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On the merger paradox and asymmetric product differentiation

Tsuyoshi Toshimitsu
(School of Economics, Kwansei Gakuin University)
Tetsuya Nakajima
(Faculty of Economics, Osaka City University)

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KWANSEI GAKUIN UNIVERSITY

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Tsuyoshi TOSHIMITSU[☆]

School of Economics, Kwansei Gakuin University

Tetsuya Nakajima[♣]

Faculty of Economics, Osaka City University

Abstract

Assuming asymmetric product differentiation, we reconsider the merger paradox in the cases of quantity-setting and price-setting games. We investigate whether emergence of the merger paradox depends on the degree of product differentiation of the outsider, irrespective of the mode of competition. In particular, being different from the result of Deneckere and Davidson (1985), we show that the merger paradox arises in the case of price-setting games if the degree of product differentiation of the outsider is sufficiently small.

Keywords: merger paradox; quantity-setting game, price-setting game, asymmetric product differentiation

JEL Classification: D43; L12; L13; L41

☆ Corresponding author: School of Economics, Kwansei Gakuin University, 1-155, Nishinomiya, 662-8501, Japan
Tel: +81 798 54 6440, Fax: +81 798 51 0944, E-mail: ttsutomu@kwansei.ac.jp
♣ E-mail: nakajima@econ.osaka-cu.ac.jp

1. Introduction

In a seminal paper, Salant et al. (1983) demonstrated that merger is unprofitable for the merging firms under Cournot (quantity) competition in a homogenous product market unless at least 80% of the firms in the industry participate. Since then, there have been many solutions proposed for the merger paradox. For example, one way to resolve the paradox is to introduce merger-related synergies and scale economies.[1]

In this paper, without assuming such merger-related synergies, we reconsider the well-known "merger paradox"; in other words, an incentive for merger and profitability of insiders (i.e., merging firms) and an external effect on the profits of outsiders (non-merging firms). In particular, we focus on the properties of products horizontally differentiated between firms.

In empirical studies of horizontal mergers with differentiated products, Baker and Bresnahan (1985, p.427), who estimate the demand system of brewing firms' mergers in the US, point out that "a merger in an industry with differentiated products increases the market power of the merged firms to the extent that their products are close substitutes

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[1] For example, Perry and Porter (1985) and Farrell and Shapiro (1990) consider the welfare (and profit) effect of horizontal mergers in a homogeneous product market, assuming scale economies, synergies, and supply-side externalities as merger efficiencies (e.g., cost advantages).

and that other firms produce only more distant substitutes."[2] Exploiting their idea,

developing an asymmetric product differentiation model, we examine the profitability

of a merger in the cases of quantity-setting (Cournot) and price-setting (Bertrand)

games.

Since Salant et al. (1983), there have been many attempts to resolve the merger

paradox. However, in this paper, we focus on the literature examining horizontal merger

with product differentiation. Hsu and Wang (2010), whose work is very close to our

model, consider horizontal merger in the case of Cournot oligopoly (i.e., competition

between four firms) with product differentiation. Assuming symmetric product

differentiation, they show that a merger is profitable for the merging firms if the degree

of product differentiation is sufficiently large and that the outsider firms always benefit

from a merger. As shown below, our result is the same result as theirs in the case of

asymmetric product differentiation with three firms. Similarly, Gelves (2014) examines

the merger paradox in the case of Cournot oligopoly (i.e., competition with n-firms)

with product differentiation and cost asymmetry. To sum up, horizontal mergers of

strategic substitutes in the case of quantity-setting games are rarely profitable. However,

if the products are sufficiently differentiated, mergers can generate at least no loss of

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[2] See Nevo (2000), who develops a new methodology to estimate mergers with
differentiated products in the case of the ready-to-eat cereal industry.

profit.

Deneckere and Davidson (1985), whose work is also very close to our model, demonstrate that price-setting games of strategic complements in a differentiated products market yield profitable mergers. They assume symmetric product differentiation. However, introducing asymmetric product differentiation—that is, the degree of product differentiation between the firms differs—we demonstrate that the merger paradox arises in the case of price-setting games if the degree of product differentiation of the outsider is sufficiently small; in other words, the product of the outsider is a close substitute to that of the insider.

