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Higher Education Subsidy Policy and R&D-based Growth

Takaaki Morimoto
Graduate School of Economics, Osaka University, Japan
Institute of Social and Economic Research, Osaka University, Japan

Ken Tabata
School of Economics, Kwansei Gakuin University, Japan

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KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

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Takaaki Morimoto

Graduate School of Economics, Osaka University, Japan

Institute of Social and Economic Research, Osaka University, Japan

Ken Tabata [*][†]

School of Economics, Kwansei Gakuin University, Japan

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Abstract

We examine how a subsidy policy for encouraging more individuals to pursue higher education affects economic growth in an overlapping generations model of R&D-based growth, including both product development and process innovation. We show that such a policy may have a negative effect on the long-run economic growth rate. When the market structure adjusts partially in the short run, the effect of an education subsidy on economic growth is ambiguous and depends on the values of the parameters. However, when the market structure adjusts fully in the long run, the education subsidy expands the number of firms but reduces economic growth. These unfavorable predictions of an education subsidy on economic growth are partly consistent with the empirical findings that mass higher education does not necessarily lead to higher economic growth.

Keywords: Higher Education, Occupational Choice, R&D, Product Development, Process Innovation

[*]Correspondence: School of Economics, Kwansei Gakuin University, 1-155 Uegahara Ichiban-cho, Nishinomiya 662-8501, Hyogo, Japan, E-mail: tabataken@kwansei.ac.jp.

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1 Introduction

Many policy makers believe that a large, highly educated workforce is key to national prosperity and that the expansion of the higher education sector is an important driver of economic growth. For example, the policy documents produced in support of the UK 2011 education financing reforms stated that "Higher education is important to growth through equipping individuals with skills that enhance their productivity in the workplace, promoting the economy's knowledge base and deriving innovations" (BIS, 2011; p.21). In fact, most developed countries provide a set of subsidy policies aimed at broadening the access to higher education, especially for the poor, and encouraging more individuals to receive higher education.[1] These policies include the public provision of low-cost tertiary institutions, means tested grants, loan and work study programs, tax credits and so on. Figure 1 shows the share of public and private funding of tertiary educational institutions in OECD countries (OECD, 2016b: Table B3.1b, p218). On average, across OECD countries, nearly 70% of all funding for tertiary educational institutions comes directly from public sources, while the share of public and private funding varies widely across countries.

Does such a subsidy policy for encouraging more individuals to receive higher education truly enhance economic growth? We theoretically examine this issue in a R&D-based growth model where skilled labor inputs matter for deriving innovations. Then, we show that a higher education subsidy policy may have a negative effect on the long-run economic growth rate. Such a policy may be inappropriate for the purpose of stimulating long-run economic growth.

The recent empirical literature on human capital and growth employs the quality measure of human capital (e.g., better test score) and suggests a positive linkage between human capital and economic growth, whereas most of the traditional studies used the quantity measure of human capital (e.g., years of schooling), of which some showed mixed evidence of the causal effect of human capital on economic growth (e.g., Krueger and Lindahl, 2001). By measuring human capital in terms of cognitive skills on international achievement tests, Hanushek and Woessmann (2011) show that these cognitive skills positively affect economic growth and can account for the differences in long-run growth performance across OECD countries.[2] Cohen and Soto (2007) show that the existing mixed evidence of the causal effect of human capital on economic growth originates from the misspecifi-

[1] State intervention for higher education is justified for several reasons (e.g., external benefits in terms of tax dividends, economic growth, social cohesion and parenting; incomplete capital markets; equal opportunities). See chapter 12 of Barr (2012) for more details regarding the optimal design of the higher education system.

[2] They also show that once the cognitive skills are included in the growth regression, the association between years of schooling and economic growth becomes statistically insignificant.

cation of human capital or poor data quality on measuring human capital. Glaeser et al. (2004) argue that the long-run growth is most fundamentally driven by human capital rather than institutions, which is in line with unified growth theories (e.g., Galor and Weil, 2000; Galor and Moav, 2004), where human capital accumulation triggers a fundamental transition, such as the industrial revolution.

However, as argued carefully in Hanushek and Woessmann (2011) and Hanushek(2016), these available test score measures (e.g., PISA test scores) are measured at the primary and secondary level of schooling. Therefore, strictly speaking, the abovementioned empirical studies suggest the importance of the primary and secondary level of education on economic growth. Hanushek and Woessmann (2011) show that when cognitive skills are accounted for, tertiary attainment *per se* is not significantly associated with long-run growth differences across OECD countries.[3] Therefore, the effect of the tertiary level of education on economic growth is still an open question. Vandenbussche et al. (2006) focuses directly on movements of the technology frontier, suggesting that tertiary education is particularly important for countries near the technology frontier, where growth requires new inventions and innovations. Using a panel data set covering 19 OECD countries between 1960 and 2000, Vandenbussche et al. (2006) find a positive linkage between the five-year growth rates and higher education, once distance from the technological frontier is controlled for.

However, Hanushek and Woessmann (2011) state that the finding of Vandenbussche et al. (2006) of a particular effect of tertiary attainment in rich countries is not robust once the focus is on long-run growth experiences and educational outcome measures are taken into account.[4] Therefore, it is still inconclusive whether a developed country should place a particular focus on tertiary education. A recent article by Holmes (2013) takes a skeptical view of the expansion of higher education on economic growth, emphasizing that there exists little concrete evidence to support the causal effect of a mass higher education on economic growth. Hanushek (2016) also states that in the absence of improved cognitive skills, the strong push toward more tertiary schooling does not look like it will consistently lead to added economic growth.

Motivated by these political debates and the recent empirical findings, we examine, in a theoretical model, how a subsidy policy for encouraging more individuals to receive higher education affects the per capita GDP growth rate of the

[3]Hanushek (2016) notes that part of this lack of impact of attainment of higher education is probably because there are no good measures of university quality.

[4]Hanushek and Woessmann (2011) note that this does not mean that leading beyond the secondary level does not matter. Rather, in the spirit of a lifecycle interpretation where early skills facilitate the development of subsequent skills, it means that outcome measures of learning in school (i.e., cognitive skills) are a good predictor for the accumulation of further skills in life and the capacity to deploy these skills effectively.

economy. Because technological progress via R&D innovation has been identified as the primary driving force of modern economic growth (e.g., Romer 1990), we are particularly interested in the effect of a higher education subsidy policy on R&D-based innovations. We develop a two-period overlapping generations (OLG) model of R&D-based growth where skilled labor inputs matter for both product development and process innovation, and the skilled labor supply is endogenously determined according to the individuals' choice of higher education. In line with the literature of the second-generation R&D-based growth model, pioneered by Peretto (1998), Segestrom (1998), and Howitt (1999), the model features two dimensions of technological progress. In the vertical dimension, incumbent production firms invest in process innovation with the objective of lowering production costs. In the horizontal dimension, the product development sector creates new product designs for firms entering the production sector. In this Schumpeterian growth model with an endogenous market structure measured by the equilibrium number of firms, we examine how a subsidy policy for encouraging more individuals to receive higher education affects the per capita GDP growth rate of the economy. Then, we show that when the market structure adjusts partially in the short run, the effect of the education subsidy on economic growth is ambiguous and depends on the values of the parameters (e.g., extent of specialization gains). However, when the market structure adjusts fully in the long run, the education subsidy expands the number of firms but reduces economic growth. A higher education subsidy policy is perhaps inappropriate for the purpose of stimulating long-run economic growth.

The intuition behind this result is explained as follows. The higher education subsidy rate increases the supply of skilled labor, lowers the employment costs of researchers, and increases incentives for both product development and process innovation, which positively affect the per capita GDP growth rate in the short run. In the long run, however, product development encourages the entry of new production firms, which in turn reduces the market size of each production firm. Given that the market size of a production firm determines its incentives for process innovation, the higher education subsidy rate decreases long-run economic growth.[5] These unfavorable predictions for an education subsidy on economic growth are partly consistent with the empirical findings that a mass higher education does not necessarily lead to higher economic growth.

Our counterintuitive hypothesis of the negative effect of education subsidies on economic growth is generated by several complicated interactions of general equilibrium effects. Therefore, it is difficult to identify all the necessary elements that help to produce our main result completely. Nevertheless, the following two

[5] Laincz and Peretto (2006) provide empirical evidence for a positive relationship between the average firm size and economic growth.

specific structures of our model play key roles in deriving our main results. First, in our model, the higher education subsidy rate increases the share of skilled workers with relatively higher wages, which positively affects the aggregate savings of the economy, and thereby, it increases the demand for shares of each production firm. These factors increase the market value of blueprints of new variety and positively affect the equilibrium number of firms, which in turn reduces the market size of each production firm in the long run. Second, in our model, we employ the Jones (1995)-type specification of research productivity of variety R&D, where the existing stock of knowledge (i.e., the number of varieties) positively affects the future research productivity, but its marginal effect on research productivity decreases with the stock of knowledge. This Jones (1995)-type specification links the steady-state number of firms to the market size (i.e., the labor size) of the economy in an intuitive manner, which enables our model to preclude the counterfactual scale effect prediction of economic growth. Consequently, similar to the conventional second-generation R&D-based growth model, it is only the vertical dimension of technological progress (i.e., process innovation) that works as a plausible engine of economic growth in the long run.[6] Under these two specific structures of our model, a higher education subsidy rate reduces the market size of each production firm, diminishes its incentives for process innovation and decreases long-run economic growth.

This paper is related to several branches of the literature. First, this paper is related to the literature on endogenous growth theories, which described human capital as the engine of growth through innovations (e.g., Aghion and Howitt, 1992; Grossman and Helpman; 1991; Romer, 1990). In particular, this paper is closely related to previous works that analyze the effects of the skill composition of the labor force on the amount of innovation in the economy (e.g., Grossman and Helpman; 1991; Vandenbussche et al., 2006). Using their seminal model of variety expansion, Grossman and Helpman (1991) show that an increase in the stock of skilled labor can be growth enhancing, while an increase in the stock of unskilled labor can be growth-depressing.[7] Vandenbussche et al.(2006) develop a model of technology catch up and show that skilled labor has a higher growth-enhancing effect closer to the technological frontier under the assumption that innovation is a relatively more skill-intensive activity than producing imitations.

Although we share numerous research interests with these studies, our research differs from them in the following respects. First, to examine the effects

[6]The second generation R&D-based growth model received substantial empirical support recently (e.g., Laincz and Peretto, 2006; Ha and Howitt, 2007; Madsen, 2008; Madsen et al., 2010; Ang and Madsen, 2011). Nevertheless, it is difficult to test the plausibility of the abovementioned two specific structures of our model precisely. In this sense, our theoretical results should be interpreted with caution.

[7]See the third section of their chapter 5.

of the skill composition of the labor force on both the vertical and horizontal dimensions of technological progress, we employ a Schumpeterian growth model with an endogenous market structure, where both growth in product variety and product quality are determined endogenously through R&D investment. This extension enables us to obtain richer theoretical implications regarding the effects of the skill composition of the labor force on R&D-based growth. To the best of our knowledge, the existing Schumpeterian growth literature has yet to analyze this issue rigorously. Second, we consider the case where the skill composition of the labor force is determined endogenously through the individuals' choice of higher education. This specification enables us to analyze the interactions among higher education subsidies, the skill composition of the labor force, and economic growth more extensively.

Second, this paper is closely related to a few pioneering theoretical studies that analyze the effects of publicly provided education targeted to high-ability workers on R&D-based growth (e.g., Grossmann, 2007; Böhm et al., 2015). Grossmann(2007) incorporates an occupational choice framework into an in-house process innovation-based growth model and shows that publicly provided education aimed at expanding the science and engineering skills of high-ability workers is unambiguously growth promoting and neutral with respect to the earnings distribution. Böhm et al.(2015) develop a numerical simulation model of directed technical change to evaluate whether economic growth triggered by the publicly provided education aimed at expanding the skills of high-ability workers eventually trickles down to low-ability workers. Then, they show that expansion of higher education is followed by rising inequality and temporarily lower wages at the bottom of the earnings distribution. Only after considerable time has elapsed does the economic situation of low-skilled workers improve and do the workers eventually become better off. In contrast to these studies, we employ a Schumpeterian growth model with an endogenous market structure. These differences in modeling strategy of technical change enable us to reveal different aspects of a higher education promoting policy that have yet to be examined in the literature. In this sense, our research complements the analyses conducted by Grossmann (2007) and Böhm et al.(2015).

Third, this paper is closely related to Chen (2015), which shows the somewhat counterintuitive negative effects of educational subsidies on economic growth. In his Diamond (1965)-type OLG model with endogenous fertility and skill acquisition, increases in educational subsidies will hamper economic growth due to the following three reasons. First, a higher income tax rate will lower the motivation to become skilled workers; second, the increased time spent on education and raising children will result in less time being available for working; and third, an increase in the average fertility rate will give rise to a capital-dilution effect, ultimately reducing the capital per worker. In contrast to Chen (2015), our analysis is based on a model that considers R&D-based innovations as the fundamental driver of economic growth. Therefore, this paper proposes an alternative theoretical mechanism that explains the long-run negative effects of educational subsidies on economic growth.

This paper is organized as follows. Section 2 presents the basic model. Section 3 investigates the dynamic equilibrium properties of the economy. Section 4 employs a numerical analysis to examine the effect of an education subsidy on economic growth. Section 5 briefly discusses the limitations of our paper. Section 6 concludes the paper.

2 Model

This section introduces a two-period OLG model of R&D-based growth with endogenous skill acquisition, productivity growth and variety expansion.[8] The economy consists of three sectors, i.e., a final goods sector, an intermediate goods sector, and a product development sector. The final goods sector produces homogeneous goods for sales in a perfectly competitive market, with a variety of imperfectly substitutable intermediate goods as inputs. The intermediate goods sector, on the other hand, consists of monopolistically competitive firms that produce differentiated product varieties for firms in the final goods sector, with both unskilled and skilled labor as inputs. Productivity growth arises as a result of process innovation undertaken by the intermediate goods firms, with the objective of lowering production costs, with solely skilled labor as an input. The product development sector creates new product designs for firms entering the intermediate goods sector, with solely skilled labor as an input.

2.1 Individuals

Individuals in this economy live for two periods, young and old. They work only in their young period and retire in the old period. In each period, young individuals who are the continuum of measure M are born with one unit of available time endowment.

In the young period, before entering the labor market, each individual chooses whether to receive a higher education. Although individuals with higher education can supply skilled labor in their young period, they must devote $1-\theta$ units of time to education according to their known ability θ. For each individual, this θ

[8] Some basic settings in our model are inherited from Tanaka and Iwaisako (2011). Tanaka and Iwaisako (2011) develop a two-period OLG model of variety expansion with endogenous skill acquisition. We extend Tanaka and Iwaisako's (2011) model by introducing the vertical dimension of technological progress.

is a random variable drawn from a distribution defined over $[\underline{\theta}, \bar{\theta}]$ with cumulative distribution function Φ, and $0 < \underline{\theta} < \bar{\theta} < 1$. For tractability, we assume Φ to be continuously differentiable, strictly increasing, with a time-invariant function, and common to all individuals. Each individual is endowed with θ before birth and takes the value as given. Therefore, higher educated individuals with ability θ can supply only $\theta \in [\underline{\theta}, \bar{\theta}]$ units of skilled labor. On the other hand, individuals who receive only a lesser education can supply one unit of unskilled labor. For simplicity, we assume that no other cost is needed for education. In fact, on average, across OECD countries, foregone earnings for a man attaining a tertiary education is US $43,700, whereas their direct costs are US $10,500 (OECD, 2016; Table B7.3b, p150). The main costs of a tertiary education are foregone earnings.

Consequently, the before-tax labor income of individuals with ability θ who become skilled workers is given by $w_t^s\theta$, whereas that of individuals who become unskilled workers is given by w_t^u, where w_t^s and w_t^u are the wage per unit of skilled and unskilled labor at period t, respectively. Explicit consideration of the costs for acquiring the basic skills to be an unskilled worker does not alter the main implications of this paper. To maintain the tractability of our model, we omit the description regarding the individuals' choice of basic education.

Moreover, higher educated individuals can obtain an education subsidy from the government. To reduce the individual's opportunity costs of acquiring skills, the government levies a tax τ_t on the labor income of all young individuals and subsidizes a fraction $s \in (0, 1)$ of $(1 - \tau_t)w_t^u$ for units of time that skilled individuals devote to acquiring skills $1 - \theta$. Note that for individuals with ability θ, the opportunity costs of acquiring skills are given by $(1 - \tau_t)w_t^u(1 - \theta)$ (i.e., the foregone unskilled labor income during education).

Most developed countries provide a set of policies aimed at improving the individuals' accessibility to higher education. These policies include the public provision of low-cost tertiary institutions, means tested grants, loan and work study programs, tax credits and so on. Broadly speaking, these policies can be interpreted as higher education subsidy policies aimed at reducing the student's opportunity costs of acquiring skills. Under such a subsidy policy, the after-tax income of individuals with ability θ who become skilled workers is given by $(1 - \tau_t)[w_t^s\theta + sw_t^u(1 - \theta)]$, whereas that of individuals who become unskilled workers is given by $(1 - \tau_t)w_t^u$.

The lifetime utility of individuals with ability θ, born in period t, is expressed as follows:

$$U_t^\theta = lnC_{1,t}^\theta + \beta lnC_{2,t+1}^\theta, \quad \beta \in (0, 1), \tag{1}$$

where $C_{1,t}^\theta$ and $C_{2,t+1}^\theta$ represent their consumption during their youth and old age, respectively, and β represents the time discount rate.

The budget constraints of these individuals are expressed as follows:

$$P_{c,t}C_{1,t}^{\theta} + S_t^{\theta} = I_t^{\theta,i}, \tag{2}$$

$$P_{c,t+1}C_{2,t+1}^{\theta} = R_{t+1}S_t^{\theta}, \tag{3}$$

where

$$I_t^{\theta,i} = \begin{cases} (1-\tau_t)\left[w_t^s\theta + sw_t^u(1-\theta)\right], & i = s, \\ (1-\tau_t)w_t^u, & i = u, \end{cases}$$

S_t^{θ} is their saving during their youth, R_{t+1} is the gross interest rate, $P_{c,t}$ and $P_{c,t+1}$ are the price of final goods at period t and $t+1$, and $I_t^{\theta,i}$ is the after-tax income of individuals with ability θ who become skilled workers ($i = s$) and unskilled workers ($i = u$), respectively.

By maximizing (1) subject to (2) and (3), we can derive the following indirect utility functions:

$$U_t^{\theta,i} = \begin{cases} ln\left[\frac{1}{(1+\beta)P_{c,t}}\right]\left[\frac{\beta R_{t+1}}{(1+\beta)P_{c,t+1}}\right]^{\beta}\left[I_t^{\theta,s}\right]^{1+\beta}, & i = s, \\ ln\left[\frac{1}{(1+\beta)P_{c,t}}\right]\left[\frac{\beta R_{t+1}}{(1+\beta)P_{c,t+1}}\right]^{\beta}\left[I_t^{\theta,u}\right]^{1+\beta}, & i = u, \end{cases}$$

where $U_t^{\theta,i}$ is the lifetime utility of individuals with ability θ who become skilled workers ($i = s$) and unskilled workers ($i = u$), respectively. Individuals with ability θ obtains a higher education, if $U_t^{\theta,s} \geq U_t^{\theta,u}$ or $I_t^{\theta,s} \geq I_t^{\theta,u}$. Notice that $I_t^{\theta,i}$ is the only variable relevant for this choice, because the rate of return on savings does not depend upon the skill level. Therefore, the equilibrium after-tax income of individuals with ability θ is given by the following expression:

$$I_t^{\theta} \equiv (1-\tau_t)\max\left\{w_t^s\theta + sw_t^u(1-\theta), w_t^u\right\}.$$

Moreover, the saving function of individuals with ability θ is described as

$$S_t^{\theta} = \frac{\beta}{1+\beta}I_t^{\theta}. \tag{4}$$

Given the definition of $\omega_t \equiv \frac{w_t^u}{w_t^s}$, the condition under which an individual with ability θ obtains a higher education is described by:

$$\theta \geq \hat{\theta}(\omega_t; s), \tag{5}$$

where

$$\hat{\theta}(\omega_t; s) = \begin{cases} \underline{\theta}, & \text{for } \omega_t \leq \omega_{min}, \\ \frac{(1-s)\omega_t}{1-s\omega_t}, & \text{for } \omega_t \in (\omega_{min}, \omega_{max}), \\ \bar{\theta}, & \text{for } \omega_t \geq \omega_{max}, \end{cases}$$

$$\omega_{min} \equiv \frac{\underline{\theta}}{(1-s)+\underline{\theta}s}, \quad \omega_{max} \equiv \frac{\bar{\theta}}{(1-s)+\bar{\theta}s}.$$

Therefore, as long as $\omega_t \in (\omega_{min}, \omega_{max})$, the relations $\hat{\theta}_\omega(\omega_t; s) > 0$ and $\hat{\theta}_s(\omega_t; s) < 0$ hold, which indicates that the skilled worker share decreases with the relative wage of unskilled/skilled workers ω_t, whereas it increases with the education subsidy rate s.

Exploiting the law of large numbers, we can compute the skilled and the unskilled labor supply as follows:

$$H(\omega_t; s) \equiv \int_{\hat{\theta}(\omega_t;s)}^{\bar{\theta}} M\theta d\Phi(\theta), \tag{6}$$

$$L(\omega_t; s) \equiv \int_{\underline{\theta}}^{\hat{\theta}(\omega_t;s)} M d\Phi(\theta). \tag{7}$$

Therefore, as long as $\omega_t \in (\omega_{min}, \omega_{max})$, as shown in Appendix A, we can confirm that the relations $H_\omega(\omega_t; s) < 0$, $H_s(\omega_t; s) > 0$, $L_\omega(\omega_t; s) > 0$ and $L_s(\omega_t; s) < 0$ hold. These results indicate that the skilled labor supply (resp., unskilled labor supply) decreases (resp., increases) with the relative wage of unskilled/skilled workers ω_t, whereas it increases (resp., decreases) with the education subsidy rate s.

2.2 Final goods sector

The final goods sector is perfectly competitive. We assume that one representative final goods firm combines n_t kinds of intermediate goods to produce the final good Y_t in period t. Following Benassy (1996) and others, we specify the technology of final goods production as follows:

$$Y_t = n_t^{\sigma+1-\frac{\eta}{\eta-1}} \left(\int_0^{n_t} x_t(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{(\eta-1)}}, \quad \eta > 1, \tag{8}$$

where $x_t(j)$ is the jth intermediate goods input. The parameter σ represents the elasticity of productivity to the mass of intermediate goods, which captures the degree of specialization in production. The elasticity of substitution between any two of the intermediate goods is equal to η. If $\sigma = 1/(\eta - 1)$, (8) reduces to a well-known Dixit-Stiglitz-type specification.

Given the price of the final good $P_{c,t}$ and those of the intermediate goods $p_t(j)$, the firm maximizes its profit. Because of the perfect competition, the final goods firm earns zero profit, that is, $P_{c,t}Y_t = \int_0^{n_t} p_t(j)x_t(j)dj$. Since the production function is in the spirit of Dixit and Stiglitz (1977), the first-order conditions for the profit maximization and the zero-profit condition yield the well-known following demand functions:

$$x_t(j) = \frac{p_t(j)^{-\eta}}{\int_0^{n_t} p_t(j)^{1-\eta} dj} P_t Y_t, \quad \forall j \in [0, n_t], \tag{9}$$

where $P_t = n_t^{\frac{1}{\eta-1}-\sigma} \left(\int_0^{n_t} p_t(j)^{1-\eta} dj \right)^{1/(1-\eta)}$ is a price index of the intermediate goods. Here, note that the relation $P_{c,t} = P_t$ holds due to the zero-profit condition.

2.3 Intermediate goods sector

Each intermediate good j is produced by monopolistically competitive firms that hold a blueprint for the intermediate good j. Each firm has the following constant-returns-to-scale production technology:

$$x_t(j) = A_t(j) \left[l_t^s(j) \right]^{\alpha} \left[l_t^u(j) \right]^{1-\alpha}, \quad \alpha \in (0, 1), \tag{10}$$

where $l_t^s(j)$ and $l_t^u(j)$ represent the skilled and unskilled labor inputs of firm j at period t, $x_t(j)$ and $A_t(j)$ are the output and productivity of firm j at period t, and α is the elasticity of skilled labor inputs to production. Let $A_t \equiv \frac{1}{n_t}(\int_0^{n_t} A_t(j) dj)$ denote the average productivity of firms in period t.

