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Education Choice and Human Capital Accumulation with Endogenous Fertility Model

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Education Choice and Human Capital Accumulation with Endogenous Fertility Model[†]

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Abstract

This paper sets an endogenous fertility model with endogenous education investment and examines determination of the share of households which select public education, income growth, income inequality, and fertility. Our paper presents consideration of policies of several types such as child allowances and education subsidies for private education and then examines how these policies affect education choice and other outcomes. Results show that a child allowance raises the share of households which select public education. Because of the tax burden, the subsidy for private education can not always raise the share of households which select private education. Furthermore, an increase in the subsidy for private education investment can not always raise the aggregate human capital accumulation even if the share of households selecting private education. The latter half of this paper presents derivation of policy allocations as a result of voting system and describes checking of the robustness of the obtained results.

Keywords: Education choice, Endogenous fertility, Income growth, Income inequality, Subsidy
JEL Classifications: J13, I22, H52

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1. Introduction

Our paper presents examination of how education choice is determined in an endogenous fertility model. Lucas (1988) sets the human capital growth model as the endogenous growth model and shows how income growth can be derived. Glomm and Ravikumar (1992) consider the human capital growth model and how differences of education systems affect income growth and income inequality. Glomm and Ravikumar (1992) present the first reported examination of the education system and human capital accumulation. Glomm and Ravikumar (1992) posit the existence of education systems of two types: public education and private education.[1] For private education, education investment is financed by household income. Therefore, inequality of education investment can exist between households. Rich households can provide large amounts of education investment. However, poor people provide only a slight amount of education investment. This situation brings about income inequality. Ray (2006) and Bar and Basu (2009) show education inequality as deriving from income inequality. These studies demonstrate that income inequality brings about education inequality. However, public education investment is financed by income taxation. It is equally provided to children. Therefore, no inequality of education investment exists. This arrangement diminishes and eliminates income inequality over time. Huw (2000) derives that public education brings benefits for households.

Among recent works, many studies have examined endogenous fertility. Galor and Weil (1996) and Apps and Rees (2004), Van Groezen, Leers and Meijdam (2003), and Van Groezen and Meijdam (2008) are the fundamental studies. In these endogenous fertility models, fertility is derived as a household maximization problem. The child allowance, as a child care policy, can raise fertility and halt the decrease of population.

Reports described above, such as those by Glomm and Ravikumar (1992), van Groezen, Leers and Meijdam and others do not include consideration of the model including both endogenous fertility and endogenous education investment. However, De la Croix and Doepke (2003) consider endogenous education investment in the endogenous fertility model and derive the result by which rich households pay a large amount of education investment and have few children, which is substantially equivalent to the results reported by Becker, Murphy and Tamura (1990): de la Croix and Doepke (2003) derive a tradeoff between education investment and fertility. Work reported by de la Croix and Doepke (2004) examines two education systems in an endogenous fertility model: public education and private education. The model of de la Croix and Doepke (2004) is considered as an endogenous fertility model formed from the Glomm and Ravikumar (1992) model.

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[1] Based on work by Glomm and Ravikumar (1992) many related studies exist. Futagami and Yanagihara (2008) consider public and private education in the model of parental time for children study time. Yasuoka, Katahira and Nakamura (2008) derive income shrinkage in the regime of private education because of the externality of education.

Some works examine how the child allowance policy and subsidy for education affect fertility and human capital accumulation. Zhang (1997), Yasuoka and Miyake (2014), and others set endogenous fertility with a human capital accumulation model and examine how fertility and human capital accumulation are determined by these policies. Results demonstrate that child allowance raises fertility and reduces education investment. However, a subsidy for education investment raises education investment and reduces fertility. This result underscores the tradeoff between quality and quantity of children. Yasuoka (2018), by greatly changing the assumption of household utility function for education investment for children and results obtained by earlier studies, demonstrates the importance of assuming a utility function to assess education investment for children.

However, these related studies described above include consideration of no case in which public education and private education exist simultaneously. In the real economy, public education and private education co-exist; households select which education system is used. Cardak (2004a, 2004b) considers the case in which public education and private education co-exist.[2] Then, depending on household income, some households select private education because the household wants to increase education investment for children. Others use public education because it entails no education costs. Empirical studies reported by Yoshida, Kogure and Ushijima (2009) demonstrate education choice as shown using theoretical analysis.

Our paper presents consideration of an endogenous fertility model with endogenous human capital accumulation that exists simultaneously in a system with both public education and private education. Then we examine how the choice of education system is affected by a child allowance, a subsidy for private education, and a subsidy for public education. Concretely explaining the model setting, we set our model based on work by Glomm and Ravikumar (1992) and de la Croix and Doepke (2004) who consider household heterogeneity.[3] Additionally, we include education choice as considered by Cardak (2004a, 2004b) into our model settings to examine qualitatively how an education subsidy affects average human capital growth and inequality of human capital within households.

