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Patent Protection and Public Capital Accumulation

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Abstract

This paper examines the balanced-growth maximizing public investment policy in a growth model where the engines of economic growth are private R&D and public capital accumulation. The government allocates tax revenue between new investment and maintenance expenditure for public capital. We consider how the balanced-growth maximizing public investment policy changes as patent protection becomes stronger, as seen in many countries. The results show that as patent protection becomes stronger, the income tax rate to finance public investment should be lower and the expenditure share of new investment should be higher. The balanced-growth maximizing policy leads to a smaller government, as patent protection becomes stronger.

Keywords: Patent Protection, Public Capital, Economic Growth, Welfare

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1 Introduction

One of the important changes that have significantly impacted the operating environments of firms in many countries is the recent strengthening of the protection of intellectual property rights, including patents. Figure 1 plots an index of patent rights on a scale of 0-5 provided by Park (2008) and shows that the strength of patent rights in all G7 countries has increased since the Trade-Related Aspects of Intellectual Property Rights (TRIPS) agreement of 1994.[1][2] It is widely believed that stronger patent protection enables the patent holders to obtain a higher rent by charging a higher price. In turn, it is likely that this process promotes innovation, thereby increasing productivity and economic growth.[3] On the other hand, many empirical studies indicate that publicly-provided infrastructure service plays a key role in sustained growth. As discussed in Agénor and Moreno-Dodson (2006) and Agénor and Neanidis (2015), productive government activities, such as building new infrastructures and extending the durability of existing infrastructures through maintenance activities, exert a strong positive impact on the productivity of private inputs, promote private investment, including R&D, and this increases economic growth. These complementary relationships among publicly-provided infrastructure, strength of patent protection and economic growth suggest that the growth implications of public investment policy depend heavily upon the degree of patent protection.

Motivated by these recent changes in the protection of intellectual property rights, this paper incorporates productive public capital into a variety expansion type R&D-based growth model and examines how stronger patent protection alters the government's public investment policy whose objective is to maximize the balanced-growth rate (i.e., the long-run growth rate) of the economy. Then, we show that as patent protection becomes stronger, the income tax rate to finance public investment should be lower and the expenditure share of new investment should be higher. The balanced-growth maximizing policy leads to a smaller government, as patent protection becomes stronger. To the best of our knowledge, existing studies have yet to analyze these issues rigorously.

Figure 2 plots the GDP share of the general government gross capital formation in G7 countries.[4] The balanced-growth maximizing policy indicates that the

[1]Park (2008) examines five categories of patent rights (patent duration, coverage, enforcement mechanism, restrictions on patent scope, and membership in international treaties) and assigns a score from 0 to 1. A larger number implies stronger protection.

[2]G7 countries accounted for 84 percent of worldwide R&D spending in 1995 (Keller, 2009) and have presumably pushed the world ' s technology frontier.

[3]This may not be the case if we consider sequential innovation. In this case, stronger patent protection may impede sequential innovation. See Chu et al. (2012a) for example.

[4]The 2019 OECD economic outlook is used to calculate the GDP share of the general government gross capital formation.

tax rate to finance public investment becomes lower as patent protection becomes stronger. This result indicates that the GDP share of public investment becomes lower as patent protection becomes stronger. However, Figure 2 shows that, except for Japan, the GDP share of public investment in most G7 countries has no clear downward trend during which the degree of patent protection becomes stronger. Consequently, the actual trends in public capital investment do not match the prediction obtained from the balanced-growth maximizing public investment policy.

Many authors have studied how publicly-provided infrastructure contributes to economic growth, both theoretically and empirically.[5] In endogenous growth settings, Barro (1990), Barro and Sala-i-Martin (1992), Futagami et al. (1993) and Turnovsky (1997), among others, theoretically investigate the balanced-growth maximizing and the optimal public investment policy, taking the infrastructure as a flow or a stock. In their studies, they assume that infrastructure improves labor productivity and show that the growth maximizing income tax rate to finance public investment is equal to the elasticity of public capital in the production. Recently, Kalaitzidakis and Kalyvitis (2004) have focused on the maintenance activities of government and shown that the growth maximizing income tax rate to finance public investment is greater than the elasticity of public capital in production when taking into account a trade-off in the allocation of tax revenue between new investment and maintenance expenditure. In their model, new government investments accumulate public capital in a one-to-one manner, while maintenance expenditures reduce the depreciation rate of public capital. Dioikitopoulos and Kalyvitis (2008) have extended Kalaitzidakis and Kalyvitis (2004)'s model by incorporating the congestion of public capital and shown that the optimal income tax rate to finance public investment is positively related to the degree of congestion.[6] By employing a two-period overlapping generations version of Kalaitzidakis and Kalyvitis (2004)'s model, Yakita (2008) has examined the effect of population aging on balanced-growth maximizing public investment policy and shown that as aging proceeds, not only should the income tax rate to finance public investment be higher but also the expenditure share of maintenance activities.

In line with Kalaitzidakis and Kalyvitis (2004), our study assumes that the government allocates the tax revenue between new investment and maintenance expenditure. In economies with substantial stocks of accumulated capital, as in developed countries, the role of maintenance and replacement investment will become more important, as pointed out by Kalaitzidakis and Kalyvitis (2004) and Yakita (2008). Moreover, in order to make our results comparable to those

[5]See Agénor and Moreno-Dodson (2006) for a more comprehensive literature review.

[6]Agénor (2009) proposes alternative model that examines the growth and welfare implications of maintenance expenditure. In his paper, maintenance spending affects both the durability and the efficiency of public capital.

of Dioikitopoulos and Kalyvitis (2008) and Yakita (2008), we assume that the government chooses the income tax rate and the allocation ratio between new investment and maintenance expenditure, so as to maximize the balanced-growth rate. However, in contrast to these existing studies, we employ a R&D-based growth model and analyze the relationship between the degree of patent protection and the growth maximizing public investment policy.[7]

This paper also relates to studies that examined the growth and welfare implications of fiscal policy in an R&D-based growth model.[8] Among them, this paper relates most closely to Iwaisako (2013) and Iwaisako (2016). Iwaisako (2013) examines the optimal degree of patent protection in a variety expansion type R&D-based growth model with productive public services and shows that as public service becomes smaller, the optimal level of patent protection is weaker. Similar to Iwaisako (2013), our study concerns the complementary relationships among publicly-provided infrastructure, strength of patent protection and economic growth. However, our study focuses on the analysis of the growth maximizing public investment policy. Moreover, since Iwaisako (2013) ignores the maintenance activities of government, the growth maximizing income tax rate to finance public investment is equal to the elasticity of public capital in production and is independent of the degree of patent protection. Therefore, this study proposes a novel framework to analyze the relationship between the degree of patent protection and the growth maximizing public investment policy. Moreover, Iwaisako (2016) examines the optimal corporate income tax and consumption tax rates in a quality ladder type R&D-based growth model and shows that as patent protection becomes stronger, the corporate tax rate should be higher and the consumption tax rate should be lower. Our study shares numerous research interests with Iwaisako (2016). However, our study considers productive public capital explicitly and focuses its analyses on the properties of the growth maximizing public investment policy. In this sense, this paper complements the analysis conducted by Iwaisako (2016).

This paper is organized as follows. Section 2 presents the basic model. Section 3 derives the balanced-growth maximizing public investment policy and examines the effect of patent protection on balanced-growth maximizing public investment policy. Section 4 briefly considers the welfare-maximizing public investment policy. Section 5 concludes the paper.

[7]In analyzing the properties of the balanced-growth maximizing public investment policy, Dioikitopoulos and Kalyvitis (2008) and Yakita (2008) did not consider the second-order sufficient condition to be a relative maximum explicitly. This may be one of the reasons why these studies exhibit many ambiguous results in their comparative static analyses. Explicit consideration of the second-order sufficient condition may be a minor contribution of this paper.

[8]See Iwaisako (2016) for a more comprehensive literature review.

2 Model

To analyze the effect of patent protection on balanced-growth maximizing public investment policy, we introduce productive public capital into a variety expansion type R&D-based growth model following Rivera-Batiz and Romer (1991). For production, there are three sectors: a final goods sector, an intermediate goods sector, and an R&D sector. The government imposes taxes on labor and corporate income and allocates tax revenue between new investment and maintenance expenditure for public capital. In accordance with Goh and Olivier (2002), we introduce patent breadth into the model.

