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Higher Education Subsidy Policy and R&D-based Growth

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Abstract

Employing a two-period overlapping generations model of R&D-based growth with both product development and process innovation, this paper examines how a subsidy policy for encouraging more individuals to receive higher education affects the per capita GDP growth rate of the economy. We show that when the market structure adjusts partially in the short run, the effect of an education subsidy on economic growth is ambiguous and depends on the values of the parameters. However, when the market structure adjusts fully in the long run, the education subsidy expands the number of firms but reduces economic growth. These unfavorable predictions for the education subsidy on economic growth are partly consistent with empirical findings that mass higher education does not necessarily lead to higher economic growth. A higher education subsidy policy is perhaps inappropriate for the purpose of stimulating long-run economic growth.

Keywords: Higher Education, Occupational Choice, R&D, Product Development, Process Innovation

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1 Introduction

It is believed by many policy makers that a large, highly educated workforce is key to national prosperity, and the expansion of the higher education sector is an important driver of economic growth. For example, the policy documents produced in support of the UK 2011 education financing reforms stated that "Higher education is important to growth through equipping individuals with skills that enhance their productivity in the workplace, promoting the economy's knowledge base and deriving innovations"(BIS, 2011; p.21). In fact, most developed countries have been providing a set of subsidy policies aimed at broadening the access to higher education, especially for the poor, and encouraging more individuals to receive higher education.[1] These policies include the public provision of low-cost tertiary institutions, means tested grants, loan and work study programs, tax credits and so on. Figure 1 shows the share of public and private funding of tertiary educational institutions in OECD countries (OECD, 2016; Table B3.1b, p218). On average, across OECD countries, nearly 70% of all funds for tertiary educational institutions comes directly from public sources, while the share of public and private funding varies widely across countries.

However, there was strikingly little evidence to support the causal effects of the quantity measures of higher education (e.g., tertiary enrollment rate, average years of tertiary schooling, and the share of workers that complete tertiary education) on economic growth. In their survey on education and economic growth, Krueger and Lindahl (2001) find that "education [is] statistically significantly and positively associated with subsequent growth only for countries with the lowest level of education." Hanushek (2016) shows that once the quality measure of pretertiary education as measured by international mathematics and science tests at earlier ages is taken into account, school attainment *per se* is unrelated to economic growth, implying that adding years of university provide no greater impact than added years of earlier schooling.[2] Moreover, a recent survey study by Holmes (2013) concludes that "neither the increase nor the initial level of higher education is found to have a statistically significant relationship with growth rates both in the OECD and world wide." These empirical findings indicate that a mass higher education does not necessarily lead to higher economic growth.

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[1]State intervention for the market of higher education is justified because higher education creates external benefits in terms of tax dividends, economic growth, social cohesion and parenting. State intervention for the finance of higher education is justified for incomplete capital markets. However, since students receive significant private benefits from their degrees, the costs of higher education should be shared, that is, there should be taxpayer subsidies but also tuition fees. See, chapter 12 of Barr (2012) for more details of the optimal design of the higher education system.

[2]Hanushek (2016) also notes that part of this lack of impact of attainment of higher education is probably because there are no good measures of university quality; thus, very different outcomes are treated the same.

Motivated by these gaps between what is asserted in the political debates and the available evidence, this paper examines theoretically how a subsidy policy for encouraging more individuals to receive higher education affects the per capita GDP growth rate of the economy. Because technological progress via R&D innovation has been identified as the primary driving force of modern economic growth (e.g., Romer 1990), we are particularly interested in the effect of a higher education subsidy policy on R&D-based innovations. This paper develops a two-period overlapping generations (OLG) model of R&D-based growth where skilled labor inputs matter for both product development and process innovation, and skilled labor supply is endogenously determined according to the individuals' choice of higher education. In line with the literature of the second-generation R&D-based growth model, pioneered by Peretto (1998), Howitt (1999), and Segestrom (2000), the model features two dimensions of technological progress. In the vertical dimension, incumbent production firms invest in process innovation with the objective of lowering production costs. In the horizontal dimension, the product development sector creates new product designs for firms entering the production sector. In this Schumpeterian growth model with endogenous market structure measured by the equilibrium number of firms, we examine how a subsidy policy for encouraging more individuals to receive higher education affects the per capita GDP growth rate of the economy. Then, we show that when the market structure adjusts partially in the short run, the effect of the education subsidy on economic growth is ambiguous and depends on the values of the parameters. However, when the market structure adjusts fully in the long run, the education subsidy expands the number of firms but reduces economic growth. A higher education subsidy policy is perhaps inappropriate for the purpose of stimulating long-run economic growth.

The intuition behind this result is explained as follows. The higher rate of education subsidy increases the supply of skilled labor, lowers the employment costs of researchers, and increases incentives for both product development and process innovation, which positively affect the per capita GDP growth rate in the short run. In the long run, however, the product development encourages the entry of new production firms, which in turn reduces the market size of each production firm. Given that the market size of a production firm determines its incentives for process innovation, the higher rate of an education subsidy decreases long-run economic growth.[3] These unfavorable predictions of an education subsidy on economic growth are partly consistent with the empirical findings that mass higher education does not necessarily lead to higher economic growth.

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[3]Laincz and Peretto (2006) provide empirical evidence for a positive relationship between the average firm size and economic growth. See also Ha and Howitt (2007), Madsen (2008), Madsen, Ang, and Banerjee (2010), and Ang and Madsen (2011) for other empirical studies that support the second-generation R&D-based growth model.

This paper is related to several branches of the literature. First, this paper is related to the literature on endogenous growth theories, which described human capital as the engine of growth through innovations (e.g., Aghion and Howitt, 1992; Grossman and Helpman; 1991; Romer, 1990). In particular, this paper is closely related to previous works that analyze the effects of the skill composition of the labor force on the amount of innovation in the economy (e.g., Grossman and Helpman; 1991; Vandenbusshe et al., 2006). Using their seminal model of a variety expansion, Grossman and Helpman (1991) show that an increase in the stock of skilled labor can be growth enhancing, while an increase in the stock of unskilled labor can be growth-depressing.[4] Vandenbusshe et al.(2006) develop a model of technology catch up and show that skilled labor has a higher growth-enhancing effect closer to the technological frontier under the assumption that innovation is a relatively more skill-intensive activity than producing imitations.[5] In contrast to these studies, this paper considers the case where the skill composition of the labor force is determined endogenously through the individuals' choice of higher education. This specification enables us to analyze the interactions among higher education subsidies, the skill composition of the labor force, and economic growth more extensively.

Second, this paper is closely related to a few pioneering theoretical studies that analyze the effects of publicly provided education targeted to high-ability workers on R&D-based growth (e.g., Grossman, 2007; Böhm et al., 2015). Grossman (2007) incorporates an occupational choice framework into an in-house process innovation-based growth model and shows that publicly provided education aimed at expanding the science and engineering skills of high-ability workers is unambiguously growth promoting and neutral with respect to the earnings distribution. Böhm et al.(2015) develop a numerical simulation model of directed technical change and show that publicly provided education aimed at expanding the skills of high-ability workers eventually trickles down to low-ability workers and serves them better than redistribution through labor income taxation or education policies targeted to the low-ability workers.

Although we share numerous research interests with these studies, our research differs from them in the following respects. First, to examine the effects of the skill composition of the labor force on both the vertical and horizontal dimensions of technological progress, we employ a Schumpeterian growth model with an endogenous market structure, where both growth in product variety and

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[4]See the third section of their chapter 5.

[5]Using a panel data set covering 19 OECD countries between 1960 and 2000, Vandenbusshe et al.(2006) find a link between the five-year growth rates and higher education, once distance from the technological frontier is controlled for. However, Hanushek (2016) shows that the point estimate on the interaction of cognitive skills and OECD countries is slightly negative, which indicates that the skills are less important in developed countries.

product quality are determined endogenously through R&D investment. This extension enables us to obtain richer theoretical implications regarding the effects of the skill composition of the labor force on R&D-based growth. To the best of our knowledge, the existing Schumpeterian growth literature has yet to analyze this issue rigorously. Second, we focus on the analysis of a higher education subsidy policy aimed at expanding the number of individuals who receive higher education, whereas existing studies focus on the analysis of a public education policy aimed at improving the skills of high-ability workers. Roughly speaking, this paper focuses on the effects of the size of the higher education sector, while existing studies focus on the effects the quality of the higher education sector. These differences in modeling strategy enable us to reveal different aspects of a higher education promoting policy that have yet to be examined in the literature. In this sense, our research complements the analyses conducted by Grossman (2007) and Böhm et al.(2015).

Third, this paper is closely related to Chen (2015), which shows the somewhat counterintuitive negative effects of educational subsidies on economic growth. In his Diamond (1965)-type OLG model with endogenous fertility and skill acquisition, increases in educational subsidies will hamper economic growth due to the following three reasons. First, a higher income tax rate will lower the motivation to become skilled workers; second, the increased time spent on education and raising children will result in less time being available for working; and third, an increase in the average fertility rate will give rise to a capital-dilution effect, ultimately reducing the capital per worker. In contrast to Chen (2015), our analysis is based on a model that considers R&D-based innovations as the fundamental driver of economic growth. Therefore, this paper proposes an alternative theoretical mechanism that explains the long-run negative effects of educational subsidies on economic growth.

This paper is organized as follows. Section 2 presents the basic model. Section 3 investigates the dynamic equilibrium properties of the economy. Section 4 examines the effect of a higher education subsidy on economic growth. Section 5 concludes the paper.

2 Model

This section introduces a two-period OLG model of R&D-based growth with endogenous skill acquisition, productivity growth and variety expansion.[6] The econ-

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[6]Some basic settings in our model are inherited from Tanaka and Iwaisako (2011), where the skilled labor supply is determined endogenously according to the individuals' choice of higher education, i.e., unskilled labor is assumed to only engage in the production activities, while skilled labor is assumed to be able to engage in the both the R&D and production activities.

omy consists of three sectors: a final goods sector, an intermediate goods sector, and a product development sector. The final goods sector produces homogeneous goods for sales in a perfectly competitive market, with a variety of imperfectly substitutable intermediate goods as inputs. The intermediate goods sector, on the other hand, consists of monopolistically competitive firms that produce differentiated product varieties for firms in the final goods sector, with both unskilled and skilled labors as inputs. Productivity growth arises as a result of process innovation undertaken by the intermediate goods firms, with the objective of lowering production costs, with solely skilled labor as an input. The product development sector creates new product designs for firms entering the intermediate goods sector, with solely skilled labor as an input.

2.1 Individuals

Individuals in this economy live for two periods, young and old. They work only in their young period and retire in the old period. In each period, young individuals who are the continuum of measure M are born with one unit of available time endowment.

In the young period, before entering the labor market, each individual chooses whether to receive a higher education. Although individuals with higher education can supply skilled labor in their young period, they must devote $\min\{1-\theta, 1\}$ units of time to education according to their known ability θ. For each individual, this θ is a random variable drawn from a distribution defined over $[0, 1]$ with cumulative distribution function Φ, which is assumed to be continuously differentiable, strictly increasing, with a time-invariant function, and common to all individuals. Each individual is endowed with θ before birth and takes the value as given. Therefore, higher educated individuals with ability θ can supply only $\max\{\theta, 0\}$ units of skilled labor. On the other hand, individuals who receive only a lesser education can supply one unit of unskilled labor. For simplicity, we assume that no other cost is needed for education. In fact, on average, across OECD countries, foregone earnings for a man attaining a tertiary education is US \$ 43,700, whereas their direct costs are US \$10,500 (OECD, 2016; Table B7.3b, p150). The main costs of a tertiary education are foregone earnings.

Consequently, the before-tax labor income of individuals with ability θ who become skilled workers is given by $w_t^s\theta$, whereas that of individuals who become unskilled workers is given by w_t^u, where w_t^s and w_t^u are the wage per unit of skilled and unskilled labor at period t, respectively. Explicit consideration of the costs for acquiring basic skills to be an unskilled worker does not alter the main implications of this paper. To maintain the tractability of our model, we omit the description regarding the individuals' choice of basic education.

Moreover, higher educated individuals can obtain an education subsidy from

the government. To reduce the individual's opportunity costs of acquiring skills, the government levies a tax τ_t on the labor income of all young individuals and subsidizes a fraction $s \in (0, 1)$ of $(1 - \tau_t)w_t^u$ for units of time that skilled individuals devote to acquiring skills $1 - \theta$. Note that for individuals with ability θ, the opportunity costs of acquiring skills are given by $(1 - \tau_t)w_t^u(1 - \theta)$ (i.e., the foregone unskilled labor income during education).

Most developed countries provide a set of policies aimed at improving the individuals' accessibility to higher education. These policies include the public provision of low-cost tertiary institutions, means tested grants, loan and work study programs, tax credits and so on. Broadly speaking, these policies can be interpreted as higher education subsidy policies aimed at reducing the student's opportunity costs of acquiring skills.

Under such a subsidy policy, the after-tax income of individuals with ability θ who become skilled workers is given by $(1 - \tau_t)[w_t^s\theta + sw_t^u(1 - \theta)]$, whereas that of individuals who become unskilled workers is given by $(1 - \tau_t)w_t^u$. Therefore, given the definition of $\omega_t \equiv \frac{w_t^u}{w_t^s}$, the condition under which an individual with ability θ obtains a higher education is described by:

$$\theta \geq \frac{(1 - s)\omega_t}{1 - s\omega_t} \equiv \hat{\omega}(\omega_t; s). \tag{1}$$

Therefore, as long as $\omega_t \leq 1$, the relations $\hat{\omega}_\omega(\omega_t; s) > 0$ and $\hat{\omega}_s(\omega_t; s) < 0$ hold, which indicates that the skilled worker share decreases with the relative wage of unskilled/skilled workers ω_t, whereas it increases with the education subsidy rate s.