Results presented herein are the following. An increase in a subsidy for public education increases the share of households which select the public education system. Conversely, because of tax burdens, an increase in the subsidy for private education can not always raise the share of households which select the private education system. These results might be intuitive. However, the effect of a child allowance raises the share of households which select public education. Moreover, our paper presents an examination of how these policies affect aggregate human capital accumulation and income

[2] Gamlath and Lahiri (2018) set a co-existence model of public and private education. However, Gamlath and Lahiri (2018) assume human capital accumulation inputted not only by public education as school education but also private education as an additional education investment.

[3] Although work by Zhang (1997) and by Zhang and Casagrande (1998) examines how education subsidies affect the human capital growth, Zhang (1997) sets the representative households economy. The model economy has no inequality. Omori (2009), Fanti and Gori (2010), and Fioroni (2010) examine public education effects in an endogenous fertility model. However, these studies include no consideration of education choice.

inequality between two education systems and within one education system. The latter half of this paper presents consideration of policy allocation by a voting system. In addition to these analyses, our manuscript checks the robustness of the obtained results by considering government budget constraints and voting systems of other types. Our paper presents consideration of a subsidy for education investment and for child care. Then these policies are provided in many OECD countries. The results reported herein have rich policy implications.

The remainder of this paper is constructed as follows. Section 2 sets the household model. Section 3 sets the government budget system. Section 4 presents derivation of the education choice. Section 5 considers the voting system and examines how the policy parameters are determined. Section 6 presents the conclusion.

2. Model with Endogenous Fertility

In this model economy, households live in two periods: young and old periods. Individuals' utility functions u_t are assumed as the following form.

$$u_t = \alpha \ln n_t h_{t+1} + (1-\alpha)\ln c_{t+1}, 0 < \alpha < 1, \tag{1}$$

This utility function is given by de la Croix and Doepke (2003) and others who consider education investment in an endogenous fertility model. Also, n_t and h_{t+1} respectively denote the quantity of children (fertility) and the quality of children (human capital stock of children). Individuals care about consumption in old period c_{t+1}. In the young period, the individuals care for the children.

Based on Glomm and Ravikumar (1992), human capital accumulation is assumed as the following form of

$$h_{t+1} = e_t^{\theta} h_t^{1-\theta}, 0 < \theta < 1, \tag{2}$$

where e_t and h_t respectively denote the education investment for children and human capital stock of individuals (parental human capital).[4]

The model economy incorporates heterogeneity of human capital stock h_t among the individuals. The human capital stock is assumed to be distributed in $[\underline{h}_t, \bar{h}_t]$. We define the density function of h_t^i as $f(h_t^i)$. i denotes the index to show the individual.

2.1 Private education

With the individual human capital h_t^i, the lifetime budget constrain can be shown for private education as

$$(1-x)e_t n_t + (z_t - q_t)n_t + \frac{c_{t+1}}{1+r} = (1-\tau-\varepsilon)wh_t^i. \tag{3}$$

[4] Glomm and Ravikumar (1992) consider the school time expended to raise human capital. However, to maintain simplicity, this paper includes no consideration of the school time.

In that equation, z_t denotes the cost to increase the quantity of children. Individuals can obtain q_t as the child allowance. Also, r and w respectively denote the interest rate and the wage rate of effective labor. As described herein, we consider this small open economy. Then r and w are constant over time. Also, x represents the subsidy rate for private education investment. The policy of the subsidy for private education and child allowance is financed by the labor income taxation at tax rate ε. Public education investment is fully financed by the labor income taxation at tax rate τ.

Then, the optimal allocations to maximize utility (1) subject to the budget constraint (3). Human capital accumulation (2) can be derived as

$$n_t^i = \frac{\alpha(1-\theta)(1-\tau-\varepsilon)wh_t^i}{z_t - q_t}, \tag{4}$$

$$e_t^i = \frac{\theta}{1-\theta}\frac{z_t - q_t}{1-x}, \tag{5}$$

$$c_{t+1}^i = (1+r)(1-\alpha)(1-\tau-\varepsilon)wh_t^i. \tag{6}$$

Therein, n_t^i, e_t^i, and c_{t+1}^i respectively represent household allocations by which human capital is h_t^i.

We assume the child care cost and child allowance respectively as $z_t = \bar{z}wh_t^i$ and $q_t = \bar{q}wh_t^i$.[5] Then, the fertility and education investment can be shown as

$$n_t = \frac{\alpha(1-\theta)(1-\tau-\varepsilon)}{\bar{z}-\bar{q}}, \tag{7}$$

$$e_t^i = \frac{\theta w}{1-\theta}\frac{\bar{z}-\bar{q}}{1-x}h_t^i. \tag{8}$$

Then, considering (2) and (8), one can obtain the growth rate of human capital stock at the *i*th household h_t^i, as shown below.

$$\frac{h_{t+1}^i}{h_t^i} = \left(\frac{\theta w}{1-\theta}\frac{\bar{z}-\bar{q}}{1-x}\right)^{\theta} \tag{9}$$

We assume $\frac{\theta w}{1-\theta}\frac{\bar{z}-\bar{q}}{1-x} > 1$ for human capital growth in the long run. Otherwise, the human capital stock converges to zero in the future.

