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## **Incomplete Contracts and Production Inefficiency in a Class Economy**

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# Incomplete Contracts and Production Inefficiency in a Class Economy<sup>1</sup>

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### **Abstract**

This paper examines whether production efficiency is attained in a capitalistic class economy where complete contracts are not available. It is shown that such an economy often fails to achieve production efficiency.

# 1 Introduction

The views and evaluations for the capitalistic production system is very different among several economics schools, in particular between the neoclassical economics and the Marxian economics. From the neoclassical economics point of view, the capitalistic production system is formed and sustained because of its comparative production efficiency.<sup>1</sup> Marxian economics, on the other hand, tries to explain this as the result of pursuit of a larger share of the pie by the owner of the assets for production (or the "means of production").<sup>2</sup> According to the latter perspective, the capitalistic production is not necessarily efficient, since the owners of the assets for production may try to maintain their power over the laborers at a sacrifice of production efficiency. There are few formal works, however, that concretely spelled out what those inefficiencies are and in what mechanisms those inefficiencies realize. The present paper attempts to give an explanation to those questions in a formal model.

In an ideal economy where complete contracting is available, like a perfectly competitive economy of the neoclassical economics type, there is no room for inefficiency in capitalistic production.<sup>3</sup> Recently, however, focus on the limitation of human rationality has shifted our research interests from such an ideal economy to the world where complete contracting is not available. In the present paper, I consider a capitalistic class economy where contracts are necessarily incomplete, and show that it is often difficult to attain production efficiency in that economy.

The construction of the rest of the paper is as follows. In the next section, I construct a simple model of a capitalistic class economy where complete contracting is not available. Then, section 3 examines whether production efficiency is attained in that economy. The model developed in sections 2 and 3 is a quite general one. So, to be concrete, section 4 is devoted to consider integration and separation of the ownership of the firm as an illustration of the inefficiency problem discussed in sections 2 and 3. The paper is concluded in section 5.

## 2 The Model

Let  $\mathcal{E} = (N, A)$  be a subeconomy where  $N = \{k, l\}$  is the set of agents and  $A$  is the set of assets for production. Agent  $k$  is the owner of the assets for production, and agent  $l$  is the laborer.

The production by  $k$  and  $l$  proceeds in the following order.

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<sup>1</sup>See, for instance, Milgrom-Roberts (1992), chapter 2.

<sup>2</sup>See, for instance, Marglin (1974) and Bowles (1985).

<sup>3</sup>See Samuelson (1957).

(P1)  $k$  and  $l$  first write and sign a contract (the initial contract) that specifies the ways of doing the production activity (if there is any that can be written).

(P2) Before production,  $k$  chooses the method of production  $\theta \in \Theta$ , and  $l$  makes the preparatory asset-specific investment  $x \in X$  that enhances  $l$ 's productivity when he works with the assets. For simplicity, I assume that  $\Theta = \{\theta_1, \theta_2\}$  and  $X = \{x_1, x_2\}$  where  $x$ 's are real numbers with  $x_1 > x_2$ . I assume that  $\theta$  is ex post observable only to  $k$  and  $l$  but not ex post observable publicly, so cannot be effectively written in the initial contract.<sup>4</sup>  $\theta$  chosen by  $k$  is assumed not to affect the final profit directly but to influence the division of the profit between  $k$  and  $l$  in case of bargaining at a later stage. Let  $t(\theta)$  be the proportion of the profit received by  $k$  in case of bargaining when  $k$  chooses  $\theta$ . Without loss of generality, let  $t(\theta_2) > t(\theta_1)$ , i.e.,  $k$ 's bargaining power will be greater at the future negotiation time if he takes  $\theta_2$  now rather than  $\theta_1$ . I assume that  $x$  is ex post observable only to  $k$  and  $l$  but not ex post observable publicly, so cannot be effectively written in the initial contract.<sup>5</sup> It costs for  $l$  to make an investment. Let  $c(x) \geq 0$  be the cost of investing  $x$ , where it is naturally assumed that  $c(x_1) > c(x_2)$ . Once investment  $x$  is done, the investment cost realizes and becomes observable only to  $k$  and  $l$  but not ex post observable publicly. Therefore, sharing the investment cost cannot be effectively written in the initial contract and so the cost is incurred by  $l$ .