In Section 2, we develop a three-firm model with asymmetric product differentiation, using a simple quadratic function. We then derive the equilibrium outcomes in the cases of premerger and merger under quantity-setting games. We demonstrate the condition under which the merger paradox arises. In Section 3, we examine the same problem in the case of price-setting games. As an application of our model, we address the choice of internal decision-making structure of a multidivisional firm and the possibility of internal competition. Finally, in Section 4, we summarize our results and present remaining issues.

2. Model

2.1 Setup: Asymmetric product differentiation

To consider the merger paradox, we assume a horizontally differentiated products market where three firms $\{i, j, k\}$ provide their products. That is, each firm provides one product; therefore, we refer to firm i (j, k) and product i (j, k) interchangeably. The three products are substitutes for each other. However, with respect to the degree of product differentiation between the products, we assume that the degree of product differentiation between products $\{i, j\}$ is different from that between products $\{i, k\}$ and from that between products $\{j, k\}$; i.e., asymmetric product differentiation (see Figure 1).

Based on the assumption mentioned above, we consider a representative consumer with the following quasi-linear utility function for three horizontally differentiated products $\{i, j, k\}$ and one numeraire product.

$$U = u\!\left(q_i, q_j, q_k\right) + q_0,$$

$$u\!\left(q_i, q_j, q_k\right) = A\!\left(q_i + q_j + q_k\right) - \frac{1}{2}\!\left(q_i{}^2 + q_j{}^2 + q_k{}^2\right) - \beta q_i q_j - \gamma q_i q_k - \gamma q_j q_k, \quad (1)$$

$$y \geq p_i q_i + p_j q_j + p_k q_k + q_0, \quad p_0 = 1,$$

where $\beta \in (0,1)$, $\gamma \in (0,1)$, and $\beta \neq \gamma$. We note that the degree of product differentiation parameters, β and γ, implies the degree of product substitution between the products. Thus, if $\beta, \gamma \Rightarrow 1\,(0)$, the degree of product differentiation is small (large); in other words, the degree of product substitution is large (small). Furthermore, if $\beta > (<)\gamma$, the degree of substitution between products $\{i, j\}$ is larger (smaller) than that between products $\{i, k\}$ and that between products $\{j, k\}$.

Based on equation (1), we derive the following inverse demand functions for three products.

$$p_i = A - q_i - \beta q_j - \gamma q_k, \tag{2.1}$$

$$p_j = A - q_j - \beta q_i - \gamma q_k, \tag{2.2}$$

$$p_k = A - q_k - \gamma q_i - \gamma q_j. \tag{2.3}$$

To simplify the analysis and to focus on the properties of asymmetric product differentiation, we assume that marginal costs of production are zero.[3] Thus, the profit functions of the firms are given by

$$\pi_i = p_i q_i = \{A - q_i - \beta q_j - \gamma q_k\}q_i, \tag{3.1}$$

$$\pi_j = p_j q_j = \{A - q_j - \beta q_i - \gamma q_k\}q_j, \tag{3.2}$$

$$\pi_k = p_k q_k = \{A - q_k - \gamma q_i - \gamma q_j\}q_k. \tag{3.3}$$

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[3] In the case of constant marginal costs, we can derive the same results as in the case of zero marginal costs.

In what follows, we consider the cases of premerger and merger under quantity-setting games. Then, comparing the equilibrium outcomes in the two cases, we examine the merger paradox.

2.2 Quantity-setting games

First, we examine the case of premerger where the firms non-cooperatively determine the outputs. Based on equations (3.1), (3.2), and (3.3), we obtain the following first-order conditions (FOC) of profit maximization.

$$\frac{\partial \pi_i}{\partial q_i} = p_i - q_i = A - 2q_i - \beta q_j - \gamma q_k = 0, \tag{4.1}$$

$$\frac{\partial \pi_j}{\partial q_j} = p_j - q_j = A - 2q_j - \beta q_i - \gamma q_k = 0, \tag{4.2}$$

$$\frac{\partial \pi_k}{\partial q_k} = p_k - q_k = A - 2q_k - \gamma q_i - \gamma q_j = 0. \tag{4.3}$$