Exploiting the law of large numbers, we can compute the skilled and the unskilled labor supply as follows:

$$H(\omega_t; s) \equiv \int_{\hat{\omega}(\omega_t; s)}^1 M\theta d\Phi(\theta), \tag{2}$$

$$L(\omega_t; s) \equiv \int_0^{\hat{\omega}(\omega_t; s)} M d\Phi(\theta). \tag{3}$$

Therefore, as long as $\omega_t \leq 1$, as shown in Appendix A, we can confirm that the relations $H_\omega(\omega_t; s) < 0$, $H_s(\omega_t; s) > 0$, $L_\omega(\omega_t; s) > 0$ and $L_s(\omega_t; s) < 0$ hold. These results indicate that the skilled labor supply (resp., unskilled labor supply) decreases (resp., increases) with the relative wage of unskilled/skilled workers ω_t, whereas it increases (resp., decreases) with the education subsidy rate s.

The lifetime utility of individuals with ability θ, born in period t, is expressed as follows:

$$U_t^\theta = lnC_{1,t}^\theta + \beta lnC_{2,t+1}^\theta, \quad \beta \in (0, 1), \tag{4}$$

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where $C_{1,t}^{\theta}$ and $C_{2,t+1}^{\theta}$ represent their consumption during their youth and old age, respectively.

The budget constraints of these individuals are expressed as follows:

$$P_{c,t}C_{1,t}^{\theta} + S_t^{\theta} = (1 - \tau_t)\max\left\{w_t^s\theta + sw_t^u(1 - \theta), w_t^u\right\}, \tag{5}$$

$$P_{c,t+1}C_{2,t+1}^{\theta} = R_{t+1}S_t^{\theta}, \tag{6}$$

where S_t^{θ} is their saving during their youth, and $P_{c,t}$ and $P_{c,t+1}$ are the price of final goods at period t and $t + 1$, respectively.

By maximizing (4) subject to (5) and (6), we obtain the following:

$$S_t^{\theta} = \frac{\beta}{1+\beta}(1 - \tau_t)\max\left\{w_t^s\theta + sw_t^u(1 - \theta), w_t^u\right\}. \tag{7}$$

2.2 Final goods sector

The final goods sector is perfectly competitive. We assume that one representative final goods firm combines n_t kinds of intermediate goods to produce the final good Y_t in period t. Following Benassy (1996) and others, we specify the technology of final goods production as follows:

$$Y_t = n_t^{\sigma+1-\frac{\eta}{\eta-1}}\left(\int_0^{n_t} x_t(j)^{\frac{\eta-1}{\eta}}dj\right)^{\frac{\eta}{(\eta-1)}}, \quad \eta > 1, \tag{8}$$

where $x_t(j)$ is the jth intermediate good input. The parameter σ captures the degree of specialization in production. The elasticity of substitution between any two of the intermediate goods is equal to η. If $\sigma = 1/(\eta - 1)$, (8) reduces to a well-known Dixit-Stiglitz-type specification.

Given the price of the final good $P_{c,t}$ and those of the intermediate goods $p_t(j)$, the firm maximizes its profit. Because of the perfect competition, the final goods firm earns zero profit, that is, $P_{c,t}Y_t = \int_0^{n_t} p_t(j)x_t(j)dj$. Since the production function is in the spirit of Dixit and Stiglitz (1977), the first-order conditions for the profit maximization and the zero-profit condition yield the well-known following demand functions:

$$x_t(j) = \frac{p_t(j)^{-\eta}}{\int_0^{n_t} p_t(j)^{1-\eta}dj}P_tY_t, \quad \forall j \in [0, n_t], \tag{9}$$

where $P_t = n_t^{\frac{1}{\eta-1}-\sigma}\left(\int_0^{n_t} p_t(j)^{1-\eta}dj\right)^{1/(1-\eta)}$ is a price index of the intermediate goods. Here, note that the relation $P_{c,t} = P_t$ holds due to the zero-profit condition.

2.3 Intermediate goods sector

Each intermediate good j is produced by monopolistically competitive firms that hold a blueprint for the intermediate good j. Each firm has the following constant-returns-to-scale production technology:

$$x_t(j) = A_t(j) \left[l_t^s(j)\right]^\alpha \left[l_t^u(j)\right]^{1-\alpha}, \quad \alpha \in (0, 1), \tag{10}$$

where $l_t^s(j)$ and $l_t^u(j)$ represent the skilled and unskilled labor inputs of firm j at period t, whereas $x_t(j)$ and $A_t(j)$ are the output and productivity of firm j at period t, respectively. Let $A_t \equiv \frac{1}{n_t}(\int_0^{n_t} A_t(j)dj)$ denote the average productivity of firms in period t.

   Intermediate good firms invest in process innovation with the aim of lowering the production cost through productivity improvements. A firm with its R&D department employs $l_t^R(j)$ units of skilled labor in process innovation. Based on Grossman (2007, 2009), the firm-level productivity $A_t(j)$ evolves according to:

$$A_t(j) = zA_{t-1}[l_t^R(j)]^\gamma, \quad z > 0, \gamma > 0, \tag{11}$$

The term A_{t-1} captures the public technological knowledge at period t, which accumulates within firms as a byproduct of process innovation. We adopt the level of the average productivity of firms in period $t-1$ as a proxy for the stock of public technological knowledge at period t. Following the process innovation framework employed by Peretto and Conolly (2007) and others, we model knowledge spillovers into process innovations among firms as a function of the average productivity of technological knowledge observable by the R&D departments of firms. [7] Each intermediate goods firm maximizes its net profit, $\pi_t(j)$. The profit maximization problem of intermediate goods firm j is as follows:

$$\pi_t(j) \equiv \max \left\{ p_t(j)x_t(j) - w_t^s l_t^s(j) - w_t^u l_t^u(j) - w_t^s l_t^R(j) \right\}, \tag{12}$$

subject to (9), (10) and (11).[8] By solving the problem, we obtain the optimal labor input as follows:

$$l_t^s = \frac{\alpha}{w_t^s} \frac{\eta - 1}{\eta} \frac{P_t Y_t}{n_t}, \tag{13}$$

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[7]An alternative formulation is that firms have to incur in-house R&D expenditure one period in advance of production, similar to in the discrete time infinite-horizon model of Young (1998). However, this assumption seems to be less plausible in an OLG model.

[8]In our model, as in Peretto and Conolly (2007), the productivity of the in-house R&D investment in period t fully depends on the level of public knowledge in period t, A_{t-1}. In addition, it is assumed that the in-house R&D investment in period t can affect the level of productivity in period t, $A_t(i)$ instantaneously. Due to these assumptions, the intermediate goods firm's market value maximization problem can be formulated as its net profit maximization problem each period. With noting (13), the market value of the jth intermediate goods firm is given by $V_t(j) = \sum_{\tau=1}^{\infty} \frac{\pi_{t+\tau}(j)}{\Pi_{i=1}^{i=\tau} R_{t+i}}$.

$$l_t^u = \frac{1-\alpha}{w_t^u} \frac{\eta-1}{\eta} \frac{P_t Y_t}{n_t}, \tag{14}$$

$$l_t^R = \frac{\gamma}{w_t^s} \frac{\eta-1}{\eta} \frac{P_t Y_t}{n_t}. \tag{15}$$

The derivations of (13) to (15) are provided in Appendix B. Hereafter, we assume $\gamma(\eta-1) < 1$ to satisfy the second-order condition for maximization. Using equations (12) to (15), we obtain the following maximum net profits for each intermediate firm in period t:

$$\pi_t = \frac{1-\gamma(\eta-1)}{\eta} \frac{P_t Y_t}{n_t}. \tag{16}$$

Because of the ex ante homogeneity of the individuals, all intermediate goods firms behave in the same way. Thus, we omit index j whenever this does not lead to confusion.

2.4 Product development sector

The invention of new variety requires skilled labor as its only private input. Between periods t and $t+1$, competitive R&D firms in the product development sector employ L_t^N efficiency units of skilled labor as researchers, develop $n_{t+1} - n_t$ new blueprints, and sell these blueprints to intermediate goods firms at their market values of V_t. Thus, given a research productivity of δ_t, output is expressed as follows:

$$n_{t+1} - n_t = \delta_t L_t^N. \tag{17}$$

Following Jones (1995), research productivity is a given for each firm but depends on the aggregate level, positively on the number of existing ideas (i.e., *the standing-on-shoulders effect*), as follows:

$$\delta_t = \bar{\delta} n_t^{\psi}, \quad \bar{\delta} > 0, \quad \psi \in [0, 1). \tag{18}$$

The specification $\psi \in [0, 1)$ implies that the marginal effect of n_t on δ_t is decreasing with n_t. The standing-on-shoulders effect arises because the creation of a new product designed adds to the existing stock of public knowledge related to product design, improving the labor productivity of future product development.

Under the assumption of free entry in the product development sector, the expected gain of $V_t \delta_t L_t^N$ from R&D must not exceed the cost of $w_t^s L_t^N$ for a finite size of R&D activities at equilibrium. Thus, we have the following conditions:

$$V_t \delta_t \begin{cases} = w_t^s, & \text{then } L_t^N > 0, \ n_{t+1} > n_t, \\ < w_t^s, & \text{then } L_t^N = 0, \ n_{t+1} = n_t. \end{cases} \tag{19}$$

We next consider no-arbitrage conditions. The market value of intermediate goods firms V_t (i.e., the market value of blueprints) is related to the risk-free interest rate R_t. Shareholders of intermediate goods firms that purchased these shares during period t obtain dividends of π_{t+1} during period $t+1$ and can sell these shares to the subsequent generation at a value of V_{t+1}. In the financial market, the total returns from holding the stock of a particular intermediate firm must be equal to the returns on the risk-free asset $R_{t+1}V_t$, which implies the following no-arbitrage condition:

$$R_{t+1} = \frac{\pi_{t+1} + V_{t+1}}{V_t}. \tag{20}$$

2.5 Government

The government levies a tax τ_t on the labor income of all young individuals and subsidizes a fraction $s \in (0, 1)$ of $(1-\tau_t)w_t^u$ for units of time that skilled individuals devote to acquiring skills $1-\theta$. Thus, its budget constraint of period t is as follows:

$$\tau_t[w_t^s H(\omega_t; s) + w_t^u L(\omega_t; s)] = (1 - \tau_t)w_t^u \int_{\hat{\omega}(\omega_t;s)}^{1} s(1 - \theta)Md\Phi(\theta), \tag{21}$$

where the left-hand side is the total tax revenue raised from all young individuals and the right-hand side is the total expenditure composed of education subsidy payments to all skilled young individuals.

2.6 Market-clearing conditions

Now, we consider the labor market conditions. Skilled labor is demanded by both intermediate goods firms and product development firms to produce intermediate goods, to conduct process innovation and to invent new products, while unskilled labor is demanded only by intermediate goods firms to produce intermediate goods. Thus, the market-clearing conditions for both skilled and unskilled laborers are described as follows:

$$n_t(l_t^s + l_t^R) + L_t^N = H(\omega_t; s), \tag{22}$$

$$n_t l_t^u = L(\omega_t; s). \tag{23}$$

Furthermore, as shown in Appendix C, we can obtain the following asset market equilibrium condition:

$$\int_0^1 S_t^\theta Md\Phi(\theta) \begin{cases} = n_{t+1}V_t, & \text{for } n_{t+1} > n_t, \\ = n_t V_t, & \text{for } n_{t+1} = n_t. \end{cases} \tag{24}$$

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This condition states that the savings of young individuals in period t must be used for investing in new inventions ($V_t(n_{t+1} - n_t)$) or purchasing existing stocks that were owned by preceding generations ($V_t n_t$). In particular, when the product development sector does not operate (i.e., $n_{t+1} = n_t$), the savings of young individuals in period t must be devoted to purchasing existing stocks that were owned by preceding generations ($V_t n_t$).

3 Equilibrium

In this section, we analyze the dynamics of the relative wages of unskilled/skilled workers, the average productivity of firms, the number of firms, and the value of GDP.

3.1 Relative wage of unskilled/skilled workers

In this subsection, we describe the determination of the relative wage of unskilled/skilled workers. We first consider the case where the product development sector operates (i.e., $n_{t+1} > n_t$). When the product development sector operates (i.e., $n_{t+1} > n_t$), from (7), (21) and (24), we can obtain the following equation:

$$n_{t+1}V_t = \frac{\beta}{1+\beta}[w_t^s H(\omega_t; s) + w_t^u L(\omega_t; s)]. \tag{25}$$

By substituting (17) and (19) into (25), equation (25) can be rewritten as follows:

$$\frac{n_t}{\delta_t} = \frac{\beta}{1+\beta}[H(\omega_t; s) + \omega_t L(\omega_t; s)] - L_t^N. \tag{26}$$

By using equations (13) to (15) and equations (22) to (25), as shown in Appendix D, we can express the skilled labor engaged in the product development sector L_t^N as follows:

$$L_t^N = H(\omega_t; s) - \frac{\alpha+\gamma}{1-\alpha}\omega_t L(\omega_t; s). \tag{27}$$

Thus, by substituting (18) and (27) into (26), we can obtain the following equation:

$$\frac{n_t^{1-\psi}}{\bar{\delta}} = \left(\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha}\right)\omega_t L(\omega_t; s) - \frac{1}{1+\beta}H(\omega_t; s). \tag{28}$$

From (28), we can see that the relative wages of unskilled/skilled workers ω_t depend on the number of firms n_t and the education subsidy rate s (i.e., $\omega_t = \omega(n_t; s)$). Appendix D shows that $\omega(n_t; s)$ satisfies the following properties: $\omega_n(n_t; s) > 0$ and $\omega_s(n_t; s) > 0$.