2.2 Public education

In the case of public education, households do not pay for education investment. Income taxation finances public education investment. Then the lifetime budget constraint is shown as the following.

[5] The assumption of the child care cost is necessary in the endogenous growth model. With a fixed child care cost, the child care cost continues to decrease, eventually reaching zero. The assumption of this child care cost in this paper is fundamentally equal to the opportunity cost of the time necessary for child care.

$$(z_t - q_t)n_t + \frac{c_{t+1}}{1+r} = (1 - \tau - \varepsilon)wh_t^i \tag{10}$$

Considering (1) and (10), we can obtain the household optimal allocations in the case of public education as presented below.

$$n_t = \frac{\alpha(1 - \tau - \varepsilon)}{\bar{z} - \bar{q}} \tag{11}$$

$$c_{t+1} = (1+r)(1-\alpha)(1 - \tau - \varepsilon)wh_t^i \tag{12}$$

Considering (2) and public education investment E_t, the human capital growth rate of public education investment can be shown as presented below.

$$\frac{h_{t+1}^i}{h_t^i} = \left(\frac{E_t}{h_t^i}\right)^\theta \tag{13}$$

As shown by (9), human capital accumulation continues to increase in private education. However, (13) shows that the human capital stock converges to the certain value in public education as long as E_t is constant. This result is the same as that reported by Cardak (2004a, 2004b). However, an increase in human capital stock in private education raises tax revenues and E_t increases. Then human capital in public education can continue to increase over time.

3. Government

Government provides public education investment for the households which select the public education system. Public education investment is financed by labor income taxation. It is provided, based on the balanced budget constraint, as

$$N_t^{pub} n_t E_t = N_t \tau w \int_{\underline{h}_t}^{\bar{h}_t} h_t^i f(h_t^i) dh_t^i \;\rightarrow\; E_t = \frac{N_t \tau w H_t}{N_t^{pub} n^{pub}}, \tag{14}$$

where $H_t = \int_{\underline{h}_t}^{\bar{h}_t} h_t^i f(h_t^i) dh_t^i$. Also, n^{pub} represents the quantity of children which households of public education have. This value is given as (11). In addition, N_t^{pub} expresses the number of households selecting public education investment. The household ratio of public education is

$$\frac{N_t^{pub}}{N_t} = F(h_t^*), \tag{15}$$

where N_t denotes the total size of households in t period. Also, $F(h_t^*)$ represents the cumulative distributive function of density function $f(h_t^*)$. Given N_t^{pri} as the household size of private education, $N_t^{pub} + N_t^{pri} = N_t$ is shown. Also, h_t^* expresses the human capital stock, which is indifferent between public education and private education. Then, the households of $[\underline{h}_t, h_t^*]$ select the public education system, as explained in the next section. However, households of $[h_t^*, \bar{h}_t]$ select the private education system.

In addition, the government provides a policy for child allowance and a subsidy for private education investment. Based on the balanced budget constraint, the budget constraint is

$$\bar{q}w\left(\int_{\underline{h}_t}^{h_t^*} h_t^i f(h_t^i)dh_t^i N_t^{pub} n^{pub} + \int_{h_t^*}^{\bar{h}_t} h_t^i f(h_t^i)dh_t^i N_t^{pri} n^{pri}\right) + N_t^{pri} n^{pri} x \int_{h_t^*}^{\bar{h}_t} e_t^i f(h_t^i)dh_t^i = N_t \varepsilon w \int_{\underline{h}_t}^{\bar{h}_t} h_t^i f(h_t^i)dh_t^i. \tag{16}$$

In that equation, n^{pri} denotes the fertility that private education households have, given as (7).

4. Education Choice

In this model, households select the education system: one for public education and the other for private education to maximize their utility.

First, if the households select private education, then the indirect utility function can be derived with (1), (7)–(9) as

$$v_t^{pri} = \alpha \ln \frac{\alpha(1-\theta)(1-\tau-\varepsilon)}{\bar{z}-\bar{q}} + \alpha\theta \ln \frac{\theta}{1-\theta}\frac{\bar{z}-\bar{q}}{1-x} w h_t^i + (1-\alpha)\ln(1+r)(1-\alpha)(1-\tau-\varepsilon)w h_t^i. \tag{17}$$

Second, if the households select the public education, then the indirect utility function can be derived with (1), and (11)–(13) as

$$v_t^{pub} = \alpha \ln \frac{\alpha(1-\tau-\varepsilon)}{\bar{z}-\bar{q}} + \alpha\theta \ln E_t + (1-\alpha)\ln(1+r)(1-\alpha)(1-\tau-\varepsilon)w h_t^i. \tag{18}$$

