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## Elderly Labor and Precautionary Saving

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## Elderly Labor and Precautionary Saving<sup>†</sup>

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### Abstract

In OECD countries, both life expectancy and elderly labor participation are increasing. The retirement age is being raised continually. The age at which pension benefits start is also being raised by governments. This paper presents an examination of how an increase in elderly labor participation affects saving and fertility in an endogenous fertility model with uncertainty of wage income during the old period. In the model with uncertainty, an increase in elderly labor time reduces savings. Thereby, fertility increases. However, without an increase in the average, an increase in the variance of wage income for elderly people increases savings and reduces fertility.

**JEL Classification:** J14, J13, J26

**Keywords:** Elderly labor, Fertility, Precautionary saving

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## 1. Introduction

In Japan, the aged population ratio is increasing. The sustainability of social security has persisted as a concern. Moreover, government policy has raised the starting age of pension benefits to 65 years old. The labor force participation rate of people older than 60 years old has therefore increased. Figure 1 portrays the labor force participation rate 60–64-year-old people in several OECD countries.

[Insert Fig. 1 around here.]

In Japan and in other OECD countries, the labor force participation rate of elderly people has increased. A worldwide trend exists by which the starting age of pension benefits has been increased. For that reason, labor participation by elderly people has increased. Japanese government policies are designed to facilitate elderly labor participation. For instance, the retirement age has been raised and wage income for elderly labor is subsidized.

An increase in elderly labor participation helps to resolve shortages of labor input in Japan. Furthermore, the government can cut pension benefits because elderly people can obtain labor income instead. This flexibility is expected to contribute to the sustainability of social security.

Some reports of the literature have described retirement and elderly labor participation. Matsuyama (2008) sets a binary choice model for the elderly labor supply. Elderly people work during the old period if the retirement benefit is smaller than the benefit from continuing labor during the old period. Gong and Liu (2012) sets a model continuous of labor supply in the old period. Using the Gong and Liu (2012) model, the time at which elderly people stop working during the old period can be derived: the retirement timing is therefore endogenized. The subsidy for elderly labor increases the benefits of working during the old period. Miyake and Yasuoka (2018) derive that the subsidy for elderly labor postpones retirement. Timing of compulsory retirement is examined by Kunze (2014). Correlation between the income growth rate and retirement is derived.

Retirement is considered along with the pension system in some reports of the relevant literature. Conde Ruiz and Galasso (2003, 2004) examine pension system effects on the retirement age. They derive that low-productivity workers choose retirement to obtain pension benefits. Maeda and Momota (2002) and Momota (2003) derive a correlation between the pension and retirement.

One paper describes that retirement timing affects fertility. An increase in life expectancy decreases fertility, as shown by relevant data and the related literature, with reports by Yakita (2001) and others. However, if retirement is postponed because of increased life expectancy, then the lifetime income increases and child care expenditures increase. Eventually, fertility can be pulled up. This result was derived by Mizuno and Yakita (2013).

This paper presents an examination of how elderly labor affects saving and fertility in an endogenous fertility model. However, different from reports of the related literature, this paper presents consideration of uncertainty related to wage income during the old period. Wage income in the old period should be regarded as uncertain because wage income during the old period is determined by the health status, economic conditions and other factors. Individuals are unable to estimate their future income correctly.

Results obtained from the analyses presented in this paper are the following. An increase in the elderly labor time decreases saving and increases fertility. An increase in the variance of wage income during the old period without an increase in the average increases precautionary saving. For that reason, fertility decreases even if the lifetime income eventually increases. Leland (1968), Caballero (1991), Liljas (1998), Picone et al. (1998), and Hemmi et al. (2007) derive precautionary saving in uncertainty related to income and assets. That uncertainty brings about precautionary saving and decreases fertility even if individuals can obtain more income *ex post*. The analyses described in this paper demonstrate that uncertainty exerts a negative effect on fertility.

