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Effects of Changes in Exchange Rate Volatility on Short-run Equilibrium in International Oligopoly

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Abstract

This paper investigates exchange rate volatility in an international oligopolistic market in a foreign country that accepts affiliate firms through foreign direct investment. The affiliate firms must procure intermediate products from their overseas parent firms. We derive a Cournot equilibrium of a market in which affiliate firms compete with local firms under foreign exchange rate uncertainty. In equilibrium, we show that affiliates aggressively expand output and the *ex-post* expected profits and *ex-ante* certainty equivalence of the affiliates' profits increase / decrease with a rise in exchange rate risk when the relative risk aversion coefficient is small / large.

Keywords: risk aversion, exchange rate volatility, short-run equilibria, and international oligopoly

JEL classification: G32, L13, L12

1. Introduction

The bubble of the Japanese economy burst in the early 1990s; since then, the country has faced a rapidly aging population and declining birth rate. Consequently, as the domestic markets for various products and services have shrunk in Japan, Japanese manufacturers have sought to expand exports to compensate for reduced domestic sales. Globalization, however, has progressed tremendously since the early 1990s. To compete with the low costs of rival overseas manufacturers, Japanese manufacturers have pursued foreign direct investments (*FDI*).

As Lahiri and Ono (2004) indicate, "many Japanese firms make foreign direct investments in China as labour costs there are much lower than those in Japan, ...in such cases the commodities produced in the host country are exported in their entirely to a third country (consuming country)."

In a 2018 regional analysis report, however, JETRO (Japan External Trade Organization) states that ASEAN member countries have become increasingly appealing both as bases of production and consumer markets for Japanese firms, and the direct investments made by Japanese firms have made a great deal of progress on both fronts. [1] In addition, another recent regional analysis report by JETRO notes that annual direct investment by Japanese firms in China has increased, reaching about 135 billion US dollars, and 60% of Japanese firms plan to expand business into China within the next two years to enhance their sales in the Chinese market.

Japanese manufacturers that make *FDI* must export intermediate production inputs to their overseas affiliates. They also remit a portion of their profits from their affiliates back

[1] See the regional analysis report by JETRO in 2018 (Fujie, 2018), for more information.

to Japan. As such, they face exchange rate uncertainty. Japanese firms must choose their outputs in overseas markets based on an *ex-ante expectation* of the exchange rate.

For parent manufacturers with a shrinking domestic market, it is important to earn an allotment of their affiliate's profits in the foreign country. To facilitate this, many governments offer manufacturers either partial or total tax exemption for remittances or dividends from overseas affiliates [2]. Currently, Japan is one of the biggest creditor countries in the world. Therefore, it has become increasingly important for the Japanese government to induce affiliates in foreign countries to remit dividends or profits to their multinational parent firms. Hasegawa and Kiyota (2017) positively explore the effect of the dividend exemption system on profit repatriation by Japanese multinational firms. They explore the effect of the recent transition by the Japanese government from a worldwide income tax system to a territorial tax system (with dividend exemption) on profit repatriation by Japanese multinational firms. By using unique confidential survey data from Japanese multinational corporations, they find that the response of Japanese multinationals to the dividend exemption has been heterogeneous. Thus, they find that foreign affiliates with a large stock of retained earnings in the previous year or before the tax reform were more responsive to the tax reform and significantly increased dividend payments to their parent firms. They also find that dividend payments by the affiliates also became more responsive to withholding tax rates on dividends levied by the government of the host country after the tax reform.

However, the trade war between the United States and China has continued. Both governments have been engaging in tit-for-tat responses to each other by levying tariffs

[2] Indeed, the Japanese government introduced a Foreign Dividend Exclusion system in 2009 that exempts dividends remitted by Japanese-owned foreign affiliates to their parent firms from home-country taxation.

on imports from the opponent country. In addition, there is a so-called "Brexit" problem; the possibility of the UK leaving the EU without a deal has risen with the birth of the Boris Jonson cabinet in the UK. If this happens, the economy of the UK may be damaged. Consequently, the businesses and economies of the world may suffer losses, and foreign exchange markets will be confused and become uncertain. Even in such an environment, parent firms need to compete through their affiliates in foreign oligopolistic markets. In such a case, the extent of relative risk aversion to exchange rate volatility by the parent firm has a crucial impact on the production strategy of its affiliate firm, which procures parts or intermediate goods from its parent firm and remits a portion of its profits to the parent firm.

Lahiri and Ono (2004) analyze trade and industrial policies, including FDI; local content requirements (LCR); and the effect of these government policies on the economic welfare of the market equilibria in a multi-country trade theoretic framework in the presence of Cournot oligopolistic interdependence in production under oligopoly. However, they do not consider the exchange rate uncertainty of the present floating exchange rate system, which affects the behaviors of firms and trade as a result of the competition among global markets.

In the literature, there are many studies that investigate the effect of exchange rate volatility on FDI decisions by multinational firms and the resultant market outcomes. For example, Owen and Perrakis (1988) consider Cournot duopoly competition in a foreign market composed of an international firm that supplies a good under perfect competition in its domestic market and also supplies the same good in a foreign market. They also consider a foreign firm that supplies the same good as the international firm does in a foreign duopolistic market. They create a two-stage model. In the first stage, *ex-ante,* both firms simultaneously choose their own outputs (the total output the international firm supplies to domestic and foreign markets and the output of the foreign firm for the foreign market) to maximize the expected their own profits before the exact value of the exchange rate random variable has been observed. In the second stage, the exchange rate variable is realized, and only the international firm allocates *ex-post* the given total output—chosen in the first stage—between the domestic and foreign markets as a function of the observed value of the exchange rate. They solve the problem backward; they solve the second stage of the game and then the first by incorporating the outcome of the second stage and the expectation with respect to the exchange rate *ex-post*. However, they do not cope with FDI explicitly or oligopoly competition; they assume that both firms are risk neutral and do not consider the effect of the extent of risk aversion by the firms on the equilibrium outcome.

