DISCUSSION PAPER SERIES

Discussion paper No. 182

Fertility, Income Growth and Inflation

Masaya Yasuoka
(School of Economics, Kwansei Gakuin University)

July 2018

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

Fertility, Income Growth and Inflation[†]

Masaya Yasuoka[‡]

Abstract

This paper sets an endogenous fertility model with human capital accumulation and monetary policy in a closed economy, with subsequent examination of how fertility, education investment for children, and the inflation rate change. Results of theoretical analysis indicate that the child allowance raises fertility and reduces educational investment. However, the effect of the subsidy for education investment on fertility and educational investment is ambiguous because of the closed economy. Because of the change of fertility and income growth, the inflation rate can be changed by the child care policy. An increase in monetary stock policy raises human capital growth because the physical capital accumulation is facilitated.

JEL Classifications: J11, J14, E31, H22

Keywords: Child care Policy, Education, Fertility, Inflation

[†] I would like to thank seminar participants for helpful comments. Research for this paper was supported financially by JSPS KAKENHI Grant Numbers 17K03746 and 17K03791. Nevertheless, any remaining errors are the author's responsibility.

[‡] Correspondence to: School of Economics, Kwansei Gakuin University, 1-155, Uegahara Ichiban-Cho, Nishinomiya, Hyogo 662-8501, Japan Tel.: +81-798-54-6993, E-mail: yasuoka@kwansei.ac.jp

1. Introduction

This paper sets an endogenous fertility model with human capital accumulation and monetary policy in a closed economy. Because of the closed economy, the child care policy effects on income growth and other indicators differ from those that occur in a small open economy. Because of a change of physical capital stock per capita, this paper presents a derivation showing that the subsidy for education investment can not raise the human capital growth rate (income growth rate) because substitution between quality and quantity of children reduces the quantity cost of children.

Moreover, this paper presents examination of the effect of child care policy on the inflation rate. The child care policy changes fertility and income growth. The inflation rate changes because the demand for money stock changes. An increase in monetary stock policy raises the human capital growth rate because of physical capital accumulation: the cost to have money as a nominal asset increases; also, the household seeks to accumulate more real assets.

Many related papers have described examinations of the effects of child care policies on fertility and education investment for children. Van Groezen, Leers and Meijdam (2003) and Yasuoka and Goto (2011) derive that a child allowance raises fertility in a small open economy. In a closed economy, a child allowance can not always raise fertility, as demonstrated by van Groezen and Meijdam (2008), Fanti and Gori (2009), and Yasuoka and Goto (2015) because of the effect that the household income is reduced by the child allowance. Miyazaki (2013) examines how the pension contribution rate affects fertility.

Subsidies for educational investment is mainly examined in human capital growth models. Glomm and Ravikumar (1992) set public education investment such that the education cost is fully covered by the government expenditure. They examine how the income growth rate and inequality. Zhang (1997) and Yasuoka and Miyake (2014) set the endogenous fertility model with human capital growth and derive the effects of a child allowance and education subsidies on fertility and the human capital growth rate: that is, income growth in the small open economy. De la Croix and Doepke (2003) examine a closed economy model that incorporates quality and quantity of children. Nevertheless, the effect of a child care policy is not examined sufficiently.

This paper presents consideration of how monetary policy affects fertility and the income growth rate. If the inflation rate is changed by the monetary policy, then the physical capital stock changes because the household changes the asset allocation for real assets as physical capital investment and nominal assets as money. Because of a change of physical capital stock, the wage rate and interest rate change and the child care cost to have children change via the child care service cost or opportunity cost. An increase in the money stock policy raises the inflation rate. Therefore, the physical capital stock increases because inflation decreases the value of money stock as an asset. Then, the income growth rate increases because the physical capital stock increases the child care cost to have children and increases educational investment raising the quality of children because of the substitution of quality and quantity of children.

Many related papers describe studies of money and inflation. Mino and Shibata (1995) derive the manner in which monetary policy affects income growth. The optimal monetary policy is derived by De Gregorio (1993) and Bhattacharya, Haslag and Martin (2009). Yasuoka (2018) demonstrates that the optimal monetary policy is determined by the level of pay-as-you-go pension. Yakita (2006) sets the endogenous growth model with monetary assets. An increase in life expectancy affects the inflation rate and the income growth rate. Fanti (2012) and Chang, Chen and Chang (2013) set an endogenous fertility model with monetary stock and derive that an increase in the money stock affects fertility. This paper presents consideration of human capital accumulation as the quality of children and derives that human capital accumulation is facilitated by increased effects of monetary policy.

The inflation rate is determined by a rate of increase of the nominal monetary stock, fertility (population growth), and human capital growth (income growth) in this model. However, the effect of child care policy on the inflation rate is generally ambiguous because, for instance, a child allowance raises fertility and reduces human capital growth, which moves conversely.

The ambiguous results obtained using the theoretical analysis are clarified by numerical examples with realistic parameters. Then, both the child allowance and the education subsidy decrease the inflation rate: a deflationary effect occurs. Therefore, monetary policy makers must consider this effect if adopting an inflation-targeting policy.

The remainder of this paper is presented follows. Section 2 presents the model. Section 3 explains derivation of the equilibrium. Section 4 examines how the child allowance, education subsidy, an increase in money stock policy and pension policy affect fertility, the human capital growth rate (income growth rate), inflation, and the physical capital stock per unit of effective labor. In section 5, numerical examples verify the result obtained in section 4 in the realistic dataset. The final section concludes this paper.

2. Model

As described in this paper, the model includes agents of three types : households, firms, and a government.

2.1 Household

Households exist in three periods: childhood, adulthood, and the old period. In childhood, individuals receive education investment from their parents in childhood. In adulthood, the individuals decide the number of children n_t, education investment for children e_t, the demand for real money stock per capita m_t, consumption in the adulthood c_{1t}, and saving s_t for consumption in the old period c_{2t+1}. Here, t denotes the period. For these analyses, we use a three-period overlapping generations model: In any t period, children, younger people and older people all co-exist. The budget constraint in adulthood is

$$s_t = w_t h_t - c_{1t} - (z_t - q_t)n_t - (1 - x_t)e_t n_t - m_t - T_t. \tag{1}$$

In that equation, z_t denotes the child care cost for a child. With child allowance q_t, the net child care cost is given as $z_t - q_t$. In the equation, x_t represents the subsidy rate for education investment. w_t and h_t respectively denote the wage rate per unit of effective unit of labor and human capital stock. T_t stands for the lump sum tax to provide for child care policies.