For equations (4.1) and (4.2), we assume that $q_i = q_j.$ Therefore, we derive the following equilibrium outputs and profits in the case of premerger (N).

$$q_i^N = q_j^N = \frac{2-\gamma}{2(2+\beta-\gamma^2)}A \text{ and } q_k^N = \frac{2+\beta-2\gamma}{2(2+\beta-\gamma^2)}A, \tag{5}$$

$$\pi_i^N = \pi_j^N = \left\{\frac{2-\gamma}{2(2+\beta-\gamma^2)}A\right\}^2 \text{ and } \pi_k^N = \left\{\frac{2+\beta-2\gamma}{2(2+\beta-\gamma^2)}A\right\}^2. \tag{6}$$

Second, we examine the case of merger (M), where firms $\{i, j\}$ are insiders (I) and

firm $\{k\}$ is an outsider (O).[4] In particular, the merged firm (i.e., insider) decides the outputs to maximize the following joint profits; i.e., equations (3.1) and (3.2).

$$\Pi^M = \pi_i + \pi_j = p_i q_i + p_j q_j = \{A - q_i - \beta q_j - \gamma q_k\}q_i + \{A - q_j - \beta q_i - \gamma q_k\}q_j. \qquad (7)$$

The outsider firm $\{k\}$ decides the output to maximize its profit; i.e., equation (3.3). In this case, we derive the following FOCs.

$$\frac{\partial \Pi^M}{\partial q_i} = p_i - q_i - \beta q_j = A - 2q_i - 2\beta q_j - \gamma q_k = 0, \qquad (8.1)$$

$$\frac{\partial \Pi^M}{\partial q_j} = p_j - q_j - \beta q_i = A - 2q_j - 2\beta q_i - \gamma q_k = 0, \qquad (8.2)$$

$$\frac{\partial \pi_k}{\partial q_k} = p_k - q_k = A - 2q_k - \gamma q_i - \gamma q_j = 0. \qquad (8.3)$$

For equations (8.1) and (8.2), we assume that $q_i = q_j$. Therefore, we derive the following equilibrium outputs and profits in the case of merger.

$$q_i^{MI} = q_j^{MI} = \frac{2 - \gamma}{2(2 + 2\beta - \gamma^2)}A \quad \text{and} \quad q_k^{MO} = \frac{1 + \beta - \gamma}{2 + 2\beta - \gamma^2}A, \qquad (9)$$

$$\pi_i^{MI} = \pi_j^{MI} = (1 + \beta)\left\{\frac{2 - \gamma}{2(2 + 2\beta - \gamma^2)}A\right\}^2 \quad \text{and} \quad \pi_k^{MO} = \left\{\frac{1 + \beta - \gamma}{2 + 2\beta - \gamma^2}A\right\}^2. \qquad (10)$$

2.3 Merger paradox under quantity-setting games

We examine the merger paradox; in other words, incentives for merger (or profitability

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[4] We denote this case as a homogenous merger. Similarly, a heterogeneous merger is where firms $\{i, k\}$ are insiders and firm $\{j\}$ is an outsider. In the Appendix, we examine that case.

of merger). Taking equations (6) and (10), with respect to profits, we derive the

following relationship.

$$\pi_i^{MI} > (<)\pi_i^{N} \Leftrightarrow (1+\beta)\left\{\frac{2-\gamma}{2(2+2\beta-\gamma^2)}A\right\}^2 > (<)\left\{\frac{2-\gamma}{2(2+\beta-\gamma^2)}A\right\}^2 \quad (11)$$

$$\Leftrightarrow \gamma^4 - 2(1+\beta)\gamma^2 + \beta(1+\beta) > (<)0 \Leftrightarrow \gamma^* > (<)\gamma,$$

where $\gamma^* \equiv \sqrt{1+\beta-\sqrt{1+\beta}}\,(<1)$.

Similarly, based on equations (6) and (10), we obtain the external effect of merger

on the profit of the outsider $\{k\}$, i.e., merger externality, as follows: $\pi_k^{MO} > \pi_k^{N}$.

Therefore, in view of equation (11), we summarize the results as the following

proposition.