Sung and Lapan (2000) investigate, in their seminal work, how exchange-rate uncertainty affects the foreign investment decision of a *risk-neutral* multinational firm (MNF) *in a monopoly setting*. They show that the MNF would open only one plant in its home country or a foreign country, under the assumption that the firm can open plants, each with decreasing average costs, in two different countries. However, under uncertainty (under a mean-preserving spread exchange rate distribution or uniform exchange rate distribution), they demonstrate that under sufficiently large exchange rate volatility, the firm can increase its expected profit by opening several plants. They also show that if the MNF faces a competitor in a foreign market, then the exchange rate risk induces the MNF to open plants in both markets, consequently preventing entry by the local competitor.

Lahiri and Mesa (2006) explore the effects of exchange rate volatility in both the host country and the parent country on host-government policy related to the local content requirement (LCR) on export-oriented foreign direct investment (FDI) in the context of an oligopolistic market in a third country using *a Brander and Spencer (1987) type trade model*. Namely, they assume that there are identical *domestic risk-neutral* firms and identical *foreign risk-averse* firms in the domestic (host) country, and they *compete in a Cournot oligopolistic market* for a homogeneous good *in a consuming third country,* where there are no producers for the good. Hence, they do not examine how a change in the volatility of exchange rates affects the behavior of affiliate and host-country firms that compete with one another and their resulting equilibrium outcomes in the host country oligopolistic market.

Under the assumption that the exchange rates follow log-normal distributions, Lahiri and Mesa (2006) show that an increase in the volatility of foreign exchange rates decreases optimal LCR levels both under free entry and exit for the foreign firm and when the number of foreign firms is fixed. They also find that the government uses a less strict LCR policy when the number of foreign firms is endogenous than when it is exogenous.

There are few studies that consider the effect of exchange rate risk on international oligopolistic competition among FDI affiliate firms in a foreign market in which affiliates procure important intermediate goods or parts from parent companies and repatriate part of their profit. Therefore, in this paper, we consider an international oligopoly model with an oligopolistic market in a foreign country. The international firms compete with local firms in a host country's oligopolistic market through FDI in affiliate firms.

Particularly, we explore how the volatility of exchange rate risk affects the behaviors of affiliate and foreign firms, depending on their level of risk aversion, in a foreign oligopolistic market. We investigate both effects in the absence of free entry and exit by affiliate firms.

We assume that the affiliates procure all intermediate goods or parts to produce their final goods from their parent firm in the home country. We suppose that they also have to repatriate a portion of the profits earned in the foreign market. We do not consider any policies by the government, such as LCR, tariffs, or production subsidies for domestic firms in the host country, that may affect the economic welfare of the equilibrium outcome.

We consider a Cournot oligopolistic market game in a host country that accepts n affiliate firms through FDI from the home country. First, we derive equilibria in the case in which the number of affiliates, n, is exogenously given under the assumption that the exchange rates follow log-normal distributions, as Lahiri and Mesa (2006) assume. We investigate how the changes in exchange rate volatility on the international oligopolistic market affect equilibria outcomes.

The rest of this paper is organized as follows. In section 2, we present our model and we derive an equilibrium in the absence of the free entry and exit of affiliate firms. We also examine the properties of the equilibrium outcomes in the equilibria. In section 3, we explore how a change in the volatility of the exchange rate affects the equilibrium outcomes in the absence of free entry and exit by affiliate firms. Finally, section 5 concludes the paper.

2. Model

Here, we consider an international oligopoly model with the oligopolistic market in a foreign country. The international home firms (*IH* firms hereafter) compete in a foreign host country (country 2) oligopolistic market through their affiliate firms (A firms hereafter) using foreign direct investment (*FDI*). Furthermore, we assume A firm i internally reserves its profit from the foreign market equilibrium at *the retained earnings rate* of s $(0 < s < 1)$ and remits its profit at *the repatriation rate* of $1 - s$ from the foreign market to the head office of its *IH* firm in the home country. We thus derive a Cournot equilibrium under foreign exchange rate uncertainty when the number of *IH* firms, n, is either exogenous or endogenous. Then, we explore the effects of foreign exchange rate volatility on equilibrium outcomes.

Assume there are two countries: country 1 (home country) and 2 (foreign country). In country 2's oligopoly, n international firms from country 1 compete in an oligopolistic market in country 2 through their affiliate companies (*A* firms) with m foreign firms (*F* firms).

Each *IH* firm in country 1 has a constant return to scale technology by $c_i^H, i = 1, \cdots, n$, indicated by the home currency.

We assume that an international firm supplies its product to both a domestic market and foreign market. Although each *IH* firm procures its parts or intermediate goods from the home country for products on its domestic market, their affiliate firms in country 2 import all parts or intermediate goods from country 1. Each foreign firm supplies its product domestically and procures all parts or intermediate goods from country 2. We only focus on *the market competition in country 2* between *A* and *F* firms, since *A* firms choose the outputs they supply to the market in the home country independently of the outputs sold to the foreign market.

Figure 1 depicts the competition among A and F firms in an oligopolistic foreign market under exchange rate uncertainty. It also shows the relationship between each parent firm in the home country and its affiliate firm.

[Insert here Figure 1]

Foreign firms have constant returns to scale technology, and their marginal and average common cost is given by $c^F \equiv c_j^F, j = 1, \cdots m$, as indicated by the foreign currency.

Each A firm in country 2 incurs marginal cost $\tilde{c}^A$ to produce its product, which is a random variable because it depends on the exchange rate between the home and host countries' currencies $\tilde{\epsilon}$. It is based on the *belief of the parent firm, IH, or its affiliate firm, A,* so it is exogenous to the model. $\tilde{\epsilon}$ is assumed to be a log-normally distributed random variable; that is, $\tilde{\epsilon} = \exp(\tilde{X}), \tilde{X} \sim N(\mu, \sigma^2)$. We also assume that $\ln 2 \approx 0.693\,15 > \sigma^2 > 0.0095$.[4] We assume that country 2's government never imposes a tariff on inputs imported from country 1.

