

# DISCUSSION PAPER SERIES

Discussion paper No.17

## OPTIMAL EXPORT POLICY IN THE PRESENCE OF R&D INVESTMENT

Tsuyoshi Toshimitsu  
Kwansei Gakuin University

September 1997



SCHOOL OF ECONOMICS  
KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho  
Nishinomiya 662-8501, Japan

# OPTIMAL EXPORT POLICY IN THE PRESENCE OF R&D INVESTMENT

Tsuyoshi TOSHIMITSU\*

Kwansei Gakuin University

## Abstract

The purpose of this paper is to analyze the optimal export policy in a two-stage game in which a domestic and a foreign firm compete in price and R&D investment. Under international Bertrand duopoly, an export subsidy directly promotes excess price competition, as delineated by Eaton and Grossman (1986). But, in the presence of international R&D rivalry, an export subsidy indirectly reduces the rival's R&D level, and thereby raises its cost. This effect offsets the negative effect of the export subsidy resulting in excess price competition. We show that an export subsidy (tax) policy is optimal if the relative return to R&D is great (small), provided that a government can precommit to an ex ante optimal export policy.

---

\* Associate Professor of Economics  
School of Economics  
Kwansei Gakuin University  
1-155, Uegahara Ichibanchyo, Nishinomiya, 662, Japan  
Tel. 0798-54-6440  
Fax. 0798-51-0944  
E-mail. z95016@kwansei.ac.jp

## 1. INTRODUCTION

There is a gap between the theoretical result of a traditional trade policy and the political economy of trade in the real world regarding an export policy. That is, it has been shown that an export tax should normally be beneficial in perfect competition. This is because a large country with a small export tax can appreciate its terms of trade. But, in real life an export subsidy is common, while an export tax is rare.<sup>1</sup>

Brander and Spencer (1985) bridged the gap. They proved that if the market structure is a Cournot duopoly, an export subsidy is optimal since it raises the profit of the domestic firm at the expense of the foreign firm.<sup>2</sup> But their model was not robust. Eaton and Grossman (1986) proved that an optimal export policy is a tax, not a subsidy in the case of a Bertrand duopoly. More recently still, this result has been challenged by Carmichel (1987). By empirical observation of practices in the real world, he pointed out two aspects: First, a subsidy is related to the price secured on an export contract, rather than the volume of export, i.e., a price subsidy, and second, the level of subsidy is determined not before, but after an export contract has been secured, i.e., an ex post policy decision. The second aspect implies that a government cannot ex ante precommit to an optimal export policy. Based on these observations, he showed that an export subsidy may be optimal when firms are price competitors. Although Neary (1991) did not deny Carmichael's interesting results, he commented as follows: If a government can precommit to its export policy, i.e., an ex ante policy decision, the optimal policy is an export tax, as shown by Eaton and Grossman. Hence, the social welfare in the ex post policy decision is worse

than that in the ex ante policy decision. On the contrary, the profit of the domestic firm under the ex post policy decision is better than that in the ex ante policy decision. Thus, Neary suggests that the ex post policy decision is optimal for the domestic firm if the "government" in this real-world situation is a specialist export credit agency, e.g., the U.S. ExIm bank, even though it is not optimal in terms of the social welfare.

Previous works have been assumed that oligopolistic firms compete in quantity or in price in the short-run. But it is well known that international oligopolistic firms compete in the R&D and capacity investments in the long-run as well as in quantity and price in the short-run. In this paper we will analyze optimal export policy under Bertrand price competition in the presence of international R&D rivalry. Assuming Cournot duopoly competition, Spencer and Brander (1983), closely related to our paper, show that an export policy is always optimal. They also comment as follows: "In particular moving to price-Nash rather quantity-Nash does not change the nature of results, provided products are slightly differentiated. [Spencer and Brander (1983, pp.717-18)]" But, as shown below, their comment should be revised. We basically follow their model, except for Bertrand duopoly. That is, our model is composed of a 3-stage game. In the first stage, the domestic government determine an export subsidy/tax level per export. This implies that the domestic government can precommit to an ex ante optimal export subsidy/tax level.<sup>3</sup> In the second stage, the firms non-cooperatively determine the cost reducing R&D investment, given the export policy. In the final stage, the firms act, a la Bertrand-Nash, in the market of the third country. We derive a subgame perfect equilibrium by backward induction.

Under an international Bertrand duopoly, an export subsidy directly promotes excess price competition as delineated by Eaton and Grossman (1986). But in the presence of international R&D rivalry, the export subsidy indirectly reduces the rival's R&D investment, and thereby raises its cost. This offsets the negative effect of the export subsidy resulting in excess price competition. We show that the optimal export policy is an export subsidy (tax) if the relative return to R&D is great (small), provided that the domestic government can precommit to its optimal export policy.<sup>3</sup>

In the next section we present a simple model. First, we derive a Bertrand-Nash equilibrium, and then a non-cooperative R&D investment equilibrium. In section 3, we analyze optimal export policy under a Bertrand duopoly in the presence of international R&D rivalry. Finally, in section 4, we summarize our results and present some remaining issues. In the Appendix, we consider cases of Cournot-quantity and Bertrand-price competition, assuming general demand and cost functions.

## 2. THE MODEL

### 2.1 Bertrand-Nash Equilibrium

Following the third market framework used by Spencer and Brander (1983), Brander and Spencer (1985), Eaton and Grossman (1986), and others, we also suppose that a domestic (= 1) and a foreign (= 2) firm compete in the market of a third country.

Let us assume the demand function of a third market as follows:

$$x_i = \alpha - \beta p_i + \gamma p_j, \quad \beta > |\gamma|, \quad i, j = 1, 2, \quad i \neq j, \quad (1)$$

where  $x_i$  is demand for firm  $i$ , and  $p_i$  is price of firm  $i$ .

Next, let us assume firm  $i$ 's marginal production cost and R&D cost functions as follows:

$$c_i = c - \delta k_i, \quad c \geq \delta k_i, \quad \delta > 0, \quad i = 1, 2, \quad (2)$$

$$B_i = (b/2)k_i^2, \quad b > 0, \quad i = 1, 2, \quad (3)$$

where  $k_i$  is firm  $i$ 's R&D investment. Thus, the profit function of both firms are given by

$$\pi_1 = (p_1 - c_1 + \theta)x_1 - B_1, \quad (4.1)$$

$$\pi_2 = (p_2 - c_2)x_2 - B_2, \quad (4.2)$$

where  $\theta$  is the per unit export subsidy (tax) charged by the domestic government, if  $\theta > (<) 0$ .

