DISCUSSION PAPER SERIES

Discussion paper No.20

ON THE ENDOGENOUS TIMING IN TRADE POLICY GAME： A GENERAL CASE

Tsuyoshi Toshimitsu
Kwansei Gakuin University

October 1997

MASTERY FOR SERVICE

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

ON THE ENDOGENOUS TIMING IN TRADE POLICY GAME: A GENERAL CASE

Tsuyoshi TOSHIMITSU*

Abstract

This paper discusses the timing and the optimal trade policy in the presence of oligopolisitic industries and free entry. Collie (1994) proved that an importing government should not commit a countervailing duty in response to a foreign export subsidy. We show that his main conclusion does not always hold, since the timing, as well as the optimal trade policy, depends on the number of firms in both countries and the characteristic of the industry, i.e. no entry or free entry.

* Associate Professor of Economics
School of Economics
Kwansei Gakuin University
1-155, Uegahara Ichibanchyo, Nishinomiya, 662, Japan
Phone (0798)54-6440
Fax (0798)51-0944
E-mail.z95016@kwansei.ac.jp

1. INTRODUCTION

A player can determine its timing (or the order of its action) as well as its action. As to this issue, Hamilton and Slutsky (1990) have proved the endogenous timing of action in the duopoly game under perfect information. Albaek (1990) has also shown that there can be a unique natural Stackelberg equilibrium under incomplete information. Applying the endogenous timing game to trade policy games, Collie (1994) has shown that there is such a unique Stackelberg equilibrium that an importing (= domestic) government, being a leader, uses an import tariff policy and an exporting (= foreign) government, being a follower, uses an export subsidy policy and that the domestic government benefits from a larger export subsidy than in a Nash equilibrium, while the foreign government benefits from a lower import tariff *than* in a Nash equilibrium. Therefore, his main suggestion is that the domestic government should not commit a countervailing duty in response to a foreign export subsidy. It means that the domestic government should be a leader, not a follower.

It is assumed in Collie (1994) that the number of firms in both countries is identical. But, it was shown in Dixit (1984) that the optimal trade policy is an export subsidy (tax) if the number of exporting firms is relatively smaller (larger) than that of importing firms. Because the slope of the reaction functions of both governments depends on the number of firms in both countries, the results about the timing, as well as the optimal trade policy, are not necessarily identical to those of Collie (1994), if the number of firms in each country is not identical, or if it is allowed to be free entry.

Generalizing the model of Collie (1994), we will reconsider the endogenous timing and the optimal trade policy. We will show as follows:

(i) In the case of no entry, the following results hold: First, if the number of exporting firms is relatively smaller than (or equal to) that of importing firms, there is such a unique Stackelberg equilibrium that the domestic government, as a leader, charges a lower import tariff than in the Nash equilibrium, and the foreign government, as a follower, charges a higher export subsidy than in the Nash equilibrium. This is the Collie case. Secondly, if the number of exporting firms exists within some range, the domestic government uses an import tariff policy and the foreign government uses an export tax policy. Then, since both countries prefer to be a leader, there is a Nash equilibrium in the simultaneous-move game. Thirdly, if the number of exporting firms is relatively larger than that of importing firms, there is such a unique Stackelberg equilibrium that the domestic government, as a follower, charges a lower import tariff, i.e. a countervailing duty, than in the Nash equilibrium, and the foreign government, as a leader, charges a lower export tax than in the Nash equilibrium. This is opposite of the Collie case.

(ii) In the case of free entry, the following results hold: First, if the rate of the export to the output in the foreign market is relatively small, the domestic government uses an import tariff policy and the foreign government uses a free trade policy. Then since both countries prefer to be a leader, there is a Nash equilibrium in the simultaneous-move game. Secondly, if the rate of the export to the output in the foreign market is relatively large, there is such a Stackelberg equilibrium that the domestic government, as a follower, charges a lower import tariff, i.e. a countervailing duty, than in the Nash equilibrium, and the foreign government, as a leader, charges an export subsidy than in the Nash equilibrium, where it uses a free trade policy. This is opposite of the Collie case.

This paper is composed as follows: The next section presents the basic model. Although our model is basically identical to the model of Collie (1994), we assume that there are oligopolistic industries in both markets and that there is the foreign market, where Cournot oligopolistic competition by only the foreign firms prevails. Section 3 discusses the optimal trade policies and the optimal timing in the case of no entry. Then Section 4 similarly analyses the case of free entry. Finally, the last section summarizes our conclusions.

2. THE MODEL

We will basically follow the model of Collie (1994), i.e. Trade Policy Game with Foreign Export Subsidy and Domestic Import Tariff, except some points as mentioned below. There are two countries, domestic and foreign. Note that foreign country variables are denoted by an asterisk (*). The industry in each country is Cournot oligopoly with N (N^*) firms located in the domestic (foreign) country. All of the firms in both countries produce a homogeneous product. Although the domestic and foreign markets are segmented, foreign firms sell their products to both markets, while domestic firms sell their products to the domestic market. We assume the inverse demand functions in the domestic and the foreign (*) market as follows:

$$P = \alpha - \beta X, \quad X = \sum_{i=1}^{N} x_i + \sum_{j=1}^{N^*} x_j^*, \tag{1}$$

$$P^* = \alpha^* - \beta Y^*, \quad Y^* = \sum_{j=1}^{N^*} y_j^*. \tag{1*}$$

Note that X is total output for the domestic market of firms in both countries, x_i (x_j^*) is output for the domestic market of a domestic (foreign) firm, Y is total output for the foreign market of foreign firms, y_j^* is output for the foreign market of a foreign firm, and P (P^*) is domestic (foreign) market price. The domestic (foreign) firms have constant marginal costs c (c^*). The domestic government charges an import tariff of t per unit, and the foreign government charges an export subsidy (or tax) of s^* ($t^* = -s^*$, if $s^* < 0$) per unit. Thus, the profit functions in the domestic (foreign) firm are given by

$$\Pi_i = (P - c)x_i - K, \quad i = 1, \cdots, N, \tag{2}$$

$$\Pi_j^* = (P - c^* - t + s^*)x_j^* + (P^* - c^*)y_j^* - K^*, \quad j^* = 1, \cdots, N^*, \tag{2*}$$

where K (K^*) is a fixed cost of the domestic (foreign) firm. Hence, we derive equilibrium in the international oligopolistic markets as follows:

$$x_i = \frac{\alpha - (N^*+1)c + N^*(c^*-s^*+t)}{\beta(F+1)} = x[s^*, t; N, N^*], \tag{3}$$

$$x_j^* = \frac{\alpha + Nc - (N+1)(c^*-s^*+t)}{\beta(F+1)} = x^*[s^*, t; N, N^*], \tag{3*}$$

$$y_j^* = \frac{\alpha^* - c^*}{\beta(N^*+1)} = y^*[N^*], \tag{4*}$$

where $F = N + N^*$. Also, the total output in each market is given by

$$X = Nx + N^*x^*, \tag{5}$$

$$Y^* = N^*y^*. \tag{5*}$$

Taking into account the above equations, we derive the welfare functions of the domestic and the foreign government as follows:

$$W = (\beta/2)X^2 + N(\beta x^2 - K) + tN^* x^*$$
$$= W[t, s^*; N, N^*], \tag{6}$$

$$W^* = (\beta/2)Y^{*2} + N^*(\beta x^{*2} + \beta y^{*2} - K^*) - s^* N^* x^*$$
$$= W^*[t, s^*; N, N^*], \text{ if } s^* > 0, \tag{6*}$$

and

$$W^* = (\beta/2)Y^{*2} + N^*(\beta x^{*2} + \beta y^{*2} - K^*) + t^* N^* x^*$$
$$= W^*[t, t^*; N, N^*], \text{ if } s^* < 0,\ t^* = -s^*. \tag{6*'}$$

3. OPTIMAL TRADE POLICIES AND ENDOGENOUS TIMING UNDER NO ENTRY

3.1 Simultaneous-move Game and Optimal Trade Policy

In this part we will derive the reaction functions of both governments in the case of no entry. Taking into account (3), (3*), (5), and (6), the first order condition of the domestic government's trade policy is given by

$$\frac{\partial W}{\partial t} = \frac{N^*}{\beta(F+1)}\{\beta Nx + \beta(N+1)x^* - (N+1)t\} = 0. \tag{7}$$