Intermediate goods firms invest in process innovation with the aim of lowering the production cost through productivity improvements. A firm with its R&D department employs $l_t^R(j)$ units of skilled labor in process innovation. Based on Grossmann (2007, 2009), the firm-level productivity $A_t(j)$ evolves according to the following:

$$A_t(j) = z A_{t-1} [l_t^R(j)]^{\gamma}, \quad z > 0, \gamma > 0, \tag{11}$$

where z is the efficiency parameter of in-house R&D and γ is the elasticity of skilled labor inputs to in-house R&D. The productivity of skilled labor employed in the in-house R&D is given by the exogenous parameter z times the average productivity of firms in period $t-1$, A_{t-1}. We adopt the level of the average productivity of firms in period $t-1$ as a proxy for the stock of public technological knowledge in period t, which accumulates within firms as a byproduct of process innovation. Following the process innovation framework employed by Peretto and Conolly (2007) and others, we model knowledge spillovers into process innovations among firms as a function of the average productivity of technological knowledge observable by the R&D departments of firms. [9]

[9] An alternative formulation is that firms have to incur in-house R&D expenditure one period in advance of production, similar to in the discrete time infinite-horizon model of Young (1998). However, this assumption seems to be less plausible in an OLG model.

The net profit of intermediate goods firm j is given as follows:

$$\pi_t(j) = p_t(j)x_t(j) - w_t^s l_t^s(j) - w_t^u l_t^u(j) - w_t^s l_t^R(j). \tag{12}$$

Each intermediate goods firm maximizes its net profit $\pi_t(j)$ subject to (9), (10) and (11).[10] As explained in Appendix B, there are two steps in the intermediate goods firms' profit maximization problem. First, the intermediate goods firms conduct process innovation, and second, they produce the intermediate goods. We solve the problems backward. In the second step, we maximize an intermediate goods firm's profit from the production given its productivity and the R&D outlay. Then, we turn back to the first step and decide how much the intermediate goods firms should invest in process innovation. These procedures yield the optimal labor input as follows:

$$l_t^s = \frac{\alpha}{w_t^s}\frac{\eta-1}{\eta}\frac{P_tY_t}{n_t}, \tag{13}$$

$$l_t^u = \frac{1-\alpha}{w_t^u}\frac{\eta-1}{\eta}\frac{P_tY_t}{n_t}, \tag{14}$$

$$l_t^R = \frac{\gamma}{w_t^s}\frac{\eta-1}{\eta}\frac{P_tY_t}{n_t}. \tag{15}$$

Hereafter, we assume $\gamma(\eta-1) < 1$ to satisfy the second-order condition for maximization. Using equations (12) to (15), we obtain the following maximum net profits for each intermediate firm in period t:

$$\pi_t = \frac{1-\gamma(\eta-1)}{\eta}\frac{P_tY_t}{n_t}. \tag{16}$$

Because of the ex-ante homogeneity of the individuals, all intermediate goods firms behave in the same way. Thus, we omit index j whenever this does not lead to confusion.

2.4 Product development sector

The invention of new variety requires skilled labor as its only private input. Between periods t and $t+1$, competitive R&D firms in the product development sector employ L_t^N efficiency units of skilled labor as researchers, develop $n_{t+1} - n_t$

[10] In our model, as in Peretto and Conolly (2007), the productivity of the in-house R&D investment in period t fully depends on the level of public knowledge in period t, A_{t-1}. In addition, it is assumed that the in-house R&D investment in period t can affect the level of productivity in period t, $A_t(i)$ instantaneously. Due to these assumptions, the intermediate goods firm's market value maximization problem can be formulated as its net profit maximization problem each period. With noting (12), the market value of the jth intermediate goods firm is given by $V_t(j) = \sum_{\tau=1}^{\infty} \frac{\pi_{t+\tau}(j)}{\Pi_{i=1}^{i=\tau} R_{t+i}}$.

new blueprints, and sell these blueprints to intermediate goods firms at their market values of V_t. Thus, given a research productivity of δ_t, output is expressed as follows:

$$n_{t+1} - n_t = \delta_t L_t^N, \tag{17}$$

Following Jones (1995), research productivity is a given for each firm but depends on the aggregate level, positively on the number of existing ideas (i.e., *the standing-on-shoulders effect*), as follows:

$$\delta_t = \bar{\delta} n_t^{\psi}, \quad \bar{\delta} > 0, \ \ \psi \in [0, 1), \tag{18}$$

where $\bar{\delta}$ is the efficiency parameter of variety R&D and ψ is an elasticity parameter of $n_(t)$. The specification $\psi \in [0, 1)$ implies that the marginal effect of n_t on δ_t is decreasing with n_t. The standing-on-shoulders effect arises because the creation of a new product designed adds to the existing stock of public knowledge related to product design, improving the labor productivity of future product development.

Under the assumption of free entry in the product development sector, the expected gain of $V_t \delta_t L_t^N$ from R&D must not exceed the cost of $w_t^s L_t^N$ for a finite size of R&D activities at equilibrium. Thus, we have the following conditions:

$$V_t \delta_t \begin{cases} = w_t^s, \ \text{then} \ L_t^N > 0, \ \ n_{t+1} > n_t, \\ < w_t^s, \ \text{then} \ L_t^N = 0, \ \ n_{t+1} = n_t. \end{cases} \tag{19}$$

We next consider no-arbitrage conditions. The market value of intermediate goods firms V_t (i.e., the market value of blueprints) is related to the risk-free gross interest rate R_t. Shareholders of intermediate goods firms that purchased these shares during period t obtain dividends of π_{t+1} during period $t + 1$ and can sell these shares to the subsequent generation at a value of V_{t+1}. In the financial market, the total returns from holding the stock of a particular intermediate firm must be equal to the returns on the risk-free asset $R_{t+1} V_t$, which implies the following no-arbitrage condition:

$$R_{t+1} = \frac{\pi_{t+1} + V_{t+1}}{V_t}. \tag{20}$$

2.5 Government

The government levies a tax τ_t on the labor income of all young individuals and subsidizes a fraction $s \in (0, 1)$ of $(1-\tau_t) w_t^u$ for units of time that skilled individuals devote to acquiring skills $1 - \theta$. Thus, its budget constraint for period t is as follows:

$$\tau_t [w_t^s H(\omega_t; s) + w_t^u L(\omega_t; s)] = (1 - \tau_t) w_t^u \int_{\hat{\theta}(\omega_t; s)}^{\bar{\theta}} s(1 - \theta) M d\Phi(\theta), \tag{21}$$

where the left-hand side is the total tax revenue raised from all young individuals and the right-hand side is the total expenditure composed of education subsidy payments to all skilled young individuals.

2.6 Market-clearing conditions

Now, we consider the labor market conditions. Skilled labor is demanded by both intermediate goods firms and product development firms to produce intermediate goods, to conduct process innovation and to invent new products, while unskilled labor is demanded only by intermediate goods firms to produce intermediate goods. Thus, the market-clearing conditions for both skilled and unskilled laborers are described as follows:

$$n_t(l_t^s + l_t^R) + L_t^N = H(\omega_t; s), \tag{22}$$

$$n_t l_t^u = L(\omega_t; s). \tag{23}$$

Furthermore, as shown in Appendix C, we can obtain the following asset market equilibrium condition:

$$\int_{\underline{\theta}}^{\bar{\theta}} S_t^{\theta} M d\Phi(\theta) \begin{cases} = n_{t+1} V_t, & \text{for } n_{t+1} > n_t, \\ = n_t V_t, & \text{for } n_{t+1} = n_t. \end{cases} \tag{24}$$

This condition states that the savings of young individuals in period t must be used for investing in new inventions ($V_t(n_{t+1} - n_t)$) or purchasing existing stocks that were owned by preceding generations ($V_t n_t$). In particular, when the product development sector does not operate (i.e., $n_{t+1} = n_t$), the savings of young individuals in period t must be devoted to purchasing existing stocks that were owned by preceding generations ($V_t n_t$).

3 Equilibrium and education subsidy policy

In this section, we first analyze the dynamic properties of the relative wages of unskilled/skilled workers, the average productivity of firms, the number of firms, and the value of GDP. Then, we examine the short-run and long-run effects of the education subsidy on the number of firms and the gross growth rate of the average productivity of firms.

3.1 Relative wage of unskilled/skilled workers

In this subsection, we describe the determination of the relative wage of unskilled/skilled workers. We first consider the case where the product development sector operates (i.e., $n_{t+1} > n_t$). When the product development sector operates (i.e., $n_{t+1} > n_t$), from (4), (21) and (24), we can obtain the following equation:

$$n_{t+1}V_t = \frac{\beta}{1+\beta}[w_t^s H(\omega_t; s) + w_t^u L(\omega_t; s)]. \tag{25}$$

By substituting (17) and (19) into (25), equation (25) can be rewritten as follows:

$$\frac{n_t}{\delta_t} = \frac{\beta}{1+\beta}[H(\omega_t; s) + \omega_t L(\omega_t; s)] - L_t^N. \tag{26}$$

By using equations (13) to (15) and equations (22) to (23), as shown in Appendix D, we can express the skilled labor engaged in the product development sector L_t^N as follows:

$$L_t^N = H(\omega_t; s) - \frac{\alpha+\gamma}{1-\alpha}\omega_t L(\omega_t; s). \tag{27}$$

Thus, by substituting (18) and (27) into (26), we can obtain the following equation:

$$\frac{n_t^{1-\psi}}{\bar{\delta}} = \left(\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha}\right)\omega_t L(\omega_t; s) - \frac{1}{1+\beta}H(\omega_t; s). \tag{28}$$

From (28), we can see that the relative wages of unskilled/skilled workers ω_t depend on the number of firms n_t and the education subsidy rate s (i.e., $\omega_t = \omega(n_t; s)$). Appendix D shows that $\omega(n_t; s)$ satisfies the following properties: $\omega_n(n_t; s) > 0$ and $\omega_s(n_t; s) > 0$.

Next, we consider the case where the product development sector does not operate (i.e., $n_{t+1} = n_t$). When the product development sector does not operate (i.e., $n_{t+1} = n_t$), from (27) and $L_t^N = 0$, we can obtain the following equation:

$$\frac{1-\alpha}{\alpha+\gamma} = \frac{\omega_t L(\omega_t; s)}{H(\omega_t; s)} \equiv \Gamma(\omega_t; s). \tag{29}$$

From (29), we can see that the relative wages of unskilled/skilled workers ω_t are given by the constant value, which is denoted as ω^*, and depends upon the education subsidy rate s (i.e., $\omega^* = \omega^*(s)$). Appendix D also shows that $\omega^*(s)$ satisfies the following property: $\omega_s^*(s) > 0$.

From (28) and (29), we denote the number of firms n_t such that it satisfies $\omega(n_t; s) = \omega^*(s)$ as $n^*(s)$. As shown in the following subsection 3-3, we can confirm that $n^*(s)$ satisfies the following property: $n_s^*(s) > 0$. From (27), when the product development sector operates (i.e., $n_{t+1} > n_t$), since $L_t^N > 0$, the following inequality $\frac{1-\alpha}{\alpha+\gamma} > \frac{\omega_t L(\omega_t;s)}{H(\omega_t;s)} \equiv \Gamma(\omega_t; s)$ holds. From (29) and $\Gamma_\omega(\omega_t; s) > 0$, this inequality implies that the relation $\omega(n_t; s) < \omega^*(s)$ holds in the case where the product development sector operates (i.e., $n_{t+1} > n_t$). With noting $\omega_n(n_t; s) > 0$ from (28), these results imply that the product development sector operates (i.e.,

$n_{t+1} > n_t$) if and only if the number of firms n_t is sufficiently small to satisfy $n_t < n^*(s)$. In contrast, suppose that $n_t \geq n^*(s)$ (i.e., $n_{t+1} = n_t$) and the product development sector does not operate; thus, the entry of new firms never occurs. Taking these results into account, the equilibrium relative wage of unskilled/skilled workers ω_t is given by the following expression:

$$\omega_t = \begin{cases} \omega(n_t; s), & \text{for } \ n_t < n^*(s), \\ \omega^*(s), & \text{for } \ n_t \geq n^*(s). \end{cases} \tag{30}$$

The solid line in Figure 2 describes the relationship between the number of firms n_t and the equilibrium relative wages of unskilled/skilled workers ω_t. Since $\omega_n(n_t; s) > 0$, the relative wages of unskilled/skilled workers increases with the number of firms n_t in the region where $n_t < n^*(s)$, but it remains constant at $\omega^*(s)$ when $n_t \geq n^*(s)$. Moreover, since $\omega_s(n_t; s) > 0$ and $\omega^*_s(s) > 0$, as shown in the upward shift of the $\omega(n_t; s)$ and $\omega^*(s)$ curves in Figure 2 (i.e., $s < s'$), given the value of n_t, the higher rate of education subsidies positively affects the relative wages of unskilled/skilled workers. The higher rate of education subsidy s increases the supply of skilled workers, which lowers their relative wages.

3.2 Dynamics of the average productivity of firms

In this subsection, we analyze the dynamics of the average productivity of firms. By substituting (14) and (23) into (15), we can obtain the following equation:

$$l^R_t = \frac{\gamma}{1-\alpha} \frac{\omega_t L(\omega_t; s)}{n_t}. \tag{31}$$

Then, using (28) and (29), the skilled labor engaged in process innovation l^R_t in (31) can be expressed as follows:

$$l^R_t = \begin{cases} \frac{\gamma}{1-\alpha} \frac{\left[\frac{1}{\bar{\delta} n_t^{\psi}} + \frac{1}{1+\beta} \frac{H(n_t; s)}{n_t}\right]}{\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha}}, & \text{for } \ n_t < n^*(s), \\ \frac{\gamma}{\alpha+\gamma} \frac{H(s)}{n_t}, & \text{for } \ n_t \geq n^*(s), \end{cases} \tag{32}$$

where

$$H(n_t; s) \equiv H\left(\omega(n_t; s); s\right),$$
$$H(s) \equiv H\left(\omega^*(s); s\right).$$

From (32), we can see that the skilled labor engaged in process innovation l^R_t depends upon the number of firms n_t and the education subsidy rate s (i.e., $l^R(n_t; s)$). Appendix E shows that the relations $H_n(n_t; s) < 0$, $H_s(n_t; s) > 0$ and $H_s(s) > 0$ hold. These results imply that $l^R(n_t; s)$ satisfies the following properties: $l^R_n(n_t; s) <$

0 and $l_s^R(n_t; s) > 0$. Since $\omega_n(n_t; s) > 0$ and $L_\omega(\omega_t; s) > 0$, equation (31) suggests that the number of firms n_t has two competing impacts upon the level of l_t^R. A larger number of firms n_t decreases each firm's market size, which negatively affects the level of l_t^R, whereas it increases the values of $\omega_t L(\omega_t; s)$, which positively affects the level of l_t^R.[11] The results obtained from (32) indicate that the former negative effect always dominates the latter positive effect because the relation $l_n^R(n_t; s) < 0$ holds. Moreover, since $l_s^R(n_t; s) > 0$, the higher rate of education subsidy s positively affects the level of skilled labor engaged in process innovation.

Using (11) and (32), the gross growth rate of the average productivity of firms is given by the following expression:

$$G_t^A \equiv \frac{A_t}{A_{t-1}} = z\left[l^R(n_t; s)\right]^\gamma \equiv G^A(n_t; s). \tag{33}$$

From (32) and (33), we can easily confirm that $G^A(n_t; s)$ satisfies the following properties: $G_n^A(n_t; s) < 0$ and $G_s^A(n_t; s) > 0$.

The solid line in Figure 3 describes the relationship between the number of firms n_t and the equilibrium gross growth rate of the average productivity of firms G_t^A. Since $G_n^A(n_t; s) < 0$, the gross growth rate of the average productivity of firms decreases with the number of firms. As the number of firms n_t increases, each firm's market size decreases, which motivates firms to invest less in process innovation and thereby lowers the gross growth rate of the average productivity of firms G_t^A. Moreover, since $G_s^A(n_t; s) > 0$, as shown in the upward shift of the $G^A(n_t; s)$ curve in Figure 3 (i.e., $s < s'$), given the value of n_t, the higher rate of education subsidy s positively affects the equilibrium gross growth rate of the average productivity of firms. The higher rate of education subsidy s increases the supply of skilled workers, lowers their relative wages, increases each firm's incentives for process innovation by reducing the employment costs of researchers and, thus, positively affects the gross growth rate of the average productivity of firms.

3.3 Dynamics of the number of firms

In this subsection, we describe the dynamics of the number of firms. When the product development sector operates (i.e., $n_{t+1} > n_t$), from (19) and (25), we can

[11] As in Peretto and Connolly (2007), the quality-adjusted gross firm size is measured by the quality-adjusted volume of production, $\frac{x_t}{A_t}$. Using (28), (A.12), (A.31) and $\hat{c}_t = \frac{(w_t^s)^\alpha (w_t^u)^{(1-\alpha)}}{\alpha^\alpha (1-\alpha)^{(1-\alpha)}}$, we obtain $\frac{x_t}{A_t} = (\frac{\alpha}{1-\alpha})^\alpha \frac{1}{\omega_t^{1-\alpha}} \frac{\frac{1}{\delta_t} + \frac{H(\omega_t; s)}{1+\beta}}{\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha}}$. Thus, with noting (18) and (30), we can easily confirm that a larger number of firms n_t decreases each firm's market size.

obtain the following equation:

$$\frac{n_{t+1}}{\delta_t} = \frac{\beta}{1+\beta}\left[H(\omega_t; s) + \omega_t L(\omega_t; s)\right]. \tag{34}$$

Then, by using equations (26) to (29) and (34), the gross growth rate of the number of firms can be expressed as follows:

$$G^n_{t+1} \equiv \frac{n_{t+1}}{n_t} = \begin{cases} \frac{1}{F(n_t;s)} \equiv G^n(n_t; s), & \text{for } n_t < n^*(s), \\ 1, & \text{for } n_t \geq n^*(s). \end{cases} \tag{35}$$

where

$$F(n_t; s) \equiv F(\Gamma(n_t; s)) = 1 - \frac{1+\beta}{\beta}\frac{1 - \frac{\alpha+\gamma}{1-\alpha}\Gamma(n_t; s)}{1 + \Gamma(n_t; s)},$$

$$\Gamma(n_t; s) \equiv \Gamma(\omega(n_t; s); s) = \frac{\omega(n_t; s)L(n_t; s)}{H(n_t; s)},$$

$$L(n_t; s) \equiv L(\omega(n_t; s); s).$$

As shown in Appendix F, $G^n(n_t; s)$ satisfies the following properties: $G^n_n(n_t; s) < 0$ and $G^n_s(n_t; s) > 0$.

The solid line in Figure 4 describes the relationship between the number of firms n_t and the gross growth rate of the number of firms G^n_{t+1}. Since $G^n_n(n_t; s) < 0$, the gross growth rate of the number of firms G^n_{t+1} decreases with the number of firms n_t. To avoid an unnecessary lexicographic explanation, as shown in Figures 2 and 4, we restrict our analyses to the case where the initial number of firms n_0 is sufficiently small to ensure the relations $\omega_0 = \omega_n(n_0; s) < \omega^*(s)$ and $G^n_1 = G^n(n_0; s) > 1$ hold. Under such assumptions, equation (35) shows that given the initial value of n_0, n_t gradually approaches its steady-state value, denoted as $n^*(s)$. On the transition path, the number of firms determines each firm's market size and the equilibrium gross growth rate of the average productivity of firms G^A_t according to (33). When n_t evolves toward its steady-state value $n^*(s)$, as described in Figure 3, G^A_t also gradually approaches its steady-state value $G^A(s)$, which is defined by $G^A(s) \equiv G^A(n^*(s); s)$.

More precisely, from (35), the dynamics of n_t are determined by the following one-dimensional difference equation:

$$n_{t+1} = \begin{cases} G^n(n_t; s)n_t, & \text{for } n_t < n^*(s), \\ n_t, & \text{for } n_t \geq n^*(s). \end{cases}$$

As shown in Appendix G, the differentiation of $G^n(n_t; s)n_t$ with respect to n_t around the steady state $n_t = n^*$ yields the following:

$$\frac{dn_{t+1}}{dn_t}\Big|_{n_t=n^*} = 1 - (1+\beta)(1-\psi)\frac{\alpha+\gamma}{\alpha+\gamma+\beta(1+\gamma)\frac{1+\frac{\omega^*(s)L_\omega(\omega^*(s);s)}{L(s)}}{1+\frac{\omega^*(s)L_\omega(\omega^*(s);s)}{L(s)}-\frac{\omega^*(s)H_\omega(\omega^*(s);s)}{H(s)}}} < 1,$$

where $L(s) \equiv L(\omega^*(s); s)$. Therefore, suppose that the stepping-on-shoulders parameter ψ is sufficiently large to satisfy the following:

$$\frac{\beta}{1+\beta}\left(1-\frac{1+\gamma}{\alpha+\gamma}\frac{1+\frac{\omega^*(s)L_\omega(\omega^*(s);s)}{L(s)}}{1+\frac{\omega^*(s)L_\omega(\omega^*(s);s)}{L(s)}-\frac{\omega^*(s)H_\omega(\omega^*(s);s)}{H(s)}}\right)\leq\psi, \tag{36}$$

We can ensure that the relation $\frac{dn_{t+1}}{dn_t}\ |_{n_t=n^*}\in[0,1)$ holds. Otherwise, we can see that the relation $\frac{dn_{t+1}}{dn_t}\ |_{n_t=n^*}<0$ holds.

The solid lines in Figures 5 and 6 illustrate the possible dynamics of n_t when the parameter conditions of (36) are satisfied and not satisfied, respectively. As shown in Figure 5, when the parameter conditions of (36) are satisfied, the dynamics of n_t are stable and n_t gradually converges to a unique positive steady-state value $n^*(s)$. However, when the parameter conditions of (36) are not satisfied, as shown in Figure 6, n_t does not necessarily converge to a unique steady-state value $n^*(s)$. Instead, the steady-state value of n_t depends upon its initial values of n_0 and may become larger than the value of $n^*(s)$.

In the following analyses, for simplicity, we restrict our analysis to the case where the parameter conditions of (36) are satisfied. Suppose that the elasticity of skilled labor input α is sufficiently small; then, the left-hand side of (36) becomes negative. In this case, the parameter conditions of (36) hold, irrespective of the values of ψ. Moreover, the numerical simulation analyses in the following section show that equation (36) holds under a wide plausible set of parameter values. The following proposition summarizes the results and derives the steady-state values $\left\{n^*(s), G^A(s)\right\}$.

Proposition 1 *Given the initial value of n_0 such that it satisfies $\omega_0 = \omega_n(n_0; s) < \omega^*(s)$, if the parameter conditions of (36) hold, the dynamics of n_t are stable and n_t gradually converges to a unique positive steady-state value. The steady-state values $\left\{n^*(s), G^A(s)\right\}$ are given by the following:*

$$n^*(s)=\left(\frac{\beta}{1+\beta}\frac{1+\gamma}{\alpha+\gamma}\bar{\delta}\right)^{\frac{1}{1-\psi}}[H(s)]^{\frac{1}{1-\psi}}, \tag{37}$$

$$G^A(s)=z\left[l^R(s)\right]^\gamma, \tag{38}$$

where

$$l^R(s)\equiv\frac{\gamma}{\alpha+\gamma}\frac{H(s)}{n^*(s)}=\frac{\gamma}{\alpha+\gamma}\frac{[H(s)]^{-\frac{\psi}{1-\psi}}}{\left(\frac{\beta}{1+\beta}\frac{1+\gamma}{\alpha+\gamma}\bar{\delta}\right)^{\frac{1}{1-\psi}}}.$$

Proof of Proposition 1 is given in Appendix G. From (37) and (38), since $H_s(s) > 0$, under the Jones (1995)-type specification of research productivity (i.e., $\psi\in$

[0, 1)), we can see that the relations $n^*_s(s) > 0$ and $G^A_s(s) \leq 0$ hold. The higher rate of education subsidy s increases the steady-state number of firms $n^*(s)$, whereas it decreases the steady-state gross growth rate of the average productivity of firms $G^A(s)$. The intuitive mechanism behind the results of proposition 1 are explained carefully in the following subsection 3-5.