(P3) Let  $v(x)$  be the profit when  $l$ 's investment is  $x$  and he works with all the assets. It is assumed that just after  $\theta$  and  $x$  are taken and  $c(x)$  realizes  $k$  and  $l$  write and sign another contract (the new contract) on the division of the profit.<sup>6</sup> Since the way of sharing the profit is not written in the initial contract, the division of the profit written in the new contract is determined through bargaining between  $k$  and  $l$  at the present stage. Then, for  $\theta$  and  $x$  taken at (P.2), the payoffs to  $k$  and  $l$  written in the new contract will be  $t(\theta)v(x)$  and  $(1 - t(\theta))v(x) - c(x)$ , respectively.

(P4) After the new contract is written, the production occurs. The production actually takes place only when  $k$  properly provides  $l$  with the assets and  $l$  properly works with the assets. By the time of recontracting, whether  $k$  properly provides the assets and whether  $l$  properly works with the assets are assumed to become contractible, and are written in the new contract. Soon after the production is done, the profit  $v(x)$  realizes. Then,  $v(x)$  is divided to  $k$  and  $l$  according to the new contract.

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<sup>4</sup>Once  $\theta$  is chosen, it is assumed not to be altered.

<sup>5</sup>Once  $x$  is chosen, it is assumed not to be altered.

<sup>6</sup>The implicit assumption here is that the payoffs received by  $k$  and  $l$  out of the total profit are verifiable. It is also assumed, however, that in the initial contract  $k$  and  $l$  cannot write a statement that depends on  $v$  in an effective way. This is considered a plausible assumption since after the initial contract is signed  $k$  can withdraw his assets from production while  $l$  can choose not to invest at all or even not to work at all, if they wish to do so. This interpretation is also seen in Hart-Moore (1990), p.1126, footnote 7.

In addition to (P1) to (P4), I set up the model as follows.

(A1) It holds that

$$v(x_1) - c(x_1) > v(x_2) - c(x_2).$$

(A2) It holds that

$$(1 - t(\theta_1))v(x_1) - c(x_1) > (1 - t(\theta_1))v(x_2) - c(x_2)$$

and

$$(1 - t(\theta_2))v(x_2) - c(x_2) > (1 - t(\theta_2))v(x_1) - c(x_1).$$

(A.1) says that the higher investment level  $x_1$  is socially more efficient than the lower investment level  $x_2$ . (A.2) says that given the smaller proportion of division  $t(\theta_1)$  to  $k$  out of the total profit,  $l$  is better off with the higher investment level  $x_1$  than with the lower investment level  $x_2$ , while given the larger proportion of division  $t(\theta_2)$  to  $k$ ,  $l$  is better off with the lower investment  $x_2$  than with the higher investment level  $x_1$ . The idea behind (A2) is that  $l$ 's investment is "held up" when  $k$  takes a large proportion out of the total profit.

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### 3 Production Inefficiency

In  $\mathcal{E}$ , production efficiency requires that  $x_1$  be taken rather than  $x_2$ . (Recall that  $k$ 's choice of  $\theta$  only affects the distribution of the profit between  $k$  and  $l$  and does not directly have to do with efficiency.) In the present section focus is put on whether  $x_1$  rather than  $x_2$  is taken and production efficiency is attained in  $\mathcal{E}$ .

The process and the outcome of the production by  $k$  and  $l$  in  $\mathcal{E}$  under incomplete contracts explained in the previous section can be formally described as a noncooperative game between  $k$  and  $l$ . In the game,  $k$  and  $l$ 's strategy spaces are  $\Theta$  and  $X$ , respectively, and when  $k$  takes  $\theta$  and  $l$  takes  $x$ , their payoffs are  $t(\theta)v(x)$  and  $(1 - t(\theta))v(x) - c(x)$ , respectively.

The structure of the game is made complete by specifying the order  $k$  and  $l$  take their strategies at (P.2). Apparently, the economic outcome depends on the order of actions by  $k$  and  $l$ . I consider three cases for the order of actions: (1)  $k$  and  $l$  move simultaneously (Table 1), (2)  $l$  moves first and  $k$  moves second (Figure 1), and (3)  $k$  moves first and  $l$  moves second (Figure 2).

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<sup>7</sup>See Williamson (1979) for the hold-up problem.

Theorem:

(1) When  $\theta$  and  $x$  are taken simultaneously, then  $(\theta_2, x_2)$  will realize as the unique Nash equilibrium (in pure strategy).<sup>8</sup>

(2) When  $x$  is taken first and  $\theta$  is taken second, then  $(\theta_2, x_2)$  will realize as the unique subgame perfect Nash equilibrium.

