DISCUSSION PAPER SERIES

Discussion paper No. 195

Note on the Excess Entry Theorem in the Presence of Network Externalities

Tsuyoshi Toshimitsu
(School of Economics, Kwansei Gakuin University)

June 2019

MASTERY FOR SERVICE

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

Note on the Excess Entry Theorem in the Presence of Network Externalities

Tsuyoshi TOSHIMITSU☆

School of Economics, Kwansei Gakuin University

Abstract

We reconsider the excess entry theorem in the presence of network externalities under Cournot oligopoly. We demonstrate that if the strength of a network externality is larger (smaller) than a half, the number of firms under free entry is socially too small (too large), based on the second-best criteria.

Keywords: Cournot oligopoly; free entry; excess entry theorem; network externality; a fulfilled equilibrium; passive expectations; responsive expectations

JEL Classifications: D21, D43, D62, L15.

☆ Corresponding author: School of Economics, Kwansei Gakuin University, 1-155, Nishinomiya, Japan, 662-8501, Tel: +81 798 54 6440, Fax: +81 798 51 0944, Email: ttsutomu@kwansei.ac.jp

1. Introduction

Since the seminal papers by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987), who established the excess entry theorem (hereafter, the theorem), there have been many studies that have generalized the theorem in various ways and extended it to industrial policies. Following the review paper by Suzumura (2012), Kagitani et al. (2016) also examined the theorem in the case of horizontally differentiated oligopoly, based on the linear demand of the Shubik and Levitan (1989) model.

In this note, we reconsider the theorem under Cournot oligopoly with network externalities. Currently, many firms are entering information and communications industries, such as telecommunications and Internet services, and are facing strong competition in these markets in which the products and services are associated with network externalities and compatibilities. Thus, we examine whether the effect of entry into such a network goods market is socially efficient. That is, we demonstrate that a social under-entry arises if the strength of a network externality is greater than a half. This result may support entry promotion and competition policy in the market.

In considering the problem, we focus on the behavior of consumer expectations of network sizes in a network goods market. That is, following the definitions by Hurkens and López (2014, p. 1007), responsive expectations means that firms first compete in quantities (or in prices), then consumers form expectations about network sizes and finally consumers make optimal purchasing decisions, given the prices and their expectations. However, passive expectations means that consumers first form expectations about network sizes and then compete in quantities (or in prices); finally, consumers make optimal purchasing decisions, given their expectations. These decisions then lead to the determination of actual market shares and network sizes. Thus, in equilibrium, realized and expected network sizes are the same (see Katz and Shapiro, 1985).[1] As mentioned below, however, differences in the consumer expectations do not change our main results.

2. The Model

2.1 Cournot oligopolistic equilibrium under passive expectations

We consider a Cournot oligopoly in a homogeneous product market with network externalities.[2] We assume that a representative consumer in the market has the following utility function:

$$u = \left\{ aQ - \frac{1}{2}Q^2 + F\left(S^E\right)Q \right\} + q_0, \quad Q = \sum_{i=1}^{n} q_i,\ i, = 1,\dots, n,$$

where a is the intrinsic market size, q_0 the amount of consumption of a numeraire product, Q the total amount of consumption of a homogeneous product associated with network externalities, q_i is the output of firm *i*, and $n(\geq 2)$ is the number of firms in the market. $F\left(S^E\right)$ denotes the network externality function where S^E represents the expected network size. We also assume the following linear network

---

[1] For example, in the case of price competition, consumers realize and expect that when one firm lowers its price it will increase its market share and become the larger network. That is, consumers must adjust their expectations in response to a price change. It is presumed that given these changed expectations, optimal purchasing decisions will cause expected and realized network sizes be equal. Thus, for all prices, expectations are required to be self-fulfilling.

[2] We also assume that the homogeneous product is perfectly compatible.

externality function, i.e., $F\left(S^E\right) = \phi S^E$, where $\phi \in [0,1)$ is the strength of a network externality. The budget constraint is $y = PQ + q_0$, $p_0 = 1$, where income y and price P are given for a representative consumer. Furthermore, we assume that the expected network size S^E is also given for a representative consumer, because the decision of each individual consumer does not affect the expected network sizes in the market.

Under these conditions, a representative consumer maximizes his/her utility with respect to the amount of consumption. The first-order condition (FOC) is given by:

$$\frac{\partial u}{\partial Q} = a - Q + F\left(S^E\right) - P = 0.$$

Thus, we derive the following inverse demand functions[3]:

$$P = a - Q + F\left(S^E\right), \quad Q = \sum_{i=1}^{n} q_i,\ i, = 1, \dots, n. \tag{1}$$

Furthermore, to simplify the analysis, we assume that production costs are zero because we observe low and even negligible marginal running costs in a network industry. Thus, the profit function of firm *i* is $\pi_i = Pq_i = \left\{a - Q + F\left(S^E\right)\right\} q_i$.