In the old period, the budget constraint is given as

$$(1 + r_{t+1})s_t + \frac{m_t}{1 + \pi_{t+1}} + p_{t+1} = c_{2t+1}. \tag{2}$$

As shown there, r_{t+1} and π_{t+1} denote the real interest rate and the inflation rate. In the old period, individuals obtain pension benefit p_{t+1}.

Considering (1) and (2), the lifetime budget constraint can be reduced.

$$\begin{aligned} w_t h_t - T_t + \frac{p_{t+1}}{1 + r_{t+1}} \\ &= \left(1 - \frac{1}{(1 + r_{t+1})(1 + \pi_{t+1})}\right) m_t + c_{1t} + \frac{c_{2t+1}}{1 + r_{t+1}} + (z_t - q_t)n_t \\ &+ (1 - x_t)e_t n_t. \end{aligned} \tag{3}$$

The human capital accumulation of children is formed by the input of education investment and parental human capital stock. It is assumed as

$$h_{t+1} = H e_t^{\varepsilon} h_t^{1-\varepsilon}, 0 < H, 0 < \varepsilon < 1. \tag{4}$$

Parents care for the number of children, human capital of children, money stock, and the consumption. The utility function is assumed as[1]

$$u_t = \alpha ln n_t h_{t+1} + (1 - \alpha) ln m_t + ln c_{1t} + \rho ln c_{2t+1}, 0 < \alpha < 1, 0 < \rho < 1. \tag{5}$$

The optimal allocations to maximize utility (5) subject to the lifetime budget constraint (3) are expressed as presented below:

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[1] This paper assumes the money in the utility model as assumed by Sidrauski (1967), Yakita (2006), Walsh (2010), and others.

$$n_t = \frac{\alpha(1-\varepsilon)}{z_t - q_t}\frac{1}{2+\rho}\left(w_t h_t - T_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{6}$$

$$e_t = \frac{\varepsilon(z_t - q_t)}{(1-\varepsilon)(1-x_t)}, \tag{7}$$

$$c_{1t} = \frac{1}{2+\rho}\left(w_t h_t - T_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{8}$$

$$c_{2t+1} = \frac{\rho(1+r_{t+1})}{2+\rho}\left(w_t h_t - T_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{9}$$

$$m_t = \frac{1-\alpha}{2+\rho}\frac{w_t h_t - T_t + \frac{p_{t+1}}{1+r_{t+1}}}{1 - \frac{1}{(1+r_{t+1})(1+\pi_{t+1})}}, \tag{10}$$

2.2 Firms

Final goods are produced by inputting physical capital stock K_t. The effective labor $L_t = N_t h_t$. N_t denotes the population size of younger people in period t. Production function Y_t is assumed as presented below:

$$Y_t = AK_t^{\theta} L_t^{1-\theta}, 0 < A, 0 < \theta < 1. \tag{11}$$

Assuming a perfectly competitive market, the wage rate and the interest rate are given by marginal productivity as

$$w_t = A(1-\theta)k_t^{\theta}, \tag{12}$$

$$1 + r_t = A\theta k_t^{\theta-1}, \tag{13}$$

where $k_t = \frac{K_t}{L_t h_t}$. For these analyses, we assume that the physical capital stock is fully depreciated in a single period.

2.3 Government

Government child care policies provide a child allowance and education subsidy. In addition to these policies, a pension benefit is provided for older people. We respectively consider a child allowance $q_t = \bar{q} w_t h_t$, education subsidy $x_t = x$, and pension benefit $p_{t+1} = \tau n_t w_{t+1} h_{t+1}$. Also, $\bar{q}$, x and τ are, respectively, constant over time. With a balanced budget, the government budget constraint is given as

$$T_t = \bar{q} w_t h_t + x e_t n_t + \tau w_t h_t. \tag{14}$$

3. Equilibrium

This section presents derivation of the equilibrium. This model includes the assumption of the child care cost as $z_t = \bar{z} w_t h_t$. Also, $\bar{z}$ is constant over time.[2] Given h_t and K_t, the growth rate of human capital $1+g$ is given as[3]

$$1+g_t = \frac{h_{t+1}}{h_t} = H\left(\frac{\varepsilon(\bar{z}-\bar{q})w_t}{(1-\varepsilon)(1-x)}\right)^{\varepsilon}. \quad (15)$$

The inflation rate π_{t+1} is given by the following equation:

$$\frac{m_{t+1}}{m_t} = \frac{1+\mu}{(1+\pi_{t+1})n_t}, \quad (16)$$

where μ represents the rate of increase of the aggregate nominal money stock.[4]

The capital market equilibrium condition is given as $K_{t+1} = N_t s_t$. Then, the following equation can be derived.

$$K_{t+1} = N_t\left(\left(1-\frac{1+\alpha}{2+\rho}-\frac{1-\alpha}{2+\rho}\frac{1}{1-\frac{1}{(1+r_{t+1})(1+\pi_{t+1})}}\right)(w_t h_t - T_t) - \frac{1}{2+\rho}\frac{p_{t+1}}{1+r_{t+1}}\left(1+\alpha+\frac{1-\alpha}{1-\frac{1}{(1+r_{t+1})(1+\pi_{t+1})}}\right)\right), \quad (17)$$

Considering $k_t = \frac{K_t}{N_t h_t}$, the dynamics of k_t is given as

$$n_t(1+g_t)k_{t+1} = \left(1-\frac{1+\alpha}{2+\rho}-\frac{1-\alpha}{2+\rho}\frac{1}{1-\frac{1}{(1+r_{t+1})(1+\pi_{t+1})}}\right)\left(w_t - \frac{T_t}{h_t}\right) - \frac{1}{2+\rho}\frac{p_{t+1}}{1+r_{t+1}}\left(1+\alpha+\frac{1-\alpha}{1-\frac{1}{(1+r_{t+1})(1+\pi_{t+1})}}\right) \quad (18)$$

Considering (6)–(10), (12)–(18), one obtains $c_{t+1}, n_t, e_t, m_t, w_t, r_{t+1}, g_t, \pi_{t+1}, k_{t+1}$ for given k_t.