(3) When  $\theta$  is taken first and  $x$  is taken second, then  $(\theta_2, x_2)$  will realize if  $t(\theta_2)v(x_2) > t(\theta_1)v(x_1)$ , and  $(\theta_1, x_1)$  will realize if  $t(\theta_1)v(x_1) > t(\theta_2)v(x_2)$ , both as the unique subgame perfect Nash equilibrium.<sup>9</sup>

(Proof)

(1) Under the assumption that  $t(\theta_2) > t(\theta_1)$  and (A.2),  $(\theta_2, x_2)$  is the unique Nash equilibrium.

(2) For any given  $x$ ,  $k$  is better off with  $\theta_2$  than with  $\theta_1$ . Given this,  $l$  is better off with  $x_2$  than with  $x_1$ . Hence, that  $l$  takes  $x_2$  and  $k$  takes  $\theta_2$  is the unique subgame perfect Nash equilibrium.

(3) If  $\theta_1$  is taken by  $k$ ,  $l$  takes  $x_1$ , and if  $\theta_2$  is taken by  $k$ ,  $l$  takes  $x_2$ . Given this, if  $t(\theta_2)v(x_2) > t(\theta_1)v(x_1)$ , then that  $k$  takes  $\theta_2$  and  $l$  takes  $x_2$  is the unique subgame perfect Nash equilibrium. If  $t(\theta_1)v(x_1) > t(\theta_2)v(x_2)$ , on the other hand, that  $k$  takes  $\theta_1$  and  $l$  takes  $x_1$  is the unique subgame perfect Nash equilibrium.

Although the order that  $\theta$  and  $x$  are taken depends on which of the specific production problems is considered, the theorem overall suggests the difficulty of attaining production efficiency in a capitalistic class economy under incomplete contracts. In fact, the hold-up problem is hard to avoid in the present production structure. In cases (1) and (2) of the theorem,  $(\theta_2, x_2)$  is chosen even if both agents would be better off with  $(\theta_1, x_1)$  than with  $(\theta_2, x_2)$ . In case (3), since  $v(x_1) - c(x_1) > v(x_2) - c(x_2)$  by (A.1), when  $t(x_2)v(x_2) > t(x_1)v(x_1)$  it follows that  $(1 - t(\theta_1))v(x_1) - c(x_1) > (1 - t(\theta_2))v(x_2) - c(x_2)$  and hence  $l$  would be better off with  $x_1$  provided that  $k$  takes  $\theta_1$ . However,  $x_1$  will not be chosen by  $l$  since  $\theta_1$  is not taken by  $k$ .<sup>10</sup>

## 4 An Example: Integration and Separation of the Ownership of the Firm

In this section I consider the integration and separation of the ownership of the firm as an illustration of the inefficiency problem discussed in sections 2 and 3. The problem here is analysed as a specific example of a more general model in

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<sup>8</sup>Here I leave mixed strategies out of considerations.

<sup>9</sup>If  $t(\theta_1)v(x_1) = t(\theta_2)v(x_2)$ ,  $k$  is indifferent between taking  $\theta_1$  and  $\theta_2$ .

<sup>10</sup>Case (3) of the theorem is the circumstance assumed in Hart-Moore (1990).

the previous two sections, so the model and notations in this section are basically the same as those in sections 2 and 3. Also the same production process as described in (P1) to (P4) of section 2 is assumed here. New concepts and notations are introduced, however, in order to deal with the specific problem of integration and separation of the ownership of the firm. In particular, I use the Shapley value as the solution concept for the division of the profit by bargaining between  $k$  and  $l$ .<sup>11</sup>

I assume that  $A = \{a_1, a_2\}$  and that  $k$  can split his ownership of the two assets into ownership of  $a_1$  and ownership of  $a_2$ . This is represented by replacing player  $k$  with players  $\{k1, k2\}$ .<sup>12</sup> I call the ownership structure in which  $k$  owns both assets integration, while the ownership structure in which  $k1$  owns  $a_1$  and  $k2$  owns  $a_2$  separation. Let  $\theta^I$  and  $\theta^S$  represent integration and separation, respectively. Which of  $\theta^I$  and  $\theta^S$  corresponds to which of  $\theta_1$  and  $\theta_2$  in sections 2 and 3, i.e., whether  $t(\theta^I) > t(\theta^S)$  or  $t(\theta^S) > t(\theta^I)$ , depends on the economic circumstances and will be briefly discussed later.

