, N_t + (X_t + Z_t I_t)\, N_t.$$

Substituting (17) and the above equation into (20), we obtain the following expression:

$$w_t L + (1 + r_t)V_{t-1}N_t = \left(c^R_{1,t} + c^R_{2,t}\right)\phi L + \left(c^P_{1,t} + c^P_{2,t}\right)(1 - \phi)L + V_t N_{t+1}.$$

Thus, by substituting (2), (3), and (19) into the above equation, we obtain

$$V_t N_{t+1} - \left[s^R_t \phi L + s^P_t (1 - \phi)L\right] = (1 + r_t)\left\{V_{t-1}N_t - \left[s^R_{t-1}\phi L + s^P_{t-1}(1 - \phi)L\right]\right\}.$$

Because initial assets are given by $V_{-1}N_0 = s^R_{-1}\phi L + s^P_{-1}(1 - \phi)L$, we can obtain the following asset market equilibrium condition:

$$V_t N_{t+1} = s^R_t \phi L + s^P_t (1 - \phi)L.$$

Appendix B: The properties of $\mu\left(N_t, \varphi_{t-1}; \tau\right)$ in (27)

By differentiating (27) with respect to N_t, φ_{t-1} and τ, we obtain

$$\mu_N\left(N_t, \varphi_{t-1}; \tau\right) = \bar{\alpha} f_N(N_t) + \beta\left[\theta(1 - \gamma) f_N\left(N_t\right) + 1\right] \frac{\hat{\alpha}\left(\varphi_{t-1}\right) + \bar{\alpha}\tau}{1 + \tau} > 0,$$

$$\mu_\varphi\left(N_t, \varphi_{t-1}; \tau\right) = \beta\left[\theta(1 - \gamma) f\left(N_t\right) + N_t\right] \frac{(\alpha^R - \alpha^P)}{1 + \tau} > 0,$$

$$\mu_\tau\left(N_t, \varphi_{t-1}; \tau\right) = \frac{\beta\left[\theta(1 - \gamma) f\left(N_t\right) + N_t\right](\alpha^R - \alpha^P)\left(\phi - \varphi_{t-1}\right)}{(1 + \tau)^2} \begin{cases} < 0, & \text{for} \quad \varphi_{t-1} > \phi, \\ = 0, & \text{for} \quad \varphi_{t-1} = \phi, \\ > 0, & \text{for} \quad \varphi_{t-1} < \phi, \end{cases}$$

where $\hat{\alpha}(\varphi_{t-1}) \equiv \alpha^R \varphi_{t-1} + \alpha^P(1 - \varphi_{t-1})$.

Appendix C: The properties of $\lambda(N_t, \varphi_{t-1}; \tau)$ in (28)

The derivation of (28)

Using the definition of φ_t, the growth in rich's savings is expressed as

$$\frac{s_t^R \phi L}{s_{t-1}^R \phi L} = \frac{1}{\varphi_{t-1}} \frac{s_t^R \phi L}{A_{t-1}}. \tag{A.1}$$

Using (5), (6), (9), (14), (17), (18), (19), (21), (22), and the definition of φ_t, the above equation can be rewritten as follows:

$$\frac{s_t^R \phi L}{s_{t-1}^R \phi L} = \frac{1}{\varphi_{t-1}} \frac{Z_t}{Z_{t-1}} \frac{\Gamma^R(N_t, \varphi_{t-1}; \tau)}{N_t}. \tag{A.2}$$

From the definition of φ_t, we obtain

$$\frac{\varphi_t}{\varphi_{t-1}} = \frac{\frac{s_t^R \phi L}{s_{t-1}^R \phi L}}{\frac{A_t}{A_{t-1}}}. \tag{A.3}$$

Thus, by substituting (17), (A.2), and $V_t N_{t+1} = A_t$ into (A.3), we obtain (28).