The remainder of this paper presents the following. Section 2 sets the model without uncertainty to show the contributions of this paper. Section 3 sets the model with uncertainty for wage income during the old period and then presents derivation of fertility and saving. This paper presents consideration of exogenous retirement timing.

The result obtained from these analyses is not different if this assumption is mitigated as endogenous retirement timing. This point is examined in Section 4. Section 5 presents discussion of a subsidy for elderly labor and how such a subsidy should be provided. The final section concludes this paper.

## 2. Model without uncertainty

First, we set the model without uncertainty related to the wage income during the old period. Individuals in households exist in two periods: young and old. During the young period, individuals work to obtain a wage income and to bring up children. The budget constraint is given as shown below.

$$zn + s = (1 - \tau)w^y \quad (1)$$

Therein,  $n$  denotes the number of children or fertility.  $z$  is necessary to bring up a child.  $s$  represents saving for later consumption during the old period.  $w^y$  denotes the wage rate of younger labor.  $\tau$  denotes the pension contribution rate.

During the old period, individuals work to obtain wages. After retirement, they can obtain pension benefits. Without the motivation of a bequest, consumption during the older period is financed by saving. Labor income and the pension are given as shown below.

$$c = (1 + r)s + lw^o + (1 - l)p \quad (2)$$

In that equation,  $c$  denotes consumption in the old period.  $r$  represents the interest rate.  $w^o$  denotes the wage rate of older labor. Defining  $l$  as the time of labor during the old period,  $1 - l$  shows the retirement period. Defining  $p$  as the pension benefit, they can obtain the pension benefit  $(1 - l)p$ , based on the retirement period.

The utility function is assumed to be the following.

$$u = \alpha \ln n + (1 - \alpha) \ln c, 0 < \alpha < 1. \quad (3)$$

The optimal allocations to maximize the utility (3) subject to (1) and (2) are shown as presented below.

$$n = \frac{\alpha \left( (1 - \tau)w^y + \frac{lw^o + (1 - l)p}{1 + r} \right)}{z} \quad (4)$$

$$c = (1 - \alpha)(1 + r) \left( (1 - \tau)w^y + \frac{lw^o + (1 - l)p}{1 + r} \right) \quad (5)$$

If  $w^o > p$  and the timing of retirement is delayed, the household lifetime income increases; fertility rises. Then, saving decreases. This result demonstrates positive correlation between fertility and elderly labor supply.

If the pension benefit financed as the pay-as-you-go system, then the pension benefit can be shown as

$$(1 - l)p = n\tau w^y. \quad (6)$$

Inserting (6) into (4), we can obtain fertility to satisfy the following equation.

$$n = \frac{\alpha \left( w^y + \frac{lw^o + \tau(n - (1 + r))w^y}{1 + r} \right)}{z} \quad (7)$$

An increase in the labor supply during the old period  $l$  raises fertility, irrespective of the level of  $w^o$ . This result reflects positive correlation between fertility and the labor supply of elderly people.

### 3. Uncertainty of wage income during the old period.

In this section, we consider the uncertainty of wage income during the old period.  $w^o$  is assumed to be distributed uniformly between  $[0, \bar{w}]$ . Then, optimal saving  $s$  is given to satisfy the following equation.

$$\frac{\alpha}{1 - \alpha} \frac{1}{(1 - \tau)w^y - s} = E \frac{1 + r}{(1 + r)s + lw^o + (1 - l)p} \quad (8)$$

Therein,  $E$  denotes the expectation operator. Because of the uniform distribution of  $w^o$ , the expected that the marginal utility of  $s$   $E \frac{1+r}{(1+r)s+lw^o+(1-l)p}$  can be shown as

$$E \frac{1 + r}{(1 + r)s + lw^o + (1 - l)p} = \frac{1 + r}{l\bar{w}} \ln \left( \frac{l\bar{w}}{(1 + r)s + (1 - l)p} + 1 \right). \quad (9)$$

Then, because of (8) and (9), optimal saving  $s$  is given for the following equation.

$$\frac{\alpha}{1 - \alpha} \frac{1}{(1 - \tau)w^y - s} = \frac{1 + r}{l\bar{w}} \ln \left( \frac{l\bar{w}}{(1 + r)s + (1 - l)p} + 1 \right) \quad (10)$$

Then, we can depict the following figure to derive optimal saving  $s$  as follows.