If $v_t^{pub} < v_t^{pri}$, then the households select the private education. The inequality of $v_t^{pub} < v_t^{pri}$ can be shown as

$$\tau < \frac{\theta(1-\theta)^{\frac{1}{\theta}-1}(1-\tau-\varepsilon)}{1-x}\frac{h_t^i}{H_t}\frac{N_t^{pub}}{N_t}, \tag{19}$$

which is

$$\frac{\tau(1-x)}{\alpha\theta(1-\theta)^{\frac{1}{\theta}-1}(1-\tau-\varepsilon)}\frac{H_t}{F(h_t^*)} < h_t^i. \tag{20}$$

One can obtain the indifference level of human capital stock to satisfy the following equation:

$$\frac{\tau(1-x)}{\alpha\theta(1-\theta)^{\frac{1}{\theta}-1}(1-\tau-\varepsilon)}\frac{H_t}{F(h_t^*)} = h_t^*. \tag{21}$$

Households of $[h_t^*, \bar{h}_t]$ select private education. Households of $[\underline{h}_t, h_t^*]$ select public education. The share of households of public education is given as (15). In the case of private education,

$$\frac{N_t^{pri}}{N_t} = 1 - F(h_t^*). \tag{22}$$

Along the balanced growth path, we obtain $\frac{H_{t+1}}{H_t} = \frac{h_{t+1}^*}{h_t^*}$ and $F(h_t^*) = F(h_{t+1}^*)$. Moreover, the human capital stock of public education converges to the same level among households because of the growth rate of human capital stock of public education (13). Then, $\frac{\underline{h}_{t+1}}{\underline{h}_t}$ and $\frac{\bar{h}_{t+1}}{\bar{h}_t}$ are given respectively as $\left(\frac{E_t}{\underline{h}_t}\right)^{\theta}$ and $\left(\frac{\theta w}{1-\theta}\frac{\bar{z}-\bar{q}}{1-x}\right)^{\theta}$ in the balanced growth path. In the next subsection, we can ascertain how policy parameters affect the education choice, income growth, and income inequality.

4.1 Increase in Public education investment

An increase in τ raises h_t^* because of (21).[6] Then, the size of households which prefer public education increases. Defining $e_t = e(h_t^*)$ as the private education investment that household h_t^* decides, we can obtain the following two cases.

$$e(h_t^*) < E_t \rightarrow \frac{\theta}{1-\theta}\frac{\bar{z}-\bar{q}}{1-x}h_t^* < \frac{\tau w H_t}{F(h_t^*)n^{pub}}, \tag{23}$$

$$e(h_t^*) \geq E_t \rightarrow \frac{\theta}{1-\theta}\frac{\bar{z}-\bar{q}}{1-x}h_t^* \geq \frac{\tau H_t}{F(h_t^*)n^{pub}}. \tag{24}$$

If inequality (23) holds, then an increase in τ raises the aggregate education investment. Aggregate human capital accumulation in $t+1$ period can always increase. Because of the existence of the transfer household from private education to public education, this household can raise education investment. However, if inequality (24) holds, then an increase in τ can not always raise the aggregate education investment and aggregate human capital in $t+1$ period. Because of the existence of the transfer household from private education to public education, this household reduces education investment. Considering (21), we can always obtain $1 < \frac{1}{(1-\theta)^{\frac{1}{\theta}}}$: (24) always holds. Then, the following proposition can be established.

Proposition 1

An increase in τ raises h_t^*: the share of households that select public education increases. Moreover, the average human capital in $t+1$ period can be pulled down.

This proposition can be derived by Cardak (2004a, 2004b). However, the results of a decrease in average human capital accumulation in $t+1$ because of (24) can be derived.

4.2 Increase in child allowance

As shown by (21), an increase in ε with $\bar{q}$ raises h_t^*. Then, the share of households which prefer

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[6] (21) can be changed to $\frac{\tau(1-x)H_t}{\theta(1-\theta)^{\frac{1}{\theta}-1}(1-\tau-\varepsilon)} = h_t^* F(h_t^*)$. The right-hand side of this equation is the increasing function of h_t^* because $F(h_t^*)$ increases with h_t^*. An increase in τ or ε or a decrease in x raises the left-hand side; then h_t^* increases.

public education rises. Then, the following proposition can be established.

Proposition 2

An increase in $\bar{q}$ raises h_t^*: the share of households selecting public education increases.

Generally, an increase in the child allowance raises private education investment. Then we can infer that the share of households which prefer private education increases because the household can afford to pay for private education. However, households consider indirect utility in both education systems. In an endogenous fertility model with education investment, the negative effect of the tax burden raises the share of household or public education. Fertility can be pulled up by the child allowance. Then, the aggregate human capital accumulation can be increased. However, as long as we consider the human capital per capita, the child allowance reduces the human capital stock per capita. Cardak (2004a, 2004b) does not consider the endogenous fertility and child allowance. However, by virtue of the education choice model with endogenous fertility, one can derive the result by which the child allowance can affect education choice.