The properties of (28)

By differentiating (28) with respect to N_t, φ_{t-1} and τ, we obtain

$$\lambda_N(N_t, \varphi_{t-1}; \tau) = \frac{\frac{\alpha^R \alpha^P \beta(\varphi_{t-1} - \phi)}{1+\tau}(1-\sigma) f(N_t)}{\mu(N_t, \varphi_{t-1}; \tau)^2} \begin{cases} > 0, & \text{for} \quad \varphi_{t-1} > \phi, \\ = 0, & \text{for} \quad \varphi_{t-1} = \phi, \\ < 0, & \text{for} \quad \varphi_{t-1} < \phi, \end{cases}$$

$$\lambda_\varphi(N_t, \varphi_{t-1}; \tau) = \frac{\frac{\alpha^R \alpha^P \beta}{(1+\tau)} [\theta(1-\gamma) f(N_t) + N_t] \{f(N_t) + \beta[\theta(1-\gamma) f(N_t) + N_t]\}}{\mu(N_t, \varphi_{t-1}; \tau)^2} > 0,$$

$$\lambda_\tau(N_t, \varphi_{t-1}; \tau) = \frac{\frac{\alpha^R \alpha^P \beta}{(1+\tau)^2} [\theta(1-\gamma) f(N_t) + N_t] \{f(N_t) + \beta[\theta(1-\gamma) f(N_t) + N_t]\} (\phi - \varphi_{t-1})}{\mu(N_t, \varphi_{t-1}; \tau)^2} \begin{cases} < 0, & \text{for} \quad \varphi_{t-1} > \phi, \\ = 0, & \text{for} \quad \varphi_{t-1} = \phi, \\ > 0, & \text{for} \quad \varphi_{t-1} < \phi. \end{cases}$$

Appendix D: The effect of an increase in τ on the NN locus

From (27), since $\mu_N(N_t, \varphi_{t-1}; \tau) > 0$, $\mu_{NN}(N_t, \varphi_{t-1}; \tau) < 0$, $\mu(0, \varphi_{t-1}; \tau) = 0$ and $\lim_{N_t \to \infty} \mu(N_t, \varphi_{t-1}; \tau) = \infty$, evaluating the value of $\mu_N(N_t, \varphi_{t-1}; \tau)$ at the NN locus, we can see that the relationship

$\mu_N(N_t, \varphi_{t-1}; \tau) \in (0,1)$ holds. Thus, by totally differentiating (29) with respect to N_t and τ and evaluating it at the NN locus, we obtain the following expression:

$$\frac{dN_t}{d\tau}|_{NN} = \frac{\mu_\tau(N_t, \varphi_{t-1}; \tau)}{1 - \mu_N(N_t, \varphi_{t-1}; \tau)}, \begin{cases} < 0, & \text{for} \quad \varphi_{t-1} > \phi, \\ = 0, & \text{for} \quad \varphi_{t-1} = \phi, \\ > 0, & \text{for} \quad \varphi_{t-1} < \phi. \end{cases}$$

These results indicate that an increase in τ holding φ_{t-1} constant, decreases (resp., increases) the value of N_t that satisfies $\Delta N_t = \mu(N_t, \varphi_{t-1}; \tau) - N_t = 0$ when $\varphi_{t-1} > \phi$ (resp., $\varphi_{t-1} < \phi$). Therefore, we obtain the following results:

$$\Gamma_\tau(N_t; \tau) \begin{cases} > 0, & \text{for} \quad \varphi_{t-1} > \phi, \\ = 0, & \text{for} \quad \varphi_{t-1} = \phi, \\ < 0, & \text{for} \quad \varphi_{t-1} < \phi. \end{cases}$$

Appendix E: The effect of an increase in τ on $\varphi\varphi$ locus

From (28), since $\lambda_\varphi(N_t, \varphi_{t-1}; \tau) > 0$, $\lambda_{\varphi\varphi}(N_t, \varphi_{t-1}; \tau) < 0$ and $0 < \lambda(N_t, 0; \tau) < \lambda(N_t, 1; \tau) < 1$, evaluating the value of $\lambda_\varphi(N_t, \varphi_{t-1}; \tau)$ at the $\varphi\varphi$ locus, we can see that the relationship $\lambda_\varphi(N_t, \varphi_{t-1}; \tau) \in (0,1)$ holds. Thus, by totally differentiating (31) with respect to φ_{t-1} and τ and evaluating it at the $\varphi\varphi$ locus, we obtain the following expression:

$$\frac{d\varphi_{t-1}}{d\tau}|_{\varphi\varphi} = \frac{\lambda_\tau(N_t, \varphi_{t-1}; \tau)}{1 - \lambda_\varphi(N_t, \varphi_{t-1}; \tau)}, \begin{cases} < 0, & \text{for} \quad \varphi_{t-1} > \phi, \\ = 0, & \text{for} \quad \varphi_{t-1} = \phi, \\ > 0, & \text{for} \quad \varphi_{t-1} < \phi. \end{cases}$$