Proposition 1

Under quantity-setting games with asymmetric product differentiation, if the degree of

product differentiation of the outsider is small (large) —in other words, the degree of

product substitution is large (small)—the merger paradox arises (does not arise).

Furthermore, the merger externality on the profit of the outsider is positive.

This result echoes that of Hsu and Wang (2010, Proposition 1).[5] In our model, if

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[5] Hsu and Wang (2010) deal with the case of symmetric product differentiation; i.e., $\gamma = \beta$. In this case, we obtain the following relationship.

$\gamma > (<)\gamma^*$, the merger is unprofitable (profitable) for the merged firms $\{i, j\}$, compared with premerger.

In a horizontally differentiated products market, the merged firms only lose a modest portion of the combined premerger market shares. Thus, if the degree of product differentiation is sufficiently small—in other words, the degree of product substitution is large, i.e., $\gamma > \gamma^*$—the market share lost by the merged firms is more than offset by the increased prices afforded by the firm's market power coming from the merger. Furthermore, because the reduction of market share of the merged firms increases the market share of the outsider by strategic substitutes, the profit of the outsider increases, compared with premerger.

Proposition 1 implies that firms $\{i, j\}$ do not have an incentive to merge if the degree of product differentiation of the outsider is small. In other words, it is a necessary condition for firms $\{i, j\}$ to merge that the degree of product differentiation of the outsider is sufficiently large. Furthermore, it is preferable for the outsider $\{k\}$ that firms $\{i, j\}$ form the merger. Thus, we examine the profitability of the insiders and the outsider, given that the merger is profitable; i.e., $\gamma^* > \gamma$.

In view of equation (10), we derive the following relationship between the profits of

$$\pi_i^{MI} > (<)\pi_i^N \Leftrightarrow \gamma^3 - 2\gamma^2 - \gamma + 1 > (<)0 \Leftrightarrow 0.555 > (<)\gamma.$$

the insiders and the outsider.

$$\pi_i^{MI} = \pi_j^{MI} < \pi_k^{MO}, \ \text{ if } \ 1 > \beta \geq \frac{1+\sqrt{17}}{8} \approx 0.64, \tag{12.1}$$

$$\pi_i^{MI} = \pi_j^{MI} > (<)\pi_k^{MO} \Leftrightarrow \gamma > (<)\gamma^{\#}, \ \text{ if } \ \frac{1+\sqrt{17}}{8} \approx 0.64 > \beta > 0, \tag{12.2}$$

where $\gamma^{\#} \equiv \dfrac{2(1+\beta) - 2(1-\beta)\sqrt{1+\beta}}{3-\beta}$.

Given equation (12.2), if the degree of product differentiation between the insiders

is sufficiently large and that of the outsider is small, the profit of the insiders is larger

than that of the outsider. Otherwise, the opposite holds.

Taking equations (11), (12.1), and (12.2), with respect to the cut-off values, i.e., γ^{*},

$\gamma^{\#}$, the following relationship holds.

$$\beta > (<)\frac{1}{3} \Leftrightarrow \gamma^{\#} > (<)\gamma^{*}. \tag{13}$$

Based on equations (12.1), (12.2), and (13), we derive the following.

If $\dfrac{1}{3} > \beta > 0$, then it holds that $\gamma < (>)\gamma^{\#} \Leftrightarrow \pi_i^{MI} = \pi_j^{MI} < (>)\pi_k^{MO}$.

If $\dfrac{1+\sqrt{17}}{8} > \beta > \dfrac{1}{3}$, then it holds that $\gamma < \gamma^{*}(<\gamma^{\#}) \Leftrightarrow \pi_i^{MI} = \pi_j^{MI} < \pi_k^{MO}$.

If $1 > \beta > \dfrac{1+\sqrt{17}}{8}$, then it holds that $\gamma < \gamma^{*} \Leftrightarrow \pi_i^{MI} = \pi_j^{MI} < \pi_k^{MO}$.

Thus, we summarize the results as the following corollary.

