DISCUSSION PAPER SERIES

Discussion paper No.205

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Kohei Daido

(School of Economics, Kwansei Gakuin University)

Tomoya Tajika

(Department of Law and Economics, Hokusei Gakuen University)

March 2020

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho

Nishinomiya 662-8501, Japan

Abstention by Loss-Averse Voters*

Kohei Daido[†] Tomoya Tajika[‡]

March 5, 2020

Abstract

This paper builds a two-candidate election model, in which voters are loss averse and face uncertainty about whether their preferred candidate is supported by a majority. Even without costs for voting, abstention may occur when voters have expectations-based reference-dependent preferences, as in Kőszegi and Rabin (2006, 2007). The model shows that loss aversion leads to the equilibrium in which abstention is more likely as an election becomes more competitive and the abstention rate of voters who prefer a minority candidate is higher than for those who prefer a majority candidate.

JEL Codes: D72, D91

Keywords: Abstention, Expectations-Based Reference-Dependent Preferences, Loss Aversion, Voting

*We thank Takeshi Murooka for helpful comments. The first author acknowledges financial support from JSPS KAKENHI Grant Number JP19K01568 and JP18H03640. The second author acknowledges financial support JSPS KAKENHI Grant Number JP19K13644.

[†]School of Economics, Kwansei Gakuin University, daido@kwansei.ac.jp

[‡]Department of Law and Economics, Hokusei Gakuen University, t-tajika@hokusei.ac.jp

1 Introduction

Abstention is a consistent feature in elections. For example, the July 2019 election of the House of Councillors in Japan achieved a voter turnout of 48.48%, which is the second lowest turnout in national elections since 1947.[1] One obvious reason for abstention is that voters incur costs from voting, thus abstaining if the marginal cost of voting is greater than the marginal benefit. This is more likely to happen in larger elections because one voter's contribution to the outcome of the election is reduced. Although the costs to voting can explain abstention, it is difficult to explain why some people vote even in large elections.

To solve this paradox, following the literature on voting participation, we provide a new rationale for voter abstention by focusing on a prominent behavioral aspect: loss aversion. Loss aversion means voters are more sensitive to losses from election outcomes than to proportional gains. We analyze a two-candidate election model in which voters have expectations-based reference-dependent preferences (EBRDPs) as in Kőszegi and Rabin (2006, 2007).

In our model, in which voting is costless, each individual is certain of their preferred candidate but uncertain about which candidate is more popular, although they receive partial information about the popularity of candidates.[2] After receiving this information, each individual decides whether to vote. It is easily demonstrable that all individuals prefer to vote rather than to abstain if they do not incur any cost for voting.[3] However, even without incurring costs, abstention could still occur if voters are loss averse in respect to a psychological loss incurred by comparing their realized outcome with their expected possible outcomes. Under loss aversion, abstention can serve as a device to alleviate voters' expected losses from uncertainty over the results of the election. In large elections, the marginal benefit of voting is low but still effective when voters do not incur any costs. However, loss aversion introduces another effect that may overweigh this marginal benefit

[1]This result was announced by the Ministry of Internal Affairs and Communications. The National Diet of Japan is composed of two houses: House of Representatives and House of Councillors.

[2]This structure follows Goeree and Großer (2007), Krasa and Polborn (2009), and Taylor and Yildirim (2010), although these authors assume a cost in voting.

[3]This is because rational individuals vote only when the marginal benefit of voting exceeds the marginal cost of voting. This holds even if we consider the above uncertainty related to the competitiveness of an election.

from voting. Loss-averse voters dislike uncertainty over the outcome of an election and feel the loss from unexpectedly losing. This feeling of loss is more severe when the election is more competitive because the uncertainty of the outcome is higher. To avoid this feeling, voters abstain.

This result contrasts sharply with that under a costly voting model, in which abstentions are rarer in competitive elections. For example, a positive relationship between competitiveness and abstention rates was observed in an official election in Mexico in 1988. Although this election was more competitive than the 1982 presidential election, the abstention rate was also higher.[4] Our comparative statics also show that the abstention tendency is higher for the individuals who prefer the losing candidate. This is because the effect of loss aversion is more severe for the voters supporting the minority candidate than those supporting the majority candidate.

Abstention has been primarily studied through costly voting models, such as by Ledyard (1981, 1984).[5] In the literature on costless voting with abstention, Feddersen and Pesendorfer (1996) focus on an asymmetric of information among voters and propose the theory of the swing voter's curse. When voters face uncertainty about which is their preferred candidate, less-informed voters abstain to prevent their poor decisions from affecting the election outcome, even when voting is costless.[6] In contrast to these approaches, our model considers the behavioral aspect of voting by incorporating voters' EBRDPs. Additionally, rather than factoring in the swing voter's curse, in which voters are uncertain about their preferences, the voters in our model know their preferences but are uncertain about the aggregate preference distribution.