Thus,

$$t = \frac{\beta Nx + \beta(N+1)x^*}{N + 1}$$
$$= \frac{(2N+1)\alpha - N(N^*-N)c + (c^*-s^*)\{NN^* - (N+1)^2\}}{N^* + 2(N+1)^2}$$
$$= t[s^*; N, N^*]\ (> 0),\ \text{if } s^* \geq 0, \tag{8}$$

and

$$= \frac{(2N+1)\alpha - N(N^*-N)c + (c^*+t^*)\{NN^* - (N+1)^2\}}{N^* + 2(N+1)^2}$$
$$= t[t^*; N, N^*]\ (> 0),\ \text{if } s^* < 0,\ t^* = -s^*. \tag{8'}$$

Thus, optimal trade policy of the domestic government is an import tariff, regardless of the foreign trade policy. Note that the slope of its reaction function curve depends on the number of firms in both countries. That is, when $s^* > 0$,

$$\frac{\partial t}{\partial s^*} = \frac{(N+1)^2 - NN^*}{N^* + 2(N+1)^2} > (<)\ 0 \leftrightarrow \frac{(N+1)^2}{N} > (<)\ N^*, \qquad (9)$$

and when $t^* = - s^* > 0$,

$$\frac{\partial t}{\partial t^*} = \frac{NN^* - (N+1)^2}{N^* + 2(N+1)^2} > (<)\ 0 \leftrightarrow \frac{(N+1)^2}{N} < (>)\ N^*. \qquad (9')$$

Next, taking into account (3), (3*), (5), (5*), and (6*)(or (6*′)), the first order condition of the foreign government's trade policy is given by

$$\frac{\partial W^*}{\partial s^*} = \frac{N^*}{\beta(F+1)}\{\beta(N+1 - N^*)x^* - (N+1)s^*\} = 0. \qquad (7*)$$

Thus, when $N + 1 > N^*$,

$$\begin{aligned} s^* &= \frac{\beta\{(N+1) - N^*\}x^*}{N+1} \\ &= \frac{\{(N+1) - N^*\}\{\alpha + Nc - (N+1)(c^*+t)\}}{2N^*(N+1)} \\ &= s^*[t;N, N^*]\ (> 0), \end{aligned} \qquad (8*)$$

and when $N + 1 < N^*$,

$$\begin{aligned} t^* &= \frac{\beta\{N^* - (N+1)\}x^*}{N+1} \\ &= \frac{\{N^* - (N+1)\}\{\alpha + Nc - (N+1)(c^*+t)\}}{2N^*(N+1)} \\ &= t^*[t;N, N^*]\ (= - s^*[t;N, N^*])\ (> 0). \end{aligned} \qquad (8*')$$

Thus, as shown in Dixit (1984), optimal trade policy of the foreign government depends on the number of firms in both countries. That is, it is an export subsidy (tax) if $N + 1 > (<) N^*$. Note that $s^* = 0$, if $N + 1 = N^*$. Hence, the foreign government chooses a free trade policy regardless of the domestic trade policy. Although we will not discuss this special case here, we will meet a similar issue again in analyzing the case of free entry.

We understand that the slope of the reaction function of the foreign government is always negative regardless of its trade policies. That is,

$$\frac{\partial s^*}{\partial t} = \frac{N^* - (N + 1)}{2N^*} < 0, \text{ if } N^* < N + 1, \tag{9*}$$

and

$$\frac{\partial t^*}{\partial t} = \frac{(N + 1) - N^*}{2N^*} < 0, \text{ if } N^* > N + 1. \tag{9*'}$$

Summing up the above discussion, we present **Lemma 1** as follows:

Lemma 1.

(a) When $\underline{N^*} < N^* < N + 1$, $s^* = s^*[t;N, N^*]$ (> 0), $t = t[s^*;N, N^*]$ (> 0), $\partial s^*/\partial t < 0$, and $\partial t/\partial s^* > 0$, where

$$\underline{N^*} = \frac{(N+1)\{6N+5 - \sqrt{32N^2+64N+33}\}}{2\ (N - 2)}, \text{ if } N > 2. \tag{10}$$

(b) When $N + 1 < N^* < (N+1)^2/N$, $t^* = t^*[t;N, N^*]$ (> 0), $t = t[t^*;N, N^*]$ (> 0), $\partial t^*/\partial t < 0$, and $\partial t/\partial t^* < 0$.

(c) When $(N+1)^2/N < N^* < \overline{N^*}$, $t^* = t^*[t;N, N^*]$ (> 0), $t = t[t^*;N, N^*]$ (> 0), $\partial t^*/\partial t < 0$, and $\partial t/\partial t^* > 0$, where

$$\overline{N^*} = \frac{3(N+1)^2}{N - 1}. \tag{11}$$

Proof. See **Appendix 1**.

Therefore, we can derive a Nash equilibrium, s^{*n} (or t^{*n}), t^n, simultaneously satisfying (8) and (8*) (or, (8′) and (8*′)).

Before deriving Stackelberg equilibria, we will show the characteristic of welfare functions of both governments in three situations shown in **Lemma 1**. As to **(a)**, from (6), we obtain

$$dW = (\partial W/\partial t)dt + (\partial W/\partial s^*)ds^*$$

$$= \frac{N^*}{\beta(F+1)}\{\beta Nx + \beta(N+1)x^* - (N+1)t\}dt$$

$$+ \frac{N^*}{\beta(F+1)}\{\beta N^* x^* - \beta Nx + (N+1)t\}ds^* . \qquad (12)$$

Susitituting (3) and (3*) into (12), the marginal rate of substitution of the isowelfare curve, dW = 0, is given by

$$-\frac{ds^*}{dt} = \lambda,$$

$$\lambda = \frac{(2N+1)\alpha - N(N^*-N)c + \{NN^*-(N+1)^2\}(c^*-s^*) - \{N^*+2(N+1)^2\}t}{(N^*-N)\alpha + N(2N^*+1)c - N^*(2N+1)(c^*-s^*) + \{(N+1)^2-NN^*\}t}.$$

Thus, since it holds that

$$\frac{d^2s^*}{dt^2} = -\frac{\partial\lambda}{\partial t} > 0, \qquad (12')$$

we can see that the isowelfare curve is convex to the t-axis. Also, from (12), since we get

$$\left.\frac{\partial W}{\partial s^*}\right|_{t=t^n} > 0, \qquad (13)$$

an increase in an export subsidy raises the level of the domestic welfare.

Similarly, we obtain

$$dW^* = (\partial W^*/\partial t)dt + (\partial W^*/\partial s^*)ds^*$$

$$= -\frac{N^*}{\beta(F+1)}(N+1)\{2\beta x^* - s^*\}dt$$

$$+ \frac{N^*}{\beta(F+1)}\{\beta(N+1 - N^*)x^* - (N+1)s^*\}ds^* . \qquad (12*)$$

Susitituting (3) and (3*) into (12*), the marginal rate of substitution of the isowelfare curve, $dW^{*} = 0$, is given by

$$\frac{dt}{ds^{*}} = \lambda^{*},$$

$$\lambda^{*} = \frac{(N+1 - N^{*})\{\alpha + Nc - (c^{*}+t)(N+1) - 2N^{*}(N+1)s^{*}}{(N+1)[2\{\alpha + Nc - (c^{*}+t)(N+1)\} + \{(N+1)-N^{*}\}s^{*}]}.$$

Thus, since it holds that

$$\frac{d^2t}{ds^{*2}} = \frac{\partial\lambda^{*}}{\partial t} < 0, \tag{12*'}$$

we can see that the isowelfare curve is concave to the s^{*}-axis. Also, from (12*), since we get

$$\frac{\partial W^{*}}{\partial t}\Big|_{s^{*}=s^{*n}} < 0, \tag{13*}$$

an increase in an import tariff lowers the level of the foreign welfare .