Corollary 1

Given that the merger is profitable (i.e., $\gamma < \gamma^{*}$), the profit of the outsider is larger than

that of the insiders, (i) if both the degree of product differentiation between the insiders

(i.e., $\frac{1}{3} > \beta > 0$) and that of the outsider is large (i.e., $\gamma < \gamma^{\#}$), and (ii) if the degree of

product differentiation between the insiders is small; i.e., $1 > \beta > \frac{1}{3}$.

We note that, given that the merger paradox arises (i.e., $\gamma > \gamma^{*}$), either if (i)

$1 > \beta > \frac{1+\sqrt{17}}{8}$ or if (ii) $\frac{1+\sqrt{17}}{8} > \beta > \frac{1}{3}$ and $\gamma^{*} < \gamma < \gamma^{\#}$, then the profit of the

outsider is larger than that of the insiders. We denote this situation as the big merger

paradox.

3. Merger Paradox under Price-setting Games

3.1 Price-setting games with asymmetric product differentiation

We proceed to analyze the merger paradox in the case of price-setting games. Taking the

linear inverse demand functions in the case of quantity-setting games—i.e., equations

(2.1), (2.2), and (2.3)—we derive the following linear demand functions for three

products.

$$q_i = \frac{1-\gamma^2}{\Delta} \left\{ \frac{1-\beta}{1+\gamma} A - p_i + \frac{\beta-\gamma^2}{1-\gamma^2} p_j + \frac{\gamma(1-\beta)}{1-\gamma^2} p_k \right\}, \tag{14.1}$$

$$q_j = \frac{1-\gamma^2}{\Delta} \left\{ \frac{1-\beta}{1+\gamma} A - p_j + \frac{\beta-\gamma^2}{1-\gamma^2} p_i + \frac{\gamma(1-\beta)}{1-\gamma^2} p_k \right\}, \tag{14.2}$$

$$q_k = \frac{1-\beta^2}{\Delta} \left\{ \frac{1+\beta-2\gamma}{1+\beta} A - p_k + \frac{\gamma}{1+\beta} p_i + \frac{\gamma}{1+\beta} p_j \right\}, \tag{14.3}$$

where $\Delta \equiv (1-\beta)(1+\beta-2\gamma^2) > 0$ and we assume that $\dfrac{1+\beta}{2} > \gamma > \gamma^2 (> 0)$.

Regarding equations (14.1) and (14.2), we address the case in which products $\{i, j\}$ are substitutes but the cross effect of prices on the demand for the other product is not necessarily that of substitutes. In particular, the effect of an increase in the price of product $\{j\}$ on the demand for product $\{i\}$ depends on the degree of the parameters;

i.e., $\dfrac{\partial q_i}{\partial p_j} = \dfrac{\beta-\gamma^2}{1-\gamma^2} > (<)0 \Leftrightarrow \sqrt{\beta} > (<)\gamma$. Furthermore, in the case of merger where firms $\{i, j\}$ are the merged firms, the reaction function of the merged firms shifts down, compared with the case of premerger if $\sqrt{\beta} < \gamma$. This is not the case of Deneckere and Davidson (1985) assuming symmetric product differentiation. In other words, as shown below, the merger paradox can arise under price-setting games in our model where we assume asymmetric product differentiation.

To examine this hypothesis, we translate the demand equations system—i.e., (14.1), (14.2), and (14.3)—to the following: $q_i = \dfrac{1-\gamma^2}{\Delta} \tilde{q}_i,$ $\quad q_j = \dfrac{1-\gamma^2}{\Delta} \tilde{q}_j,$ and

$$q_k = \frac{1-\beta^2}{\Delta}\widetilde{q}_k, \quad \text{where}$$

$$\widetilde{q}_i = \alpha - p_i + \delta p_j + \varepsilon p_k, \tag{15.1}$$

$$\widetilde{q}_j = \alpha - p_j + \delta p_i + \varepsilon p_k, \tag{15.2}$$

$$\widetilde{q}_k = \phi - p_k + \varphi p_i + \varphi p_j. \tag{15.3}$$

Furthermore, regarding the parameters, we assume that $\alpha \equiv \dfrac{1-\beta}{1+\gamma}A > 0$,