The notion of reference dependence was originally investigated by Kahneman and Tversky (1979) and models of EBRDPs are developed by Kőszegi and Rabin (2006, 2007). Although many studies have applied Kőszegi and Rabin's (2006, 2007) approach, to the best of our knowledge, few researchers have studied the influence of voters' loss aversion in competitive elections based on

[4] See Larmer (1988) for more details.

[5] Although much of the literature uses pivotal voter models, several studies employ an ethical voter model, in which some citizens vote to make their group optimal (Harsanyi (1980), Feddersen and Sandroni (2006), and Ali and Lin (2013)). In addition, Feddersen (2004) surveys the theories of why people vote or abstain in economic models.

[6] As subsequent extension models of the swing voter's curse, see, for example, Feddersen and Pesendorfer (1999), McMurray (2013), and Herrera et al. (2019).

EBRDPs.[7] One of the most noticeable exceptions is Grillo's (2016) model, in which two candidates state their preferences, which remain private information, and voters prefer politically attractive candidates. Grillo (2016) shows that voters' loss aversion makes truthful communication possible at equilibrium.[8] In contrast to Grillo (2016), we find the condition for an equilibrium where abstention occurs due to voters' loss aversion.

The rest of the paper is organized as follows. Section 2 introduces our model. In section 3, we analyze our equilibrium. Section 4 extends the analysis provided in section 3 to the case of asymmetric voters, and section 5 concludes the paper.

2 The Model

The model includes two candidates, A and B. The electorate has $2n + 1$ individuals and is divided into two types, where each type has an identical preference and the preferences of each type are diametrically opposed. One type (type A) prefers A to B and the other type (type B) prefers B to A. An individual's type is private information held by that individual. A given individual is type A with probability ω. There are two states with respect to ω: $\omega \in \{\omega_a, \omega_b\}$ and the prior of $\omega = \omega_a$ is $\frac{1}{2}$. Before voting, each individual receives a signal θ on the true state. This signal depends on the true state and is assumed to be an independent and identically distributed random variable with cumulative distribution F_i and density f_i for each $i \in \{a, b\}$, where $f_i > 0$ for all θ. $\Pr(\omega = \omega_a \mid \theta)$ denotes the probability of $\omega = \omega_a$ when the individual receives signal θ. Then, $\Pr(\omega = \omega_a \mid \theta) = 1 - \Pr(\omega = \omega_b \mid \theta) = \frac{f_a(\theta)}{f_a(\theta)+f_b(\theta)} = \theta$. Each individual chooses $e \in \{0, 1\}$, where $e = 1$ indicates that the individual votes for their preferred candidate and $e = 0$ indicates they abstain. Elections are decided by a simple plurality with some tie-breaking rule, such as a coin toss.

[7] The EBREPs model developed by Kőszegi and Rabin (2006, 2007) has been applied to the models of Heidhues and Kőszegi (2008, 2014), Herweg and Mierendorff (2013), Karle and Peitz (2014), Rosato (2016), Karle and Schumacher (2017), and research conducted by Heidhues and Kőszegi (2018) on industrial organization, as well as by Herweg, Muller, and Weinschenk (2010), Daido and Murooka (2016), Dato, Muller, and Grunewald (2018), and research conducted by Kőszegi (2014) on contract theory.

[8] Although their formulation of reference points differs from that of Kőszegi and Rabin (2006, 2007), Alesina and Passarelli (2019) build a political model that incorporates voters' loss aversion and show that loss aversion leads to significant differences from the results that do not consider voters' loss aversion.

The payoff of an individual, $x \in \{x_w, x_l\}$, depends on the outcome of the election: x_w is the payoff for winning and x_l that for losing. We assume that $x_w > x_l$ without loss of generality. Denoting p_e as the probability of winning when an individual chooses e, we represent the expected payoff of the individual as follows:

$$u(e) = p_e x_w + (1 - p_e) x_l. \tag{1}$$

We consider loss-averse voters that exhibit EBRDPs á la Kőszegi and Rabin (2006, 2007).[9] A key assumption of EBRDPs is that each individual's overall utility comprises of intrinsic payoffs and psychological gain-loss payoffs. In our model, the individual has one payoff dimension from the result of an election and experiences a psychological gain or loss by comparing a realized with a reference payoff.