Taking into account (1), (4.1), and (4.2), the first order conditions are given by

$$\frac{\partial \pi_1}{\partial p_1} = \alpha - 2\beta p_1 + \gamma p_2 + \beta(c_1 - \theta) = 0, \quad (5.1)$$

$$\frac{\partial \pi_2}{\partial p_2} = \alpha - 2\beta p_2 + \gamma p_1 + \beta c_2 = 0. \quad (5.2)$$

From (5.1) and (5.2), the prices at a Bertrand-Nash equilibrium are given by

$$p_1 = \frac{(2\beta + \gamma)\alpha + 2\beta^2(c_1 - \theta) + \beta\gamma c_2}{(2\beta + \gamma)(2\beta - \gamma)} = p_1[k_1, k_2, \theta], \quad (6.1)$$

$$p_2 = \frac{(2\beta + \gamma)\alpha + 2\beta^2 c_2 + \beta\gamma(c_1 - \theta)}{(2\beta + \gamma)(2\beta - \gamma)} = p_2[k_1, k_2, \theta]. \quad (6.2)$$

(6.1) and (6.2) show that an increase in a firm's R&D investment or an increase in an export subsidy reduces the price of both firms.

For the following analysis, taking into account (1), (6.1), and (6.2), the direct effect of an export subsidy/tax to the export

of firm  $i$  ( $i, j = 1, 2, i \neq j$ ) are given by

$$\frac{dx_1}{d\theta} = \frac{\partial x_1}{\partial p_1} \frac{\partial p_1}{\partial \theta} + \frac{\partial x_1}{\partial p_2} \frac{\partial p_2}{\partial \theta} = \frac{\beta(2\beta^2 - \gamma^2)}{(2\beta + \gamma)(2\beta - \gamma)} > 0, \quad (7.1)$$

$$\begin{aligned} \frac{dx_2}{d\theta} &= \frac{\partial x_2}{\partial p_1} \frac{\partial p_1}{\partial \theta} + \frac{\partial x_2}{\partial p_2} \frac{\partial p_2}{\partial \theta} \\ &= - \frac{\beta^2 \gamma}{(2\beta + \gamma)(2\beta - \gamma)} > (<) 0 \leftrightarrow \gamma < (>) 0. \end{aligned} \quad (7.2)$$

Also, taking into account (1), (2), (6.1), and (6.2), the effects of the firms' R&D on the export of firm  $i$  ( $i, j = 1, 2, i \neq j$ ), are given by

$$\frac{dx_i}{dk_i} = \frac{\partial x_i}{\partial p_i} \frac{\partial p_i}{\partial k_i} + \frac{\partial x_i}{\partial p_j} \frac{\partial p_j}{\partial k_i} = \frac{\beta(2\beta^2 - \gamma^2)\delta}{(2\beta + \gamma)(2\beta - \gamma)} > 0, \quad (8.1)$$

$$\begin{aligned} \frac{dx_i}{dk_j} &= \frac{\partial x_i}{\partial p_i} \frac{\partial p_i}{\partial k_j} + \frac{\partial x_i}{\partial p_j} \frac{\partial p_j}{\partial k_j} \\ &= - \frac{\beta^2 \gamma \delta}{(2\beta + \gamma)(2\beta - \gamma)} > (<) 0 \leftrightarrow \gamma < (>) 0. \end{aligned} \quad (8.2)$$

Thus, an increase in the domestic firm's R&D level, or in an export subsidy, increases its own export and decreases (increases) the rival's export, if their products are substitute (complement). With the case of an export tax, the opposite results hold.

Before discussing our analysis, we will verify the proposition of Eaton and Grossman (1986). From (4.1), the welfare function of the domestic government is given by

$$W = \pi_1 [p_1, p_2, k_1, \theta] - \theta x_1 [p_1, p_2]. \quad (9)$$

Note that both firms' R&D investments have been done at this stage, in which the domestic government decides an optimal export/tax level. Thus, the domestic export policy does not affect their R&D investment levels. Taking into account (5.1), (5.2), (7.1), (7.2), and (9), we derive the first order condition

as follows:

$$\begin{aligned}\frac{dW}{d\theta} &= \frac{\partial \pi_1}{\partial p_2} \frac{\partial p_2}{\partial \theta} + \frac{\partial \pi_1}{\partial \theta} - x_1 - \theta \frac{dx_1}{d\theta} \\ &= (p_1 - c_1 + \theta) \frac{\partial x_1}{\partial p_2} \frac{\partial p_2}{\partial \theta} - \theta \frac{dx_1}{d\theta},\end{aligned}\quad (10)$$

where we use  $\partial \pi_1 / \partial \theta = x_1$ . The first term of (10) shows the negative effect of an export subsidy resulting in excess price competition. Taking into account  $x_1 = \beta(p_1 - c_1 + \theta)$ , (10) can be rewritten by

$$\frac{dW}{d\theta} = - \frac{\beta \gamma^2}{(2\beta + \gamma)(2\beta - \gamma)} x_1 - \theta \frac{\beta(2\beta^2 - \gamma^2)}{(2\beta + \gamma)(2\beta - \gamma)} = 0,$$

thus, we have

$$\theta = - \frac{\gamma^2}{\beta(2\beta^2 - \gamma^2)} x_1 < 0, \quad (11)$$

where we note that  $x_1 = x_1 [p_1 [k_1, k_2, \theta], p_2 [k_1, k_2, \theta]]$ .

Therefore, an optimal export subsidy is negative. We can see that an export tax is optimal under Bertrand price competition, i.e., **Proposition 2** of Eaton and Grossman (1986).

## 2.2 Non-cooperative R & D Investment

Here we will discuss the second stage, in which both firms non-cooperatively determine the level of R&D investment, given the export policy of the domestic government.