$$\tilde{c}^A = c^H / \tilde{\epsilon} \tag{1}$$

Then, it is well known that the mean and variance of $\tilde{\epsilon}$ are given by

$$\mu_{\tilde{\epsilon}} = \exp(\mu + \sigma^2/2) \tag{2}$$

and

$$\sigma_{\tilde{\epsilon}}^2 = \mu_{\tilde{\epsilon}}^2 (e^{\sigma^2} - 1) \approx 9.\,545\,3 \times 10^{-3} \times \mu_{\tilde{\epsilon}}^2 < \mu_{\tilde{\epsilon}}^2 \text{ for } \ln 2 \approx 0.693\,15 > \sigma^2 > 0.0095. \tag{3}$$

We assume that the mean of exchange rate $\tilde{\epsilon}$,[5]

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[4] We estimate μ and σ^2 from the exchange rate of the Japanese Yen per unit of a foreign currency (i.e., the Chinese yuan, Indian rupee, Thai baht, Malaysian ringgit, or Korean 100 won), as $\tilde{\epsilon} = \exp(\tilde{X}), \tilde{X} \sim N(\mu, \sigma^2)$ using the Monthly Foreign Exchange Quotation Data published by the Mizuho Bank Corporation in Japan. All of the estimated sample variances, $\widehat{\sigma^2}$, of these currencies are included in open interval $(0.01, 0.03)$. Namely, under our assumption that $\tilde{\epsilon}$ is log-normally distributed, we can estimate that the variance of $\ln \tilde{\epsilon}$ is very small. Hence, we assume that $0.0095 < \sigma^2 < \ln 2$ to satisfy the assumption $\boldsymbol{a}_\epsilon > 0$. Thereafter, we add an essential parameter, $\boldsymbol{a}_\epsilon$. The Monthly Foreign Exchange Quotation Data published by the Mizuho Bank Corporation in Japan is available on its website: https://www.mizuhobank.co.jp/market/csv/m_quote.csv.

[5] We estimate $\widehat{\mu_\epsilon}$, using the same data as the estimations for μ and σ^2. All of the estimated sample means of exchange rate $\widehat{\mu_\epsilon}$ for these currencies are included in open interval $(2, 29)$. Therefore, we out the assumption.

$$\mu_{\tilde{\epsilon}} > 2. \tag{4}$$

We can easily derive

$$E_{\tilde{\epsilon}}[1/\tilde{\epsilon}] = \exp(-\mu + \sigma^2/2) = \frac{\mu_{\tilde{\epsilon}}}{e^{2\mu}}, \tag{5}$$

$$Q^{FM} \equiv_{i=1}^{n} q_i^A +_{j=n+1}^{n+m} q_j^F = Q^A + Q^F, \tag{6}$$

$$p^F = a^F - Q^{FM} = a^F -_{i=1}^{n} q_i^A -_{j=1}^{m} q_j^F. \tag{7}$$

As mentioned in the introduction, the *A* firm remits a $(1-s)$ portion of its profit to the head office in home country 1. Therefore, the head office is interested in the amount of the expected remittance from its affiliate. Hence, we can define the amount of remittance to the head office of *IH* firm from its affiliate in foreign country 2 as

$$\pi_i^{IH} \equiv (1-s)\tilde{\epsilon}(p^F - \tilde{c}^A)q_i^A$$

$$= (1-s)\tilde{\epsilon}(a^F -_{i=1}^{n} q_i^A -_{j=1}^{m} q_j^F - \tilde{c}^A)q_i^A$$

$$= (1-s)\tilde{\epsilon}(a^F - Q^A - Q^F - c^H/\tilde{\epsilon})q_i^A, i = 1, \dots, n. \tag{8}$$

From (6), (7), and (8), the certainty equivalence of the profit of *A* firm i is given by

$$E_{\tilde{\epsilon}}[CE\pi_i^{IH}] = (1-s)E_{\tilde{\epsilon}}[CE\pi_i^A]$$

$$= (1-s)\{E_{\tilde{\epsilon}}[\tilde{\epsilon}(p^F - \tilde{c}^A)q_i^A] - \gamma SD_{\tilde{\epsilon}}[\tilde{\epsilon}(p^F - \tilde{c}^A)q_i^A]\}$$

$$= (1-s)[\boldsymbol{a}_{\epsilon}\{a^F - Q^A - Q^F\} - (1-\gamma)c^H]q_i^A, \tag{9}$$

where $E(\cdot)$ and $SD(\cdot)$ stand for expectation and standard deviation operators, respectively, and

$$\boldsymbol{a}_{\epsilon} = \mu_{\varepsilon} - \gamma\sigma_{\epsilon},$$

where γ is a relative risk averse coefficient. Throughout this paper, we assume that $0 <$

$\gamma < 1$.[6] We can interpret $\boldsymbol{a}_\epsilon$ as *the home currency compensation coefficient against the exchange rate risk* because it devaluates the mean of the exchange rate of one unit of foreign currency, as indicated by home currency, by the relative risk averse coefficient of the head office of the affiliate firm.

We assume that

$$\boldsymbol{a}_\epsilon = \mu_{\tilde{\epsilon}} - \gamma\sigma_{\tilde{\epsilon}} \ > 0. \tag{10}$$

From (3), this assumption is equivalent to

$$0 < \gamma < (e^{\sigma^2} - 1)^{-1/2} \equiv \overline{\gamma}(\sigma^2), \tag{11}$$

because $\boldsymbol{a}_\epsilon = \mu_{\tilde{\epsilon}} - \gamma\sigma_{\tilde{\epsilon}} = \mu_{\tilde{\epsilon}}(1 - \gamma(e^{\sigma^2} - 1)^{1/2})$.[7]

3. Equilibrium Derivation

In this section, we first derive a Cournot game equilibrium in a foreign market, where free entry or exit by affiliate firms in not available due to regulation, such as a foreign direct investment control by the host government, and the number of affiliates in the foreign market is exogenously given. We also examine properties of the equilibrium outcome.

