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On a Stackelberg leader's incentive to invite entry into horizontally differentiated oligopolies with network externalities: A reexamination

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Abstract
We develop a model of horizontally differentiated oligopolies with network externalities and reconsider a Stackelberg leader's incentive to invite entry, a problem previously examined by Economides (1996) and Kim (2002). We demonstrate that a Stackelberg leader has (does not have) an incentive to invite entry if the degree of network externalities is larger (smaller) than that of the product substitutability, such that a follower's profit increases (decreases).

JEL classifications: D21; D43; D62; L13

Keywords: Network externality; Horizontally differentiated oligopoly; Stackelberg competition; Entry; Passive expectation; Responsive expectation

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1. Introduction

In a seminal paper, Economides (1996) considered a Stackelberg competition in a homogeneous product market with network externalities and demonstrated that an incumbent (the Stackelberg leader) has an incentive to invite new entry (followers) if the strength of the network externalities is sufficiently large (Proposition 2, p. 221). Subsequently, Kim (2002) considered a Stackelberg competition in a horizontally differentiated product market with network externalities and demonstrated the following. First, if products are homogeneous, the Stackelberg leader never has an incentive to invite entry regardless of the strength of the network externalities (Proposition 1, p.398). Second, for differentiated products, the Stackelberg leader (never) has an incentive to invite entry if the strength of the network externalities is *small* (*large*) (Proposition 2, p. 399, emphasis added).

Following these studies, we reconsider the Stackelberg leader's incentive to invite entry into a market with network externalities. In analyzing this market, we need to consider the role of consumer expectations about network size. In particular, we examine the following cases: the case of passive (responsive) expectations where consumers form their expectations of network sizes before (after) firms decide their outputs. In other words, under passive expectations, firms (i.e., the leader and followers in a Stackelberg competition) determine their outputs given expectations. In contrast, under responsive expectations, firms can commit to their outputs, and thus, consumers believe the levels announced. In this case, the expected network sizes equal the actual outputs.

The remainder of the paper is organized as follows. In Section 2, we develop a model of horizontally differentiated oligopolies with network externalities and derive a

Stackelberg equilibrium in the case of passive expectations. We then demonstrate that the Stackelberg leader's incentive to invite entry depends on the strength of the network externalities. In Section 3, we summarize the results and present some remaining issues, and in the Appendix we confirm whether these main results hold in the case of responsive expectations.

2. The Model

2.1 Preliminary

We consider an oligopoly model ($n+1$ firms) in a horizontally differentiated products market with network externalities where firms compete in a quantity setting competition. Applying the frameworks of Economides (1996) and Häckner (2000), we assume the following linear inverse demand function with network externalities for firm *i*'s product:

$$p_i = A - q_i - \gamma Q_{-i} + F\left(S_i^E\right), \quad Q_{-i} = \sum_{-i=0}^{n} q_{-i}, \; i, -i = 0, \ldots, n, i \neq -i, \tag{1}$$

where A is the intrinsic market size, q_i is the output of firm *i*, $\gamma \in (0,1)$ is the degree of product substitutability, and $n(\geq 1)$ is the number of firms in the market. $F\left(S_i^E\right)$ is the network function where S_i^E is an expected network size of firm *i*'s product.[1] The expected network size is given by:

[1] In our model, the expected network size relates to total output in the market and not the number of consumers or users. For example, we assume that the expected network size relates to the frequency of times of use of an Internet service.

$$S_i^E = q_i^E + \phi Q_{-i}^E, \quad Q_{-i}^E = \sum_{-i=0}^{n} q_{-i}^E, \; i, -i = 0, ..., n, i \neq -i, \tag{2}$$

where $\phi \in [0,1]$ denotes the degree of product i's compatibility with product $-i$. To simplify the analysis, hereafter, we assume perfect compatibility, i.e., $\phi = 1$. We also assume that a linear network function is given by: $F\left(S_i^E\right) = eS_i^E$, where $e \in [0,1)$ is the degree of network externalities.

In the following analysis, we assume that firm *0* is a leader and firm *j* $(=1,...,n)$. is a follower. Using equations (1) and (2), we derive the following inverse demand function of follower *j*:

$$p_j = A - q_j - \gamma\left(q_0 + Q_{-j}\right) + F\left(S_j^E\right), \tag{3}$$

where $Q_{-j} = \sum_{-j=1}^{n} q_{-j}$, $S_j^E = q_j^E + q_0^E + Q_{-j}^E$ and $Q_{-j}^E = \sum_{-j=1}^{n} q_{-j}^E$, $j, -j = 1, ..., n, j \neq -j$.