The balanced growth path can be given as $k_{t+1} = k_t = k$. Then, the growth rate of human capital g_t is constant rate g. Without policy parameters, that is, $\bar{q} = 0$, $x = 0$

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2 This paper assumes that the child care cost is proportional to the wage rate. This is a consistent assumption because the child care service is provided by nursing labor. The wage rate is a cost of providing childcare. Yasuoka and Miyake (2010) consider the labor market of child care services explicitly and derive that child care service costs depend on the wage rate.

3 In this model, the growth rate of human capital coincides with the income growth rate.

4 Defining P_t as the price level and M_t as the aggregate nominal money stock, respectively, we obtain $m_t = \frac{M_t}{N_t P_t}$.

and $\tau = 0$, the growth rate of human capital in the balanced growth path is given as

$$1 + g = H\left(\frac{\varepsilon \bar{z} w}{1-\varepsilon}\right)^{\varepsilon}. \tag{19}$$

where

$$w = A(1-\theta)k^{\theta}. \tag{20}$$

In the balanced growth path, $\frac{m_{t+1}}{m_t} = 1 + g$ holds. Considering (16), the inflation rate in the balanced growth path π is

$$1 + \pi = \frac{1+\mu}{(1+g)n}, \tag{21}$$

where

$$n = \frac{\alpha(1-\varepsilon)}{\bar{z}} \frac{1}{2+\rho}. \tag{22}$$

The interest rate in the balanced growth path is given as shown below.

$$1 + r = A\theta k^{\theta-1}. \tag{23}$$

Substituting (19)–(23) into (18), the capital stock per unit of effective unit of labor can be derived such that the following equation holds:[5]

$$\frac{\alpha H(1-\varepsilon)}{(2+\rho)\bar{z}^{1-\varepsilon}A^{1-\varepsilon}(1-\theta)^{1-\varepsilon}}\left(\frac{\varepsilon}{1-\varepsilon}\right)^{\varepsilon} k^{1-\theta(1-\varepsilon)}$$
$$= 1 - \frac{1+\alpha}{2+\rho} - \frac{1-\alpha}{2+\rho} \frac{1}{1 - \frac{\frac{\alpha H(1-\varepsilon)}{\bar{z}^{1-\varepsilon}(2+\rho)}\left(\frac{\varepsilon A(1-\theta)}{1-\varepsilon}\right)^{\varepsilon} k^{1-\theta(1-\varepsilon)}}{A\theta(1+\mu)}}. \tag{24}$$

Considering (19)–(24), we obtain w, r, g, π, k, n in the balanced growth path.

4. Policy Effect

In this section, this paper presents an examination of how policies such as an increase in money stock, child allowance, education subsidy and pension benefit affect the income growth rate, fertility and inflation.

4.1 Increase in the Money Stock

This subsection presents an examination of the effect of an increase in money stock on the income growth rate, inflation rate, and other factors.

Defining the left-hand side of (24) as L and the right-hand side of (24) as R, respectively, one can obtain the unique steady state as shown in Fig. 1.

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[5] See Appendix for the detail proof of local stability condition.

[Insert Fig. 1 around here.]

R shifts up and the physical capital stock per unit of effective labor increases because the intersection moves to right if the rate of an increase in aggregate nominal money stock μ rises as the monetary policy. Then, with (19) and (20), the human capital growth rate or income growth rate g rises. However, the effect on the inflation rate is ambiguous.

Substituting (19), (20), and (22) into (21), with total differentiation with respect to $\pi, \mu, k,$ one obtains $\frac{d\pi}{d\mu}$ as

$$\frac{d\pi}{d\mu} = \frac{1}{n(1+g)}\left(1 - \frac{\varepsilon\theta(1+\mu)}{k}\right). \tag{25}$$

With $k > \varepsilon\theta(1+\mu)$, one can obtain $\frac{d\pi}{d\mu} > 0$: that is, an increase in money stock policy raises the inflation rate. Consequently, the following proposition can be established.

Proposition 1

If the rate of an increase in aggregate money stock rises, then the growth rate of human capital and the physical capital stock per unit of effective labor increase. The inflation rate increases if $k > \varepsilon\theta(1+\mu)$.

With an increase in money stock, the education investment for children increases because an increase in the wage rate caused by an increase in capital stock per unit of effective labor raises the cost to increase the number of children. As a result, because of substitution between the quality of children and the quantity of children, education investment increases and the human capital growth rate or income growth rate increases. In this policy, fertility does not change because both the child care cost and wage income increase. These increases are cancelled out.

4.2 Child Allowance

This subsection presents an examination of the child allowance effect. Then, the government budget constraint (14) changes to the following equation as

$$T_t = \bar{q} w_t h_t. \tag{26}$$

Considering (6) and (26), we obtain the following fertility. An increase in child allowance level $\bar{q}$ raises fertility.

$$n=\frac{\frac{\alpha(1-\varepsilon)}{2+\rho}}{\bar{z}-\left(1-\frac{\alpha(1-\varepsilon)}{2+\rho}\right)\bar{q}}. \tag{27}$$

With total differentiation of (15) with respect to $g, \bar{q}, k$, one obtains the following:

$$\frac{dg}{d\bar{q}}=-\frac{\varepsilon(1+g)}{\bar{z}}+\frac{\varepsilon\theta(1+g)}{k}\frac{dk}{d\bar{q}}. \tag{28}$$

The physical capital stock is given such that the following equation holds.

$$n(1+g)k=\left(1-\frac{1+\alpha}{2+\rho}-\frac{1-\alpha}{2+\rho}\frac{1}{1-\frac{1}{(1+r)(1+\pi)}}\right)(1-\bar{q}n)w. \tag{29}$$

Regarding the variables therein, $1+g, w, 1+\pi, 1+r, n$ are given respectively as (15), (20), (21), (23), and (27). From total differentiation of (15), (20), (21), (23), (27), and (29) with respect to $k, 1+g, w, 1+\pi, 1+r, n, \bar{q}$, one can obtain $\frac{dk}{d\bar{q}}$ as[6]

$$\frac{dk}{d\bar{q}} = -\frac{n\left(\left(1-\frac{\alpha(1-\varepsilon)}{2+\rho}-\varepsilon\right)\frac{1+g}{\bar{z}}\left(\frac{k}{w}+\frac{(1-\alpha)\left(1-\frac{n(1+g)}{(1+r)(1+\mu)}\right)^{-2}}{(2+\rho)(1+\mu)(1+r)}\right)+\left(1-\frac{1+\alpha}{(2+\rho)}-\frac{1-\alpha}{2+\rho}\frac{1}{1-\frac{1}{(1+r)(1+\pi)}}\right)\right)}{X_1} \tag{30}$$

where

$$X_1=n(1+g)\big(1-\theta(1-\varepsilon)\big)\left(\frac{1}{w}+\frac{(1-\alpha)}{(2+\rho)(1+\mu)A\theta k^{\theta}}\left(1-\frac{n(1+g)}{(1+r)(1+\mu)}\right)^{-2}\right)>0\ . \tag{31}$$