We denote an individual's reference point for their outcome as r. For a deterministic reference point, if their actual outcome is x, then their overall utility is given by

$$\underbrace{x}_{\text{intrinsic payoff}} + \underbrace{\mu(x - r)}_{\text{gain-loss payoff}},$$

where $\mu(\cdot)$ is a gain-loss function that corresponds to Kahneman and Tversky's (1979) value function. We assume $\mu(\cdot)$ is a piecewise linear function in order to focus on the effect of loss aversion. We then define the gain-loss function when the intrinsic payoff is x and the reference point is r as

$$\mu(x - r) = \begin{cases} \eta(x - r) & \text{if } x - r \geqslant 0, \\ \eta\lambda(x - r) & \text{if } x - r < 0, \end{cases}$$

where $\eta \geqslant 0$ represents the weight on the gain-loss payoff and $\lambda \geqslant 1$ represents the degree of loss aversion.[10]

Following Kőszegi and Rabin (2006, 2007), we assume that the reference point is determined by rational beliefs on outcomes and that the reference point itself is stochastic if the outcome is

[9] O'Donoghue and Sprenger (2018) provide a concise, but detailed introduction to EBRDPs and its related topics.
[10] The individual is loss neutral when $\lambda = 1$.

stochastic. Each individual experiences a gain or loss by comparing every possible outcome with every reference point. To describe the concept of stochastic reference points in a simple manner, suppose that the individual expects to choose $e = 1$. This individual expects to win and receive x_w with probability p_1 and lose and receive x_l with probability $1 - p_1$. If they choose $e = 1$ and the candidate that they vote for wins, the individual receives x_w. Because this occurs with probability p_1 and they expected to receive x_w with probability p_1 or x_l with probability $1 - p_1$, they experience no gain or loss with probability $p_1 \times p_1$ but experience a gain of $x_w - x_l$ with probability $p_1 \times (1 - p_1)$. However, if they choose $e = 1$ and the candidate that they vote for loses, the individual receives x_l. Because this occurs with probability $1 - p_1$ and they compare this result with their expectations, they experience a loss of $x_w - x_l$ with probability $(1 - p_1) \times p_1$ but experience no gain-loss with probability $(1 - p_1) \times (1 - p_1)$. Consequently, the individual correctly anticipates all off the above cases. Their expected gain-loss utility, when their reference choice is $e = 1$ and their actual choice is $e = 1$, is $-p_1(1 - p_1)\eta(\lambda - 1)(x_w - x_l)$.

In our model, the individual's overall expected utility when their reference choice is $\widehat{e}$, their actual choice is e, and the corresponding probabilities are $p_{\widehat{e}}$ and p_e, respectively, can be represented as follows:

$$\begin{aligned} u(e \mid \widehat{e}) &= p_e x_w + (1 - p_e)x_l \\ &\quad + p_{\widehat{e}}[p_e \mu(x_w - x_w) + (1 - p_e)\mu(x_l - x_w)] + (1 - p_{\widehat{e}})[p_e \mu(x_w - x_l) + (1 - p_e)\mu(x_l - x_l)] \\ &= \underbrace{p_e x_w + (1 - p_e)x_l}_{\text{intrinsic payoff}} - \underbrace{p_{\widehat{e}}(1 - p_e)\eta\lambda(x_w - x_l)}_{\text{loss payoff}} + \underbrace{(1 - p_{\widehat{e}})p_e \eta(x_w - x_l)}_{\text{gain payoff}}. \end{aligned} \tag{2}$$

3 Equilibrium Analysis

We focus on a symmetric case where $\omega_a = 1 - \omega_b = \rho > \frac{1}{2}$ and $F_a(\theta) = 1 - F_b(1 - \theta)$ for all θ.

Considering the behavior of a type A individual, say i, we suppose that the number of individuals except i who vote for A is n_A and that this figure is n_B for B. Then, for individual i, $p_1 = [\Pr(n_A + 1 >$

$n_B) + \frac{1}{2}\Pr(n_A + 1 = n_B)] \equiv p_1^A$ and $p_0 = [\Pr(n_A > n_B) + \frac{1}{2}\Pr(n_A = n_B)] \equiv p_0^A$.[11] For analytical simplicity, we assume $x_w = 1$ and $x_l = 0$.

First, we briefly confirm the choice of individual i, who is loss neutral ($\lambda = 1$). The utility of the loss-neutral individual i depends on their choice $e_i \in \{0, 1\}$, as follows:

$$u(e_i) = \begin{cases} p_1^A & \text{if} \quad e_i = 1, \\ p_0^A & \text{if} \quad e_i = 0. \end{cases}$$

$e_i = 1$ always dominates $e_i = 0$ because our model does not consider a direct cost in voting.