These results indicate that an increase in τ holding N_t constant, decreases (resp., increases) the value of φ_{t-1} that satisfies $\Delta\varphi_{t-1} = \lambda(N_t, \varphi_{t-1}; \tau) - \varphi_{t-1} = 0$ for $\varphi_{t-1} > \phi$ (resp., $\varphi_{t-1} < \phi$). Therefore, we obtain the following results:

$$Q_\tau(N_t, \varphi_{t-1}; \tau) \begin{cases} < 0, & \text{for} \quad \varphi_{t-1} > \phi, \\ = 0, & \text{for} \quad \varphi_{t-1} = \phi, \\ > 0, & \text{for} \quad \varphi_{t-1} < \phi. \end{cases}$$

In equilibrium, since the relationship $\varphi_{t-1} > \phi$ holds for all t, we can confirm that the relationship $Q_\tau(N_t, \varphi_{t-1}; \tau) < 0$ holds.

Appendix F: Proof of Proposition 1

The characteristic equation of the Jacobian matrix J is given by

$$\Phi(\omega) \equiv \omega^2 - tr(J)\,\omega + |J| = 0, \tag{A.4}$$

where $tr(J)$ and $|J|$ denote the trace and determinant of matrix J. The eigenvalues of the matrix J, ω_1 and ω_2, are given by the solution to this quadratic equation. Calculating $tr(J)$ and $|J|$, we obtain

$$tr(J) = \mu_N^* + \lambda_\varphi^* \in (0, 2),$$

$$|J| = \mu_N^* \lambda_\varphi^* - \mu_\varphi^* \lambda_N^*.$$

In addition, the discriminant of the characteristic equation Δ is given by

$$\Delta \equiv tr(J)^2 - 4|J| = \left(\mu_N^* - \lambda_\varphi^*\right)^2 + 4\mu_\varphi^* \lambda_N^* > 0.$$

Hence, the eigenvalues are necessarily real.

The steady-state E (N^*, φ^*) is a sink, if both eigenvalues of the matrix J, ω_1 and ω_2, have modulus less than 1 (i.e., $\omega_1, \omega_2 \in (-1, 1)$). From (34), suppose that

$$\Phi(1) = 1 - tr(J) + |J| = (1 - \omega_1)(1 - \omega_2) > 0, \tag{A.5}$$

then both eigenvalues, ω_1 and ω_2, must fall on the same side of 1. Since $tr(J) = \omega_1 + \omega_2 \in (0, 2)$, this result means that if $\Phi(1) > 0$, both eigenvalues, ω_1 and ω_2, must lie in $(-1, 1)$ (i.e., $\omega_1, \omega_2 \in (-1, 1)$). Therefore, $\Phi(1) > 0$ ensures that the steady-state E (N^*, φ^*) is a sink.

From (29) and (31), the slope of the NN locus evaluated at steady-state E (N^*, φ^*) is given by $\frac{d\varphi_{t-1}}{dN_t}\mid_{NN}= \frac{1-\mu_N^*}{\mu_\varphi^*}$, whereas the slope of the $\varphi\varphi$ locus is given by $\frac{d\varphi_{t-1}}{dN_t}\mid_{\varphi\varphi}= \frac{\lambda_N^*}{1-\lambda_\varphi^*}$. As depicted in Figure 3, the slope of the NN locus is steeper than that of the $\varphi\varphi$ locus. Thus we have the following inequality:

$$\frac{1 - \mu_N^*}{\mu_\varphi^*} > \frac{\lambda_N^*}{1 - \lambda_\varphi^*}. \tag{A.6}$$

This equation can be rewritten as follows:

$$1 - tr(J) + |J| > 0.$$

Hence, from (A.5), $\Phi(1) > 0$ holds and the steady-state E (N^*, φ^*) is a sink.