[Insert Fig. 2 around here.]

For simplicity, we consider the model economy without the pension. With an increase in  $l$ , the labor supply of elderly people, the  $R$  curve shifts upward or downward. By differentiating  $L$  with respect to  $l$ , one can obtain the following.

$$\begin{aligned}
 & -\frac{\ln\left(1 + \frac{l\bar{w}}{(1+r)s}\right)}{l^2} + \frac{1}{l} \frac{\frac{\bar{w}}{(1+r)s}}{1 + \frac{l\bar{w}}{(1+r)s}} \\
 & = \frac{\ln\left(1 + \frac{l\bar{w}}{(1+r)s}\right)}{l^2} \left( \frac{\frac{d\ln\left(1 + \frac{l\bar{w}}{(1+r)s}\right)}{\frac{dl}{l}}}{\frac{\ln\left(1 + \frac{l\bar{w}}{(1+r)s}\right)}{\frac{dl}{l}} - 1} \right) \tag{11}
 \end{aligned}$$

Therein,  $\frac{\frac{d\ln\left(1 + \frac{l\bar{w}}{(1+r)s}\right)}{\frac{dl}{l}}}{\frac{\ln\left(1 + \frac{l\bar{w}}{(1+r)s}\right)}{\frac{dl}{l}}}$  represents the elasticity related to how  $\ln\left(1 + \frac{l\bar{w}}{(1+r)s}\right)$  changes in

the change of  $l$ . Defining  $\frac{\frac{d\ln\left(1 + \frac{l\bar{w}}{(1+r)s}\right)}{\frac{dl}{l}}}{\frac{\ln\left(1 + \frac{l\bar{w}}{(1+r)s}\right)}{\frac{dl}{l}}} = \varepsilon$ , inelasticity case  $\varepsilon < 1$  makes  $R$  shift down.

Saving decreases. However, with inelasticity case  $\varepsilon > 1$ , an increase in  $l$  raises saving; consequently, fertility decreases.

An increase in  $l$  signifies an increase in wage income in the older period. Consequently, the average and variance of wage income  $lw^o$  in the older period increase. Because of increased variance, they deal with low wage income during the old period and then increase saving as precautionary saving. However, an increase in the average decreases saving for consumption in the old period. Considering the following figure, we can obtain the result that an increase in  $l$  decreases saving as shown in Fig. 3.<sup>1</sup>

[Insert Fig. 3 around here.]

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<sup>1</sup> This paper assumes a log utility function. However, as shown by Fig. 3, given the assumption of  $u' > 0, u'' < 0$  and additive separability, we can obtain the same result using the log utility assumption.

In the figure, A and B respectively show the utility level at  $(1+r)s$  and  $(1+r)s + l\bar{w}$ . The expected marginal utility of  $(1-\alpha)lnc$  is given as the middle point of the line AB. Then, at the optimal allocations, the level of expected marginal utility of  $(1-\alpha)lnc$  is equal to the level of marginal utility  $alnn$ . An increase in  $l$  raises the expected marginal utility of  $(1-\alpha)lnc$ . We obtain  $\frac{\alpha}{n} > E \frac{1-\alpha}{c}$  for saving before an increase in  $l$ . Therefore, individuals reduce savings such that  $\frac{\alpha}{n} = E \frac{1-\alpha}{c}$  holds. Then, the following proposition can be established.

**Proposition 1**

An increase in the labor supply time during the old period decreases saving. Then, fertility increases.

An increase in  $l$  raises the variance of wage income during the old period. Then, the precautionary saving can be pulled up. Simultaneously, the average of wage income during the old period rises. This effect decreases saving. These two effects determine the change of saving. However, one can obtain the results by which an increase in the variance decreases saving and by which the positive correlation between elderly labor supply and fertility is obtainable.