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[6] A representative CRRA (Constant Relative Risk Averse) utility function of wealth, w, is $u(w) = w^{1-\gamma}$ for $0 < \gamma < 1$. We can ascertain that $u(w)$ is a concave increasing fuction in w for $0 < \gamma < 1$, and the RRA (Relative Risk Averse) measure is $-\frac{u''(w)w}{u'(w)} = \gamma$. If $\gamma > 1$, a representative CRRA utility function of w is $u(w) = w_0 - w^{-(\gamma-1)}$ for $\gamma > 1$, where w_0 is initial wealth. We also ascertain that $u(w)$ is a concave increasing fuction in $w^{-(\gamma-1)}$ for $\gamma > 1$, and RRA=$-\frac{u''(w)w}{u'(w)} = \gamma$. However, an organization or individual with this type of utility function is a public utility foundation or a person living on unearned income. Here, we assume that a parent private IH firm and its affiliate firm are constant relative risk averse, so it seems natural that they have the former CRRA utility function, $u(w) = w^{1-\gamma}$ for $0 < \gamma < 1$.

[7] Note that the upper bound of γ is obtained by assumption (11), which guarantees $\boldsymbol{a}_\epsilon > 0$ for $0.0095 < \sigma^2 < \ln 2$. $0 < \overline{\gamma}(\sigma^2) < 10.23$ because $\overline{\gamma}(\sigma^2)$ is decreasing in σ^2 and $\sigma^2 > 0.0095$, as will be shown in section 3. However, we assume that $0 < \gamma < 1$ because the parent IH firm and its affiliate firm have the constant relative risk averse utility described in footnote 6.

Here, A firm i chooses q_i^A to maximize the certainty equivalence of *ex-ante* profit $E_{\tilde{\epsilon}}[CE\pi_i^A]$. The first order condition for q_i^A of A firm i is given by

$$\frac{\partial E_{\tilde{\epsilon}}[CE\pi_i^A]}{\partial q_i^A} = \boldsymbol{a}_\epsilon\{a^F - Q^A - Q^F - q_i^A - (1-\gamma)c^H\} = 0,\ i = 1,\ldots,n. \tag{12}$$

The profit of foreign firm j is defined by

$$\pi_j^F = (p^F - c^F)q_i^F. \tag{13}$$

From (6) and (13), the first order condition for q_j^F of foreign firm j is given by

$$\frac{\partial \pi_j^F}{\partial q_j^F} = a^F - Q^A - Q^F - c^F - q_j^F = 0,\ j = 1,\ldots,m. \tag{14}$$

Adding (12) and (14) on i and j, respectively, we obtain

$$(1-s)\boldsymbol{a}_\epsilon\{na^F - (n+1)Q^A - nQ^F\} - (1-s)n(1-\gamma)c^H = 0$$

and

$$ma^F - mQ^A - (m+1)Q^F - mc^F = 0.$$

Solving the above two equations w.r.t. Q^A and Q^F, we obtain

$$Q^{*A} = \frac{n}{m+n+1}\{a^F + mc^F - (m+1)(1-\gamma)c^H/\boldsymbol{a}_\epsilon\} \tag{15}$$

and

$$Q^{*F} = \frac{m}{m+n+1}\{a^F - (n+1)c^F + n(1-\gamma)c^H/\boldsymbol{a}_\epsilon\}. \tag{16}$$

Since q_i^A and q_j^F are symmetrical in $i = 1,\ldots,n$ and $j = 1,\ldots,m$, respectively, from (15) and (16), we get

$$q^{*A} \equiv q_i^{*A} = \frac{1}{m+n+1}\{a^F + mc^F - (m+1)(1-\gamma)c^H/\boldsymbol{a}_\epsilon\} \tag{17}$$

and

$$q^{*F} \equiv q_j^{*F} = \frac{1}{m+n+1}\{a^F - (n+1)c^F + n(1 - \gamma)c^H/\boldsymbol{a}_\epsilon\}. \tag{18}$$

Substituting (15) and (16) into (6), we obtain equilibrium prices in the foreign market:

$$p^{*F} = \frac{1}{m+n+1}\{a^F + mc^F + n(1-\gamma)c^H/\boldsymbol{a}_\epsilon\}. \tag{19}$$

From (9), (17), and (19), the certainty equivalent of the expected profit of affiliate firm i in the foreign market and IH firm i in the home country at equilibrium are given by

$$E_{\tilde{\epsilon}}[CE\pi_i^{*A}] = E_{\tilde{\epsilon}}[\tilde{\varepsilon}(p^{*F} - \tilde{c}^A)q_i^{*A}] - \gamma SD_{\tilde{\epsilon}}[\tilde{\varepsilon}(p^{*F} - \tilde{c}^A)q_i^{*A}]$$
$$= \boldsymbol{a}_\epsilon(q^{*A})^2, \tag{20}$$

and

$$E_{\tilde{\epsilon}}[CE\pi_i^{*IH}] = (1-s)E_{\tilde{\epsilon}}[CE\pi_i^{*A}], \tag{21}$$

where $\boldsymbol{a}_\epsilon = \mu_{\tilde{\epsilon}} - \gamma\sigma_{\tilde{\epsilon}}$, $\sigma_{\tilde{\epsilon}}$ and γ stand for the standard deviation of $\tilde{\epsilon}$ and the relative risk aversion coefficient, respectively.

The *ex-post* expected profit of affiliate firm i in the foreign market and *IH* firm i in the home country market at the short-run equilibrium are

$$E_{\tilde{\epsilon}}[\pi_i^{*A}] = E_{\tilde{\epsilon}}[\tilde{\varepsilon}(p^{*F} - \tilde{c}^A)q_i^{*A}]$$
$$= E_{\tilde{\epsilon}}[\tilde{\varepsilon}(a^F - Q^{*A} - Q^{*F} - c^H/\tilde{\epsilon})q_i^{*A}]$$
$$= [\frac{\mu_\epsilon}{m+n+1}(a^F + mc^F + n(1-\gamma)c^H/\boldsymbol{a}_\epsilon) - c^H] \times q^{*A}, \tag{22}$$

and

$$E_{\tilde{\epsilon}}[\pi_i^{*IH}] = (1-s)E_{\tilde{\epsilon}}[\pi_i^{*A}], \tag{23}$$

The *ex-post* expected profit of firm j in the foreign market at the short-run equilibrium is given by

$$\pi^{*F} = (p^{*F} - c^F)q_j^{*F} = (q_j^{*F})^2. \quad (24)$$

The proof of the proposition is provided in the appendix.