$\phi \equiv \dfrac{1+\beta-2\gamma}{1+\beta}A > 0$, $\quad \delta \equiv \dfrac{\beta-\gamma^2}{1-\gamma^2} > (<)0 \Leftrightarrow \beta > (<)\gamma^2$, $\qquad \varepsilon \equiv \dfrac{\gamma(1-\beta)}{1-\gamma^2} > 0$, and

$\varphi \equiv \dfrac{\gamma}{1+\beta} > 0$. Similarly for the profit functions, we have $\pi_i = \dfrac{1-\gamma^2}{\Delta}p_i\widetilde{q}_i$,

$\pi_j = \dfrac{1-\gamma^2}{\Delta}p_j\widetilde{q}_j$, and $\pi_k = \dfrac{1-\beta^2}{\Delta}p_k\widetilde{q}_k$. In what follows, we use these profit

functions.

3.2 Merger paradox and negative externality

By the same procedure as that under quantity-setting games, we derive the following

outcomes in the cases of premerger and merger.

In the case of premerger under price-setting games (*PN*), we derive the following.

$$p_i^{PN} = p_j^{PN} = \frac{2\alpha + \varepsilon\phi}{D} \quad \text{and} \quad p_k^{PN} = \frac{2\varphi\alpha + (2-\delta)\phi}{D}, \tag{16}$$

$$\pi_i^{PN} = \pi_j^{PN} = \frac{1-\gamma^2}{\Delta}\left\{\frac{2\alpha + \varepsilon\phi}{D}\right\}^2 \quad \text{and} \quad \pi_k^{PN} = \frac{1-\beta^2}{\Delta}\left\{\frac{2\varphi\alpha + (2-\delta)\phi}{D}\right\}^2, \tag{17}$$

where $D \equiv 2\{2-\delta-\varphi\varepsilon\} = \dfrac{2\{(2-\beta)(1+\beta)-2\gamma^2\}}{(1-\gamma^2)(1+\beta)} > 0.$

In the case of merger under price-setting games (*PM*), we derive the following.

$$p_i^{PMI} = p_j^{PMI} = \frac{2\alpha + \varepsilon\phi}{D^{PM}} \quad \text{and} \quad p_k^{PMO} = \frac{2\varphi\alpha + 2(1-\delta)\phi}{D^{PM}}, \tag{18}$$

$$\pi_i^{PMI} = \pi_j^{PMI} = \frac{1-\gamma^2}{\Delta}(1-\delta)\left\{\frac{2\alpha + \varepsilon\phi}{D^{PM}}\right\}^2 \quad \text{and} \quad \pi_k^{PMO} = \frac{1-\beta^2}{\Delta}\left\{\frac{2\varphi\alpha + 2(1-\delta)\phi}{D^{PM}}\right\}^2, \tag{19}$$

where $D^{PM} \equiv 2\{2 - 2\delta - \varphi\varepsilon\} = \dfrac{2(1-\beta)\{2(1+\beta) - \gamma^2\}}{(1-\gamma^2)(1+\beta)} > 0.$

Using equations (17) and (19), we derive the profitability of merger under price-setting games as follows.

$$\pi_i^{PMI} > (<)\pi_i^{PN} \Leftrightarrow \frac{1-\gamma^2}{\Delta}(1-\delta)\left\{\frac{2\alpha + \varepsilon\phi}{D^{PM}}\right\}^2 > (<)\frac{1-\gamma^2}{\Delta}\left\{\frac{2\alpha + \varepsilon\phi}{D}\right\}^2$$

$$\Leftrightarrow \delta X > (<)0,$$

where $X \equiv (2 - \delta - \varphi\varepsilon)(\delta + \varphi\varepsilon) - \delta > 0.$ [6] Therefore, we have

$$\pi_i^{PMI} > (<)\pi_i^{PN} \Leftrightarrow \sqrt{\beta} > (<)\gamma. \tag{20}$$

Similarly, for the merger externality, we derive the following.

$$\pi_k^{PMO} > (<)\pi_k^{PN} \Leftrightarrow p_k^{PMO} > (<)p_k^{PN}$$

$$\Leftrightarrow \delta\varphi(2\alpha + \varepsilon\phi) > (<)0 \Leftrightarrow \delta > (<)0,$$

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[6] Regarding X in equation (18), we derive the following relationship.