Taking into account (2), (3), (5.1), (5.2), (8.1), and (8.2), from (4.1) and (4.2), we can derive the first order condition for the optimal R&D investment as follows:

$$\begin{aligned}\frac{\partial \pi_1}{\partial k_1} &= \frac{\partial \pi_1}{\partial p_2} \frac{\partial p_2}{\partial k_1} + \frac{\partial \pi_1}{\partial k_1} \\ &= (p_1 - c_1 + \theta) \frac{\partial x_1}{\partial p_2} \frac{\partial p_2}{\partial k_1} - \frac{\partial c_1}{\partial k_1} x_1 - \frac{\partial B_1}{\partial k_1}\end{aligned}$$



$$= \frac{2\delta(2\beta^2-\gamma^2)}{(2\beta+\gamma)(2\beta-\gamma)} x_1 - bk_1 = 0, \quad (12.1)$$

where  $x_1 = x_1 [p_1 [k_1, k_2, \theta], p_2 [k_1, k_2, \theta]]$ . Similarly for the foreign firm, we have

$$\frac{\partial \pi_2}{\partial k_2} = \frac{2\delta(2\beta^2-\gamma^2)}{(2\beta+\gamma)(2\beta-\gamma)} x_2 - bk_1 = 0, \quad (12.2)$$

where  $x_2 = x_2 [p_1 [k_1, k_2, \theta], p_2 [k_1, k_2, \theta]]$ . The second order condition is given by

$$\frac{\partial^2 \pi_i}{\partial k_i^2} = 2\beta Z^2 - b < 0, \quad i, j = 1, 2, \quad (13)$$

where  $Z = \delta(2\beta^2-\gamma^2)/\{(2\beta+\gamma)(2\beta-\gamma)\}$ .

For the following analysis, let us define the parameter as follows:

$$\eta = 2\beta Z^2/b < 1. \quad (14)$$

This parameter,  $\eta$ , measures the relative return to R&D (see Leahy and Neary (1996)).<sup>4</sup> In addition, the effect of the rival's R&D investment on the marginal profit ( $i, j = 1, 2, i \neq j$ ) is given by

$$\frac{\partial^2 \pi_i}{\partial k_i \partial k_j} = -2\beta Z^2 \frac{\beta\gamma}{2\beta^2-\gamma^2} > (<) 0 \leftrightarrow \gamma < (>) 0. \quad (15)$$

Thus, if the firms' products are substitute (complement), the R&D competition is strategic substitute (complement), although the price competition is strategic complement (substitute).

From (13) and (15), the stability condition of R&D competition is given by

$$D(\eta) = b^2 \left( 1 - \eta \frac{(2\beta+\gamma)(\beta-\gamma)}{2\beta^2-\gamma^2} \right) \left( 1 - \eta \frac{(2\beta-\gamma)(\beta+\gamma)}{2\beta^2-\gamma^2} \right) > 0. \quad (16)$$

From (13) and (16), the R&D investment equilibrium is stable and satisfies the second order conditions, if and only if

$$\tilde{\eta} > \eta > 0, \quad (17)$$

where  $\tilde{\eta} = (2\beta^2 - \gamma^2) / \{(2\beta - \gamma)(\beta + \gamma)\} < 1$ .

Next, let us show the effects of an export subsidy/tax of the domestic government on the firms' R&D investment. Taking into account (12.1), (12.2), (13), (15), (16), and (17), we have

$$\frac{dk_1}{d\theta} = \frac{\eta b^2}{\delta D(\eta)} \left( 1 - \eta \frac{(4\beta^2 - \gamma^2)(\beta^2 - \gamma^2)}{(2\beta^2 - \gamma^2)^2} \right) > 0, \quad (18.1)$$

$$\frac{dk_2}{d\theta} = - \frac{\eta b^2}{\delta D(\eta)} \frac{\beta\gamma}{2\beta^2 - \gamma^2} > (<) 0 \leftrightarrow \gamma < (>) 0. \quad (18.2)$$

Thus, an increase in an export subsidy increases the domestic firm's R&D investment. However, it decreases (increases) the foreign firm's R&D investment, if their products are substitute (complement). With the case of an export tax, the opposite results hold.

### 3. OPTIMAL EXPORT POLICY IN THE PRESENCE OF R&D INVESTMENT

Here let us characterize the optimal export policy of the domestic government, which directly and indirectly affects both firms' R&D investments, the prices, and the amount of exports. The welfare function of the domestic government at the first stage is given by

$$W = \pi_1 [p_1, p_2, k_1, \theta] - \theta x_1 [p_1, p_2], \quad (19)$$

where  $p_i = p_i [k_1, k_2, \theta]$ , and  $k_i = k_i [\theta]$ ,  $i = 1, 2$ .

Taking into account (5.1) and (12.1), the first order condition

is given by

$$\begin{aligned}
\frac{dW}{d\theta} &= \frac{\partial \pi_1}{\partial p_2} \left( \frac{\partial p_2}{\partial k_2} \frac{\partial k_2}{\partial \theta} + \frac{\partial p_2}{\partial \theta} \right) + \frac{\partial \pi_1}{\partial \theta} - x_1 \\
&\quad - \theta \left[ \frac{\partial x_1}{\partial p_1} \left( \frac{\partial p_1}{\partial k_1} \frac{\partial k_1}{\partial \theta} + \frac{\partial p_1}{\partial k_2} \frac{\partial k_2}{\partial \theta} + \frac{\partial p_1}{\partial \theta} \right) \right. \\
&\quad \left. + \frac{\partial x_1}{\partial p_2} \left( \frac{\partial p_2}{\partial k_1} \frac{\partial k_1}{\partial \theta} + \frac{\partial p_2}{\partial k_2} \frac{\partial k_2}{\partial \theta} + \frac{\partial p_2}{\partial \theta} \right) \right] \\
&= (p_1 - c_1 + \theta) \frac{\partial x_1}{\partial p_2} \left( \frac{\partial p_2}{\partial k_2} \frac{\partial k_2}{\partial \theta} + \frac{\partial p_2}{\partial \theta} \right) \\
&\quad - \theta \left[ \frac{\partial x_1}{\partial p_1} \left( \frac{\partial p_1}{\partial k_1} \frac{\partial k_1}{\partial \theta} + \frac{\partial p_1}{\partial k_2} \frac{\partial k_2}{\partial \theta} + \frac{\partial p_1}{\partial \theta} \right) \right. \\
&\quad \left. + \frac{\partial x_1}{\partial p_2} \left( \frac{\partial p_2}{\partial k_1} \frac{\partial k_1}{\partial \theta} + \frac{\partial p_2}{\partial k_2} \frac{\partial k_2}{\partial \theta} + \frac{\partial p_2}{\partial \theta} \right) \right], \tag{20}
\end{aligned}$$

where we use  $\partial \pi_1 / \partial \theta = x_1$ .