Similarly, the inverse demand fucntion of leader *0* is given by:

$$p_0 = A - q_0 - \gamma Q_j + F\left(S_0^E\right), \tag{4}$$

where $Q_j = \sum_{j=1}^{n} q_j$, $j = 1,..., n$, $S_0^E = q_0^E + Q_j^E$, and $Q_j^E = \sum_{j=1}^{n} q_j^E$, $j = 1, ..., n$.

Furthermore, we assume that production costs are zero. This is because we observe low and even negligible marginal running costs in network industries, e.g., telecommunication and Internet businesses.

Finally, following Economides (1996) and Kim (2002), we assume the case of free license (i.e., no charge on entry). Thus, the following profit functions of leader *0* and follower *j* are respectively given by: $\pi_0 = p_0 q_0 = \left\{A - q_0 - \gamma Q_j + F\left(S_0^E\right)\right\} q_0$ and

$$\pi_j = p_j q_j = \{A - q_j - \gamma(q_0 + Q_{-j}) + F(S_j^E)\} q_j,\ j = 1,\dots,n.$$

2.2 A fulfilled Stackelberg equilibrium under passive expectations

Given the expected network sizes and the outputs of leader *0* and the other followers $-j$, follower j decides that output to maximize profit. The first-order condition (FOC) of profit maximization is given by:

$$\frac{\partial \pi_j}{\partial q_j} = p_j - q_j = A - 2q_j - \gamma(q_0 + Q_{-j}) + F(S_j^E) = 0, \quad j = 1,\dots,n.$$

Thus, the reaction function of follower j is expressed as: $q_j = \dfrac{A + F(S_j^E)}{2} - \dfrac{\gamma}{2}(q_0 + Q_{-j})$.

Assuming symmetric followers, i.e., $q_j = q_{-j}, \quad j, -j = 1,\dots,n, j \neq -j,$ we derive

$$q_j = \frac{A + F(S_j^E)}{\Gamma} - \frac{\gamma}{\Gamma} q_0, \tag{5}$$

where $\Gamma \equiv 2 - \gamma + \gamma n > 0.$ Given equation (5), it holds that $\dfrac{\partial q_j}{\partial q_0} = -\dfrac{\gamma}{\Gamma} < 0, \quad j = 1,\dots,n,$ which implies strategic substitutes.

The FOC for the profit maximization of leader *0* is then given by:

$$\frac{\partial \pi_0}{\partial q_0} = p_0 - q_0 - \gamma \sum_{j=1}^{n} \left(\frac{\partial q_j}{\partial q_0} \right) q_0 = A - 2q_0 - \gamma Q_j + \frac{\gamma^2 n}{\Gamma} q_0 + F\left(S_0^E\right) = 0,\ j = 1,\dots,n,$$

Given symmetric followers, i.e., $q_j = q_{-j},\ j, -j = 1,\dots,n, j \neq -j,$ the above equation can be rewritten as:

$$A - \frac{(2-\gamma)(2+\gamma n)}{\Gamma} q_0 - \gamma n q_j + F(S_0^E) = 0. \tag{6}$$

In a fulfilled Stackelberg equilibrium, i.e., $q_0^E = q_0 = q_L, \quad q_j^E = q_j = q_F,\ j = 1,\dots, n,$

where subscript L (F) denotes a leader (follower), equations (5) and (6) can be rewritten as:

$$\{2-\gamma+(\gamma-e)n\}q_F+(\gamma-e)q_L=A, \tag{7}$$

$$\left(2-e-\frac{\gamma^2 n}{\Gamma}\right)q_L+(\gamma-e)nq_F=A. \tag{8}$$

Thus, we derive the following fulfilled Stackelberg equilibrium.

$$q_F=\frac{(2-\gamma)\Gamma-\gamma^2 n}{\Delta}A, \tag{9}$$

$$q_L=\frac{(2-\gamma)\Gamma}{\Delta}A, \tag{10}$$

where $\Delta\equiv\{(2-e)\Gamma-\gamma^2 n\}\{2-\gamma+(\gamma-e)n\}-\Gamma(\gamma-e)^2 n>0.$[2] Taking the FOCs for profit maximization, the prices of a leader and followers are expressed as: $p_L=\left(\frac{\Gamma-\gamma^2 n}{\Gamma}\right)q_L=\frac{(2-\gamma)(\Gamma-\gamma^2 n)}{\Delta}A$ and $p_F=q_F$, respectively.