Appendix G: Proof of Proposition 2

By totally differentiating (27) and (28) with respect to N^*, φ^*, and τ, we obtain

$$\frac{dN^*}{d\tau} = \frac{\mu_\tau^*(1 - \lambda_\varphi^*) + \lambda_\tau^* \mu_\varphi^*}{\Phi(1)} < 0,$$

$$\frac{d\varphi^*}{d\tau} = \frac{\lambda_\tau^*(1 - \mu_N^*) + \mu_\tau^* \lambda_N^*}{\Phi(1)} < 0,$$

where $\mu_\tau^* \equiv \mu_\tau(N^*, \varphi^*; \tau) < 0$, $\lambda_\tau^* \equiv \lambda_\tau(N^*, \varphi^*; \tau) < 0$ and $\Phi(1) = 1 - tr(J) + |J| = 1 - (\mu_N^* + \lambda_\varphi^*) + \mu_N^* \lambda_\varphi^* - \lambda_N^* \mu_\varphi^* > 0$.

References

[1] Adams, James and Adam Jaffe (1996) Bounding the effects of R&D: An Investigation using matched establishment-firm data, *RAND Journal of Economics* 27(4), 700-721.

[2] Aghion, Philippe and Peter Howitt (2005) Growth with quality-improving innovations: An integrated framework. in Philippe Aghion & Steven Durlauf (eds.) *Handbook of Economic Growth*, 67-110, Elsevier.

[3] Alan, S., , K. Atalay and T. F. Crossley (2015) Do the rich save more? Evidence from Canada, *Review of Income and Wealth* 61(4), 739-758.

[4] Alesina, Alberto and Dani Rodrik (1994) Distributive politics and economic growth, *The Quarterly Journal of Economics* 109(2), 465-490.

[5] Barro, Robert J., and Xavier Sala-i-Martin (2004) *Economic Growth. 2nd edition.* Cambridge, MA: MIT Press.

[6] Becker, Robert A. (1980) On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households, *The Quarterly Journal of Economics* 95 (2), 375–382.

[7] Bozio, Antoine, Carl Emmerson, Cormac O'Dea and Gemma Tetlow (2013) Savings and wealth of the lifetime rich: Evidence from the UK and US, *IFS (Institute for Fiscal Studies) Working Papers* W13/30.

[8] Breyer, Friedrich and Martin Straub (1993) Welfare effects of unfunded pension systems when labor supply is endogenous, *Journal of Public Economics* 50(1), 77-91.

[9] Broda, Christian, Joshua Greenfield and David Weinstein (2006) From groundnuts to globalization: A structural estimate of trade and growth, *NBER Working Papers* 12512, National Bureau of Economic Research.

[10] Benabou, Roland (2000) Unequal societies: Income distribution and the social contract, *American Economic Review* 90(1), 96-129.

[11] Benabou, Roland (2002) Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency?, *Econometrica* 70(2), 481-517.

[12] Berg, Andrew, Jonathan D. Ostry, Charalambos G. Tsangarides, Yorbol Yakhshilikov (2018) Redistribution, inequality, and growth: New evidence, *Journal of Economic Growth* 23(3), 259-305.

[13] Bleaney, Michael, Gemmell Norman and Kneller Richard (2001) Testing the endogenous growth model: public expenditure, taxation, and growth over the long run, *Canadian Journal of Economics* 34(1), 36-57.

[14] Bossmann, Martin, Christian Kleiber and Klaus Wälde (2007) Bequests, taxation and the distribution of wealth in a general equilibrium model, *Journal of Public Economics* 91(7–8) 1247-1278.

[15] Broda, Christian, Joshua Greenfield and David Weinstein (2006) From groundnuts to globalization: A structural estimate of trade and growth, *NBER Working Papers* 12512, National Bureau of Economic Research.

[16] Brunnschweiler, Christa N., Pietro F. Peretto and Simone Valente (2017) Wealth creation, wealth dilution, and population dynamics, *Economic Research Initiatives at Duke (ERID) Working Paper* 258, Duke University.

[17] Caucutt, Elizabeth, Selahattin, Imrohoroğlu and Krishna Kumar (2003) Growth and welfare analysis of tax progressivity in a heterogeneous-agent model, *Review of Economic Dynamics* 6(3), 546-577.