Proposition 1

Suppose *that* $c^H < \boldsymbol{a}_\epsilon c^F$.[8] *The equilibrium output of the affiliate firm is greater than the equilibrium output of the foreign firm, that is,* $q^{*A} > q^{*F}$. *Suppose that the exchange risk is very small; thus,* $0 < \sigma^2 < \ln 2 \approx 0.693\,15$. Then, the following relationship holds among *the equilibrium ex-post expected profit, the equilibrium certainty equivalence of the affiliate firm and equilibrium ex-post expected profit of the foreign firm: If* $0 < \gamma < 1 < \overline{\gamma}(\sigma^2)$ or $\ln 2 \le \sigma^2$, $0 < \gamma < \overline{\gamma}(\sigma^2) \le 1$, *and* $a^F + mc^F > (m + n + 1)(\overline{\gamma}(\sigma^2) - \gamma)c^H/\boldsymbol{a}_\epsilon$, *then* $\pi^{*F} < E_{\tilde{\epsilon}}[CE\pi_i^{*A}] < E_{\tilde{\epsilon}}[\pi_i^{*A}]$.

From (17) and (18), we see that the affiliate firm *A* expands its output more aggressively when parent firm *IH* has a larger value of γ (the extent of relative risk aversion to exchange rate volatility). Thus, $(1-\gamma)/\boldsymbol{a}_\epsilon$ becomes smaller when the exchange risk σ^2 ($\sigma^2 < \ln 2 \approx 0.693\,15$) is so small because the numerator $(1-\gamma)$ decreases faster than the denominator, $\boldsymbol{a}_\epsilon$, of the ratio $(1-\gamma)/\boldsymbol{a}_\epsilon$ as the value of γ becomes larger. Then, foreign firm *F* shrinks its output due to the strategic substitute property and the *ex-ante* certainty equivalence of the profit of affiliate firm *A* also decreases as $\boldsymbol{a}_\epsilon$ decreases. However, $E_{\tilde{\epsilon}}[\pi_i^{*A}]$, the *ex-post* expected profit of affiliate firm *A*, and $E_{\tilde{\epsilon}}[\pi_i^{*IH}]$, the profit of its parent firm *IH*, increases from (23). Consequently,

[8] If this inequality does not hold, then *IH* firms would not want to buy intermediate inputs from home country 1 under the exchange rate risk.

we see that $E_{\tilde{\epsilon}}[\pi_i^{*A}] > E_{\tilde{\epsilon}}[CE\pi_i^{*A}] > \pi^{*F}$.

4. Effect of Change in Exchange Rate Risk on Equilibrium Outcomes

In the following, we explore how a change in the volatility of the exchange rate affects the equilibrium outcome.

We posit a lemma before the presenting results. From (2) and (3), we have

$\frac{\partial \mu_{\tilde{\epsilon}}}{\partial \sigma^2} = \exp(\mu + \sigma^2/2)/2 = \mu_{\tilde{\epsilon}}/2 > 0$

$\frac{\partial \sigma_{\tilde{\epsilon}}^2}{\partial \sigma^2} = e^{\sigma^2} \exp(2\mu + \sigma^2) + (e^{\sigma^2} - 1) \exp(2\mu + \sigma^2) = (2e^{\sigma^2} - 1) \exp(2\mu + \sigma^2) > 0.$

Hence, when $\tilde{\epsilon}$ is a log-normally distributed random variable, that is, $\tilde{\epsilon} = \exp(\tilde{X})$, $\tilde{X} \sim N(\mu, \sigma^2)$, we see that $\frac{\partial \mu_{\tilde{\epsilon}}}{\partial \sigma^2} > 0$, $\frac{\partial \sigma_{\tilde{\epsilon}}^2}{\partial \sigma^2} > 0$. That is, an increase in the exchange rate risk increases the mean and variance of the exchange rate.

We also have

$\frac{\partial \boldsymbol{a}_\epsilon}{\partial \sigma^2} = \mu_{\tilde{\epsilon}}(1 - \gamma(2e^{\sigma^2} - 1)(e^{\sigma^2} - 1)^{-1/2})/2 \gtreqless 0 \Leftrightarrow \gamma \ \ (e^{\sigma^2} - 1)^{1/2}/(2e^{\sigma^2} - 1) \equiv \gamma^*(\sigma^2)$.

From (10), we can easily derive the next lemma without proof.

Lemma 1 *For any* $0.0095 < \sigma^2 < \ln 2$, *if* $\overline{\gamma}(0.0095) = (\exp(0.0095) - 1)^{-1/2} \approx 10.23 > \gamma \geq \gamma^*(\sigma^2)$, *then* $\frac{\partial}{\partial \sigma^2} \boldsymbol{a}_\epsilon \leq 0$. *If* $0 < \gamma < \gamma^*(\sigma^2)$, *then* $\frac{\partial}{\partial \sigma^2} \boldsymbol{a}_\epsilon > 0$.

From Lemma 1, an increase in exchange rate risk increases (decreases) the home currency compensation coefficient against exchange rate risk $\boldsymbol{a}_\epsilon$ when the relative risk averse coefficient, γ, is small (large).

We denote, by $\overline{\gamma}(\sigma^2) \equiv (e^{\sigma^2} - 1)^{-1/2}$, the upper bound of γ, given by the assumption (11). We can easily show that $\gamma^*(\sigma^2) < 1 < \overline{\gamma}(\sigma^2) \equiv (e^{\sigma^2} - 1)^{-1/2} < \overline{\gamma}(0.0095) \approx 10.23$ for any σ^2 such that $\ln 2 > \sigma^2 > 0.0095$. The last inequality is from (11).