Comparing (10) and (20), we see that (20) contains the indirect effect of an export subsidy/tax on prices as a result of its effect on R&D investments. For example, an increase in an export subsidy reduces the foreign firm's R&D level, and thereby raises its marginal cost. Thus, the bracket of the first term of (20) shows that an increase in an export subsidy raises the foreign firm's price, while it directly reduces it. This implies that the negative effect of an increase in an export subsidy resulting in excess price competition is offset. (This deduction was suggested by an anonymous referee. We will discuss optimal export policy in a general case in the **Appendix**.) As shown below, we can also prove that the sign of the second term of (20) is positive. Therefore, if the indirect effect of an export subsidy on the foreign firm's price is greater than the direct effect, then the optimal export subsidy is positive. Otherwise, it is negative.

Taking into account (1), (6.1), (6.2), (18.1), and (18.2), from (20), we derive the optimal export subsidy/tax as follows:

$$\theta = \frac{\gamma^2 H(\eta)}{\beta F(\eta)} x_1 [k_1(\theta), k_2(\theta), \theta] > (<) 0 \leftrightarrow H(\eta) > (<) 0, \quad (21)$$

where

$$H(\eta) = \frac{\eta b^2}{D(\eta)} \frac{2\beta^2}{2\beta^2 - \gamma^2} - 1, \quad (22.1)$$

$$F(\eta) = \frac{\eta b^2 (2\beta^2 - \gamma^2)}{D(\eta)} \left( 1 - \eta \frac{(4\beta^2 - \gamma^2)(\beta^2 - \gamma^2)}{(2\beta^2 - \gamma^2)^2} \right) + \frac{\eta b^2}{D(\eta)} \frac{\beta^2 \gamma^2}{2\beta^2 - \gamma^2} + 2\beta^2 - \gamma^2 > 0. \quad (22.2)$$

Whether an optimal export subsidy is positive or not depends on the sign of  $H(\eta)$ , since  $F(\eta)$  is always positive.

The second order condition is given by

$$\frac{\partial^2 W}{\partial \theta^2} = \frac{\beta F(\eta)}{4\beta^2 - \gamma^2} \left( \frac{\gamma^2 H(\eta)}{4\beta^2 - \gamma^2} - 1 \right) < 0. \quad (23)$$

Thus, if and only if  $H(\eta) < (4\beta^2/\gamma^2) - 1$ , the second order condition is satisfied.

Taking into account (21), (22.1), and (23), we derive our main result as follows:

**Proposition.** As to the relative return to R&D,  $\eta$ , (1) if  $\eta^* < \eta < \bar{\eta}$  ( $< \tilde{\eta}$ ), then the optimal export policy is an export subsidy, and (2) if  $0 < \eta < \eta^*$ , then the optimal export policy is an export tax.

**Proof.**

First, from (22.1) and (23), to satisfy the second order condition, the following has to hold:

$$f(\eta) < 0, \quad (24.1)$$

where

$$f(\eta) = \frac{b^2 \gamma^2 \eta}{2(2\beta^2 - \gamma^2)} - D(\eta).$$

Hence, we can derive  $\bar{\eta} = \{\eta | f(\eta) = 0\}$ . Since  $f(\eta)' > 0$  for  $0 < \eta < \tilde{\eta}$ ,  $f(\tilde{\eta}) > 0$ , and  $f(0) < 0$ , we see that  $0 < \bar{\eta} < \tilde{\eta}$ . Thus, if and only if  $\eta < \bar{\eta}$ , the second order condition holds.

Next, from (16) and (22.1), it holds that

$$H(\eta) > (<) 0 \leftrightarrow h(\eta) > (<) 0, \quad (24.2)$$

where

$$h(\eta) = \frac{2\beta^2 b^2 \eta}{2\beta^2 - \gamma^2} - D(\eta).$$

Hence, we can derive  $\eta^* = \{\eta | h(\eta) = 0\}$ . Since  $h(\eta)' > 0$  for  $0 < \eta < \tilde{\eta}$ ,  $h(\tilde{\eta}) > 0$ , and  $h(0) < 0$ , we see that  $0 < \eta^* < \tilde{\eta}$ . Also, since it holds that  $h(\eta) > f(\eta)$  for  $0 < \eta < \tilde{\eta}$ , we see that  $\eta^* < \bar{\eta}$ . Thus, if  $0 < \eta < \eta^*$ , then  $h(\eta) < 0$ , and if  $\eta^* < \eta < \bar{\eta}$ ,  $h(\eta) > 0$ .

Therefore, if  $0 < \eta < \eta^*$ , then  $H(\eta) < 0$ , and if  $\eta^* < \eta < \bar{\eta}$ ,  $H(\eta) > 0$ . We prove our **Proposition.** |

Suppose that the relative return to R&D,  $\eta$ , is great. This implies that the extent of cost reduction,  $\delta$ , is great, and/or that the R&D cost,  $b$ , is small, given the parameter of product differentiation,  $\beta$ ,  $\gamma$ . Hence, we note that the indirect effect is larger than the direct effect. Then the domestic government provides the optimal export subsidy for the domestic firm. As mentioned above, the export subsidy reduces the foreign firm's R&D level, and thereby increases its cost. This offsets the

negative effect of the export subsidy directly resulting in excess price competition. Thus, the domestic firm gets more profit, and the domestic government gets more producer surplus than it would have, if the domestic government did not intervene.

On the contrary, suppose that the relative return to R&D is small. That is, the indirect effect is smaller than the direct effect. So the domestic government charges the optimal export tax on the domestic firm. In this case the export tax increases the foreign firm's R&D level, and thereby decreases its cost. This implies that the export tax indirectly promotes price competition. But, its extent does not exceed the direct effect of the export tax raising the foreign firm's price. Thus, as delineated by Eaton and Grossman (1986), since both firms' prices rise in a third market due to an export tax by the domestic government, the domestic firm gets more profit, and the domestic government gets more producer surplus than it would have, if the domestic government did not intervene.