[18] Caucutt, Elizabeth, Selahattin, Imrohoroğlu and Krishna Kumar (2006) Does the progressivity of income taxes matter for human capital and growth?, *Journal of Public Economic Theory* 8(1), 95-118.

[19] Chou, Chien-Fu and Gabriel Talmain (1996) Redistribution and growth: Pareto improvements, *Journal of Economic Growth* 1(4), 505-523.

[20] Chu, Angus C., Yuichi Furukawa and Lei Ji (2016) Patents, R&D subsidies, and endogenous market structure in a Schumpeterian economy, *Southern Economic Journal* 82(3), 809-825.

[21] Chu, Angus C. and Lei Ji (2016) Monetary policy and endogenous market structure in a Schumpeterian economy, *Macroeconomic Dynamics* 20(5), 1127-1145.

[22] Dynan, Karen E., Jonathan Skinner and Stephen P. Zeldes (2004) Do the rich save more?, *Journal of Political Economy* 112(2), 397-444l.

[23] Easterly, William (2007) Inequality does cause underdevelopment: Insights from a new instrument, *Journal of Development Economics* 84(2), 755-776.

[24] Galor, Oded and Joseph Zeira (1993) Income distribution and macroeconomics, *Review of Economic Studies* 60 (1), 35-52.

[25] García-Peñelosa, Cecilia and Jean-François Wen (2008) Redistribution and entrepreneurship with Schumpeterian growth, *Journal of Economic Growth* 13(1), 57-80.

[26] Grossmann, Volker (2007) How to promote R&D-based growth? Public education expenditure on scientists and engineers versus R&D subsidies, *Journal of Macroeconomics* 29(4), 891-911.

[27] Grossmann, Volker (2009) Entrepreneurial innovation and economic growth, *Journal of Macroeconomics* 31(4), 602-613.

[28] Ha, Joonkyung and Peter Howitt (2007) Accounting for trends in productivity and R&D: A Schumpeterian critique of semi-endogenous growth theory, *Journal of Money, Credit and Banking* 39(4), 733-774.

[29] Halter, Daniel, Oechslin Manuel and Josef Zweimüller (2014), Inequality and growth: the neglected time dimension, *Journal of Economic Growth* 19(1), 81-104.

[30] Harrison, Glenn W., Morten I. Lau and Melonie B. Williams (2002) Estimating individual discount rates in Denmark: a field experiment, *American Economic Review* 92(5), 1606-1617.

[31] Howitt, Peter (1999) Steady endogenous growth with population and R&D inputs growing, *Journal of Political Economy* 107(4), 715-30.

[32] Iacopetta, Maurizio, Raoul Minetti and Pietro F. Peretto (2019) Financial markets, industry dynamics and growth, *Economic Journal* 129(621), 2192-2215.

[33] Ji, Lei, Juin-Jen Chang and Chien-Yu Huang (2016) Unionization, market structure, and economic growth, *Southern Economic Journal* 82(3), 935-951.

[34] Laincz, Christopher and Pietro F. Peretto (2006) Scale effects in endogenous growth theory: an error of aggregation not specification, *Journal of Economic Growth* 11(3), 263-288.

[35] Lawrance, Emily C. (1991) Poverty and the rate of time preference: evidence from panel data, *Journal of Political Economy* 99(1), 54-77.

[36] Lindert, Peter (2004). *Growing public: social spending and economic growth since the eighteenth century*, Cambridge, U.K. University Press.

[37] Madsen, Jakob (2008) Semi-endogenous versus Schumpeterian growth models: Testing the knowledge production function using international data, *Journal of Economic Growth* 13(1),1-26.

[38] Madsen, Jakob, James Ang and Rajabrata Banerjee (2010) Four centuries of British economic growth: The roles of technology and population, *Journal of Economic Growth* 15(4), 263-290.

[39] Maebayashi, Noritaka and Kunihiko Konishi. (2021) Sustainability of public debt and inequality in a general equilibrium model, *Macroeconomic Dynamics* 25(4),874-895.

[40] Morimoto, Takaaki and Ken Tabata (2020) Higher education subsidy policy and R&D-based growth, *Macroeconomic Dynamics* 24(8), 2129-2168.

[41] Néstor, Gandelman (2017) Do the rich save more in Latin America?, *The Journal of Economic Inequality* 15(1), 75-92.