4.3 Increase in private education subsidy

An increase in the subsidy rate for private education x raises the tax rate ε. If the subsidy effect is larger than the tax effect, that is, $\frac{\tau(1-x)}{\theta(1-\theta)^{\frac{1}{\theta}-1}(1-\tau-\varepsilon)}$ of the left side hand of (21) is decreased by an increase in x and ε, then h_t^* decreases. The share of households which prefer the public education system decreases. These are intuitive results. The subsidy for private education reduces the education burden in private education system. Then, the aggregate human capital accumulation can be pulled up because the private education investment per capita increases. The following proposition can be established.

Proposition 3

An increase in x can reduce h_t^* as long as the tax burden effect is small.

Total differentiation with respect to h_t^*, x, and ε. At the approximation of $x = 0$, one can obtain the following:

$$\frac{dh_t^*}{dx} = \frac{-1 + \frac{1}{1-\tau}\frac{d\varepsilon}{dx}}{\frac{1}{h_t^*} + \frac{f(h_t^*)}{F(h_t^*)}}. \tag{25}$$

The condition to have a positive sign $\frac{dh_t^*}{dx} < 0$ is given as

$$\frac{\int_{h_t^*}^{\bar{h}_t} h_t^i f(h_t^i) dh_t^i}{H_t} < \frac{1}{\alpha\theta\left(1 - F(h_t^*)\right)}. \tag{26}$$

If τ is close to zero, then $\frac{\int_{h_t^*}^{\bar{h}_t} h_t^i f(h_t^i) dh_t^i}{H_t}$ and $F(h_t^*)$ are, respectively, close to one and zero. Then, the above inequality holds.

The aggregate human capital in $t+1$ period $\widetilde{H}_{t+1}$ can be shown as

$$\widetilde{H}_{t+1} = N_t \left(F(h_t^*) n^{pub} \int_{\underline{h}_t}^{h_t^*} (E_t)^\theta (h_t^i)^{1-\theta} dh_t^i + \left(1 - F(h_t^*)\right) n^{pri} \int_{h_t^*}^{\bar{h}_t} \left(\frac{\theta w}{1-\theta} \frac{\bar{z} - \bar{q}}{1-x} h_t^i \right)^\theta (h_t^i)^{1-\theta} dh_t^i \right) \tag{27}$$

$$= N_t \left(F(h_t^*) n^{pub} (E_t)^\theta \int_{\underline{h}_t}^{h_t^*} (h_t^i)^{1-\theta} dh_t^i + \left(1 - F(h_t^*)\right) n^{pri} \left(\frac{\theta w}{1-\theta} \frac{\bar{z} - \bar{q}}{1-x} \right)^\theta \int_{h_t^*}^{\bar{h}_t} h_t^i dh_t^i \right)$$

The population dynamics is

$$N_{t+1} = N_t \left(F(h_t^*) n^{pub} + \left(1 - F(h_t^*)\right) n^{pri} \right). \tag{28}$$

The average human capital in $t+1$ H_{t+1} is

$$H_{t+1} = F(h_t^*) \int_{\underline{h}_t}^{h_t^*} (E_t)^\theta (h_t^i)^{1-\theta} dh_t^i + \left(1 - F(h_t^*)\right) \int_{h_t^*}^{\bar{h}_t} (e_t^i)^\theta (h_t^i)^{1-\theta} dh_t^i$$

$$= F(h_t^*) \frac{\tau w H_t (\bar{z} - \bar{q})}{F(h_t^*) \alpha (1 - \tau - \varepsilon)} \int_{\underline{h}_t}^{h_t^*} (h_t^i)^{1-\theta} dh_t^i + \left(1 - F(h_t^*)\right) \left(\frac{\theta w}{1-\theta} \frac{\bar{z} - \bar{q}}{1-x} \right) \int_{h_t^*}^{\bar{h}_t} h_t^i dh_t^i,$$

that is,

$$\frac{H_{t+1}}{H_t} = \left((\bar{z} - \bar{q})\right) w \left(\frac{\tau}{\alpha(1 - \tau - \varepsilon)} \int_{\underline{h}_t}^{h_t^*} (h_t^i)^{1-\theta} dh_t^i + \frac{1 - F(h_t^*)}{H_t} \frac{\theta}{(1-\theta)(1-x)} \int_{h_t^*}^{\bar{h}_t} h_t^i dh_t^i \right), \tag{29}$$

Zhang (1997) shows that the child allowance reduces human capital in $t+1$ period. Our study can provide the same result as that reported by Zhang (1997). However, Zhang (1997) does not consider the endogenous education choice. This study derives the results by which the child allowance affects education choice. Cardak (2004a, 2004b) considers the education choice. However, Cardak (2004a, 2004b) does not consider endogenous fertility. If child care cost $\bar{z}$ or wage rate w increases, then the average human capital growth rate rises because of a relative decrease in the cost of education as shown by (29).