3.4 The value of GDP

In this subsection, we describe the equilibrium value of GDP. In the R&D-based growth model, the value of GDP is not necessarily equivalent to that of Y_t.[12] The correct value of GDP is defined as follows:

$$GDP_t = \begin{cases} Y_t + \frac{V_t}{P_{c,t}} (n_{t+1} - n_t), & \text{for } n_t < n^*(s), \\ Y_t, & \text{for } n_t \geq n^*(s). \end{cases} \tag{39}$$

Appendix H shows that GDP_t in (39) can be rewritten as follows:

$$GDP_t = \alpha^\alpha (1-\alpha)^{1-\alpha} A_t n_t^\sigma \Lambda_t. \tag{40}$$

where

$$\Lambda_t \equiv \begin{cases} \frac{\eta-1}{\eta} \frac{H(n_t;s)}{\omega(n_t;s)^{1-\alpha}} + \frac{1-(\alpha+\gamma)\frac{\eta-1}{\eta}}{1-\alpha} \frac{\omega(n_t;s)L(n_t;s)}{\omega(n_t;s)^{1-\alpha}} \equiv \Lambda(n_t;s), & \text{for } n_t < n^*(s), \\ \frac{1}{1-\alpha} \frac{\omega^*(s)L(s)}{\omega^*(s)^{1-\alpha}} \equiv \Lambda(s), & \text{for } n_t \geq n^*(s). \end{cases}$$

Equation (40) indicates that the value of GDP depends upon the average productivity of firms A_t, the number of firms n_t, and the other market factors Λ_t. Intuitively, the term Λ_t in equation (40) captures the effects of skilled and unskilled labor allocations on the value of GDP in period t. Since these allocations of labors are determined by market equilibrium conditions through changes in market prices such as the relative wage of unskilled/skilled workers and the price index of intermediate goods, the term Λ_t reflects the degree of static efficiency of production given the existing technologies and factor inputs. Appendix H shows that the relation $\Lambda_s(s) < 0$ holds, which indicates that the education subsidy policy deteriorates the steady-state static efficiency of production.

From (40), the gross growth rate of GDP is given by the following expression:

$$G_t^{GDP} \equiv \frac{GDP_t}{GDP_{t-1}} = \frac{A_t}{A_{t-1}} \left(\frac{n_t}{n_{t-1}}\right)^\sigma \frac{\Lambda_t}{\Lambda_{t-1}} = G_t^A \left(G_t^n\right)^\sigma G_t^\Lambda. \tag{41}$$

From (41), we can see that the gross growth rate of GDP depends on the gross growth rate of the average productivity of firms G_t^A, the number of firms G_t^n, and

[12] See Appendix H for details.

the other market factors G_t^{Λ}. The productivity growth rate is based on both the production efficiency improvement and variety expansion, and the degree of specialization parameter σ determines the relative importance of the variety expansion on productivity growth.

In the steady-state equilibrium where the relations $n_t = n_{t-1} = n^*(s)$ and $\Lambda_t = \Lambda_{t-1} = \Lambda(s)$ hold, the gross growth rate of GDP becomes equivalent to the gross growth rate of the average productivity of firms, as follows:

$$G^{GDP*} \equiv G_t^{GDP} |_{n_t=n^*(s)} = G^A(s). \tag{42}$$

Since $G_s^A(s) \leq 0$ from (38), we can easily confirm that the steady-state gross growth rate of GDP decreases with the education subsidy rate s.

3.5 The short-run and long-run effects of the education subsidy policy

In this subsection, we summarize the implications of both the short-run and long-run effects of the education subsidy on the number of firms and the gross growth rate of the average productivity of firms.

First, we summarize the effects of the education subsidy on the number of firms. Since $G_s^n(n_t; s) > 0$ from (35), as described in the upward shift of the $G^n(n_t; s)$ curve in Figure 4 (i.e., $s < s'$), the initial impact of a higher education subsidy rate on the gross growth rate of the number of firms G_{t+1}^n is positive. Given the value of n_t, the higher education subsidy rate s increases the supply of skilled workers, lowers their relative wage, enhances the entry of new firms by reducing the employment costs of researchers and, thus, increases the gross growth rate of the number of firms for some periods. However, as the number of firms increases, the gross growth rate of the number of firms decreases steadily, and the equilibrium number of firms gradually converges to its new steady value. Since $n_s^*(s) > 0$ from (37), as described in Figure 5, the number of firms attained in the new steady state equilibrium becomes larger than that attained in the original steady-state equilibrium (i.e., $n_s^*(s) < n_s^*(s')$). These results are summarized in the following Proposition 2.

Proposition 2 *The higher rate of the education subsidy enhances the entry of new firms, increases the gross growth rate of the number of firms for some periods and increases the steady-state number of firms.*

Equation (37) indicates that the Jones (1995)-type specification of research productivity (i.e., $\psi \in [0, 1)$) plays a crucial role in deriving the result of $n_s^*(s) > 0$. Suppose that we consider an alternative specification of $\psi = 1$; then, our model will generate the counterfactual scale effect prediction of economic growth. Therefore, we find the parameter specification $\psi \in [0, 1)$ to be a more reasonable specification. Moreover, the intuition behind the result of $n_s^*(s) > 0$ is explained as follows. In our model, the higher education subsidy rate increases the share of skilled workers with relatively higher wages, which positively affects the aggregate savings of the economy, and it thereby increases the demand for shares of each intermediate goods firm. These factors increase the market value of blueprints of new variety, which positively affects the equilibrium number of firms in the long run.

Next, we summarize the effect of the education subsidy on the gross growth rate of the average productivity of firms. Since $G_s^A(n_t; s) > 0$ from (33), as described in the upward shift of the $G^A(n_t; s)$ curve in Figure 3 (i.e., $s < s'$), the initial impact of a higher rate of the education subsidy on the gross growth rate of the average productivity of firms G_t^A is positive. Given the value of n_t, the higher rate of education subsidy s increases the supply of skilled workers, lowers their relative wages, increases each firm's incentives for process innovation by reducing the employment costs of researchers and, thus, positively affects the gross growth rate of the average productivity of firms. We denote this positive effect of the education subsidy on the gross growth rate of the average productivity of firms as the "cost reduction effect". In the long run, however, the market structure is endogenous, and the number of firms adjusts. The increased supply of skilled workers enhances the entry of new firms, which in turn reduces each firm's market size and decreases incentives for process innovation. We denote this negative effect of the education subsidy on the gross growth rate of the average productivity of firms as the "entry effect". Since $G_s^A(s) \leq 0$ from (38), as described in Figure 3, this negative "entry effect" dominates the positive "cost reduction effect" in the long run (i.e., $G^A(s) > G^A(s')$). Therefore, allowing for the endogeneity of the market structure, we can find opposite short-run and long-run predictions with regard to the effects of the education subsidy on the gross growth rate of the average productivity of firms. These results are summarized in the following Proposition 3.

Proposition 3 *The initial effect of a higher education subsidy rate on the gross growth rate of the average productivity of firms is positive as a result of an increased supply of skilled workers. However, in the long run, the increased supply of skilled workers enhances the entry of new firms and reduces the market size of each firm. The smaller market size decreases incentives for process innovation and decreases the steady-state gross growth rate of the average productivity of firms.*

Equation (38) indicates that the Jones (1995)-type specification of research productivity (i.e., $\psi \in [0, 1)$) plays a crucial role in deriving the result of $G_s^A(s) \leq$

0. Moreover, the standing-on-shoulders effect parameter ψ determines the relative magnitude of the entry and cost reduction effects.

4 Numerical Analysis

In this section, to obtain further insights with respects to the effects of the education subsidy on the per capita GDP growth rate, we resort to numerical simulations of our model. We choose the parameters of the model such that a hypothetical steady-state economy replicates the average values of key macroeconomic variables observed across OECD countries. Then, we assess the likely impact of an education subsidy on the per capita GDP growth rate. However, the main objective of these numerical exercises is not to calibrate our simple model to actual data but to supplement the qualitative results of our theoretical model. Although we chose the parameter values carefully, the quantitative results obtained in this paper should be interpreted with caution.

4.1 The model parameterization

Our first objective is to choose the parameters of the model such that the steady-state economy where the education subsidy rate s is given by zero replicates the average values of key macroeconomic variables observed across OECD countries. Panel A in Table 1 considers a list of four endogenous variables for which we set target values from available data or empirical evidence. The target value of R&D propensity, $\frac{w^s n^* l^R}{PY} = 0.022$, is given by the average ratio between R&D expenditures and gross domestic product observed in OECD countries during the 1995-2015 period (OECD, 2017). Note that the relations $n_t = n^*$ and $GDP = Y$ hold in the steady-state equilibrium. The target value of the mass of firms relative to population, $\frac{n^*}{M} = 0.0327$, equals the OECD-average numbers of firms in 2013, $n^* = 1,181,040$, divided by the average population in the same year, $M = 365,525,680$. [13] Since the target values of these variables are already calculated by Brunnschweiler et al. (2017: Table 1, p39), we employ their calculated values as our target values. The target value of the relative wage of unskilled/skilled workers, $\omega^* = 0.667$, is given by the inverse of the relative wages of those with a university degree relative to a high school education across OECD

[13] According to the United Nations (2015), the target population, i.e., $M = 365,525,680$, matches the average population of OECD countries in 2013. Moreover, by summing the data reported in OECD (2016a: Ch2, Table 2.1) across countries, the aggregate number of enterprises in 2013 in OECD economies is 46,060,568 (all sizes and sectors). Dividing this number by 39 countries, we obtain the OECD-average numbers of firms in 2013, $n^* = 1,181,040$. See Appendix E of Brunnschweiler et al. (2017) for further details.

countries (OECD, 2016 :Table A6.1, p125) [14]. The target value of the net per capita GDP growth rate, $g^{GDP*} = 0.02$, approximates the average per capita GDP growth rate of developed countries over a century (Barro and Sala-i-Martin, 2004: Table I.1, p13) [15].

Table 2 lists our preset parameters, the values of which reflect available data or empirical estimates or existing numerical studies, except for some policy and scaling parameters. To investigate the effect of the education subsidy policy, we set the education subsidy rate s to 0 in the base case simulation and changed it from 0 to 0.9 in increments of 0.1. In addition, the initial values of both the average productivity of firms A_0 and the number of firms n_0 are normalized to 1. As explained later, the starting point of our numerical simulation exercise is the steady-state equilibrium where the education subsidy rate s is given by zero. Therefore, as long as the relation $n_0 < n^*(s)$ holds, our numerical simulation results do not change even when we consider the alternative values of A_0 and n_0.

To parameterize the model, we require an explicit distribution function of individual ability θ. For tractability, we assume that the ability θ is distributed uniformly over the interval $[\underline{\theta}, \bar{\theta}]$. Under such an assumption, the skilled and unskilled labor supply is given by $H(\omega_t; s) = \frac{M[\bar{\theta}^2 - \hat{\theta}(\omega_t; s)^2]}{2\Delta}$ and $L(\omega_t; s) = \frac{M[\hat{\theta}(\omega_t; s) - \underline{\theta}]}{\Delta}$, where $\Delta \equiv \bar{\theta} - \underline{\theta}$. We set the population size M to 365,525,680, the baseline value of $\underline{\theta}$ to 0, and the value of $\bar{\theta}$ to 1. Then, we check the robustness of our results under alternative values of $\underline{\theta}$ and $\bar{\theta}$. In particular, we examine how the mean preserving spread of individual ability θ affects our simulation results.

The model is calibrated under the assumption that one period has a length of 30 years. The discount factor, $\beta = (0.99)^{120}$ (0.99 per quarter), is standard in the real-business-cycle literature. The value of the elasticities of substitution across intermediates, $\eta = 4.3$, implies a mark-up for monopolistic firms equal to 1.3, in the middle of the range of 1.2-1.4 suggested by international evidence (Britton et al., 2000; Gali et al., 2007). The standing-on-shoulders effect parameter, $\psi = 0.35$, reflects the baseline simulation parameter of Strulik et al. (2013 : p429) that is well-known as a numerical study of the R&D-based OLG model. Then, we check the robustness of our results under alternative values of ψ in the range 0.05-0.65.

The value of σ represents the elasticity of productivity to the mass of intermediate goods. The closest empirical counterpart to our σ is the "elasticity of productivity to variety" calculated by Broda et al. (2006), which ranges from 0.05 to 0.2. However, as stressed by Brunnschweiler et al. (2017), the empirical estimates of "gains from variety" that come from empirical studies of international trade are not fully consistent with the notion of "gains from differentiation" originally emphasized by Romer (1990). Therefore, as in Brunnschweiler et al. (2017),

[14] That is $\omega^* = 1/1.5 = 0.667$.

[15] Note that $g^{GDP*} \equiv G^{GDP*} - 1$.

we adopt a conservative approach by setting the baseline value of σ on the low end, $\sigma = 0.05$, and then perform a sensitivity analysis with alternative values of σ in the range 0-0.2.

Given the preset parameters, we calibrate the remaining 4 parameters listed in Panel B of Table 1 to match the 4 target values of the endogenous variables listed in Panel A. These four parameters consist of the elasticity of skilled labor inputs to in-house R&D (γ), the elasticity of skilled labor inputs to production (α), the efficiency parameter of variety R&D ($\bar{\delta}$), and the efficiency parameter of in-house R&D (z). Let us first consider the determination of the value of γ. We identify the value of γ by imposing the target value of $\frac{w^s n^* l^R}{PY}$ in equation (15). Using (15), the steady-state R&D propensity is given by $\frac{w^s n^* l^R}{PY} = \frac{\gamma(\eta-1)}{\eta}$. Since the value of η is already determined in Table 2, the value of γ is set to match the target value of $\frac{w^s n^* l^R}{PY}$. This procedure yields the calibrated value of γ reported in Panel B of Table 1. Note that the second order condition $\gamma(\eta - 1) < 1$ is satisfied in our baseline parameterization. Let us next consider the determination of the values of $(\bar{\delta}, \alpha, z)$. We identify the values of $(\bar{\delta}, \alpha, z)$ by imposing the target values of $\frac{n^*}{M}$, ω^* and g^{GDP*} in equations (29), (37) and (38). Note that $\frac{n^*}{M}$ is determined by (37), ω^* is determined by (29), and g^{GDP*} is determined by (38). Intuitively, we adjust the value of $\bar{\delta}$ to match the target value of $\frac{n^*}{M}$, whereas the value of α is adjusted to match the target value of ω^*. Moreover, we adjust the value of z to match the target value of g^{GDP*}. These procedures yield the calibrated values of $(\bar{\delta}, \alpha, z)$ reported in Panel B of Table 1. Then, we check the robustness of our results under alternative values of $\bar{\delta}$. The second row of Panel A in Table 1 reports the simulated steady-state values of our target variables under the baseline parameterization.

To examine the robustness of our numerical results, as mentioned above, we conduct several sensitivity analyses. However, in the following subsection, due to space limitations, we only show some results under the baseline parameterization. In Appendix I, we show some results of our sensitivity analyses and confirm that the qualitative features of the model continue to hold for a relatively wide range of parameter values.

4.2 Effects of the education subsidy

The calibrated model in the previous subsection allows us to evaluate both the short-run and long-run effects of the education subsidy on the per capita GDP growth rate. In the following numerical exercise, we consider the case where the economy is initially in the steady-state equilibrium where the education subsidy rate s is given by zero (i.e., $s_k = 0$ for all period $k < 3$). Then, our hypothetical steady-state "average OECD economy" introduces the education subsidy policy from period 3. More concretely, the education subsidy rate in period 3 and subsequent periods is increased from 0 to 0.3 (i.e., $s_k = 0.3$ for all periods $k \geq 3$). Although we focus our analysis on the case where the education subsidy rate is increased from 0 to 0.3, the sensitivity analyses in Appendix I shows that the effect of an education subsidy on economic growth remains unchanged qualitatively, even when we consider an alternative value of the education subsidy rate.

Figures 7-1 to 7-6 show the numerical examples of the transition path of the relative wage of unskilled/skilled workers ω_t (Figure 7-1), the number of firms per capita n_t/M (Figure 7-2), the net growth rate of the number of firms g_t^n (Figure 7-3), the net growth rate of the average productivity of firms g_t^A (Figure 7-4), other market factors Λ_t (Figure 7-5), and the net growth rate of GDP g_t^{GDP} (Figure 7-6) under different values of the degree of specialization parameter (i.e., $\sigma = 0, 0.05, 0.15$ and 0.2). From (28), (29), (33), (35) and (40), we can confirm that the value of σ is irrelevant to the equilibrium values of ω_t, n_t, g_t^n, g_t^A and Λ_t, respectively. Consequently, from (41), as shown in Figure 7-6, only the transition path of the net growth rate of GDP g_t^{GDP} is influenced by the changes in the value of σ.

As shown in Figures 7-1 to 7-4, the introduction of the education subsidy policy in period 3 increases the relative wages of unskilled/skilled workers in period 3 (Figure 7-1), enhances the entry of new firms in period 4 (Figure 7-2), increases the net growth rate of the number of firms in period 4 (Figure 7-3) and increases the net growth rate of the average productivity of firms in period 3 (Figure 7-4). However, as the number of firms increases, the net growth rate of the number of firms begins to decline from period 5 (Figure 7-3), and the equilibrium number of firms gradually converges to its new steady-state value, which is larger than that in the original steady-state equilibrium (Figure 7-2). Therefore, the education subsidy policy expands the equilibrium number of firms. Moreover, since the negative "entry effect" dominates the positive "cost reduction effect" in the long run, the net growth rate of the average productivity of firms begins to decline from period 4, and it gradually converges to its new steady-state value, which is lower than that in the original steady-state equilibrium (Figure 7-4). Therefore, the education subsidy policy positively affects the net growth rate of the average productivity of firms in the short run, but it negatively affects the net growth rate of the average productivity of firms in the long run.

The education subsidy policy encourages more firms to enter the market with new products, which strengthens the horizontal competition among firms. It is this strengthening of horizontal competition that gives rise to the negative "entry effect" of the education subsidy on the net growth rate of the average productivity of firms. In our model, the relative magnitudes of the entry and cost reduction effects depend on the value of the standing-on-shoulders effect parameter ψ.

Equation (41) indicates that the net growth rate of GDP shown in Figure 7-6 depends on the net growth rate of the number of firms g_t^n in Figure 7-3, the average productivity of firms g_t^A in Figure 7-4 and the other market factors Λ_t in Figure 7-5. It is difficult to examine analytically all the dynamic properties of the other market factor Λ_t and the value of GDP G_t^{GDP}. Therefore, we only show numerical examples of them. Figure 7-6 shows the transition path of g_t^{GDP} when the degree of specialization parameter σ is changed from 0 to 0.2, which corresponds to the empirical estimates of σ by Broda et al. (2006). Suppose that $\sigma = 0$, and the productivity growth rate is solely based on the production efficiency improvement. Peretto and Conolly (2007) provide a theoretical justification that a production efficiency (or quality) improvement is the only plausible engine of economic growth in the long run.

The introduction of the education subsidy policy in period 3 deteriorates the static efficiency of production in period 3, which leads to the lower level of the other market factors in period 3 (Figure 7-5). The evolutions of g_t^A in Figure 7-4 and Λ_t in Figure 7-5 indicate that the education subsidy policy in period 3 provides two competing impacts upon the net growth rate of GDP in period 3 (Figure 7-6). On the one hand, as shown in Figure 7-4, the rise in the net growth rate of the average productivity of firms in period 3 positively affects the net growth rate of GDP in period 3. On the other hand, as shown in Figure 7-5, the decline in the level of the other market factors in period 3 negatively affects the net growth rate of GDP in period 3. In our baseline simulation, since the latter negative effect dominates the former positive effect, the net growth rate of GDP in period 3 becomes lower than that in the original steady-state equilibrium. The sensitivity analyses indicate that this prediction holds for a wide range of plausible parameter values that satisfy both (36) and $\gamma(\eta - 1) < 1$. Therefore, irrespective of the values of σ, the net growth rate of GDP in period 3 under education subsidy becomes lower than that in the original steady-state equilibrium.

However, as shown in Figures 7-2 and 7-3, the number of firms starts to increase from period 4, which increases the net growth rate of the number of firms in period 4. In addition, as shown in Figure 7-5, the static efficiency of production improves gradually from period 4. Therefore, the net GDP growth rate increases in period 4 and reaches its highest value. Then, if the degree of specialization parameter σ is sufficiently large, as described in the $\sigma = 0.15$ and $\sigma = 0.2$ lines in Figure 7-6, the net growth rate of GDP in period 4 lies sufficiently above that in the original steady-state equilibrium. However, if the degree of specialization parameter σ is sufficiently small, as described in the $\sigma = 0$ and $\sigma = 0.05$ lines in Figure 7-6, the net growth rate of GDP in period 4 lies slightly above that in the original steady-state equilibrium.[16] After period 4, the net growth rate of GDP decreases gradually.

[16]The net growth rate of GDP in period 4 becomes higher than that in period 3 irrespective of the values of σ because the negative growth effect of the decline in Λ_t occurs only in period 3.

These results imply that the short-run effect of the education subsidy on economic growth is generally ambiguous and depends on the values of the parameters. Suppose that the degree of specialization parameter σ is sufficiently large; then, the length of periods for which the net growth rate of GDP under education subsidy is beyond that in the original steady-state is relatively long. Therefore, education subsidy may positively affect the net growth rate of GDP in the short run. However, suppose that the degree of specialization parameter σ is sufficiently small; the length of periods for which the net growth rate of GDP under education subsidy is beyond that in the original steady-state is relatively short. Therefore, the short-run effect of education subsidy on economic growth is ambiguous. Since it is difficult to obtain a reliable estimate value of σ, the short-run effect of the education subsidy on economic growth remains inconclusive.

After period 4, the economy gradually converges to its new steady-state equilibrium. During this transition process, the net growth rate of the number of firms and the other market factors gradually approach zero. Therefore, in the steady-state equilibrium, the net growth rate of GDP becomes equivalent to the net growth rate of the average productivity of firms. Consequently, as shown in Figures 7-4 and 7-6, the net growth rate of GDP in the new steady-state equilibrium becomes lower than that in the original steady-state equilibrium. Therefore, irrespective of the values of σ, the long-run effect of the education subsidy on economic growth is negative. These numerical simulation results indicate that when the market structure adjusts partially in the short run, the growth effect of the education subsidy is ambiguous and depends upon the values of the parameters. However, when market structure adjusts fully in the long run, the education subsidy expands the number of firms but reduces economic growth.

5 Discussion

To clarify our main arguments, we employ a simple, tractable growth model with some restrictive specifications and ignore various important elements of higher education, such as credit constraints due to family income inequality and uncertainty of educational outcomes. Although these specifications enable us to obtain an intuitive clear-cut prediction regarding the effect of the higher education subsidy on R&D-based growth, some of them are overly restrictive from an empirical perspective. Therefore, the application of our simple framework to assess the likely impact of policy reform is obviously limited. Here, we note several limitations of our specifications and discuss directions for future research.

First, we remark on our modeling way of the education subsidy. For analytical simplicity, we assume that government gives subsidies on opportunity costs of higher education (i.e., foregone wages as unskilled worker). However, there is no way that governments can actually do subsidize such opportunity costs directly. They rather give subsidies on direct costs of education such as tuition fees and living expenses. In this sense, our modeling way of the education subsidy is at odd as a straightforward approximation of the realities. In Appendix J, to check the robustness of our main predictions, we generalize our basic model by incorporating the direct costs of education (e.g., tuition fee) and examine how education subsidy for such direct costs of education affects the per capita GDP growth rate. Numerical simulation analyses in Appendix J show that the effects of education subsidy on economic growth remain unchanged qualitatively even when we consider this alternative modeling way of education subsidy. Appendix J also shows that the introduction of the direct costs of education make the analytical treatment of the model difficult, which prevents us from showing the main predictions of our paper clearly. Therefore, for clarity of our main arguments, we employ a rather abstract modeling way of the education subsidy in the main paper. The model developed in Appendix J still remains very primitive. Further investigation is thus required.

Second, for clarity of our main arguments, we focus our analyses on the education subsidy policy and do not discuss other interesting issues. However, the previous studies of endogenous growth with heterogeneous households (e.g., García-Peñalosa and Turnovsky, 2006, 2011) stress how the extent of household's heterogeneity affects income inequality and economic growth. Moreover, the relationships among the standing-on-shoulders effect parameter ψ, the efficiency parameter of variety R&D $\bar{\delta}$, the equilibrium number of firms and economic growth are of high concern in the literature on R&D-based growth. In Appendix K, to reflect these concerns briefly, we examine how changes in the extent of individual heterogeneity in ability θ, the value of the standing-on-shoulders effect parameter ψ and the efficiency parameter of variety R&D $\bar{\delta}$ affect the per capita GDP growth rate. However, these numerical analyses conducted in Appendix K still remain very tentative. Further investigation is thus required.