Next, we consider the case where the product development sector does not operate (i.e., $n_{t+1} = n_t$). When the product development sector does not operate (i.e., $n_{t+1} = n_t$), from (27) and $L_t^N = 0$, we can obtain the following equation:

$$\frac{1-\alpha}{\alpha+\gamma} = \frac{\omega_t L(\omega_t; s)}{H(\omega_t; s)} \equiv \Gamma(\omega_t; s). \tag{29}$$

From (29), we can see that the relative wages of unskilled/skilled workers ω_t are given by the constant value, which is denoted as ω^*, and depends upon the education subsidy rate s (i.e., $\omega^* = \omega^*(s)$). Appendix D also shows that $\omega^*(s)$ satisfies the following property: $\omega_s^*(s) > 0$.

From (28) and (29), we denote the number of firms n_t such that it satisfies $\omega(n_t; s) = \omega^*(s)$ as $n^*(s)$. As shown in the following subsection 3-3, we can confirm that $n^*(s)$ satisfies the following property: $n_s^*(s) > 0$. From (27), when the product development sector operates (i.e., $n_{t+1} > n_t$), since $L_t^N > 0$, the following inequality $\frac{1-\alpha}{\alpha+\gamma} > \frac{\omega_t L(\omega_t; s)}{H(\omega_t; s)} \equiv \Gamma(\omega_t; s)$ holds. From (29) and $\Gamma_\omega(\omega_t; s) > 0$, this inequality implies that the relation $\omega(n_t; s) < \omega^*(s)$ holds in the case where the product development sector operates (i.e., $n_{t+1} > n_t$). With noting $\omega_n(n_t; s) > 0$ from (28), these results imply that the product development sector operates (i.e., $n_{t+1} > n_t$) if and only if the number of firms n_t is sufficiently small to satisfy $n_t < n^*(s)$. In contrast, suppose that $n_t \geq n^*(s)$ (i.e., $n_{t+1} = n_t$) and the product development sector does not operate; thus, the entry of new firms never occurs. Taking these results into account, the equilibrium relative wage of unskilled/skilled workers ω_t is given by the following expression:

$$\omega_t = \begin{cases} \omega(n_t; s), & \text{for } n_t < n^*(s), \\ \omega^*(s), & \text{for } n_t \geq n^*(s). \end{cases} \tag{30}$$

The solid line in Figure 2 describes the relationship between the number of firms n_t and the equilibrium relative wages of unskilled/skilled workers ω_t. Since $\omega_n(n_t; s) > 0$, the relative wages of unskilled/skilled workers increases with the number of firms n_t in the region where $n_t < n^*(s)$, but it remains constant at $\omega^*(s)$ when $n_t \geq n^*(s)$. Moreover, since $\omega_s(n_t; s) > 0$ and $\omega_s^*(s) > 0$, as shown in the upward shift of the $\omega(n_t; s)$ and $\omega^*(s)$ curves in Figure 2 (i.e., $s < s'$), given the value of n_t, the higher rate of education subsidies positively affects the relative wages of unskilled/skilled workers. The higher rate of education subsidy s increases the supply of skilled workers, which lowers their relative wages.

3.2 Dynamics of the average productivity of firms

In this subsection, we analyze the dynamics of the average productivity of firms. By substituting (14) and (23) into (15), we can obtain the following equation:

$$l_t^R = \frac{\gamma}{1 - \alpha} \frac{\omega_t L(\omega_t; s)}{n_t}. \tag{31}$$

Then, using (28) and (29), the skilled labor engaged in process innovation l_t^R in (31) can be expressed as follows:

$$l_t^R = \begin{cases} \dfrac{\gamma}{1-\alpha} \dfrac{\left[\frac{1}{\delta n_t^{\psi}} + \frac{1}{1+\beta} \frac{\hat{H}(n_t; s)}{n_t}\right]}{\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha}}, & \text{for } n_t < n^*, \\[4ex] \dfrac{\gamma}{\alpha+\gamma} \dfrac{\hat{H}^*(s)}{n_t}, & \text{for } n_t \geq n^*, \end{cases} \tag{32}$$

where

$$\hat{H}(n_t; s) \equiv H\left(\omega(n_t; s); s\right),$$

$$\hat{H}^*(s) \equiv H\left(\omega^*(s); s\right).$$

From (32), we can see that the skilled labor engaged in process innovation l_t^R depends upon the number of firms n_t and the education subsidy rate s (i.e., $l^R(n_t; s)$). Appendix E shows that the relations $\hat{H}_n(n_t; s) < 0$, $\hat{H}_s(n_t; s) > 0$ and $\hat{H}_s^*(s) > 0$ hold. These results imply that $l^R(n_t; s)$ satisfies the following properties: $l_n^R(n_t; s) < 0$ and $l_s^R(n_t; s) > 0$. Since $\omega_n(n_t; s) > 0$ and $L_\omega(\omega_t; s) > 0$, equation (31) suggests that the number of firms n_t has two competing impacts upon the level of l_t^R. A larger number of firms n_t decreases each firm's market size, which negatively affects the level of l_t^R, whereas it increases the values of $\omega_t L(\omega_t; s)$, which positively affects the level of l_t^R.[9] The results obtained from (32) indicate that the former negative effect always dominates the latter positive effect because the relation $l_n^R(n_t; s) < 0$ holds. Moreover, since $l_s^R(n_t; s) > 0$, the higher rate of education subsidy s positively affects the level of skilled labor engaged in process innovation.

Using (11) and (32), the gross growth rate of the average productivity of firms is given by the following expression:

$$G_t^A \equiv \frac{A_t}{A_{t-1}} = z\left[l^R(n_t; s)\right]^\gamma \equiv G^A(n_t; s). \tag{33}$$

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[9]As in Peretto and Connolly (2007), the quality-adjusted gross firm size is measured by the quality-adjusted volume of production, $\frac{x_t}{A_t}$. Using (28), (A.12), (A.31) and $\hat{c}_t = \frac{(w_t^s)^\alpha (w_t^u)^{(1-\alpha)}}{\alpha^\alpha (1-\alpha)^{(1-\alpha)}}$, we obtain $\frac{x_t}{A_t} = \left(\frac{\alpha}{1-\alpha}\right)^\alpha \frac{1}{\omega_t^{1-\alpha}} \frac{\frac{1}{\delta_t} + \frac{H(\omega_t; s)}{1+\beta}}{\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha}}$. Thus, with noting (18) and (30), we can easily confirm that a larger number of firms n_t decreases each firm's market size.

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From (32) and (33), we can easily confirm that $G^A(n_t; s)$ satisfies the following properties: $G_n^A(n_t; s) < 0$ and $G_s^A(n_t; s) > 0$.

The solid line in Figure 3 describes the relationship between the number of firms n_t and the equilibrium gross growth rate of the average productivity of firms G_t^A. Since $G_n^A(n_t; s) < 0$, the gross growth rate of the average productivity of firms decreases with the number of firms. As the number of firms n_t increases, each firm's market size decreases, which motivates firms to invest less in process innovation and thereby lowers the gross growth rate of the average productivity of firms G_t^A. Moreover, since $G_s^A(n_t; s) > 0$, as shown in the upward shift of the $G^A(n_t; s)$ curve in Figure 3 (i.e., $s < s'$), given the value of n_t, the higher rate of education subsidy s positively affects the equilibrium gross growth rate of the average productivity of firms. The higher rate of education subsidy s increases the supply of skilled workers, lowers their relative wages, increases each firm's incentives for process innovation by reducing the employment costs of researchers and, thus, positively affects the gross growth rate of the average productivity of firms.

3.3 Dynamics of the number of firms

In this subsection, we describe the dynamics of the number of firms. When the product development sector operates (i.e., $n_{t+1} > n_t$), from (19) and (25), we can obtain the following equation:

$$\frac{n_{t+1}}{\delta_t} = \frac{\beta}{1+\beta} \left[H(\omega_t; s) + \omega_t L(\omega_t; s) \right]. \tag{34}$$

Then, by using equations (26) to (29) and (34), the gross growth rate of the number of firms can be expressed as follows:

$$G_{t+1}^n \equiv \frac{n_{t+1}}{n_t} = \begin{cases} \frac{1}{F(n_t; s)} \equiv G^n(n_t; s), & \text{for } n_t < n^*(s), \\ 1, & \text{for } n_t \geq n^*(s). \end{cases} \tag{35}$$

where

$$F(n_t; s) \equiv F\left(\hat{\Gamma}(n_t; s)\right) = 1 - \frac{1+\beta}{\beta} \frac{1 - \frac{\alpha+\gamma}{1-\alpha}\hat{\Gamma}(n_t; s)}{1 + \hat{\Gamma}(n_t; s)},$$

$$\hat{\Gamma}(n_t; s) \equiv \Gamma\left(\omega(n_t; s); s\right) = \frac{\omega(n_t; s)\hat{L}(n_t; s)}{\hat{H}(n_t; s)},$$

$$\hat{L}(n_t; s) \equiv L\left(\omega(n_t; s); s\right).$$

As shown in Appendix F, $G^n(n_t; s)$ satisfies the following properties: $G_n^n(n_t; s) < 0$ and $G_s^n(n_t; s) > 0$.

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The solid line in Figure 4 describes the relationship between the number of firms n_t and the gross growth rate of the number of firms G_{t+1}^n. Since $G_n^n(n_t; s) < 0$, the gross growth rate of the number of firms G_{t+1}^n decreases with the number of firms n_t. To avoid an unnecessary lexicographic explanation, as shown in Figures 2 and 4, we restrict our analyses to the case where the initial number of firms n_0 is sufficiently small to ensure the relations $\omega_0 = \omega_n(n_0; s) < \omega^*(s)$ and $G_1^n = G^n(n_0; s) > 1$ hold. Under such assumptions, equation (35) shows that given the initial value of n_0, n_t gradually approaches its steady-state value, denoted as $n^*(s)$. On the transition path, the number of firms determines each firm's market size and the equilibrium gross growth rate of the average productivity of firms G_t^A according to (33). When n_t evolves toward its steady-state value $n^*(s)$, as described in Figure 3, G_t^A also gradually approaches its steady-state value $G^{A*}(s)$, which is defined by $G^{A*}(s) \equiv G^A(n^*(s); s)$.

More precisely, from (35), the dynamics of n_t are determined by the following one-dimensional difference equation:

$$n_{t+1} = \begin{cases} G^n(n_t; s)n_t, & \text{for } n_t < n^*(s), \\ n_t, & \text{for } n_t \geq n^*(s). \end{cases}$$

As shown in Appendix G, the differentiation of $G^n(n_t; s)n_t$ with respect to n_t around the steady state $n_t = n^*$ yields the following:

$$\frac{dn_{t+1}}{dn_t}\Big|_{n_t=n^*} = 1 - (1+\beta)(1-\psi)\frac{\alpha+\gamma}{\alpha+\gamma+\beta(1+\gamma)\frac{1+\frac{\omega^*(s)L_\omega(\omega^*(s);s)}{\hat{L}(s)}}{1+\frac{\omega^*(s)L_\omega(\omega^*(s);s)}{\hat{L}(s)}-\frac{\omega^*(s)H_\omega(\omega^*(s);s)}{\hat{H}(s)}}} < 1,$$

where $\hat{L}(s) \equiv \hat{L}(\omega^*(s); s)$. Therefore, suppose that the stepping-on-shoulders parameter ψ is sufficiently large to satisfy the following:

$$\frac{\beta}{1+\beta}\left(1 - \frac{1+\gamma}{\alpha+\gamma}\frac{1+\frac{\omega^*(s)L_\omega(\omega^*(s);s)}{\hat{L}(s)}}{1+\frac{\omega^*(s)L_\omega(\omega^*(s);s)}{\hat{L}(s)}-\frac{\omega^*(s)H_\omega(\omega^*(s);s)}{\hat{H}(s)}}\right) \leq \psi, \tag{36}$$

We can ensure that the relation $\frac{dn_{t+1}}{dn_t}\Big|_{n_t=n^*} \in [0,1)$ holds. Otherwise, we can see that the relation $\frac{dn_{t+1}}{dn_t}\Big|_{n_t=n^*} < 0$ holds.

The solid lines in Figures 5 and 6 illustrate the possible dynamics of n_t when the parameter conditions of (36) are satisfied and not satisfied, respectively. As shown in Figure 5, when the parameter conditions of (36) are satisfied, the dynamics of n_t are stable and n_t gradually converges to a unique positive steady-state value $n^*(s)$. However, when the parameter conditions of (36) are not satisfied, as shown in Figure 6, n_t does not necessarily converge to a unique steady-state value

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$n^*(s)$. Instead, the steady-state value of n_t depends upon its initial values of n_0 and may become larger than the value of $n^*(s)$.