In the case of  $(1-l)p \neq 0$ , an increase in  $l$  decreases pension benefit  $(1-l)p$ . This effect increases saving because of the consumption in the old period. Finally, two effects of an increase in the variance in wage income during the old period and a decrease in pension benefit should be regarded as examining the effects on saving if we consider the reform case in which the elderly labor supply increases instead of the pension cut. A decrease in  $(1-l)p$  increases saving. The negative effect of an increase in the variance of wage income during the old period on saving is therefore weakened.

Now, we consider the other uniform distribution. The range is assumed to be  $[(1-\sigma)\bar{w}, (1+\sigma)\bar{w}]$ ,  $0 < \sigma < 1$ . An increase in  $\sigma$  raises the variance of wage income during the old period. However, the average does not change. The density function is given as  $\frac{1}{2\sigma\bar{w}}$ . Then, (10) changes the following equation.

$$\frac{\alpha}{1-\alpha} \frac{1}{w^y - s} = \frac{1+r}{2l\bar{w}} \ln \left( \frac{(1+r)s + (1+\sigma)\bar{w}}{(1+r)s + (1-\sigma)\bar{w}} \right) \quad (12)$$

Considering the following figure, we can obtain the result demonstrating that an increase in the variance increases saving.

[Insert Fig. 4 around here.]

As shown by Fig. 4, for fixed  $s$  at the optimal level before increasing the variance, we can obtain  $\frac{\alpha}{n} < E \frac{1-\alpha}{c}$ . The levels of  $E \frac{1-\alpha}{c}$  and  $\frac{\alpha}{n}$  are shown respectively by  $C$  and  $D$  in Fig. 4. Because of  $\frac{\alpha}{n} < E \frac{1-\alpha}{c}$ , individuals increase saving such that  $\frac{\alpha}{n} = E \frac{1-\alpha}{c}$ .

### 3. Binary choice model of labor and pension

This section presents consideration of the model by which individuals choose labor during the old period or to obtain the pension benefit instead of quitting the labor force during the old period. If they choose labor during the old period, then  $l = 1$ . Otherwise,  $l = 0$ . In the case of  $l = 1$ , the optimal saving is given by the following equation as

$$\frac{\alpha}{1-\alpha} \frac{1}{(1-\tau)w^y - s} = \frac{1+r}{\bar{w}} \ln \left( 1 + \frac{\bar{w}}{(1+r)s} \right). \quad (13)$$

For  $l = 0$ , the optimal saving is given by the following equation:

$$\frac{\alpha}{1-\alpha} \frac{1}{(1-\tau)w^y - s} = \frac{1+r}{(1+r)s + p}. \quad (14)$$

Then, defining the right-hand side of (13) and (14) as  $R1$  and  $R0$ , respectively, the optimal saving is given by the intersection of  $L$  and  $R$  in Fig. 5.<sup>2</sup>

[Insert Fig. 5 around here.]

Actually,  $L$  can be depicted as a solid line if  $\alpha$  is small. Then, the optimal saving in the cases of  $l = 1$  and  $l = 0$  as  $s^1$  and  $s^0$ , respectively; we can obtain  $s^1 < s^0$ . Then, the

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<sup>2</sup> We can verify that  $R0$  is always above  $R1$ , as shown by the numerical check if the pension benefit is zero or small. Therefore, this paper includes consideration of the case of a non-small pension benefit.

case of labor during the old period brings about more saving. However, fertility remains low in the case of  $s^0$ .

Consequently,  $L$  can be depicted as a dashed line if  $\alpha$  is large. We can obtain  $s^1 > s^0$ . Fertility in the case of  $l = 0$  is less than in the case of  $l = 1$ . Then, the following proposition can be established.

**Proposition 2**

With a small preference for rearing children, saving in the case of labor during the old period is smaller than the case of pension without labor during the old period. However, in the case of large preference for rearing children, the order of the level of saving changes.