Third, for deriving our main predictions, the negative relationship between the relative supply of skilled workers and their relative wages plays a crucial role. However, in his celebrated paper, Acemoglu (2002) points out that a sharp rise in the relative wages between skilled and unskilled workers is coupled with an increase in the relative supply of skilled workers in the U.S. after the late 1970s. Obviously, our paper's finding stands in contrast with the Acemoglu (2002)'s practical observation. The introduction of a directed technological change framework could potentially change this negative relationship between the relative supply of skilled workers and their relative wages. Therefore, it is interesting to check whether our main predictions hold even when we consider a directed technological change framework explicitly. However, the introduction of a directed technological change framework produces more complex dynamics of technologies, which prevent us from showing the dynamic properties of our model clearly. So far, we have yet to derive any intuitive predictions from this extension. Nevertheless, the introduction of a directed technological change framework that generates more realistic relative wage dynamics is a promising direction for future research.

Fourth, for analytical tractability, this paper assumes that the product development firms that are inventing new varieties have to incur an R&D expenditure one period in advance of production, whereas intermediate goods firms can improve their efficiency of production through their in-house process innovation instantaneously. This asymmetric specification of product development and process innovation improve the tractability of the model greatly without altering the main predictions of this paper. Nevertheless, it will be interesting to consider alternative specifications of R&D activities.

Fifth, since this paper uses a two-period OLG framework, if we employ a very straightforward interpretation, one period in our model is interpreted as approximately 30 years. The concept of "short-run" in our model does not match the concept of "short-run" in the real world, which makes the comparison of our theoretical results with actual data slightly difficult. Moreover, this paper only focuses on the growth implications of an education subsidy policy and cannot propose a reasonable framework to analyze the welfare and distributional implications of a higher education subsidy policy. Therefore, to evaluate the likely impact of a higher education subsidy on economic growth and income inequality more precisely, it is necessarily to develop a more elaborate numerical version of the large-scale OLG model with various important elements of higher education, such as credit constraints due to family income inequality and uncertainty of educational outcomes.

6 Concluding Remarks

Employing a two-period overlapping generations model of R&D-based growth with both product development and process innovation, we examined how a subsidy policy for encouraging more individuals to receive higher education affects the per capita GDP growth rate of the economy. We showed that when the market structure adjusts partially in the short run, the effect of the education subsidy on economic growth is ambiguous and depends on the values of the parameters (e.g., extent of specialization gains). However, when the market structure adjusts fully in the long run, the education subsidy expands the number of firms but reduces economic growth. These unfavorable predictions regarding the impact of the education subsidy on economic growth are partly consistent with empirical findings that mass higher education does not necessarily lead to higher economic growth. A higher education subsidy policy is perhaps inappropriate for the purpose of stimulating long-run economic growth.

Appendix

Appendix A: Properties of $H(\omega_t; s)$ and $L(\omega_t; s)$

Differentiating $H(\omega_t; s)$ with respect to ω_t and s yields:

$$\frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)} = -\frac{\omega_t}{H(\omega_t; s)} M\hat{\theta}_t \phi(\hat{\theta}_t) \frac{1-s}{(1-s\omega_t)^2} < 0, \tag{A.1}$$

$$\frac{sH_s(\omega_t; s)}{H(\omega_t; s)} = \frac{s}{H(\omega_t; s)} M\hat{\theta}_t \phi(\hat{\theta}_t) \frac{\omega_t(1-\omega_t)}{(1-s\omega_t)^2} > 0, \tag{A.2}$$

where $\hat{\theta}_t = \hat{\theta}(\omega_t; s)$ and $\phi(\hat{\theta}_t)$ is the value of the probability density function of θ, $\phi(\theta)$, evaluated at $\theta = \hat{\theta}_t$. Analogously, differentiating $L(\omega_t; s)$ with respect to ω_t and s yields

$$\frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)} = \frac{\omega_t}{L(\omega_t; s)} M\phi(\hat{\theta}_t) \frac{1-s}{(1-s\omega_t)^2} > 0, \tag{A.3}$$

$$\frac{sL_s(\omega_t; s)}{L(\omega_t; s)} = -\frac{s}{L(\omega_t; s)} M\phi(\hat{\theta}_t) \frac{\omega_t(1-\omega_t)}{(1-s\omega_t)^2} < 0. \tag{A.4}$$

Appendix B: Intermediate goods firms' profit maximization

Second step

The jth intermediate goods firm's profit maximization problem in the second step can be written as follows:

$$\hat{\pi}_t(j) \equiv \max \left\{ p_t(j)x_t(j) - [w_t^s l_t^s(j) + w_t^u l_t^u(j)] \right\},$$

subject to (9) and (10), where $\hat{\pi}_t(j)$ is a profit function in this step of the problem, given its productivity $A_t(j)$. We first minimize the production costs of $w_t^s l_t^s(j) + w_t^u l_t^u(j)$ subject to (10), which yields the following cost function as well as both skilled and unskilled labor demand functions of intermediate goods firm j:

$$c_t(j) = \frac{\hat{c}_t x_t(j)}{A_t(j)},$$

$$l_t^s(j) = \alpha \frac{\hat{c}_t x_t(j)}{w_t^s A_t(j)}, \tag{A.5}$$

$$l_t^u(j) = (1-\alpha) \frac{\hat{c}_t x_t(j)}{w_t^u A_t(j)}, \tag{A.6}$$

where $\hat{c}_t \equiv \frac{(w_t^s)^\alpha (w_t^u)^{(1-\alpha)}}{\alpha^\alpha (1-\alpha)^{(1-\alpha)}}$. Thus, the jth intermediate goods firm's profit maximization problem in this step can be rewritten as follows:

$$\hat{\pi}_t(j) \equiv \max \left\{ [p_t(j) - \frac{\hat{c}_t}{A_t(j)}] x_t(j) \right\},$$

subject to (9). The first-order condition yields the optimal price and output of firm j given its productivity and prices and productivities of other firms as follows:

$$p_t(j) = \frac{\eta}{\eta - 1} \frac{\hat{c}_t}{A_t(j)}, \tag{A.7}$$

$$x_t(j) = \frac{\eta - 1}{\eta} \frac{A_t(j)^\eta}{\hat{c}_t \int_0^{n_t} A_t(j)^{\eta-1} dj} P_t Y_t. \tag{A.8}$$

Substituting (A.7) and (A.8) into $\hat{\pi}_t(j)$ yields the profit function as follows:

$$\hat{\pi}_t(j) = \frac{1}{\eta} \frac{A_t(j)^{\eta-1}}{\int_0^{n_t} A_t(j)^{\eta-1} dj} P_t Y_t. \tag{A.9}$$

First step

Then, turn back to the first step. The object of this step is to maximize the intermediate goods firm's net profits $\pi_t(j)$ defined in (12). Using the results obtained in the second step, the jth intermediate goods firm's profit maximization problem in the first step can be written as follows:

$$\pi_t(j) \equiv \max \left\{ \hat{\pi}_t(j) - w_t^s l_t^R(j) \right\},$$

subject to (11) and (A.9). The first-order condition with respect to $l_t^R(j)$ is:

$$\gamma(\eta - 1) \frac{\hat{\pi}_t(j)}{l_t^R(j)} = w_t^s. \tag{A.10}$$

Equation (A.10) implies that $l_t^R(j)$ is independent of j and so are $A_t(j)$, $p_t(j)$ and $x_t(j)$. Therefore, we can omit the index j, and thus, equations (A.7) to (A.9) are rewritten as follows:

$$p_t = \frac{\eta}{\eta - 1} \frac{\hat{c}_t}{A_t}, \tag{A.11}$$

$$x_t = \frac{\eta - 1}{\eta} \frac{A_t}{\hat{c}_t} \frac{P_t Y_t}{n_t}, \tag{A.12}$$

$$\hat{\pi}_t = \frac{1}{\eta} \frac{P_t Y_t}{n_t}. \tag{A.13}$$

By substituting (A.12) into (A.5) and (A.6) and rearranging them, we can obtain the optimal level of skilled and unskilled labor inputs for intermediate goods production as (13) and (14). Moreover, substituting (A.13) into (A.10) yields the optimal level of skilled labor engaged in process innovation as (15).

Appendix C: The market-clearing condition for assets (This Appendix is not intended for publication)

Due to perfect competition in the final goods market, the value of the final goods output is expressed as follows:

$$P_{c,t}Y_t = n_t p_t x_t.$$

Thus, using the profits of intermediate goods firms $\pi_t = p_t x_t - w_t^s l_t^s - w_t^u l_t^u - w_t^s l_t^R$ and equations (20), (22) and (23), the above equation can be rewritten as follows:

$$P_{c,t}Y_t = w_t^s H(\omega_t; s) + w_t^u L(\omega_t; s) - w_t^s L_t^N + n_t(V_{t-1}R_t - V_t).$$

Using (21), the above equation can be rewritten as follows:

$$P_{c,t}Y_t = (1-\tau_t)\int_{\hat{\theta}(\omega_t;s)}^{\bar{\theta}} \left[w_t^s\theta + sw_t^u(1-\theta)\right] Md\Phi(\theta) + (1-\tau_t)w_t^u L(\omega_t; s) - w_t^s L_t^N + n_t(V_{t-1}R_t - V_t).$$

Therefore, the market-clearing condition for final goods is expressed in the following manner:

$$\sum_{i=1,2}\int_{\underline{\theta}}^{\bar{\theta}} P_{c,t}C_{i,t}^{\theta} Md\Phi(\theta) = (1-\tau_t)\int_{\hat{\theta}(\omega_t;s)}^{\bar{\theta}} \left[w_t^s\theta + sw_t^u(1-\theta)\right] Md\Phi(\theta)$$
$$+ (1-\tau_t)w_t^u L(\omega_t; s) - w_t^s L_t^N + n_t(V_{t-1}R_t - V_t).$$

In the case of $V_t\delta_t = w_t^s$, $L_t^N > 0$ and $n_{t+1} > n_t$

With respect to (19), consider the case of $V_t\delta_t = w_t^s$ in which the product development sector functions, i.e., $L_t^N > 0$; and $n_{t+1} > n_t$. By substituting (6), (7), (2), (3), (17) and $V_t\delta_t = w_t^s$ into the market-clearing condition for final goods, we obtain the following expression:

$$\int_{\underline{\theta}}^{\bar{\theta}} S_t^{\theta} Md\Phi(\theta) - V_t n_{t+1} = R_t\left[\int_{\underline{\theta}}^{\bar{\theta}} S_{t-1}^{\theta} Md\Phi(\theta) - V_{t-1}n_t\right].$$

Because initial assets are given by $\int_{\underline{\theta}}^{\bar{\theta}} S_{-1}^{\theta} Md\Phi(\theta) = V_{-1}n_0$, we can obtain the following asset market equilibrium condition:

$$V_t n_{t+1} = \int_{\underline{\theta}}^{\bar{\theta}} S_t^{\theta} Md\Phi(\theta), \quad \text{for } V_t\delta_t = w_t^s.$$

In the case of $V_t\delta_t < w_t^s$, $L_t^N = 0$ and $n_{t+1} = n_t$

With respect to (19), consider the case of $V_t\delta_t < w_t^s$ in which the product development sector does not function, i.e., $L_t^N = 0$ and $n_{t+1} = n_t$. By substituting (6), (7), (2), (3) and $L_t^N = 0$ into the market-clearing condition for final goods, we obtain the following expression:

$$\int_{\underline{\theta}}^{\bar{\theta}} S_t^{\theta} M d\Phi(\theta) - V_t n_t = R_t \left[\int_{\underline{\theta}}^{\bar{\theta}} S_{t-1}^{\theta} M d\Phi(\theta) - V_{t-1} n_t \right].$$

Because the initial assets are given by $\int_{\underline{\theta}}^{\bar{\theta}} S_{-1}^{\theta} M d\Phi(\theta) = V_{-1} n_0$, we obtain the following asset market equilibrium condition:

$$V_t n_t = \int_{\underline{\theta}}^{\bar{\theta}} S_t^{\theta} M d\Phi(\theta), \quad \text{for } V_t\delta_t < w_t^s.$$

Appendix D: Properties of $\omega(n_t; s)$ and $\omega^*(s)$

Derivations of (27)

Using (13), (15) and (22), we obtain the following equation:

$$H(\omega_t; s) - L_t^N = n_t(l_t^s + l_t^R) = (\alpha + \gamma)\frac{\eta - 1}{\eta}\frac{P_t Y_t}{w_t^s}. \tag{A.14}$$

Furthermore, substituting (14) into (23), we obtain the following equation:

$$L(\omega_t; s) = (1 - \alpha)\frac{\eta - 1}{\eta}\frac{P_t Y_t}{w_t^u}. \tag{A.15}$$

Using (A.14) and (A.15), L_t^N can be expressed as (27).

Properties of $\omega(n_t; s)$

From (28), by differentiating ω_t with respect to n_t and s, noting that $\left(\frac{n_t}{\delta_t} + \frac{1}{1+\beta}H(\omega_t; s)\right)\frac{1}{\omega_t} = \left(\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha}\right)L(\omega_t; s)$, we obtain:

$$\frac{n_t\omega_n(n_t; s)}{\omega(n_t; s)} = \frac{(1-\psi)\frac{n_t}{\delta_t}}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)}\right) + \frac{H(\omega_t;s)}{1+\beta}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)} - \frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)}\right)} > 0, \tag{A.16}$$

$$\frac{s\omega_s(n_t; s)}{\omega(n_t; s)} = -\frac{\left(\frac{n_t}{\delta_t} + \frac{H(\omega_t;s)}{1+\beta}\right)\frac{sL_s(\omega_t;s)}{L(\omega_t;s)} - \frac{H(\omega_t;s)}{1+\beta}\frac{sH_s(\omega_t;s)}{H(\omega_t;s)}}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)}\right) + \frac{H(\omega_t;s)}{1+\beta}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)} - \frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)}\right)} > 0, \tag{A.17}$$

where $\omega_t = \omega(n_t; s)$.

Properties of $\omega^*(s)$

From (29), the differentiation of $\Gamma(\omega_t; s)$ with respect to ω_t and s yields:

$$\frac{\omega_t \Gamma_\omega(\omega_t; s)}{\Gamma(\omega_t; s)} = 1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)} - \frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)} > 0, \tag{A.18}$$

$$\frac{s\Gamma_s(\omega_t; s)}{\Gamma(\omega_t; s)} = \frac{sL_s(\omega_t; s)}{L(\omega_t; s)} - \frac{sH_s(\omega_t; s)}{H(\omega_t; s)} < 0. \tag{A.19}$$

Hence, by differentiating $\omega^*(s)$ with respect to s, we obtain:

$$\frac{s\omega^*_s(s)}{\omega^*(s)} = -\frac{\frac{sL_s(\omega^*;s)}{L(\omega^*;s)} - \frac{sH_s(\omega^*;s)}{H(\omega^*;s)}}{1 + \frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)} - \frac{\omega^* H_\omega(\omega^*;s)}{H(\omega^*;s)}} > 0, \tag{A.20}$$

where $\omega^* = \omega^*(s)$.

Appendix E: Properties of $H(n_t; s)$ and $H(s)$

Properties of $H(n_t; s)$

From (32), differentiating $H(n_t; s)$ with respect to n_t yields:

$$\frac{n_t H_n(n_t; s)}{H(n_t; s)} = \frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)} \frac{n_t \omega_n(n_t; s)}{\omega(n_t; s)} < 0, \tag{A.21}$$

where $\omega_t = \omega(n_t; s)$. Note that the relations $\frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)} < 0$ and $\frac{n_t \omega_n(n_t;s)}{\omega(n_t;s)} > 0$ hold from (A.1) and (A.16).

Analogously, from (32), differentiating $H(n_t; s)$ with respect to s yields:

$$\frac{sH_s(n_t; s)}{H(n_t; s)} = \frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)} \frac{s\omega_s(n_t; s)}{\omega(n_t; s)} + \frac{sH_s(\omega_t; s)}{H(\omega_t; s)},$$

where $\omega_t = \omega(n_t; s)$. Then, by substituting (A.17) into the above equation and rearranging it, we obtain:

$$\frac{sH_s(n_t; s)}{H(n_t; s)} = \frac{\left(\frac{n_t}{\delta_t} + \frac{H(\omega_t;s)}{1+\beta}\right)\left[\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)}\right)\frac{sH_s(\omega_t;s)}{H(\omega_t;s)} - \frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)}\frac{sL_s(\omega_t;s)}{L(\omega_t;s)}\right]}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)}\right) + \frac{H(\omega_t;s)}{1+\beta}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)} - \frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)}\right)},$$

where $\omega_t = \omega(n_t; s)$. From equations (A.1) to (A.4), we can see that the relations $\frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)} = -\frac{\omega_t H_\omega(\omega_t;s)}{L(\omega_t;s)\hat{\theta}_t}$ and $\frac{sL_s(\omega_t;s)}{L(\omega_t;s)} = -\frac{sH_s(\omega_t;s)}{L(\omega_t;s)\hat{\theta}_t}$ hold, where $\hat{\theta}_t = \hat{\theta}(\omega_t; s)$. Thus, by substituting these relations into the above equation, we obtain:

$$\frac{sH_s(n_t; s)}{H(n_t; s)} = \frac{\left(\frac{n_t}{\delta_t} + \frac{H(\omega_t;s)}{1+\beta}\right)\frac{sH_s(\omega_t;s)}{H(\omega_t;s)}}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)}\right) + \frac{H(\omega_t;s)}{1+\beta}\left(1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)} - \frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)}\right)} > 0, \tag{A.22}$$

where $\omega_t = \omega(n_t; s)$.

Properties of $H(s)$

From (32), differentiating $H(s)$ with respect to s yields:

$$\frac{sH_s(s)}{H(s)} = \frac{\omega^* H_\omega(\omega^*; s)}{H(\omega^*; s)} \frac{s\omega_s^*(s)}{\omega^*} + \frac{sH_s(\omega^*; s)}{H(\omega^*; s)},$$

where $\omega^* = \omega^*(s)$. Then, by substituting (A.20) into the above equation and rearranging it, we obtain:

$$\frac{sH_s(s)}{H(s)} = \frac{\left(1 + \frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)}\right) \frac{sH_s(\omega^*)}{H(\omega^*;s)} - \frac{\omega^* H_\omega(\omega^*;s)}{H(\omega^*;s)} \frac{sL_s(\omega^*;s)}{L(\omega^*;s)}}{1 + \frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)} - \frac{\omega^* H_\omega(\omega^*;s)}{H(\omega^*;s)}},$$

where $\omega^* = \omega^*(s)$. From equations (A.1) to (A.4), we can see that the relations $\frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)} = -\frac{\omega^* H_\omega(\omega^*;s)}{L(\omega^*;s)\hat{\theta}}$ and $\frac{sL_s(\omega^*;s)}{L(\omega^*;s)} = -\frac{sH_s(\omega^*;s)}{L(\omega^*;s)\hat{\theta}}$ hold, where $\hat{\theta} \equiv \hat{\theta}(\omega^*; s)$. Thus, by substituting these relations into the above equation, we obtain:

$$\frac{sH_s(s)}{H(s)} = \frac{\frac{sH_s(\omega^*;s)}{H(\omega^*;s)}}{1 + \frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)} - \frac{\omega^* H_\omega(\omega^*;s)}{H(\omega^*;s)}} > 0, \tag{A.23}$$

where $\omega^* = \omega^*(s)$.

Appendix F: Properties of $G^n(n_t; s)$

Properties of $\Gamma(n_t; s)$

From (35), differentiating $\Gamma(n_t; s)$ with respect to n_t yields:

$$\frac{n_t \Gamma_n(n_t; s)}{\Gamma(n_t; s)} = \frac{\omega_t \Gamma_\omega(\omega_t; s)}{\Gamma(\omega_t; s)} \frac{n_t \omega_n(n_t; s)}{\omega(n_t; s)} > 0, \tag{A.24}$$

where $\omega_t = \omega(n_t; s)$. Note that the relations $\frac{\omega_t \Gamma_\omega(\omega_t;s)}{\Gamma(\omega_t;s)} > 0$ and $\frac{n_t \omega_n(n_t;s)}{\omega_t} > 0$ hold from (A.16) and (A.18). Further, by substituting (A.16) and (A.18) into (A.24), noting that $\frac{n_t}{\delta_t} = H(\omega_t; s)[(\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha})\Gamma(\omega_t; s) - \frac{1}{1+\beta}]$ from (28), we obtain:

$$\frac{n_t \Gamma_n(n_t; s)}{\Gamma(n_t; s)} = \frac{1 - \psi}{\frac{1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)}}{1 + \frac{\omega_t L_\omega(\omega_t;s)}{L(\omega_t;s)} - \frac{\omega_t H_\omega(\omega_t;s)}{H(\omega_t;s)}} + \frac{1}{1+\beta} \frac{1}{\left(\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha}\right)\Gamma(\omega_t;s) - \frac{1}{1+\beta}}}, \tag{A.25}$$

where $\omega_t = \omega(n_t; s)$. Similarly, from (35), differentiating $\Gamma(n_t; s)$ with respect to s yields:

$$\frac{s\Gamma_s(n_t; s)}{\Gamma(n_t; s)} = \frac{\omega_t \Gamma_\omega(\omega_t; s)}{\Gamma(\omega_t; s)} \frac{s\omega_s(n_t; s)}{\omega(n_t; s)} + \frac{s\Gamma_s(\omega_t; s)}{\Gamma(\omega_t; s)},$$

where $\omega_t = \omega(n_t; s)$. Then, by substituting equations (A.17) to (A.19) into the above equation and rearranging it, we obtain:

$$\frac{s\Gamma_s(n_t; s)}{\Gamma(n_t; s)} = \frac{\frac{n_t}{\delta_t}\left[\frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)}\frac{sL_s(\omega_t; s)}{L(\omega_t; s)} - \left(1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)}\right)\frac{sH_s(\omega_t; s)}{H(\omega_t; s)}\right]}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)}\right) + \frac{H(\omega_t; s)}{1+\beta}\left(1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)} - \frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)}\right)},$$

where $\omega_t = \omega(n_t; s)$. From equations (A.1) to (A.4), we can see that the relations $\frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)} = -\frac{\omega_t H_\omega(\omega_t; s)}{L(\omega_t; s)\hat{\theta}_t}$ and $\frac{sL_s(\omega_t; s)}{L(\omega_t; s)} = -\frac{sH_s(\omega_t; s)}{L(\omega_t; s)\hat{\theta}_t}$ hold, where $\hat{\theta}_t = \hat{\theta}(\omega_t; s)$. Thus, by substituting these relations into the above equation, we obtain:

$$\frac{s\Gamma_s(n_t; s)}{\Gamma(n_t; s)} = -\frac{\frac{n_t}{\delta_t}\frac{sH_s(\omega_t; s)}{H(\omega_t; s)}}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)}\right) + \frac{H(\omega_t; s)}{1+\beta}\left(1 + \frac{\omega_t L_\omega(\omega_t; s)}{L(\omega_t; s)} - \frac{\omega_t H_\omega(\omega_t; s)}{H(\omega_t; s)}\right)} < 0, \quad \text{(A.26)}$$

where $\omega_t = \omega(n_t; s)$.

The relationship between $F(\Gamma(n_t; s))$ and $\Gamma(n_t; s)$

From (35), the differentiation of $F(\Gamma(n_t; s))$ with respect to $\Gamma(n_t; s)$ yields:

$$\frac{\Gamma(n_t; s)F_\Gamma(\Gamma(n_t; s))}{F(\Gamma(n_t; s))} = \frac{\frac{1+\beta}{\beta}\frac{1+\gamma}{1-\alpha}\frac{\Gamma(n_t; s)}{(1+\Gamma(n_t; s))^2}}{F(\Gamma(n_t; s))} > 0. \quad \text{(A.27)}$$

Properties of $G^n(n_t; s)$

From (35), differentiating $G^n(n_t; s)$ with respect to n_t yields:

$$\frac{n_t G^n_n(n_t; s)}{G^n(n_t; s)} = -\frac{\Gamma(n_t; s)F_\Gamma(\Gamma(n_t; s))}{F(\Gamma(n_t; s))}\frac{n_t\Gamma_n(n_t; s)}{\Gamma(n_t; s)} < 0, \quad \text{(A.28)}$$

where the relations $\frac{\Gamma(n_t; s)F_\Gamma(\Gamma(n_t; s))}{F(\Gamma(n_t; s))} > 0$ and $\frac{n_t\Gamma_n(n_t; s)}{\Gamma(n_t; s)} > 0$ hold from (A.24) and (A.27).