$$X = (2 - \delta - \varphi\varepsilon)(\delta + \varphi\varepsilon) - \delta > (<)0 \Leftrightarrow \frac{(1-\beta)x}{(1-\gamma^2)^2(1+\beta)^2} > (<)0,$$

where $x \equiv \beta(1+\beta)^2 + (1+\beta)(1-3\beta)\gamma^2 - (1-\beta)\gamma^4$. In this case, we can rewrite as

follows: $x = \beta(1+\beta)(1+\beta - 2\gamma^2) + (1-\beta)(1+\beta - \gamma^2)\gamma^2$. Given the assumptions,

because it holds that $x > 0$, we obtain $X > 0$.

where $\varphi(2\alpha + \varepsilon\phi) > 0$. Thus, we have

$$\pi_k^{PMO} > (<)\pi_k^{PN} \Leftrightarrow \sqrt{\beta} > (<)\gamma. \tag{21}$$

Therefore, in view of equations (20) and (21), we summarize the results as the following proposition.

Proposition 2

Under price-setting games with asymmetric product differentiation, if the degree of product differentiation of the outsider is small (large)—in other words, the degree of product substitution is large (small), i.e., $\sqrt{\beta} < (>)\gamma$—the merger paradox arises (does not arise). Furthermore, the merger externality on the profit of the outsider is negative (positive).

Deneckere and Davidson (1985) show that the merger paradox does not arise in the case of oligopolistic price competition. However, assuming asymmetric product differentiation, the result in the case of price competition is the same as in the case of quantity competition. That is, if the degree of product differentiation of the outsider is small—in other words, the degree of product substitution of the outsider is large—the merger paradox arises.

16

Furthermore, although the externality on the profit of the outsider in the case of

quantity competition is always positive, the effect depends on the degree of the

parameters in the case of price competition. In particular, if the degree of product

differentiation of the outsider is small, the merger reduces the profit of the outsider,

compared with the case of premerger.

To sum up, in the case of price-setting games, the strategic complements hold in the

cases of premerger and merger. However, given asymmetric product differentiation, if

the degree of product differentiation of the outsider is small, the reaction curves of the

merged firms can shift down, so that the merger paradox and negative externality arise.

3.3 Application

We can apply our model to analyze an internal organization of a multidivisional firm.[7]

That is, we suppose that a multidivisional firm providing products $\{i, j\}$ and an

outsider providing product $\{k\}$ in a market. In this case, we examine the problem that

the multidivisional firm decides on an optimal decision-making structure; i.e.,

centralization or decentralization. In view of equations (11) and (20), if the degree of

product differentiation of the outsider is sufficiently small, the multidivisional firm

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[7] Regarding this issue, for example, see Creane and Davidson (2004).

should choose the decentralization of the quantity (price) decision. That is, it maximizes the total profit of the multidivisional firm in which divisions $\{i\}$ and $\{j\}$ non-cooperatively decide their quantity (price). This implies internal competition in the multidivisional firm. Otherwise, the multidivisional firm itself should decide the quantities (prices) of its two divisions to maximize its joint profits.

4. Concluding Remarks

Assuming asymmetric product differentiation, we considered the merger paradox and externality in the cases of quantity-setting and price-setting games. In particular, we demonstrated that, irrespective of the mode of games, the merger paradox arises if the degree of product differentiation of the outsider is sufficiently small; in other words, the property of the outsider's product is similar to that of the merging firms. Conversely, even if the products of the merging firms are homogeneous (identical) to each other, if the product of the outsider is sufficiently different from their products, the merger paradox does not arise. In this case, the merger externality on the profit of the outsider is positive.

The contribution of our paper is to show that the merger paradox can arise in the case of price competition. This result is different from that of Deneckere and Davidson (1985), although it depends on the assumption of asymmetric product differentiation. It is shown in the Appendix that the firm does not have an incentive to form a merger if its product is similar to that of the outsider. In other words, the firms producing similar products are likely to form a merger.

Our result has some limitations because the model is based on specific assumptions; e.g., linear functions, three firms, and no production costs. In future research, we intend to discuss more general cases, relaxing the assumptions and extending the model to oligopolistic competition. Furthermore, we should consider the welfare effects of mergers.