Under passive expectations, when deciding their output, each firm takes S^E and thus $F\left(S^E\right)$ as given. The FOC of profit maximization is given by:

$$\frac{\partial \pi_i}{\partial q_i} = P - q_i = a - Q + F\left(S_E\right) - q_i = 0, \quad i = 1, \dots, n. \tag{2}$$

Based on equation (2), in a fulfilled expected and symmetric equilibrium, i.e., $S^E = Q$ and $q_i = q_{-i} = q^P,\ i, -i = 1, \dots, n, i \neq -i$, we derive the following equilibrium output per firm.

[3] See equation (1) of Economides (1996).

$$q^p = \frac{a}{(1-\phi)n+1}, \tag{3}$$

where superscript p denotes passive expectations. Using equations (2) and (3), equilibrium profit is given by:

$$\pi^p = \left(q^p\right)^2 = \left\{\frac{a}{(1-\phi)n+1}\right\}^2. \tag{4}$$

2.2 Responsive expectations

We assume that consumers form expectations of network size after firms' output decisions.[4] As mentioned in the Introduction, this implies that the firms can commit to their output levels, so that consumers believe the output levels and then, based on these active beliefs, form expectations about the network size, i.e., $S^E = Q$. Thus, the inverse demand function is changed as follows:

$$P = a - (1-\phi)Q, \quad Q = \sum_{i=1}^{n} q_i, \ i, = 1,..., n. \tag{5}$$

By following the same procedure as in Section 2.1, we derive the following equilibrium output and profit per firm:

$$q^r = \frac{a}{(1-\phi)(n+1)}, \tag{6}$$

$$\pi^r = (1-\phi)\left(q^r\right)^2 = \frac{1}{(1-\phi)}\left\{\frac{a}{n+1}\right\}^2. \tag{7}$$

where superscript r denotes responsive expectations.

2.3 Comparison: The role of consumers' expectations

[4] See Appendix of Katz and Shapiro (1985).

For a given number of firms in the short-run, we compare the equilibrium output and profit under passive and responsive expectations.

Using equations (3) and (6), we derive $q^r > q^p$ and thus $Q^r > Q^p$. This result implies that, regarding consumer surplus, $CS^r > CS^p$, where $CS^k = \frac{(Q^k)^2}{2}$, $k = p, r$. That is, the firm outputs, total output, and consumer surplus under responsive expectations are larger than those under passive expectations. This is because under responsive expectations, where firms can commit to their output levels in advance, firms have incentives to increase their output levels compared with under passive expectations. Thus, competition is more intense under responsive expectations compared with under passive expectations. Accordingly, output, total output, and thus, consumer surplus under responsive expectations are larger than those under passive expectations.

Furthermore, using equations (4) and (7), with respect to the profits, we derive the following relationship:

$$\pi^r > (<) \pi^p \Leftrightarrow \frac{\sqrt{1-\phi}}{1-\phi} > (<) n, \tag{8}$$

Given $n(\geq 2)$ and using equation (8), we obtain the following result. If $\frac{3}{4} < \phi < 1$, then it holds that $\pi^r > (<) \pi^p \Leftrightarrow \frac{\sqrt{1-\phi}}{1-\phi} > (<) n$. Otherwise, i.e., $0 \leq \phi \leq \frac{3}{4}$, then it holds that $\pi^r < \pi^p$. That is, if the strength of network externalities is sufficiently large, the profit under responsive expectations is larger than that under passive expectations. Conversely, if the strength is small, the reverse is true. In other words, unless either the strength of network externalities is significantly large, or unless the number of firms is small, competition is severe and thus the profit is lower under responsive expectations than under passive expectations.

3. Free Entry and Excess Entry Theorem In the presence of Network Externalities

3.1 Free entry equilibrium under passive and responsive expectations

Before considering the theorem, we examine the long-run equilibrium with free entry where the zero-profit condition arises. We assume that the entry cost per firm is $g > 0$. The zero-profit condition can be expressed as: $\pi^k - g = 0, \quad k = p, r$. Thus, using equations (4) and (7), with respect to the cases of passive and responsive expectations, we obtain the number of firms under free entry as follows.

$$n^p = \frac{a - \sqrt{g}}{(1-\phi)\sqrt{g}}, \tag{9}$$

$$n^r = \frac{a - \sqrt{(1-\phi)g}}{\sqrt{(1-\phi)g}}, \tag{10}$$

where we assume $a > \sqrt{g}$. In this case, by assuming that the number of firms in the case of non-network externalities, i.e., $\phi = 0$, is given by $n^0 = \dfrac{a - \sqrt{g}}{\sqrt{g}} \geq 2$, and based on equations (9) and (10), it holds that $n^p > n^r$. That is, the number of firms under passive expectations is larger than that under responsive expectations. This is because firms have an incentive to increase their output more under responsive expectations, compared with under passive expectations, and thus competition is stronger.