Let the production possibility in  $\mathcal{E}$  be represented by a function

$$u : 2^{\{a_1, a_2, l\}} * X \rightarrow R$$

with  $u(\emptyset, x) = 0$  and  $u(T, x) \geq u(T_1, x) + u(T_2, x)$  where  $T \subseteq \{a_1, a_2, l\}$  and  $\{T_1, T_2\}$  constitute a partition of  $T$  (i.e.,  $T_1 \cup T_2 = T$  and  $T_1 \cap T_2 = \emptyset$ ).<sup>13 14</sup> Let

$$u(\{a_1, a_2, l\}, x) = v(x) \quad (1)$$

in the notation of sections 2 and 3.

As in the previous sections, I assume that the investment  $x$  is productivity-enhancing only when the laborer who has invested  $x$  works with the assets. This naturally leads to assume that

$$u(\{a_i\}, x) = r_i \quad (i = 1, 2) \quad (2)$$

and

$$u(\{l\}, x) = w \quad (3)$$

where  $r_i$  and  $w$  are nonnegative constants. An interpretation for this is that  $r_i$  is the rental fee of  $a_i$  and  $w$  is the wage at the market outside of  $\mathcal{E}$ . It is

<sup>11</sup>The following results will not change even if other standard solution concepts are used.

<sup>12</sup>For instance, both of  $k1$  and  $k2$  are  $k$  himself where  $k1$  stands for  $k$  as the owner-manager of firm 1 and  $k2$  stands for him as the owner-manager of firm 2. Another interpretation may be such that  $k1$  is  $k$  himself and  $k2$  is, e.g., a member of his family (his wife or his son, etc.), so that the agency cost is negligible.

<sup>13</sup>This is so-called the superadditivity property of the characteristic function  $u$ .

<sup>14</sup>For instance,  $u(\{a_i, l\}, x)$  shows the profit obtained when  $l$  who has invested  $x$  works with asset  $a_i$ , while  $u(\{a_i\}, x)$  shows the profit when  $a_i$  is available but there is no laborer who works with the asset. In the latter case, investment  $x$  does not have to do with the productivity of  $a_i$ , as assumed in (P.2) in section 2.



also natural to assume that

$$u(A, x) = r_1 + r_2 \quad (4)$$

i.e., without the laborer who has made the asset-specific investment there is no synergy between the two assets.

Under integration, the bargaining on profit-sharing at the new contract is made in a game  $(\{k, l\}, u^I)$  with  $u^I : 2^{\{k, l\}} * X \rightarrow R$  where

$$u^I(\emptyset, x) = u(\emptyset, x)$$

$$u^I(\{k\}, x) = u(\{a_1, a_2\}, x)$$

$$u^I(\{l\}, x) = u(\{l\}, x)$$

and

$$u^I(\{k, l\}, x) = u(\{a_1, a_2, l\}, x).$$

Let  $p_h^I(x)$  denote the payoff to  $h \in N$  under integration. Let  $\psi$  be the Shapley value.<sup>15</sup> Then, by (1) to (4),

$$\begin{aligned} p_k^I(x) &= \psi_k(u^I, x) \\ &= \frac{1}{2}v(x) - \frac{1}{2}w + \frac{1}{2}(r_1 + r_2) \end{aligned} \quad (5)$$

and

$$\begin{aligned} p_l^I(x) &= \psi_l(u^I, x) - c(x) \\ &= \frac{1}{2}v(x) - \frac{1}{2}(r_1 + r_2) + \frac{1}{2}w - c(x). \end{aligned} \quad (6)$$

Under separation, the bargaining on profit-sharing is made in a game  $(\{k1, k2, l\}, u^S)$  with  $u^S : 2^{\{k1, k2, l\}} * X \rightarrow R$  where

$$u^S(\emptyset, x) = u(\emptyset, x)$$

$$u^S(\{ki\}, x) = u(\{a_i\}, x) \quad (i = 1, 2)$$

$$u^S(\{l\}, x) = u(\{l\}, x)$$

$$u^S(\{k1, k2\}, x) = u(\{a_1, a_2\}, x)$$

$$u^S(\{ki, l\}, x) = u(\{a_i, l\}, x) \quad (i = 1, 2)$$

and

$$u^S(\{k1, k2, l\}, x) = u(\{a_1, a_2, l\}, x).$$

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<sup>15</sup>See Owen (1982), chapter 10, for the Shapley value.