In the following analyses, for simplicity, we restrict our analysis to the case where the parameter conditions of (36) are satisfied. Suppose that the elasticity of skilled labor input α is sufficiently small; then, the left-hand side of (36) becomes negative. In this case, the parameter conditions of (36) hold, irrespective of the values of ψ. Moreover, the numerical simulation analyses in the following section show that equation (36) holds under a wide plausible set of parameter values. The following proposition summarizes the results and derives the steady-state values $\{n^*(s), G^{A*}(s)\}$.

Proposition 1 *Given the initial value of n_0 such that it satisfies $\omega_0 = \omega_n(n_0; s) < \omega^*(s)$, if the parameter conditions of (36) hold, the dynamics of n_t are stable and n_t gradually converges to a unique positive steady-state value. The steady-state values $\{n^*(s), G^{A*}(s)\}$ are given by the following:*

$$n^*(s) = \left(\frac{\beta}{1+\beta} \frac{1+\gamma}{\alpha+\gamma} \bar{\delta} \right)^{\frac{1}{1-\psi}} \left[\hat{H}^*(s) \right]^{\frac{1}{1-\psi}}, \qquad (37)$$

$$G^{A*}(s) = z \left[l^{R*}(s) \right]^{\gamma}, \qquad (38)$$

where

$$l^{R*}(s) \equiv \frac{\gamma}{\alpha+\gamma} \frac{\hat{H}^*(s)}{n^*(s)} = \frac{\gamma}{\alpha+\gamma} \frac{\left[\hat{H}^*(s)\right]^{-\frac{\psi}{1-\psi}}}{\left(\frac{\beta}{1+\beta} \frac{1+\gamma}{\alpha+\gamma} \bar{\delta} \right)^{\frac{1}{1-\psi}}}.$$

Proof of Proposition 1 is given in Appendix G. From (37) and (38), since $\hat{H}^*_s(s) > 0$, we can see that the relations $n^*_s(s) > 0$ and $G^{A*}_s(s) \leq 0$ hold. The higher rate of education subsidy s increases the steady-state number of firms $n^*(s)$, whereas it decreases the steady-state gross growth rate of the average productivity of firms $G^{A*}(s)$. The intuitive mechanism behind the results of proposition 1 are explained carefully in the following Section 4.

3.4 The value of GDP

In this subsection, we describe the equilibrium value of GDP. In the R&D-based growth model, the value of GDP is not necessarily equivalent to that of Y_t.[10] The correct value of GDP is defined as follows:

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[10]Gross domestic income (GDI) is calculated by

$$GDI_t = \frac{(1-\tau_t) \int_{\hat{\omega}(\omega_t;s)}^1 \left[w_t^s \theta + s w_t^u (1-\theta) \right] M d\Phi(\theta) + (1-\tau_t) w_t^u L(\omega_t; s) + \pi_t n_t}{P_{c,t}}$$

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$$GDP_t = \begin{cases} Y_t + \frac{V_t}{P_{c,t}}(n_{t+1} - n_t), & \text{for } n_t < n^*, \\ Y_t, & \text{for } n_t \geq n^*. \end{cases} \quad (39)$$

Appendix I shows that GDP_t in (39) can be rewritten as follows:

$$GDP_t = \alpha^\alpha (1 - \alpha)^{1-\alpha} A_t n_t^\sigma \Lambda_t. \quad (40)$$

where

$$\Lambda_t \equiv \begin{cases} \frac{\eta-1}{\eta} \frac{\hat{H}(n_t;s)}{\omega(n_t;s)^{1-\alpha}} + \frac{1-(\alpha+\gamma)\frac{\eta-1}{\eta}}{1-\alpha} \frac{\omega(n_t;s)\hat{L}(n_t;s)}{\omega(n_t;s)^{1-\alpha}} \equiv \Lambda(n_t;s), & \text{for } n_t < n^*, \\ \frac{1}{1-\alpha} \frac{\omega^*(s)\hat{L}^*(s)}{\omega^*(s)^{1-\alpha}} \equiv \Lambda^*(s), & \text{for } n_t \geq n^*. \end{cases}$$

Equation (40) indicates that the value of GDP depends upon the average productivity of firms A_t, the number of firms n_t, and the other market factors Λ_t. In particular, the term Λ_t captures the degree of static efficiency of production given the existing technologies and factor inputs. Appendix H shows that the relation $\Lambda_s^*(s) < 0$ holds, which indicates that the education subsidy policy deteriorates the steady-state static efficiency of production.

From (40), the gross growth rate of GDP is given by the following expression:

$$G_t^{GDP} \equiv \frac{GDP_t}{GDP_{t-1}} = \frac{A_t}{A_{t-1}} \left(\frac{n_t}{n_{t-1}}\right)^\sigma \frac{\Lambda_t}{\Lambda_{t-1}} = G_t^A (G_t^n)^\sigma G_t^\Lambda. \quad (41)$$

From (41), we can see that the gross growth rate of GDP depends on the gross growth rate of the average productivity of firms G_t^A, the number of firms G_t^n, and the other market factors G_t^Λ. The productivity growth rate is based on both the production efficiency improvement and variety expansion, and the degree of specialization parameter σ determines the relative importance of the variety expansion on productivity growth.

From (16) and (21), we have

$$GDI_t = \frac{w_t^s [H(\omega_t;s) + \omega_t L(\omega_t;s)]}{P_{c,t}} + \frac{[1-\gamma(\eta-1)]}{\eta} Y_t.$$

Further, from (A.14) and (A.15), we obtain

$$H(\omega_t;s) + \omega_t L(\omega_t;s) = (1+\gamma)\frac{\eta-1}{\eta} \frac{P_{c,t}Y_t}{w_t^s} + L_t^N.$$

By combining the above two equations, with noting (17) and (19), GDI_t is given as follows:

$$GDI_t = Y_t + \frac{V_t}{P_{c,t}}(n_{t+1} - n_t).$$

Therefore, we can confirm that the value of GDI is equivalent to that of GDP.

In the steady-state equilibrium where the relations $n_t = n_{t-1} = n^*$ and $\Lambda_t = \Lambda_{t-1} = \Lambda^*(s)$ hold, the gross growth rate of GDP becomes equivalent to the gross growth rate of the average productivity of firms, as follows:

$$G^{GDP*} \equiv G_t^{GDP} \mid_{n_t=n^*} = G^{A*}(s). \tag{42}$$

Since $G_s^{A*}(s) \leq 0$ from (38), we can easily confirm that the steady-state gross growth rate of GDP decreases with the education subsidy rate s.

4   Education subsidy policy

In this section, we summarize the implications of both the short-run and long-run effects of the education subsidy on the number of firms, the gross growth rate of the average productivity of firms, and economic growth.

4.1   Effects of the education subsidy on the number of firms and the gross growth rate of the average productivity of firms

First, we consider the effects of the education subsidy on the number of firms. Since $G_s^n(n_t; s) > 0$ from (35), as described in the upward shift of the $G^n(n_t; s)$ curve in Figure 4 (i.e., $s < s'$), the initial impact of a higher rate of the education subsidy on the gross growth rate of the number of firms G_{t+1}^n is positive. Given the value of n_t, the higher rate of education subsidy s increases the supply of skilled workers, lowers the relative wage of them, enhances the entry of new firms by reducing the employment costs of researchers and, thus, increases the gross growth rate of the number of firms for some periods. However, as the number of firms increases, the gross growth rate of the number of firms decreases steadily, and the equilibrium number of firms gradually converges to its new steady value. Since $n_s^*(s) > 0$ from (37), as described in Figure 5, the number of firms attained in the new steady state equilibrium becomes larger than that attained in the original steady-state equilibrium (i.e., $n_s^*(s) < n_s^*(s')$). These results are summarized in the following Proposition 2.

Proposition 2 *The higher rate of the education subsidy enhances the entry of new firms, increases the gross growth rate of the number of firms for some periods and increases the steady-state number of firms.*

Next, we consider the effect of the education subsidy on the gross growth rate of the average productivity of firms. Since $G_s^A(n_t; s) > 0$ from (33), as described in the upward shift of the $G^A(n_t; s)$ curve in Figure 3 (i.e., $s < s'$), the initial impact of a higher rate of the education subsidy on the gross growth rate of the average

productivity of firms G_t^A is positive. Given the value of n_t, the higher rate of education subsidy s increases the supply of skilled workers, lowers their relative wages, increases each firm's incentives for process innovation by reducing the employment costs of researchers and, thus, positively affects the gross growth rate of the average productivity of firms. We denote this positive effect of the education subsidy on the gross growth rate of the average productivity of firms as the "cost reduction effect". In the long run, however, market structure is endogenous, and the number of firms adjusts. The increased supply of skilled workers enhances the entry of new firms, which in turn reduces each firm's market size and decreases incentives for process innovation. We denote this negative effect of the education subsidy on the gross growth rate of the average productivity of firms as the "entry effect". Since $G_s^{A*}(s) \leq 0$ from (38), as described in Figure 3, this negative "entry effect" dominates the positive "cost reduction effect" in the long run (i.e., $G^{A*}(s) > G^{A*}(s')$). Therefore, allowing for the endogeneity of the market structure, we can find the opposite short-run and long-run predictions with regard to the effects of the education subsidy on the gross growth rate of the average productivity of firms. These results are summarized in the following Proposition 3.

Proposition 3 *The initial effect of a higher rate of the education subsidy on the gross growth rate of the average productivity of firms is positive as a result of an increased supply of skilled workers. However, in the long run, the increased supply of skilled workers enhances the entry of new firms and reduces the market size of each firm. The smaller market size decreases incentives for process innovation and decreases the steady-state gross growth rate of the average productivity of firms.*

Figures 7-1 to 7-4 show the numerical examples of the transition path of the relative wage of unskilled/skilled workers ω_t (Figure 7-1), the number of firms n_t (Figure 7-2), the net growth rate of the number of firms g_t^n (Figure 7-3) and the net growth rate of the average productivity of firms g_t^A (Figure 7-4), when the education subsidy rate in period 5 and subsequent periods is increased from 0 to 0.3 (i.e., $s_t = 0.3$ for all periods $t \geq 5$). In the simulation, for simplicity, we assume that the economy is initially in the steady-state equilibrium where the education subsidy rate s is given by zero (i.e., $s_t = 0$ for all period $t < 5$). In addition, the ability θ is assumed to be distributed uniformly over the interval $[0, 1]$. Under such an assumption, the skilled and unskilled labor supply is given by $H(\omega_t; s) = \frac{M}{2} \left[1 - \hat{\omega}(\omega_t; s)^2 \right]$ and $L(\omega_t; s) = M\hat{\omega}(\omega_t; s)$. The parameters used in the baseline simulations are given in footnote 11, and its explanation is provided in Appendix I. [11] Note that the objective of these numerical examples is not to

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[11]$\beta = (0.98)^{30}$, $\eta = 6$, $\sigma = 1/(\eta - 1)$, $\alpha = 0.3$, $\gamma = 0.15$, $z = 2.177$, $\psi = 0.35$, $\bar{\delta} = 1$, $M = 1$, $s = 0$, $A_0 = 1$, $n_1 = 0.1$.

calibrate our simple model to actual data but to supplement the qualitative results. The quantitative results obtained in this paper should be interpreted with caution.

As shown in Figures 7-1 to 7-4, the introduction of the education subsidy policy in period 5 increases the relative wages of unskilled/skilled workers in period 5 (Figure 7-1), enhances the entry of new firms in period 6 (Figure 7-2), increases the net growth rate of the number of firms in period 6 (Figure 7-3) and increases the net growth rate of the average productivity of firms in period 5 (Figure 7-4). However, as the number of firms increases, the net growth rate of the number of firms starts to decline from period 7 (Figure 7-3), and the equilibrium number of firms gradually converges to its new steady-state value, which is larger than that in the original steady-state equilibrium (Figure 7-2). Therefore, the education subsidy policy expands the equilibrium number of firms. Moreover, since the negative "entry effect" dominates the positive "cost reduction effect" in the long run, the net growth rate of the average productivity of firms starts to decline from period 6, and it gradually converges to its new steady-state value, which is lower than that in the original steady-state equilibrium (Figure 7-4). Therefore, the education subsidy policy positively affects the net growth rate of the average productivity of firms in the short run, but it negatively affects the net growth rate of the average productivity of firms in the long run.

The education subsidy policy encourages more firms to enter the market with new products, which strengthens horizontal competition among firms. It is this strengthening of horizontal competition that gives rise to the negative "entry effect" of the education subsidy on the gross growth rate of the average productivity of firms. In our model, the relative magnitude of the entry and cost reduction effects depends on the specifications of the research productivity in the product development sector. In this study, we have followed Jones (1995) to assume that the research productivity in the product development sector δ_t depends positively on the number of existing ideas n_t, but the marginal effect of n_t on δ_t is decreasing with n_t (i.e., $\psi \in [0, 1)$). Equations (37) and (38) indicate that this parameter specification plays a crucial role in deriving the results of $n_s^*(s) > 0$ and $G_s^{A*}(s) \leq 0$. Suppose that we consider an alternative specification of $\psi = 1$; then, our model will generate the counter factual scale effect prediction of economic growth. Therefore, we find the parameter specification $\psi \in [0, 1)$ to be a more reasonable specification.

4.2   Effects of the education subsidy on economic growth

In this subsection, we consider the effect of the education subsidy on economic growth. Equation (41) shows that the gross growth rate of GDP depends on the gross growth rate of the average productivity of firms G_t^A, the number of firms G_t^n, and the other market factors G_t^Λ. However, it is difficult to examine analytically all

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the dynamic properties of the other market factor Λ_t and the value of GDP G_t^{GDP}. Therefore, in this subsection, we only show numerical examples of them.