We will illustrate the effects of export subsidy/tax in the price space. First, see Figure 1.1. The export subsidy directly shifts the domestic firm's reaction curve downward, i.e., from N to Z. This results in excess price competition. On the other hand, the export subsidy indirectly shifts the foreign firm's reaction function upward, i.e., from Z to S, by raising the foreign firm's cost. The indirect effect offsets the direct negative effect. Consequently, a Bertrand-Nash price equilibrium under the optimal export subsidy is at S, in which the domestic firm's profit is better off. Next, see Figure 1.2. The export tax directly shifts the domestic firm's reaction function curve upward, i.e., from N to Z. This reduces price competition. The export tax indirectly shifts the foreign firm's reaction function

downward, i.e., from Z to T, by reducing the foreign firm's cost. This is because the export tax increases the foreign firm's R&D level, and thereby reduces its cost. This indirect effect promotes price competition. Consequently, the domestic firm's profit is better off at T.

#### 4. CONCLUDING REMARKS

We have shown that an export subsidy can be optimal in the presence of international R&D rivalry, even under Bertrand price competition. In other words, the government should provide an export subsidy for the domestic firm in the long-run, if the government can precommit to an ex ante optimal export subsidy level. But, as mentioned above, the optimal policy is an export tax in the short-run, i.e., given the R&D levels.

We have to mention some important remaining issues. First, we assume that the government can precommit to its export policy. But, as discussed in Carmichael (1987), Neary (1991), Goldberg (1995), and Leahy and Neary (1996), the government cannot necessarily precommit to its policy.<sup>5</sup> It has been shown in this paper that the export tax can be optimal, if the government cannot precommit to its policy. Thus, our next paper will try to analyze the case without government commitment. Second, we treated output subsidy/tax. But, as Carmichael (1987) pointed out, a subsidy/tax may be practically related to the price secured on an export contract. Thirdly, we did not analyze the R&D policy. But, as Spencer and Brander (1983), Goldberg (1995), and Leahy and Neary (1996) discuss, an industrial policy such as R&D subsidies in a high-technology industry can be more important

than an export subsidy/tax under the WTO system. Finally, although we assume that there is complete information of the firms' cost structure, or the R&D investment decision, we should analyze the case of asymmetric information. We should show what export policy the domestic government, i.e., principal, designs for the domestic firm, i.e., agent (see, Qiu (1994) and others).



## Appendix.

Here we discuss the optimal export policy in the presence of an international R&D rivalry. In what follows, we assume that the R&D investment reduces marginal cost, and that the differentiated products are substitute.<sup>6</sup>

First, we confirm that an export subsidy is optimal in the case of Cournot duopoly, as shown in Spencer and Brander (1983). Second, we analyze optimal export policy in the case of Bertrand duopoly.

### 1. Quantity, R&D, and Optimal Export Policy under Cournot duopoly

First, we will derive a Nash equilibrium at the final stage in the case of Cournot duopoly. The profit function of the domestic firm (= 1) is given by

$$\pi_1 = (p_1 - c_1[k_1] + \theta)x_1 - B_1[k_1], \quad (\text{A.1})$$

where it is assumed that  $c_1' < 0$ , and  $B_1' > 0$ .

The inverse demand function is assumed to be

$$p_i = p_i[x_1, x_2], \quad i = 1, 2. \quad (\text{A.2})$$

From (A.1) and (A.2), the first order condition is

$$\frac{\partial \pi_1}{\partial x_1} = p_1 - c_1[k_1] + \theta + \frac{\partial p_1}{\partial x_1} x_1 = 0. \quad (\text{A.3})$$

It is the same for the foreign firm. Thus, we derive the exports of both firms at a Cournot-Nash equilibrium as follows:

$$x_i = x_i[k_1, k_2, \theta], \quad i = 1, 2, \quad (\text{A.4})$$

where the exports are strategic substitute. We can show that a firm's Nash equilibrium level of export is increasing in its R&D

and decreasing in the other firm's R&D, and that an export subsidy increases the exports of the domestic firm, but reduces that of the foreign firm. As to these proofs, see Spencer and Brander (1983, pp.709-10).

Secondly, we will analyze the second stage, i.e., the R&D investment decision stage. Taking into account (A.1), (A.2), and (A.4), the profit function of the domestic firm can be rewritten by

$$\pi_1 = \pi_1 [x_1 [k_1, k_2, \theta], x_2 [k_1, k_2, \theta], k_1, \theta], \quad (\text{A.5})$$

From (A.5), we can derive the first order condition to hold a firm's Nash equilibrium in R&D levels as follows:

$$\frac{\partial \pi_1}{\partial x_2} \frac{\partial x_2}{\partial k_1} + \frac{\partial \pi_1}{\partial k_1} = 0. \quad (\text{A.6})$$

where we use (A.3). The condition for the foreign firm is similar. Thus, we derive that R&D investments are strategic substitute. We can also show that the domestic (foreign) firm's Nash equilibrium of R&D level is increasing (decreasing) in the export subsidy of the domestic government, i.e.,  $k_i = k_i[\theta]$ ,  $i = 1, 2$ ,  $\partial k_1 / \partial \theta > 0$ , and  $\partial k_2 / \partial \theta < 0$ , for  $\theta > 0$ . The opposite result holds with the case of an export tax. As to these proofs, see Spencer and Brander (1983, pp.719-20).

Thirdly, we will show that an export subsidy is always optimal in the case of Cournot duopoly. The welfare function of the domestic government is given by

$$W = \pi_1 [x_1 [k_1, k_2, \theta], x_2 [k_1, k_2, \theta], k_1, \theta] - \theta x_1 [k_1, k_2, \theta], \quad (\text{A.7})$$

where  $k_i = k_i[\theta]$ ,  $i = 1, 2$ . From (A.7), the first order condition is

$$\frac{\partial W}{\partial \theta} = \frac{\partial \pi_1}{\partial x_2} \left( \frac{\partial x_2}{\partial k_2} \frac{\partial k_2}{\partial \theta} + \frac{\partial x_2}{\partial \theta} \right) - \theta \left( \frac{\partial x_1}{\partial k_1} \frac{\partial k_1}{\partial \theta} + \frac{\partial x_1}{\partial k_2} \frac{\partial k_2}{\partial \theta} + \frac{\partial x_1}{\partial \theta} \right). \quad (\text{A.8})$$

where we use (A.4), (A.6), and  $\partial \pi_1 / \partial \theta = x_1$ .