Let  $p_h^S(x)$  denote the payoff to  $h \in N$  under separation. Then,

$$\begin{aligned} p_k^S(x) &= \psi_{k1}(u^S, x) + \psi_{k2}(u^S, x) \\ &= \frac{2}{3}v(x) - \frac{1}{6}u(\{a_1, l\}, x) - \frac{1}{6}u(\{a_2, l\}, x) + \frac{1}{2}(r_1 + r_2) - \frac{1}{3}w \quad (7) \end{aligned}$$

and

$$\begin{aligned} p_l^S(x) &= \psi_l(u^S, x) - c(x) \\ &= \frac{1}{3}v(x) + \frac{1}{6}u(\{a_1, l\}, x) + \frac{1}{6}u(\{a_2, l\}, x) - \frac{1}{2}(r_1 + r_2) + \frac{1}{3}w - c(x) \quad (8) \end{aligned}$$

Definition: For a pair of nonempty subsets  $T_1$  and  $T_2$  of  $\{a_1, a_2, l\}$  with  $T_1 \cap T_2 = \emptyset$ ,  $T_1$  and  $T_2$  are called *synergistic* iff  $u(T_1 \cup T_2, x) > u(T_1, x) + u(T_2, x)$  for both  $x \in X$ , and  $T_1$  and  $T_2$  are called *independent* iff  $u(T_1 \cup T_2, x) = u(T_1, x) + u(T_2, x)$  for both  $x \in X$ .

Seeing assets and labor as all inputs for production, the meaning of the definition would be clear. In what follows I consider the two cases.

(Case 1) For both  $x \in X$ ,

$$u(\{a_i, l\}, x) = u(\{a_i\}, x) + u(\{l\}, x) \quad (i = 1, 2) \quad (9)$$

and

$$u(\{a_1, a_2, l\}, x) > \max\{u(\{a_1, l\}, x) + u(\{a_2\}, x), u(\{a_2, l\}, x) + u(\{a_1\}, x)\}. \quad (10)$$

(Case 2) For both  $x \in X$ ,

$$u(\{a_i, l\}, x) > u(\{a_i\}, x) + u(\{l\}, x) \quad (i = 1, 2) \quad (11)$$

and

$$u(\{a_1, a_2, l\}, x) = \max\{u(\{a_1, l\}, x) + u(\{a_2\}, x), u(\{a_2, l\}, x) + u(\{a_1\}, x)\}. \quad (12)$$

Case 1 is when  $\{a_1\}$  and  $\{l\}$ , and  $\{a_2\}$  and  $\{l\}$ , are independent, but  $\{a_1, l\}$  and  $\{a_2\}$ , and  $\{a_2, l\}$  and  $\{a_1\}$ , are synergistic.<sup>16</sup> Conversely, Case 2 is when

<sup>16</sup>When  $\{a_1\}$  and  $\{l\}$ , and  $\{a_2\}$  and  $\{l\}$ , are independent, and  $\{a_1, l\}$  and  $\{a_2\}$  are synergistic, it is always true that  $\{a_2, l\}$  and  $\{a_1\}$  are synergistic. Suppose that  $u(\{a_i, l\}, x) = u(\{a_i\}, x) + u(\{l\}, x)$  for  $i = 1, 2$  and  $u(\{a_1, a_2, l\}, x) > u(\{a_1, l\}, x) + u(\{a_2\}, x)$  but that  $u(\{a_1, a_2, l\}, x) = u(\{a_2, l\}, x) + u(\{a_1\}, x)$ . Then  $u(\{a_1\}, x) + u(\{a_2\}, x) + u(\{l\}, x) = u(\{a_2, l\}, x) + u(\{a_1\}, x) = u(\{a_1, a_2, l\}, x) > u(\{a_1, l\}, x) + u(\{a_2\}, x) = u(\{a_1\}, x) + u(\{a_2\}, x) + u(\{l\}, x)$ , which is a contradiction.