As in Figures 7-1 to 7-4, Figure 7-5 and Figure 7-6 show the numerical example of the transition path of the other market factors Λ_t (Figure 7-5) and the net growth rate of GDP g_t^{GDP} (Figure 7-6), when the education subsidy rate s in period 5 and subsequent periods are increased from 0 to 0.3. The solid line in Figure 7-6 shows the transition path of g_t^{GDP} when the degree of specialization parameter σ is set to $\sigma = 1/(\eta - 1)$ to match the Dixit-Stiglitz type specification, whereas the dashed line in Figure 7-6 shows the transition path of g_t^{GDP} when the degree of specialization parameter σ is set to 0. Suppose that $\sigma = 0$, and the productivity growth rate is solely based on the production efficiency improvement. Peretto and Conolly (2007) provide a theoretical justification that a production efficiency (or quality) improvement is the only plausible engine of economic growth in the long run.

The introduction of the education subsidy policy in period 5 deteriorates the static efficiency of production in period 5, which leads to the lower level of the other market factors in period 5 (Figure 7-5). The evolutions of g_t^A in Figure 7-4 and Λ_t in Figure 7-5 indicate that the education subsidy policy in period 5 provides two competing impacts upon the net growth rate of GDP in period 5 (Figure 7-6). On the one hand, as shown in Figure 7-4, the rise in the net growth rate of the average productivity of firms in period 5 positively affects the net growth rate of GDP in period 5. On the other hand, as shown in Figure 7-5, the decline in the level of the other market factors in period 5 negatively affects the net growth rate of GDP in period 5. In our baseline simulation, since the latter negative effect dominates the former positive effect, the net growth rate of GDP in period 5 becomes lower than that in the original steady-state equilibrium. The sensitivity analyses indicate that this prediction holds for a wide range of plausible parameter values that satisfy both (36) and $\gamma(\eta - 1) < 1$.

However, as shown in Figure 7-2 and 7-3, the number of firms starts to increase from period 6, which increases the net growth rate of the number of firms in period 6. Therefore, if the degree of specialization parameter σ is sufficiently large, as described in the solid line in Figure 7-6, the net growth rate of GDP in period 6 becomes higher than that in the original steady-state equilibrium. However, if the degree of specialization parameter σ is sufficiently small, as described in the dashed line in Figure 7-6, the net growth rate of GDP in period 6 becomes lower than that in the original steady-state equilibrium.[12] These results imply that the short-run effect of the education subsidy on economic growth is generally ambiguous and depends on the values of the parameters. Suppose that the

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[12]The net growth rate of GDP in period 6 becomes higher than that in period 5 irrespective of the values of σ because the negative growth effect of the decline in Λ_t occurs only in period 5.

degree of specialization parameter σ is sufficiently large; then, the education subsidy may positively affect the net growth rate of GDP in the short run. However, suppose that the degree of specialization parameter σ is sufficiently small; then, the short-run effect of education subsidy on economic growth is negative. Since it is difficult to obtain a reliable estimate value of σ, the short-run effect of the education subsidy on economic growth remains inconclusive.

After period 6, the economy gradually converges to its new steady-state equilibrium. During this transition process, the net growth rate of the number of firms and the other market factors gradually approach zero. Therefore, in the steady-state equilibrium, the net growth rate of GDP becomes equivalent to the net growth rate of the average productivity of firms. Consistent with the results of (42), Figure 7-6 shows that the net growth rate of GDP in the new steady-state equilibrium becomes lower than that in the original steady-state equilibrium. Therefore, irrespective of the value of σ, the long-run effect of the education subsidy on economic growth is negative.

These numerical simulation results indicate that when the market structure adjusts partially in the short run, the growth effect of the education subsidy is ambiguous and depends upon the values of the parameters. However, when market structure adjusts fully in the long run, the education subsidy expands the number of firms but reduces economic growth.

5   Concluding Remarks

Employing a two-period overlapping generations model of R&D-based growth with both product development and process innovation, this paper examined how a subsidy policy for encouraging more individuals to receive higher education affects the per capita GDP growth rate of the economy. We showed that when the market structure adjusts partially in the short run, the effect of the education subsidy on economic growth is ambiguous and depends on the values of the parameters. However, when the market structure adjusts fully in the long run, the education subsidy expands the number of firms but reduces economic growth. These unfavorable predictions of the education subsidy on economic growth are partly consistent with empirical findings that mass higher education does not necessarily lead to higher economic growth. A higher education subsidy policy is perhaps inappropriate for the purpose of stimulating long-run economic growth.

To clarify our main arguments, we employ a simple, tractable growth model with some restrictive specifications and ignore various important elements of higher education, such as credit constraints due to family income inequality and uncertainty of educational outcomes. Although these specifications enable us to obtain an intuitive clear-cut prediction regarding the effect of the higher education sub-

sidy on R&D-based growth, some of them are overly restrictive from an empirical perspective. Therefore, the application of our simple framework to assess the likely impact of policy reform is obviously limited. Here, we note several limitations of our specifications and briefly discuss directions for future research. First, for analytical tractability, this paper assumes that the product development firms inventing new varieties have to incur R&D expenditure one period in advance of production, whereas intermediate goods firms can improve their efficiency of production through their in-house process innovation instantaneously. This asymmetric specification of product development and process innovation improve the tractability of the model greatly without altering the main predictions of this paper. Nevertheless, it will be interesting to consider alternative specifications of R&D activities. Second, since this paper uses a two-period OLG framework, if we employ a very straightforward interpretation, one period in our model is interpreted as approximately 30 years. The concept of "short run" in our model does not match the concept of "short run" in the real world, which makes the comparison of our theoretical results with actual data slightly difficult. Therefore, to evaluate the likely impact of a higher education subsidy on economic growth more precisely, it is necessarily to develop a more elaborate numerical version of the large-scale OLG model. Third, this paper only focuses on the growth implications of an education subsidy policy and cannot propose a reasonable framework to analyze the welfare and distributional implications of a higher education subsidy policy. The investigation of these issues in a more elaborate version of a R&D-based growth model that fully accounts for the important properties of higher education is a promising direction for future research.

Appendix

Appendix A: Properties of $H(\omega_t; s)$ and $L(\omega_t; s)$

Differentiating $H(\omega_t; s)$ with respect to ω_t and s yields:

$$\frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)} = -\frac{\omega_t}{H(\omega_t; s)} M\hat{\omega}_t\phi(\hat{\omega}_t)\frac{1-s}{(1-s\omega_t)^2} < 0, \qquad (A.1)$$

$$\frac{sH_s(\omega_t; s)}{H(\omega_t; s)} = \frac{s}{H(\omega_t; s)} M\hat{\omega}_t\phi(\hat{\omega}_t)\frac{\omega_t(1-\omega_t)}{(1-s\omega_t)^2} > 0, \qquad (A.2)$$

where $\hat{\omega}_t = \hat{\omega}(\omega_t; s)$ and $\phi(\hat{\omega}_t)$ is the value of the probability density function of θ, $\phi(\theta)$, evaluated at $\theta = \hat{\omega}_t$. Analogously, differentiating $L(\omega_t; s)$ with respect to ω_t and s yields

$$\frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)} = \frac{\omega_t}{L(\omega_t; s)} M\phi(\hat{\omega}_t)\frac{1-s}{(1-s\omega_t)^2} > 0, \qquad (A.3)$$

$$\frac{sL_s(\omega_t; s)}{L(\omega_t; s)} = -\frac{s}{L(\omega_t; s)}M\phi(\hat{\omega}_t)\frac{\omega_t(1 - \omega_t)}{(1 - s\omega_t)^2} < 0. \tag{A.4}$$

Appendix B: Intermediate goods firms' profit maximization

There are two steps in the intermediate goods firms' profit maximization problem. First, the intermediate goods firms conduct process innovation, and second, they produce the intermediate goods. We solve the problems backward. In the second step, we maximize an intermediate goods firm's profit from the production given its productivity and the R&D outlay. Then, we turn back to the first step and decide how much the intermediate goods firms should invest in process innovation.

Second step

The jth intermediate goods firm's profit maximization problem in the second step can be written as follows:

$$\hat{\pi}_t(j) \equiv \max \{p_t(j)x_t(j) - [w_t^s l_t^s(j) + w_t^u l_t^u(j)]\},$$

subject to (9) and (10), where $\hat{\pi}_t(j)$ is a profit function in this step of the problem, given its productivity $A_t(j)$. We first minimize the production costs of $w_t^s l_t^s(j) + w_t^u l_t^u(j)$ subject to (10), which yields the following unit cost function as well as both skilled and unskilled labor demand functions of intermediate goods firm j:

$$c_t(j) = \frac{\hat{c}_t}{A_t(j)},$$

$$l_t^s(j) = \alpha \frac{\hat{c}_t x_t(j)}{w_t^s A_t(j)}, \tag{A.5}$$

$$l_t^u(j) = (1 - \alpha)\frac{\hat{c}_t x_t(j)}{w_t^u A_t(j)}, \tag{A.6}$$

where $\hat{c}_t \equiv \frac{(w_t^s)^\alpha (w_t^u)^{(1-\alpha)}}{\alpha^\alpha (1-\alpha)^{(1-\alpha)}}$. Thus, the jth intermediate goods firm's profit maximization problem in this step can be rewritten as follows:

$$\hat{\pi}_t(j) \equiv \max \left\{ [p_t(j) - \frac{\hat{c}_t}{A_t(j)}]x_t(j) \right\},$$

subject to (9). The first-order condition yields the optimal price and output of firm j given its productivity and prices and productivities of other firms as follows:

$$p_t(j) = \frac{\eta}{\eta - 1}\frac{\hat{c}_t}{A_t(j)}, \tag{A.7}$$

$$x_t(j) = \frac{\eta - 1}{\eta} \frac{A_t(j)^\eta}{\hat{c}_t \int_0^{n_t} A_t(j)^{\eta-1} dj} P_t Y_t. \tag{A.8}$$

Substituting (A.7) and (A.8) into $\hat{\pi}_t(j)$ yields the profit function as follows:

$$\hat{\pi}_t(j) = \frac{1}{\eta} \frac{A_t(j)^{\eta-1}}{\int_0^{n_t} A_t(j)^{\eta-1} dj} P_t Y_t. \tag{A.9}$$

First step

Then, turn back to the first step. The object of this step is to maximize the inter-mediate goods firm's net profits $\pi_t(j)$ defined in (12). Using the results obtained in the second step, the jth intermediate goods firm's profit maximization problem in the first step can be written as follows:

$$\pi_t(j) \equiv \max \left\{ \hat{\pi}_t(j) - w_t^s l_t^R(j) \right\},$$

subject to (11) and (A.9). The first-order condition with respect to $l_t^R(j)$ is:

$$\gamma(\eta - 1) \frac{\hat{\pi}_t(j)}{l_t^R(j)} = w_t^s. \tag{A.10}$$

Equation (A.10) implies that $l_t^R(j)$ is independent on j and so are $A_t(j)$, $p_t(j)$ and $x_t(j)$. Therefore, we can omit the index j, and thus, equations (A.7) to (A.9) are rewritten as follows:

$$p_t = \frac{\eta}{\eta - 1} \frac{\hat{c}_t}{A_t}, \tag{A.11}$$

$$x_t = \frac{\eta - 1}{\eta} \frac{A_t}{\hat{c}_t} \frac{P_t Y_t}{n_t}, \tag{A.12}$$

$$\hat{\pi}_t = \frac{1}{\eta} \frac{P_t Y_t}{n_t}. \tag{A.13}$$

By substituting (A.12) into (A.5) and (A.6) and rearranging them, we can obtain the optimal level of skilled and unskilled labor inputs for intermediate goods production as (13) and (14). Moreover, substituting (A.13) into (A.10) yields the optimal level of skilled labor engaged in process innovation as (15).