As to **(b)** and **(c)**, by a similar use of the previous analysis, we can derive that the welfare function of the domestic (foreign) government is concave to the $t(t^{*})$-axis. Also, since it holds that

$$\frac{\partial W}{\partial t^{*}}\Big|_{t=t^{n}} < 0, \text{ and } \frac{\partial W^{*}}{\partial t}\Big|_{t^{*}=t^{*n}} < 0, \tag{14}$$

we can see that an increase in an import tariff (export tax) lowers the level of the foreign (domestic) welfare.

3.2 Stackelberg equilibrium and Endogenous timing

Since we have obtained the characteristic of welfare functions in three situations, we will consider Stackelberg equilibria and the endogenous timing of trade policies.

(a) $\underline{N}^* < N^* < N + 1$

First, we suppose that the domestic (foreign) government is a leader (follower). Taking into account (8*), the welfare function of the domestic government is given by

$$W = W[t,\ s^*[t;N,\ N^*];N,\ N^*]. \tag{15}$$

Thus, the first order condition is given by

$$\frac{\partial W}{\partial t} + \frac{\partial W}{\partial s^*}\frac{\partial s^*}{\partial t} = 0. \tag{16}$$

There is a Stackelberg equilibrium, t^l, s^{*f}, satisfiying (7*) and (16).

Similarly, in the case that the foreign (domestic) government is a leader (follower), taking into account (8), the welfare function of the foreign government is given by

$$W^* = W[s^*,\ t[s^*;N,N^*];N,N^*]. \tag{15*}$$

Thus, the first order condition is given by

$$\frac{\partial W^*}{\partial s^*} + \frac{\partial W^*}{\partial t}\frac{\partial t}{\partial s^*} = 0. \tag{16*}$$

There is a Stackelberg equilibrium, t^f, s^{*l}, satisfiying (7) and (16*).

Therefore, from **Lemma 1 (a)**, (16), and (16*), we derive **Lemma 2** as follows:

Lemma 2. When $\underline{N}^* < N^* < N + 1$, it holds that

$$t^l < t^n, \quad t^f < t^n, \tag{17}$$

and

$$s^{*f} > s^{*n} > 0 > s^{*l}, \quad t^{*l} = - s^{*l}. \tag{17*}$$

Proof. See **Appendix 2.**

Regardless of the order of its action, the level of an import tariff in the Stackelberg equilibrium is lower than in the Nash equilibirum of the simultaneous-move game. On the other hand, when the foreign government is a follower, the level of an export subsidy in the Stackelberg equilibrium is higher than that in the Nash equilibrium of the simultaneous-move game. As shown in Collie (1991), however, when the foreign government is a leader, the optimal export policy is an export tax.

We will confirm whether the domestic (foreign) government prefers to be a leader (follower) or not. We present **Lemma 3** as follows:

Lemma 3. When $\underline{N}^* < N^* < N + 1$, it holds that

$$W^l > W^n > W^f, \tag{18}$$

and

$$W^{*f} > W^{*n}, \quad W^{*l} > W^{*n}. \tag{18*}$$

Proof. See **Appendix 3.**

(18) implies that taking a first move is a strictly dominating strategy for the domestic government. On the other hand, (18*) means that the foreign government prefers playing the sequential-move game to playing the simultaneous-move game, and wants to

take a first move. Therefore, taking into account **Lemmata 2, 3** and Theorem IV in Hamilton and Slutsky (1990), we derive **Proposition 1** as follows:

Proposition 1. When $\underline{N}^* < N^* < N + 1$, there is such a unique Stackelberg equilibrium that the domestic government, as a leader, charges a lower import tariff than in the Nash equilibrium, and the foreign government, as a follower, charges a higher export subsidy than in the Nash equilibrium.

Proof. Omit.

Proposition 1, which is identical to Proposition 2 in Collie (1994), implies that the domestic government should not commit a countervailing duty in response to the forign export subsidy. Rather, it should set a lower tariff than in the Nash equilibrium before the foreign export subsidy policy. As a result, both governments are better off than in the Nash equilibirum in the simultaneous-move game, or than in the Stackelberg equilibirum in which the domestic (foreign) government is a follower (leader). See **S** in **Figure 1 (a)**.

We have understood that Collie's conclusion holds if the number of firms in both countries satisfies **(a)**. If not, however, his conclusion must be modified. Because we will have discussions about **(b)** and **(c)** similar to those in **(a)**, we will present important results as lemmata and propositions to save space.

(b) $N + 1 < N^* < (N+1)^2/N$

We obtain **Lemma 4** as follows:

Lemma 4. When $N + 1 < N^{*} < (N+1)^2/N$, it holds that

$$t^{l} > t^{n} > t^{f}, \tag{19}$$

and

$$t^{*l} > t^{*n} > t^{*f}. \tag{19*}$$

Proof. Omit, since we can prove it, taking into account **Lemma 1 (b)**, and by a similar way to the proof of **Lemma 2**.

If the government is a leader (follower) in the Stackelberg equilibrium, the level of its tariff or tax is higher (lower) than in the Nash equilibrium. Moreover, we have **Lemma 5** as follows:

Lemma 5. When $N + 1 < N^{*} < (N+1)^2/N$, it holds that

$$W^{l} > W^{n} > W^{f}, \tag{20}$$

and

$$W^{*l} > W^{*n} > W^{*f}. \tag{20*}$$

Proof. Omit, since we can prove it, taking into account **Lemma 4**, and by a similar way to the proof of **Lemma 3**.

(20) and (20*) imply that both governments prefer being a leader to being a follower or plyaing a simultaneous-move game. In other words, taking a first move is a strictly dominating strategy for both governments. Therefore, we derive **Proposition 2** as follows:

Proposition 2. When $N + 1 < N^{*} < (N+1)^2/N$, there is a unique Nash equilibirum in the simultaneous-move game.

Proof. Omit, since we can prove it, by taking into account **Lemma 5** and Theorem II in Hamilton and Slutsky (1990).

Proposition 2 means that although both governments take a first move and charge a higher tariff or tax than in the Nash equilibrium, prisoners' dilemma happens, and that consequently the Nash equilibrium of the simultaneous-move game holds. See **N** in **Figure 1 (b)**.

(c) $(N+1)^2/N < N^* < \overline{N^*}$

We can obtain **Lemma 6** as follows:

Lemma 6. When $(N+1)^2/N < N^* < \overline{N^*}$, it holds that

$$t^l > t^n > t^f, \tag{21}$$

and

$$t^{*n} > t^{*f},\ t^{*n} > t^{*l}. \tag{21*}$$

Proof. Omit, since we can prove it, taking into account **Lemma 1 (c)**, and by a similar way to the proof of **Lemma 2**.

If the domestic government is a leader (follower) in the Stackelberg equilibrium, the level of an import tariff is higher (lower) than in the Nash equilibrium. On the other hand, when the foreign government charges a higher export tax in the Nash equilibirum of the simultaneous-move game than in the Stackelberg equilibrium. Moreover, we present **Lemma 7** as follows:

Lemma 7. When $(N+1)^2/N < N^* < \overline{N^*}$, it holds that

$$W^l > W^n, \ W^f > W^n, \tag{22}$$

and

$$W^{*l} > W^{*n} > W^{*f}. \tag{22*}$$

Proof. Omit, since we can prove it, taking into account **Lemma 6**, and by a similar way to the proof of **Lemma 3**.

(22) implies that the domestic government reluctantly takes a second move, since it prefers playing the sequential-move game to playing the simultaneous-move game, although it wants to take a first move. On the other hand, (22*) implies that taking a first move is a strictly dominating strategy for the foreign government. Thus, we can see that the domestic government is forced to be a follower, since it knows that the foreign government must be a leader, even if the follower's welfare is lower than the leader's. Therefore, we can derive **Proposition 3** as follows:

Proposition 3. When $(N+1)^2/N < N^* < \overline{N^*}$, there is such a unique Stackelberg equilibirum that the domestic government, as a follower, charges a lower import tariff than in the Nash equilibrium, and the foreign government, as a leader, charges a lower export tax than in the Nash equilibrium.

Proof. It can be proved by taking into account **Lemmata 6, 7** and Theorem IV in Hamilton and Slutsky (1990).