Analogously, from (35), differentiating $G^n(n_t; s)$ with respect to s yields:

$$\frac{sG^n_s(n_t; s)}{G^n(n_t; s)} = -\frac{\Gamma(n_t; s)F_\Gamma(\Gamma(n_t; s))}{F(\Gamma(n_t; s))}\frac{s\Gamma_s(n_t; s)}{\Gamma(n_t; s)} > 0, \quad \text{(A.29)}$$

where the relations $\frac{\Gamma(n_t; s)F_\Gamma(\Gamma(n_t; s))}{F(\Gamma(n_t; s))} > 0$ and $\frac{s\Gamma_s(n_t; s)}{\Gamma(n_t; s)} < 0$ hold from (A.26) and (A.27).

Appendix G: Proof of Proposition 1

The steady state values $\left\{n^*, G^A\right\}$

When the product development sector does not operate (i.e., $n_{t+1} = n_t$), from (29), we can see that the relation $\omega^* L(\omega^*; s) = \frac{1-\alpha}{\alpha+\gamma}H(\omega^*; s)$ holds. Therefore, substituting $\omega^* L(\omega^*; s) = \frac{1-\alpha}{\alpha+\gamma}H(\omega^*; s)$ and $\omega^* = \omega^*(s)$ into (28) yields the steady-state number of firms as (37). Moreover, substituting (32) and (37) into (33) yields the steady-state gross growth rate of the average productivity of firms as (38).

Local stability

By differentiating $G^n(n_t; s)n_t$ with respect to n_t and evaluating it at $n_t = n^*$, we obtain:

$$\frac{dn_{t+1}}{dn_t}\mid_{n_t=n^*} = 1 + \frac{n^* G^n_n(n^*; s)}{G^n(n^*; s)}\mid_{n_t=n^*},$$

where the relation $G^n(n^*; s) = 1$ holds. Therefore, by substituting (A.25), (A.27) and (A.28) into the above equation and evaluating it at $n_t = n^*$, noting that $\Gamma(n^*; s) = \frac{\omega^* L(\omega^*;s)}{H(\omega^*;s)} = \frac{1-\alpha}{\alpha+\gamma}$, and $F(\Gamma(\omega^*; s)) = 1$, we obtain:

$$\frac{dn_{t+1}}{dn_t}\mid_{n_t=n^*} = 1 - (1+\beta)(1-\psi)\frac{\alpha+\gamma}{\alpha+\gamma+\beta(1+\gamma)\frac{1+\frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)}}{1+\frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)}-\frac{\omega^* H_\omega(\omega^*;s)}{H(\omega^*;s)}}}, \tag{A.30}$$

where $\omega^* = \omega^*(s)$. From (A.30), suppose that the parameter conditions of (36) hold; then, we can confirm that the relation $\frac{dn_{t+1}}{dn_t}\mid_{n_t=n^*} \in [0, 1)$ holds.

Appendix H: Properties of GDP_t

Equivalence of GDI and GDP (This Appendix is not intended for publication)

Gross domestic income (GDI) is calculated by

$$GDI_t = \frac{(1-\tau_t)\int_{\hat{\theta}(\omega_t;s)}^{1}\left[w^s_t\theta + s w^u_t(1-\theta)\right] M d\Phi(\theta) + (1-\tau_t)w^u_t L(\omega_t; s) + \pi_t n_t}{P_{c,t}}$$

From (16) and (21), we have

$$GDI_t = \frac{w^s_t\left[H(\omega_t; s) + \omega_t L(\omega_t; s)\right]}{P_{c,t}} + \frac{\left[1-\gamma(\eta-1)\right]}{\eta}Y_t.$$

Further, from (A.14) and (A.15), we obtain

$$H(\omega_t; s) + \omega_t L(\omega_t; s) = (1+\gamma)\frac{\eta-1}{\eta}\frac{P_{c,t}Y_t}{w^s_t} + L^N_t.$$

By combining the above two equations, noting (17) and (19), GDI_t is given as follows:

$$GDI_t = Y_t + \frac{V_t}{P_{c,t}}\left(n_{t+1} - n_t\right).$$

Therefore, we can confirm that the value of GDI is equivalent to that of GDP.

Derivations of (40)

From (A.15), we obtain:

$$Y_t = \frac{\eta}{(1-\alpha)(\eta-1)}\frac{w_t^s}{P_t}\omega_t L(\omega_t; s). \tag{A.31}$$

Further, substituting (A.11) into $P_t = n_t^{\frac{1}{\eta-1}-\sigma}\left(\int_0^{n_t} p_t(j)^{1-\eta}dj\right)^{1/(1-\eta)}$ yields:

$$\frac{w_t^s}{P_t} = \frac{\eta-1}{\eta}\alpha^\alpha(1-\alpha)^{1-\alpha}\frac{A_t n_t^\sigma}{\omega_t{}^{1-\alpha}}. \tag{A.32}$$

Using (A.31) and (A.32), we obtain:

$$Y_t = \frac{1}{1-\alpha}\frac{\omega_t L(\omega_t; s)}{\omega_t^{1-\alpha}}\alpha^\alpha(1-\alpha)^{1-\alpha}A_t n_t^\sigma. \tag{A.33}$$

Furthermore, substituting (17), (19) (A.14) and $P_{c,t} = P_t$ into $\frac{V_t}{P_{c,t}}(n_{t+1}-n_t)$ yields:

$$\frac{V_t}{P_{c,t}}(n_{t+1}-n_t) = \frac{w_t^s}{P_t}H(\omega_t; s) - (\alpha+\gamma)\frac{\eta-1}{\eta}Y_t. \tag{A.34}$$

Therefore, by substituting equations (A.32) to (A.34) into the definition of GDP in (39), we obtain:

$$GDP_t = \left[\frac{\eta-1}{\eta}\frac{H(\omega_t; s)}{\omega_t^{1-\alpha}} + \frac{1-(\alpha+\gamma)\frac{\eta-1}{\eta}}{1-\alpha}\frac{\omega_t L(\omega_t; s)}{\omega_t^{1-\alpha}}\right]\alpha^\alpha(1-\alpha)^{1-\alpha}A_t n_t^\sigma. \tag{A.35}$$

Note that substituting (30) into (A.33) and (A.35) yields (40).

Properties of $\Lambda(s)$

In the steady-state equilibrium, from (29), the following relation holds:

$$\omega^*(s)L(s) = H(s)\frac{1-\alpha}{\alpha+\gamma}.$$

Thus, $\Lambda(s)$ in (40) can be rewritten as follows:

$$\Lambda(s) = \frac{1}{\alpha+\gamma}\frac{H(s)}{\omega^*(s)^{1-\alpha}}.$$

By differentiating the above equation with respect to s, we obtain:

$$\frac{s\Lambda_s(s)}{\Lambda(s)} = \frac{sH_s(s)}{H(s)} - (1-\alpha)\frac{s\omega_s^*(s)}{\omega^*(s)}.$$

Substituting (A.20) and (A.23) into the above equation yields:

$$\frac{s\Lambda_s(s)}{\Lambda(s)} = \frac{(1-\alpha)\frac{sL_s(\omega^*;s)}{L(\omega^*;s)} + \alpha\frac{sH_s(\omega^*;s)}{H(\omega^*;s)}}{1 + \frac{\omega^* L_\omega(\omega^*;s)}{L(\omega^*;s)} - \frac{\omega^* H_\omega(\omega^*;s)}{H(\omega^*;s)}},$$

where $\omega^* = \omega^*(s)$. From equations (A.2) and (A.4), we can see that the relation $\frac{sL_s(\omega^*;s)}{L(\omega^*;s)} = -\frac{sH_s(\omega^*;s)}{L(\omega^*;s)\hat{\theta}}$ holds, where $\hat{\theta} \equiv \hat{\theta}(\omega^*; s)$. Thus, by substituting (29) and $\frac{sL_s(\omega^*;s)}{L} = -\frac{sH_s(\omega^*;s)}{L\hat{\theta}}$ into the above equation, we obtain:

$$\frac{s\Lambda_s(s)}{\Lambda(s)} = \frac{\left[-(\alpha+\gamma)\frac{\omega^*}{\hat{\theta}} + \alpha\right]\frac{sH_s(\omega^*;s)}{H}}{1 + \frac{\omega^* L_\omega(\omega^*;s)}{L} - \frac{\omega^* H_\omega(\omega^*;s)}{H}} < 0,$$

where $\omega^* = \omega^*(s)$, and the relation $\frac{\omega^*}{\hat{\theta}} = \frac{1-s\omega^*}{1-s} > 1$ holds from (5).

Appendix I: Robustness Check (This Appendix is not intended for publication)

In this appendix, to show the robustness of our results regarding the effect of an education subsidy on economic growth, we show some results of our sensitivity analyses. As in Section 4-2, we consider the case where the economy is initially in the steady-state equilibrium where the education subsidy rate s is given by zero (i.e., $s_k = 0$ for all period $k < 3$). Then, we introduce the education subsidy policy from period 3.

First, we check whether the effects of an education subsidy on economic growth differ qualitatively with the value of the education subsidy rate s. Figure 8-1 shows the numerical examples of the transition path of the net growth rate of GDP g_t^{GDP}, when the education subsidy rate in period 3 and subsequent periods is increased from 0 to 0.1, 0.3, 0.5, and 0.7, (i.e., $s_k = 0.1, 0.3, 0.5$ and 0.7 for all periods $k \geq 3$). As shown in Figure 8-1, irrespective of the value of s, the dynamics of the net growth rate of GDP are qualitatively similar to those in the benchmark case shown in Figure 7-6. The net growth rate of GDP decreases once in period 3, and then it increases in period 4 and reaches its highest value, which is higher than that in the original steady-state equilibrium. However, after period 4, the net growth rate of GDP decreases gradually and reaches its new steady-state value, which is lower than that in the original steady-state equilibrium. These results indicate that the effects of an education subsidy on economic growth remain unchanged qualitatively irrespective of the value of the education subsidy rate s. [17]

[17] Figure 8-1 also shows that the higher education subsidy rate strengthens both the positive and negative effects of the education subsidy on economic growth.

Next, we check whether the effects of an education subsidy on economic growth differ qualitatively with the extent of individual heterogeneity in ability θ, the value of the standing-on-shoulders effect parameter ψ, and the value of the efficiency parameter of variety R&D $\bar{\delta}$. To this end, we set the mean value of individual ability μ to $\frac{1}{2}$. Since the ability θ is distributed uniformly over the interval $[\underline{\theta}, \bar{\theta}]$, it yields the following equation: $\mu = \frac{\bar{\theta}+\underline{\theta}}{2} = \frac{1}{2}$. Then, we change the value of $\bar{\theta}$ from 0.6 to 1 and adjust the value of $\underline{\theta}$ so that the relation $\mu = \frac{\bar{\theta}+\underline{\theta}}{2} = \frac{1}{2}$ holds. Under this specification, since the standard deviation of ability θ is given by $\sqrt{\frac{(\bar{\theta}-\mu)^2}{3}}$, the larger value of $\bar{\theta}$ implies a higher standard deviation of ability θ with the fixed mean value of $\mu = \frac{1}{2}$. Therefore, the larger value of $\bar{\theta}$ leads to the mean-preserving spread of individual ability θ.

As in Section 4-2, we focus our analyses on the case where the education subsidy rate in period 3 and subsequent periods is increased from 0 to 0.3 (i.e., $s_k = 0.3$ for all periods $k \geq 3$). Figure 8-2 shows the numerical examples of the transition path of the net growth rate of GDP g_t^{GDP} under different values of $\bar{\theta}$ (i.e., $\bar{\theta} = 0.7, 0.8, 0.9, 1$). Irrespective of the value of $\bar{\theta}$, the dynamics of the net growth rate of the GDP are similar qualitatively to those in the benchmark case shown in Figure 7-6. Therefore, the effects of an education subsidy on economic growth remain unchanged qualitatively, irrespective of the extent of individual heterogeneity in ability θ.

Table 3 and Table 4 show the numerical examples of the transition path of the net growth rate of GDP g_t^{GDP} under different values of ψ (i.e., $\psi = 0.2, 0.35, 0.5, 0.65$) and $\bar{\delta}$ (i.e., $\bar{\delta} = \delta^b, 2\times\delta^b, 3\times\delta^b, 4\times\delta^b$), respectively. Here, δ^b represents the baseline parameter value of $\bar{\delta}$ in Table 1 (i.e., $\delta^b = 14,655 * 10^{-7}$), and we consider the case where the value of $\bar{\delta}$ increases by a factor of 2, 3, and 4. Since the quantitative impacts of changes in ψ and $\bar{\delta}$ on economic growth are substantially larger than those of changes in the education subsidy rate s, for exposition, we omit the graphical expression. Both Table 3 and Table 4 show that, irrespective of the values of ψ and $\bar{\delta}$, the dynamics of the net growth rate of GDP are similar qualitatively to those in the benchmark case shown in Figure 7-6. Therefore, the effects of an education subsidy on economic growth remain unchanged qualitatively, irrespective of the value of the standing-on-shoulders effect parameter ψ and the value of the efficiency parameter of variety R&D $\bar{\delta}$.

Appendix J: Alternative education subsidy policy (This Appendix is not intended for publication)

In this appendix, to check the robustness of our main predictions, we generalize our basic model by incorporating the direct costs of education (e.g., tuition fee) and examine how an education subsidy for such direct costs of education affects the per capita GDP growth rate.

Individuals

To become a skilled worker, individuals with ability θ incur not only the opportunity costs of education but also the direct cost of education λ_t (e.g., tuition fee). We assume that the direct cost of education λ_t is proportional to the skilled wage rate w_t^s (e.g., teacher salary) and is given by $\lambda_t = \lambda w_t^s$, where $\lambda > 0$ is an educational cost parameter.

To reduce the individual's costs of education, the government levies a tax τ_t on the labor income of all young individuals and subsidizes a fraction $s \in (0, 1)$ of the direct cost of education λ_t that skilled individuals devote to acquiring skills. The government also makes private education expenses deductible from income tax (e.g., educational deductions).

Under such a subsidy policy, the after-tax income of individuals with ability θ who become skilled workers is given by $(1-\tau_t)[w_t^s - (1-s)\lambda_t]$, whereas that of individuals who become unskilled workers is given by $(1-\tau_t)w_t^u$. Therefore, given the definition of $\omega_t \equiv \frac{w_t^u}{w_t^s}$ and $\lambda_t = \lambda w_t^s$, the condition under which an individual with ability θ obtains a higher education is described by:

$$\theta \geq \tilde{\theta}(\omega_t; s), \tag{A.36}$$

where

$$\tilde{\theta}(\omega_t; s) \begin{cases} = \underline{\theta}, & \text{for } \omega_t \leq \tilde{\omega}_{min}, \\ = (1-s)\lambda + \omega_t, & \text{for } \omega_t \in (\tilde{\omega}_{min}, \tilde{\omega}_{max}), \\ = \bar{\theta}, & \text{for } \omega_t \geq \tilde{\omega}_{max}, \end{cases}$$

$$\tilde{\omega}_{min} \equiv \underline{\theta} - (1-s)\lambda, \quad \tilde{\omega}_{max} \equiv \bar{\theta} - (1-s)\lambda.$$

As long as $\omega_t \in (\tilde{\omega}_{min}, \tilde{\omega}_{max})$, the relations $\tilde{\theta}_\omega(\omega_t; s) > 0$ and $\tilde{\theta}_s(\omega_t; s) < 0$ hold, which indicates that the skilled worker share decreases with the relative wage of unskilled/skilled workers ω_t, whereas it increases with the education subsidy rate s. Therefore, the skilled and unskilled labor supply in our generalized model are given by

$$\tilde{H}(\omega_t; s) \equiv \int_{\tilde{\theta}(\omega_t;s)}^{\bar{\theta}} M\theta d\Phi(\theta), \tag{A.37}$$

$$\tilde{L}(\omega_t; s) \equiv \int_{\underline{\theta}}^{\tilde{\theta}(\omega_t;s)} M d\Phi(\theta). \tag{A.38}$$

By differentiating (A.37) and (A.38) with respect to ω_t and n_t, we obtain

$$\frac{\omega_t \tilde{H}_\omega(\omega_t; s)}{\tilde{H}(\omega_t; s)} = -\frac{\omega_t}{\tilde{H}(\omega_t; s)} M\tilde{\theta}_t \phi(\tilde{\theta}_t) < 0,$$

$$\frac{s\tilde{H}_s(\omega_t; s)}{\tilde{H}(\omega_t; s)} = \frac{s}{\tilde{H}(\omega_t; s)} M\tilde{\theta}_t \phi(\tilde{\theta}_t)\lambda > 0,$$

$$\frac{\omega_t \tilde{L}_\omega(\omega_t; s)}{\tilde{L}(\omega_t; s)} = \frac{\omega_t}{\tilde{L}(\omega_t; s)} M\phi(\tilde{\theta}_t) > 0,$$

$$\frac{s\tilde{L}_s(\omega_t; s)}{\tilde{L}(\omega_t; s)} = -\frac{s}{\tilde{L}(\omega_t; s)} M\phi(\tilde{\theta}_t)\lambda < 0,$$

where $\tilde{\theta}_t = \tilde{\theta}(\omega_t; s)$ and $\phi(\tilde{\theta}_t)$ are the values of the probability density function of θ, $\phi(\theta)$, evaluated at $\theta = \tilde{\theta}_t$. Therefore, as long as $\omega_t \in (\tilde{\omega}_{min}, \tilde{\omega}_{max})$, like the functions $H(\omega_t; s)$ and $L(\omega_t; s)$ defined in (6) and (7) in the basic model, the functions $\tilde{H}(\omega_t; s)$ and $\tilde{L}(\omega_t; s)$ defined in (A.37) and (A.38) satisfy the following properties: $\tilde{H}_\omega(\omega_t; s) < 0$, $\tilde{H}_s(\omega_t; s) > 0$, $\tilde{L}_\omega(\omega_t; s) > 0$ and $\tilde{L}_s(\omega_t; s) < 0$.

Moreover, the saving function of individuals with ability θ is written as follows:

$$S_t^\theta = \frac{\beta}{1+\beta} I_t^\theta, \tag{A.39}$$

where

$$I_t^\theta \equiv (1-\tau_t)\max\left\{w_t^s\left[\theta - (1-s)\lambda\right], w_t^u\right\}.$$

Governments

The government budget constraint in period t is written as follows:

$$\tau_t \left\{ \int_{\tilde{\theta}(\omega_t; s)}^{\bar{\theta}} \left[w_t^s \theta - (1-s)\lambda_t\right] M d\Phi(\theta) + w_t^u \tilde{L}(\omega_t; s) \right\} = s\lambda_t \int_{\tilde{\theta}(\omega_t; s)}^{\bar{\theta}} M d\Phi(\theta), \tag{A.40}$$

where the left-hand side is the total tax revenue raised from all young individuals, and the right-hand side is the total expenditure composed of education subsidy payments to all skilled young individuals. Note that private education expenses $(1-s)\lambda_t$ are deductible from income tax.

Relative wage of unskilled/skilled workers

The production side of the economy is the same as the basic model. The labor market clearing conditions of (22) and (23) are rewritten as follows:

$$n_t(l_t^s + l_t^R) + L_t^N = \tilde{H}(\omega_t; s), \tag{A.41}$$

$$n_t l_t^u = \tilde{L}(\omega_t; s). \tag{A.42}$$

Therefore, when the product development sector operates (i.e., $n_{t+1} > n_t$), from (24), (A.39) and (A.40), we can obtain the following equation:

$$n_{t+1} V_t = \frac{\beta}{1+\beta} \left\{ w_t^s \tilde{H}(\omega_t; s) + w_t^u \tilde{L}(\omega_t; s) - \lambda w_t^s \left[M - \tilde{L}(\omega_t; s) \right] \right\}. \tag{A.43}$$

The term $\lambda w_t^s \left[M - \tilde{L}(\omega_t; s) \right]$ represents the aggregate direct costs of education, which negatively affects the aggregate level of savings. By substituting (17) and (19) into (A.43), equation (A.43) can be rewritten as follows:

$$\frac{n_t}{\delta_t} = \frac{\beta}{1+\beta} \left\{ \tilde{H}(\omega_t; s) + \omega_t \tilde{L}(\omega_t; s) - \lambda \left[M - \tilde{L}(\omega_t; s) \right] \right\} - L_t^N. \tag{A.44}$$

By using equations (13) to (15) and equations (A.41) to (A.42), we can express the skilled labor engaged in the product development sector L_t^N as follows:

$$L_t^N = \tilde{H}(\omega_t; s) - \frac{\alpha + \gamma}{1 - \alpha} \omega_t \tilde{L}(\omega_t; s). \tag{A.45}$$

Thus, by substituting (18) and (A.45) into (A.44), we can obtain the following equation:

$$\frac{n_t^{1-\psi}}{\bar{\delta}} = \left(\frac{\beta}{1+\beta} + \frac{\alpha+\gamma}{1-\alpha} \right) \omega_t \tilde{L}(\omega_t; s) - \frac{1}{1+\beta} \tilde{H}(\omega_t; s) - \frac{\beta\lambda}{1+\beta} \left[M - \tilde{L}(\omega_t; s) \right]. \tag{A.46}$$

From (A.46), we can see that the relative wages of unskilled/skilled workers ω_t depend on the number of firms n_t and the education subsidy rate s (i.e., $\omega_t = \tilde{\omega}(n_t; s)$). In addition, by differentiating ω_t with respect to n_t and s, we obtain:

$$\frac{n_t \tilde{\omega}_n(n_t; s)}{\tilde{\omega}(n_t; s)} = \frac{(1-\psi)\frac{n_t}{\delta_t}}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t \tilde{L}_\omega(\omega_t;s)}{\tilde{L}(\omega_t;s)}\right) + \frac{\tilde{H}(\omega_t;s)}{1+\beta}\left(1 + \frac{\omega_t \tilde{L}_\omega(\omega_t;s)}{\tilde{L}(\omega_t;s)} - \frac{\omega_t \tilde{H}_\omega(\omega_t;s)}{\tilde{H}(\omega_t;s)}\right) + \frac{\beta\lambda}{1+\beta}\left(M - \tilde{L}(\omega_t; s) + \frac{M\omega_t \tilde{L}_\omega(\omega_t;s)}{\tilde{L}(\omega_t;s)}\right)} > 0,$$

$$\frac{s \tilde{\omega}_s(n_t; s)}{\tilde{\omega}(n_t; s)} = -\frac{\left(\frac{n_t}{\delta_t} + \frac{\tilde{H}(\omega_t;s)}{1+\beta}\right)\frac{s\tilde{L}_s(\omega_t;s)}{\tilde{L}(\omega_t;s)} - \frac{\tilde{H}(\omega_t;s)}{1+\beta}\frac{s\tilde{H}_s(\omega_t;s)}{\tilde{H}(\omega_t;s)}}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t \tilde{L}_\omega(\omega_t;s)}{\tilde{L}(\omega_t;s)}\right) + \frac{\tilde{H}(\omega_t;s)}{1+\beta}\left(1 + \frac{\omega_t \tilde{L}_\omega(\omega_t;s)}{\tilde{L}(\omega_t;s)} - \frac{\omega_t \tilde{H}_\omega(\omega_t;s)}{\tilde{H}(\omega_t;s)}\right) + \frac{\beta\lambda}{1+\beta}\left(M - \tilde{L}(\omega_t; s) + \frac{M\omega_t \tilde{L}_\omega(\omega_t;s)}{\tilde{L}(\omega_t;s)}\right)} > 0,$$

where $\omega_t = \tilde{\omega}(n_t; s)$. Therefore, like the function $\omega(n_t; s)$ defined in (28) in the basic model, when the product development sector operates (i.e., $n_{t+1} > n_t$), the function $\tilde{\omega}(n_t; s)$ defined in (A.46) satisfies the following properties: $\tilde{\omega}_n(n_t; s) > 0$ and $\tilde{\omega}_s(n_t; s) > 0$.