Appendix C: The market-clearing condition for assets

Due to perfect competition in the final goods market, the value of final goods output is expressed as follows:

$$P_{c,t} Y_t = n_t p_t x_t.$$

Thus, using the profits of intermediate goods firms $\pi_t = p_t x_t - w_t^s l_t^s - w_t^u l_t^u - w_t^s l_t^R$ and equations (20), (22) and (23), the above equation can be rewritten as follows:

$$P_{c,t}Y_t = w_t^s H(\omega_t; s) + w_t^u L(\omega_t; s) - w_t^s L_t^N + n_t(V_{t-1}R_t - V_t).$$

Using (21), the above equation can be rewritten as follows:

$$P_{c,t}Y_t = (1-\tau_t) \int_{\hat{\omega}(\omega_t;s)}^1 [w_t^s \theta + s w_t^u(1-\theta)] M d\Phi(\theta) + (1-\tau_t)w_t^u L(\omega_t;s) - w_t^s L_t^N + n_t(V_{t-1}R_t - V_t).$$

Therefore, the market-clearing condition for final goods is expressed in the following manner:

$$\sum_{i=1,2} \int_0^1 P_{c,t}C_{i,t}^\theta M d\Phi(\theta) = (1-\tau_t) \int_{\hat{\omega}(\omega_t;s)}^1 [w_t^s \theta + s w_t^u(1-\theta)] M d\Phi(\theta)$$
$$+ (1-\tau_t)w_t^u L(\omega_t;s) - w_t^s L_t^N + n_t(V_{t-1}R_t - V_t).$$

In the case of $V_t\delta_t = w_t^s$, $L_t^N > 0$ and $n_{t+1} > n_t$

With respect to (19), consider the case of $V_t\delta_t = w_t^s$ in which the product development sector functions: $L_t^N > 0$; and $n_{t+1} > n_t$. By substituting (2), (3), (5), (6), (17) and $V_t\delta_t = w_t^s$ into the market-clearing condition for final goods, we obtain the following expression:

$$\int_0^1 S_t^\theta M d\Phi(\theta) - V_t n_{t+1} = R_t \left[\int_0^1 S_{t-1}^\theta M d\Phi(\theta) - V_{t-1}n_t \right].$$

Because initial assets are given by $\int_0^1 S_{-1}^\theta M d\Phi(\theta) = V_{-1}n_0$, we can obtain the following asset market equilibrium condition:

$$V_t n_{t+1} = \int_0^1 S_t^\theta M d\Phi(\theta), \quad \text{for } V_t\delta_t = w_t^s.$$

In the case of $V_t\delta_t < w_t^s$, $L_t^N = 0$ and $n_{t+1} = n_t$

With respect to (19), consider the case of $V_t\delta_t < w_t^s$ in which the product development sector does not function; $L_t^N = 0$ and $n_{t+1} = n_t$. By substituting (2), (3), (5), (6) and $L_t^N = 0$ into the market-clearing condition for final goods, we obtain the following expression:

$$\int_0^1 S_t^\theta M d\Phi(\theta) - V_t n_t = R_t \left[\int_0^1 S_{t-1}^\theta M d\Phi(\theta) - V_{t-1}n_t \right].$$

Because the initial assets are given by $\int_0^1 S_{-1}^\theta M d\Phi(\theta) = V_{-1} n_0$, we obtain the following asset market equilibrium condition:

$$V_t n_t = \int_0^1 S_t^\theta M d\Phi(\theta), \quad \text{for } V_t \delta_t < w_t^s.$$

Appendix D: Properties of $\omega(n_t; s)$ and $\omega^*(s)$

Derivations of (27)

Using (13), (15) and (22), we obtain the following equation:

$$H(\omega_t; s) - L_t^N = n_t(l_t^s + l_t^R) = (\alpha + \gamma)\frac{\eta - 1}{\eta}\frac{P_t Y_t}{w_t^s}. \tag{A.14}$$

Furthermore, substituting (14) into (23), we obtain the following equation:

$$L(\omega_t; s) = (1 - \alpha)\frac{\eta - 1}{\eta}\frac{P_t Y_t}{w_t^u}. \tag{A.15}$$

Using (A.14) and (A.15), L_t^N can be expressed as (27).

Properties of $\omega(n_t; s)$

From (28), by differentiating ω_t with respect to n_t and s, with noting $\left(\frac{n_t}{\delta_t} + \frac{1}{1+\beta}H(\omega_t; s)\right)\frac{1}{\omega_t} = \left(\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha}\right)L(\omega_t; s)$, we obtain:

$$\frac{n_t \omega_n(n_t; s)}{\omega(n_t; s)} = \frac{(1 - \psi)\frac{n_t}{\delta_t}}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)}\right) + \frac{H(\omega_t; s)}{1+\beta}\left(1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)} - \frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)}\right)} > 0, \tag{A.16}$$

$$\frac{s \omega_s(n_t; s)}{\omega(n_t; s)} = -\frac{\left(\frac{n_t}{\delta_t} + \frac{H(\omega_t; s)}{1+\beta}\right)\frac{sL_s(\omega_t; s)}{L(\omega_t; s)} - \frac{H(\omega_t; s)}{1+\beta}\frac{sH_s(\omega_t; s)}{H(\omega_t; s)}}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)}\right) + \frac{H(\omega_t; s)}{1+\beta}\left(1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)} - \frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)}\right)} > 0, \tag{A.17}$$

where $\omega_t = \omega(n_t; s)$.

Properties of $\omega^*(s)$

From (29), the differentiation of $\Gamma(\omega_t; s)$ with respect to ω_t and s yields:

$$\frac{\omega_t \Gamma_\omega(\omega_t; s)}{\Gamma(\omega_t; s)} = 1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)} - \frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)} > 0, \tag{A.18}$$

$$\frac{s \Gamma_s(\omega_t; s)}{\Gamma(\omega_t; s)} = \frac{sL_s(\omega_t; s)}{L(\omega_t; s)} - \frac{sH_s(\omega_t; s)}{H(\omega_t; s)} < 0. \tag{A.19}$$

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Hence, by differentiating $\omega^*(s)$ with respect to s, we obtain:

$$\frac{s\omega_s^*(s)}{\omega^*(s)} = -\frac{\frac{sL_s(\omega^*;s)}{L(\omega^*;s)} - \frac{sH_s(\omega^*;s)}{H(\omega^*;s)}}{1 + \frac{\omega^*L_\omega(\omega^*;s)}{L(\omega^*;s)} - \frac{\omega^*H_\omega(\omega^*;s)}{H(\omega^*;s)}} > 0, \qquad (A.20)$$

where $\omega^* = \omega^*(s)$.

Appendix E: Properties of $\hat{H}(n_t; s)$ and $\hat{H}^*(s)$

Properties of $\hat{H}(n_t; s)$

From (32), differentiating $\hat{H}(n_t; s)$ with respect to n_t yields:

$$\frac{n_t\hat{H}_n(n_t; s)}{\hat{H}(n_t; s)} = \frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)} \frac{n_t\omega_n(n_t; s)}{\omega(n_t; s)} < 0, \qquad (A.21)$$

where $\omega_t = \omega(n_t; s)$. Note that the relations $\frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)} < 0$ and $\frac{n_t\omega_n(n_t;s)}{\omega(n_t;s)} > 0$ hold from (A.1) and (A.16).

Analogously, from (32), differentiating $\hat{H}(n_t; s)$ with respect to s yields:

$$\frac{s\hat{H}_s(n_t; s)}{\hat{H}(n_t; s)} = \frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)} \frac{s\omega_s(n_t; s)}{\omega(n_t; s)} + \frac{sH_s(\omega_t; s)}{H(\omega_t; s)},$$

where $\omega_t = \omega(n_t; s)$. Then, by substituting (A.17) into above equation and rearranging it, we obtain:

$$\frac{s\hat{H}_s(n_t; s)}{\hat{H}(n_t; s)} = \frac{\left(\frac{n_t}{\delta_t} + \frac{H(\omega_t;s)}{1+\beta}\right)\left[\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)}\right)\frac{sH_s(\omega_t;s)}{H(\omega_t;s)} - \frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)}\frac{sL_s(\omega_t;s)}{L(\omega_t;s)}\right]}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)}\right) + \frac{H(\omega_t;s)}{1+\beta}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)} - \frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)}\right)},$$

where $\omega_t = \omega(n_t; s)$. From equations (A.1) to (A.4), we can see that the relations $\frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)} = -\frac{\omega_t H_\omega(\omega_t;s)}{L(\omega_t;s)\hat{\omega}_t}$ and $\frac{sL_s(\omega_t;s)}{L(\omega_t;s)} = -\frac{sH_s(\omega_t;s)}{L(\omega_t;s)\hat{\omega}_t}$ hold, where $\hat{\omega}_t = \omega(\omega_t; s)$. Thus, by substituting these relations into the above equation, we obtain:

$$\frac{s\hat{H}_s(n_t; s)}{\hat{H}(n_t; s)} = \frac{\left(\frac{n_t}{\delta_t} + \frac{H(\omega_t;s)}{1+\beta}\right)\frac{sH_s(\omega_t;s)}{H(\omega_t;s)}}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)}\right) + \frac{H(\omega_t;s)}{1+\beta}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)} - \frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)}\right)} > 0, \quad (A.22)$$

where $\omega_t = \omega(n_t; s)$.

Properties of $\hat{H}^*(s)$

From (32), differentiating $\hat{H}^*(s)$ with respect to s yields:

$$\frac{s\hat{H}^*(s)}{\hat{H}^*(s)} = \frac{\omega^* H_\omega(\omega^*; s)}{H(\omega^*; s)} \frac{s\omega_s^*(s)}{\omega^*} + \frac{sH_s(\omega^*; s)}{H(\omega^*; s)},$$

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where $\omega^* = \omega^*(s)$. Then, by substituting (A.20) into the above equation and rearranging it, we obtain:

$$\frac{s\hat{H}^*(s)}{\hat{H}^*(s)} = \frac{\left(1 + \frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)}\right)\frac{sH_s(\omega^*)}{H(\omega^*;s)} - \frac{\omega^* H_\omega(\omega^*;s)}{H(\omega^*;s)}\frac{sL_s(\omega^*;s)}{L(\omega^*;s)}}{1 + \frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)} - \frac{\omega^* H_\omega(\omega^*;s)}{H(\omega^*;s)}},$$

where $\omega^* = \omega^*(s)$. From equations (A.1) to (A.4), we can see that the relations $\frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)} = -\frac{\omega^* H_\omega(\omega^*;s)}{L(\omega^*;s)\hat{\omega}}$ and $\frac{sL_s(\omega^*;s)}{L(\omega^*;s)} = -\frac{sH_s(\omega^*;s)}{L(\omega^*;s)\hat{\omega}}$ hold, where $\hat{\omega} \equiv \hat{\omega}(\omega^*; s)$. Thus, by substituting these relations into above equation, we obtain:

$$\frac{s\hat{H}^*(s)}{\hat{H}^*(s)} = \frac{\frac{sH_s(\omega^*;s)}{H(\omega^*;s)}}{1 + \frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)} - \frac{\omega^* H_\omega(\omega^*;s)}{H(\omega^*;s)}} > 0, \tag{A.23}$$

where $\omega^* = \omega^*(s)$.

Appendix F: Properties of $G^n(n_t; s)$

Properties of $\hat{\Gamma}(n_t; s)$

From (35), differentiating $\hat{\Gamma}(n_t; s)$ with respect to n_t yields:

$$\frac{n_t\hat{\Gamma}_n(n_t; s)}{\hat{\Gamma}(n_t; s)} = \frac{\omega_t\Gamma_\omega(\omega_t; s)}{\Gamma(\omega_t; s)}\frac{n_t\omega_n(n_t; s)}{\omega(n_t; s)} > 0, \tag{A.24}$$

where $\omega_t = \omega(n_t; s)$. Note that the relations $\frac{\omega_t\Gamma_\omega(\omega_t;s)}{\Gamma(\omega_t;s)} > 0$ and $\frac{n_t\omega_n(n_t;s)}{\omega_t} > 0$ hold from (A.16) and (A.18). Further, by substituting (A.16) and (A.18) into (A.24), with noting $\frac{n_t}{\delta_t} = H(\omega_t; s)[(\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha})\Gamma(\omega_t; s) - \frac{1}{1+\beta}]$ from (28), we obtain:

$$\frac{n_t\hat{\Gamma}_n(n_t; s)}{\hat{\Gamma}(n_t; s)} = \frac{1 - \psi}{\frac{1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)}}{1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)} - \frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)}} + \frac{1}{1+\beta}\frac{1}{\left(\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha}\right)\Gamma(\omega_t;s) - \frac{1}{1+\beta}}}, \tag{A.25}$$

where $\omega_t = \omega(n_t; s)$. Similarly, from (35), differentiating $\hat{\Gamma}(n_t; s)$ with respect to s yields:

$$\frac{s\hat{\Gamma}_s(n_t; s)}{\hat{\Gamma}(n_t; s)} = \frac{\omega_t\Gamma_\omega(\omega_t; s)}{\Gamma(\omega_t; s)}\frac{s\omega_s(n_t; s)}{\omega(n_t; s)} + \frac{s\Gamma_s(\omega_t; s)}{\Gamma(\omega_t; s)},$$

where $\omega_t = \omega(n_t; s)$. Then, by substituting equations (A.17) to (A.19) into the above equation and rearranging it, we obtain:

$$\frac{s\hat{\Gamma}_s(n_t; s)}{\hat{\Gamma}(n_t; s)} = \frac{\frac{n_t}{\delta_t}\left[\frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)}\frac{sL_s(\omega_t;s)}{L(\omega_t;s)} - \left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)}\right)\frac{sH_s(\omega_t;s)}{H(\omega_t;s)}\right]}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)}\right) + \frac{H(\omega_t;s)}{1+\beta}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)} - \frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)}\right)},$$

where $\omega_t = \omega(n_t; s)$. From equations (A.1) to (A.4), we can see that the relations $\frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)} = -\frac{\omega_t H_\omega(\omega_t; s)}{L(\omega_t; s)\hat{\omega}_t}$ and $\frac{sL_s(\omega_t; s)}{L(\omega_t; s)} = -\frac{sH_s(\omega_t; s)}{L(\omega_t; s)\hat{\omega}_t}$ hold, where $\hat{\omega}_t = \omega(\omega_t; s)$. Thus, by substituting these relations into the above equation, we obtain:

$$\frac{s\hat{\Gamma}_s(n_t; s)}{\hat{\Gamma}(n_t; s)} = -\frac{\frac{n_t}{\delta_t} \frac{sH_s(\omega_t; s)}{H(\omega_t; s)}}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)}\right) + \frac{H(\omega_t; s)}{1+\beta}\left(1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)} - \frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)}\right)} < 0, \quad \text{(A.26)}$$

where $\omega_t = \omega(n_t; s)$.