Proposition 3 means that the domestic government should charge a countervailing duty corresponding to the foreign export subsidy. As a result, however, both governments are better off than in the Nash equilibrium of the simultaneous-move game or than in the Stackelberg equilibrium of the sequential-move game, in which the domestic (foreign) government is a leader (follower). See **S*** in **Figure 1 (c)**. This result is opposite to that of Collie (1994).

4. OPTIMAL TRADE POLICIES AND ENDOGENOUS TIMING UNDER FREE ENTRY

4.1 Market Equilibrium and Optimal Trade Policies

We will assume free entry where the profit of each firm will not be made. Taking into account (2), (2*), (3), (3*), and (4*), we have

$$\beta x^2[t, s^*, N, N^*] = K, \tag{23}$$

$$\beta x^{*2}[t, s^*, N, N^*] + \beta y^{*2}[N^*] = K^*. \tag{23*}$$

Given the trade policies of both governments, (3), (3*), (4*), (23), and (23*) determine the variables x, x^*, y^*, N, and N^*. Hence, taking into account the zero-profit condition, the welfare functions of both governments are given by

$$\begin{aligned} W &= (\beta/2)X^2 + tN^*x^* \\ &= W[t, s^*], \end{aligned} \tag{24}$$

$$\begin{aligned} W^* &= (\beta/2)Y^{*2} - s^*N^*x^* \\ &= W^*[t, s^*]. \end{aligned} \tag{24*}$$

Note that if $s^* < 0$, $s^* = -t^*$, then $W^* = (\beta/2)Y^{*2} + t^*N^*x^* = W^*[t, t^*]$.

First, taking into account (3), (3*), (4*), (23), (23*), and (24), the first order condition of the domestic government's trade policy is given by

$$\frac{\partial W}{\partial t} = N^{*}x^{*} + t\left(\frac{dx^{*}}{dt}N^{*} + \frac{dN^{*}}{dt}x^{*}\right)$$

$$= N^{*}x^{*} - t\frac{dN}{dt}x = 0, \tag{25}$$

where

$$\frac{dN}{dt}x = -\left(\frac{dx^{*}}{dt}N^{*} + \frac{dN^{*}}{dt}x^{*}\right)(> 0),$$

$$\frac{dx^{*}}{dt} = -\frac{1}{\beta}, \text{ and } \frac{dN^{*}}{dt}x^{*} = -\frac{(N^{*}+1)x^{*2}}{\beta y^{*2}}. \tag{25'}$$

Thus, regardless of the foreign trade policy, optimal trade policy of the domestic government is an import tariff, which is given by

$$t = \frac{N^{*}x^{*}}{(dN/dt)x} \tag{26}$$

$$= t[s^{*}] \ (> 0).$$

Before analyzing the slope of the reaction function, we will verify the second order condidion:

$$\frac{\partial^{2}W}{\partial t^{2}} = -x\left(2\frac{dN}{dt} + \frac{d^{2}N}{dt^{2}}t\right), \tag{27}$$

where

$$\frac{d^{2}N}{dt^{2}} = -\frac{3(N^{*}+1)x^{*2}}{\beta^{2}xy^{*2}}\left(1 + \frac{x^{*2}}{y^{*2}}\right) \ (< 0). \tag{27'}$$

Thus, SOC holds if and only if

$$2\frac{dN}{dt} + \frac{d^{2}N}{dt^{2}}t > 0. \tag{28}$$

Substituting (25´), (26), and (27´) into (28), the left-hand side of (28) is given by

$$\text{LHS} = 2N^{*2} + N^{*}(N^{*}+1)z - (N^{*}+1)(N^{*}-2)z^{2},$$

where $z = x^{\cdot 2}/y^{\cdot 2}$ (> 0). Thus, SOC always holds if $N^{\cdot} \leq 2$. When $N^{\cdot} > 2$, SOC holds if and only if $z < \bar{z}$, where

$$\bar{z} = \frac{N^{\cdot}\{N^{\cdot}+1 + \sqrt{(N^{\cdot}+1)(9N^{\cdot}-15)}\}}{2(N^{\cdot}+1)(N^{\cdot}-2)}.$$

We will confirm the slope of the reaction function of the doemstic government. From (26), we derive

$$\frac{dt}{ds^{\cdot}} = \frac{\dfrac{dN}{dt} + \dfrac{d^2N}{dt^2}t}{2\dfrac{dN}{dt} + \dfrac{d^2N}{dt^2}t}. \tag{29}$$

Taking into account (28), the sign of (29) is given by

$$\frac{dt}{ds^{\cdot}} > (<)\ 0 \leftrightarrow \frac{dN}{dt} + \frac{d^2N}{dt^2}t > (<)\ 0. \tag{29$'$}$$

From (25′) and (27′), (29′) is rewritten by

$$N^{\cdot 2} - N^{\cdot}(N^{\cdot}+1)z - (N^{\cdot}+1)(2N^{\cdot}-1)z^2 > (<)\ 0.$$

Thus, it holds that

$$\frac{dt}{ds^{\cdot}} > (<)\ 0 \leftrightarrow z < (>)\ z^{\cdot}, \tag{30}$$

where

$$z^{\cdot} = \frac{N^{\cdot}\{\sqrt{(N^{\cdot}+1)(9N^{\cdot}-3)} - (N^{\cdot}+1)\}}{2(N^{\cdot}+1)(2N^{\cdot}-1)}\ (< \bar{z}).$$

To analyze below, we will suppose two situations as follows:

(d) $0 < z < z^{\cdot}$,

(e) $z^{\cdot} < z < \bar{z}$.

Secondly, we will consider optimal export policy of the foreign government, given the import tariff policy. Taking into account (3), (3*), (4*), (23), (23*), and (24*), the first order condition is given by

$$\frac{\partial W^*}{\partial s^*} = - s^* \left(\frac{dx^*}{ds^*} N^* + \frac{dN^*}{ds^*} x^* \right)$$

$$= - s^* \frac{dN}{dt} x = 0, \tag{25*}$$

where

$$\frac{dN}{dt} x = \left(\frac{dx^*}{ds^*} N^* + \frac{dN^*}{ds^*} x^* \right) (> 0).$$

(25*) implies that the reaction function of the foreign government is identical to the t-axis. The optimal export policy is a free trade, if the foreign government is a follower, or if it plays the simultaneous-move game. In other words, as shown below, if it will be a leader, the optimal export policy is not necessarily a free trade. Also, although we will omit the proof, SOC of (25*) always holds.

Therefore, we have derived the reaction functions of both governments. We can obtain a Nash equilibrium in the case of free entry, s^{*n} (= 0), t^n, simultaneously satisfying with (25) and (25*). Also, we can show that the Nash equilibrium is stable. As mentioned above, it holds that $s^{*f} = 0$, given the import tariff of the doemestic government as a leader, and that $t^n = t^l$, in which the domestic government, as a leader, charges the same import tariff as that in the Nash eqiulibrium. That is, the Stackelberg equilibrium, in which the domestic (foreign) government is a leader (follower), is identical to the Nash equilibrium of the simultaneous-move game.

Before discussing the Stackelberg equilibrium, in which the foreign (domestic) government is a leader (follower), and the endogenous timing, we will show the characteristic of welfare functions of both governments in the case of free entry.

By a similar procedure in 3.1, from (24),

$$dW = (\partial W/\partial t)dt + (\partial W/\partial s^{*})ds^{*}$$
$$= \left(N^{*}x^{*} - t\frac{dN}{dt}x\right)dt + \left(t\frac{dN}{dt}x\right)ds^{*} \qquad (32)$$

Thus, the marginal rate of substitution of the isowelfare curve, $dW = 0$, is given by

$$-\frac{ds^{*}}{dt} = \mu,$$

where

$$\mu = \frac{N^{*}x^{*} - t\dfrac{dN}{dt}x}{t\dfrac{dN}{dt}x}.$$

Thus, since it holds that

$$\frac{d^2 s^{*}}{dt^2} = -\frac{\partial \mu}{\partial t} > 0, \qquad (32')$$

we can see that the isowelfare curve is convex to the t-axis. Also, from (32) and (25′), since we can get

$$\frac{\partial W}{\partial s^{*}}\bigg|_{t=t^n} > 0, \qquad (33)$$

an increase in an export subsidy raises the level of the domestic welfare.