2.1 Household

There is a unit continuum of identical households. The population size of each household is given by L, which is assumed to be constant over time. Each member of households supplies one unit of labor inelastically. The lifetime utility of a household is given by:

$$U = \int_0^\infty \frac{c_t^{1-\sigma} - 1}{1-\sigma} L e^{-\rho t} dt, \quad \sigma > 0,\ \rho > 0, \tag{1}$$

where c_t is the level of per capita consumption, ρ is the subjective discount rate, and σ is the inverse of the elasticity of intertemporal substitution. We assume that labor income is taxed at rate $\tau \in (0, 1)$. The household supplies L units of labors to earn wages and makes consumption-saving decisions to maximize lifetime utility subject to the following asset-accumulation equation:

$$\dot{A}_t = r_t A_t + (1-\tau) w_t L - c_t L, \tag{2}$$

where A_t is the real value of assets, r_t is the real interest rate, w_t is the real wage rate, and τ is the tax rate on wage income. As in Grossman and Helpman (1991), all assets are held in the form of the shares of monopolistic firms. From standard dynamic optimization, the Eular equation is given by

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(r_t - \rho). \tag{3}$$

In addition, the following transversality condition holds:

$$\lim_{t\to\infty} c_t^{-\sigma} A_t e^{-\rho t} = 0.$$

2.2 Final Goods Sector

The final goods Y_t are produced by perfectly competitive firms using the following technology:

$$Y_t = AL_{Y,t}^{1-\alpha} \int_0^{N_t} (h_t x_{i,t})^\alpha \, di, \quad A > 0, \alpha \in (0, 1), \tag{4}$$

where

$$h_t = G_t^\epsilon N_t^{1-\epsilon}, \quad \epsilon \in (0, 1), \tag{5}$$

$A > 0$ is a productivity parameter, $L_{Y,t}$ is labor input in the final goods sector, $x_{i,t}$ is the input of intermediate good i, N_t is the number of intermediate goods, and G_t is the aggregate stock of public capital. Following Chatterjee and Turnovsky (2012), the composite externality h_t represents a combination of the role of knowledge spillover from the existing stock of variety N_t, as in Benassy (1998), together with productive public capital services G_t, as in Barro (1990).[9] The government can increase the productivity of private firms by providing public capital services. Under these specifications, the output elasticity of public capital is given by $\alpha\epsilon$, whereas the price elasticity of intermediate goods is given by $\frac{1}{1-\alpha}$.[10] We take the final output as numeraire.

Given the price of the intermediate goods $p_{i,t}$ and wage rate w_t, the profit maximization yields

$$w_t = (1 - \alpha)\frac{Y_t}{L_{Y,t}}, \tag{6}$$

$$p_{i,t} = \alpha A L_{Y,t}^{1-\alpha} h_t^\alpha x_{i,t}^{\alpha-1}. \tag{7}$$

2.3 Intermediate Goods Sector

There is a continuum of intermediate goods $i \in [0, N_t]$. One unit of intermediate goods is produced with one unit of labor. A single firm holding the patent monopolistically supplies each intermediate good i. The profit function of each intermediate good firm is $\pi_{i,t} = (p_{i,t} - w_t)\, x_{i,t}$. The familiar unconstrained profit-maximizing price is $p_{i,t} = \frac{w_t}{\alpha}$. Here we follow Goh and Olivier (2002) to introduce

[9] More precisely, Chatterjee and Turnovsky (2012) consider the composite externality from physical capital, as in Romer (1986), and productive public spending, as in Barro (1990), and conduct numerical analyses including growth and welfare effects. Our application of (5) is in line with Chatterjee and Turnovsky (2012), because in our model, the stock of N_t replaces the role of physical capital (i.e., the engine of economic growth and the source of spillover). Since the pioneering work by Romer (1986), the positive social returns to variety have been applied in many R&D-based growth models (e.g., Aghion and Howitt, 1998; Benassy, 1998; Perretto, 2007).

[10] The main implications of this paper do not change qualitatively even if we consider the case where there is no knowledge spillover from the existing stock of variety (i.e., $\epsilon = 1$).

patent breadth $\beta > 1$ as a policy variable, such that $p_{i,t} = \max\left\{\beta, \frac{1}{\alpha}\right\} w_t$.[11] We focus on the interesting case in which $\beta \in (1, \frac{1}{\alpha})$. Consequently, a broader patent breadth β enables the monopolistic firms to charge a higher markup capturing Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price".[12] Substituting $p_{i,t} = p_t = \beta w_t$ into (7) and $\pi_{i,t} = (p_{i,t} - w_t)\, x_{i,t}$ shows that the relations $x_{i,t} = x_t$ and $\pi_{i,t} = \pi_t$ for all $i \in [0, N_t]$. Therefore, henceforth, we can omit the index i. Under these specifications, the profit of each intermediate good firm satisfies

$$\pi_t = \frac{\beta - 1}{\beta} p_t x_t = \frac{\beta - 1}{\beta} \frac{\alpha Y_t}{N_t}, \tag{8}$$

where the second equality follows from (7) and $Y_t = A L_{Y,t}^{1-\alpha} N_t \,(h_t x_t)^{\alpha}$. Moreover, substituting $p_t = \beta w_t$ and (6) into (8) yields

$$x_t = \frac{(\alpha/\beta)}{1 - \alpha} \frac{L_{Y,t}}{N_t}. \tag{9}$$

2.4 R&D Sector

Denote $V_{i,t}$ as the value of the patent on variety $i \in [0, N_t]$. $\pi_{i,t} = \pi_t$ from (8) implies that $V_{i,t} = V_t$ for all $i \in [0, N_t]$. Here, we assume that the profit of firms is taxed at rate $\tau \in (0, 1)$. If households possess one unit of stock in the time interval dt, they can obtain a profit of $(1 - \tau)\pi_t$ and a capital gain or loss of $\dot{V}_t$. Alternatively, they can invest V_t units of funds in the risk-free asset. Therefore, in equilibrium, the no-arbitrage condition for V_t is

$$r_t V_t = (1 - \tau)\pi_t + \dot{V}_t. \tag{10}$$

Competitive entrepreneurs employ R&D inputs for innovation. In accordance with Rivera-Batiz and Romer (1991), we consider the lab equipment type R&D specification. Devoting a units of the final good, R&D firms can invent one unit of intermediate goods. Given the value of the patent on variety V_t, the zero profit condition yields

$$V_t = a. \tag{11}$$

Combining (8), (10) and (11) yields $r_t = \frac{1-\tau}{a} \frac{\beta-1}{\beta} \frac{\alpha Y_t}{N_t}$.

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[11]Generally, governments control the degree of patent protection through patent length and breadth. In this paper, for simplicity, we assume that the patent length is fixed and infinite and that governments control the degree of patent protection using only patent breadth.

[12]Specifically, we assume that the broader the government makes patent breadth, the more difficult it is to produce imitative goods. We specify the unit cost of producing imitative goods as βw_t. Each firm that produces an intermediate good charges a price such that producers of imitative goods cannot earn positive profits, as follows: $p_{i,t} = \beta w_t$.

2.5 Government

The government imposes taxes on labor and corporate income at the constant tax rate $\tau \in (0,1)$, and allocates tax revenue between new investment and maintenance expenditure for public capital. Denoting new investment and maintenance expenditure by $I_{G,t}$ and $I_{M,t}$, respectively, the budget constraint of the government can be written as

$$I_{G,t} + I_{M,t} = \tau\left(w_t L + \pi_t N_t\right) = \tau Y_t, \tag{12}$$

where the second equality follows from (6), (7), (15) and $\pi_t = (p_t - w_t)x_t$.[13] The tax rate τ can be interpreted as the share of output allocated to public expenditure (i.e., the size of government). Denoting the expenditure share of new investment as $\lambda \in [0,1]$, we have

$$I_{G,t} = \lambda\tau Y_t, \quad I_{M,t} = (1-\lambda)\,\tau Y_t. \tag{13}$$

As in Kalaitzidakis and Kalyvitis (2004), we assume that the ratio of the maintenance expenditure to final output reduces the depreciation rate of public capital. An increase in maintenance expenditure for given aggregate economic activity reduces the depreciation rate, while a higher economic activity in the economy for a given maintenance activity accelerates depreciation. The evolution of public capital stock can be given as

$$\dot{G}_t = I_{G,t} - \delta\left(\frac{I_{M,t}}{Y_t}\right)G_t, \tag{14}$$

where $\delta(\cdot)$ denotes the depreciation rate function, which is assumed to satisfy the following conditions: $\delta(\cdot) \in (0,1)$, $\delta'(\cdot) < 0$, $\delta''(\cdot) > 0$, $\lim_{\frac{I_{M,t}}{Y_t}\to\infty}\left(\frac{I_{M,t}}{Y_t}\right) = \underline{\delta} > 0$ and $\lim_{\frac{I_{M,t}}{Y_t}\to 0}\left(\frac{I_{M,t}}{Y_t}\right) = \bar{\delta} < 1$. The higher ratio of maintenance expenditure to final output reduces the depreciation rate of public capital, but its marginal effect on the depreciation rate decreases with the ratio of maintenance expenditure to final output.