We consider inequality using this model. In a private education system, we consider the ith household and jth household, which respectively select private education. We define $h_t^{pri,i}$ and $h_t^{pri,j}$ respectively as the human capital in t period in households of the two types. Then, considering (9), we can follow the equation as an inequality within private education.

$$\frac{h_{t+1}^{pri,i}}{h_{t+1}^{pri,j}} = \frac{h_t^{pri,i}}{h_t^{pri,j}} \tag{30}$$

Then, the income inequality within private education does not shrink over time. However, considering (13), income inequality within public education shrinks over time as shown below.

$$\frac{h_{t+1}^{pub,i}}{h_{t+1}^{pub,j}} = \frac{E_t^{\theta}\left(h_t^{pub,i}\right)^{1-\theta}}{E_t^{\theta}\left(h_t^{pub,j}\right)^{1-\theta}} = \left(\frac{h_t^{pub,i}}{h_t^{pub,j}}\right)^{1-\theta} \tag{31}$$

We respectively define $h_t^{pub,i}$ and $h_t^{pub,j}$ as the human capital stock in t period of ith and jth household that select public education. Compared with public education, the inequality of human capital is constant over time in private education because education investment in public education is equally distributed within the group. However, in private education, education investment depends on the household income. Therefore, a household that has more income can give children more education investment.

Moreover, considering (9) and (13), one can obtain income inequality between private education and public education as

$$\frac{h_{t+1}^{pri,i}}{h_{t+1}^{pub,j}} = \frac{\left(\frac{\theta w}{1-\theta}\frac{\bar{z}-\bar{q}}{1-x}\right)^{\theta} h_t^{pri,i}}{\left(\frac{E_t}{h_t^{pub,j}}\right)^{\theta} h_t^{pub,j}} = \frac{\left(\frac{\theta w}{1-\theta}\frac{\bar{z}-\bar{q}}{1-x}\right)^{\theta}}{\left(\frac{E_t}{h_t^{j}}\right)^{\theta}}\frac{h_t^{pri,i}}{h_t^{pub,j}} = \left(\frac{\alpha\theta}{1-\theta}\frac{1-\tau-\varepsilon}{\tau(1-x)}\frac{h_t^{pub,j}}{H_t}\right)^{\theta}. \tag{32}$$

With $\frac{\alpha\theta}{1-\theta}\frac{1-\tau-\varepsilon}{\tau(1-x)}\frac{h_t^{pub,j}}{H_t} > 1$, $\frac{h_{t+1}^{pri,i}}{h_{t+1}^{pub,j}}$ increases over time. Then income inequality between public education and private education rises over time. Consequently, the following proposition can be established.

Proposition 4

With $\frac{\alpha\theta}{1-\theta}\frac{1-\tau-\varepsilon}{\tau(1-x)}\frac{h_t^{pub,j}}{H_t} > 1$, the inequality between private education and public education $\frac{h_{t+1}^{pri,i}}{h_{t+1}^{pub,j}}$ can be magnified.

This is an intuitive result. If H_t is large, then inequality $\frac{h_{t+1}^{pri,i}}{h_{t+1}^{pub,j}}$ shrinks because the fund of public education investment is large and the amount of E_t can be large. Without a child allowance or subsidy for private education, $\frac{\alpha\theta}{1-\theta}\frac{1-\tau-\varepsilon}{\tau(1-x)}\frac{h_t^{pub,j}}{H_t} > 1$ changes to $\frac{\alpha\theta}{1-\theta}\frac{1-\tau}{\tau}\frac{h_t^{pub,j}}{H_t} > 1$; that is, $\tau < \frac{\frac{\alpha\theta}{1-\theta}\frac{h_t^{pub,j}}{H_t}}{1+\frac{\alpha\theta}{1-\theta}\frac{h_t^{pub,j}}{H_t}}$.

Defining τ^* as $\tau = \frac{\frac{\alpha\theta}{1-\theta}\frac{h_t^{pub,j}}{H_t}}{1+\frac{\alpha\theta}{1-\theta}\frac{h_t^{pub,j}}{H_t}}$, then, $\tau < \tau^*$ can obtain $\tau < \frac{\frac{\alpha\theta}{1-\theta}\frac{h_t^{pub,j}}{H_t}}{1+\frac{\alpha\theta}{1-\theta}\frac{h_t^{pub,j}}{H_t}}$. That is, the income inequality $\frac{h_{t+1}^{pri,i}}{h_{t+1}^{pub,j}}$ is magnified. Otherwise, the government expenditure for public education is too large for $\tau > \tau^*$ to hold; income inequality shrinks.