Similarly, we have

$$dW^{*} = (\partial W^{*}/\partial t)dt + (\partial W^{*}/\partial s^{*})ds^{*}$$
$$= \left(-N^{*}x^{*} + s^{*}\frac{dN}{dt}x\right)dt - \left(s^{*}\frac{dN}{dt}x\right)ds^{*} \qquad (32*)$$

Thus, the marginal rate of substitution of the isowelfare curve, $dW^{*} = 0$, is given by

$$\frac{dt}{ds^*} = \mu^*, \qquad (32*)$$

where

$$\mu^* = \frac{s^* \dfrac{dN}{dt} x}{- N^* x^* + s^* \dfrac{dN}{dt} x}.$$

Thus, since it holds that

$$\frac{d^2 t}{ds^{*2}} = \frac{\partial \mu^*}{\partial t} < 0, \qquad (32*')$$

we can see that the isowelfare curve is concave to the s^*-axis. Also, from (32*), since we can get

$$\frac{\partial W^*}{\partial t}\Big|_{s^* = s^{*n} (= 0)} < 0, \qquad (33*)$$

an increase in an import tariff lowers the level of the foreign welfare.

4.2 Stackelberg Equilibrium and Endogenous Timing

First, we will show the Stackelberg equilibrium when the foreign (domestic) government is a leader (follower) in two situations.

(d) $0 < z < z^*$

Taking into account (15*), the welfare function of the foreign government as a leader in the case of free entry is given by

$$W^* = W[s^*, t[s^*]], \qquad (34*)$$

where $t = t[s^*]$ is given by (26) and it holds that $dt/ds^* > 0$ in this situation. Then, the first order condition is

$$\frac{\partial W^*}{\partial s^*} + \frac{\partial W^*}{\partial t}\frac{\partial t}{\partial s^*} = 0. \tag{35*}$$

There is a Stackelberg equilibrium, t^f, s^{*l}, satisfiying with (25) and (35*). Therefore, we obtain **Lemma 8** as follows:

Lemma 8. When $0 < z < z^*$, it holds that

$$t^l = t^n > t^f \ (> 0), \tag{36}$$

and

$$s^{*n} = s^{*f} = 0 > s^{*l}, \ s^{*l} = -t^{*l}. \tag{36*}$$

Proof. See **Appendix 4**.

(36*) means that if the foreign government is a leader, it will use an export tax policy. Otherwise, it will use a free trade policy. Before showing the endogenous timing of the trade policy game in this situation, we present **Lemma 9** as follows:

Lemma 9. When $0 < z < z^*$, it holds that

$$W^l = W^n > W^f, \tag{37}$$

and

$$W^{*l} > W^{*n} = W^{*f}. \tag{37*}$$

Proof. Omit. We can prove it by a similar way to the proof of **Lemma 3**, taking into account (32´), (33), (32*´), (33*), and **Lemma 8**.

(37) and (37*) imply that taking a first move is a weakly dominating strategy for both governments. Therefore, from **Lemma 9** and Theorem II in Hamilton and Slutsky (1990), we have **Proposition 4** as follows:

Proposition 4. When $0 < z < z^*$, there is a Nash equilibrium in the simultaneous-move game.

Proof. Omit.

Since in this situation the Nash equilibrium is identical to the Stackelberg equilibrium, where the domestic (foreign) government as a leader (follower) uses an import tariff (a free trade) policy, it is indifferent for both governments to play the simultaneous-move game and to take their move in the sequential-move game. **Proposition 4** may weakly support Collie's conclusion that the domestic government should not commit a countervailing duty, although the foreign government uses a free trade policy. See **N** (= **S**) in **Figure 2 (d)**.

(e) $z^* < z < \bar{z}$

Next, in this situation, we have **Lemma 10** as follows:

Lemma 10. When $z^* < z < \bar{z}$, it holds that

$$t^l = t^n > t^f \ (> 0), \tag{38}$$

and

$$s^{*n} = s^{*f} = 0 < s^{*l}. \tag{38*}$$

Proof. Omit. We can prove it by a similar way to the proof of **Lemma 8**, taking into account (30), (33*), and (35*).

(38*) means that if the foreign government is a leader, it will use an export subsidy policy. Otherwise, it will use a free trade policy. Before showing the endogenous timing of trade policy game in this situation, we present **Lemma 11** as follows:

Lemma 11. When $z^{*} < z < \bar{z}$, it holds that

$$W^{l} = W^{n} < W^{f}, \tag{39}$$

and

$$W^{*l} > W^{*n} = W^{*f}. \tag{39*}$$

Proof. Omit. We can prove it by a similar procedure on the proof of **Lemma 3**, taking into account (32´), (33), (32*´), (33*), and **Lemma 10**.

(39) implies that taking a second move is a weakly dominating strategy for the domestic government. On the other hand, (39*) implies that taking a first move is a weakly dominating strategy for the foreign government. Therefore, from **Lemma 11** and Theorem IV in Hamilton and Slutsky (1990), we have **Proposition 5** as follows:

Proposition 5. When $z^{*} < z < \bar{z}$, there is such a Stackelberg equilibrium that the foreign (domestic) government as a leader (follower) uses an export subsidy (import tariff).

Proof. Omit.

Proposition 5 is opposite to Collie's conclusion. That is, if it is allowed to be a free entry, and if the proportion of the export to the output in the foreign market is relatively large, the domestic government should commit a lower countervailing duty than in the Nash equilibrium in response to a foreign export subsidy. See S^{*} in **Figure 2 (e)**.

5. CONCLUSION

It has been already shown in another model of Collie (1991) that corresponding to a countervailing duty, the optimal export policy is an export tax. Note that this result is shown in **Lemmata 2, 4, 6** of our model. But it is assumed in this model that the order of their move is exogenously given by the fact that the domestic (foreign) government is a follower (leader). Introducing a game in which the order of the trade policy decision is endogenously decided, Collie (1994) has insisted that the domestic government should not commit a countervailing duty corresponding to a foreign export policy. Rather, it should charge a lower import tariff before the foreign export subsidy policy, and thereby both governments are better off than in the Nash equilibrium of the simultaneous-move game. That is, the domestic government, as a leader, charges a lower import tariff and the foreign government, as a follower, charges a larger export subsidy. Hence, the domestic government benefits from a larger export subsidy while the foreign government benefits from a lower import tariff.

Generalizing the model of Collie (1994), we have shown that his main conclusions hold if the number of exporting firms is equal to or less than that of importing firms in the case of no entry (see **Proposition 1**). Otherwise, his conclusions do not necessarily hold. That is, if the number of exporting firms is sufficiently larger than that of importing firms in the case of no entry, or if the reaction function of the domestic government is downward-sloping in the case of free entry, the domestic government, being a follower, should charge a countervailing duty responding to the foreign export tax or subsidy (see **Propositions** 3 and 5). In addition, there can be a case where both governments prefer to play the simultaneous-move game (see **Propositions 2** and 4).

Appendix 1. Proof of **Lemma 1.**

We will omit the proof of the optimal trade policies and the slope of reaction functions, since we have shown them above. Here we will show (10) and (11) which are derived from the stability condition. That is, the stability condition is given by

$$\left|\frac{\partial s^{*}}{\partial t}\right|\left|\frac{\partial t}{\partial s^{*}}\right| < 1.$$

Thus, taking into acount (9) and (9*), we can have (10). By similar way to the above, taking into acount (9´), (9*´), and the stability condition, i.e.

$$\left|\frac{\partial t^{*}}{\partial t}\right|\left|\frac{\partial t}{\partial t^{*}}\right| < 1,$$

we obtain (11). Note that we can see the second order conditions in three situations always hold, given the number of the firms in both countries. Q.E.D.

Appendix 2. Proof of Lemma 2.

First, evaluating (16) at a Nash equilibrium, and taking into account (9*) and (13), (16) is negative. Thus, $t^{l} < t^{n}$. Similarly, evaluating (16*) at a Nash equilibrium, and taking into account (9) and (13*), (16*) is negative. Thus, $s^{*n} > s^{*l}$. Moreover, evaluating (16*) at $s^{*} = 0$, then (16*) is negative. Thus, $s^{*n} > 0 > s^{*l}$, $t^{*l} = - s^{*l}$.