Given a small preference for rearing children, saving is large because the preference for consumption during the old period is large. In this case, uncertainty of the wage income during the old period decreases the marginal utility of an increase in saving. Then, the individuals reduce saving and raise fertility in the young period without uncertainty.

However, given a large preference for children, saving is small. Then, the uncertainty raises the marginal utility of saving.

**4. Endogenous labor supply and uncertainty**

This section presents consideration of the case of endogenous labor supply in the older period. In the preceding section, the labor supply during the old period is exogenous. Now, we consider the following utility function as

$$u = \alpha \ln n + E v, 0 < \alpha < 1. \tag{15}$$

Therein, the following variable is used:

$$v = (1 - \alpha) \ln c + \ln(1 - l). \tag{16}$$

The budget constraints are given as (1) and (2). In this section, we derive the optimal allocations by a backward solution.

In the old period, older people maximize utility function (16) subject to (2). In the old

period, saving is regarded as given. The wage rate in the older period is regarded as a certain variable. Then, the labor supply during the old period to maximize utility (16) is shown such that the following equation holds:

$$\frac{\partial v}{\partial l} = (1 - \alpha) \frac{w^o - p}{(1 + r)s + lw^o + (1 - l)p} - \frac{1}{1 - l} = 0. \quad (17)$$

Therefore, we can obtain the labor supply during the old period as

$$l = \frac{(1 - \alpha)w^o - (2 - \alpha)p - (1 + r)s}{(2 - \alpha)(w^o - p)}. \quad (18)$$

Inserting (16) and (18) into (15), one derive the optimal saving to maximize the utility function (15). Optimal saving is shown according to the following equation:

$$\frac{1}{(1 - \tau)w^y - s} = \frac{1 + r}{\bar{w}} \ln \frac{\bar{w} + (1 + r)s}{(1 + r)s}. \quad (19)$$

Defining the left-hand side and the right-hand side of (19) respectively as  $L$  and  $R$ , the optimal saving is obtainable, as shown by Fig. 1. During the young period, the individuals undertake precautionary saving for the uncertainty of wage income during the old period. However, the wage income during the old period is determined; uncertainty is eliminated if they live during the old period. Then, the individuals undertake over-consumption because they have no bequest motivation. Therefore, to decrease over-consumption, the individuals reduce working time and raise their leisure time. However, the wage income during the old period is an opportunity cost associated with having leisure. Therefore, an effect exists by which the working time of elderly people increases if the wage income during the old period is larger.

## 5. Discussion

In OECD countries, the labor participation rate of elderly people increases. The labor participation rate in Japan is at a high level in OECD countries. These results are presented in Fig. 1. The subsidy for elderly people is provided to facilitate the labor participation of elderly people. The subsidy raises the wage income during the old period and raises the opportunity cost to have leisure. However, simultaneously, the variance of wage income during the old period rises because of an increase in wage income that

elderly people can obtain. As shown by Fig. 3, saving decreases and fertility rises because the subsidy for the wage income raises the average and the variance of the wage income during the old period.

However, if the variance of wage income during the old period increases without an increase in the average, then precautionary saving can be facilitated and fertility decreases. This outcome leads to the fewer children problem. Therefore, if government policy is intended to solve not only the shortage of labor supply in the aging society but also the problem posed by fewer children, a government should alleviate uncertainty. As a policy to eliminate the wage income uncertainty in the old period, wage income insurance can be considered. The insurance provides income to meet an average income level if the elderly person obtains a less than average wage income. Instead of this insurance, if the older person can obtain more than the average income level, then the income that is more than the average income is collected for insurance. Then, older people can always obtain the average income and no uncertainty arises. By virtue of mitigating uncertainty, precautionary saving is reduced. Fertility can thereby be pulled up.

## **6. Conclusions**

This paper presents an examination of how household saving is determined according to uncertainty of wage income during the old period in the endogenous fertility model. Without uncertainty, an increase in wage income during the old period decreases saving and fertility increases. This result is obtainable in the uncertainty case that the average and the variance of the wage income in the old period increase. However, an increase in the variance of wage income during the old period without an increase in the average raises saving and decreases fertility.