2.6 Market Clearing Condition

Labor is demanded by both final goods firms and intermediate goods firms. The equilibrium condition for the labor market is given as

$$L = L_{Y,t} + N_t x_t. \tag{15}$$

[13]Combining (6) and (7) yields $Y_t = w_t L_{Y,t} + N_t p_t x_t$. Using (15) and $\pi_t = (p_t - w_t)x_t$, this equation can be rewritten as follows: $Y_t = w_t L + N_t(p_t - w_t)x_t = w_t L + N_t\pi_t$. Thus, we can confirm that the relation $\tau\left(w_t L + \pi_t N_t\right) = \tau Y_t$ holds.

Combining (9) and (15) yield $L_{Y,t} = \frac{1-\alpha}{1-\alpha+(\alpha/\beta)}L$ and $x_t N_t = \frac{(\alpha/\beta)}{1-\alpha+(\alpha/\beta)}L$.

The final goods are used for household consumption, new investment and maintenance expenditure for public capital and R&D investment. Thus, the final goods market clearing condition becomes

$$Y_t = c_t L + I_{G,t} + I_{M,t} + a\dot{N}_t. \tag{16}$$

In addition, the asset market clearing condition is given as

$$A_t = V_t N_t.$$

3 Balanced-growth and Equilibrium Dynamics

3.1 Equilibrium and Dynamics

The production function of the final goods can be written as $Y_t = AL_{Y,t}^{1-\alpha} N_t (h_t x_t)^{\alpha}$. Substituting (5), $L_{Y,t} = \frac{1-\alpha}{1-\alpha+(\alpha/\beta)}L$ and $x_t N_t = \frac{(\alpha/\beta)}{1-\alpha+(\alpha/\beta)}L$ into the production function, we obtain the final output as follows:

$$Y_t = AG_t^{\alpha\epsilon} N_t^{1-\alpha\epsilon} \Gamma(\beta) L, \tag{17}$$

where

$$\Gamma(\beta) \equiv \frac{(1-\alpha)^{(1-\alpha)}(\alpha/\beta)^{\alpha}}{1-\alpha+(\alpha/\beta)}.$$

Since $\Gamma'(\beta) = -\frac{\alpha(1-\alpha)(\beta-1)}{\beta[\beta(1-\alpha)+\alpha]}\Gamma(\beta) < 0$, we can see that a larger patent breadth negatively affects the volume of production through its distortional effects on labor allocations.

The dynamic system of the economy for a given patent breadth β is illustrated by the following equations:

$$\frac{\dot{N}_t}{N_t} = \frac{L}{a}\left[(1-\tau)\Gamma(\beta)A\left(\frac{G_t}{N_t}\right)^{\alpha\epsilon} - \frac{c_t}{N_t}\right], \tag{18}$$

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}\left[\frac{L}{a}(1-\tau)\alpha A\left(\frac{G_t}{N_t}\right)^{\alpha\epsilon}\frac{\beta-1}{\beta}\Gamma(\beta) - \rho\right], \tag{19}$$

$$\frac{\dot{G}_t}{G_t} = \lambda\tau\Gamma(\beta)AL\left(\frac{G_t}{N_t}\right)^{\alpha\epsilon-1} - \delta\left((1-\lambda)\tau\right), \tag{20}$$

where (18) is obtained from (12), (16) and (17); (19) is obtained from (3), (17) and $r_t = \frac{1-\tau}{a}\frac{\beta-1}{\beta}\frac{\alpha Y_t}{N_t}$; (20) is obtained from (13), (14) and (17), respectively.

Summarizing equations (18) to (20), we obtain the following dynamic equations

$$\dot{x}_t = \left\{ \lambda\tau\Gamma(\beta)ALx_t^{\alpha\epsilon-1} - \delta\left((1-\lambda)\tau\right) - \frac{L}{a}\left[(1-\tau)\Gamma(\beta)Ax_t^{\alpha\epsilon} - z_t\right] \right\} x_t, \tag{21}$$

$$\dot{z}_t = \left\{ \frac{1}{\sigma}\left[\frac{L}{a}(1-\tau)\alpha Ax_t^{\alpha\epsilon}\frac{\beta-1}{\beta}\Gamma(\beta) - \rho\right] - \frac{L}{a}\left[(1-\tau)\Gamma(\beta)Ax_t^{\alpha\epsilon} - z_t\right] \right\} z_t, \tag{22}$$

where $x_t \equiv \frac{G_t}{N_t}$ and $z_t \equiv \frac{c_t}{N_t}$.

3.2 Steady-State Equilibrium

In the steady-state equilibrium, from (21) and (22), the relations $\dot{x}_t = \dot{z}_t = 0$ hold (i.e., $x_t = x$ and $z_t = z$). Therefore, the per capita consumption, public capital and the number of intermediate goods grow at the same balanced-growth rate g (i.e., $\frac{\dot{c}_t}{c_t} = \frac{\dot{G}_t}{G_t} = \frac{\dot{N}_t}{N_t} = g$). Noting that the relations $g = \frac{\dot{N}_t}{N_t}$ and $x = \frac{G_t}{N_t}$ hold, by solving (20) for x, we obtain

$$x = \left[\frac{\lambda\tau\Gamma(\beta)AL}{g + \delta\left((1-\lambda)\tau\right)}\right]^{\frac{1}{1-\alpha\epsilon}}. \tag{20'}$$

By substituting (20') into (19), the equilibrium balanced-growth rate g is implicitly determined by the following equation:

$$\Omega(g;\lambda,\tau) = \lambda^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}(LA)^{\frac{1}{1-\alpha\epsilon}}\frac{\alpha}{a}(1-\tau)\tau^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}\Gamma(\beta)^{\frac{1}{1-\alpha\epsilon}}\frac{\beta-1}{\beta}, \tag{23}$$

where

$$\Omega(g;\lambda,\tau) \equiv (\sigma g + \rho)\left[g + \delta\left((1-\lambda)\tau\right)\right]^{\frac{\alpha\epsilon}{1-\alpha\epsilon}},$$

$\Omega_g(g;\lambda,\tau) > 0$, $\Omega(0;\lambda,\tau) = \rho\left[\delta\left((1-\lambda)\tau\right)\right]^{\frac{\alpha\epsilon}{1-\alpha\epsilon}} > 0$, $\lim_{g\to\infty}\Omega(g;\lambda,\tau) = \infty$. Equation (23) indicates that a unique balanced-growth equilibrium exists, if the parameter conditions $\Omega(0;\lambda,\tau) < \lambda^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}(LA)^{\frac{1}{1-\alpha\epsilon}}\frac{\alpha}{a}(1-\tau)\tau^{\frac{\alpha\epsilon}{1-\alpha\epsilon}}\Gamma(\beta)^{\frac{1}{1-\alpha\epsilon}}\frac{\beta-1}{\beta}$ are satisfied. Thus, we can obtain the following proposition.

Proposition 1 *Suppose that the parameter conditions*

$$\frac{\delta\left((1-\lambda)\tau\right)}{\lambda\tau} < \left[\frac{1}{\rho}(LA)^{\frac{1}{1-\alpha\epsilon}}\frac{\alpha}{a}(1-\tau)\Gamma(\beta)^{\frac{1}{1-\alpha\epsilon}}\frac{\beta-1}{\beta}\right]^{\frac{1-\alpha\epsilon}{\alpha\epsilon}} \tag{24}$$

hold, a unique balanced-growth equilibrium exists.

Proposition 1 indicates that the two fiscal instruments, τ and λ, are relevant for the existence of balanced-growth equilibrium. In particular, as underlined by Dioikitopoulos and Kalyvitis (2008), the composition of government expenditure may matter for a positive balanced-growth rate. Given the value of tax rate $\tau \in (0,1)$, suppose that all tax revenues are devoted to maintenance expenditure (i.e., $\lambda = 0$), equation (24) never holds, implying that certain amounts of new investments are necessary for a positive balanced-growth rate. That is, the condition $\lambda > \lambda_l$ must hold for a positive balanced-growth rate, where $\lambda_l \in (0,1)$ is the minimum expenditure share of new investments that ensures a positive balanced-growth rate for a given $\tau \in (0,1)$. On the other hand, given the value of tax rate $\tau \in (0,1)$, suppose that all tax revenues are devoted to new investments (i.e., $\lambda = 1$), if the parameter conditions $\frac{\delta(0)}{\tau} > \left[\frac{1}{\rho}(LA)^{\frac{1}{1-\alpha\epsilon}}\frac{\alpha}{a}(1-\tau)\Gamma(\beta)^{\frac{1}{1-\alpha\epsilon}}\frac{\beta-1}{\beta}\right]^{\frac{1-\alpha\epsilon}{\alpha\epsilon}}$ are satisfied, equation (24) never holds. In this case, due to the convexity of the depreciation function $\delta(\cdot)$, as shown in Dioikitopoulos and Kalyvitis (2008), the conditions $\lambda \in (\lambda_l, \lambda_u)$ must hold for a positive balanced-growth rate, where $\lambda_u \in (\lambda_l, 1)$ is the maximum expenditure share of new investments that ensures a positive balanced-growth rate for given $\tau \in (0,1)$. These results indicate that under certain parameter conditions, both new investment and maintenance expenditure are necessary for a positive balanced-growth rate.[14] In the following analysis, to clarity our main arguments, we restrict our analyses to the case where the parameter condition of (24) is satisfied.