Thus, we see that

$$\frac{\partial}{\partial \sigma^2}\gamma^*(\sigma^2) = \frac{e^{\sigma^2}(3 - 2e^{\sigma^2})}{2(2e^{\sigma^2} - 1)^2(e^{\sigma^2} - 1)^{1/2}} \gtreqless 0 \Leftrightarrow$$

$$\sigma \quad \ln 3 - \ln 2 = 0.405\,47 \text{ for } \ln 2 > \sigma^2 > 0.0095,$$

and

$$\frac{\partial}{\partial \sigma^2}\overline{\gamma}(\sigma^2) = \frac{\partial}{\partial \sigma^2}((e^{\sigma^2} - 1)^{-1/2}) = -\frac{1}{2}\frac{e^{\sigma^2}}{\left(e^{\sigma^2} - 1\right)^{\frac{3}{2}}} < 0,$$

$$\text{for } \ln 2 > \sigma^2 > 0.0095.$$

Therefore, the upper bound of γ, $\overline{\gamma}(\sigma^2)$, that guarantee $\boldsymbol{a}_\epsilon > 0$ does not affect the optimal choice of output q^{*A}. Only $\gamma^*(\sigma^2)$ affects it because we assume that $0 < \gamma < 1$.

Now, we present the next lemma.

Lemma 2 *The upper bound of γ given by assumption (11), $\overline{\gamma}(\sigma^2)$, is decreasing in exchange risk σ^2, but the threshold for the direction of the change of the home currency compensation coefficient against exchange rate risk $\boldsymbol{a}_\epsilon$ in Lemma 1, $\gamma^*(\sigma^2)$, is inceasing / decreasing in exchange risk σ^2,* when $0.0095 < \sigma^2 < \ln 3 - \ln 2 \approx 0.405\,47$ / $\ln 3 - \ln 2 < \sigma^2 < \ln 2$.

Figure 2 illustrates the properties of $\gamma^*(\sigma^2)$ and $\overline{\gamma}(\sigma^2)$ in Lemma 2. Note that the difference between $\overline{\gamma}(\sigma^2)$ and $\gamma^*(\sigma^2)$ narrows as exchange rate risk σ^2 increases.

[Insert here Figure 2]

A firm with a very small relative risk estimate does not have a high home currency compensation coefficient against the exchange risk, but a firm with a large relative risk estimate does have a high home currency coefficient when the exchange rate risk becomes large enough. Considering the results of the two lemmata together, we can conclude that an increase in the exchange risk increases the extent of risk averseness by the firm, which shrinks the firm's estimate of the home currency compensation coefficient against the exchange risk.

By using Lemma 1, we conduct a comparative statics analysis of the equilibrium outputs, expected firm profits, and the certain equivalence of profits and price on the volatility of exchange rate σ^2.

Next, we conduct a comparative analysis of the equilibrium outcome derived above and volatility exchange rate σ^2. We begin with a comparative statics analysis of exchange rate volatility σ^2.

We present the next proposition. The proof of the proposition is provided in the appendix..

Proposition 2

Suppose that the exchange risk is distributed within a smaller range; thus, $0.0095 < \sigma^2 < \ln\ 2 \approx 0.693\,15$.

If the parent IH firm is strongly risk averse (i.e., $0 < \gamma^*(\sigma^2) < \gamma < 1 \leq \overline{\gamma}(\sigma^2)$*) / weakly risk averse (i.e. ,*$0 < \gamma \leq \gamma^*(\sigma^2) < 1 < \overline{\gamma}(\sigma^2)$*), then the equilibrium output of the affiliate A firm (*q^{*A}*) and the total output of affiliate firms (*Q^{*A}*) decrease / do not* *decrease as the volatility of the exchange rate (σ^2) increases.*

If the parent IH firm is strongly risk averse, (i.e., $0 < \gamma^*(\sigma^2) < \gamma < 1 < \overline{\gamma}(\sigma^2)$) / weakly risk averse, (i.e., $0 < \gamma \leq \gamma^*(\sigma^2) < 1 < \overline{\gamma}(\sigma^2)$), then the equilibrium output of the foreign F firm (q^{*F}), total output of foreign firms (Q^{*F}), and equilibrium price in the foreign market (p^{*F}) increase / do not increase as the volatility of the exchange rate (σ^2) increases.

From the above proposition, when the relative risk aversion coefficient is smaller than a threshold, $\gamma^*(\sigma^2)$, we see that if the volatility of the exchange rate increases, then each *A* firm aggressively expands its output into the foreign market. Therefore, Q^{*A} increases enough so that the equilibrium price decreases if foreign *F* firms decrease their outputs, q^{*F}, to mitigate the reduction in price because the number of A firms, n, is given in this case. Hence, Q^{*F} decreases because they are strategic substitute in Cournot competition. As the increase effect of Q^{*A} surpasses that of the decrease effect of Q^{*F}, the total equilibrium output, $Q^{*A} + Q^{*F}$, increases and p^{*F} decreases.

In the following, we examine how the change in the exchange rate risk affects the *ex-post* expected profits of firms as well as their certainty equivalence.

We can present the next proposition. The proof of the proposition is provided in the appendix.

Proposition 3

Suppose that the exchange risk is distributed within a smaller range; thus, $0.0095 < \sigma^2 < \ln 2 \approx 0.693\,15$. Suppose also that the market share of the foreign firms is relatively larger than that of the affiliate firms: $m+1 \geq n$ and $a^F + mc^H > (m+1)(1-\gamma)c^H/\boldsymbol{a}_\epsilon$.

If the relative risk averse coefficient, γ, *of the parent IH firm is small (i.e.,* $0 < \gamma \leq \gamma^*(\sigma^2) < 1 < \overline{\gamma}(\sigma^2)$*), then the equilibrium ex-post expected profits of the affiliate A firm and the parent IH firm,* $E_{\tilde{\epsilon}}[\pi_i^{*A}]$ and $E_{\tilde{\epsilon}}[\pi_i^{*IH}]$*, increase as the volatility of the exchange rate,* σ^2*, increases. However, if the relative risk averse coefficient,* γ*, of the parent IH firm is large (i.e.,* $\frac{3(m+1)-n}{2(m+1)-n} \cdot \gamma^*(\sigma^2) < \gamma < 1$*), then the equilibrium ex-post expected profits of the affiliate A firm and parent IH firm,* $E_{\tilde{\epsilon}}[\pi_i^{*IH}]$ *and* $E_{\tilde{\epsilon}}[\pi_i^{*A}]$*, decrease as the volatility of the exchange rate,* σ^2*, increases.*