The sign of $\frac{dk}{d\bar{q}}$ is always negative $\frac{dk}{d\bar{q}}<0$. The child allowance reduces the capital stock per unit of effective labor. Because of $\frac{dk}{d\bar{q}}$, one can obtain $\frac{dg}{d\bar{q}}<0$. The human capital growth rate decreases. Actually, the child allowance reduces educational investment directly because of a decrease in child care cost to increase in the number of children. This result is the same as that reported by Zhang (1997). Even extending the model for the closed economy, we obtain the same result that the child allowance reduces the human capital growth rate.

From (21), the effects on the inflation rate are

[6] See the Appendix for total differentiation.

$$\frac{d\pi}{d\bar{q}} = (1+\pi)\left(\frac{-\left(1-\frac{\alpha(1-\varepsilon)}{2+\rho}\right)+\varepsilon}{\bar{z}} - \frac{\varepsilon\theta}{k}\frac{dk}{d\bar{q}}\right). \tag{32}$$

The sign of $\frac{d\pi}{d\bar{q}}$ is ambiguous. The child allowance increases fertility, which reduces the inflation rate. The term of $\frac{\varepsilon}{\bar{z}} - \frac{\varepsilon\theta}{k}\frac{dk}{d\bar{q}}$ shows the positive effect on the inflation rate via the change of the human capital growth rate. Consequently, the following proposition can be established.

Proposition 2

The child allowance can raise fertility and reduce the capital stock per unit of effective labor and the human capital growth rate. The inflation rate effect is ambiguous.

Child allowance effects on fertility are reported in many related papers such as those by van Groezen, Leers and Meijdam (2008), and Yasuoka and Goto (2011). Considering the closed economy, the negative effect of the child allowance on the human capital growth rate is magnified because the wage rate, regarded as the child care cost to have children decreases. As a result of $\frac{dk}{d\bar{q}} < 0$, substitution of the quantity and quality of children occurs.

4.3 Education Subsidy

This subsection presents derivation of how the education subsidy affects the capital stock per unit of effective labor, the human capital growth rate, fertility, and the inflation rate. The government budget constraint (14) is given by the following:

$$T_t = xe_t n_t = \frac{x\varepsilon\bar{z}wh_t}{(1-\varepsilon)(1-x)}n. \tag{33}$$

Then, fertility is given as

$$n = \frac{\alpha(1-\varepsilon)}{\bar{z}(2+\rho)}\left(1-\frac{x\varepsilon\bar{z}}{(1-\varepsilon)(1-x)}n\right) \rightarrow n = \frac{\alpha(1-\varepsilon)}{\bar{z}(2+\rho)\left(1+\frac{\alpha\varepsilon x}{(2+\rho)(1-x)}\right)}. \tag{34}$$

Therefore, we obtain $\frac{dn}{dx} < 0$, i.e., the education subsidy decreases fertility. Zhang (1997) shows that fertility decreases because of the substitution between the quality and quantity of children. However, the analyses presented herein yield the same result because of the tax burden imposed to support education policy.

Substituting (33) and $p_{t+1}=0$ into (18) and performing total differentiation with respect to x,k at the approximation of $x=0$, one obtains $\frac{dk}{dx}$. The numerator of $\frac{dk}{dx}$ is[7]

$$\frac{dk}{dx}=\frac{X_2}{X_1}<0, \tag{35}$$

where

$$X_2=-\left(1-\frac{\alpha}{2+\rho}\right)\left(\frac{1-\alpha}{(2+\rho)(1+r)(1+\mu)}\left(1-\frac{1}{(1+r)(1+\pi)}\right)^{-2}+\frac{k}{w}\right)\varepsilon n(1+g)$$
$$-\frac{\alpha\varepsilon}{2+\rho}\left(1-\frac{1+\alpha}{(2+\rho)}-\frac{1-\alpha}{2+\rho}\frac{1}{1-\frac{1}{(1+r)(1+\pi)}}\right)<0. \tag{36}$$

Then, the sign of (36) is negative. The subsidy for education investment reduces the physical capital stock per unit of effective labor. Then, human capital accumulation does not always increase because

$$\frac{dg}{dx}=\varepsilon(1+g)\left(\frac{\theta}{k}\frac{dk}{dx}+1\right). \tag{37}$$

Because of $\frac{dk}{dx}<0$, the sign of (37) is ambiguous even if the subsidy for education investment has a direct positive effect on the human capital growth rate. Moreover, the effect on the inflation rate is ambiguous because

$$\frac{d\pi}{dx}=\varepsilon(1+\pi)\left(\frac{\alpha}{2+\rho}-\frac{\theta}{k}\frac{dk}{dx}-1\right). \tag{38}$$

The decrease in fertility raises the inflation rate, as shown by the first term. The second and third terms represent the effect of income growth on the inflation rate. An increase in the income growth rate reduces the inflation rate. However, because $\frac{dk}{dx}<0$, the negative effect of income growth rate on the inflation rate is weakened. Then, the following proposition can be established.

Proposition 3

The subsidy for education decreases fertility. The physical capital stock per unit of effective labor decreases. Therefore, because $\frac{dk}{dx}<0$, the subsidy for education investment can not always raise the human capital growth rate. The effect on the

[7] See the Appendix for total differentiation.

inflation rate is ambiguous.

As shown by Zhang (1997), Yasuoka and Miyake (2014), and others, the subsidy for education investment can raise the human capital growth rate in a small open economy for which physical capital accumulation is not considered. However, considering physical capital accumulation as shown by my paper, the human capital growth rate does not always increase because the cost to increase the number of children changes. This cost affects education investment for children.