The sign of the first term of (A.8) is always positive. That is, an increase in the export subsidy not only directly decreases the foreign firm's export, but also indirectly reduces it by reducing its R&D level. A decrease in the foreign firm's export increases the profit of the domestic firm. We can see that the bracket of the second term is almost positive. Thus, from (A.8), we confirm that an export subsidy policy is optimal.<sup>7</sup>

## 2. Price, R&D and Optimal Export Policy under Bertrand duopoly

In a way similar to the above, we will first derive a Nash equilibrium at the final stage in the case of Bertrand duopoly. The profit function of the domestic firm (= 1) is given by

$$\pi_1 = (p_1 - c_1[k_1] + \theta)x_1 - B_1[k_1], \quad (\text{B.1})$$

where it is assumed that  $c_1' < 0$ , and  $B_1' > 0$ . The demand function is assumed to be

$$x_i = x_i[p_1, p_2], \quad i = 1, 2. \quad (\text{B.2})$$

From (B.1) and (B.3), the first order condition is

$$\frac{\partial \pi_1}{\partial p_1} = x_1 + (p_1 - c_1[k_1] + \theta) \frac{\partial x_1}{\partial p_1} = 0. \quad (\text{B.3})$$

From (B.3), the second order condition is

$$\frac{\partial^2 \pi_1}{\partial p_1^2} = 2 \frac{\partial x_1}{\partial p_1} + (p_1 - c_1[k_1] + \theta) \frac{\partial^2 x_1}{\partial p_1^2} < 0. \quad (\text{B.4})$$

The effect of the foreign firm's price on the marginal profit of the domestic firm is

$$\frac{\partial^2 \pi_1}{\partial p_1 \partial p_2} = \frac{\partial x_1}{\partial p_2} + (p_1 - c_1 [k_1] + \theta) \frac{\partial^2 x_1}{\partial p_1 \partial p_2}, \quad (\text{B.5})$$

where (B.5) can be positive. Similar conditions hold for the foreign firm. Thus, the prices are strategic complement.

We assume that own effects of price on marginal profit dominate cross effects, giving rise to the following condition:

$$D_p = \pi_1^{11} \pi_2^{22} - \pi_1^{12} \pi_2^{21} > 0, \quad (\text{B.6})$$

where  $\pi_i^{ii} = \partial^2 \pi_i / \partial p_i^2$ ,  $\pi_i^{ij} = \partial^2 \pi_i / \partial p_i \partial p_j$ ,  $i, j = 1, 2$ ,  $i \neq j$ .

Hence, we can obtain as follows ( $i, j = 1, 2$ ,  $i \neq j$ ):

$$dp_i / dk_i = c_i' (\partial x_i / \partial p_i) \pi_j^{jj} / D_p < 0, \quad (\text{B.7.1})$$

$$dp_j / dk_i = - c_i' (\partial x_i / \partial p_i) \pi_i^{ij} / D_p < 0, \quad (\text{B.7.2})$$

$$dp_i / d\theta = - (\partial x_i / \partial p_i) \pi_j^{jj} / D_p < 0, \quad (\text{B.8.1})$$

$$dp_j / d\theta = (\partial x_i / \partial p_i) \pi_j^{ji} / D_p < 0. \quad (\text{B.8.2})$$

That is, a firm's Nash equilibrium level of price is decreasing in its R&D, the other firms' R&D, and the export subsidy of the domestic government. We can present the price of both firms at Bertrand-Nash equilibrium as follows:

$$p_i = p_i [k_1, k_2, \theta], \quad i = 1, 2, \quad (\text{B.9})$$

Secondly, we will analyze the second stage, i.e., the R&D investment decision stage. Taking into account (B.1), (B.2), and (B.9), the profit function of the domestic firm can be rewritten by

$$\pi_1 = \pi_1 [p_1 [k_1, k_2, \theta], p_2 [k_1, k_2, \theta], k_1, \theta]. \quad (\text{B.10})$$

From (B.10), we can derive the first order condition to hold a firm's Nash equilibrium in R&D levels as follows:

$$\frac{\partial \pi_1}{\partial p_2} \frac{\partial p_2}{\partial k_1} + \frac{\partial \pi_1}{\partial k_1} = 0, \quad (\text{B.11})$$

where we use (B.3).

From (B.11), the second order condition is

$$\begin{aligned} \frac{\partial^2 \pi_1}{\partial k_1^2} &= \frac{\partial \pi_1}{\partial p_2} \frac{\partial^2 p_2}{\partial k_1^2} + \frac{\partial p_2}{\partial k_1} \left( \frac{\partial^2 \pi_1}{\partial p_2 \partial p_1} \frac{\partial p_1}{\partial k_1} + \frac{\partial^2 \pi_1}{\partial p_2^2} \frac{\partial p_2}{\partial k_1} + \frac{\partial^2 \pi_1}{\partial p_2 \partial k_1} \right) \\ &+ \frac{\partial^2 \pi_1}{\partial k_1 \partial p_1} \frac{\partial p_1}{\partial k_1} + \frac{\partial^2 \pi_1}{\partial k_1 \partial p_2} \frac{\partial p_2}{\partial k_1} + \frac{\partial^2 \pi_1}{\partial k_1^2} < 0. \end{aligned} \quad (\text{B.12})$$

The effect of the foreign firm's R&D on the marginal profit of the domestic firm is

$$\begin{aligned} \frac{\partial^2 \pi_1}{\partial k_1 \partial k_2} &= \frac{\partial \pi_1}{\partial p_2} \frac{\partial^2 p_2}{\partial k_1 \partial k_2} + \frac{\partial p_2}{\partial k_1} \left( \frac{\partial^2 \pi_1}{\partial p_2 \partial p_1} \frac{\partial p_1}{\partial k_2} + \frac{\partial^2 \pi_1}{\partial p_2^2} \frac{\partial p_2}{\partial k_2} + \frac{\partial^2 \pi_1}{\partial p_2 \partial k_2} \right) \\ &+ \frac{\partial^2 \pi_1}{\partial k_1 \partial p_1} \frac{\partial p_1}{\partial k_2} + \frac{\partial^2 \pi_1}{\partial k_1 \partial p_2} \frac{\partial p_2}{\partial k_2}, \end{aligned} \quad (\text{B.13})$$

where (B.13) can be negative. Similar conditions hold for the foreign firm. Thus, the R&D investments are strategic substitute.