[42] OECD (2016) *Entrepreneurship at a Glance 2016* Organisation for Economic Co-operation and Development.

[43] Ostry, Jonathan D., Andrew Berg, Charalambos G. Tsangarides, Yorbol Yakhshilikov (2014) Redistribution, inequality, and growth, *IMF Staff Discussion Note* 14/02.

[44] Pagano Patrizio and Fabiano Schivardi (2003) Firm size distribution and growth, *Scandinavian Journal of Economics* 105(2), 255-274

[45] Peretto, Pietro F. (1998) Technological change and population growth, *Journal of Economic Growth* 3(4), 283-311.

[46] Peretto, Pietro F. (2011) The growth and welfare effects of deficit- financed dividend tax cuts, *Journal of Money, Credit and Banking* 43(5), 835-869.

[47] Peretto, Pietro F. (2015) From Smith to Schumpeter: A theory of take-off and convergence to sustained growth, *European Economic Review* 78(C), 1-26.

[48] Persson, Torsten and Guido Tabellini (2007) Is inequality harmful for growth?, *The American Economic Review* 84(3), 600-621.

[49] Romer, Paul M. (1990) Endogenous technological change, *Journal of Political Economy* 98(5), 71-102.

[50] Saint-Paul, Gilles and Thierry Verdier (1993) Education, democracy and growth, *Journal of Development Economics* 42(2), 399-407.

[51] Saint-Paul, Gilles and Thierry Verdier (1997) Power, distributive conflicts, and multiple growth paths, *Journal of Economic Growth* 2(2), 155-168.

[52] Segerstrom, Paul S. (1998) Endogenous growth without scale effects, *American Economic Review* 88(5), 1290-1310.

[53] Tanzi, Vito and Howell H. Zee (1997) Fiscal policy and long-run growth, *IMF Staff Papers* 44(2), 179-209.

[54] United Nations (2015) *World population prospects: The 2015 Revision, DVD Edition*. United Nations, Department of Economic and Social Affairs, Population Division.

[55] Wigger, Berthold U. (2001) Pareto-improving intergenerational transfers, *Oxford Economic Papers* 53(2), 260-280.

Table 1: Preset parameters

Parameter	Description	Value
ϕ	Population ratio of rich	0.5
β	Relative importance of consumption/bequest	0.3
α^R	Intertemporal preference parameter of rich	0.45
α^P	Intertemporal preference parameter of poor	0.25
$1-\gamma$	Degree of technology spillovers	0.833
σ	Social return to variety	0.2 (0.2-0.8)

Table 2: Calibration of baseline parameters and steady-state results

A. Targeted Variables	Firms/Population $\frac{N^*}{L}$	Per capita output growth G^{y*}
Target values	0.0327	1.02
Baseline simulation results	0.0327	1.02
B. Calibrated Parameters	δ	k
Baseline simulation Parameters	31.722	0.263015
(Identifications)	(Firms/Population)	(Per capita output growth)