The relationship between $F\left(\hat{\Gamma}(n_t; s)\right)$ and $\hat{\Gamma}(n_t; s)$

From (35), the differentiation of $F\left(\hat{\Gamma}(n_t; s)\right)$ with respect to $\hat{\Gamma}(n_t; s)$ yields:

$$\frac{\hat{\Gamma}(n_t; s)F_{\hat{\Gamma}}\left(\hat{\Gamma}(n_t; s)\right)}{F\left(\hat{\Gamma}(n_t; s)\right)} = \frac{\frac{1+\beta}{\beta}\frac{1+\gamma}{1-\alpha}\frac{\hat{\Gamma}(n_t; s)}{(1+\hat{\Gamma}(n_t; s))^2}}{F\left(\hat{\Gamma}(n_t; s)\right)} > 0. \quad \text{(A.27)}$$

Properties of $G^n(n_t; s)$

From (35), differentiating $G^n(n_t; s)$ with respect to n_t yields:

$$\frac{n_t G_n^n(n_t; s)}{G^n(n_t; s)} = -\frac{\hat{\Gamma}(n_t; s)F_{\hat{\Gamma}}\left(\hat{\Gamma}(n_t; s)\right)}{F\left(\hat{\Gamma}(n_t; s)\right)}\frac{n_t\hat{\Gamma}_n(n_t; s)}{\hat{\Gamma}(n_t; s)} < 0, \quad \text{(A.28)}$$

where the relations $\frac{\hat{\Gamma}(n_t; s)F_{\hat{\Gamma}}\left(\hat{\Gamma}(n_t; s)\right)}{F\left(\hat{\Gamma}(n_t; s)\right)} > 0$ and $\frac{n_t\hat{\Gamma}_n(n_t; s)}{\hat{\Gamma}(n_t; s)} > 0$ hold from (A.24) and (A.27).

Analogously, from (35), differentiating $G^n(n_t; s)$ with respect to s yields:

$$\frac{sG_s^n(n_t; s)}{G^n(n_t; s)} = -\frac{\hat{\Gamma}(n_t; s)F_{\hat{\Gamma}}\left(\hat{\Gamma}(n_t; s)\right)}{F\left(\hat{\Gamma}(n_t; s)\right)}\frac{s\hat{\Gamma}_s(n_t; s)}{\hat{\Gamma}(n_t; s)} > 0, \quad \text{(A.29)}$$

where the relations $\frac{\hat{\Gamma}(n_t; s)F_{\hat{\Gamma}}\left(\hat{\Gamma}(n_t; s)\right)}{F\left(\hat{\Gamma}(n_t; s)\right)} > 0$ and $\frac{s\hat{\Gamma}_s(n_t; s)}{\hat{\Gamma}(n_t; s)} < 0$ hold from (A.26) and (A.27).

Appendix G: Proof of Proposition 1

The steady state values $\left\{n^*, G^{A*}\right\}$

When the product development sector does not operate (i.e., $n_{t+1} = n_t$), from (29), we can see that the relation $\omega^* L(\omega^*; s) = \frac{1-\alpha}{\alpha+\gamma}H(\omega^*; s)$ holds. Therefore, substituting $\omega^* L(\omega^*; s) = \frac{1-\alpha}{\alpha+\gamma}H(\omega^*; s)$ and $\omega^* = \omega^*(s)$ into (28) yields the steady-state number of firms as (37). Moreover, substituting (32) and (37) into (33) yields the steady-state gross growth rate of the average productivity of firms as (38).

Local stability

By differentiating $G^n(n_t; s)n_t$ with respect to n_t and evaluating it at $n_t = n^*$, we obtain:

$$\frac{dn_{t+1}}{dn_t}\Big|_{n_t=n^*} = 1 + \frac{n^* G_n^n(n^*; s)}{G^n(n^*; s)}\Big|_{n_t=n^*},$$

where the relation $G^n(n^*; s) = 1$ holds. Therefore, by substituting (A.25), (A.27) and (A.28) into the above equation and evaluating it at $n_t = n^*$ with noting $\hat{\Gamma}(n^*; s) = \frac{\omega^* L(\omega^*; s)}{H(\omega^*; s)} = \frac{1-\alpha}{\alpha+\gamma}$, and $\Gamma(\omega^*; s) = 1$, we obtain:

$$\frac{dn_{t+1}}{dn_t}\Big|_{n_t=n^*} = 1 - (1+\beta)(1-\psi)\frac{\alpha+\gamma}{\alpha+\gamma+\beta(1+\gamma)\frac{1+\frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)}}{1+\frac{\omega^* L_\omega(\omega^*;s)}{(\omega^*;s)L} - \frac{\omega^* H_\omega(\omega^*;s)}{H(\omega^*;s)}}}, \tag{A.30}$$

where $\omega^* = \omega^*(s)$. From (A.30), suppose that the parameter conditions of (36) hold; then, we can confirm that the relation $\frac{dn_{t+1}}{dn_t}\Big|_{n_t=n^*} \in [0, 1)$ holds.

Appendix H: Properties of GDP_t

Derivations of (40)

From (A.15), we obtain:

$$Y_t = \frac{\eta}{(1-\alpha)(\eta-1)}\frac{w_t^s}{P_t}\omega_t L(\omega_t; s). \tag{A.31}$$

Further, substituting (A.11) into $P_t = n_t^{\frac{1}{\eta-1}-\sigma}\left(\int_0^{n_t} p_t(j)^{1-\eta}dj\right)^{1/(1-\eta)}$ yields:

$$\frac{w_t^s}{P_t} = \frac{\eta-1}{\eta}\alpha^\alpha(1-\alpha)^{1-\alpha}\frac{A_t n_t^\sigma}{\omega_t^{1-\alpha}}. \tag{A.32}$$

Using (A.31) and (A.32), we obtain:

$$Y_t = \frac{1}{1-\alpha}\frac{\omega_t L(\omega_t; s)}{\omega_t^{1-\alpha}}\alpha^\alpha(1-\alpha)^{1-\alpha}A_t n_t^\sigma. \tag{A.33}$$

Furthermore, substituting (17), (19) (A.14) and $P_{c,t} = P_t$ into $\frac{V_t}{P_{c,t}}(n_{t+1}-n_t)$ yields:

$$\frac{V_t}{P_{c,t}}(n_{t+1}-n_t) = \frac{w_t^s}{P_t}H(\omega_t; s) - (\alpha+\gamma)\frac{\eta-1}{\eta}Y_t. \tag{A.34}$$

Therefore, by substituting equations (A.32) to (A.34) into the definition of GDP in (39), we obtain:

$$GDP_t = \left[\frac{\eta-1}{\eta}\frac{H(\omega_t; s)}{\omega_t^{1-\alpha}} + \frac{1-(\alpha+\gamma)\frac{\eta-1}{\eta}}{1-\alpha}\frac{\omega_t L(\omega_t; s)}{\omega_t^{1-\alpha}}\right]\alpha^\alpha(1-\alpha)^{1-\alpha}A_t n_t^\sigma. \tag{A.35}$$

Note that substituting (30) into (A.33) and (A.35) yields (40).

Properties of $\Lambda^*(s)$

In the steady-state equilibrium, from (29), the following relation holds:

$$\omega^*(s)\hat{L}^*(s) = \hat{H}^*(s)\frac{1-\alpha}{\alpha+\gamma}.$$

Thus, $\Lambda^*(s)$ in (40) can be rewritten as follows:

$$\Lambda^*(s) = \frac{1}{\alpha+\gamma}\frac{\hat{H}^*(s)}{\omega^*(s)^{1-\alpha}}.$$

By differentiating the above equation with respect to s, we obtain:

$$\frac{s\Lambda_s^*(s)}{\Lambda^*(s)} = \frac{s\hat{H}_s^*(s)}{\hat{H}^*(s)} - (1-\alpha)\frac{s\omega_s^*(s)}{\omega^*(s)}.$$

Substituting (A.20) and (A.23) into the above equation yields:

$$\frac{s\Lambda_s^*(s)}{\Lambda^*(s)} = \frac{(1-\alpha)\frac{sL_s(\omega^*;s)}{L(\omega^*;s)} + \alpha\frac{sH_s(\omega^*;s)}{H(\omega^*;s)}}{1 + \frac{\omega^*L_\omega(\omega^*;s)}{L(\omega^*;s)} - \frac{\omega^*H_\omega(\omega^*;s)}{H(\omega^*;s)}},$$

where $\omega^* = \omega^*(s)$. From equations (A.2) and (A.4), we can see that the relation $\frac{sL_s(\omega^*;s)}{L(\omega^*;s)} = -\frac{sH_s(\omega^*;s)}{L(\omega^*;s)\hat{\omega}}$ holds, where $\hat{\omega} \equiv \hat{\omega}(\omega^*;s)$. Thus, by substituting (29) and $\frac{sL_s(\omega^*;s)}{L} = -\frac{sH_s(\omega^*;s)}{L\hat{\omega}}$ into the above equation, we obtain:

$$\frac{s\Lambda_s^*(s)}{\Lambda^*(s)} = \frac{\left[-(\alpha+\gamma)\frac{\omega^*}{\hat{\omega}} + \alpha\right]\frac{sH_s(\omega^*;s)}{H}}{1 + \frac{\omega^*L_\omega(\omega^*;s)}{L} - \frac{\omega^*H_\omega(\omega^*;s)}{H}} < 0,$$

where $\omega^* = \omega^*(s)$, and the relation $\frac{\omega^*}{\hat{\omega}} = \frac{1-s\omega^*}{1-s} > 1$ holds from (1).

Appendix I: The parameters for simulation

We set the discount factor (β) to $(0.98)^{30}$, since the one period in this model is assumed to be approximately 30 years. According to Alvarez-Pelaez and Groth (2005), markup estimates in the US industry range between 1.05 and 1.40. Based on this estimate, the substitution parameter (η) is set to 6. We also set the degree of specialization parameter (σ) to $1/(\eta-1)$ to match the Dixit-Stiglitz-type specification. Further, to satisfy the second-order condition for maximization $\gamma(\eta-1) < 1$, we set the in-house R&D efficiency parameter (γ) to 0.15.

According to the OECD (2015), the average skill premium for the OECD measured as the relative wages of those with a university degree relative to a high

school education is approximately 1.5. Based on this estimate, the elasticity of skilled labor inputs (α) is set to 0.3. In addition, in accordance with the example of Strulik et al. (2013), we set the standing-on-shoulders effect parameter (ψ) to 0.35 and normalize the scaling parameter of R&D production ($\bar{\delta}$) to 1.

The population size (M) and the initial value of average productivity of firms (A_0) are normalized to 1, whereas the initial value of the number of firms (n_0) is set to 0.1 to ensure that the relation $\omega_0 = \omega_n(n_0; s) < \omega^*$ holds.

To achieve an approximately 2.5% balanced GDP growth rate at an education subsidy rate (s) of 0, we adjust the value of z to $z = 2.177$. In addition, to investigate the effect of the education subsidy policy, we set the education subsidy rate (s) to 0 in the base case simulation and changed it from 0 to 0.9 in increments of 0.1.