$\{a_1\}$  and  $\{l\}$ , and  $\{a_2\}$  and  $\{l\}$ , are synergistic, but either  $\{a_1, l\}$  and  $\{a_2\}$ , or  $\{a_2, l\}$  and  $\{a_1\}$ , is independent.<sup>17 18</sup>

In words, case 1 occurs when the two assets are complementary, while case 2 does when the two assets are substitutable. Suppose, for instance, that the two assets are a personal computer and a printer. Then, an office worker cannot be productive with either a personal computer or a printer, but he becomes productive with both of the personal computer and the printer. In this case, the personal computer and the printer are considered complementary. On the other hand, suppose that the two assets are an IBM computer set (i.e., a pair of the computer and the printer) and a Macintosh computer set. Then, an office worker is already productive with either computer set, and he does not become more productive even if another computer set becomes available. In this case, the IBM and Macintosh computer sets are considered substitutable.

(A3) The following relations hold:

$$p_i^f(x) > p_i^s(x) \text{ for both } x \in X \Rightarrow p_i^f(x_1) > p_i^f(x_2) \text{ and } p_i^s(x_2) > p_i^s(x_1),$$

$$p_i^s(x) > p_i^f(x) \text{ for both } x \in X \Rightarrow p_i^s(x_1) > p_i^s(x_2) \text{ and } p_i^f(x_2) > p_i^f(x_1).$$

(A.3) is a representation of the hold-up circumstance in the present context, where each of the two relations of (A.3) corresponds to (A.2) in section 2. In the first relation of (A.3), for given  $x$ , suppose that  $l$ 's bargaining power is stronger under integration than under separation, and hence  $l$  can get greater share under integration than under separation. Then, it pays  $l$  to make the high level investment under integration, while it doesn't pay him to do so under separation. The second relation of (A.3) is explained in the similar way.<sup>19</sup>

<sup>17</sup>When  $\{a_2\}$  and  $\{l\}$  are synergistic and  $\{a_1, l\}$  and  $\{a_2\}$  are independent, it is always true that  $\{a_1\}$  and  $\{l\}$  are synergistic. Suppose that  $u(\{a_2, l\}, x) > u(\{a_2\}, x) + u(\{l\}, x)$  and  $u(\{a_1, a_2, l\}, x) = u(\{a_1, l\}, x) + u(\{a_2\}, x)$  but that  $u(\{a_1, l\}, x) = u(\{a_1\}, x) + u(\{l\}, x)$ . Then  $u(\{a_1, a_2, l\}, x) = u(\{a_1, l\}, x) + u(\{a_2\}, x) = u(\{a_1\}, x) + u(\{l\}, x) + u(\{a_2\}, x) < u(\{a_1\}, x) + u(\{a_2, l\}, x)$ , which contradicts superadditivity of  $u$ .

<sup>18</sup>In Case 2, when  $u(\{a_1, a_2, l\}, x) = u(\{a_1, l\}, x) + u(\{a_2\}, x)$ , it is allowed that  $u(\{a_1, a_2, l\}, x) > u(\{a_2, l\}, x) + u(\{a_1\}, x)$  so  $\{a_2, l\}$  and  $\{a_1\}$  are synergistic, according to the definition. But since  $u(\{a_1, a_2, l\}, x) = u(\{a_1, l\}, x) + u(\{a_2\}, x)$ , in fact, it is between  $\{a_1\}$  and  $\{l\}$ , not between  $\{a_1\}$  and  $\{a_2\}$  (with  $l$ ), that there is a synergy. In words,  $l$  is better at using  $a_1$  than  $a_2$ , and so when  $a_1$  is already available  $a_2$  is no longer necessary for  $l$ .

<sup>19</sup>It is easy to make a plausible example in which a larger payoff to  $l$  induces him to make a higher investment. Let  $\psi_l$  and  $c$  be functions defined over  $x \in X$  with  $X$  being an interval such that  $l$ 's payoff is given by  $\psi_l(x) - c(x)$ . Let  $\psi_l' > 0$ ,  $\psi_l'' < 0$ ,  $c' > 0$  and  $c'' > 0$ . Then, a greater payoff to  $l$  is tied to an incentive for  $l$  to make a higher investment. Hart-Moore (1990) used this setting. In the present paper I omit the derivation of the relation between greater payoff and higher investment for the compactness of the paper.

Lemma: In Case 1, it holds that  $p_k^S(x) > p_k^I(x)$  and  $p_l^I(x) > p_l^S(x)$  for both  $x \in X$ . In Case 2, it holds that  $p_k^I(x) > p_k^S(x)$  and  $p_l^S(x) > p_l^I(x)$  for both  $x \in X$ .