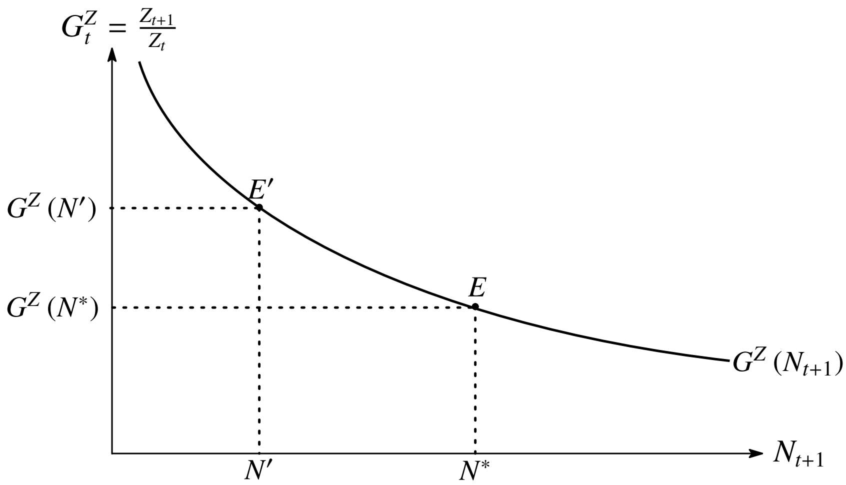

Figure 1: The relationship between N_{t+1} and G_t^Z

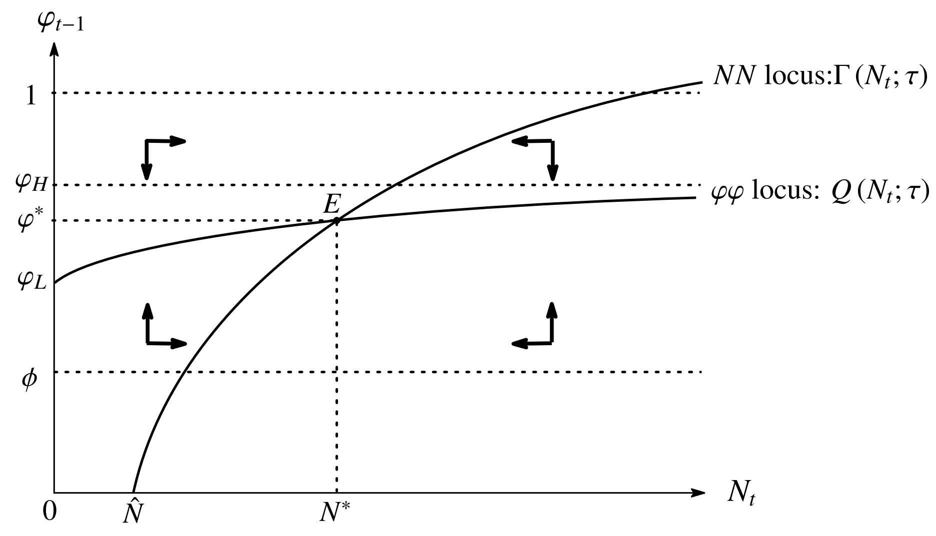

Figure 2: Phase diagram on the (N_t, φ_{t-1}) plane

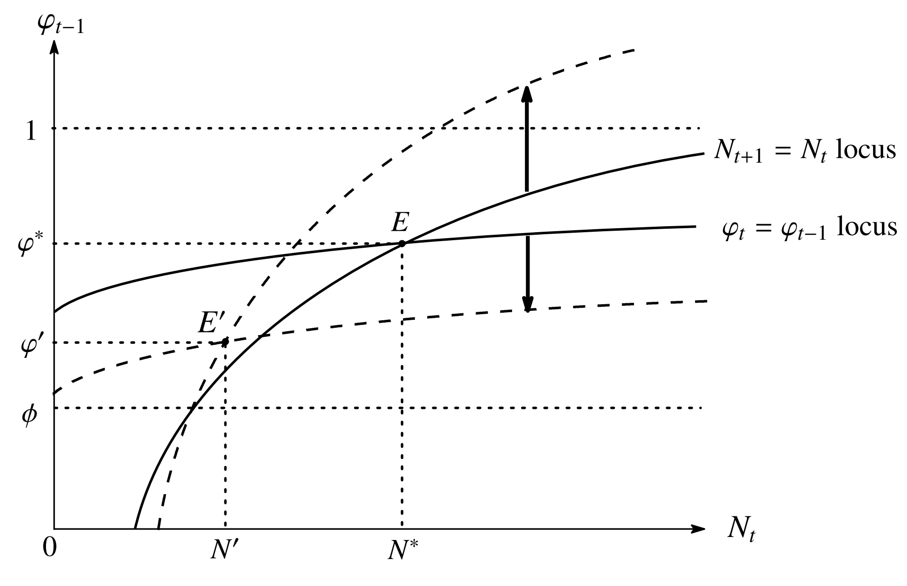