Next, we study the decisions of loss-averse voters. We derive the optimal voting behavior based on the *choice-acclimating personal equilibrium* (CPE) defined by Kőszegi and Rabin (2007). Under CPE, the individual's reference point is acclimated to the chosen action. This is plausible when the action is determined well in advance of realizing the outcome. And, hence, they modify their belief to the action they chose before the outcome is realized. Because the individual knows that their belief will change based on their chosen action before the outcome is realized, they take this change into account when choosing their action. Hence, each individual's action determines their reference point under CPE. The condition for the individual to choose to vote under CPE is:

$$u(1 \mid 1) \geqslant u(0 \mid 0). \tag{CPE}$$

We confine our analysis to pure strategies and focus on symmetric equilibria.[12]

For individual i who prefers A, the condition of voting under (CPE) is represented as follows:

$$\begin{aligned} u(1 \mid 1) \geqslant u(0 \mid 0) &\Rightarrow (p_1^A - p_0^A)[1 - \eta(\lambda - 1)(1 - p_1^A - p_0^A)] \geqslant 0 \\ &\Rightarrow p_1^A + p_0^A \geqslant 1 - \frac{1}{\eta(\lambda - 1)} \equiv \gamma. \end{aligned} \tag{CPE-A}$$

[11] The corresponding probabilities for the individual whose preferred candidate is B are p_1^B and p_0^B, respectively.

[12] As shown by Dato, Müller, and Grunewald (2017), each individual will never prefer to randomize their own choices under the solution concept of CPE. Hence, without loss of generality, we can focus on pure strategies.

This shows that when $\gamma > 0$ (i.e., $\eta(\lambda - 1) > 1$) abstention can occur.[13] Because voters are loss averse, they have first-order risk aversion to uncertainty over winning the election, which is qualitatively different from a standard concave utility. To confirm this, suppose that voters are loss neutral ($\eta = 0$) but have a concave utility function, $v(x)$. The condition to vote is $(p_1^A - p_0^A)v(1) \geqslant 0$. This condition always holds and abstention never occurs.

We can rewrite the left-hand side of (CPE-A) when individual i receives a signal θ_i, as follows:

$$p_1^A + p_0^A = \underbrace{\frac{\rho\theta_i}{\rho\theta_i + (1-\rho)(1-\theta_i)}}_{\text{Posterior probability that } \omega = \omega_a}[p_1^A(\omega_a) + p_0^A(\omega_a)] + \underbrace{\frac{(1-\rho)(1-\theta_i)}{\rho\theta_i + (1-\rho)(1-\theta_i)}}_{\text{Posterior probability that } \omega = \omega_b}[p_1^A(\omega_b) + p_0^A(\omega_b)],$$

where we denote $p_{e_i}^K(\omega_j)$ as the probability that the candidate K wins when a type K individual chooses e_i under state ω_j.

Rearranging (CPE-A), we obtain the following condition regarding θ:

$$\bar{\theta}_A = \frac{(1-\rho)(\gamma - [p_1^A(\omega_b) + p_0^A(\omega_b)])}{\rho[p_1^A(\omega_a) + p_0^A(\omega_a)] - (1-\rho)[p_1^A(\omega_b) + p_0^A(\omega_b)] - \gamma(2\rho - 1)} < \theta_i. \tag{3}$$

When this condition holds, the individual chooses to vote rather than abstain. $v_K(\omega_j)$ denotes the probability that an individual is type K and they vote for K in state ω_j. Then, respectively,

$$v_A(\omega_a) = \rho(1 - F_a(\bar{\theta}_A)) \text{ and } v_A(\omega_b) = (1-\rho)(1 - F_b(\bar{\theta}_A)).$$

Correspondingly, the condition of a type B individual voting for B is

$$\bar{\theta}_B = \frac{(1-\rho)(\gamma - [p_1^B(\omega_a) + p_0^B(\omega_a)])}{\rho[p_1^B(\omega_b) + p_0^B(\omega_b)] - (1-\rho)[p_1^B(\omega_a) + p_0^B(\omega_a)] - \gamma(2\rho - 1)} < 1 - \theta_i. \tag{4}$$

[13]Estimating λ in an EBRDPs model, Dreyfuss et al. (2019) find a mean of $\lambda = 1.6-3.0$ with substantial heterogeneity in a matching model. For more experimental results on the estimation of λ, see the references in Dreyfuss et al. (2019).