(Proof) For any given  $x$ , since  $p_k^I(x) + p_l^I(x) = p_k^S(x) + p_l^S(x) = v(x)$ ,  $p_k^S(x) > (<) p_k^I(x)$  is equivalent to  $p_l^I(x) > (<) p_l^S(x)$ . Hence, it suffices to show the inequality for  $l$  in each case. In Case 1, by (1) to (4) and (9),

$$p_l^I(x) - p_l^S(x) = \frac{1}{6}[u(\{a_1, a_2, l\}, x) - u(\{a_1, l\}, x) - u(\{a_2\}, x)]$$

which is positive by (10). In case 2, without loss of generality, suppose that  $u(A \cup \{l\}, x) = u(\{a_1, l\}, x) + u(\{a_2\}, x)$  in (12). Then, by (1) to (4),

$$p_l^S(x) - p_l^I(x) = \frac{1}{6}[u(\{a_2, l\}, x) - u(\{a_2\}, x) - u(\{l\}, x)]$$

which is positive by (11).  $\square$

The intuition behind the lemma is as follows. In Case 1 where the two assets are complementary, since both assets are necessary for  $l$  to be productive, it is advantageous for  $k$  to split his ownership and make it difficult for  $l$  to access both assets at the same time. Conversely, in Case 2 where the two assets are independent, since either asset is enough for  $l$  to be productive, it is advantageous for  $k$  to merge his ownership and forcibly make it necessary for  $l$  to access both assets even though in fact either asset is unnecessary to  $l$ .

In Case 1,  $k$ 's payoff is greater under separation than under integration, so, for any given  $x$ ,  $\theta^I = \theta_1$  and  $\theta^S = \theta_2$  (in the notation of sections 2 and 3), while  $l$ 's payoff is greater under integration than under separation, so by (A.3)  $l$  would choose  $x_1$  under integration and  $x_2$  under separation. Similarly, in case 2,  $\theta^S = \theta_1$  and  $\theta^I = \theta_2$ , while  $l$  would take  $x_1$  under separation and  $x_2$  under integration.

Proposition:

(1) Consider the case when  $\theta$  and  $x$  are taken simultaneously. In Case 1,  $(\theta^S, x_2)$  will realize as the unique Nash equilibrium. In Case 2,  $(\theta^I, x_2)$  will realize as the unique Nash equilibrium.

(2) Consider the case when  $x$  is taken first and  $\theta$  second. In Case 1,  $(\theta^S, x_2)$  will realize as the unique subgame perfect Nash equilibrium. In Case 2,  $(\theta^I, x_2)$  will realize as the unique subgame perfect Nash equilibrium.

(3) Consider the case where  $\theta$  is taken first and  $x$  second. In Case 1,  $(\theta^S, x_2)$  will realize if  $p_k^S(x_2) > p_k^I(x_1)$ , and  $(\theta^I, x_1)$  will realize if  $p_k^I(x_1) > p_k^S(x_2)$ , both as the unique subgame perfect Nash equilibrium. In Case 2,  $(\theta^I, x_2)$  will realize if  $p_k^I(x_2) > p_k^S(x_1)$ , and  $(\theta^S, x_1)$  will realize if  $p_k^S(x_1) > p_k^I(x_2)$ , both as the unique subgame perfect Nash equilibrium.

(Proof)

(1) By Lemma together with (A.3), the results are obtained.

(2) In Case 1, given any  $x \in X$ , since  $p_k^S(x) > p_k^I(x)$  by Lemma,  $k$  takes  $\theta^S$ . Given that  $k$  takes  $\theta^S$ , since  $p_l^S(x_2) > p_l^S(x_1)$  by (A.3),  $l$  takes  $x_2$ . In Case 2, given any  $x \in X$ , since  $p_k^I(x) > p_k^S(x)$  by Lemma,  $k$  takes  $\theta^I$ . Given that  $k$  takes  $\theta^I$ , since  $p_l^I(x_2) > p_l^I(x_1)$  by (A.3),  $l$  takes  $x_2$ .