Consequently, the incentive to enter into a market where consumers have responsive expectations is weak for firms, compared with a market where consumers have passive expectations.

Furthermore, regarding the output and total output levels in the long-run, it also holds that $q_f^p = \sqrt{g} < \sqrt{\frac{g}{1-\phi}} = q_f^r$ and $Q_f^p = \frac{a-\sqrt{g}}{1-\phi} < \frac{a-\sqrt{(1-\phi)g}}{1-\phi} = Q_f^r$, where subscript f denotes free entry. Thus, in the case of free entry in the long-run, outputs, total outputs, and thus consumer surplus under responsive expectations are larger than those under passive expectations. This result is similar to that in the case of the short-run, given the number of firms.

3.2 The Excess Entry Theorem reconsidered

We next examine the theorem in the second-best criteria. Considering an entry cost, social welfare can be expressed as:

$$W^k = W\left[n, q^k(n)\right] = CS^k + PS^k - ng = \frac{\left(Q^k\right)^2}{2} + n\left\{\pi^k - g\right\}, \quad k = p, r.$$

Thus, we define the second-best socially optimal number of firms as follows:

$$\begin{aligned} n^{k*} &\equiv \arg\max W\left[n, q^k(n)\right] \\ &= \left\{ n \geq 2 \middle| \frac{\partial W^k}{\partial n} = \frac{\partial CS^k}{\partial n} + n\frac{\partial \pi^k}{\partial n} + \pi^k - g = 0 \right\}, \quad k = p, r. \end{aligned} \tag{11}$$

In this case, with respect to the cases of passive and responsive expectations, we respectively consider whether free entry is excess or not, compared with the second-best socially optimal number of firms.

First, let us examine the case of passive expectations. Using equations (3), (4), (11), and a zero-profit condition, we obtain $\left.\frac{\partial W^r}{\partial n}\right|_{\pi^r - g = 0} = (q^r)^2 \frac{n(-1+2\phi)}{(1-\phi)n+1}$. Thus, we obtain the following relationship.

$$\left.\frac{\partial W^r}{\partial n}\right|_{\pi^r - g = 0} > (<) 0 \Leftrightarrow \phi > (<) \frac{1}{2}. \tag{12}$$

Second, by a similar procedure in the case of responsive expectations, we can derive the following relationship:

$$\left.\frac{\partial W^p}{\partial n}\right|_{\pi^p - g = 0} > (<) 0 \Leftrightarrow \phi > (<) \frac{1}{2}. \tag{13}$$

Therefore, in view of equations (12) and (13), we summarize the results as the following proposition.

Proposition

Regardless of the types of consumer expectations, if the strength of a network externality is larger (smaller) than a half, i.e., $\phi > (<) \frac{1}{2}$, *the number of firms in the case of free entry is socially too low (high), based on the second-best criteria, i.e.,* $n^{k*} > (<) n^k$, $k = p, r$.

Thus, if the strength of a network externality is sufficiently large, the theorem does not holds in the case of Cournot oligopolistic competition with free entry. In view of equations (3) and (6), output per firm is higher in the case of positive network externalities, compared with that in the case of non-network externalities, i.e., $\phi = 0$, where the theorem holds. As a result, the number of firms under free entry is lower than the socially second-best number of firms.

4. Conclusion

Although our model is very specific, e.g., linear functions, we have demonstrated that the presence of excess quantity competition and entry that is socially too low when significant network externalities exist. In addition to generalization of the model and analysis of the case of price competition, we should examine competition and entry policy in network industries.

References

Economides, N. (1996) Network externalities, complementarities, and invitations to enter, *European Journal of Political Economy*, 12, pp. 211−233.

Hurkens, S. and López, A. L. (2014) Mobile termination, network externalities and consumer expectations, *Economic Journal*, 124, pp. 1005-1039.

Kagitani, K., Ohkawa, T. and Okamura, M. (2016) Does the excess entry theorem hold in a differentiated oligopoly? *The Manchester School*, 84 (3), pp. 340–362.

Katz, M. and Shapiro, C. (1985) Network externalities, competition, and compatibility, *American Economic Review*, 75 (3), pp. 424–440.

Mankiw, G. N. and Whinston, M. D. (1986) Free entry and social inefficiency. *Rand Journal of Economics*, 17 (1), pp. 48–58.

Shubik, M. and Levitan, R, (1980) *Market Structure and Behavior*, Cambridge, MA, Harvard university Press.

Suzumura, K. (2012) Excess entry theorems after 25 years. *Japanese Economic Review*, 63 (2), pp. 152–170.

Suzumura, K. and Kiyono, K. (1987) Entry barriers and economic welfare. *Review of Economic Studies*, 54 (1), pp. 157–167.