Because of precautionary saving, expenditures during the young period decrease. These expenditures include not only consumption but also expenditures for child care. Because of the uncertainty of wage income during the old period, expenditures for child care decrease; then fertility decreases.

As a policy supporting pension system sustainability, OECD governments pull up the

starting age of pension benefits. Then, the elderly labor supply increases. However, if the variance of wage income during the old period increases, then precautionary saving can be facilitated and fertility is thereby decreased. The problem of fewer children is a severer problem. The sustainability of social security is thus weakened.

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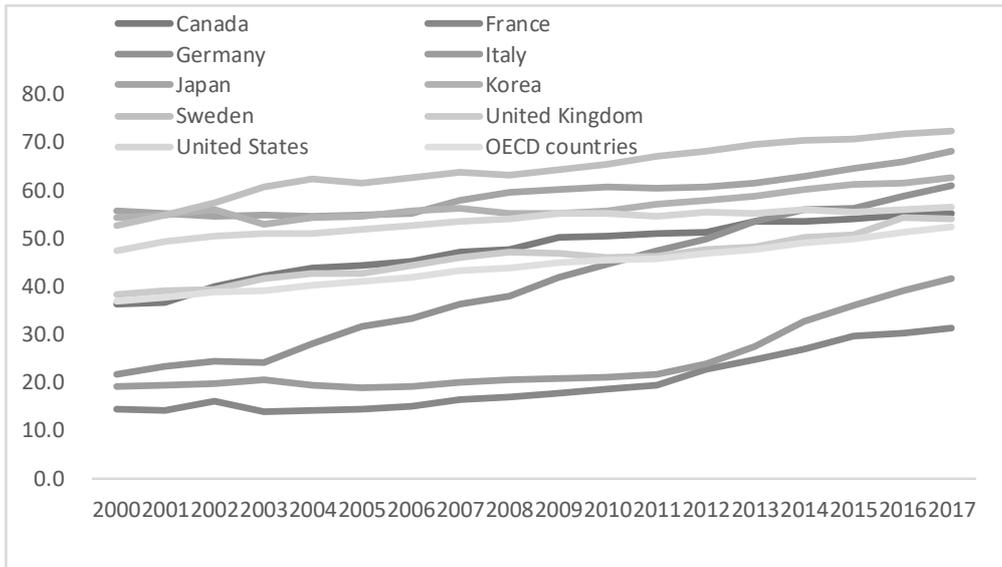


Fig. 1: Labor Force Participation Rate of 60–64-year-old People (Data: OECD Statistics).