When the product development sector does not operate (i.e., $n_{t+1} = n_t$), from (A.45) and $L_t^N = 0$, the relative wages of unskilled/skilled workers are determined by the following implicit function of ω_t:

$$\frac{1-\alpha}{\alpha+\gamma} = \frac{\omega_t \tilde{L}(\omega_t; s)}{\tilde{H}(\omega_t; s)} \equiv \tilde{\Gamma}(\omega_t; s). \tag{A.47}$$

We denote the value of ω_t such that it satisfies (A.47) as $\tilde{\omega}^*(s)$. By differentiating (A.47) with respect to ω_t and s, we obtain the following:

$$\frac{s\tilde{\omega}_s^*(s)}{\tilde{\omega}^*(s)} = -\frac{\frac{s\tilde{L}_s(\tilde{\omega}^*;s)}{\tilde{L}(\tilde{\omega}^*;s)} - \frac{s\tilde{H}_s(\tilde{\omega}^*;s)}{\tilde{H}(\tilde{\omega}^*;s)}}{1 + \frac{\tilde{\omega}^*\tilde{L}_\omega(\tilde{\omega}^*;s)}{\tilde{L}(\tilde{\omega}^*;s)} - \frac{\tilde{\omega}^*\tilde{H}_\omega(\tilde{\omega}^*;s)}{\tilde{H}(\tilde{\omega}^*;s)}} > 0,$$

where $\tilde{\omega}^* = \tilde{\omega}^*(s)$. Therefore, like the function $\omega^*(s)$ defined in (29) in the basic model, when the product development sector does not operate (i.e., $n_{t+1} = n_t$), the function $\tilde{\omega}^*(s)$ defined in (A.47) satisfies the following property: $\tilde{\omega}_s^*(s) > 0$. We also denote the number of firms n_t such that it satisfies $\tilde{\omega}(n_t; s) = \omega^*(s)$ as $\tilde{n}^*(s)$.

Following the same arguments in the basic model, from (A.46) and (A.47), the equilibrium relative wage of unskilled/skilled workers ω_t is given by the following expression:

$$\omega_t = \begin{cases} \tilde{\omega}(n_t; s), & \text{for } n_t < \tilde{n}^*(s), \\ \tilde{\omega}^*(s), & \text{for } n_t \geq \tilde{n}^*(s). \end{cases} \tag{A.48}$$

From (A.48), the equilibrium relationship between the number of firms n_t and the relative wages of unskilled/skilled workers ω_t is described as the solid line in Figure 2 in the basic model. Moreover, since $\tilde{\omega}_s(n_t; s) > 0$ and $\tilde{\omega}_s^*(s) > 0$ from (A.46) and (A.47), given the value of n_t, the higher rate of education subsidies positively affects the relative wages of unskilled/skilled workers. These results indicate that the effects of an education subsidy on the relative wages of unskilled/skilled workers are similar qualitatively to those obtained in the basic model.

However, as shown in the following subsection, in contrast to the results obtained in the basic model, the effect of an education subsidy on the steady-state number of firms $\tilde{n}^*(s)$ is generally ambiguous and depends upon the values of the parameters. We will discuss these points more rigorously later.

Dynamics of the average productivity of firms

By substituting (14) and (A.42) into (15), the skilled labor engaged in process innovation l_t^R is given by $l_t^R = \frac{\gamma}{1-\alpha}\frac{\omega_t \tilde{L}(\omega_t;s)}{n_t}$. Then, using (A.46) and (A.47), l_t^R can be expressed as follows:

$$l_t^R = \begin{cases} \frac{\gamma}{1-\alpha} \frac{\left\{\frac{1}{\delta n_t^{\psi}} + \frac{1}{1+\beta} \frac{\tilde{H}(n_t;s)+\beta\lambda\left[M-\tilde{L}(n_t;s)\right]}{n_t}\right\}}{\frac{\beta}{1+\beta}+\frac{\alpha+\gamma}{1-\alpha}}, & \text{for } n_t < \tilde{n}^*(s), \\ \frac{\gamma}{\alpha+\gamma}\frac{\tilde{H}(s)}{n_t}, & \text{for } n_t \geq \tilde{n}^*(s), \end{cases} \tag{A.49}$$

where

$$\tilde{H}(n_t; s) \equiv \tilde{H}\left(\tilde{\omega}(n_t; s); s\right),$$
$$\tilde{L}(n_t; s) \equiv \tilde{L}\left(\tilde{\omega}(n_t; s); s\right),$$
$$\tilde{H}(s) \equiv \tilde{H}\left(\tilde{\omega}^*(s); s\right).$$

From (A.49), we can see that the skilled labor engaged in process innovation l_t^R depends upon the number of firms n_t and the education subsidy rate s (i.e., $\tilde{l}^R(n_t; s)$). By differentiating $\tilde{H}(n_t; s)$, $\tilde{L}(n_t; s)$ and $\tilde{H}(s)$ with respect to n_t and s, we obtain:

$$\frac{n_t\tilde{H}_n(n_t; s)}{\tilde{H}(n_t; s)} = \frac{\omega_t\tilde{H}_\omega(\omega_t; s)}{\tilde{H}(\omega_t; s)}\frac{n_t\tilde{\omega}_n(n_t; s)}{\tilde{\omega}(n_t; s)} < 0,$$

$$\frac{s\tilde{H}_s(n_t; s)}{\tilde{H}(n_t; s)} = \frac{\left\{\frac{n_t}{\delta_t} + \frac{\tilde{H}(\omega_t;s)}{1+\beta} + \frac{\beta\lambda}{1+\beta}\left[M - \tilde{L}(\omega_t; s)\right]\right\}\frac{s\tilde{H}_s(\omega_t;s)}{\tilde{H}(\omega_t;s)}}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t\tilde{L}_\omega(\omega_t;s)}{\tilde{L}(\omega_t;s)}\right) + \frac{\tilde{H}(\omega_t;s)}{1+\beta}\left(1 + \frac{\omega_t\tilde{L}_\omega(\omega_t;s)}{\tilde{L}(\omega_t;s)} - \frac{\omega_t\tilde{H}_\omega(\omega_t;s)}{\tilde{H}(\omega_t;s)}\right) + \frac{\beta\lambda}{1+\beta}\left(M - \tilde{L}(\omega_t; s) + \frac{M\omega_t\tilde{L}_\omega(\omega_t;s)}{\tilde{L}(\omega_t;s)}\right)} > 0,$$

$$\frac{n_t\tilde{L}_n(n_t; s)}{\tilde{L}(n_t; s)} = \frac{\omega_t\tilde{L}_\omega(\omega_t; s)}{\tilde{L}(\omega_t; s)}\frac{n_t\tilde{\omega}_n(n_t; s)}{\tilde{\omega}(n_t; s)} > 0,$$

$$\frac{s\tilde{L}_s(n_t; s)}{\tilde{L}(n_t; s)} = \frac{-\left\{\frac{n_t}{\delta_t} + \frac{\tilde{H}(\omega_t;s)}{1+\beta} + \frac{\beta\lambda}{1+\beta}\left[M - \tilde{L}(\omega_t; s)\right]\right\}\frac{s\tilde{H}_s(\omega_t;s)}{\tilde{\theta}_t\tilde{L}(\omega_t;s)}}{\frac{n_t}{\delta_t}\left(1 + \frac{\omega_t\tilde{L}_\omega(\omega_t;s)}{\tilde{L}(\omega_t;s)}\right) + \frac{\tilde{H}(\omega_t;s)}{1+\beta}\left(1 + \frac{\omega_t\tilde{L}_\omega(\omega_t;s)}{\tilde{L}(\omega_t;s)} - \frac{\omega_t\tilde{H}_\omega(\omega_t;s)}{\tilde{H}(\omega_t;s)}\right) + \frac{\beta\lambda}{1+\beta}\left(M - \tilde{L}(\omega_t; s) + \frac{M\omega_t\tilde{L}_\omega(\omega_t;s)}{\tilde{L}(\omega_t;s)}\right)} < 0,$$

$$\frac{s\tilde{H}_s(s)}{\tilde{H}(s)} = \frac{\frac{s\tilde{H}_s(\tilde{\omega}^*;s)}{\tilde{H}(\tilde{\omega}^*;s)}}{1 + \frac{\tilde{\omega}^*\tilde{L}_\omega(\tilde{\omega}^*;s)}{\tilde{L}(\tilde{\omega}^*;s)} - \frac{\tilde{\omega}^*\tilde{H}_\omega(\tilde{\omega}^*;s)}{\tilde{H}(\tilde{\omega}^*;s)}} > 0$$

where $\omega_t = \tilde{\omega}(n_t; s)$ and $\tilde{\omega}^* = \tilde{\omega}^*(s)$. Like the function $l^R(n_t; s)$ defined in (32) in the basic model, these results imply that $\tilde{l}^R(n_t; s)$ defined in (A.49) satisfies the following properties: $\tilde{l}_n^R(n_t; s) < 0$ and $\tilde{l}_s^R(n_t; s) > 0$.

Using (11) and (A.49), the gross growth rate of the average productivity of firms is given by the following expression:

$$G_t^A \equiv \frac{A_t}{A_{t-1}} = z\left[\tilde{l}^R(n_t; s)\right]^\gamma \equiv \tilde{G}^A(n_t; s). \tag{A.50}$$

From (A.49) and (A.50), like the function $G^A(n_t; s)$ defined in (33) in the basic model, we can easily confirm that the function $\tilde{G}^A(n_t; s)$ defined in (A.50) satisfies the following properties: $\tilde{G}_n^A(n_t; s) < 0$ and $\tilde{G}_s^A(n_t; s) > 0$. Equation (A.50) indicates that the equilibrium relationship between the number of firms n_t and the gross growth rate of the average productivity of firms G_t^A is described as the solid line in Figure 3 in the basic model. Moreover, since $\tilde{G}_s^A(n_t; s) > 0$, given the value of n_t, the higher education subsidy rate s positively affects the equilibrium gross growth rate of the average productivity of firms. These results indicate that the effects of an education subsidy on the dynamics of the average productivity of firms are similar qualitatively to those obtained in the basic model.

Dynamics of the number of firms

When the product development sector operates (i.e., $n_{t+1} > n_t$), from (19) and (A.43), we can obtain the following equation:

$$\frac{n_{t+1}}{\delta_t} = \frac{\beta}{1+\beta}\left\{\tilde{H}(\omega_t; s) + \omega_t \tilde{L}(\omega_t; s) - \lambda\left[M - \tilde{L}(\omega_t; s)\right]\right\}. \tag{A.51}$$

Then, by using equations (A.44) to (A.47) and (A.51), the gross growth rate of the number of firms can be expressed as follows:

$$G_{t+1}^n \equiv \frac{n_{t+1}}{n_t} = \begin{cases} \frac{1}{\tilde{F}(n_t;s)} \equiv \tilde{G}^n(n_t; s), & \text{for } n_t < \tilde{n}^*(s), \\ 1, & \text{for } n_t \geq \tilde{n}^*(s). \end{cases} \tag{A.52}$$

where

$$\tilde{F}(n_t; s) \equiv 1 - \frac{1+\beta}{\beta}\frac{1 - \frac{\alpha+\gamma}{1-\alpha}\tilde{\Gamma}(n_t; s)}{1 + \tilde{\Gamma}(n_t; s) - \frac{\lambda\left[M - \tilde{L}(n_t;s)\right]}{\tilde{H}(n_t;s)}},$$

$$\tilde{\Gamma}(n_t; s) \equiv \tilde{\Gamma}\left(\tilde{\omega}(n_t; s); s\right) = \frac{\tilde{\omega}(n_t; s)\tilde{L}(n_t; s)}{\tilde{H}(n_t; s)}.$$

From (A.52), like (35) in the basic model, we can easily confirm that the dynamics of n_t are determined by the one-dimensional first order difference equation. However, it is difficult to examine analytically the precise properties of the function $\tilde{G}^n(n_t; s)$ defined in (A.52). As a result, we cannot discuss the dynamic properties of our generalized model in a precise manner. Moreover, since we cannot show analytically the sign of $\tilde{\Gamma}_s(n_t; s)$ in an intuitive way, the effect of an education subsidy on the gross growth rate of the number of firms is generally ambiguous and depends upon the values of the parameters. However, the numerical simulation analyses in the following subsection suggest that the dynamics of n_t are stable and gradually converge to a unique steady-state value $\tilde{n}^*(s)$ under a wide plausible set of parameter values.

From (A.46) and (A.47), the steady-state value of $\tilde{n}^*(s)$ is determined by the following implicit function of n:

$$\frac{n^{1-\psi}}{\bar{\delta}} = \frac{\beta}{1+\beta}\left[\tilde{H}(s)\frac{1+\gamma}{\alpha+\gamma} - \lambda\frac{M - \tilde{L}(s)}{\tilde{H}(s)}\right] \equiv RH(s), \tag{A.53}$$

where

$$\tilde{L}(s) \equiv \tilde{L}\left(\tilde{\omega}^*(s); s\right).$$

By differentiating $\tilde{L}(s)$ with respect to s, we obtain:

$$\frac{s\tilde{L}_s(s)}{\tilde{L}(s)} = -\frac{\frac{s\tilde{H}_s(\tilde{\omega}^*;s)}{\tilde{\theta}\tilde{L}(\tilde{\omega}^*;s)}}{1 + \frac{\tilde{\omega}^*\tilde{L}_\omega(\tilde{\omega}^*;s)}{\tilde{L}(\tilde{\omega}^*;s)} - \frac{\tilde{\omega}^*\tilde{H}_\omega(\tilde{\omega}^*;s)}{\tilde{H}(\tilde{\omega}^*;s)}} < 0,$$

where $\tilde{\theta} = \tilde{\theta}(\tilde{\omega}^*; s)$ and $\tilde{\omega}^* = \tilde{\omega}^*(s)$.

From (A.53), suppose that there were no direct costs of education (i.e., $\lambda = 0$), the steady-state value of $\tilde{n}^*(s)$ is given by $\tilde{n}^*(s) = \left(\frac{\beta}{1+\beta}\frac{1+\gamma}{\alpha+\gamma}\bar{\delta}\right)^{\frac{1}{1-\psi}}\left[\tilde{H}(s)\right]^{\frac{1}{1-\psi}}$, which corresponds to the steady-state value of $n^*(s)$ defined in (37) in the basic model.

By differentiating (A.53) with respect to n_t and s, and evaluating them at $n_t = \tilde{n}^*(s)$, we obtain:

$$\tilde{n}_s^*(s) = \frac{RH_s(s)}{\frac{1-\psi}{\bar{\delta}\tilde{n}^*(s)^\psi}}, \tag{A.54}$$

where

$$RH_s(s) \equiv \frac{\beta}{1+\beta}\left\{\frac{1+\gamma}{\alpha+\gamma}\tilde{H}_s(s) + \frac{\lambda}{\tilde{H}(s)}\left[\tilde{L}_s(s) + \frac{M-\tilde{L}(s)}{\tilde{H}(s)}\tilde{H}_s(s)\right]\right\}.$$

From (A.54), the sign of $\tilde{n}_s^*(s)$ equals to the sign of $RH_s(s)$. Therefore, when the direct cost of education is sufficiently small (i.e., $\lambda \approx 0$), since $\tilde{H}_s(s) > 0$, the relation $\tilde{n}_s^*(s) > 0$ is more likely to hold, which indicates that the higher education subsidy rate increases the steady-state number of firms. However, when the direct cost of education is sufficiently large, since the sign of the term $\frac{\lambda}{\tilde{H}(s)}\left[\tilde{L}_s(s) + \frac{M-\tilde{L}(s)}{\tilde{H}(s)}\tilde{H}_s(s)\right]$ is indeterminate a priori, we cannot determine the sign of $\tilde{n}_s^*(s)$ analytically. Therefore, the overall effects of an education subsidy on the steady-state number of firms are generally ambiguous and depend upon the values of the parameters. Intuitively, the higher education subsidy rate increases the share of skilled workers with higher wages, which positively affects the aggregate savings, and thereby positively affects the steady-state number of firms. However, the higher share of skilled workers increases the aggregate direct costs of education, which negatively affects the aggregate savings, and thereby negatively affects the steady-state number of firms. Explicit consideration of the direct costs of education adds a new channel through which an education subsidy negatively affects the steady-state number of firms. Consequently, in contrast to the results obtained in the basic model, the effect of an education subsidy on the steady-state number of firms is generally ambiguous and depends upon the values of the parameters.

From (A.49) and (A.50), the steady-state gross growth rate of the average productivity of firms is given by the following:

$$\tilde{G}^A(s) = z\left[\tilde{l}^R(s)\right]^\gamma, \tag{A.55}$$

where

$$\tilde{l}^R(s) \equiv \frac{\gamma}{\alpha+\gamma}\frac{\tilde{H}(s)}{\tilde{n}^*(s)}.$$

Since the sign of $\tilde{n}_s^*(s)$ is indeterminate a priori, the sign of $\tilde{G}_s^A(s)$ is also difficult to clarify analytically. Therefore, the overall effects of an education subsidy on the steady-state gross growth rate of the average productivity of firms $\tilde{G}^A(s)$ are generally ambiguous and depend upon the values of the parameters.

The value of GDP

Following the same arguments given in Appendix G, GDP in our generalized model can be written as follows:

$$GDP_t = \alpha^\alpha(1-\alpha)^{1-\alpha}A_t n_t^\sigma \tilde{\Lambda}_t. \tag{A.56}$$

where

$$\tilde{\Lambda}_t \equiv \begin{cases} \frac{\eta-1}{\eta}\frac{\tilde{H}(n_t;s)}{\tilde{\omega}(n_t;s)^{1-\alpha}} + \frac{1-(\alpha+\gamma)\frac{\eta-1}{\eta}}{1-\alpha}\frac{\tilde{\omega}(n_t;s)\tilde{L}(n_t;s)}{\tilde{\omega}(n_t;s)^{1-\alpha}} \equiv \tilde{\Lambda}(n_t;s), & \text{for } n_t < \tilde{n}^*(s), \\ \frac{1}{1-\alpha}\frac{\tilde{\omega}^*(s)\tilde{L}(s)}{\tilde{\omega}^*(s)^{1-\alpha}} \equiv \tilde{\Lambda}(s), & \text{for } n_t \geq \tilde{n}^*(s). \end{cases}$$

Equation (A.56) indicates that the value of GDP depends upon the average productivity of firms A_t, the number of firms n_t, and the other market factors $\tilde{\Lambda}_t$. The term $\tilde{\Lambda}_t$ captures the degree of static efficiency of production given the existing technologies and factor inputs. By differentiating $\tilde{\Lambda}(s)$ with respect to s, we obtain

$$\frac{s\tilde{\Lambda}_s(s)}{\tilde{\Lambda}(s)} = \frac{\left[-(\alpha+\gamma)\frac{\tilde{\omega}^*}{\tilde{\theta}} + \alpha\right]\frac{sH_s(\tilde{\omega}^*;s)}{H(\tilde{\omega}^*;s)}}{1 + \frac{\omega^* L_\omega(\omega^*;s)}{L(\tilde{\omega}^*;s)} - \frac{\omega^* H_\omega(\omega^*;s)}{H(\tilde{\omega}^*;s)}} \begin{cases} > 0, & \text{for } \frac{\tilde{\omega}^*}{\tilde{\theta}} < \frac{\alpha}{\alpha+\gamma}, \\ < 0, & \text{for } \frac{\tilde{\omega}^*}{\tilde{\theta}} > \frac{\alpha}{\alpha+\gamma}. \end{cases}$$

where $\tilde{\omega}^* = \tilde{\omega}^*(s)$, and the relation $\frac{\tilde{\omega}^*}{\tilde{\theta}} = \frac{\omega^*}{(1-s)\lambda+\omega^*} < 1$ holds from (A.36). Therefore, the education subsidy policy deteriorates (resp., improves) the steady-state static efficiency of production when the direct costs of education λ are sufficiently small (resp., large) to satisfy the relation $\frac{\tilde{\omega}^*}{\tilde{\theta}} = \frac{\omega^*}{(1-s)\lambda+\omega^*} > \frac{\alpha}{\alpha+\gamma}$ (resp.,$\frac{\tilde{\omega}^*}{\tilde{\theta}} = \frac{\omega^*}{(1-s)\lambda+\omega^*} < \frac{\alpha}{\alpha+\gamma}$).

From (A.56), the gross growth rate of GDP is given by the following expression:

$$G_t^{GDP} \equiv \frac{GDP_t}{GDP_{t-1}} = \frac{A_t}{A_{t-1}}\left(\frac{n_t}{n_{t-1}}\right)^\sigma \frac{\tilde{\Lambda}_t}{\tilde{\Lambda}_{t-1}} = \tilde{G}_t^A\left(\tilde{G}_t^n\right)^\sigma \tilde{G}_t^{\tilde{\Lambda}}. \tag{A.57}$$

Like the basic model, we can see that the gross growth rate of GDP depends on the gross growth rate of the average productivity of firms $\tilde{G}_t^A$, the number of firms $\tilde{G}_t^n$, and the other market factors $\tilde{G}_t^{\tilde{\Lambda}}$. Moreover, in the steady-state equilibrium where the relations $n_t = n_{t-1} = \tilde{n}^*(s)$ and $\Lambda_t = \Lambda_{t-1} = \tilde{\Lambda}(s)$ hold, the gross growth rate of

GDP becomes equivalent to the gross growth rate of the average productivity of firms, as follows:

$$\tilde{G}^{GDP*} \equiv G_t^{GDP} \mid_{n_t=\tilde{n}^*(s)} = \tilde{G}^A(s). \tag{A.58}$$

Numerical Analysis

In our generalized model, it is difficult to examine analytically not only the short-run effects of an education subsidy on the per capita GDP growth rate but also the long-run effects of an education subsidy on the number of firms and the average productivity growth rate of firms. Therefore, to obtain further insights, we resort to numerical simulation analysis. For comparison, we employ the same parameter values as the basic model.

To calibrate the value of the direct cost of education λ, we include one more endogenous variable, for which we set the target value from available data. The target value of education expenditure share per GDP, $\frac{\lambda w^s[M-\tilde{L}(s)]}{PY} = 0.052$, is given by the OECD average expenditure on educational institutions from primary to tertiary education as a percentage of GDP in 2013 (OECD, 2016b: Table B2.1, p205). Using (A.15), the steady-state education expenditure share per GDP is given by $\frac{\lambda w^s[M-\tilde{L}(s)]}{PY} = \frac{\lambda(1-\alpha)(\eta-1)}{\eta}\frac{M-\tilde{L}(s)}{\tilde{\omega}^*\tilde{L}(s)}$. Since the other parameter values are set as in the basic model, the value of λ is set to match the target value of $\frac{\lambda w^s[M-\tilde{L}(s)]}{PY}$. This procedure yields $\lambda = 0.1463$.

As in the basic model, we assume that the economy is initially in the steady-state equilibrium where the education subsidy rate s is given by zero (i.e., $s_k = 0$ for all period $k < 3$). Then, the education subsidy rate in period 3 and subsequent periods is increased from 0 to 0.3 (i.e., $s_k = 0.3$ for all periods $k \geq 3$). Figures 9-1 to 9-6 show the numerical examples of the transition path of the relative wage of unskilled/skilled workers ω_t (Figure 9-1), the number of firms per capita n_t/M (Figure 9-2), the net growth rate of the number of firms g_t^n (Figure 9-3), the net growth rate of the average productivity of firms g_t^A (Figure 9-4), the other market factors Λ_t (Figure 9-5), and the net growth rate of GDP g_t^{GDP} (Figure 9-6) under different values of the degree of specialization parameter (i.e., $\sigma = 0, 0.05, 0.15, 0.2$).

As shown in Figures 9-1 to 9-5, the introduction of the education subsidy policy in period 3 increases the relative wages of unskilled/skilled workers in period 3 (Figure 9-1), enhances the entry of new firms in period 4 (Figure 9-2), increases the net growth rate of the number of firms in period 4 (Figure 9-3), increases the net growth rate of the average productivity of firms in period 3 (Figure 9-4), and deteriorates the static efficiency of production in period 3 (Figure 9-5). However, as the number of firms increases, the net growth rate of the number of firms starts to decline from period 5 (Figure 9-3), and the equilibrium number of firms gradually converges to its new steady-state value, which is larger than that in the original steady-state equilibrium (Figure 9-2). Therefore, the education subsidy policy expands the equilibrium number of firms. Moreover, since the negative "entry effect" dominates the positive "cost reduction effect" in the long run, the net growth rate of the average productivity of firms begins to decline from period 4, and it gradually converges to its new steady-state value, which is lower than that in the original steady-state equilibrium (Figure 9-4). Therefore, the education subsidy policy positively affects the net growth rate of the average productivity of firms in the short run, but it negatively affects the net growth rate of the average productivity of firms in the long run. Furthermore, the static efficiency of production starts to improve from period 4, and it gradually converges to its new steady-state value, which is slightly higher than that in the original steady-state equilibrium.