We assume that own effects of price on marginal profit dominate cross effects, giving rise to the following condition:

$$D_k = \pi_{1k^11} \pi_{2k^22} - \pi_{1k^12} \pi_{2k^21} > 0, \quad (\text{B.14})$$

where  $\pi_{ik^ii} = \partial^2 \pi_i / \partial k_i^2$ ,  $\pi_{ik^ij} = \partial^2 \pi_i / \partial k_i \partial k_j$ ,  $i, j = 1, 2$ ,  $i \neq j$ .

Taking into account (B.12), (B.13), and (B.14), we can obtain as follows:

$$dk_1 / d\theta = - S \pi_{2k^22} / D_k > 0, \quad (\text{B.15.1})$$

$$dk_2 / d\theta = S \pi_{1k^12} / D_k < 0, \quad (\text{B.15.2})$$

where

$$S = \frac{\partial \pi_1}{\partial p_2} \frac{\partial^2 p_2}{\partial k_1 \partial \theta} + \frac{\partial p_2}{\partial k_1} \left( \frac{\partial^2 \pi_1}{\partial p_2 \partial p_1} \frac{\partial p_1}{\partial \theta} + \frac{\partial^2 \pi_1}{\partial p_2^2} \frac{\partial p_2}{\partial \theta} + \frac{\partial^2 \pi_1}{\partial p_2 \partial \theta} \right) + \frac{\partial^2 \pi_1}{\partial k_1 \partial p_1} \frac{\partial p_1}{\partial \theta} + \frac{\partial^2 \pi_1}{\partial k_1 \partial p_2} \frac{\partial p_2}{\partial \theta} + \frac{\partial^2 \pi_1}{\partial k_1 \partial \theta}, \quad (B.16)$$

where (B.16) can be positive. Thus, we derive that the domestic (foreign) firm's Nash equilibrium of R&D level is increasing (decreasing) in the export subsidy of the domestic government. The opposite result holds with the case of an export tax.

Thirdly, we will show an optimal export policy under Bertrand duopoly in the presence of international R&D rivalry. The welfare function of the domestic government is given by

$$W = \pi_1 [p_1 [k_1, k_2, \theta], p_2 [k_1, k_2, \theta], k_1, \theta] - \theta x_1 [p_1 [k_1, k_2, \theta], p_2 [k_1, k_2, \theta]], \quad (B.17)$$

where  $k_i = k_i [\theta]$ ,  $i = 1, 2$ . From (B.17), the first order condition is

$$\frac{\partial W}{\partial \theta} = \frac{\partial \pi_1}{\partial p_2} \left( \frac{\partial p_2}{\partial k_2} \frac{\partial k_2}{\partial \theta} + \frac{\partial p_2}{\partial \theta} \right) - \theta \left[ \frac{\partial x_1}{\partial p_1} \left( \frac{\partial p_1}{\partial k_1} \frac{\partial k_1}{\partial \theta} + \frac{\partial p_1}{\partial k_2} \frac{\partial k_2}{\partial \theta} + \frac{\partial p_1}{\partial \theta} \right) + \frac{\partial x_1}{\partial p_2} \left( \frac{\partial p_2}{\partial k_1} \frac{\partial k_1}{\partial \theta} + \frac{\partial p_2}{\partial k_2} \frac{\partial k_2}{\partial \theta} + \frac{\partial p_2}{\partial \theta} \right) \right] = 0, \quad (B.18)$$

where we use (B.3), (B.11), and  $\partial \pi_1 / \partial \theta = x_1$ .

Although the first term of (A.8) is positive, the sign of the first term of (B.18) is ambiguous. The first term of (B.18) is composed of two effects: The first is that an export subsidy reduces the foreign firm's R&D level, and thereby increases the foreign firm's cost, hence, the price (see (B.7.1) and (B.15.1)). The second is that an export subsidy directly reduces the price of the foreign firm (see (B.8.2)). The first effect offsets the

negative effect of an export subsidy resulting in excess price competition. Thus, if the first effect exceeds the second, then the sign of the first term is positive, and vice versa. We can see that the bracket of the second term can be positive. Thus, from (B.18), we can show that an export subsidy (tax) can be optimal, if the sign of the first term is positive (negative).<sup>8</sup>





## Notes

1. In the case of a small country, no type of trade intervention can be first best. Itoh and Kiyono (1987) show that an export subsidy can be beneficial even in competitive circumstances. This is because while an export subsidy directly worsens the terms of trade of the subsidized product, it may indirectly improve the terms of trade in the markets for related products.

2. As shown in Dixit (1984), an optimal export policy depends on the number of firms in the case of international oligopolistic industries. That is, if the domestic country has a large number of domestic firms, it will choose to tax the export of its firms.

3. In other words, the domestic government act as a Stackelberg leader vis-a-vis both domestic and foreign firms in setting an export subsidy/tax. Also, in this paper we will treat output subsidy/tax, but not price subsidy/tax.

4. The relative return to R&D, originally defined by Leahy and Neary (1996), is a measure composed of three parameters as follows: The extent of product differentiation, the extent of cost reduction, and the R&D cost. Thus, given the extent of product differentiation, the greater the extent of cost reduction, and/or the smaller the R&D cost, the greater the relative return to R&D.

5. Goldberg (1995), and Leahy and Neary (1996) discuss a similar problem to that of Spencer and Brander (1983) without the government commitment, assuming Cournot quantity competition.