2.3 The entry effect: The Stackelberg leader's incentive to invite entry

Before examining this problem, using equation (10), we derive the following effects of an increase in the number of firms on the output and price of the leader.

$$\frac{dq_L}{dn}=\frac{(2-\gamma)}{\Delta^2}A\left\{\gamma\Delta-\Gamma\frac{d\Delta}{dn}\right\}, \tag{11}$$

$$\frac{dp_L}{dn}=\frac{(2-\gamma)}{\Delta^2}A\left\{\gamma(1-\gamma)\Delta-(\Gamma-\gamma^2 n)\frac{d\Delta}{dn}\right\}, \tag{12}$$

[2] For the following analysis, we rewrite this as follows:

$$\Delta=2\gamma(1-\gamma)(\gamma-e)n^2+2(2-\gamma)\{\gamma(2-\gamma)-e\}n+(2-\gamma)^2(2-e),$$

where $n\geq 1$.

where $\frac{d\Delta}{dn} = 4\gamma(1-\gamma)(\gamma-e)n + 2(2-\gamma)\{\gamma(2-\gamma)-e\}$.

We investigate the entry effect on the leader's profit, i.e., $\pi_L = p_L q_L$. In particular, we have $\frac{d\pi_L}{dn} = q_L \frac{dp_L}{dn} + p_L \frac{dq_L}{dn} = \frac{q_L}{\Gamma}\left\{\Gamma \frac{dp_L}{dn} + (\Gamma - \gamma^2 n)\frac{dq_L}{dn}\right\}$. Using equations (11) and (12), we derive the following relationship:

$$\begin{aligned} &\frac{d\pi_L}{dn} > (<) 0 \Leftrightarrow \gamma\Delta\{(2-\gamma)\Gamma - \gamma^2 n\} - 2\Gamma(\Gamma - \gamma^2 n)\frac{d\Delta}{dn} > (<) 0 \\ &\Leftrightarrow H_L(n) > (<) 0, \end{aligned} \tag{13}$$

where $\begin{aligned} H_L(n) \equiv\, &(e-\gamma)\{4\gamma^3(1-\gamma)^2 n^3 + 6\gamma^2(1-\gamma)(2-\gamma)^2 n^2 + 2\gamma(2-\gamma)^2(6-6\gamma+\gamma^2)n\} \\ &+(2-\gamma)^3\{[2(2-\gamma)+\gamma^2]e - 2\gamma(2-\gamma)\}. \end{aligned}$

In view of equation (13), we obtain the following proposition.

Proposition 1.

If the degree of network externalities is sufficiently larger (smaller) than that of the product substitutability, an increase in entry increases (decreases) the leader's profit.

Proof.

Based on equation (13), if $e > \gamma$, it holds that $H_L(n) > 0$ because $\gamma > \frac{2\gamma(2-\gamma)}{2(2-\gamma)+\gamma^2}$. Thus, the entry effect on the leader's profit is positive. Alternatively, if $\gamma > \frac{2\gamma(2-\gamma)}{2(2-\gamma)+\gamma^2} > e$, then it holds that $H_L(n) < 0$. Thus, the entry effect is negative. However, if $\gamma > e > \frac{2\gamma(2-\gamma)}{2(2-\gamma)+\gamma^2}$, the entry effect depends on the number of firms.

Proposition 1 implies that in a horizontally differentiated products market with sufficiently *strong* (*weak*) network externalities, the Stackelberg leader (i.e., the incumbent monopolist) will *invite* (*deter*) entry, by granting a free license (emphasis added). This lies opposite to Proposition 2 shown by Kim (2002). Furthermore, in the case of a homogeneous product market, i.e., $\gamma = 1,$ it holds that $\gamma = 1 > e.$ Accordingly, the Stackelberg leader never has an incentive to invite entry. This result is the same as Proposition 1 shown by Kim (2002).