4.4 Pension Policy

The final subsection presents examination of the effects of a pension on fertility, the human capital growth rate, the physical capital stock per capita, and the inflation rate. From (14), the government budget constraint is given as

$$T_t = \tau w_t h_t. \quad (39)$$

The pension benefit is given by $p_t = \tau n w_t h_t$. Then, fertility (6) is

$$n = \frac{\alpha(1-\varepsilon)}{\bar{z}(2+\rho)}\left(1-\tau+\frac{\tau n(1+g)}{1+r}\right). \quad (40)$$

Also, $\frac{dn}{d\tau}$ can be derived as

$$\frac{dn}{d\tau} = \frac{\alpha(1-\varepsilon)}{\bar{z}(2+\rho)}\left(\frac{n(1+g)}{1+r}-1\right). \quad (41)$$

With $\frac{n(1+g)}{1+r}-1>0$, an increase in τ raises the household lifetime income by virtue of an increase in pension benefit: fertility increases.[8]

Substituting (26) and $p_{t+1}=0$ into (18), with total differentiation with respect to τ, k at the approximation of $\tau = 0$, $\frac{dk}{d\tau}$ can be obtained as[9]

$$\frac{dk}{d\tau} = -\frac{X_3}{X_1}. \quad (42)$$

where

[8] This result can be obtained generally. The condition of $\frac{n(1+g)}{1+r}-1>0$ dictates that the pension benefit is greater than the interest rate for saving and dictates the lifetime income increases. By virtue of the increase in lifetime income, fertility increases.

[9] See the Appendix for total differentiation.

$$X_3 = 1 - \frac{1+\alpha}{(2+\rho)} - \frac{1-\alpha}{2+\rho}\frac{1}{1-\frac{1}{(1+r)(1+\pi)}} + \frac{n(1+g)\left(1+\alpha+\frac{1-\alpha}{1-\frac{1}{(1+r)(1+\pi)}}\right)}{(1+r)(2+\rho)} \tag{43}$$

$$+n(1+g)\left(\frac{n(1+g)}{1+r}-1\right)\left(\frac{k}{w}+\frac{1-\alpha}{(1+r)(1+\mu)(2+\rho)}\right)\left(1-\frac{1}{(1+r)(1+\pi)}\right)^{-2}.$$

Then, the sign of (42) is ambiguous. One can obtain $\frac{dk}{d\tau} < 0$ because fertility increases and the physical capital stock per unit of effective labor decreases if $\frac{n(1+g)}{1+r} - 1 > 0$. Then, human capital accumulation does not always increase because of

$$\frac{dg}{d\tau} = \frac{\varepsilon\theta}{k}\frac{dk}{d\tau}. \tag{44}$$

As long as $\frac{dk}{d\tau} < 0$, the sign of (44) is negative. A decrease in the physical capital stock per unit of effective labor decreases the child care cost to increase the number of children. The household reduces educational investment because of substitution between quality and quantity of children. Moreover, the effect on the inflation rate is ambiguous because

$$\frac{d\pi}{d\tau} = -(1+\pi)\left(\frac{n(1+g)}{1+r} - 1 + \frac{\varepsilon\theta}{k}\frac{dk}{d\tau}\right). \tag{45}$$

An increase in fertility raises the inflation rate, as shown by the term of $\frac{n(1+g)}{1+r} - 1$. The term of $\frac{\varepsilon\theta}{k}\frac{dk}{d\tau}$ is the effect of income growth on the inflation rate. Then, the following proposition can be established.

Proposition 4

In the case of $\frac{n(1+g)}{1+r} - 1 > 0$, an increase in contribution rate τ decreases the physical capital stock per unit of effective labor. Fertility increases and the human capital growth rate decreases. The effect on the inflation rate is ambiguous.

One might state from intuition that the condition to increase fertility and the income growth rate depends on $\frac{n(1+g)}{1+r} - 1 > 0$. However, because fertility and the income growth rate change conversely to each other, the effect on the inflation rate is ambiguous. Fanti and Gori (2010), Miyazaki (2013), and others examine how the contribution rate of a pay-as-you-go pension affects the lifetime income. Given the parameter conditions, an increase in the contribution rate raises the pension benefit and lifetime income. An increase in fertility occurs because an increase in lifetime income deriving from an increase in the contribution rate reduces the physical capital stock per unit of effective labor. A decrease in physical capital stock per unit of effective labor reduces the child care cost to have children. The educational investment decreases because of the high cost of raising the quality of children.

My paper presents consideration of a closed economy in which physical capital accumulation is considered. Being different from the small open economy, the effects of the policies on the endogenous variables are complicated. Therefore, we can not consider the intuitive policy effects. In the next section, we examine policy effects using numerical examples.

5. Numerical Example

This section presents numerical examples with the model based on realistic parameters. The model economy is given by the following equations.

・Capital market (18)

$$n(1+g)k = \left(1-\frac{1+\alpha}{2+\rho}-\frac{1-\alpha}{2+\rho}\frac{1}{1-\frac{1}{(1+r)(1+\pi)}}\right)\left(w-\frac{T}{h_t}\right) - \frac{\tau n w(1+g)}{(2+\rho)(1+r)}\left(1+\alpha+\frac{1-\alpha}{1-\frac{1}{(1+r)(1+\pi)}}\right). \quad (46)$$

・Fertility rate (6)

$$n = \frac{\alpha(1-\varepsilon)}{\bar{z}-\bar{q}}\frac{1}{2+\rho}\left(1-\frac{T}{wh_t}+\frac{\tau n(1+g)}{1+r}\right). \quad (47)$$

・Government budget constraint (14)

$$\frac{T}{h_t} = \left(\bar{q}+x\frac{\varepsilon(\bar{z}-\bar{q})}{(1-\varepsilon)(1-x)}n+\tau\right)w. \quad (48)$$

・Income growth rate (15)

$$1+g = \frac{h_{t+1}}{h_t} = H\left(\frac{\varepsilon(\bar{z}-\bar{q})w}{(1-\varepsilon)(1-x)}\right)^{\varepsilon}. \quad (49)$$

・Wage rate (20)

$$w = A(1-\theta)k^{\theta}. \quad (50)$$

・Inflation rate (21)

$$1+\pi=\frac{1+\mu}{(1+g)n}, \tag{51}$$

・Interest rate (23)

$$1+r=A\theta k^{\theta-1}. \tag{52}$$

We set the parameter as the following table.

[Insert Table 1 around here.]