3.3 Local Stability

When the balanced-growth rate g is determined uniquely, the associated steady-state values of (x, z) are also determined uniquely from (18) and (20'). [15] Linearizing (21) and (22) around the balanced-growth equilibrium (x, z), we obtain:

$$\begin{pmatrix} \dot{x}_t \\ \dot{z}_t \end{pmatrix} = \begin{pmatrix} -\Gamma(\beta)\alpha\epsilon A x^{\alpha\epsilon-1} L\left\{\lambda\tau\frac{1-\alpha\epsilon}{\alpha\epsilon} + \frac{1-\tau}{a}x\right\} & \frac{L}{a}x \\ -\frac{\sigma-\frac{\beta-1}{\beta}\alpha}{\sigma} z\frac{L}{a}(1-\tau)\Gamma(\beta)\alpha\epsilon A x^{\alpha\epsilon-1} & \frac{L}{a}z \end{pmatrix} \begin{pmatrix} x_t - x \\ z_t - z \end{pmatrix}. \tag{25}$$

Calculating the determinants of this coefficient matrix J, we obtain

$$detJ = -\Gamma(\beta)\alpha\epsilon A x^{\alpha\epsilon-1}(\frac{L}{a})^2 z\left\{\lambda\tau a\frac{1-\alpha\epsilon}{\alpha\epsilon} + \frac{x}{\sigma}\frac{\beta-1}{\beta}\alpha(1-\tau)\right\} < 0$$

Therefore, one of the eigenvalues of the coefficient matrix is positive and the other is negative, that is, the balanced-growth equilibrium is a saddle point. This means

[14]See Proposition 1 of Dioikitopoulos and Kalyvitis (2008) for more details.

[15]Noting that $g = \frac{\dot{N}_t}{N_t} = \frac{\dot{G}_t}{G_t}$, $x = \frac{G_t}{N_t}$ and $z = \frac{c_t}{N_t}$, we obtain that $x = \left[\frac{\lambda\tau\Gamma(\beta)AL}{g+\delta((1-\lambda)\tau)}\right]^{\frac{1}{1-\alpha\epsilon}}$ from (20') and that $z = (1-\tau)\Gamma(\beta)Ax^{\alpha\epsilon} - \frac{ag}{L}$ from (18).

that there is a one-dimensional stable manifold. Since the initial value of $z_0 = \frac{c_0}{N_0}$ is not predetermined, we can choose a unique initial value of z_0 on this stable manifold for a given initial value of $x_0 = \frac{G_0}{N_0}$. Thus, as in Futagami et al. (1993), we can obtain the following proposition:

Proposition 2 *When there is a unique balanced-growth equilibrium, a unique stable path converging to the balanced-growth equilibrium exists.*

4 Balanced-Growth Maximizing Public Investment Policy

In this section, we examine the effects of patent protection on balanced-growth maximizing public investment policy. We assume that initially, the government chooses two fiscal instruments, τ and λ, so as to maximize the balanced-growth rate g, taking the strength of patent protection β as given, and then examines how the balanced-growth maximizing public investment policy changes as patent protection becomes stronger.

4.1 Income Tax Rate and Expenditure Share

In this subsection, we analyze the balanced-growth maximizing tax rate and expenditure share of new investment. The problem for the government is to choose τ and λ so as to maximize g for a given β. From (23), we can see that the balanced-growth rate g depends upon the income tax rate τ, the expenditure share of new investment λ and the strength of patent protection β (i.e., $g = g(\lambda, \tau; \beta)$). Therefore, by totally differentiating (23) with respect to λ and τ, and rearranging them, we obtain the following first order conditions:

$$g_\lambda(\tau, \lambda; \beta) = \frac{\frac{\alpha\epsilon}{1-\alpha\epsilon}\left[\frac{1}{\lambda} + \frac{\delta'((1-\lambda)\tau)\tau}{g+\delta((1-\lambda)\tau)}\right]}{\frac{\sigma}{\sigma g+\rho} + \frac{\alpha\epsilon}{1-\alpha\epsilon}\frac{1}{g+\delta((1-\lambda)\tau)}} = 0, \tag{26}$$

$$g_\tau(\tau, \lambda; \beta) = \frac{\frac{\alpha\epsilon}{1-\alpha\epsilon}\left[\frac{1}{\tau} - \frac{\delta'((1-\lambda)\tau)(1-\lambda)}{g+\delta((1-\lambda)\tau)}\right] - \frac{1}{1-\tau}}{\frac{\sigma}{\sigma g+\rho} + \frac{\alpha\epsilon}{1-\alpha\epsilon}\frac{1}{g+\delta((1-\lambda)\tau)}} = 0. \tag{27}$$

See Appendix A for the deviation of equations (26) and (27). From (23), (26) and (27), we can see that the balanced-growth maximizing public investment policy (τ^*, λ^*) and the associated balanced-growth rate g^* must satisfy the following conditions:

$$\tau^* = \frac{\alpha\epsilon}{\alpha\epsilon + (1-\alpha\epsilon)\lambda^*}, \tag{28}$$

$$\frac{\delta'\left((1-\lambda^*)\tau^*\right)}{g^*+\delta\left((1-\lambda^*)\tau^*\right)}=-\frac{1}{\lambda^*\tau^*}, \tag{29}$$

$$\Omega(g^*;\lambda^*,\tau^*)=\lambda^{*\frac{\alpha\epsilon}{1-\alpha\epsilon}}(LA)^{\frac{1}{1-\alpha\epsilon}}\frac{\alpha}{a}(1-\tau^*)\tau^{*\frac{\alpha\epsilon}{1-\alpha\epsilon}}\Gamma(\beta)^{\frac{1}{1-\alpha\epsilon}}\frac{\beta-1}{\beta}. \tag{30}$$

From equations (28) to (30), we can see that the patent parameter β is included only in (30). This result implies that stronger patent protection (i.e., the broader patent breadth β) affects the balanced-growth maximizing public investment policy (τ^*, λ^*) only though its effect on the balanced-growth rate g^*.

Conditions (28) and (29) are consistent with those found in Kalaitzidakis and Kalyvitis (2004). Let us first explain the condition (28). Condition (28) indicates that as long as the government engages in maintenance activities of public capital, the balanced-growth maximizing income tax rate τ^* is greater than the elasticity of public capital in the output production function $\alpha\epsilon$. Consequently, the balanced-growth maximizing income tax rate τ^* is positively (resp., negatively) related to the balanced-growth maximizing expenditure share of maintenance activities $1-\lambda^*$ (resp., new investment λ^*).

Let us next explain the condition (29). Since $g^*+\delta\left((1-\lambda^*)\tau^*\right)=\lambda^*\tau^*\Gamma(\beta)ALx^{*\alpha\epsilon-1}$ from (20), equation (29) can be rewritten as

$$-\delta'\left((1-\lambda^*)\tau^*\right)=\Gamma(\beta)ALx^{*\alpha\epsilon-1}. \tag{29'}$$

As depicted in Figure 3, the condition (29') indicates that the balanced-growth maximizing public investment policy (λ^*, τ^*) is determined so that the marginal expenditures of new investment and maintenance have the same contribution to public capital accumulation.

4.2 Effects of Patent Protection

In this subsection, we examine the effect of patent protection on the balanced-growth maximizing public investment policy described in the previous subsection.