5. Voting

We consider the following welfare function for voting system as the probabilistic voting problem[7] as

$$W = \Omega v^{pub} + (1-\Omega)v^{pri}$$
$$= \ln(1-\tau-\varepsilon) - ln(\bar{z}-\bar{q}) + \Omega\alpha\theta\ln E_t + \alpha\theta(1-\Omega)\ln\frac{\bar{z}-\bar{q}}{1-x}, 0<\Omega<1. \tag{33}$$

Therein, Ω denotes the preference parameter in considering social welfare. Now, because we consider the conflict between public education and private education in a voting system, we omit policy parameter $\bar{q}$. If the government budget constraint is given as (14) and (16), then the optimal policy allocations E_t and x can be derived as presented below.[8]

$$\frac{1}{1-\tau} = \frac{\Omega\alpha\theta}{E_t}\frac{dE_t}{d\tau} \tag{34}$$

$$\frac{1}{1-\varepsilon} = \frac{(1-\Omega)\alpha\theta}{1-x}\frac{dx}{d\varepsilon} \tag{35}$$

The left-hand-side of (34), and (35) show a marginal welfare loss because of the tax burden. However, the right-hand-side of (34), and (35) show marginal welfare gain because of the subsidy. An increase in Ω raises public education E_t and reduces x because of an increase or decrease in the marginal welfare gain.

Concretely, (34) and (35) can be shown by the following equations.

$$\tau = \frac{\Omega\alpha\theta}{1+\Omega\alpha\theta} \tag{36}$$

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[7] If setting $\Omega = F(h_t^*)$, then we consider the welfare function as Benthamian welfare function. Also, (33) is regarded as the more general form.

[8] In this section, we consider h_t^* as fixed variables to derive (34)–(37). If we consider the case in which the policy variables affects h_t^*, for instance, (34) changes to $\frac{1}{1-\tau} = \frac{\Omega\alpha\theta}{E_t}\frac{wH_t}{F(h_t^*)n^{pub}}\left(1-\frac{E_t}{wH_t}\left(n^{pub}\frac{dF}{dh^*}\frac{dh_t^*}{d\tau}+F(h_t^*)\frac{dn^{pub}}{d\tau}\right)\right)$. As long as $1-\frac{E_t}{wH_t}\left(n^{pub}\frac{dF}{dh^*}\frac{dh_t^*}{d\tau}+F(h_t^*)\frac{dn^{pub}}{d\tau}\right)>0$, we can same result in increase in Ω. However, in this section, to avoid complicated analysis, we consider h_t^* as fixed variables. If one considers that h^* is affected by x, then $\frac{dx}{d\varepsilon}$ is given not by $\frac{\varepsilon wH_t}{(1-F(h_t^*))n^{pri}E_t^{Pri}}$, but $\frac{1}{(1-F(h_t^*))n^{pri}E_t^{Pri}}\left(wH_t + xn^{pri}E_t^{Pri}\frac{dF(h_t^*)}{d\varepsilon} - x(1-F(h_t^*))E_t^{Pri}\frac{dn^{pri}}{d\varepsilon} - x(1-F(h_t^*))n^{pri}\frac{dE_t^{Pri}}{d\varepsilon}\right)$. E_t^{Pri} denotes the average private education investment.

$$x = \frac{1 - \frac{\Omega\alpha\theta}{1+\Omega\alpha\theta}}{(1-(1-\Omega)\alpha\theta)\left(1 - \frac{\Omega\alpha\theta}{1+\Omega\alpha\theta}\right) + (1-\Omega)\alpha\theta + \frac{H_t(1-\Omega)\bar{z}\theta}{\left(1-F(h_t^*)\right)\int_{h_t^*}^{\bar{h}_t} h_t^i dh_t^i (1-\theta)}} \tag{37}$$

If the preference for public education, that is, Ω increases, then public education investment is pulled up. This outcome can be checked using (34) and (36). However, although we can obtain the reduced form of optimal x to maximize the welfare function (33), (37) is complicated. An increase in child care cost $\bar{z}$ reduces the subsidy for private education in voting preference. With large $\bar{z}$, education investment reaches a high level. Therefore, because the marginal utility of education investment is small, households do not prefer a subsidy for private education.

Now, we consider public education to maximize the growth rate of average human capital stock $\frac{H_{t+1}}{H_t}$. Without $\bar{q}$, x $\frac{H_{t+1}}{H_t}$ is given as

$$\frac{H_{t+1}}{H_t} = 1 + g = \frac{\tau w\bar{z}}{\alpha(1-\tau)}\int_{\underline{h}_t}^{h_t^*}\left(h_t^i\right)dh_t^i + \frac{1-F(h_t^*)}{H_t}\frac{\theta w\bar{z}}{1-\theta}\int_{h_t^*}^{\bar{h}_t} h_t dh_t^i. \tag{38}$$

The tax rate τ to maximize g can be given as $\frac{dg}{d\tau}$,

$$(1-\tau)^2 = \frac{\alpha(1-\theta)\frac{dh_t^{*1-\theta}}{d\tau}}{H_t\theta\left(-\frac{dF(h_t^*)}{d\tau}\frac{h_t^{*2}}{2} + \left(1-F(h_t^*)\right)h_t^*\frac{dh_t^*}{d\tau}\right)}, \tag{39}$$

that is,

$$\tau = 1 - \sqrt{\frac{\alpha(1-\theta)\frac{dh_t^{*1-\theta}}{d\tau}}{H_t\theta\left(-\frac{dF(h_t^*)}{d\tau}\frac{h_t^{*2}}{2} + \left(1-F(h_t^*)\right)h_t^*\frac{dh_t^*}{d\tau}\right)}}. \tag{40}$$

An increase in α raises the tax rate of public education to maximize the human capital growth rate. This result is the same as that of (36). However, generally speaking, tax rate (40) is higher than that of (36) because (40) does not incorporate consideration of the utility obtained using consumption and fertility.