If the relative risk averse coefficient, γ*, of the parent IH firm is small (i.e.,* $\gamma^*(\sigma^2) < \gamma < 1$ *) / large (i.e.,* $\gamma^*(\sigma^2) < \gamma < 1 < \overline{\gamma}(\sigma^2)$ *), then the equilibrium certainty equivalence of their profits,* $E_{\tilde{\epsilon}}[CE\pi_i^{*IH}]$ and $E_{\tilde{\epsilon}}[CE\pi_i^{*A}]$*, increase / decrease as the volatility of the exchange rate,* σ^2*, increases. If the relative risk averse coefficient,* γ*, of the parent IH firm is large (i.e.,* $\gamma^*(\sigma^2) < \gamma < 1 < \overline{\gamma}(\sigma^2)$*) / small (i.e.,* $0 < \gamma \leq \gamma^*(\sigma^2) < 1 \leq \overline{\gamma}(\sigma^2)$*), then the equilibrium ex-post expected profit of the foreign F firm,* π^{*F}*, increases / does not increases as the volatility of the exchange rate,* σ^2*, increases.*

The proposition states that exchange rate volatility σ^2 affects the equilibrium ex-post expected profits, equilibrium certainty equivalence of the affiliate firms' profits, and equilibrium expected profits of foreign firms (when the number of foreign firms is relatively more than the affiliate firms, i.e., $m+1 \geq n$). Thus, the total market share of the affiliate firms will be smaller than that of the foreign firms.

Note that the signs for $\frac{\partial}{\partial\sigma^2} E_{\tilde{\epsilon}}[\pi_i^{*IH}]$ and $\frac{\partial}{\partial\sigma^2} E_{\tilde{\epsilon}}[\pi_i^{*A}]$, and those for $\frac{\partial}{\partial\sigma^2} E_{\tilde{\epsilon}}[CE\pi_i^{*IH}]$

and $\frac{\partial}{\partial \sigma^2} E_{\tilde{\varepsilon}}[CE\pi_i^{*A}]$, are positive / negative when the relative risk averse coefficient, γ, of the parent IH firm is small (i.e., $0 < \gamma \leq \gamma^*(\sigma^2) < 1$) / large *(*i.e. $\gamma^*(\sigma^2) < \gamma < 1 < \overline{\gamma}(\sigma^2)$*)* in the above proposition in the equilibrium. The ex post expected profit of foreign firms decrease / increase when the relative risk averse coefficient, γ, of the parent IH firm is small (i.e., $0 < \gamma \leq \gamma^*(\sigma^2) < 1$) / large *(*i.e. $\gamma^*(\sigma^2) < \gamma < 1 < \overline{\gamma}(\sigma^2)$*)* as the volatility of the exchange rate, σ^2, increases.

5. Conclusion

We considered an international oligopoly model with an oligopolistic market in a foreign country. The international parent firms (*IH* firms) compete in a foreign host country's oligopolistic market through their affiliates using *FDI*. Furthermore, we assume *A* firm (affiliate firm) i internally reserves its profit at foreign market equilibrium at the ratio of s $(0 < s < 1)$ and remits its profit at the ratio of $1 - s$ from the foreign market to the head office of its parent firm *IH* in the home country. We derived a Cournot equilibrium under foreign exchange rate uncertainty for when the number of *IH* firms, n, is exogenously given. Then, we explored the effects of foreign exchange rate volatility on the equilibrium outcome.

In the equilibrium, we show that if the measure of the relative risk aversion of *IH* firms is small affiliate firms *A* aggressively expand their outputs, while the foreign *F* firms defensively decrease their outputs and equilibrium price on the foreign market. However, if γ is large, *IH* firms are severely reluctant to expand due to the increase on the exchange rate risk. Further, when the total number of foreign firms is more than that of the affiliates firms minus one and the relative risk aversion coefficient is small/large at equilibrium, the *ex-post* expected profits and the *ex-ante* certainty equivalence of profits for the affiliate firms increase/decrease, but the expected profits for the foreign firms decrease/increase.