Appendix: The Heterogeneous Merger Case

To simplify the analysis, we suppose that $\beta = 1 \geq \gamma$. In this case, with respect to the homogeneous merger case, based on equations (11), (12.1), and (12.2), we derive the following results.

(1.i) $\pi_i^{MI} = \pi_j^{MI} > (<)\pi_i^N = \pi_j^N \Leftrightarrow \sqrt{2 - \sqrt{2}} \approx 0.765 > (<)\gamma.$

(1.ii) $\pi_k^{MO} > \pi_k^N.$

(1.iii) $\pi_k^{MO} > \pi_i^{MI} = \pi_j^{MI}.$

Thus, if the degree of product differentiation of the outsider is large, the big merger paradox arises.

We examine the heterogeneous merger case where firms $\{i,k\}$ form a merger and firm $\{j\}$ is an outsider. In this case, we can easily derive the following outcomes at the equilibrium (*HM*).

$$q_i^{HMI} = \frac{2-\gamma}{6(1+\gamma)}A, \quad q_k^{HMI} = \frac{1}{2(1+\gamma)}A, \quad \text{and} \quad q_j^{HMO} = \frac{1}{3}A, \qquad (A.1)$$

$$\pi_i^{HMI} = \frac{2-\gamma}{18(1+\gamma)}A^2, \quad \pi_k^{HMI} = \frac{3-\gamma}{12(1+\gamma)}A^2, \quad \text{and} \quad \pi_j^{HMO} = \frac{1}{9}A^2. \qquad (A.2)$$

Based on equations (6) and (A.2), we obtain the following results.

(2.i) $\pi_i^{HMI} < \pi_i^N$, $\pi_k^{HMI} > \pi_k^N$, and $\pi_k^{HMI} > \pi_i^{HMI}.$

(2.ii) $\pi_j^{HMO} > \pi_j^N.$

(2.iii) $\pi_j^{HMO} > \pi_i^{HMI}$ and $\pi_k^{HMI} > (<)\pi_j^{HMO} \Leftrightarrow \frac{5}{7} > (<)\gamma.$

As for (2.i), the product of the outsider $\{j\}$ is identical to that of the insider $\{i\}$ of the merged firms. As shown by Salant et al. (1983), this case is the same as the case of quantity-setting games in a homogeneous product market. On the other hand, because the product of the insider $\{k\}$ is horizontally differentiated, its profit increases compared with the case of premerger. Taking the results in the case of homogeneous merger, (2.i) implies that incentives to form a merger differ between the firms. In other words, whether a firm has an incentive to form a merger depends on the degree of product differentiation of the outsiders, not that of the insiders. For (2.ii), we have the same result as in the homogenous merger case; i.e., (1.ii). Finally, for (2.iii), the profit of the insider $\{i\}$ is smaller than that of the outsider $\{j\}$ because their products are identical. However, whether the profit of the insider $\{k\}$ increases depends on the degree of product differentiation of the insider $\{k\}$. In particular, if $\frac{5}{7} < \gamma \Rightarrow 1$, as mentioned above, the profit of the insider $\{k\}$ is smaller than that of the outsider $\{j\}$. On the contrary, if $\gamma < \frac{5}{7}$, the profit of the insider $\{k\}$ is larger than that of the outsider $\{j\}$ because the degree of product differentiation increases.

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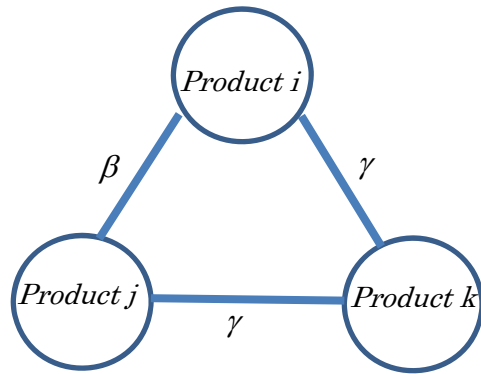
Figure 1: The properties of product differentiation between three products

$$\beta \in (0,1) \quad \text{and} \quad \gamma \in (0,1)$$