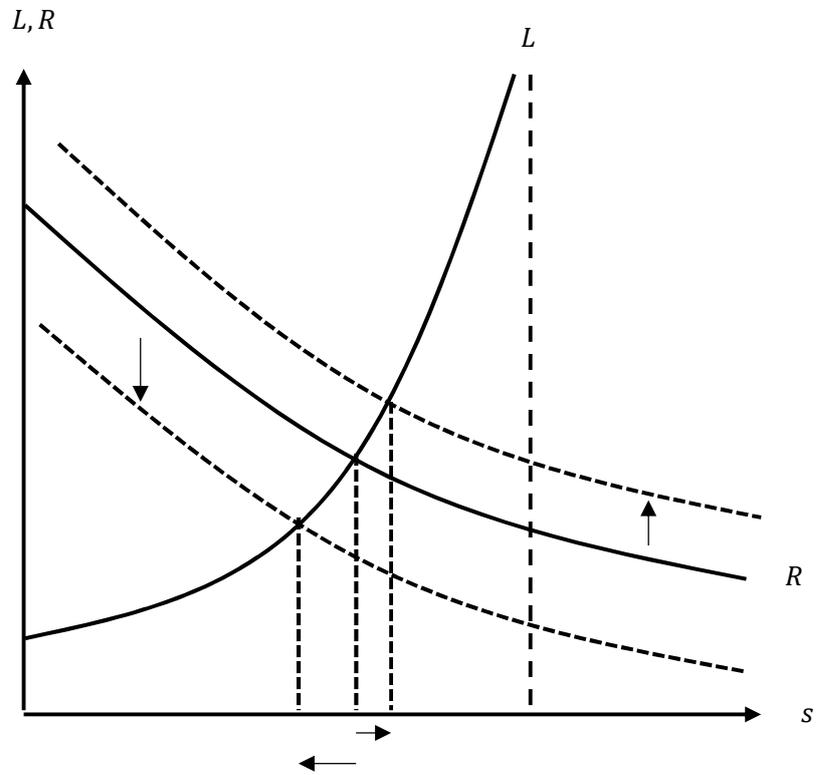


Fig. 2: Optimal Saving.

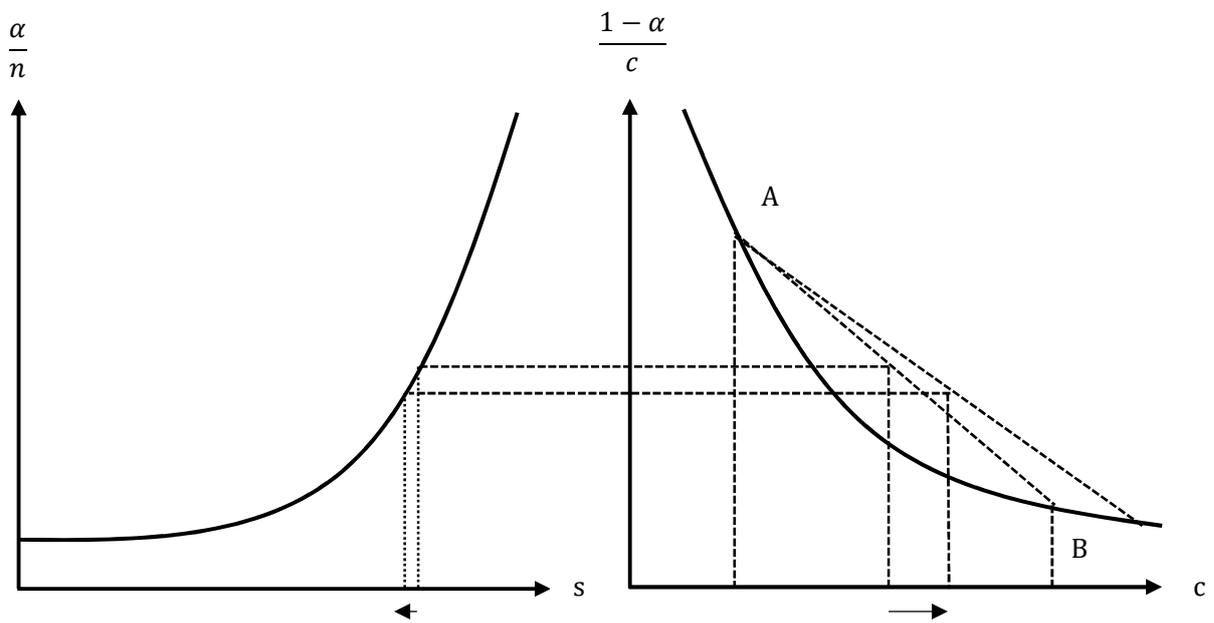


Fig. 3: Optimal Saving (Case of a Decrease in Saving).