Furthermore, the entry effect on the follower's profit, i.e., $\pi_F = \left(q_F\right)^2$, is expressed as: $\dfrac{d\pi_F}{dn} = 2q_F \dfrac{dq_F}{dn} > (<)0 \Leftrightarrow \dfrac{dq_F}{dn} > (<)0.$ Taking equation (9), we derive the following relationship:

$$\begin{aligned} &\frac{d\pi_F}{dn} > (<)0 \Leftrightarrow 2\gamma(1-\gamma)\Delta - \left\{(2-\gamma)\Gamma - \gamma^2 n\right\}\frac{d\Delta}{dn} > (<)0 \\ &\Leftrightarrow H_F(n) > (<)0 \Leftrightarrow e > (<)\gamma, \end{aligned} \tag{14}$$

where $H_F(n) \equiv (e-\gamma)\left\{4\gamma^2(1-\gamma)^2 n^2 + 4\gamma(1-\gamma)(2-\gamma)^2 n + 2(2-\gamma)^2(2-\gamma+\gamma^2)\right\}.$ Thus, in view of equation (14), we summarize the result as follows.

Proposition 2.

If the degree of network externalities is sufficiently larger (smaller) than that of the product substitutability, an increase in entry increases (decreases) the follower's profit.

We address the implications of Propositions 1 and 2 as follows. The effects of an increase in entry include a competitive effect through product substitutability and a network externalities effect. In view of equation (4), the former shifts the leader's inverse demand function downward, i.e., the leader's market-reduction effect, whereas the latter shifts it upward, i.e., the leader's market-enlargement effect. In particular, the parameter γ implies the marginal competitive effect and parameter e the marginal network externalities effect. Thus, if the network externalities effect exceeds the competitive effect, an increase in entry increases the leader's market and thus its profit, and vice versa. Based on equation (3), there is a similar effect on the follower's profit.

In the Appendix, we demonstrate that Propositions 1 and 2 hold in the case of responsive expectations.[3]

3. Concluding Remarks

In this paper, we examined a problem previously considered by Economides (1996) and Kim (2002), i.e., a Stackelberg leader's incentive to invite entry assuming a horizontally differentiated oligopoly with network externalities. We demonstrated that a Stackelberg leader has (does not have) an incentive to invite entry if the degree of network externalities is sufficiently larger (smaller) than that of the product substitutability. In this case, a follower's profit increases (decreases).

In addition to the generalization of the specific assumptions in the model, e.g., linear inverse demand and network functions, perfect and symmetric compatibility, there are

[3] Given equations (A.1) and (A.4), it is clear that if $e > (<) \gamma$ an increase in entry increases (decreases) the profits of the leader and followers.

some remaining issues.[4] For example, we should investigate whether the result depends on the mode of competition, i.e., the Stackelberg leader's incentive to invite entry in the case of price competition. We have also not examined welfare effects. Relating to this, we should examine optimal entry regulation in the case of a Stackelberg competition with network externalities.

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[4] In the case of responsive expectations, we demonstrate that Propositions 1 and 2 hold even with partial compatibility.

Appendix: The case of responsive expectations

1. Stackelberg equilibrium

We confirm whether the main results, i.e., Propositions 1 and 2, hold in the case of responsive expectations. From equation (3), it holds that $q_0^E = q_0,\ q_j^E = q_j,$ and $Q_{-j}^E = Q_{-j},\quad j,-j=1,...,n,\ j \neq -j.$ Thus, we derive the following inverse demand function of follower j:

$$p_j = A-(1-e)q_j-(\gamma-e)\{q_0+Q_{-j}\},\quad j,-j=1,...,n,\ j\neq -j. \tag{A.1}$$

In view of equation (A.1), we assume that the own-price effect exceeds the cross-price effect, i.e., $\left|\frac{\partial p_j}{\partial q_j}\right| > \left|\frac{\partial p_j}{\partial q_k}\right|,$ where $k \neq j,\quad k=0,1,...,n.$ This implies that $1-e > |\gamma - e|.$

The FOC for the profit maximizing of flower j is given by:

$$\frac{\partial \pi_j}{\partial q_j} = p_j-(1-e)q_j = A-2(1-e)q_j-(\gamma-e)\{q_0+Q_{-j}\}=0. \tag{A.2}$$

Given equation (A.2), we present the reaction function for follower j as follows:

$$q_j = \frac{A}{2(1-e)} - \frac{\gamma-e}{2(1-e)}\{q_0+Q_{-j}\},\quad j,-j=1,...,n,\ j\neq -j.$$

Assuming symmetric followers, i.e., $q_j = q_{-j},\ j,-j=1,...,n,\ j\neq -j,$ we obtain the following reaction function for follower j.

$$q_j = \frac{A}{2(1-e)+(\gamma-e)(n-1)} - \frac{\gamma-e}{2(1-e)+(\gamma-e)(n-1)}q_0. \tag{A.3}$$

Thus, it holds that $\frac{\partial q_j}{\partial q_0} = -\frac{\gamma-e}{2(1-e)+(\gamma-e)(n-1)} > (<)0 \Leftrightarrow e > (<)\gamma.$ Unlike the case of passive expectations, the strategic relationship depends on the degree of network externalities and of product differentiation.