Next, taking into account (9*) and $s^{*n} > 0 > s^{*l}$, it holds that $t^{f} < t^{n}$. Similarly, taking into account (9) and $t^{l} < t^{n}$, it holds that $s^{*f} > s^{*n} > 0$. Q.E.D.

Appendix 3. Proof of Lemma 3.

First, it will be denoted that $W^{l} = W[t^{l}, s^{*f}]$, and $W^{n} = W[t^{n},$

$s^{*n}]$ (Note that N, N^* is omited). We will give $W' = W[t^n, s^{*f}]$. Then, from (13) and (17*), it holds that $W' > W^n$. Also, taking into account that (16) is negative at a Nash equilibrium, and (17), $W^l > W'$. Thus, $W^l > W^n$. Next, from (8), it will be denoted that $W^f = W[t^f, s^{*l}] = W[t[s^{*l}], s^{*l}]$, and that $W^n = W[t^n, s^{*n}] = W[t[s^{*n}], s^{*n}]$. Thus, taking into account (7) (or (8)), (9), (13), and (17*), it holds that $W^n > W^f$. Therefore, we obtain (18).

Similarly, as to (18*), first, it will be denoted that $W^{*l} = W^*[s^{*l}, t^f]$, and $W^{*n} = W*[s^{*n}, t^n]$. We will give $W^{*\prime} = W^*[s^{*n}, t^f]$. Then, from (13*) and (17), it holds that $W^{*\prime} > W^{*n}$. Also, taking into account that (16*) is negative at a Nash equilibrium, and (17*), $W^{*l} > W^{*\prime}$. Thus, $W^{*l} > W^{*n}$. Next, from (8*), it will be denoted that $W^{*f} = W^*[s^{*f}, t^l] = W^*[s^*[t^l], t^l]$, and that $W^{*n} = W^*[s^{*n}, t^n] = W^*[s^*[t^n], t^n]$. Thus, taking into account (7*) (or (8*)), (9*), (13*), and (17), it holds that $W^{*f} > W^{*n}$. Therefore, we obtain (18*). Q.E.D.

Appendix 4. Proof of Lemma 8.

As to (36*), first, taking into account (30), and (33*), and evaluating (35*) at the Nash equilibirum, the sign of (35*) is negative. Thus, s^{*n} $(= 0) > s^{*l}$. Also, if the foreign government is a follower, as shown above, it will use a free trade policy as in the Nash equilibrium. Thus, $s^{*n} = s^{*f} = 0$.

Secondly, as to (36), it has been shown that $t^l = t^n$. Also, taking into account (30) and s^{*n} $(= 0) > s^{*l}$, it holds that $t^n = t[s^{*n}]$, and $t^f = t[s^{*l}]$. Thus, $t^n > t^f$. Q.E.D.

References

Albaek, Svend, 1990, Stackelberg Leadership as a Natural Solution under Cost Uncertainty, Journal of Industrial Economics, 38, 355-347.

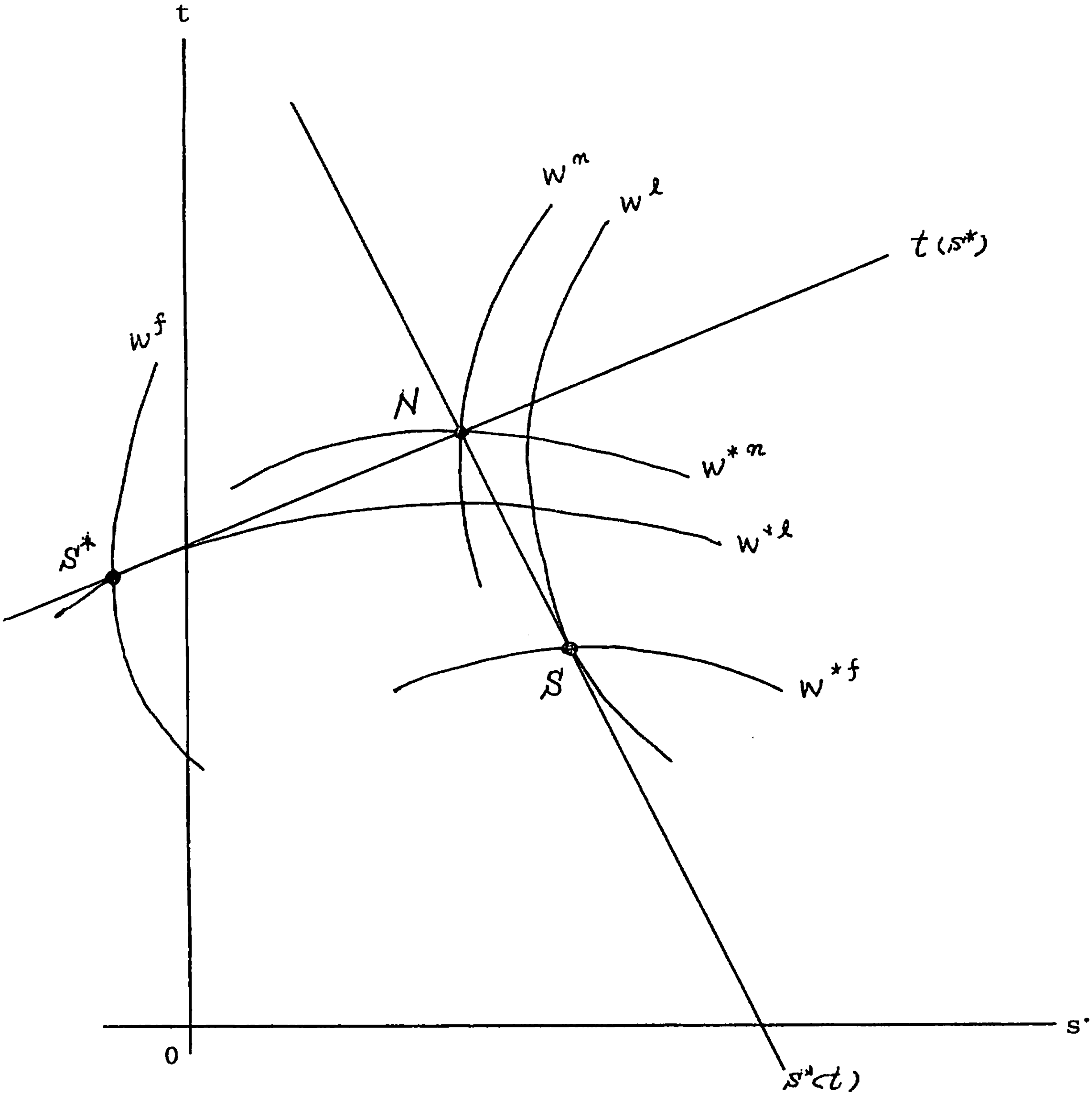
Collie, David, 1991, Export Subsidies and Countervailing Tariffs, Journal of International Economics, 31, 309-324.

Collie, David, 1994, Endogenous Timing in Trade Policy Games: Should Governments Use Countervailing Duties ? Weltwirtschaftliches Archiv, 130, 191-209.

Dixit, A. K., 1984, International Trade Policy for Oligopolistic Industries, Economic Journal, 94, 1-16.