Then, respectively,

$$v_B(\omega_b) = \rho F_b(1-\bar{\theta}_B) \text{ and } v_B(\omega_a) = (1-\rho)F_a(1-\bar{\theta}_B).$$

Since we assume symmetry between types A and B, we focus on the equilibria in which type A and B individuals adopt the same cutoff, so that $\bar{\theta}_A = \bar{\theta}_B = \bar{\theta}$. By assuming symmetry, $F_b(1-\bar{\theta}_B) = 1 - F_a(\bar{\theta}_B)$ and $F_a(1-\bar{\theta}_B) = 1 - F_b(\bar{\theta}_B)$. Therefore, if $\bar{\theta}_A = \bar{\theta}_B = \bar{\theta}$, we have

$$v_A(\omega_a) = v_B(\omega_b) > v_B(\omega_a) = v_A(\omega_b). \tag{5}$$

Then, $p^B_{e_i}(\omega_a) = p^A_{e_i}(\omega_b)$ and $p^B_{e_i}(\omega_b) = p^A_{e_i}(\omega_a)$ for each $e_i \in \{0,1\}$. If $\bar{\theta}_A = \bar{\theta}_B = \bar{\theta}$, $\bar{\theta}$ is defined as:

$$\bar{\theta} = \frac{(1-\rho)(\gamma - [p^A_1(\omega_b) + p^A_0(\omega_b)])}{\rho[p^A_1(\omega_a) + p^A_0(\omega_a)] - (1-\rho)[p^A_1(\omega_b) + p^A_0(\omega_b)] - \gamma(2\rho - 1)}. \tag{6}$$

We then prove the existence of an equilibrium and characterize $\bar{\theta}$ in the limit.

Proposition 1. *For a sufficiently large n, a symmetric equilibrium exists. The equilibrium cutoff point in the limit $n \to \infty$ is given by:*

$$\theta^* \equiv \frac{(1-\rho)\gamma}{\rho(2-\gamma) + (1-\rho)\gamma}. \tag{7}$$

Proof of Proposition 1. We first prove the existence of an equilibrium. Note that $\bar{\theta}$ is self-mapping because each $p^K_{e_i}(\omega_j)$ is a function of n and $v_K(\omega_j)$, where $K \in \{A,B\}$, $j \in \{a,b\}$,and $v_K(\omega_j)$ is a function of $\bar{\theta} = \bar{\theta}_A = \bar{\theta}_B$. Letting $G_n(\bar{\theta})$ be defined as the right-hand side of (6), finding a fixed point for G_n proves the existence of an equilibrium. We first check the continuity of G_n. By (5), we can show that $p^A_{e_i}(\omega_a) > 1/2 > p^A_{e_i}(\omega_b)$ for $e_i = 0, 1$. Then, the denominator of G_n becomes

$$\begin{aligned}\rho[p^A_1(\omega_a) + p^A_0(\omega_a)] - (1-\rho)[p^A_1(\omega_b) + p^A_0(\omega_b)] - \gamma(2\rho - 1)\\ > \rho - (1-\rho) - \gamma(2\rho-1) = (1-\gamma)(2\rho-1) > 0.\end{aligned}$$

Since the denominator takes a positive value for each $\bar{\theta}$, G_n is well defined. The continuity immediately follows from the continuity of F_j for $j \in \{a, b\}$.

Note that, when n is sufficiently large and if $\bar{\theta} = 0$, the probability that A wins under the state of ω_a converges to 1 and the probability that A wins under the state of ω_b converges to 0 because of (5) and the law of large numbers. Hence, $G_n(0) \approx \frac{(1-\rho)\gamma}{2\rho(1-\gamma)+\gamma} > 0$. Conversely, if $\bar{\theta} = 1$, $G_n(1) = \frac{(1-\rho)(\gamma-1)}{(1-\gamma)(2\rho-1)} < 0$ because individuals abstain. Then, by the continuity of G_n, for a sufficiently large n, the intermediate value theorem implies the existence of the fixed point.

Now, we can characterize the fixed point of G_n when $n \to \infty$. As long as $\bar{\theta} < 1$, for a sufficiently large n, $p^A_{e_i}(\omega_a) \approx 1$ and $p^A_{e_i}(\omega_b) \approx 0$ for $e_i = 1, 0$ because of (5) and the law of large numbers. Then,

$$\begin{aligned}\lim_{n\to\infty} G_n(\bar{\theta}) &:= \lim_{n\to\infty} \frac{(1-\rho)(\gamma - [p^A_1(\omega_b) + p^A_0(\omega_b)])}{\rho[p^A_1(\omega_a) + p^A_0(\omega_a)] - (1-\rho)[p^A_1(\omega_b) + p^A_0(\omega_b)] - \gamma(2\rho-1)}\\ &= \frac{(1-\rho)\gamma}{\rho(2-\gamma) + (1-\rho)\gamma}.\end{aligned}$$

Therefore, the fixed point also converges to θ^*. □

Since θ^* is defined by (7), as long as $\gamma \in (0, 1)$, the fixed point is within $(0, 1/2)$. In this case, the rate of abstention in state ω_a at the limit is

$$\rho F_a(\theta^*) + (1-\rho)(1 - F_a(1-\theta^*)). \tag{8}$$

In our symmetric case, the rate of abstention in the state of ω_b at the limit has the same value. The comparative statics show the properties of the limit abstention rate.