(3) First consider Case 1. Given  $\theta^I$ , since  $p_l^I(x_1) > p_l^I(x_2)$  by (A.3),  $l$  takes  $x_1$ . Given  $\theta^S$ , since  $p_l^S(x_2) > p_l^S(x_1)$  by (A.3),  $l$  takes  $x_2$ . Given this, if  $p_k^S(x_2) > p_k^I(x_1)$ ,  $k$  takes  $\theta^S$ , and if  $p_k^I(x_1) > p_k^S(x_2)$ ,  $k$  takes  $\theta^I$ . Next consider Case 2. Given  $\theta^I$ , since  $p_l^I(x_2) > p_l^I(x_1)$ ,  $l$  takes  $x_2$ . Given  $\theta^S$ , since  $p_l^S(x_1) > p_l^S(x_2)$  by (A.3),  $l$  takes  $x_1$ . Given this, if  $p_k^I(x_2) > p_k^S(x_1)$ ,  $k$  takes  $\theta^I$ , and if  $p_k^S(x_1) > p_k^I(x_2)$ ,  $k$  takes  $\theta^S$ .  $\square$

It has been noted in the previous sections that the order  $\theta$  and  $x$  are taken depends on cases. As for the integration and separation of the ownership of the firm, it seems that while it usually takes for years for employees to make a firm-specific investment, merger and separation plans are often disclosed suddenly and they are done pretty soon after the announcement. If it is the case, the second case in Proposition seems most likely, or, if information on ownership and investment is not revealed easily, the first case may be plausible. In either case, the production outcome is inefficient.

## 5 Conclusion

In this paper I considered whether production efficiency is attained in a capitalistic class economy under incomplete contracts. It was shown that overall the production efficiency is hard to be achieved in such an economy. As a concrete example, I took the problem of integration and separation of the ownership of the firm and illustrated the difficulty of attaining production efficiency in the economy of this type.

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	$x_1$	$x_2$
$\theta_1$	$(t(\theta_1)v(x_1), (1 - t(\theta_1))v(x_1) - c(x_1))$	$(t(\theta_1)v(x_2), (1 - t(\theta_1))v(x_2) - c(x_2))$
$\theta_2$	$(t(\theta_2)v(x_1), (1 - t(\theta_2))v(x_1) - c(x_1))$	$(t(\theta_2)v(x_2), (1 - t(\theta_2))v(x_2) - c(x_2))$

Table 1:  $k$  and  $l$  move simultaneously.

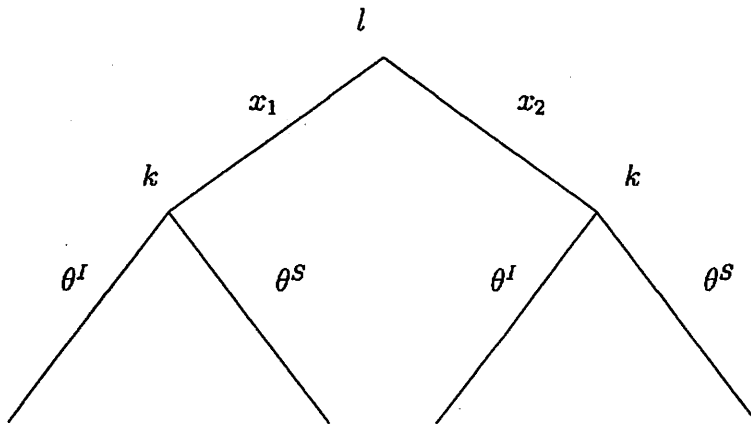


Figure 1:  $l$  moves first and  $k$  moves second. (The payoffs are the same as in Table 1.)

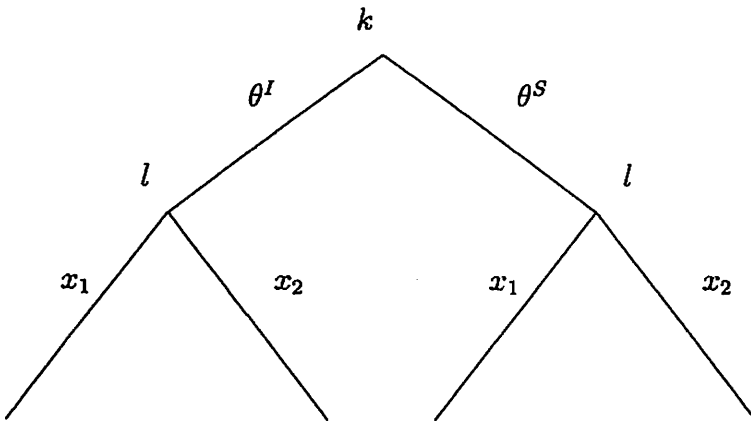


Figure 2:  $k$  moves first and  $l$  moves second. (The payoffs are the same as in Table 1.)