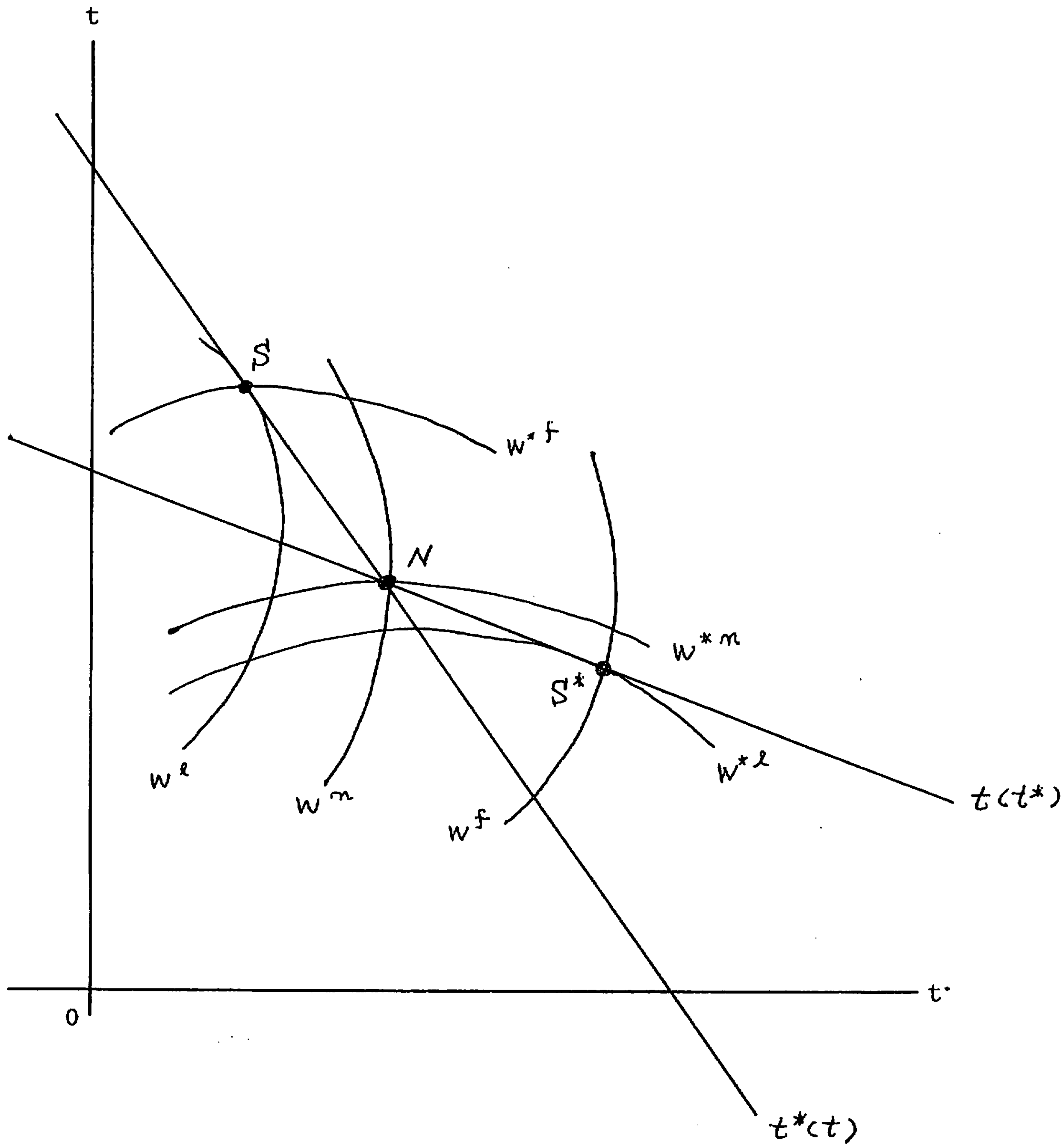
Hamilton, Jonathan H. and Steven M. Slutsky, 1990, Endogenous Timing in Duopoly Games: Stackelberg or Cournot Equilibria, Games and Economic Behavior, 2, 29-46.

Figure 1 (a). Collie Case



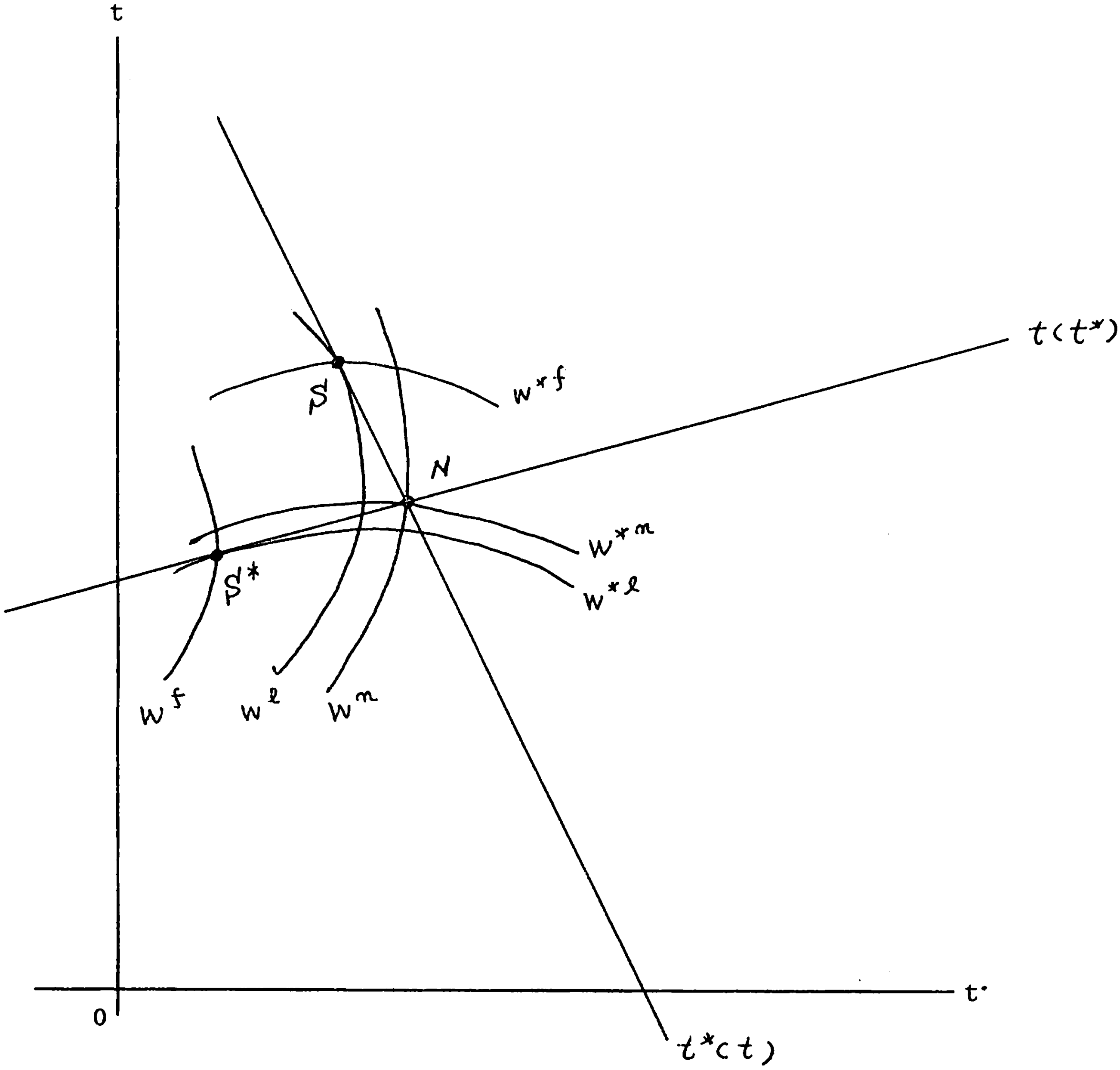
N: Nash equilibrium in the simultaneous-move game
S: Stackelberg equilibrium in the sequential-move game in which the domestic is a leader and the foreign is a follower
S*: Stackelberg equilibrium in the sequential-move game in which the domestic is a follower and the foreign is a leader

Figure 1 (b).