Proposition 2. *Suppose that $\eta(\lambda - 1) > 1$. (i) The limit abstention rate is increasing in η and λ.*

Additionally, if $F_a(1/2) < 1/2$, (ii) the limit abstention rate is higher in the losing group than in the winning group, and (iii) the limit abstention rate is decreasing in ρ.

Proof of Proposition 2. We first check the properties of θ^* as follows:[14]

$$\frac{\partial \theta^*}{\partial \gamma} = \frac{\rho(2-(\rho(2-\gamma)+(1-\rho)\gamma))}{[\rho(2-\gamma)+(1-\rho)\gamma]^2} > 0, \tag{9}$$

$$\frac{\partial \theta^*}{\partial \rho} = -\frac{2(1-\gamma)+\gamma[\rho(2-\gamma)+(1-\rho)\gamma]}{[\rho(2-\gamma)+(1-\rho)\gamma]^2} < 0. \tag{10}$$

First, using (9), we can show that $R_\varnothing$ is increasing in γ. Since γ increases in η and λ, this completes the proof of (i). Next, under the state of ω_a, type A individuals are in the winning group, while type B individuals are in the losing group. When $\theta^* < 1/2$ and $F_a(1/2) < 1/2$, $F_a(\theta^*) \leqslant F_a(1/2) < 1 - F_a(1/2) \leqslant 1 - F_a(1-\theta^*)$. This means that the abstention rate of Group A ($F_a(\theta^*)$) is less than that of Group B ($1 - F_a(1-\theta^*)$), thus completing the proof of (ii). Finally, because $F_a(\theta^*) > 1 - F_a(1-\theta^*)$ and (10), we verify that the differentiation of the limit abstention rate by ρ is negative, thus completing the proof of (iii). □

In Proposition 2, assumption $F_a(1/2) < 1/2$ implies that more than half of voters receive signals informing them that state ω_a is likelier than ω_b when the state is ω_a. What causes our abstention result is voters' loss aversion. A voter's feeling of loss from unexpectedly losing in the election is greater than any proportional gain from unexpectedly winning. We can understand this effect by scrutinizing (CPE-A).

From (CPE-A), the individual does not vote when

$$\underbrace{(p_1^A - p_0^A)}_{\text{the standard effect}} - \underbrace{\eta(\lambda-1)(p_1^A - p_0^A)(1-p_1^A-p_0^A)]}_{\text{the gain-loss effect}} < 0. \tag{11}$$

(11) implies that although the standard effect is present and the individual always votes if they are loss-neutral ($\lambda = 0$), the loss-averse individual may abstain when the gain-loss effect outweighs the standard effect. To hold (11), it is necessary that $1 - p_1^A - p_0^A > 0$. This is satisfied when

[14]Note that $\gamma > 0 \Leftrightarrow \eta(\lambda-1) > 1$. Additionally, by the definition of γ, $\gamma < 1$ when $\lambda > 1$ and $\eta > 0$.

the probability of experiencing a gain-loss from voting is higher than that from not voting, where $p_1^A(1-p_1^A) - p_0^A(1-p_0^A) > 0$. If $1 - p_1^A - p_0^A > 0$ holds, then (11) holds when $\eta(\lambda - 1) > \frac{1}{1-p_1^A-p_0^A}$. As a result, the abstention rate increases in both η and λ (Proposition 2 (i)).

Because individuals receive an informative signal about which candidate is the majority's preference, the expected probability of winning for the minority group (p_e^{min}) is smaller than for the majority group (p_e^{maj}) and, thus, $1 - p_1^{min} - p_0^{min} > 1 - p_1^{maj} - p_0^{maj}$. This implies that the marginal gain-loss effect by voting for the minority group is higher than that for the majority group.[15] As a result, the members of the minority group are more likely to abstain than the members of the majority group (Proposition 2 (ii)).

The smaller ρ means that the election is more competitive and the probability of winning is smaller. In this case, the gain-loss effect is more significant because $1 - p_1^K - p_0^K > 0$ for $K = A, B$ is more plausible (Proposition 2 (iii)). This result is counterintuitive. The situation with high ρ implies a close election, in which a candidate wins by a narrow margin. One would believe that in a close election, people realize the value of their vote, leading the abstention rate to drop. In fact, even when the marginal benefit of voting is almost negligible in a large election, people still prefer to vote instead of staying at home as long as they do not care about cost. However, when people are loss averse, they stay at home if the gain-loss effect overweighs this standard effect of voting.

4 Asymmetric case

In this section, we study the asymmetric case, where we assume that $\omega_a > 1/2 > \omega_b$ instead of $\omega_a = 1 - \omega_b$. We also weaken the assumption $F_a(\theta) = 1 - F_b(1 - \theta)$.