6. Concluding Remarks

This paper sets an endogenous fertility model with endogenous education investment and examines how the share of households which select a public education system, income growth, income inequality, and fertility are determined. Our paper presents consideration of policies of some types as child allowance, and of education subsidy for public and private education. Moreover, we examine how these policies affect education choice and other outcomes. Results show that a child allowance raises the share of households which select public education. Because of the tax burden, the subsidy for private education can not always raise the share of households which select private education. In addition, an increase in the subsidy for education investment can not always raise the average amount of human capital accumulation.

Policy parameters derived by the voting system represent some interesting results. Intuitively speaking, if the household has no interest for education investment for children, then the public education investment for children decreases because of the voting equilibrium. This intuitively obtained result reflects that the public education investment is at a low level in an aging population because of a decrease in preferences for quality and quantity of children.

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Appendix

A. Integrated Government Budget Constraint

Our paper sets the separate government budget constraint between public education investment and other policies. However, we can consider the government budget constraint that all policies are included in the same government budget constraint. Then, the one policy expenditure increases and the other policy expenditure can be reduced because of the constant tax revenue. This is tradeoff in policies. If we consider the integrated government budget constraint, then the following budget constraint can be shown as

$$\bar{q}w\left(\int_{\underline{h}_t}^{h_t^*} h_t^i f(h_t^i)dh_t^i N_t^{pub} n^{pub} + \int_{h_t^*}^{\bar{h}_t} h_t^i f(h_t^i)dh_t^i N_t^{pri} n^{pri}\right) + N_t^{pri} n^{pri} x \int_{h_t^*}^{\bar{h}_t} e_t^i f(h_t^i)dh_t^i + N_t^{pub} n^{pub} E_t = N_t \tau w \int_{\underline{h}_t}^{\bar{h}_t} h_t^i f(h_t^i)dh_t^i \tag{A.1}$$

The condition of (19) can be changed as

$$\frac{\tau}{1-\tau} < \frac{1}{wH_t}\left(\frac{\theta(1-\theta)^{\frac{1}{\theta}} wF(h_t^*)h_t^*}{1-x} + X\right). \tag{A.2}$$

In that equation,

$$X = \bar{q}w\left(\int_{\underline{h}_t}^{h_t^*} h_t^i f(h_t^i)dh_t^i F(h_t^*) n^{pub} + \int_{h_t^*}^{\bar{h}_t} h_t^i f(h_t^i)dh_t^i \left(1-F(h_t^*)\right) n^{pri}\right) + \left(1-F(h_t^*)\right) n^{pri} x \int_{h_t^*}^{\bar{h}_t} e_t^i f(h_t^i)dh_t^i. \tag{A.3}$$

Considering (A.2) and (A.3), an increase in τ, $\bar{q}$ and x raises h_t^*. The results are the same as those obtained in the case of separated government budget constraint.

B. Median Voter

As described in this paper, we consider the probabilistic voting problem. However, we can consider the other type of voting problem as the median voter problem.

We derive how the policy parameters are determined by the median voting system. First, we consider only the tax rate for public education τ for simplicity. Then, if the median voter prefers the public education, τ can be reduced as follows to maximize utility (18):

$$\tau = \frac{\theta}{\alpha+\theta}. \tag{B.1}$$

The tax rate for public education is decreased using a decrease in the preference for quantity and quality of children α. An increase in θ raises the income tax rate τ for public education investment.

Next, we consider median voting to set the tax rate for public education τ and tax rate for child allowance ε. Then, we can obtain the following, as

$$\tau = \frac{\alpha\theta(\bar{z} - A)}{\alpha A + (1 + \alpha\theta)(\bar{z} - A)}, \tag{B.2}$$

$$\varepsilon = \frac{\alpha A}{\alpha A + (1 + \alpha\theta)(\bar{z} - A)}, \tag{B.3}$$

where

$$A = \frac{N_t \varepsilon \int_{\underline{h}_t}^{\overline{h}_t} h_t^i f(h_t^i) dh_t^i}{\int_{\underline{h}_t}^{h_t^*} h_t^i f(h_t^i) dh_t^i \, N_t^{pub} n^{pub} + \int_{h_t^*}^{\overline{h}_t} h_t^i f(h_t^i) dh_t^i \, N_t^{pri} n^{pri}}. \tag{B.4}$$