Using equations (28) to (30), we obtain

$$d\lambda^*=-\frac{\alpha\epsilon+(1-\alpha\epsilon)\lambda^*}{\tau^*(1-\alpha\epsilon)}d\tau^*, \tag{31}$$

$$dg^*=-\left[\delta'\left((1-\lambda^*)\tau^*\right)+\frac{(1-\tau^*)\alpha\epsilon\delta''\left((1-\lambda^*)\tau^*\right)}{(1-\alpha\epsilon)^2}\right]d\tau^*, \tag{32}$$

$$\left[\frac{\sigma}{\sigma g^*+\rho}+\frac{\alpha\epsilon}{1-\alpha\epsilon}\frac{1}{g^*+\delta\left((1-\lambda^*)\tau^*\right)}\right]dg^*=\left\{\frac{\beta(1-\alpha)[1-\alpha\beta+\alpha+\alpha(1-\epsilon)]+\alpha^2(1-\epsilon)}{(\beta-1)\beta[\beta(1-\alpha)+\alpha](1-\alpha\epsilon)}\right\}d\beta \tag{33}$$

The deviation of equations (31) to (33) is shown in Appendix B. Appendix C shows that the second-order sufficient condition for g^* to be a relative maximum is satisfied when

$$\begin{aligned} &\delta'((1-\lambda^*)\tau^*) + \frac{\alpha\epsilon(1-\tau^*)\delta''((1-\lambda^*)\tau^*)}{(1-\alpha\epsilon)^2} > 0, \\ &\Leftrightarrow \quad -\frac{\delta''((1-\lambda^*)\tau^*)(1-\lambda^*)\tau^*}{\delta'((1-\lambda^*)\tau^*)} > \frac{(1-\alpha\epsilon)^2(1-\lambda^*)\tau^*}{\alpha\epsilon(1-\tau^*)}. \end{aligned} \tag{34}$$

The sufficient condition for g^* to be a relative maximum requires that the elasticity of the change in the public capital depreciation rate with respect to the share of maintenance expenditure to final output be large enough to satisfy (34).[16] Hereafter, we restrict our analysis to the parameter regions that ensure that the condition (34) holds. Under the condition of (34), from equations (31) to (33), we can obtain the effect of patent protection on the balanced-growth maximizing public investment policy (τ^*, λ^*) and the associated balanced-growth rate g^* as follows.

$$\frac{d\tau^*}{d\beta} = -\frac{\frac{\beta(1-\alpha)[1-\alpha\beta+\alpha+\alpha(1-\epsilon)]+\alpha^2(1-\epsilon)}{(\beta-1)\beta[\beta(1-\alpha)+\alpha](1-\alpha\epsilon)}}{\left[\frac{\sigma}{\sigma g+\rho} + \frac{\alpha\epsilon}{1-\alpha\epsilon}\frac{1}{g^*+\delta((1-\lambda^*)\tau^*)}\right]\left[\delta'((1-\lambda^*)\tau^*) + \frac{(1-\tau^*)\alpha\epsilon\delta''((1-\lambda^*)\tau^*)}{(1-\alpha\epsilon)^2}\right]} < 0, \tag{35}$$

$$\frac{d\lambda^*}{d\beta} = -\frac{\alpha\epsilon + (1-\alpha\epsilon)\lambda^*}{\tau^*(1-\alpha\epsilon)}\frac{d\tau^*}{d\beta} > 0, \tag{36}$$

$$\frac{dg^*}{d\beta} = \frac{\frac{\beta(1-\alpha)[1-\alpha\beta+\alpha+\alpha(1-\epsilon)]+\alpha^2(1-\epsilon)}{(\beta-1)\beta[\beta(1-\alpha)+\alpha](1-\alpha\epsilon)}}{\frac{\sigma}{\sigma g^*+\rho} + \frac{\alpha\epsilon}{1-\alpha\epsilon}\frac{1}{g^*+\delta((1-\lambda^*)\tau^*)}} > 0, \tag{37}$$

where (35) is obtained from (32) and (33); (36) is obtained from (31) and (35); (37) is obtained from (33), respectively. Thus, we can obtain the following proposition:

Proposition 3 *Under condition (34), the following statements hold.*

(1). If patent protection is stronger, the balanced-growth maximizing expenditure share of new investment is higher, whereas the balanced-growth maximizing income tax rate is lower.

(2). Both with and without balanced-growth maximizing public investment policy, stronger patent protection enhances economic growth. The balanced-growth maximizing public investment policy always strengthens the direct growth-enhancing effect of patent protection.

[16] Analogous parameter conditions are briefly discussed in Yakita (2008).

In order to understand the intuition behind the results of Proposition 3, we first consider the effect of patent protection β on the steady-state public capital/number of intermediate goods ratio x^* under the balanced-growth maximizing public investment policy. Using (20'), (28), (29) and (32), we obtain the effect of β on x^* under the balanced-growth maximizing public investment policy as follows:

$$\frac{1}{x^*}\frac{dx^*}{d\beta} = \frac{1}{(1-\alpha\epsilon)}\left\{\left[\frac{1}{\lambda^*\tau^*} + \frac{\delta'\left((1-\lambda^*)\tau^*\right) + \frac{\alpha\epsilon(1-\tau^*)\delta''((1-\lambda^*)\tau^*)}{(1-\alpha\epsilon)^2}}{g^* + \delta\left((1-\lambda^*)\tau^*\right)}\right]\frac{d\tau^*}{d\beta} - \frac{\alpha(1-\alpha)(\beta-1)}{\beta(1-\alpha)+\alpha}\right\} < 0 \tag{38}$$

The deviation of equation (38) is shown in Appendix D.

From (38), we can see that stronger patent protection, together with the associated public investment policy changes (i.e., $\frac{d\tau^*}{d\beta} < 0$ and $\frac{d\lambda^*}{d\beta} > 0$), lowers the steady-state public capital/number of intermediate goods ratio. Intuitively, on the one hand, as shown in Appendix E, given the value of τ and λ (i.e., without the associated changes in public investment policy), stronger patent protection increases the share of income that goes to monopolistic profits, enhances innovations and thus negatively affects the steady-state public capital/number of intermediate goods ratio (i.e., "direct effect"). On the other hand, as is inferred from (20'), the effects of the associated public investment policy changes (i.e., $\frac{d\tau^*}{d\beta} < 0$ and $\frac{d\lambda^*}{d\beta} > 0$) on x^* are generally ambiguous, because the effect of the decline in τ^* on x^* and that of the rise in λ^* on x^* offset each other (i.e., "policy change effect"). Consequently, the former "direct effect" dominates the latter "policy change effect", and the stronger patent protection, together with the associated public investment policy changes, lowers the steady-state public capital/number of intermediate goods ratio.

Let us next explain the intuition behind the results of Proposition 3-1 (i.e., $\frac{d\lambda^*}{d\beta} > 0$ and $\frac{d\tau^*}{d\beta} < 0$). We first consider the result shown in (36) (i.e., $\frac{d\lambda^*}{d\beta} > 0$). From (29'), the optimal expenditure share of new investment λ^* is determined so that the marginal expenditures of new investment and maintenance make the same contribution to public capital accumulation. From Figure 3, since $\Gamma'(\beta) < 0$ and $\frac{d\tau^*}{d\beta} < 0$ from (35), stronger patent protection and the associated decrease in tax rate shifts the $\Gamma(\beta)ALx^{*\alpha\epsilon-1}$ line downward and $-\delta'\left((1-\lambda^*)\tau^*\right)$ curve upward. These effects negatively affect the balanced-growth maximizing expenditure share of new investment (i.e., "direct effect" and "tax change effect"). However, since $\frac{dx^*}{d\beta} < 0$ from (38), the associated decrease in the steady-state public capital/number of intermediate goods ratio shifts the $\Gamma(\beta)ALx^{*\alpha\epsilon-1}$ line upward. This effect positively affects the balanced-growth maximizing expenditure share of new investment (i.e., "public capital effect"). The result shown in (36) (i.e., $\frac{d\lambda^*}{d\beta} > 0$) indicates that the latter "public capital effect" dominates the former "direct effect" and "tax change effect". Consequently, stronger patent protection, together with the associated public investment policy changes, eventually shift the $\Gamma(\beta)ALx^{*\alpha\epsilon-1}$ upward, as described in Figure 3, and increases the balanced-growth maximizing expenditure share of new investment.

Next, we consider the result shown in (35) (i.e., $\frac{d\tau^*}{d\beta} < 0$). From (28), the balanced-growth maximizing income tax rate τ^* is negatively related to the balanced-growth maximizing expenditure share of new investment λ^*. As shown in (36), stronger patent protection increases the balanced-growth maximizing expenditure share of new investment. Therefore, from (28) and (36), stronger patent protection, together with the associated increase in expenditure share of new investment, lowers the balanced-growth maximizing income tax rate.

Lastly, let us explain the intuition behind the results of Proposition 3-2. From (37), we can see that the stronger patent protection, together with the associated public investment policy changes (i.e., $\frac{d\tau^*}{d\beta} < 0$ and $\frac{d\lambda^*}{d\beta} > 0$), increase the balanced-growth rate. Moreover, as shown in Appendix E, even if the values of τ and λ are held constant (i.e., without the associated changes in public investment policy), stronger patent protection increases the balanced growth rate. Note that under the balanced-growth maximizing public investment policy, fiscal instruments τ and λ are adjusted to maximize the balanced-growth rate for each value of β. Consequently, the balanced-growth maximizing public investment policy always strengthens the direct growth-enhancing effect of patent protection.