[15] Note that because the difference of the probabilities between voting and abstaining, $p_1^K - p_0^K$, is sufficiently small for $K = A, B$ in a large election, we focus on the effect caused by $1 - p_1^K - p_0^K$.

Letting $p \in (0,1)$ denote that probability of $\omega = \omega_a$, then

$$\begin{aligned} p_1^A + p_0^A = & \frac{p\omega_a\theta_i}{p\omega_a\theta_i + (1-p)\omega_b(1-\theta_i)}[p_1^A(\omega_a) + p_1^A(\omega_a)] \\ & + \frac{(1-p)\omega_b(1-\theta_i)}{p\omega_a\theta_i + (1-p)\omega_b(1-\theta_i)}[p_1^A(\omega_b) + p_0^A(\omega_b)]. \end{aligned}$$

A type A individual votes for A if and only if $\theta_i > \theta_A$, where

$$\theta_A = \frac{(1-p)\omega_\beta(\gamma - [p_1^A(\omega_b) + p_0^A(\omega_b)])}{p\omega_a[p_1^A(\omega_a) + p_0^A(\omega_a)] - (1-p)\omega_b[p_1^A(\omega_b) + p_0^A(\omega_b)] - \gamma(p(\omega_a + \omega_b) - \omega_b)}. \tag{12}$$

In the same way, a type B individual votes for B if and only if $\theta_i < 1 - \theta_B$, where

$$\theta_B = \frac{(p(1-\omega_a)(\gamma - [p_1^B(\omega_a) + p_0^B(\omega_a)])}{(1-p)(1-\omega_b)[p_1^B(\omega_b) + p_0^B(\omega_b)] - p(1-\omega_a)[p_1^B(\omega_a) + p_0^B(\omega_a)] - \gamma(p(\omega_a + \omega_b - 2) + 1 - \omega_b)}. \tag{13}$$

Note that if $2 \geqslant p_1^A(\omega_a) + p_0^A(\omega_a) - (p_1^A(\omega_b) + p_0^A(\omega_b)) \geqslant 1 > \gamma$ and $2 \geqslant p_1^B(\omega_b) + p_0^B(\omega_b) - (p_1^B(\omega_a) + p_0^B(\omega_a)) \geqslant 1 > \gamma$, both of the denominators of the right-hand sides of (12) and (13) are positive.

Letting $\bar{\theta}_A = \max\{\min\{1, \theta_A\}, 0\}$ and $\bar{\theta}_B = \max\{\min\{1, \theta_B\}, 0\}$, then the probability that a given type A individual votes for A is

$$\Pr(v = A \mid \omega_a) = \omega_a(1 - F_a(\bar{\theta}_A)), \quad \Pr(v = A \mid \omega_b) = \omega_b(1 - F_b(\bar{\theta}_A)).$$

Similarly, the probability that a given type B individual votes for B is

$$\Pr(v = B \mid \omega_b) = (1 - \omega_b)F_b(1 - \bar{\theta}_B), \quad \Pr(v = B \mid \omega_a) = (1 - \omega_a)F_a(1 - \bar{\theta}_B).$$

For each given $(p_e^K(\omega_j))_{e,j,K}$, $\bar{\theta}_A$ and $\bar{\theta}_B$ are computed.[16] Using these, the probability that a given individual votes for their preferred candidate is derived. Subsequently, each $(p_e^K(\omega_j))_{e,j,K}$ is also derived. Thus, the relationship between the original $(p_e^K(\omega_j))_{e,j,K}$ and the derived $(p_e^K(\omega_j))_{e,j,K}$ is written as the function $\Gamma_n : [0,1]^8 \to [0,1]^8$. In an equilibrium, each $(p_e^K(\omega_j))_{e,j,K}$ is a fixed point

[16] Here, $(p_e^K(\omega_j))_{e,j,K}$ represents $p_e^K(\omega_j)$ for $e = 0, 1$, $j = a, b$, and $K = A, B$.

of Γ_n.