In OECD countries, no population growth exists: $n=1$. The income growth rate is about 2.0% per year. Considering a period in the overlapping generations model as thirty years: $1+g=1.81$. Furthermore, the interest rate is about 1.0% per year in OECD countries, that is, $1+r=1.34$.[10] The inflation rate is about 2.0% per year, that is, $1+\pi=1.81$.[11] Therefore, the remaining parameters that are consistent with these data are set as presented in Table 1.

[Insert Fig. 2-5 around here.]

Figure 2 portrays the monetary policy effects. We consider the policy parameter μ range as $[2.2761,\ 1.1\times 2.2761]$. As shown by Fig. 2, an increase in the money stock raises the inflation rate. However, other variables do not change because the preference for money is very small in this realistic economy model.

Figure 3 presents effects of the child allowance. We consider the policy parameter $\bar{q}$ range as $[0,\ 0.1\bar{z}]$. As shown by Fig. 3, the child allowance raises fertility. However, the human capital growth rate decreases. The inflation rate decreases thanks to an increase in fertility.

Figure 4 portrays the effects of an education subsidy. We consider the policy parameter x range as [0, 0.1]. As shown by Fig. 4, an education subsidy raises the human capital growth rate and reduces fertility. Because of the strong effect on the human capital growth rate, the inflation rate decreases.

Figure 5 shows the effects of pension benefit increase. We consider the policy parameter range τ as [0, 0.1]. As shown by Fig. 5, the contribution rate raises fertility and reduces the human capital growth rate. The inflation rate increases.

As shown in Fig. 2 – Fig. 5, the results obtained by numerical examples are

[10] Data: OECD Statistics.

[11] The 2% target inflation policy is adopted by some OECD countries such as Japan and others.

consistent with the results obtained using the theoretical analysis.

6. Conclusions

This paper sets an endogenous fertility model with human capital accumulation and monetary policy in the closed economy. Because of the closed economy, the effects of the child care policy on the income growth and others differ from the case of small open economy. Because of a change of physical capital stock per unit of effective labor, this paper presents derivation by which the subsidy for education investment can not raise the human capital growth rate (income growth rate) because the substitution between quality and quantity of children reduces the cost of quantity for children.

Moreover, this paper presents an examination of the effect of child care policy on the inflation rate. The child care policy changes fertility and income growth. Then, the inflation rate changes because the demand for money stock changes overtime. Therefore, if the government considers an inflation-targeting policy, then the effect of child care policies on the inflation rate via the effect on fertility and income growth rate should be considered.

An increase in the nominal monetary stock policy raises the income growth rate because the investment for the physical capital stock increases as a consequence of a decrease in the cost of having a monetary stock. However, in the numerical examples, the preference for the monetary stock is extremely low. Consequently, the inflation effect on real assets is small. The change of the income growth rate is also very small. Therefore, child care policies should be adopted if government policy is designed to increase fertility or income growth to mitigate the effects of an aging society with fewer children.

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Appendix

Dynamics of $\boldsymbol{k}$

Total differentiation of (18) with respect to k_t and k_{t+1} at the balanced growth path without policy is given as

$$\frac{dk_{t+1}}{dk_t}=\frac{n(1+g)\theta(1-\varepsilon)-\dfrac{(1-\alpha)n(1+g)(1-\theta)}{(2+\rho)(1+\mu)A\theta k^{\theta}}\left(1-\dfrac{1}{(1+r)(1+\pi)}\right)^{-2}}{n(1+g)+\dfrac{(1-\alpha)n(1+g)(1-\theta)}{(2+\rho)(1+\mu)A\theta k^{\theta}}\left(1-\dfrac{1}{(1+r)(1+\pi)}\right)^{-2}}. \qquad \text{(A.1)}$$

With $-1<\frac{dk_{t+1}}{dk_t}<1$, the balanced growth path is locally stable.

Derivation of $\frac{dk}{d\bar{q}}$

From (27), one can obtain

$$dn=\frac{\left(1-\dfrac{\alpha(1-\varepsilon)}{2+\rho}\right)n}{\bar{z}}d\bar{q}. \qquad \text{(A.2)}$$

Considering (20), (21), (23), (28), and (A.2), the left-hand-side of (29) is changed by total differentiation as

$$\frac{(1+g)nk}{w\bar{z}}\left(1-\frac{\alpha(1-\varepsilon)}{2+\rho}-\varepsilon\right)d\bar{q}+\frac{n(1+g)\big(1-\theta(1-\varepsilon)\big)}{w}dk. \qquad \text{(A.3)}$$

Total differentiation of the right-hand-side of (29) is reduced to the following:

$$\begin{aligned}&-\frac{(1-\alpha)\alpha(1-\varepsilon)}{(2+\rho)^2(1+\mu)A\theta}\left(1-\frac{n(1+g)}{(1+r)(1+\pi)}\right)^{-2}\left(\frac{\big(1-\theta(1-\varepsilon)\big)(1+g)k^{-\theta}}{\bar{z}}dk\right.\\&\qquad\left.+\frac{(1+g)k^{1-\theta}}{\bar{z}^2}\left(1-\frac{\alpha(1-\varepsilon)}{2+\rho}-\varepsilon\right)d\bar{q}\right)\\&\qquad+n\left(1-\frac{1+\alpha}{(2+\rho)}-\frac{1-\alpha}{2+\rho}\frac{1}{1-\dfrac{1}{(1+r)(1+\pi)}}\right)d\bar{q}.\end{aligned} \qquad \text{(A.4)}$$

From (A.3) and (A.4), one can obtain $\frac{dk}{d\bar{q}}$.

Derivation of $\frac{dk}{dx}$

From (34), one can obtain the following

$$dn=-\frac{\alpha\varepsilon n}{(2+\rho)}d\bar{q}=-\frac{\alpha^2\varepsilon(1-\varepsilon)}{\bar{z}(2+\rho)^2}d\bar{q}. \qquad \text{(A.5)}$$

Considering (15), (18), (20), (21), (23), (33), and (A.5), the left-hand-side of (18) is changed by the total differentiation at the approximation of $T_t = 0$ with respect to x, k as

$$\frac{\varepsilon(1+g)nk}{w}\left(1-\frac{\alpha}{2+\rho}\right)dx+\frac{n(1+g)\big(1-\theta(1-\varepsilon)\big)}{w}dk. \tag{A.6}$$

Total differentiation of the right-hand-side of (18) is reduced to

$$-\frac{(1-\alpha)\alpha(1-\varepsilon)n(1+g)}{(2+\rho)(1+\mu)}\left(1-\frac{n(1+g)}{(1+r)(1+\pi)}\right)^{-2}\left(\frac{1-\theta(1-\varepsilon)}{A\theta k^{\theta}}dk\right.$$
$$\left.+\frac{\varepsilon}{1+r}\left(1-\frac{\alpha}{2+\rho}\right)dx\right) \tag{A.7}$$
$$-\frac{\alpha\varepsilon}{2+\rho}\left(1-\frac{\alpha}{(2+\rho)}-\frac{1-\alpha}{2+\rho}\frac{1}{1-\frac{1}{(1+r)(1+\pi)}}\right)dx.$$

From (A.6) and (A.7), one can obtain $\frac{dk}{dx}$.