The solid lines in Figures 4-1 to 4-3 show the numerical examples of the relationship between patent protection β and the expenditure share of new investment λ^* (Figure 4-1), the income tax rate τ^* (Figure 4-2) and the balanced-growth rate g^* (Figure 4-3) under the balanced-growth maximizing public investment policy, whereas the dashed line in Figure 4-3 shows the balanced-growth rate g^{fix} that is achieved when λ and τ are held constant at their fixed values (i.e., $\lambda = \lambda^{fix} = 0.8004$ and $\tau = \tau^{fix} = 0.2380$). As shown in Figures 4-1 and 4-2, the relations $\lambda^* = \lambda^{fix}$ and $\tau^* = \tau^{fix}$ hold only when $\beta = 0.3$. Since the quantitative impact of public investment policy changes (i.e., $\frac{d\lambda^*}{d\beta} > 0$ and $\frac{d\tau^*}{d\beta} < 0$) on economic growth is relatively small in our baseline simulation, the difference between g^* and g^{fix} is invisible in Figure 4-3. Therefore, Figure 4-4 shows the difference between g^* and g^{fix} for several different values of $\beta \in [1.15, 1.45]$. To parameterize the model, we need to specify the functional form of the depreciation rate of public capital. Following Yakita (2008), we assume that $\delta\left(\frac{I_{M,t}}{Y_t}\right) = H\left[1 - (\frac{I_{M,t}}{Y_t})^\eta\right]$, where $H > 0$ and $\eta \in (0, 1)$. The parameter values used to derive the results of Figures 4-1 to 4-4 are summarized in Table 1, and explanations of these parameter values are provided in Appendix F. [17]

[17] These numerical examples satisfy both conditions of (24) and (34). The objective of this numerical analysis is not to calibrate our simple model to actual data but to supplement the quantitative results of Proposition 3. Although we chose the parameter values carefully, these quantitative results should be interpreted with caution.

As shown in Figures 4-1 to 4-3, as patent protection becomes stronger (i.e., as patent breadth β increases), the balanced-growth maximizing expenditure share of new investment is higher (Figure 4-1) and the balanced-growth maximizing income tax rate is lower (Figure 4-2). In addition, both with and without balanced-growth maximizing public investment policy, stronger patent protection increases the balanced growth rate (Figure 4-3). These numerical simulation results are consistent with the results obtained in Propositions 3-1 and 3-2. Moreover, Figure 4-4 shows that the relation $g^* - g^{fix} \geq 0$ holds for $\beta \in [1.15, 1.45]$ implying that the balanced-growth maximizing public investment policy always strengthens the direct growth enhancing effect of stronger patent protection. Therefore, Figure 4-4 confirms the second statement of Proposition 3-2.

5 Welfare Maximizing Public Investment Policy

In this section, we consider the welfare-maximizing public investment policy along the balanced-growth path (BGP). As shown in Appendix G, the lifetime utility of a representative household along the BGP is expressed as

$$U = \frac{L}{1-\sigma}\left[\frac{c_0^{1-\sigma}}{\rho - g(1-\sigma)} - \frac{1}{\rho}\right]. \tag{39}$$

where

$$c_0 = \frac{aN_0}{L}\frac{(\sigma - \frac{\beta-1}{\beta}\alpha)g + \rho}{\frac{\beta-1}{\beta}\alpha}, \tag{40}$$

Here, c_0 is the initial per capita consumption. For $c_0 > 0$, the relation $[\sigma - \frac{\beta-1}{\beta}\alpha]g + \rho > 0$ must hold. Equations (39) and (40) indicate that public investment policy (τ, λ) affects the lifetime utility of a representative household along the BGP only through its effect on the balanced-growth rate g. Thus, by differentiating (39) with respect to g, we obtain

$$\frac{\partial U}{\partial g} = Lc_0^{1-\sigma}\frac{\left[(\sigma - \frac{\beta-1}{\beta}\alpha)g + \rho\right]\sigma + \rho(1 - \frac{\beta-1}{\beta}\alpha)}{\left[(\sigma - \frac{\beta-1}{\beta}\alpha)g + \rho\right][\rho - g(1-\sigma)]^2} > 0. \tag{41}$$

As shown in Appendix H, the transversality condition ensures that the relations $\rho - g(1-\sigma) > 0$ and $[\sigma - \frac{\beta-1}{\beta}\alpha]g + \rho > 0$ hold. From (41), the lifetime utility of a representative household along the BGP is monotonically increasing in g, which implies that the balanced-growth maximizing public investment policy is also the welfare maximizing public investment policy along the BGP. Thus, we obtain the following proposition:

Proposition 4 *The balanced-growth growth maximizing public investment policy is equivalent to the welfare maximizing public investment policy along the balanced-growth path.*

Proposition 4 relies on the assumption that the economy always rides on the balanced-growth path. However, if we explicitly consider the transitional process to balanced-growth equilibrium, the balanced-growth growth maximizing public investment policy does not necessarily equal the welfare maximizing public investment policy (e.g., Futagami et al, 1993; Turnovsky, 1997). Unfortunately, we have not yet obtained any new policy implications from the analysis of the welfare maximizing public investment policy with explicit considerations of the transition process. Therefore the complete analysis of the welfare maximizing public investment policy with explicit considerations of the transition process may be a promising direction for future research.

6 Concluding Remarks

This paper examined the balanced-growth maximizing public investment policy in a growth model where the engines of economic growth are private R&D and public capital accumulation. In the model, the government allocates tax revenue between new investment and maintenance expenditure for public capital. Then, we considered how the balanced-growth maximizing public investment policy changes as patent protection becomes stronger, as seen in many countries. The results showed that as patent protection becomes stronger, the income tax rate to finance public investment should be lower and the expenditure share of new investment should be higher. That is, the balanced-growth maximizing policy leads to a smaller government, as patent protection becomes stronger.

Appendix

Appendix A

By totally differentiating (23) with respect to λ, τ and β, we obtain

$$\begin{aligned}\left[\frac{\sigma}{\sigma g+\rho}+\frac{\alpha\epsilon}{1-\alpha\epsilon}\frac{1}{g+\delta\left((1-\lambda)\tau\right)}\right]dg = &\frac{\alpha\epsilon}{1-\alpha\epsilon}\left[\frac{1}{\lambda}+\frac{\delta'\left((1-\lambda)\tau\right)\tau}{g+\delta\left((1-\lambda)\tau\right)}\right]d\lambda\\ &+\left\{\frac{\alpha\epsilon}{1-\alpha\epsilon}\left[\frac{1}{\tau}-\frac{\delta'\left((1-\lambda)\tau\right)(1-\lambda)}{g+\delta\left((1-\lambda)\tau\right)}\right]-\frac{1}{1-\tau}\right\}d\tau\\ &+\left\{\frac{\beta(1-\alpha)[1-\alpha\beta+\alpha+\alpha(1-\epsilon)]+\alpha^2(1-\epsilon)}{(\beta-1)\beta[\beta(1-\alpha)+\alpha](1-\alpha\epsilon)}\right\}d\beta\end{aligned} \tag{A.1}$$

The above equation yields (26) and (27) immediately.

Appendix B

By totally differentiating (28) with respect to λ and τ, we obtain (31). Analogously, by totally differentiating (29) with respect to λ and τ, and rearranging it using (31), we obtain (32). Moreover, by substituting (26) and (27) into (A.1), we obtain (33).