We also add a restriction on $[0,1]^8$, as follows:

$$\begin{aligned} 2-\gamma &\geqslant \tfrac{p_1^A(\omega_a)+p_0^A(\omega_a)+(p_1^A(\omega_b)+p_0^A(\omega_b))}{2} \geqslant \gamma, \\ 2-\gamma &\geqslant \tfrac{p_1^B(\omega_b)+p_0^B(\omega_b)+(p_1^B(\omega_a)+p_0^B(\omega_a))}{2} \geqslant \gamma. \end{aligned} \tag{14}$$

That is, we consider the following set of equations.

$$\Pi = \{(p_e^K(\omega_j))_{(e,j,K)\in\{1,0\}\times\{a,b\}\times\{A,B\}} \mid (14) \text{ is satisfied}\}$$

This set is compact and convex, and if $(p_e^K(\omega_j))_{e,j,K} \in \Pi$, both θ_A and θ_B are less than or equal to $1/2$. Then, in the state ω_a, the probability that a given individual votes for A is greater than or equal to $\omega_\alpha(1-F_\alpha(1/2))$. Conversely, the probability that a given individual votes for B is less than or equal to $1-\omega_\alpha$. Then, we can verify that if the following inequality holds, at the state ω_a, the probability of voting for A is greater than that for B.

$$\omega_a(1-F_a(1/2)) > 1-\omega_a \iff \omega_a > \frac{1}{2-F_a(1/2)}.$$

Similarly, if

$$(1-\omega_b)F_b(1/2) > \omega_b \iff \omega_b < \frac{F_b(1/2)}{1+F_b(1/2)},$$

the probability of voting for B is greater than that for A at the state ω_b.

Under these conditions, for each point in Π, as for the output of Γ_n, and for sufficiently large n, $p_e^A(\omega_a)$ and $p_e^B(\omega_b)$ are close to 1, while $p_e^A(\omega_a)$ and $p_e^B(\omega_b)$ are close to 0. Then, for any point Π, the denominators of the definitions of θ_A and θ_B are positive and $\Gamma_n(\Pi) \subset \Pi$. Because Γ_n is continuous and Π is compact and convex, Brouwer's fixed point theorem implies that Γ_n has a fixed point.

If the fixed point of Γ_n satisfies (14) and is an interior point of Π, the fixed point $(p_e^K(\omega_j))_{e,j,K}$

is the tuple of equilibrium probabilities of winning. As $n \to \infty$, the equilibrium probabilities of winning $p_e^A(\omega_a)$ and $p_e^B(\omega_b)$ converge to 1, and $p_e^A(\omega_a)$ and $p_e^B(\omega_b)$ converge to 0. Then, the cutoff levels are the following.

$$
\begin{aligned}
\theta_A &= \frac{(1-p)\omega_b\gamma}{p\omega_a(2-\gamma)+(1-p)\omega_b\gamma} \\
\theta_B &= \frac{p(1-\omega_a)\gamma}{(1-p)(1-\omega_b)(2-\gamma)+p(1-\omega_a)\gamma}
\end{aligned}
\tag{15}
$$

If $\gamma > 0$, both are within $(0, 1/2)$ and the derived $(p_e^K(\omega_j))_{e,j,K}$ satisfies (14). In sum, we have the following result.

Proposition 3. *Suppose that the following inequalities hold:*

$$
\omega_a > \frac{1}{2-F_a(1/2)}, \quad \omega_b < \frac{F_b(1/2)}{1+F_b(1/2)}.
$$

Then, for a sufficiently large n, an equilibrium in which some voters abstain exists. As $n \to \infty$, the abstention rate converges to the following values.

$$
\begin{aligned}
&\omega_a F_a(\theta_A) + (1-\omega_a)(1-F_a(1-\theta_B)) \quad \textit{if} \quad \omega = \omega_a, \\
&\omega_b F_b(\theta_A) + (1-\omega_b)(1-F_b(1-\theta_B)) \quad \textit{if} \quad \omega = \omega_b,
\end{aligned}
$$

where θ_A and θ_B are defined in (15).

Notice that the abstention rate of type A individuals is $F_k(\theta_A)$ and for type B individuals is $1 - F_k(1-\theta_B)$ when $\omega = \omega_k$. Comparative statics show that $F_k(\theta_A)$ is decreasing in p while $1 - F_k(1-\theta_B)$ is increasing in p, where p is the probability that A is the more popular candidate. This result implies that individuals who support the candidate who is more likely to lose also are more likely to abstain. We can therefore confirm that the result of the symmetric case is still valid in the asymmetric case.

5 Conclusions

This study develops a voting model that shows the existence of abstention behavior when individuals have EBRDPs. At equilibrium, abstention is more likely to occur in a close election. Individuals who support minority candidates are also more likely to abstain.

One concern of our assumption is that individuals cannot vote for a candidate they do not support. The motivation for abstention is that individuals who severely dislike the unexpected losses reduce their uncertainty by lowering the probability of winning. However, the reduction in the probability of winning is larger when individuals vote for the opposing candidate. One explanation is that individuals may have a behavioral cost of voting for the opponent, but this appears to rather be ad hoc. Further analysis of this issue is left for future research.

Other potential directions for future research include extending our model by introducing preference uncertainty á la Feddersen and Pesendorfer (1999) and determining whether loss-aversion deteriorates swing voters' curse and information aggregation.

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