From equation (4), the inverse demand function for leader *0* is given by:

$$p_0 = A - (1-e)q_0 - n(\gamma - e)q_j. \tag{A.4}$$

Given equation (A.4), we obtain the following FOC for the profit maximization of leader *0*:

$$\begin{aligned}\frac{\partial \pi_0}{\partial q_0} &= p_0 - (1-e)q_0 - n(\gamma - e)\left(\frac{\partial q_j}{\partial q_0}\right)q_0 \\ &= A - 2(1-e)q_0 - n(\gamma - e)q_j + \frac{(\gamma - e)^2 n}{2(1-e) + (\gamma - e)(n-1)}q_0 = 0.\end{aligned} \tag{A.5}$$

In the Stackelberg equilibrium, i.e., $q_0 = q_l$ and $q_j = q_f$, where subscript l (f) denotes a leader (follower), equations (A.2) and (A.5) can be rewritten as:

$$\{2(1-e) + (\gamma - e)(n-1)\}q_f + (\gamma - e)q_l = A, \tag{A.6}$$

$$\left\{2(1-e) - \frac{(\gamma - e)^2 n}{2(1-e) + (\gamma - e)(n-1)}\right\}q_l + (\gamma - e)nq_f = A. \tag{A.7}$$

Therefore, we have the following outputs in the Stackelberg equilibrium:

$$q_l = \frac{2 - \gamma - e}{2D}A, \tag{A.8}$$

$$q_f = \frac{(2-\gamma-e)^2 + 2(\gamma - e)(1-\gamma)n}{2\{2-\gamma-e+(\gamma-e)n\}D}A, \tag{A.9}$$

where $D \equiv (1-e)(2-\gamma-e) + (\gamma-e)(1-\gamma)n > 0$. Furthermore, the prices in the Stackelberg equilibrium can be represented as: $p_f = (1-e)q_f$ and

$$p_l = \left(\frac{D}{2(1-e) - (\gamma - e) + (\gamma - e)n}\right)q_l = \frac{2-\gamma-e}{2\{2-\gamma-e+(\gamma-e)n\}}A. \tag{A.10}$$

2. The entry effect

Based on equations (A.8) and (A.10), the entry effects on the output and price of the leader are given by:

$$\frac{dq_l}{dn} = \frac{(e-\gamma)(1-\gamma)}{D} q_l > (<) 0 \Leftrightarrow e > (<) \gamma, \tag{A.11}$$

$$\frac{dp_l}{dn} = \frac{e-\gamma}{\{2-\gamma-e+(\gamma-e)n\}} p_l > (<) 0 \Leftrightarrow e > (<) \gamma. \tag{A.12}$$

Using equations (A.11) and (A.12), we derive the following entry effect on the profit of the leader:

$$\frac{d\pi_l}{dn} = \frac{dp_l}{dn} q_l + \frac{dq_l}{dn} p_l > (<) 0 \Leftrightarrow e > (<) \gamma. \tag{A.13}$$

Therefore, we confirm Proposition 1.

Furthermore, using equation (A.3), we derive the following entry effect on the follower's output:

$$\frac{dq_f}{dn} = \frac{e-\gamma}{2-\gamma-e+(\gamma-e)n}\left[\frac{dq_l}{dn} + q_f\right]. \tag{A.14}$$

In view of equation (A.14), we derive the following relationships.

(i) If $e > \gamma$ then $\frac{dq_l}{dn} > 0$. Thus, it holds that $\frac{dq_f}{dn} > 0$ and $\frac{d\pi_f}{dn} > 0$.

(ii) If $e < \gamma$ then $\frac{dq_l}{dn} < 0$. Because $\frac{dq_l}{dn} + q_f > 0$ holds, we have $\frac{d\pi_f}{dn} < 0$.

Therefore, we confirm Proposition 2.