Figure 3: The effect of Changes in tax rate τ

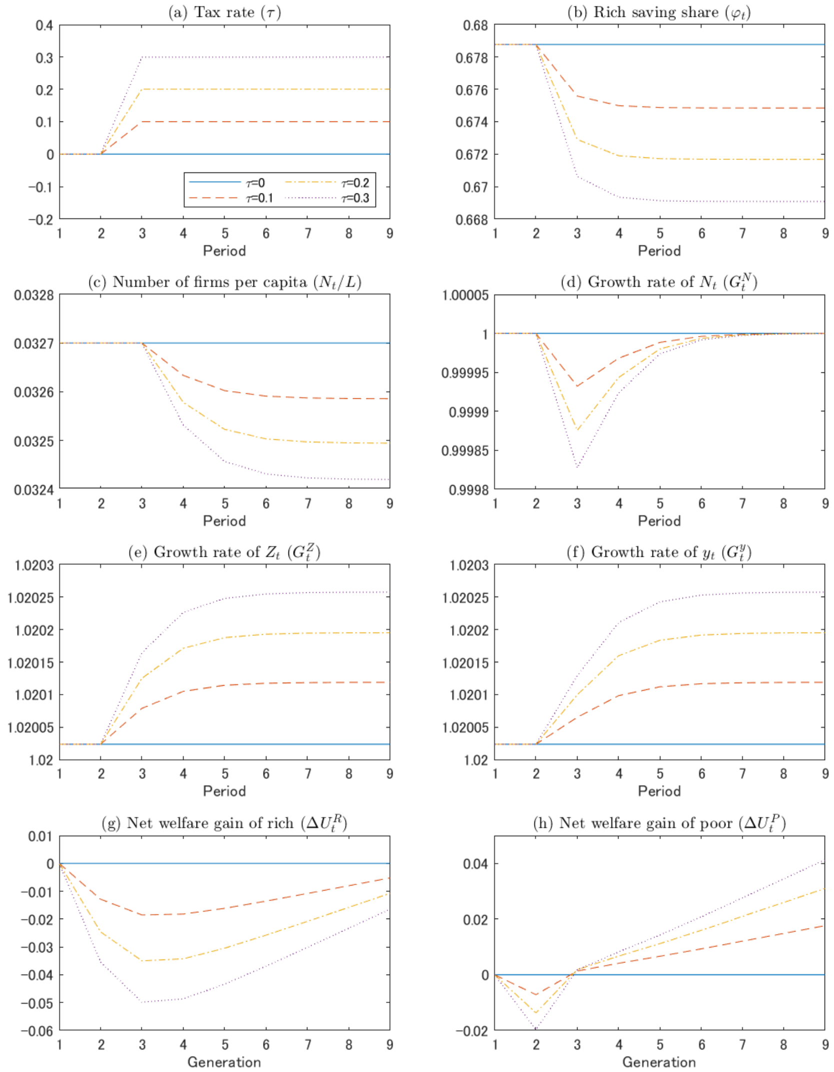

Figure 4: Changes in tax rate τ from 0 to 0.1, 0.2 and 0.3 under σ=0.2

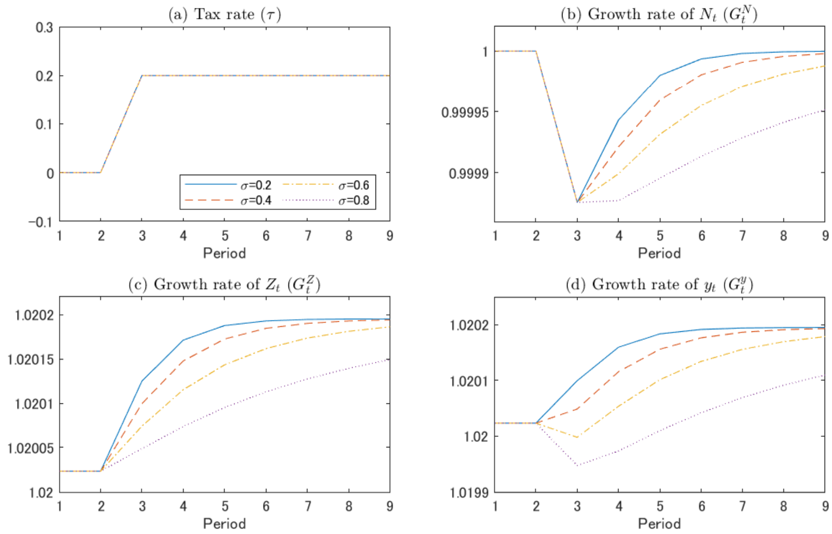

Figure 5: Changes in tax rate τ from 0 to 0.2 under σ=0.2, 0.4, 0.6 and 0.8