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Figure1 Distribution of public and private expenditure on tertiary educational institutions (2013)
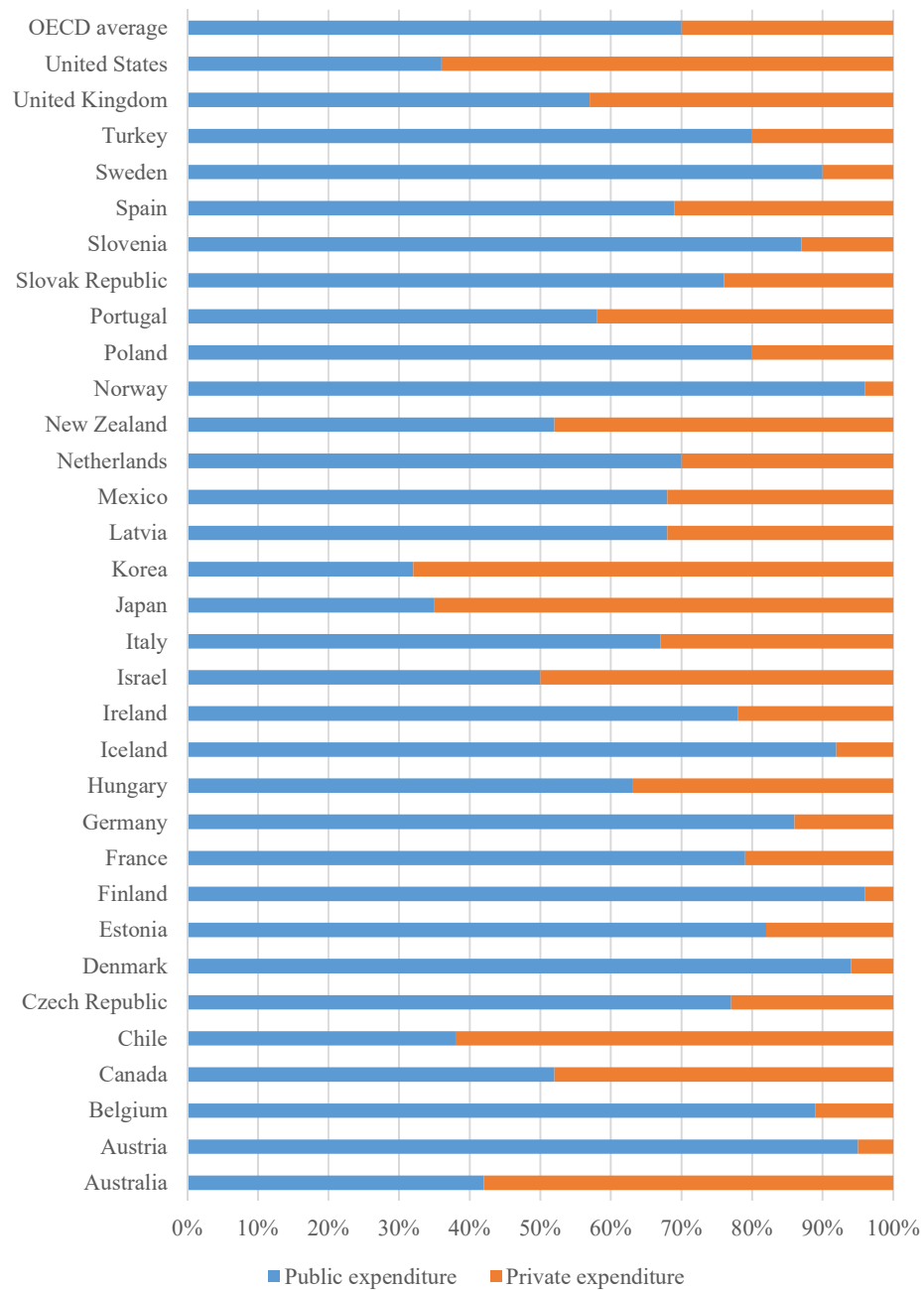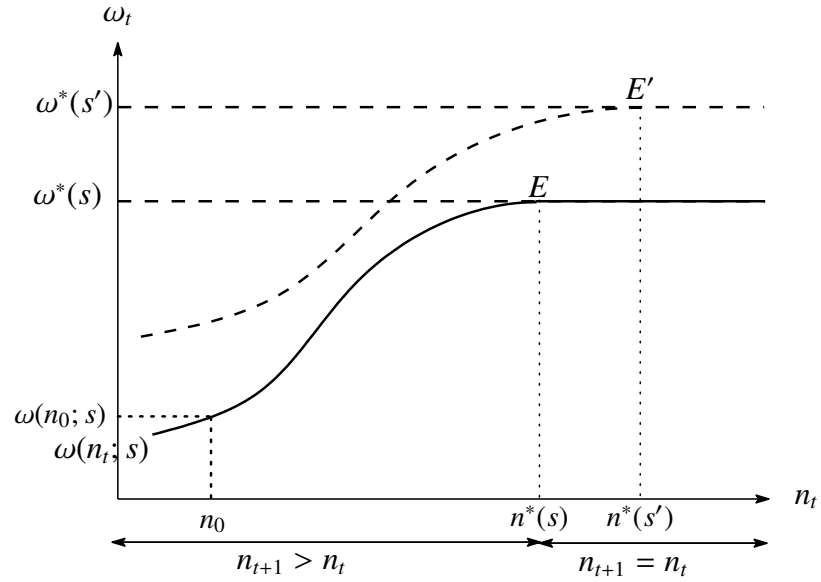
Figure 2: The relationship between the number of firms and the relative wage of unskilled/skilled workers ($s < s'$)
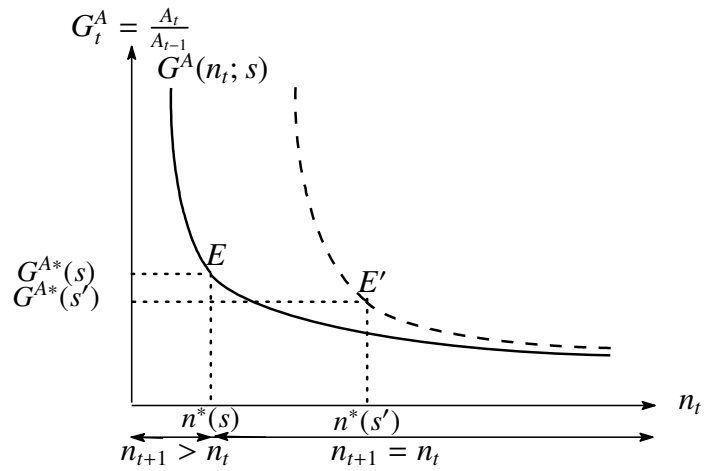

Figure 3: The relationship between the number of firms and the gross growth rate of the average productivity of firms ($s < s'$)
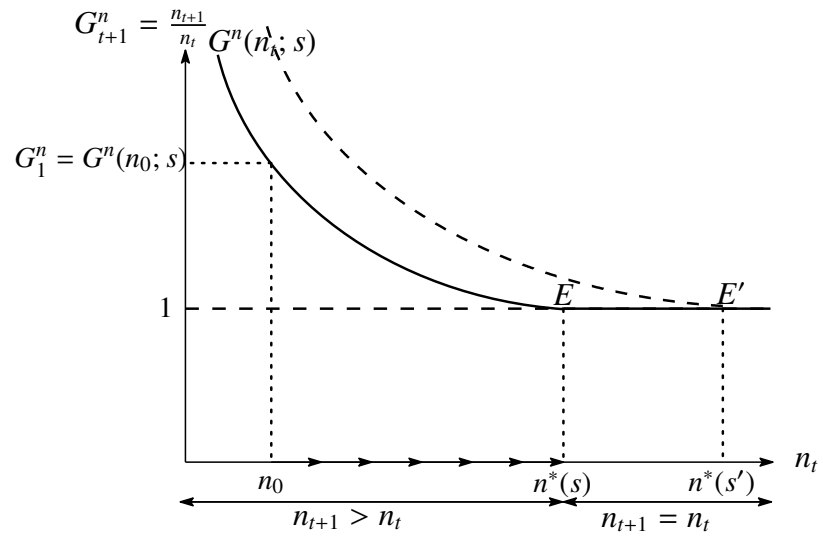
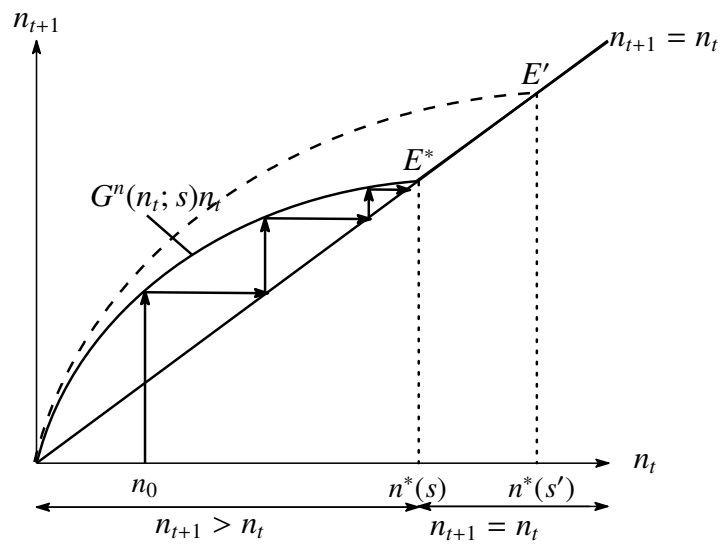Figure 4: The relationship between the number of firms and the gross growth rate of the number of firms ($s < s'$)

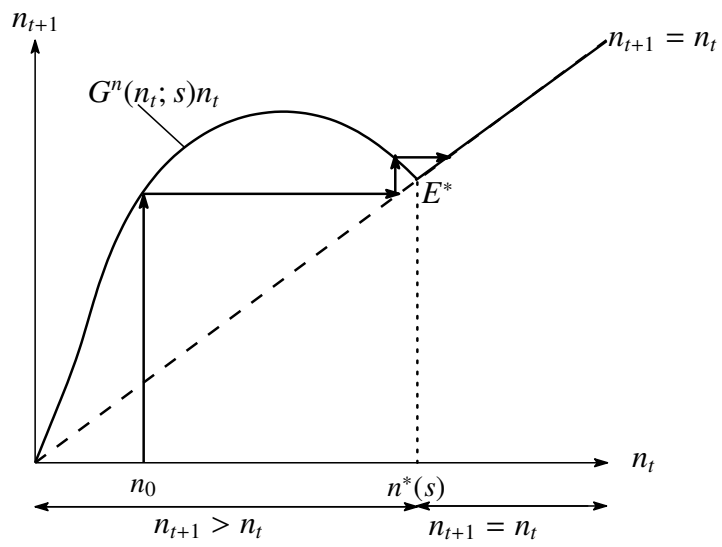Figure 5: The possible dynamics of n_t when the parameter conditions of (36) are satisfied ($s < s'$)

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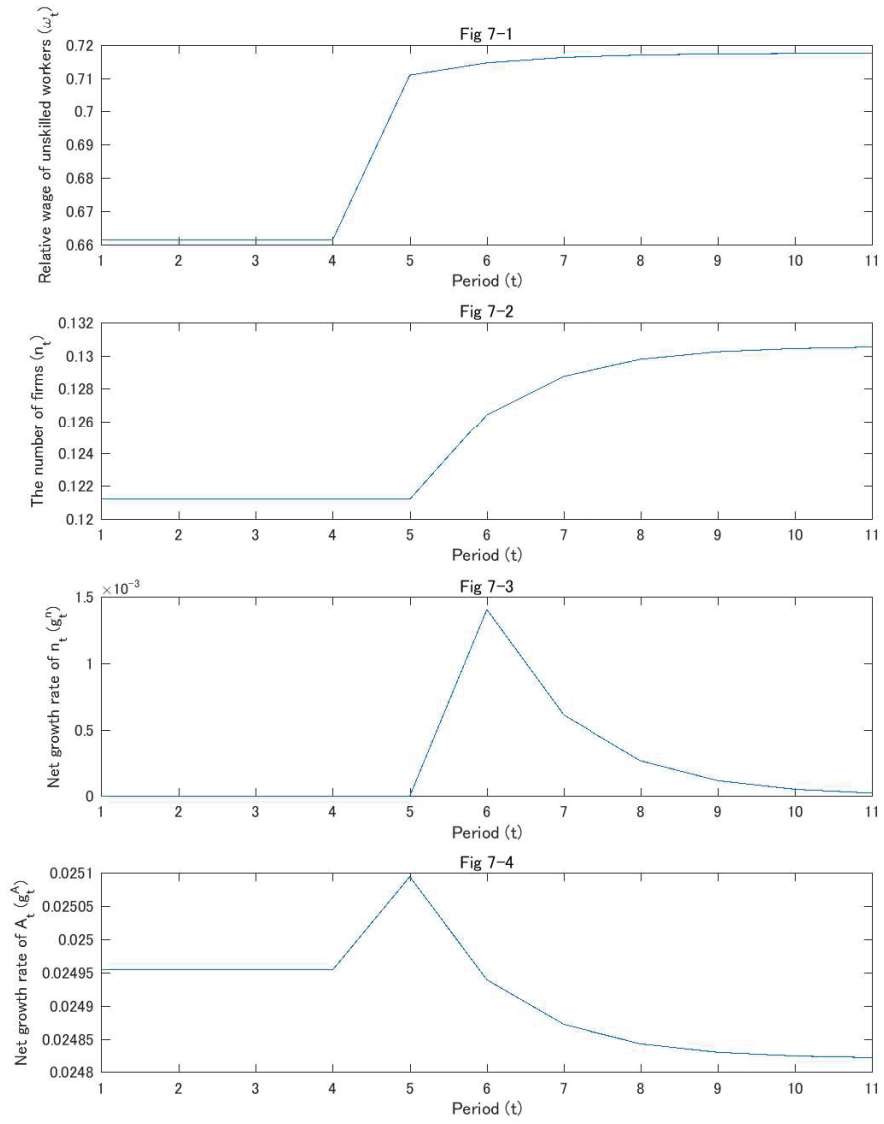Figure 6: The possible dynamics of n_t when the parameter conditions of (36) are not satisfied

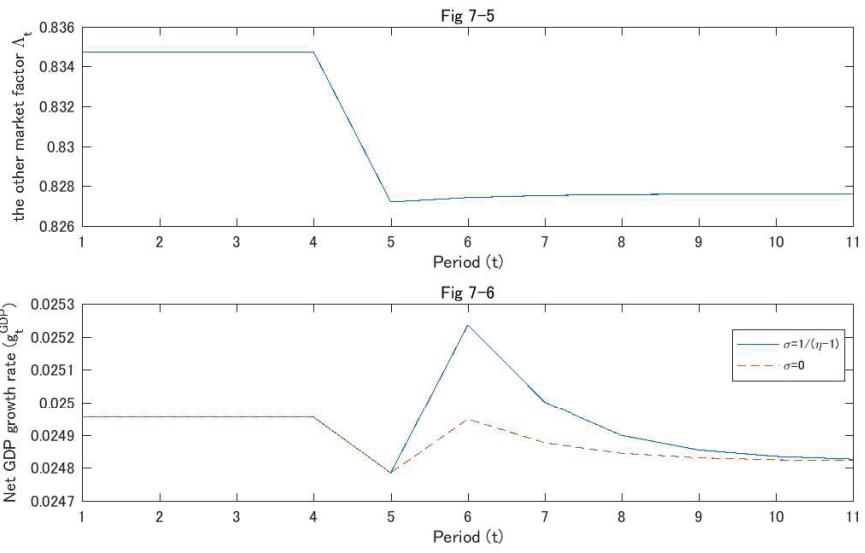Figure 7: The effects of a rise in education subsidy rate *s*

Figure 7: The effects of a rise in education subsidy rate *s* (Cont.)