Derivation of $\frac{dk}{d\tau}$

Considering (18), (19), (20), (21), (23), (39), and (41), the left-hand-side of (18) is changed by total differentiation at the approximation of $T_t = 0$ with respect to τ, k as follows:

$$\frac{n(1+g)\big(1-\theta(1-\varepsilon)\big)}{w}dk+\frac{n(1+g)k\left(\frac{n(1+g)}{1+r}-1\right)}{w}d\tau. \tag{A.8}$$

Total differentiation of the right-hand-side of (18) is reduced to

$$-\left(1-\frac{\alpha}{(2+\rho)}-\frac{1-\alpha}{2+\rho}\frac{1}{1-\frac{1}{(1+r)(1+\pi)}}\right)d\tau-\frac{n(1+g)\left(1+\alpha+\frac{1-\alpha}{1-\frac{1}{(1+r)(1+\pi)}}\right)}{(1+r)(2+\rho)}d\tau$$
$$-\frac{\alpha(1-\varepsilon)(1-\alpha)\big(1-\theta(1-\varepsilon)\big)(1+g)}{A\theta(1+\mu)(2+\rho)^2k^{\theta}}\left(1-\frac{\alpha(1-\varepsilon)(1+g)}{(1+r)(1+\mu)(2+\rho)\bar{z}}\right)dk \tag{A.9}$$
$$-\frac{\alpha(1-\varepsilon)(1-\alpha)(1+g)}{(1+r)(1+\mu)(2+\rho)^2\bar{z}}\left(1-\frac{1}{1-\frac{1}{(1+r)(1+\pi)}}\right)^{-2}\left(\frac{n(1+g)}{1+r}-1\right)d\tau.$$

From (A.8) and (A.9), one can obtain $\frac{dk}{d\tau}$.

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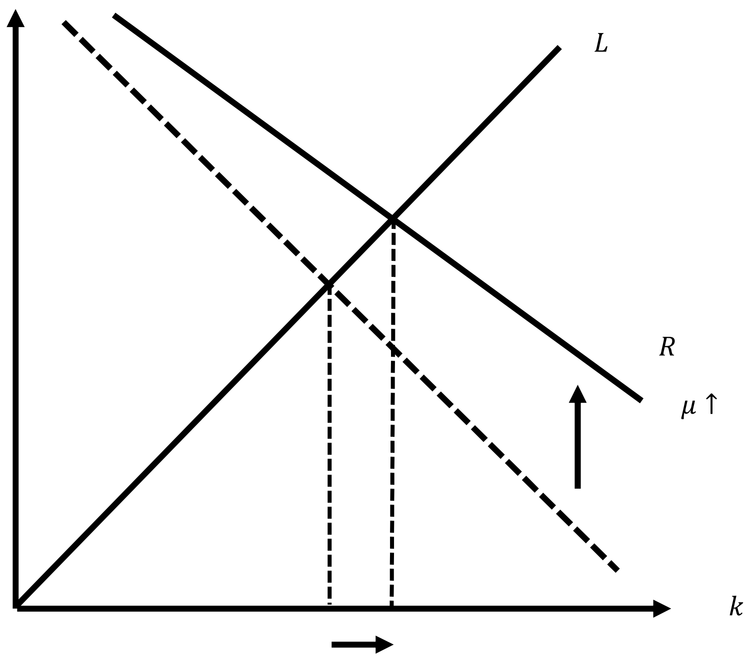

Fig. 1: Effect on k of an increase in μ.

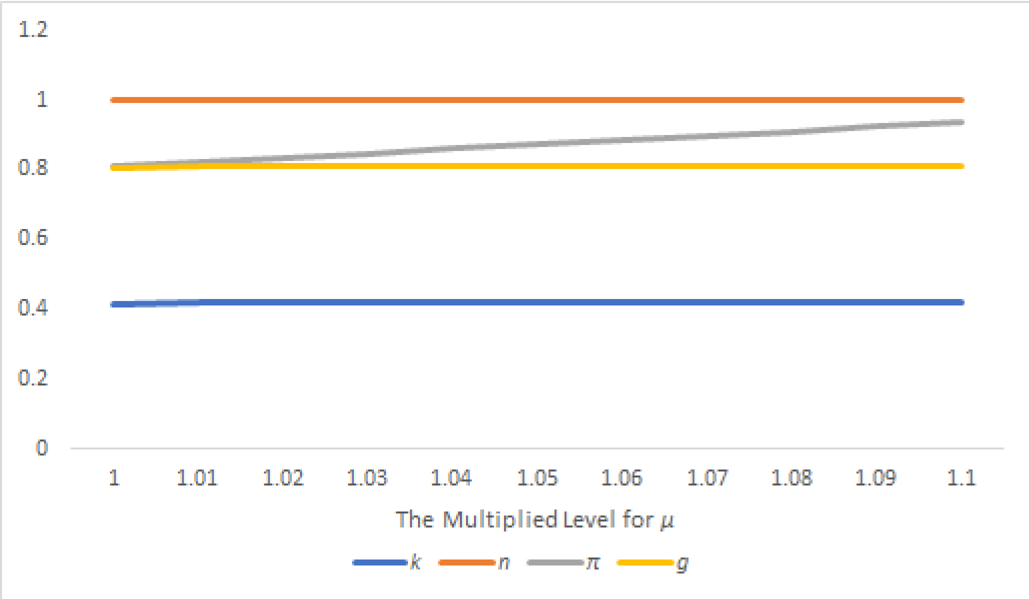

Fig. 2: Monetary policy effect.