Appendix C

From (26) and (27), the Hessian matrix of second-order derivatives is defined as follows:

$$H \equiv \begin{pmatrix} g^*_{\lambda\lambda} & g^*_{\lambda\tau} \\ g^*_{\tau\lambda} & g^*_{\lambda\lambda} \end{pmatrix}, \tag{A.2}$$

where

$$g^*_{\lambda\lambda} \equiv g_{\lambda\lambda}(\tau^*,\lambda^*) = -\Delta\tau^2\delta''\left((1-\lambda^*)\tau^*\right) < 0,$$

$$g^*_{\lambda\tau} \equiv g_{\lambda\tau}(\tau^*,\lambda^*) = \Delta\left[\frac{(1-\alpha\epsilon)\tau}{\alpha\epsilon(1-\tau^*)}\delta'\left((1-\lambda^*)\tau^*\right)+\frac{\tau^*-\alpha\epsilon}{1-\alpha\epsilon}\delta''\left((1-\lambda^*)\tau^*\right)\right],$$

$$g^*_{\tau\tau} \equiv g_{\tau\tau}(\tau^*,\lambda^*) = \Delta\left\{\frac{2\alpha\epsilon-\tau}{\alpha\epsilon\tau^*(1-\tau^*)}\delta'\left((1-\lambda^*)\tau^*\right)-\left[\frac{\tau^*-\alpha\epsilon}{(1-\alpha\epsilon)\tau^*}\right]^2\delta''\left((1-\lambda^*)\tau^*\right)\right\} < 0,$$

$g^*_{\lambda\tau} = g^*_{\tau\lambda}$ and $\Delta \equiv \frac{\frac{\alpha\epsilon}{1-\alpha\epsilon}\frac{1}{g+\delta((1-\lambda^*)\tau^*)}}{\frac{\sigma}{\sigma g^*+\rho}+\frac{\alpha\epsilon}{1-\alpha\epsilon}\frac{1}{g^*+\delta((1-\lambda^*)\tau^*)}} > 0$. Moreover, we obtain

$$g^*_{\lambda\lambda}g^*_{\tau\tau} - g^*_{\lambda\tau}g^*_{\tau\lambda} = -\Delta^2 \frac{\tau^{*2}\delta'\left((1-\lambda^*)\tau^*\right)(1-\alpha\epsilon)^2}{[\alpha\epsilon(1-\tau^*)]^2}\left[\delta'\left((1-\lambda^*)\tau^*\right) + \frac{\alpha\epsilon(1-\tau^*)\delta''\left((1-\lambda^*)\tau^*\right)}{(1-\alpha\epsilon)^2}\right]. \tag{A.3}$$

The second-order sufficient condition for $g(\lambda^*, \tau^*; \beta)$ to be a relative maximum is given by $g^*_{\lambda\lambda} < 0$ and $g^*_{\lambda\lambda}g^*_{\tau\tau} - g^*_{\lambda\tau}g^*_{\tau\lambda} > 0$. From (A.3), the relation $g^*_{\lambda\lambda}g^*_{\tau\tau} - g^*_{\lambda\tau}g^*_{\tau\lambda} > 0$ holds when the condition (34) is satisfied.

Appendix D

By totally differentiating (20') with respect to λ, τ and β, we obtain

$$\begin{aligned}\frac{dx}{x} =& -\frac{1}{(1-\alpha\epsilon)[g+\delta\left((1-\lambda)\tau\right)]}dg + \frac{1}{\tau(1-\alpha\epsilon)}\left[1 - \frac{\tau(1-\lambda)\delta'\left((1-\lambda)\tau\right)}{g+\delta\left((1-\lambda)\tau\right)}\right]d\tau \\ &+ \frac{1}{\lambda(1-\alpha\epsilon)}\left[1 + \frac{\lambda\tau\delta'\left((1-\lambda)\tau\right)}{g+\delta\left((1-\lambda)\tau\right)}\right]d\lambda - \frac{1}{1-\alpha\epsilon}\frac{\alpha(1-\alpha)(\beta-1)}{\beta(1-\alpha)+\alpha}d\beta.\end{aligned} \tag{A.4}$$

Substituting (28) and (29) into (A.4) yields

$$\begin{aligned}\frac{dx^*}{x^*} =& -\frac{1}{(1-\alpha\epsilon)[g^*+\delta\left((1-\lambda^*)\tau^*\right)]}dg^* + \frac{1}{\tau^*(1-\alpha\epsilon)\lambda^*}d\tau^* \\ &- \frac{1}{1-\alpha\epsilon}\frac{\alpha(1-\alpha)(\beta-1)}{\beta(1-\alpha)+\alpha}d\beta.\end{aligned} \tag{A.5}$$

By substituting (32) into (A.5), and rearranging them, we obtain (38).

Appendix E

Given the value of τ and λ (i.e., $d\tau = d\lambda = 0$), from (A.1) and (A.4), we obtain

$$\frac{dg}{d\beta} = \frac{\frac{\beta(1-\alpha)[1-\alpha\beta+\alpha+\alpha(1-\epsilon)]+\alpha^2(1-\epsilon)}{(\beta-1)\beta[\beta(1-\alpha)+\alpha](1-\alpha\epsilon)}}{\frac{\sigma}{\sigma g+\rho}+\frac{\alpha\epsilon}{1-\alpha\epsilon}\frac{1}{g+\delta((1-\lambda)\tau)}} > 0, \tag{A.6}$$

$$\frac{1}{x}\frac{dx}{d\beta} = -\frac{1}{(1-\alpha\epsilon)[g+\delta\left((1-\lambda)\tau\right)]}\frac{dg}{d\beta} - \frac{1}{1-\alpha\epsilon}\frac{\alpha(1-\alpha)(\beta-1)}{\beta(1-\alpha)+\alpha} < 0. \tag{A.7}$$

Appendix F

We set the discount rate (ρ) to 0.02 and the inverse of the elasticity of intertemporal substitution (σ) to 1.5 according to Jones et al. (1993). The value of labor size (L) and the unit cost of R&D (a) are normalized to 1.

According to international evidence (Britton et al., 2000; Gali et al., 2007), the markup estimates in developed countries range from 1.2 to 1.4. Based on these estimates, we set the share of intermediate goods inputs in output (α) to 2/3 so that the highest markup rate ($\frac{1}{\alpha}$) is given by 1.5. We also set the patent breadth parameter (β) to 1.3 in the baseline simulation, and changed it from 1.15 to 1.45.

The empirical estimates of output elasticity of infrastructure vary substantially among existing studies. However, recent empirical studies indicate that the output elasticity of infrastructure lies in the 0.1-0.2 range on average (e.g., Shioji, 2001; Kamps, 2006; Bom and Ligthart, 2014). In particular, the meta-regression analysis by Bom and Ligthart (2014) finds that the long-run output elasticity of core infrastructure at the regional/local level of government is 0.193. Based on these estimates, we set the weight parameter of the externality effect from the public capital (ϵ) to 0.3 so that the output elasticity of public capital ($\alpha\epsilon$) is given by 0.2.

Published data on maintenance are very scarce, due to inherent problems in the measurement of this type of expenditures. To the best of our knowledge, there has been only one source of long-term data on maintenance expenditures, namely, the Canadian survey on Capital and Repair Expenditures. Using this dataset, Kalaitzidakis and Kalyvitis (2005) find that 21% of total public capital spending went toward maintenance and repair expenditures. Based on this finding, we set the scaling parameter of the depreciation rate function (H) to 0.62, and the elasticity parameter of the depreciation rate function (η) to 0.1 so that the expenditure share of new investment in the baseline simulation is approximately given by 0.8. Furthermore, to achieve a 2% balanced growth rate in the baseline simulation, we adjust the productivity parameter in the final goods production sector (A) to 0.955.

Appendix G

On the balanced-growth path, per capita consumption c_t grows at a rate g with its time path given by $c_t = c_0 e^{gt}$. By substituting $c_t = c_0 e^{gt}$ into (1) and rearranging them, we obtain (39). Noting that $\frac{\dot{c}_t}{c_t} = \frac{\dot{N}_t}{N_t} = g$ and $x = \frac{G_t}{N_t}$, from (18) and (19), we obtain

$$c_0 = \left[(1-\tau)\Gamma(\beta)Ax^{\alpha\epsilon} - \frac{a}{L}g\right]N_0, \tag{A.8}$$

$$(1-\tau)\Gamma(\beta)Ax^{\alpha\epsilon} = \frac{a}{L}\frac{1}{\frac{\beta-1}{\beta}\alpha}(\sigma g + \rho). \tag{A.9}$$

By substituting (A.9) into (A.8), and rearranging them, we obtain (40).

Appendix H

Substituting (11) into asset market clearing condition yields $A_t = aN_t$. Hence, on the balanced-growth path, the real value of asset A_t grows at a rate g with its time path given by $A_t = A_0 e^{gt}$. By substituting $A_t = A_0 e^{gt}$, $c_t = c_0 e^{gt}$ into the transversality condition, we obtain

$$\lim_{t\to\infty} c_0^{-\sigma} A_0 e^{-[\rho-(1-\sigma)g]t} = 0, \tag{A.10}$$

which implies that the relation $\rho - (1-\sigma)g > 0$ must hold. Since $\frac{\beta-1}{\beta}\alpha < 1$, we obtain the following relationship:

$$\begin{aligned} 0 &< \rho - (1-\sigma)g \\ &< \rho - (\frac{\beta-1}{\beta}\alpha - \sigma)g = [\sigma - \frac{\beta-1}{\beta}\alpha]g + \rho. \end{aligned}$$

Therefore, the transversality condition ensures that the relations $\rho - g(1-\sigma) > 0$ and $[\sigma - \frac{\beta-1}{\beta}\alpha]g + \rho > 0$ hold.