These dynamic properties of ω_t, n_t, g_t^n, g_t^A and $\tilde{\Lambda}_t$ are similar qualitatively to those obtained in the basic model shown in Figures 7-1 to 7-5.[18] Therefore, as shown in Figure 9-6, the effects of an education subsidy on economic growth are similar qualitatively to those obtained in the basic model shown in Figure 7-6. From (A.57), the net growth rate of GDP shown in Figure 9-6 depends upon the net growth rate of the number of firms g_t^n in Figure 9-3, the average productivity of firms g_t^A in Figure 9-4, and the static efficiency of production $\tilde{\Lambda}_t$ in Figure 9-5. The introduction of the education subsidy policy in period 3 decreases the net growth rate of GDP in period 3, because the negative growth effect caused by the decline in Λ_t in period 3 (Figure 9-5) dominates the positive growth effect caused by the rise in g_t^A in period 3 (Figure 9-4). However, as shown in Figures 9-2 and 9-3, the number of firms begins to increase from period 4, which increases the net growth rate of the number of firms in period 4. In addition, the static efficiency of production begins to improve from period 4. Therefore, the net GDP growth rate increases in period 4 and reaches its highest value. Then, if the degree of specialization parameter σ is sufficiently large, as described in the $\sigma = 0.15$ and $\sigma = 0.2$ lines in Figure 9-6, the net growth rate of GDP in period 4 lies sufficiently above that in the original steady-state equilibrium. However, if the degree of specialization parameter σ is sufficiently small, as described in the $\sigma = 0$ and $\sigma = 0.05$ lines in Figure 9-6, the net growth rate of GDP in period 4 lies slightly above that in the original steady-state equilibrium. Following the same arguments in the basic model, these results imply that the short-run effect of the education subsidy on economic growth is generally ambiguous and depends on the values of the parameters.

[18] Strictly speaking, the dynamic properties of $\tilde{\Lambda}_t$ in the generalized model are slightly different from those of Λ_t in the basic model. As shown in Figure 9-5, in contrast to the result obtained in the basic model shown in Figure 7-5, the value of the other market factor in the new steady-state equilibrium becomes larger than that in the original steady-state equilibrium.

After period 4, the economy gradually converges to its new steady-state equilibrium. During this transition process, the net growth rate of the number of firms and the other market factors gradually approach zero. Therefore, in the steady-state equilibrium, the net growth rate of GDP becomes equivalent to the net growth rate of the average productivity of firms. Consequently, as shown in Figures 9-4 and 9-6, the net growth rate of GDP in the new steady-state equilibrium becomes lower than that in the original steady-state equilibrium. Therefore, when the market structure adjusts partially in the short run, the growth effect of the education subsidy is ambiguous and depends upon the values of the parameters. However, when the market structure adjusts fully in the long run, the education subsidy expands the number of firms but reduces economic growth. These results indicate that the effects of an education subsidy on economic growth remain unchanged qualitatively, even when we consider this alternative method of modeling the education subsidy.

Appendix K: Sensitivity analyses (This Appendix is not intended for publication)

In this appendix, we examine how changes in the extent of individual heterogeneity in ability θ, the value of the standing-on-shoulders effect parameter ψ and the value of the efficiency parameter of variety R&D $\bar{\delta}$ affect the per capita GDP growth rate. For comparison, we employ the same parameter values in Section 4.

The extent of individual heterogeneity in ability θ

In this subsection, we consider the effect of the mean-preserving spread of individual ability θ. To this end, as in Appendix I, we set the mean value of individual ability μ to $\frac{1}{2}$, change the value of $\bar{\theta}$ from 0.6 to 1, and adjust the value of $\underline{\theta}$ so that the relation $\mu = \frac{\bar{\theta}+\underline{\theta}}{2}$ holds. Under this specification, the larger value of $\bar{\theta}$ leads to the mean-preserving spread of individual ability θ.

We assume that the economy is initially in the steady-state equilibrium, where the value of $\bar{\theta}$ is given by 0.6 (i.e., $\bar{\theta}_k = 0.6$ for all period $k < 3$). Then, the shocks hit our steady-state economy in period 3, and the value of $\bar{\theta}$ in period 3 and subsequent periods is increased from 0.6 to 0.7, 0.8, 0.9 and 1, respectively (i.e., $\bar{\theta}_k = 0.7, 0.8, 0.9$ and 1 for all periods $k \geq 3$). Figures 10-1 to 10-6 show the numerical examples of the transition path of the relative wage of unskilled/skilled workers ω_t (Figure 10-1), the number of firms per capita n_t/M (Figure 10-2), the net growth rate of the number of firms g_t^n (Figure 10-3), the net growth rate of the average productivity of firms g_t^A (Figure 10-4), the other market factors Λ_t (Figure 10-5), and the net growth rate of GDP g_t^{GDP} (Figure 10-6), respectively.

As shown in Figures 10-1 to 10-5, the mean-preserving spread of individual ability θ (i.e., the rise in the value of $\bar{\theta}$) in period 3 increases the share of skilled workers in period 3, increases the relative wages of unskilled/skilled workers in period 3 (Figure 10-1), enhances the entry of new firms in period 4 (Figure 10-2), increases the net growth rate of the number of firms in period 4 (Figure 10-3), increases the net growth rate of the average productivity of firms in period 3 (Figure 10-4), and improves the static efficiency of production in period 3 (Figure 10-5). However, as the number of firms increase, the net growth rate of the number of firms begins to decline from period 5 (Figure 10-3), and the equilibrium number of firms gradually converges to its new steady-state value, which is larger than that in the original steady-state equilibrium (Figure 10-2). Therefore, the mean-preserving spread of individual ability θ expands the equilibrium number of firms. Moreover, the net growth rate of the average productivity of firms begins to decline from period 4, and it gradually converges to its new steady-state value, which is lower than that in the original steady-state equilibrium (Figure 10-4). Therefore, the mean-preserving spread of individual ability θ positively affects the net growth rate of the average productivity of firms in the short run, but it negatively affects the net growth rate of the average productivity of firms in the long run.[19]

The mean-preserving spread of individual ability θ encourages more firms to enter the market with new products, which strengthens horizontal competition among firms. It is this strengthening of horizontal competition that gives rise to the negative "entry effect" of the mean-preserving spread of individual ability θ on the net growth rate of the average productivity of firms. Figures 10-2 and 10-4 also show that a larger spread of individual ability θ (i.e., the larger increase in the value of $\bar{\theta}$) encourages more firms to enter the market with new products, which strengthens both the short run positive and the long-run negative effects of the mean-preserving spread of individual ability θ on the net growth rate of the average productivity of firms.

From (41), the net growth rate of GDP shown in Figure 10-6 depends upon the net growth rate of the number of firms g_t^n in Figure 10-3, the average productivity of firms g_t^A in Figure 10-4, and the static efficiency of production Λ_t in Figure 10-5. As shown in Figure 10-6, the mean-preserving spread of individual ability θ in period 3 increases the net growth rate of GDP in period 3 due to the surge in the both values of g_t^A and Λ_t in period 3. However, as shown in Figures 10-4 and 10-5, the net growth rate of the average productivity of firms g_t^A starts to decline from period 4, and the static efficiency of production Λ_t improves only slightly in period 4. Therefore, while the net growth rate of the number of firms g_t^N increases

[19] A simple calculation shows that most of these predictions hold in the case where the equilibrium threshold value of ability $\hat{\theta}(\omega_t; s)$ lies above the mean value of individual ability μ (i.e., $\hat{\theta}(\omega_t; s) > \mu$).

temporally in period 4 (Figure 10-3), the net growth rate of GDP starts to decline from period 4. After period 4, the economy gradually converges to its new steady-state equilibrium. During this transition process, the net growth rate of the number of firms and the other market factors gradually approach zero. Therefore, in the steady-state equilibrium, the net growth rate of GDP becomes equivalent to the net growth rate of the average productivity of firms. Consequently, as shown in Figures 10-4 and 10-6, the net growth rate of GDP in the new steady-state equilibrium becomes lower than that in the original steady-state equilibrium. Therefore, when the market structure adjusts partially in the short run, the growth effect of the mean-preserving spread of individual ability θ is positive. However, when the market structure adjusts fully in the long run, the mean-preserving spread of individual ability θ expands the number of firms but reduces economic growth.

The value of the standing-on-shoulders effect parameter ψ

In this subsection, we consider the effect of the rise in the value of the standing-on-shoulders effect parameter ψ. We assume that the economy is initially in the steady-state equilibrium where the value of the standing on shoulders parameter ψ is given by 0.05 (i.e., $\psi_k = 0.05$ for all period $k < 3$). Then, the shocks hit our steady-state economy in period 3, and the value of ψ in period 3 and subsequent periods is increased from 0.05 to 0.15, 0.25, 0.35 and 0.45 (i.e., $\psi_k = 0.15, 0.25, 0.35$ and 0.45 for all periods $k \geq 3$). Figures 11-1 to 11-6 show the numerical examples of the transition path of the relative wage of unskilled/skilled workers ω_t (Figure 11-1), the number of firms per capita n_t/M (Figure 11-2), the net growth rate of the number of firms g_t^n (Figure 11-3), the net growth rate of the average productivity of firms g_t^A (Figure 11-4), the other market factors Λ_t (Figure 11-5), and the net growth rate of GDP g_t^{GDP} (Figure 11-6).

As shown in Figures 11-1 to 11-5, the rise in the value of the standing-on-shoulders effect parameter ψ in period 3 decreases the relative wages of unskilled/skilled workers in period 3 (Figure 11-1), enhances the entry of new firms in period 4 (Figure 11-2), increases the net growth rate of the number of firms in period 4 (Figure 11-3), decreases the net growth rate of the average productivity of firms in period 3 (Figure 11-4), and deteriorates the static efficiency of production in period 3 (Figure 11-5). However, as the number of firms increases, the net growth rate of the number of firms begins to decline from period 5 (Figure 11-3), and the equilibrium number of firms gradually converges to its new steady-state value, which is larger than that in the original steady-state equilibrium (Figure 11-2). Therefore, the rise in the value of the standing-on-shoulders effect parameter ψ expands the equilibrium number of firms. Moreover, the net growth rate of the average productivity of firms continues to decline in period 4 and in subsequent periods, and it gradually converges to its new steady-state value, which is lower than that in the original steady-state equilibrium (Figure 11-4). Therefore, the rise in the value of the standing-on-shoulders effect parameter ψ negatively affects the net growth rate of the average productivity of firms not only in the short run but also in the long run.[20]

The rise in the value of the standing-on-shoulders effect parameter ψ encourages more firms to enter the market with new products, which strengthens horizontal competition among firms. It is this strengthening of horizontal competition that gives rise to the negative "entry effect" of the rise in the value of the standing-on-shoulders effect parameter ψ on the net growth rate of the average productivity of firms. Figures 11-2 and 11-4 also show that the larger increase in the value of the standing-on-shoulders effect parameter ψ encourages more firms to enter the market with new products, which strengthens the long-run negative effects of the rise in the value of the standing-on-shoulders effect parameter ψ on the net growth rate of the average productivity of firms.

From (41), the net growth rate of GDP shown in Figure 11-6 depends upon the net growth rate of the number of firms g_t^n in Figure 11-3, the average productivity of firms g_t^A in Figure 11-4, and the static efficiency of production Λ_t in Figure 11-5. As shown in Figure 11-6, the rise in the value of the standing-on-shoulders effect parameter ψ in period 3 decreases the net growth rate of GDP in period 3 due to the declines in both the values of g_t^A and Λ_t in period 3. However, as shown in Figures 11-2 and 11-3, the number of firms begins to increase from period 4, which increases the net growth rate of the number of firms in period 4. In addition, as shown in Figure 11-5, the static efficiency of production improves gradually from period 4. Therefore, while the net growth rate of the average productivity of firms g_t^A continues to decline in period 4 (Figure 11-4), the net growth rate of GDP increases temporarily in period 4. After period 4, the economy gradually converges to its new steady-state equilibrium. During this transition process, the net growth rate of the number of firms and the other market factors gradually approach zero. Therefore, in the steady-state equilibrium, the net growth rate of GDP becomes equivalent to the net growth rate of the average productivity of firms. Consequently, as shown in Figures 11-4 and 11-6, the net growth rate of GDP in the new steady-state equilibrium becomes lower than that in the original steady-state equilibrium. Therefore, when the market structure adjusts partially in the short run, the growth effect of the rise in the value of the standing-on-shoulders effect parameter ψ is generally ambiguous. However, when the market structure adjusts fully in the long run, the rise in the value of the standing-on-shoulders effect parameter ψ expands the number of firms but reduces economic growth.

[20]Simple calculation shows that most of these predictions hold in the case where the equilibrium number of firms is more than unity (i.e., $n_t > 1$).

The value of the efficiency parameter of variety R&D $\bar{\delta}$

In this subsection, we consider the effect of the rise in the value of the efficiency parameter of variety R&D $\bar{\delta}$. As explained below, the effects of the rise in $\bar{\delta}$ on economic growth are similar qualitatively to those of the rise in ψ on economic growth. We assume that the economy is initially in the steady-state equilibrium where the efficiency parameter of variety R&D $\bar{\delta}$ is given by its baseline parameter value in Table 1 (i.e., $\bar{\delta}_k = \delta^b = 14,655 * 10^{-7}$ for all period $k < 3$). Then, the shocks hit our steady-state economy in period 3, and the value of $\bar{\delta}$ in period 3 and subsequent periods increases by a factor of 2, 3, 4 and 5 (i.e., $\bar{\delta}_k = 2\delta^b, 3\delta^b, 4\delta^b$ and $5\delta^b$ for all periods $k \geq 3$). Figures 12-1 to 12-6 show the numerical examples of the transition path of the relative wage of unskilled/skilled workers ω_t (Figure 12-1), the number of firms per capita n_t/M (Figure 12-2), the net growth rate of the number of firms g_t^n (Figure 12-3), the net growth rate of the average productivity of firms g_t^A (Figure 12-4), the other market factors Λ_t (Figure 12-5), and the net growth rate of GDP g_t^{GDP} (Figure 12-6).

As shown in Figures 12-1 to 12-5, the rise in the value of the efficiency parameter of variety R&D $\bar{\delta}$ in period 3 decreases the relative wages of unskilled/skilled workers in period 3 (Figure 12-1), enhances the entry of new firms in period 4 (Figure 12-2), increases the net growth rate of the number of firms in period 4 (Figure 12-3), decreases the net growth rate of the average productivity of firms in period 3 (Figure 12-4), and deteriorates the static efficiency of production in period 3 (Figure 12-5). However, as the number of firms increases, the net growth rate of the number of firms begins to decline from period 5 (Figure 12-3), and the equilibrium number of firms gradually converges to its new steady-state value, which is larger than that in the original steady-state equilibrium (Figure 12-2). Therefore, the rise in the value of the efficiency parameter of variety R&D $\bar{\delta}$ expands the equilibrium number of firms. Moreover, the net growth rate of the average productivity of firms continue to decline in period 4 and subsequent periods, and it gradually converges to its new steady-state value, which is lower than that in the original steady-state equilibrium (Figure 12-4). Therefore, the rise in the value of the efficiency parameter of variety R&D $\bar{\delta}$ negatively affects the net growth rate of the average productivity of firms not only in the short run but also in the long run.

The rise in the value of the efficiency parameter of variety R&D $\bar{\delta}$ encourages more firms to enter the market with new products, which strengthens the horizontal competition among firms. It is this strengthening of horizontal competition that gives rise to the negative "entry effect" of the rise in the value of the efficiency parameter of variety R&D $\bar{\delta}$ on the net growth rate of the average productivity of firms. Figures 12-2 and 12-4 also show that a larger increase in the value of the efficiency parameter of variety R&D $\bar{\delta}$ encourages more firms to enter the market with new products, which strengthens the long-run negative effects of the rise in the value of the efficiency parameter of variety R&D $\bar{\delta}$ on the net growth rate of the average productivity of firms.

From (41), the net growth rate of GDP shown in Figure 12-6 depends upon the net growth rate of the number of firms g_t^n in Figure 12-3, the average productivity of firms g_t^A in Figure 12-4, and the static efficiency of production Λ_t in Figure 12-5. As shown in Figure 12-6, the rise in the value of the efficiency parameter of variety R&D $\bar{\delta}$ in period 3 decreases the net growth rate of GDP in period 3 due to the declines in both the values of g_t^A and Λ_t in period 3. However, as shown in Figures 12-2 and 12-3, the number of firms begins to increase from period 4, which increases the net growth rate of the number of firms in period 4. In addition, as shown in Figure 12-5, the static efficiency of production improves gradually from period 4. Therefore, while the net growth rate of the average productivity of firms g_t^A continues to decrease in period 4 (Figure 12-4), the net growth rate of GDP increases temporarily in period 4. After period 4, the economy gradually converges to its new steady-state equilibrium. During this transition process, the net growth rate of the number of firms and the other market factors gradually approach zero. Therefore, in the steady-state equilibrium, the net growth rate of GDP becomes equivalent to the net growth rate of the average productivity of firms. Consequently, as shown in Figures 12-4 and 12-6, the net growth rate of GDP in the new steady-state equilibrium becomes lower than that in the original steady-state equilibrium. Therefore, when the market structure adjusts partially in the short run, the growth effect of the rise in the value of the efficiency parameter of variety R&D $\bar{\delta}$ is generally ambiguous. However, when the market structure adjusts fully in the long run, the rise in the value of the efficiency parameter of variety R&D $\bar{\delta}$ expands the number of firms but reduces economic growth. These results indicate that the effects of the rise in $\bar{\delta}$ on economic growth are similar qualitatively to those of the rise in ψ on economic growth.

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Table 1: Calibration of baseline parameters and steady-state results

A. Targeted Variables	R&D Propensity $\frac{w^s n^* l^R}{PY}$	Firms/Population $\frac{n^*}{M}$	Relative wage of unskilled ω^*	Per capita GDP growth g^{GDP*}
Target values	0.022	0.0327	0.667	0.02
Baseline simulation results	0.022	0.0327	0.667	0.02
B. Calibrated Parameters	γ	$\bar{\delta}$	α	z
Baseline simulation Parameters	0.0287	$14,655 \times 10^{-7}$	0.3669	1.83665
(Identifications)	(R&D Propensity)	(Firms/Population)	(Relative wage of unskilled)	(Per capita GDP growth)

Table 2: Preset parameters

Parameter	Description	Value
s	Education subsidy rate	0 (0-0.9)
A_0	Initial average productivity of firms	1
n_0	Initial number of firms	1
M	Population size	36,525,680
$\underline{\theta}$	Lowest value of ability	0
$\bar{\theta}$	Highest value of ability	1
β	Discount factor	$(0.99)^{120}$
η	Elasticities of substitution across intermediates	4.3
ψ	Standing-on-shoulders effect parameter	0.35 (0.05-0.65)
σ	Degree of specialization parameter	0.05 (0-0.2)

Table 3: Changes in education subsidy rate s from 0 to 0.3 for different values of ψ (%)

Period	1	2	3	4	5	6	7	8	9
$\psi = 0.2$	2.2560	2.2560	2.2423	2.2697	2.2589	2.2559	2.2551	2.2549	2.2548
$\psi = 0.35$	2.0000	2.0000	1.9864	2.0123	2.0035	1.9999	1.9985	1.9979	1.9976
$\psi = 0.5$	1.5917	1.5917	1.5781	1.6026	1.5955	1.5916	1.5895	1.5884	1.5877
$\psi = 0.65$	0.8377	0.8377	0.8243	0.8472	0.8415	0.8375	0.8348	0.8330	0.8318

Table 4: Changes in education subsidy rate s from 0 to 0.3 for different values of $\bar{\delta}$ (%)

Period	1	2	3	4	5	6	7	8	9
$\bar{\delta} = \delta^b$	2.0000	2.0000	1.9864	2.0123	2.0035	1.9999	1.9985	1.9979	1.9976
$\bar{\delta} = 2 \times \delta^b$	1.8961	1.8961	1.8825	1.9084	1.8996	1.8961	1.8946	1.8940	1.8937
$\bar{\delta} = 3 \times \delta^b$	1.8354	1.8354	1.8218	1.8476	1.8389	1.8353	1.8339	1.8333	1.8330
$\bar{\delta} = 4 \times \delta^b$	1.7923	1.7923	1.7787	1.8046	1.7959	1.7923	1.7908	1.7902	1.7900

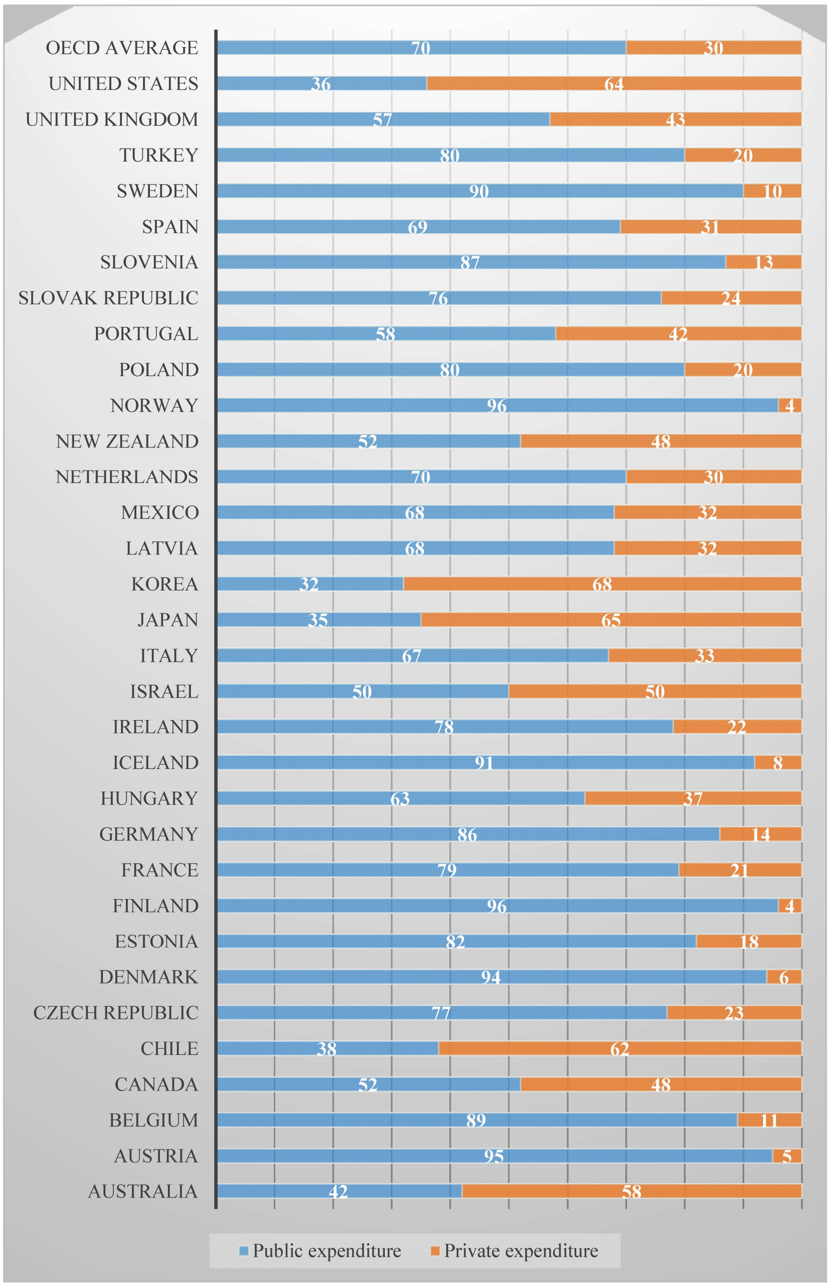

Figure 1: Distribution of public and private expenditure on tertiary educational institutions (2013)

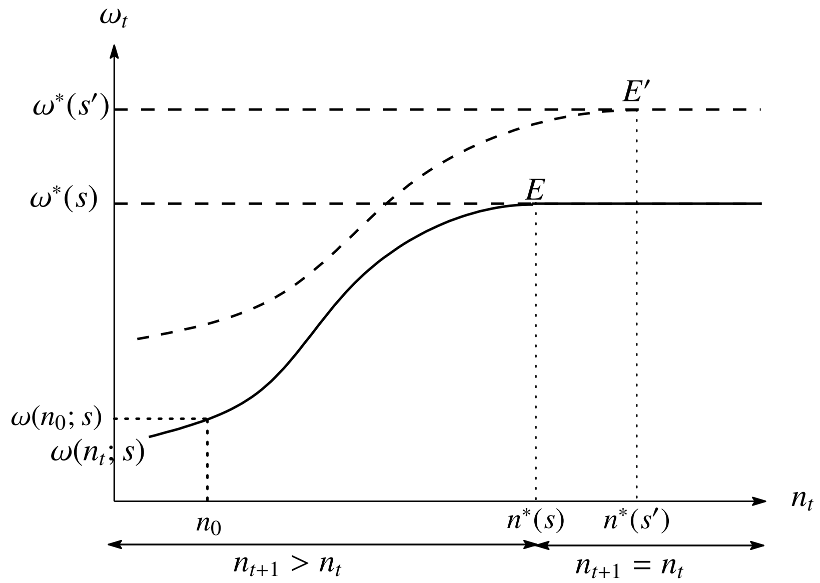

Figure 2: The relationship between the number of firms and the relative wage of unskilled/skilled workers ($s < s'$)