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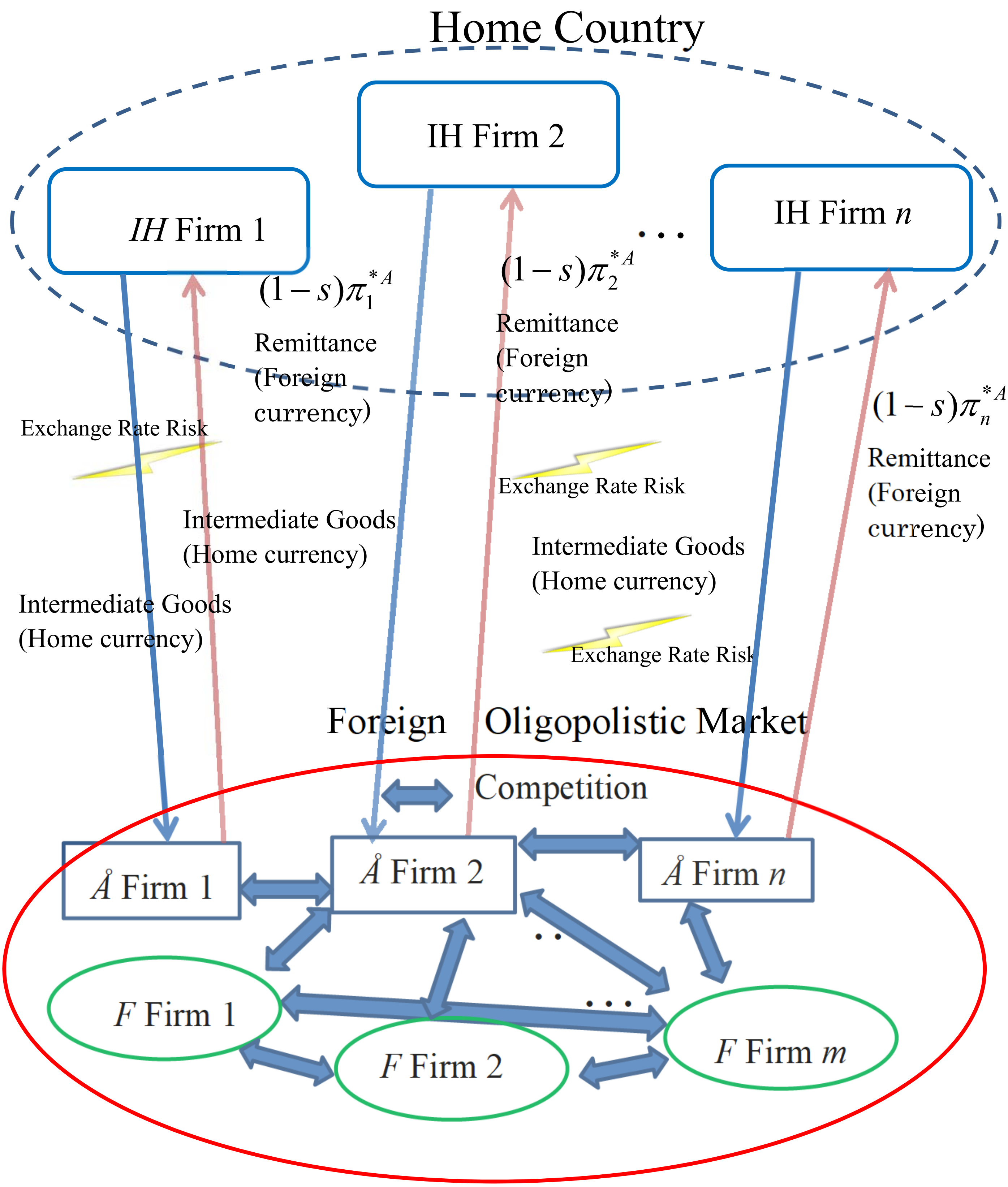
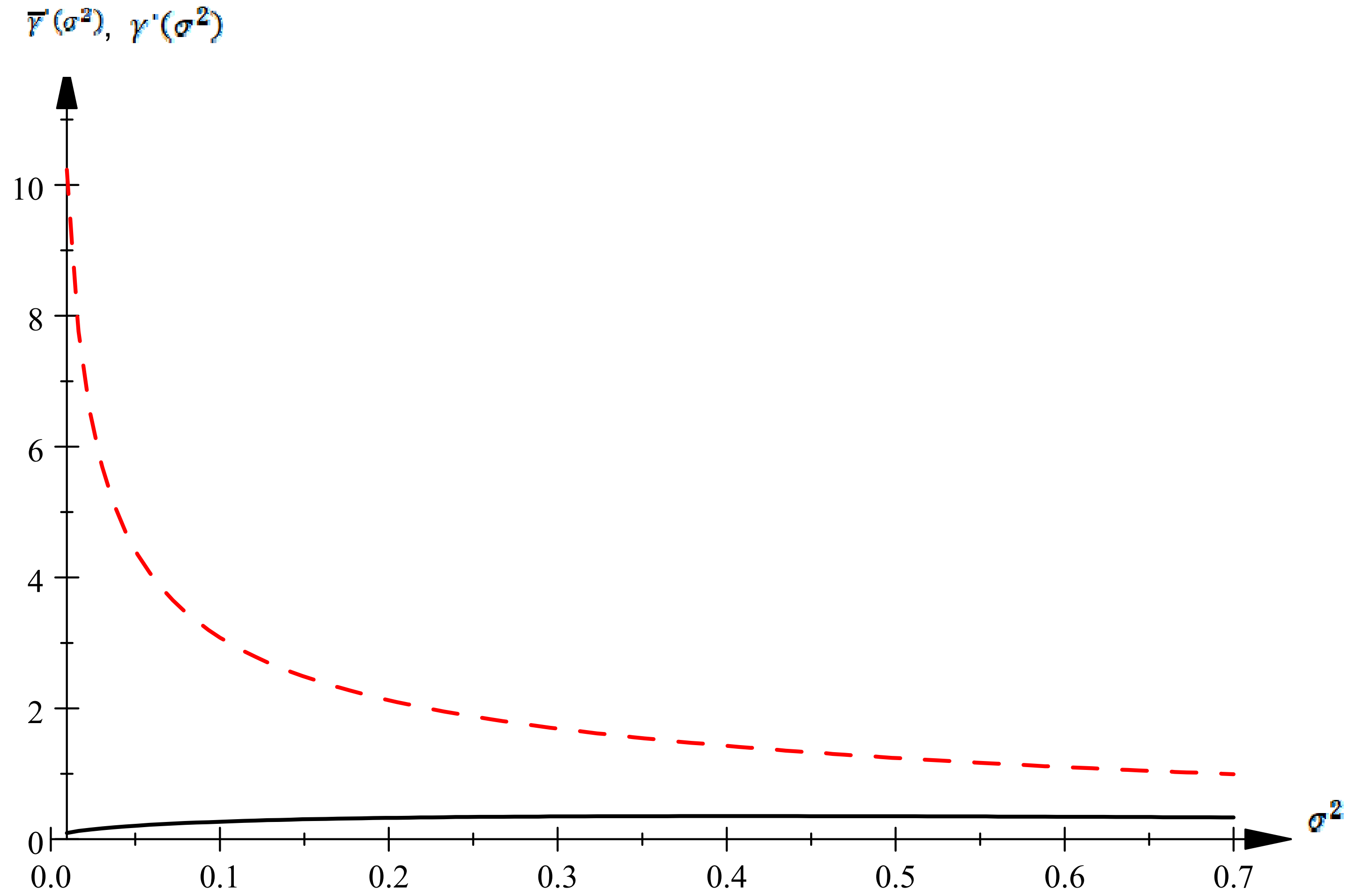


Figure 1 Oligopolistic Competition among Affiliate and Foreign firms in a Foreign Market



$\gamma^*(\sigma)$ Black solid line, $\gamma^*(\sigma) = (\exp(\sigma) - 1)^{1/2}/(2\exp(\sigma) - 1)$, $\overline{\gamma}(\sigma) = (\exp(\sigma) - 1)^{-1/2}$ Dotted Light Red line,

Figure 2 $\overline{\gamma}(\sigma^2), \gamma^*(\sigma^2)$