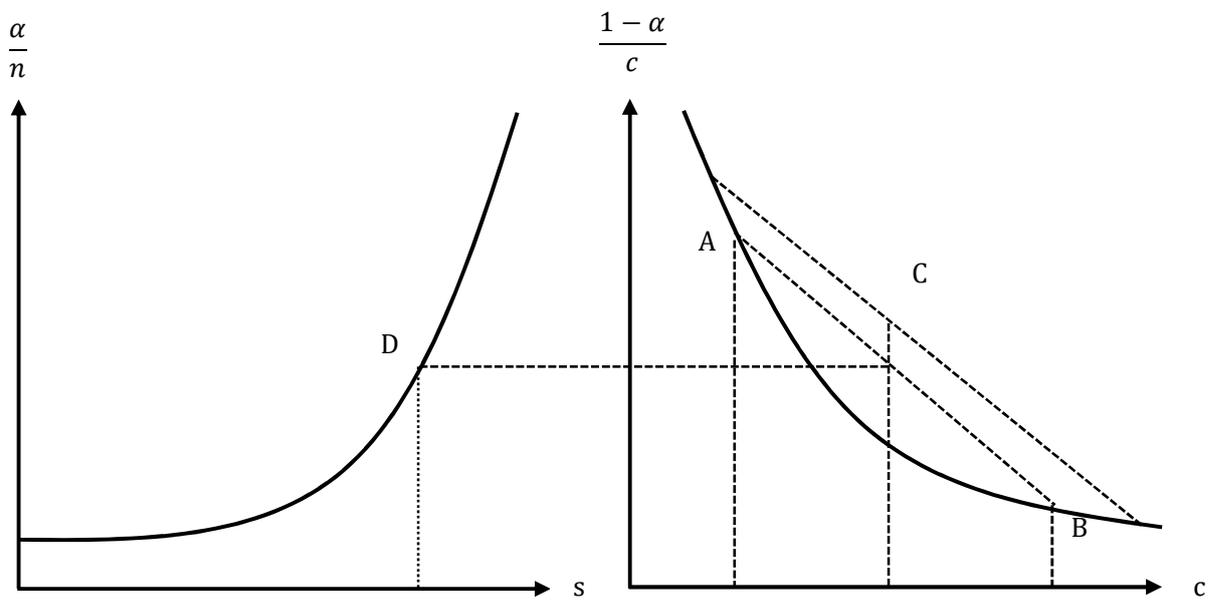


Fig. 4: Optimal Saving (Case of an Increase in Saving).

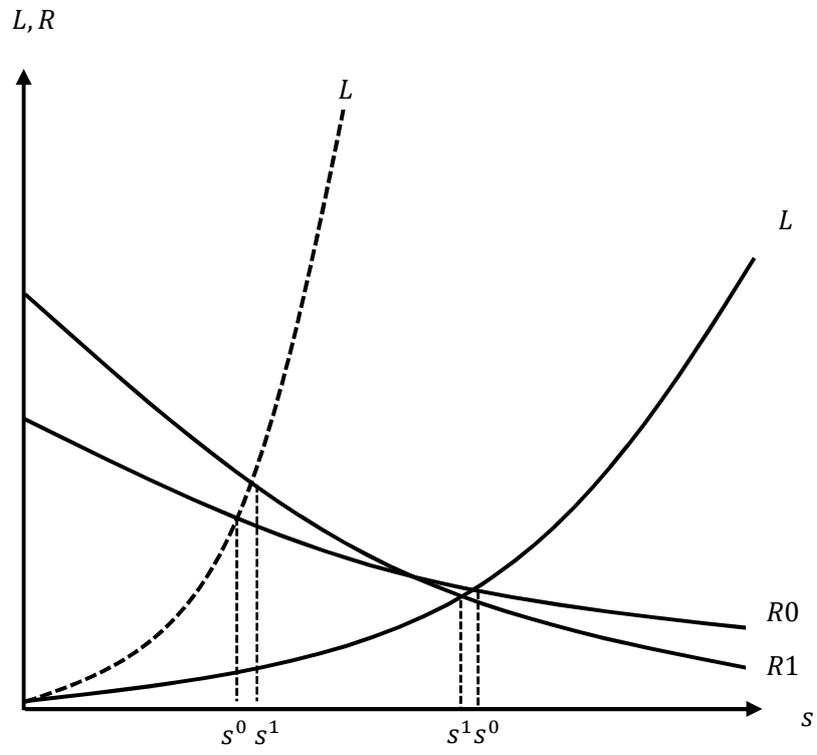


Fig. 5: Optimal Saving.