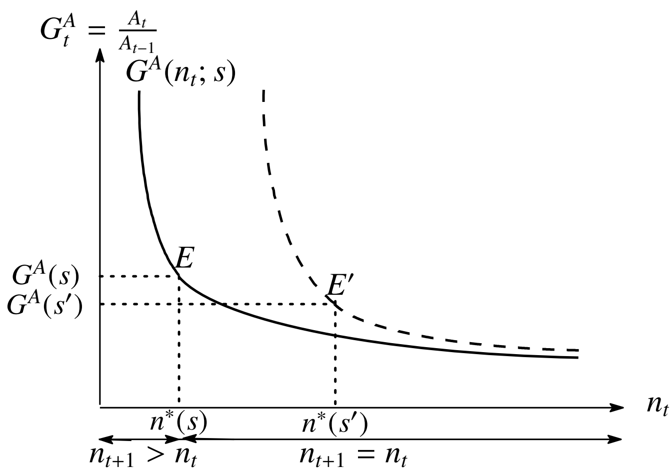

Figure 3: The relationship between the number of firms and the gross growth rate of the average productivity of firms ($s < s'$)

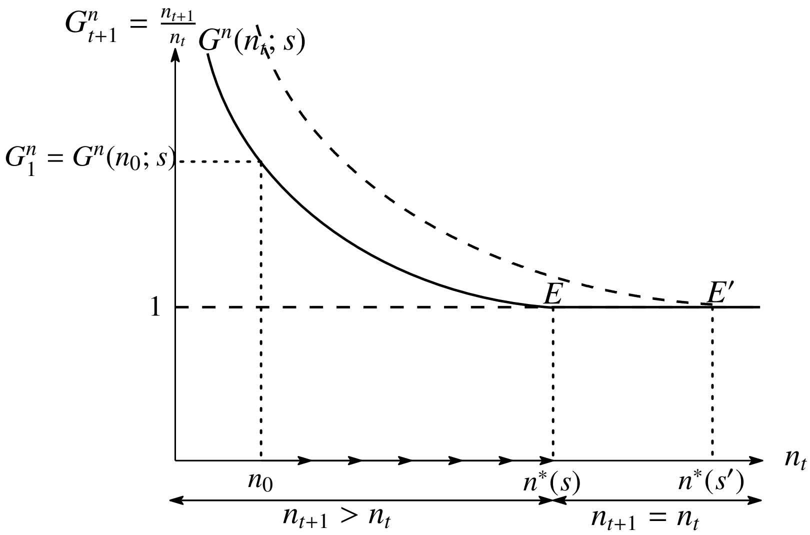

Figure 4: The relationship between the number of firms and the gross growth rate of the number of firms ($s < s'$)

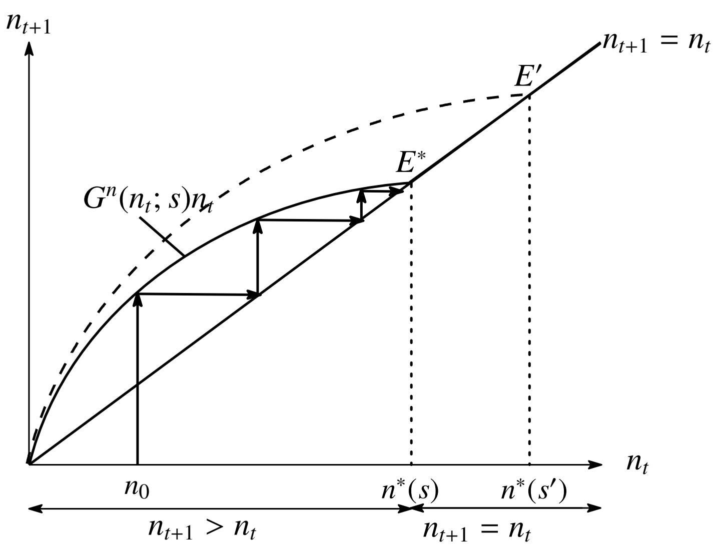

Figure 5: The possible dynamics of n_t when the parameter conditions of (36) are satisfied ($s < s'$)

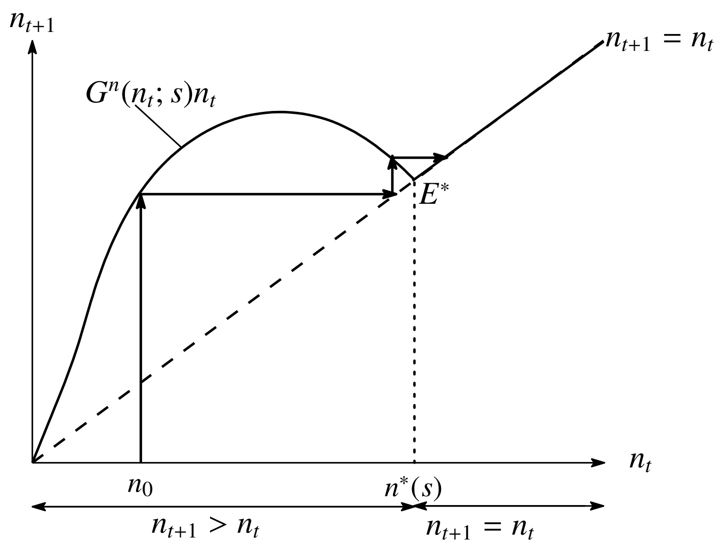

Figure 6: The possible dynamics of n_t when the parameter conditions of (36) are not satisfied

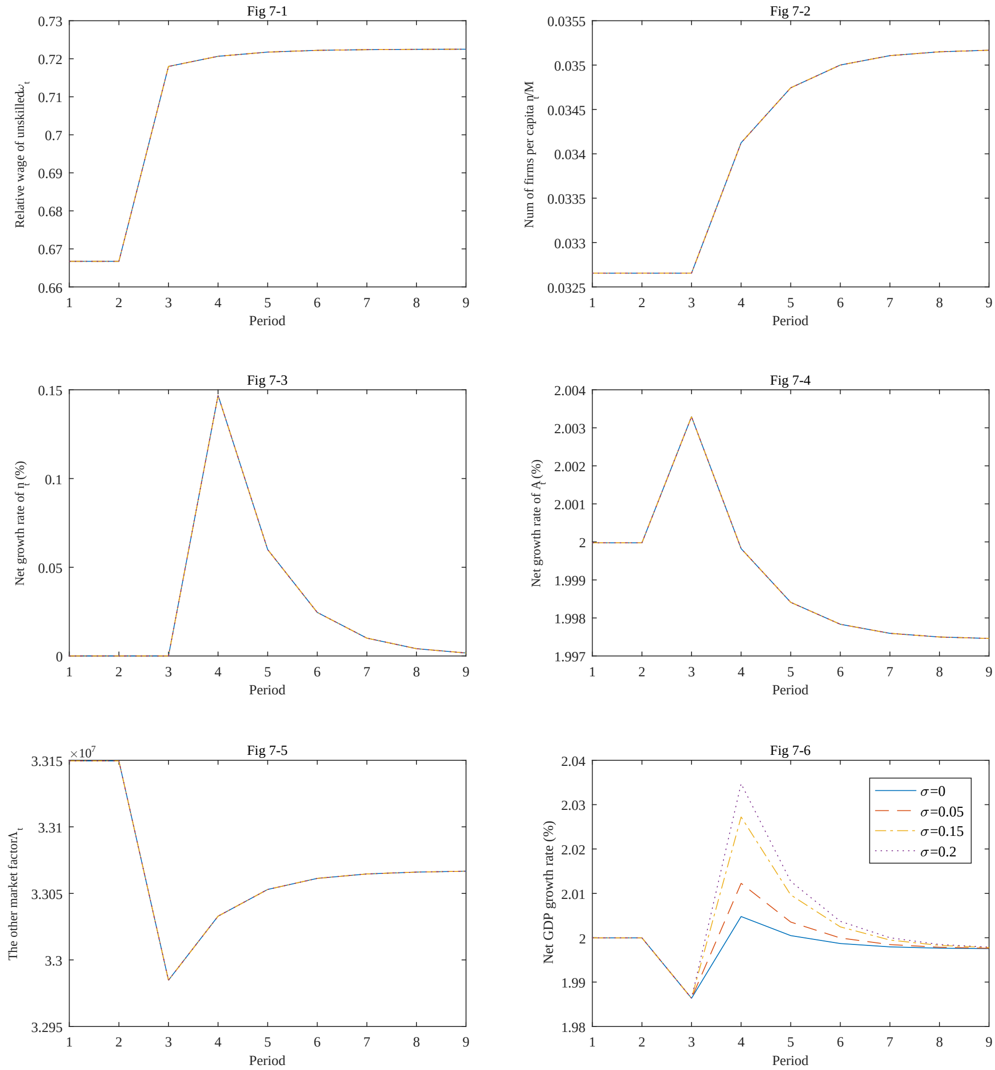

Figure 7: Changes in education subsidy rate *s* from 0 to 0.3

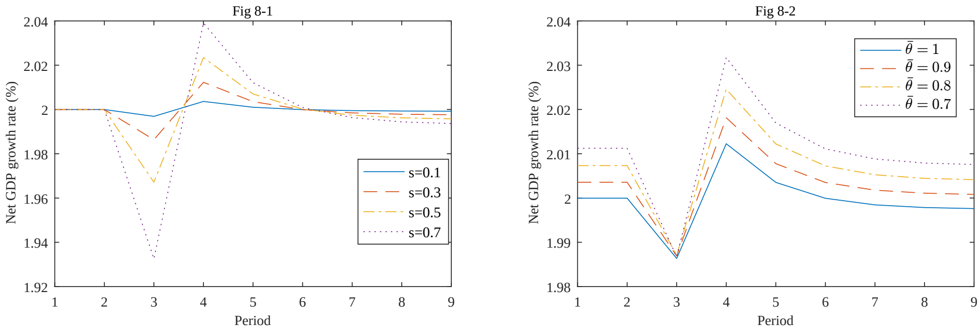

Figure 8: Robustness check for different values of education subsidy rate s and the extent of individual heterogeneity in ability $\bar{\theta}$

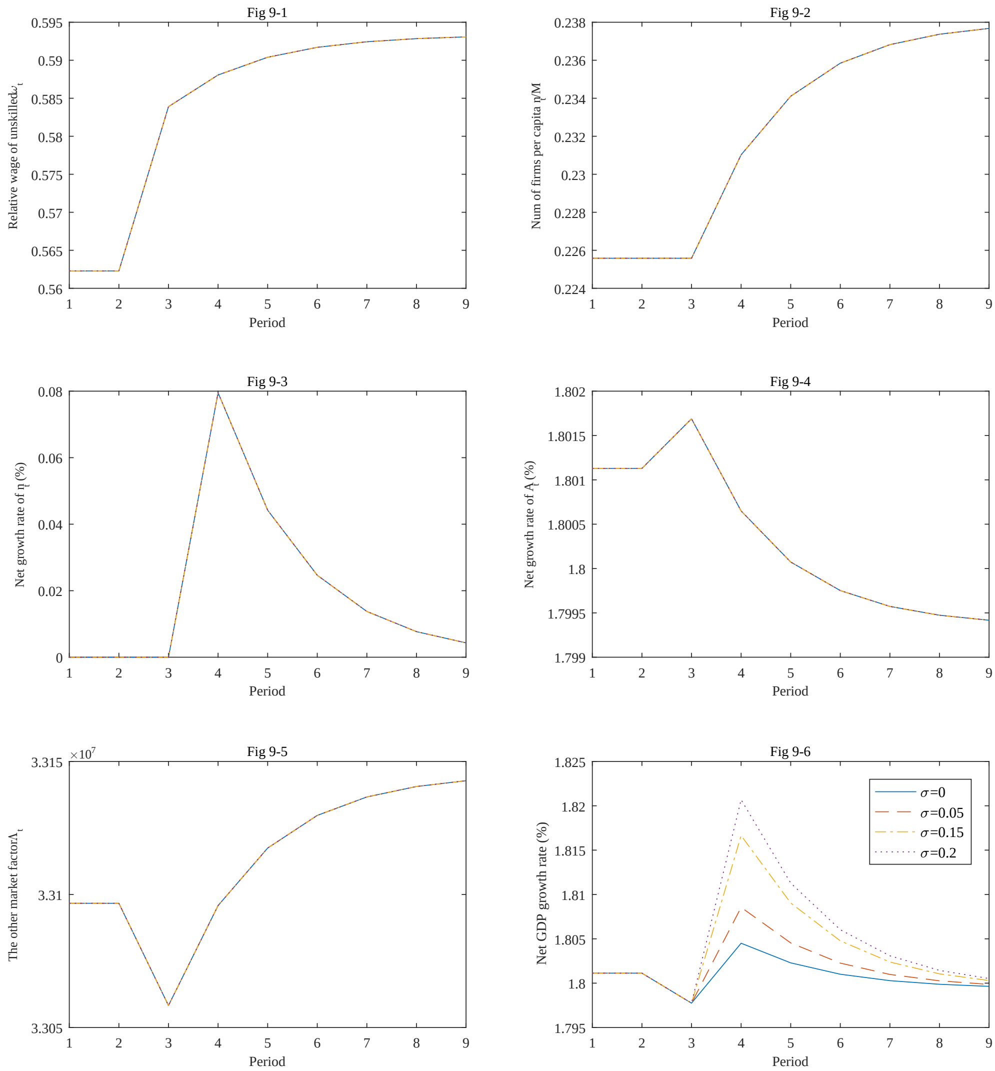

Figure 9: Changes in education subsidy rate s from 0 to 0.3: direct costs of education

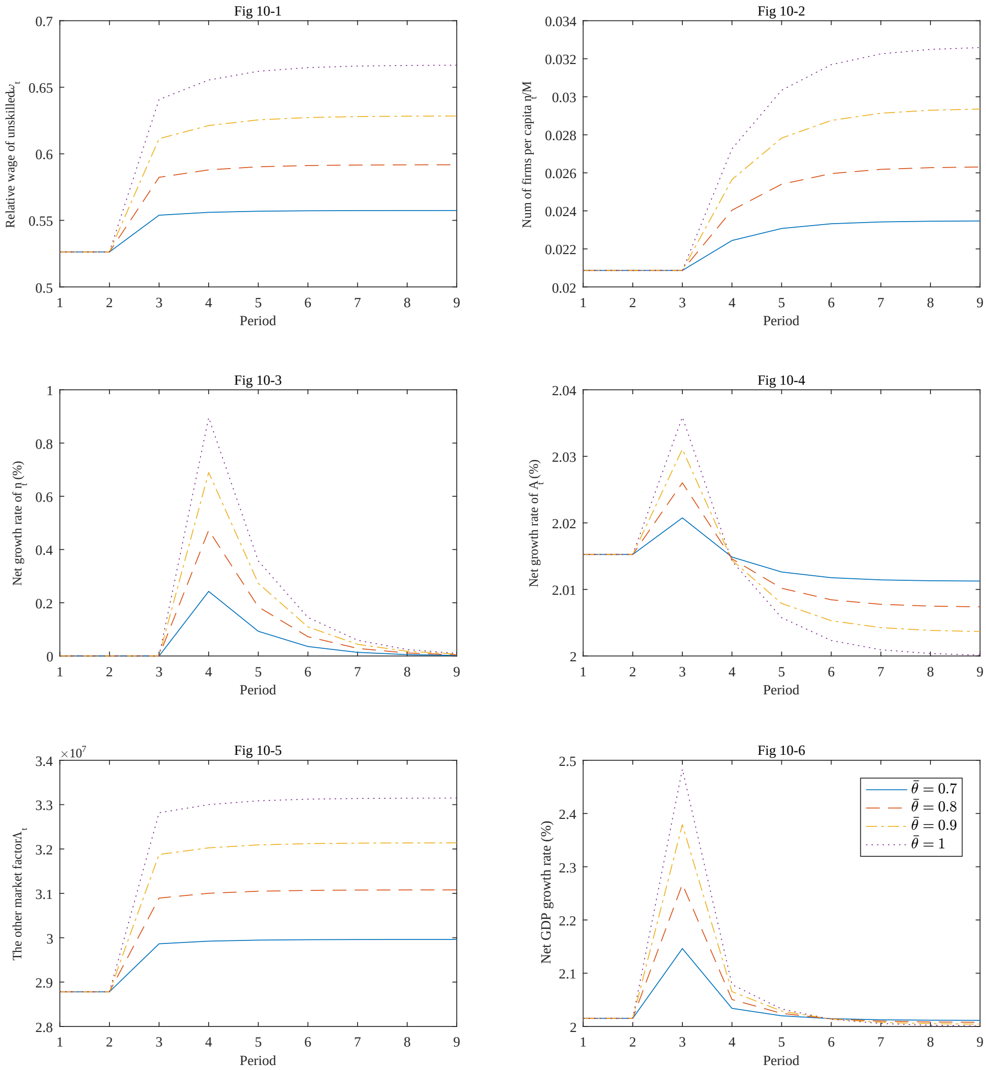

Figure 10: Changes in the extent of individual's ability $\bar{\theta}$ from 0.6 to 1

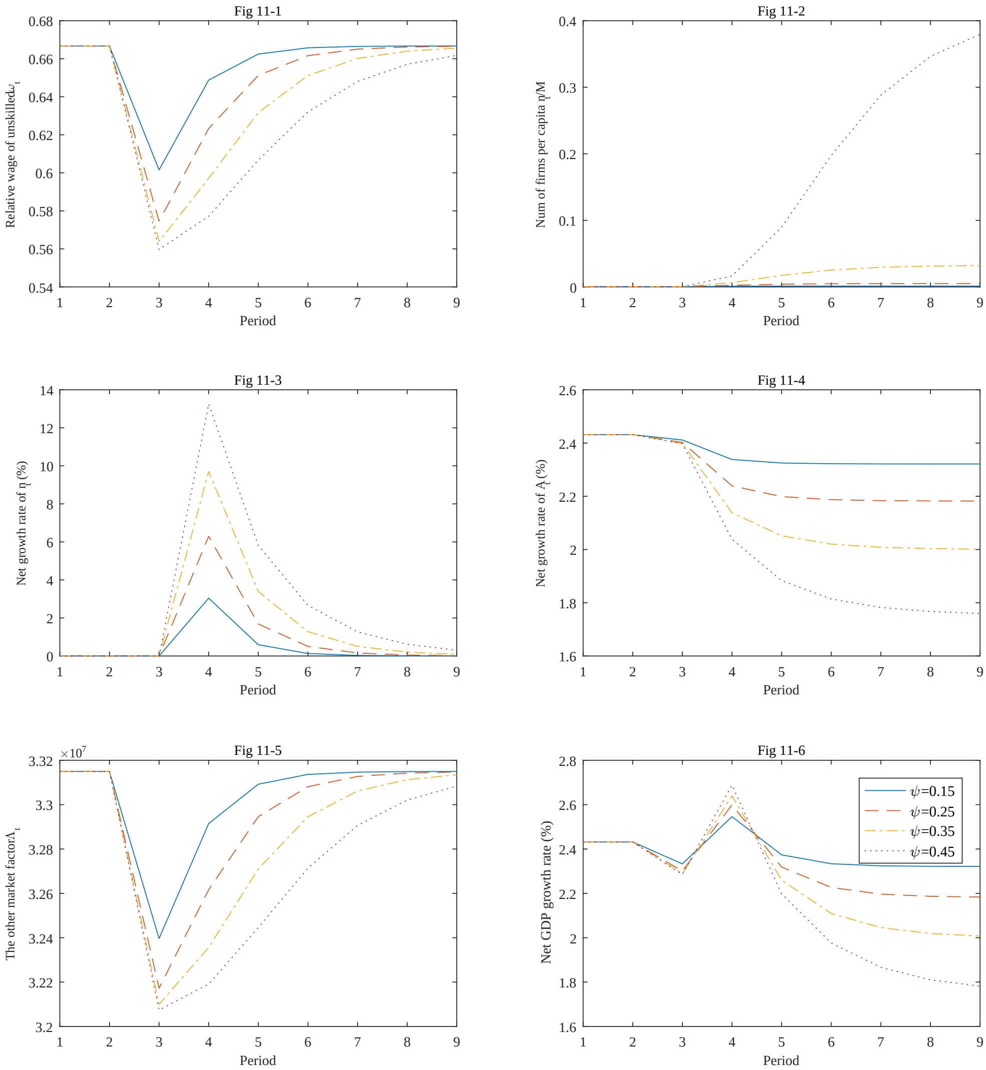

Figure 11: Changes in the standing-on-shoulders parameter ψ from 0.05 to 0.45

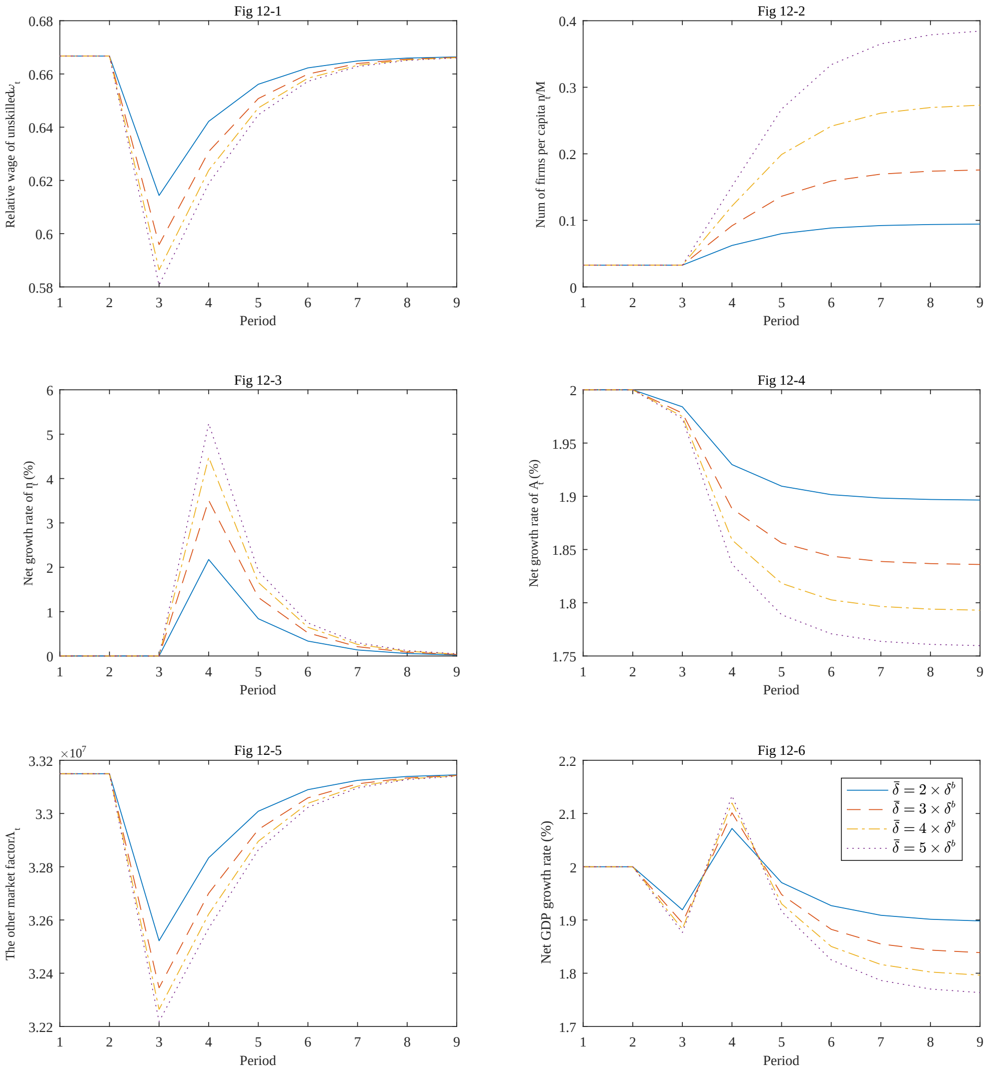

Figure 12: Changes in the efficiency parameter of variety R&D $\bar{\delta}$ from δ^b to $5 \times \delta^b$