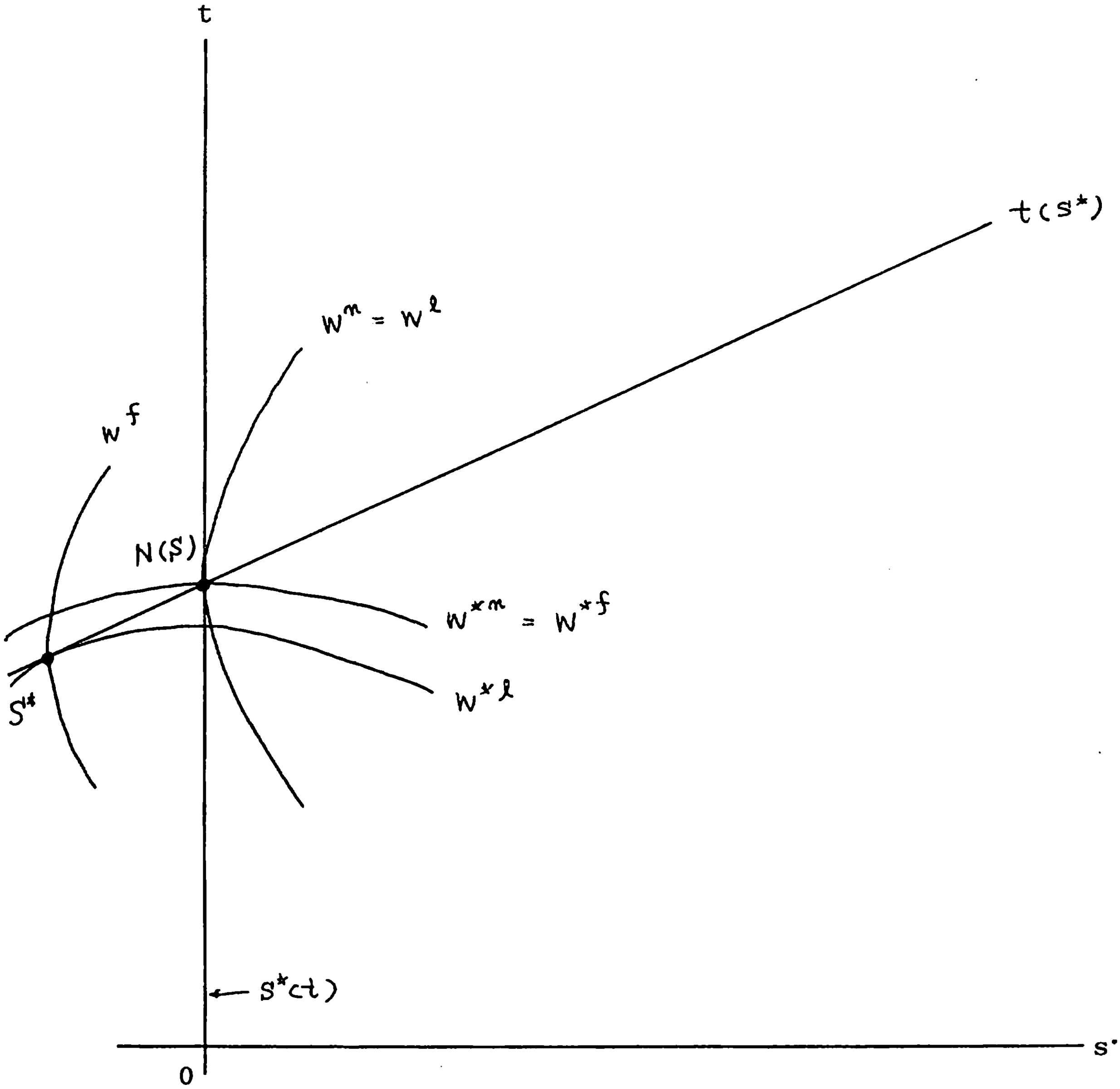
N: Nash equilibrium in the simultaneous-move game
S: Stackelberg equilibrium in the sequential-move game in which the domestic is a leader and the foreign is a follower
S^*: Stackelberg equilibrium in the sequential-move game in which the domestic is a follower and the foreign is a leader

Figure 1 (c).



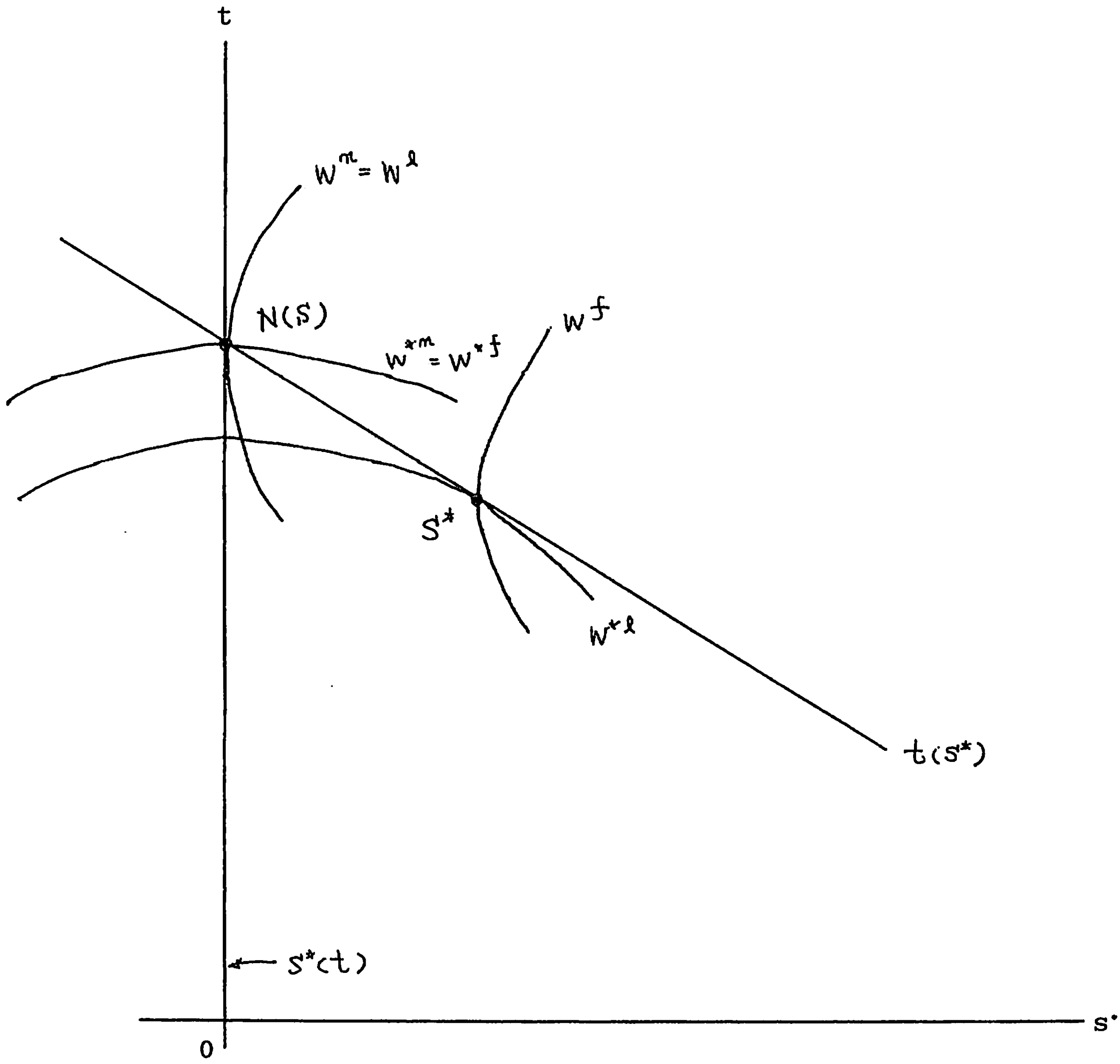
N: Nash equilibrium in the simultaneous-move game
S: Stackelberg equilibrium in the sequential-move game in which the domestic is a leader and the foreign is a follower
S*: Stackelberg equilibrium in the sequential-move game in which the domestic is a follower and the foreign is a leader

Figure 2(d).



N: Nash equilibrium in the simultaneous-move game
S: Stackelberg equilibrium in the sequential-move game in which the domestic is a leader and the foreign is a follower
S': Stackelberg equilibrium in the sequential-move game in which the domestic is a follower and the foreign is a leader

Figure 2(e).



N: Nash equilibrium in the simultaneous-move game
S: Stackelberg equilibrium in the sequential-move game in which the domestic is a leader and the foreign is a follower
S^*: Stackelberg equilibrium in the sequential-move game in which the domestic is a follower and the foreign is a leader