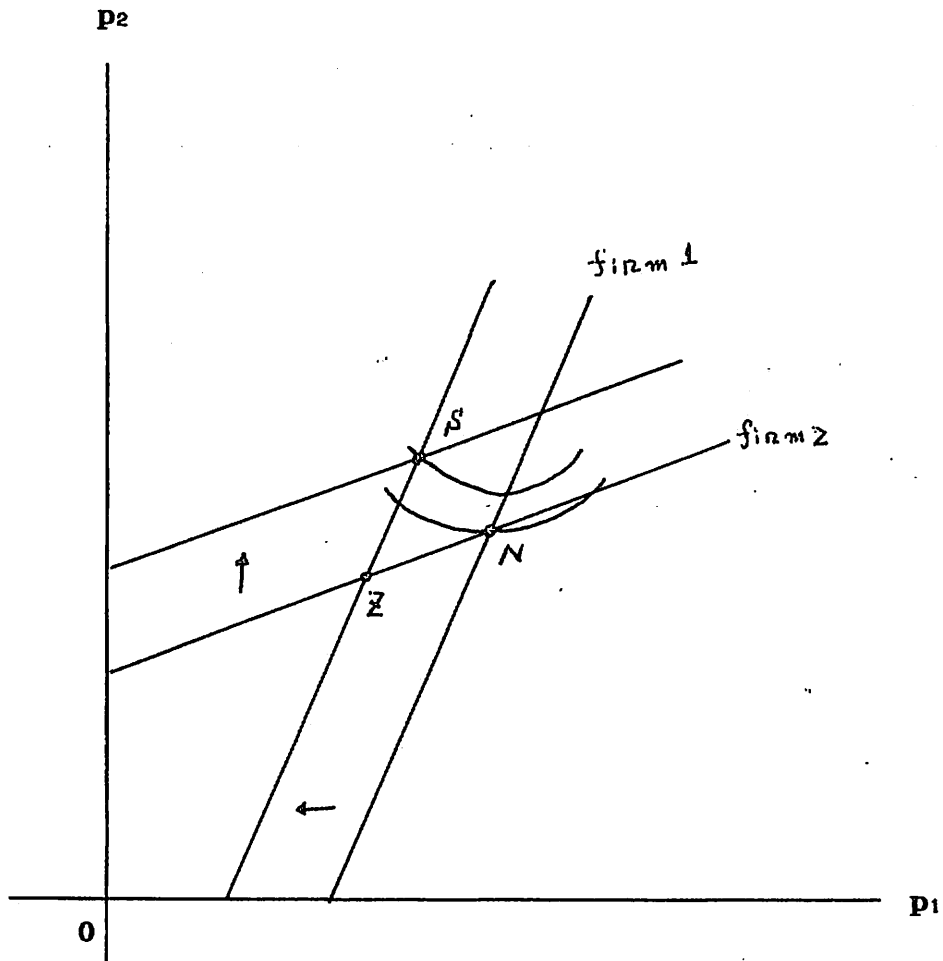
6. We can derive the same result in the case of Cournot duopoly even with homogenous products.

7. Evaluating at  $\theta = 0$ , (A.8) is positive. That is, the domestic welfare is better off by an increase in the export subsidy.

8. Evaluating at  $\theta = 0$ , (B.17) is ambiguous. That is, the domestic welfare is better off by an increase in export subsidy (tax), if it holds that

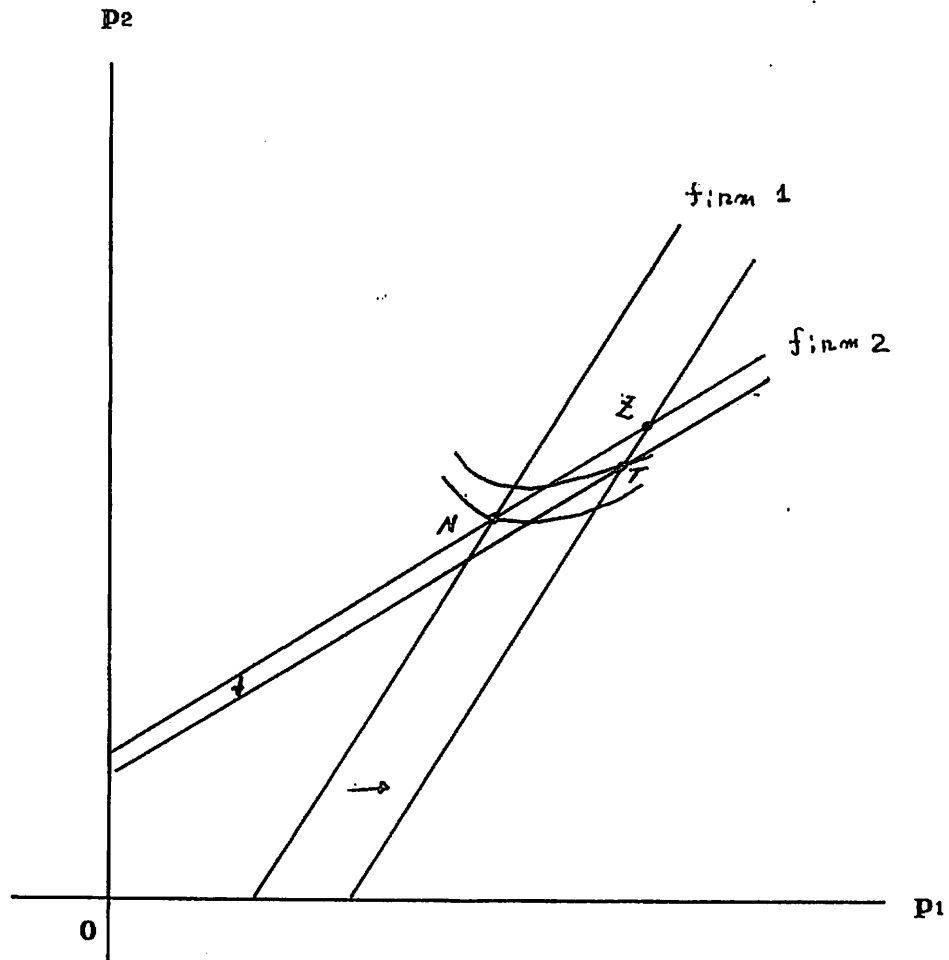
$$\frac{\partial p_2}{\partial k_2} \frac{\partial k_2}{\partial \theta} > (<) - \frac{\partial p_2}{\partial \theta}.$$

Figure 1.1 Export Subsidy Case



N: No Intervention Equilibrium  
S: Export Subsidy Policy Equilibrium

Figure 1.2 Export Tax Case



**N: No Intervention Equilibrium**  
**T: Export Tax Policy Equilibrium**