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Table 1: Parameter values

Parameter	Description	Value
ρ	Discount rate	0.02
σ	The inverse of the elasticity of intertemporal substitution	1.5
L	Labor size	1
a	A unit cost of R&D	1
α	The share of intermediate goods inputs in output	2/3
β	The strength of patent protection	1.3 (1.05-1.45)
ϵ	The weight parameter of the externality effect from the public capital	0.3
H	The scaling parameter of the depreciation rate function	0.62
η	The elasticity parameter of the depreciation rate function	0.1
A	The productivity parameter of the final good production	0.955

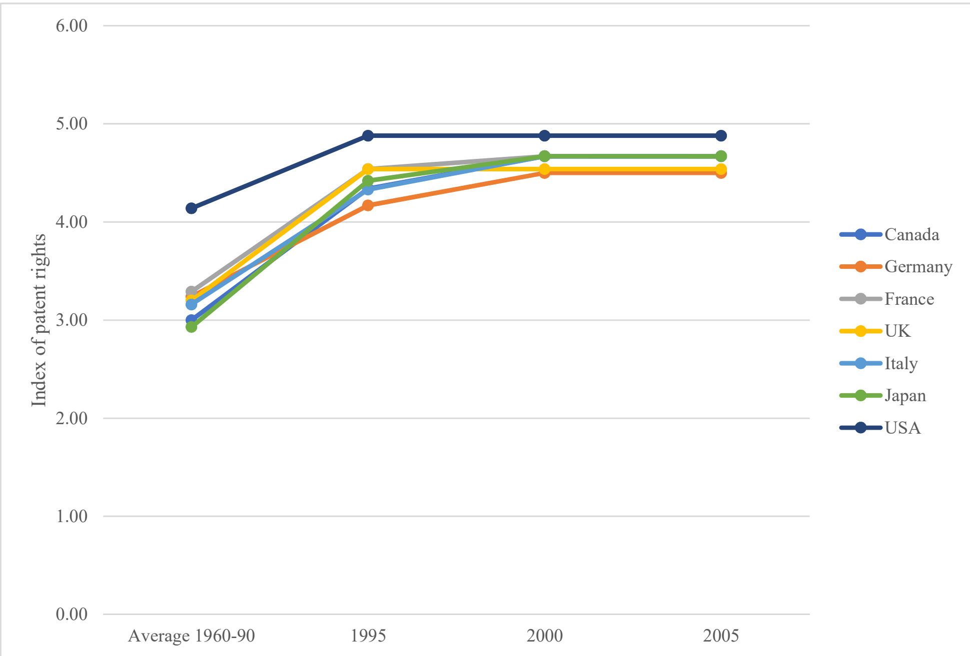

Figure 1: Index of patent rights 1960-2005 in G7 countries

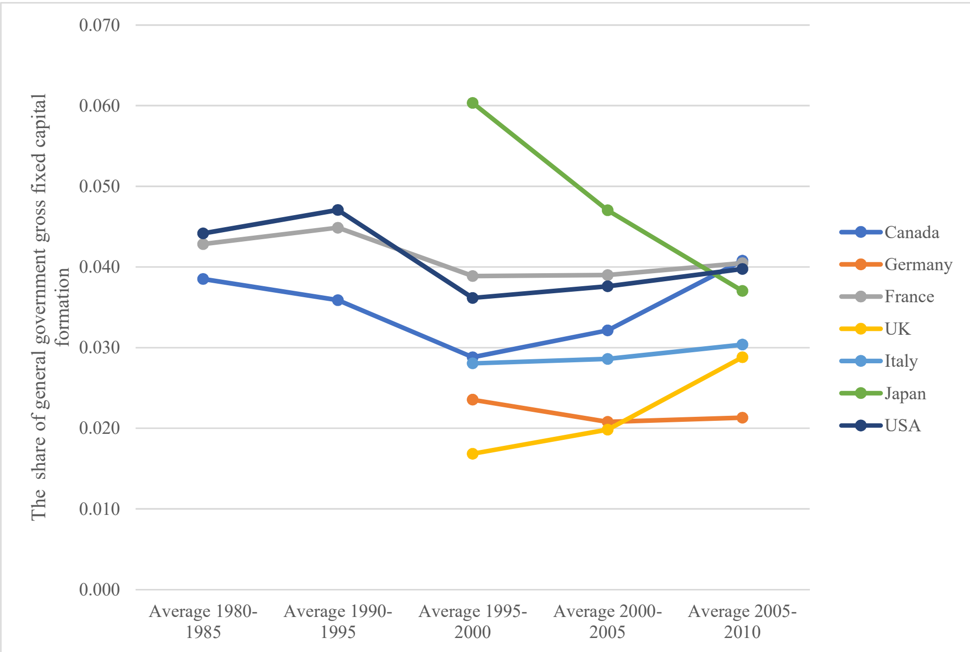

Figure 2: The GDP share of general government gross fixed capital formation 1980-2010 in G7 countries

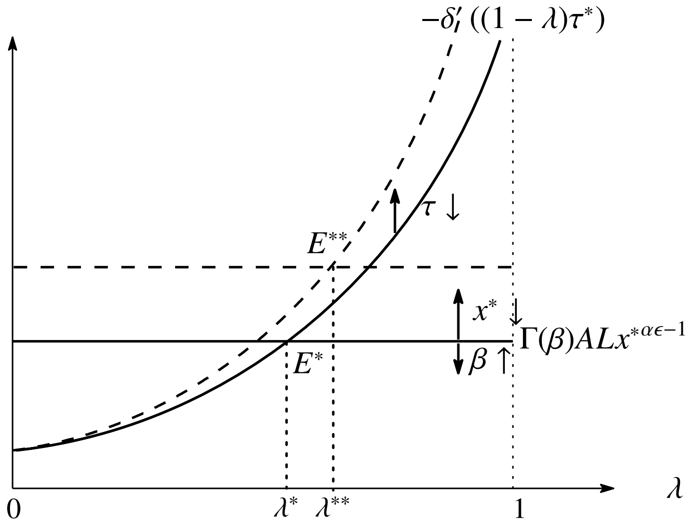

Figure 3: The effect of β on the balanced-growth maximizing expenditure share of new investment λ^*

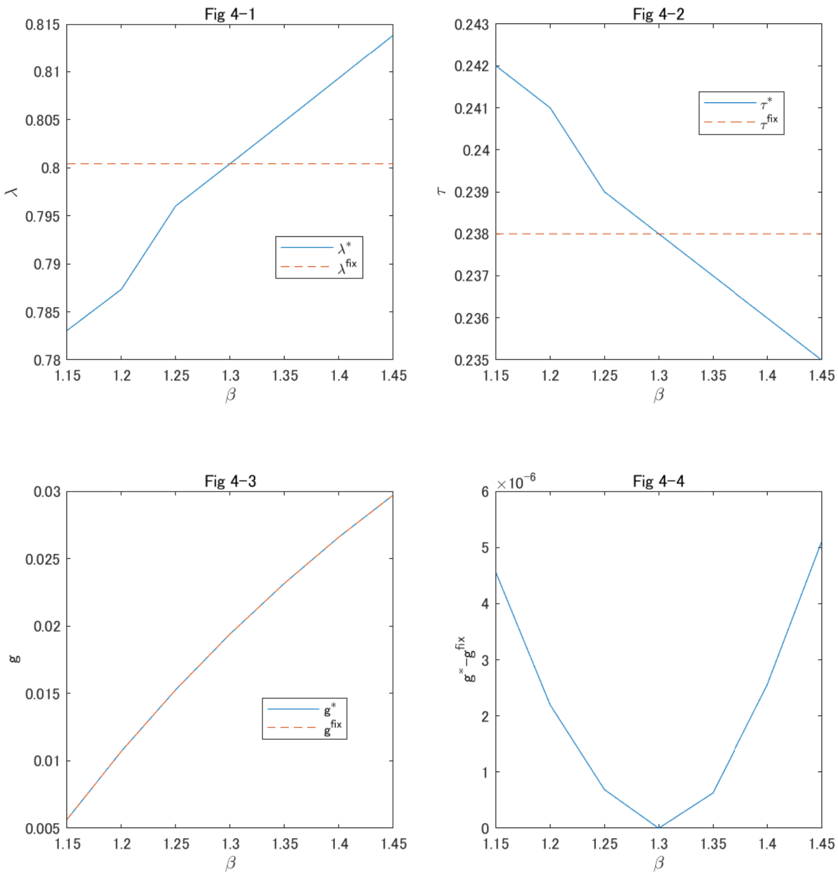

Figure 4: Patent protection and growth maximizing public investment policy