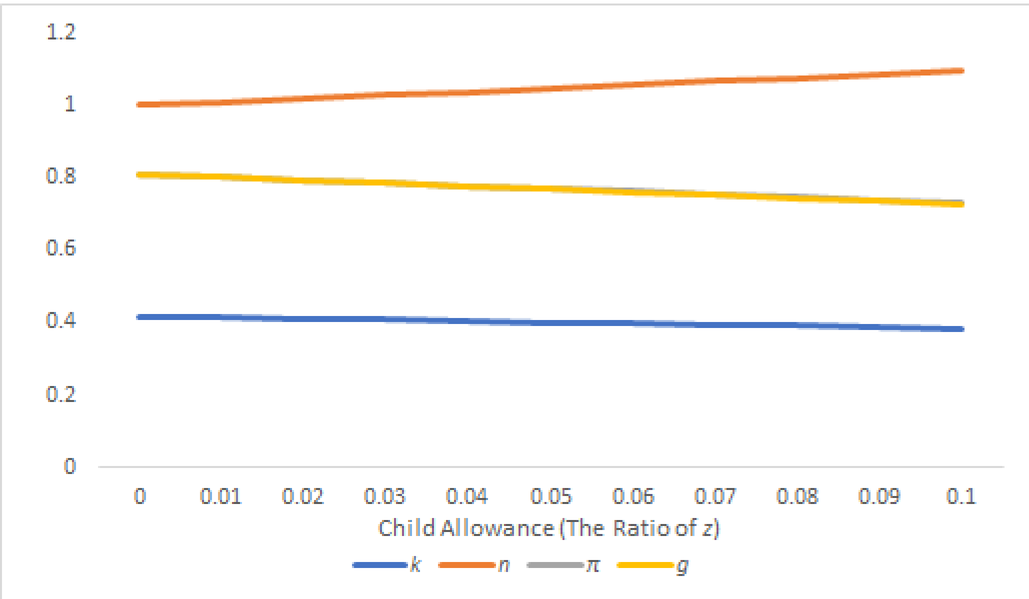

Fig. 3: Child allowance effect.

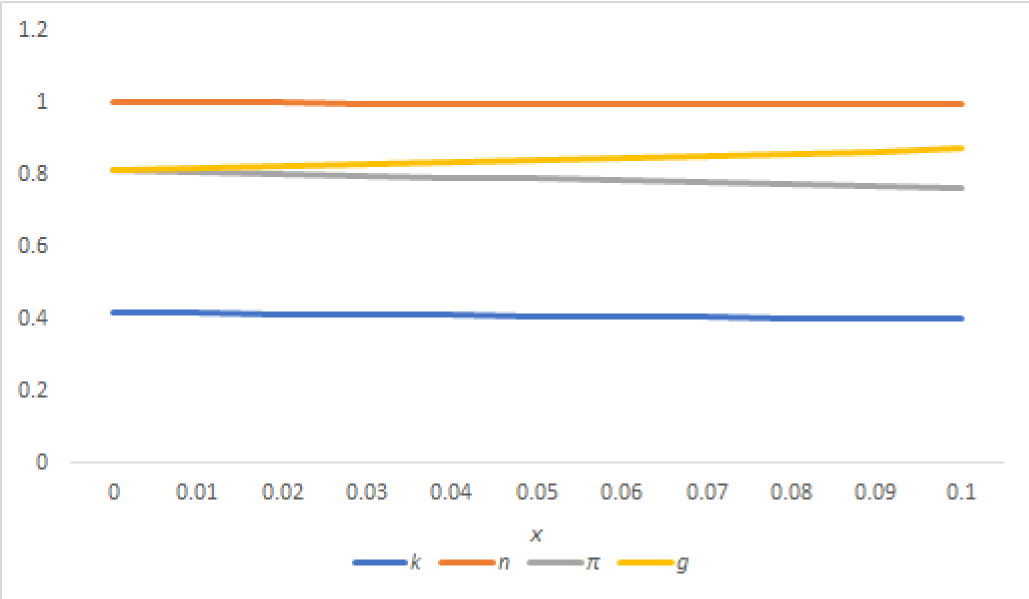

Fig. 4: Education subsidy effect.

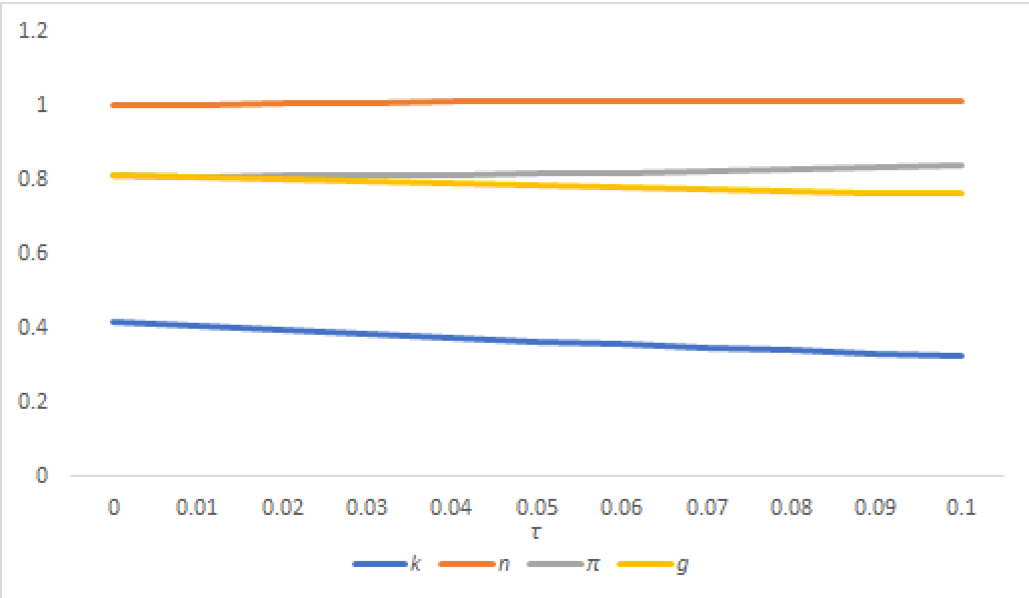

Fig. 5: Pension effect.

Table 1: Parameter settings

α	0.87828
ε	0.35928
ρ	2.95215
θ	0.3
A	2.42461
H	4.42175
$\bar{z}$	0.11363
μ	2.2761