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Asset Bubbles, Technology Choice, and Financial Crises

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Abstract

How does an economy fall into depression after an asset bubble bursts? To address this question, we extend Matsuyama's (2007) overlapping-generations model with multiple technologies to a dynamic general equilibrium model with infinitely lived agents. Our analysis focuses on a case of two technologies: one with high productivity and another with low productivity. The *crowd-in* effect that asset bubbles have on capital accumulation occurs in equilibrium, in which the high interest rates resulting from asset bubbles crowd out low-productivity technology. When asset bubbles with high-productivity technology collapse, a depression follows.

Keywords: Asset bubbles; Crowd-in effect; Matsuyama model; Infinitely-lived agents; Financial crises.
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1 Introduction

Since the beginning of modern monetary and financial systems in the 17th century, the world has seen numerous financial crises. A financial crisis often occurs without warning; macroeconomic variables such as growth and inflation rates may give no sign of a coming crisis. Before a financial crisis, output and asset prices often grow synchronously at a very high rate, asset prices deviate from their fundamental values, and asset bubbles emerge. During the synchronized growth of output and asset prices, the presence of asset bubbles seems to promote economic growth. History has repeatedly seen serious economic depressions following a crash in asset prices. Therefore, financial crises seem to originate from the bursting of asset bubbles.[1] Motivated by these facts observed in our history, we investigate both how asset bubbles promote economic growth and how an economy falls into depression after the collapse of asset bubbles by constructing a dynamic general equilibrium model with infinitely lived agents and multiple technologies.

An asset bubble is defined as the difference between an asset's fundamental value and its market value. Asset bubbles cannot exist in equilibrium in standard dynamic general equilibrium models with an infinitely lived representative agent. To understand this more concretely, consider a dynamic general equilibrium model with an infinitely lived representative agent and no frictions. In this economy, a unique equilibrium path converging to a steady state exists under mild conditions, and the steady-state interest rate is always greater than the economic growth rate. Due to the latter property, the transversality condition rules out any bubbly dynamic paths. In our model, however, potential entrepreneurs face borrowing constraints. When borrowing constraints are binding in the steady state without asset bubbles, the market interest rate becomes lower than the economic growth rate, deviating downward from the marginal product of capital. In such a case, the sum of the present values of future output becomes infinite, and asset bubbles can appear in equilibrium despite that consumers' preferences are locally nonsatiated and their lifetime budget constraints are binding.

Our analytical framework is based on Matsuyama's (2007) overlapping-generations model with credit market imperfections and multiple technologies. Matsuyama considers credit multiplier mechanisms, as in the literature on the macroeconomics of credit market imperfections (e.g., Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). He emphasizes the composition effect of credit that shifts across investment projects with different productivity levels and derives various dynamic economic behaviors that reflect the relationships among capital accumulation, technology choice, and the composition of credit

[1] Brunnermeier and Oehmke (2012) provide a historical overview of asset bubbles and financial crises. According to them, early examples of asset bubbles include the Dutch tulip mania from 1634 to 1637, the Mississippi Bubble from 1719 to 1720, and the South Sea Bubble in 1720. Recent examples include the bursting of the Japanese asset price bubble in 1990, the Asian financial crisis in 1997, and the US subprime loan crisis from 2007 to 2009.

allocated to investment projects. In the present paper, we extend the Matsuyama model to an infinitely lived agent model and introduce a bubbly asset into the model. In our model, bubbles create the *crowd-in* effect, which stimulates capital accumulation. The key elements in our extension to induce the asset bubble *crowd-in* effect are the *net worth effect* of bubbles, which is stressed by Hirano et al. (2015) and Hirano and Yanagawa (2016), and endogenous *technology choice*, which is an important element in Matsuyama's model to produce rich dynamics.

In Matsuyama's two-period settings, the wealth effect that asset bubbles have on borrowing constraints does not appear, and thus, there is no *crowd-in* effect. In fact, Matsuoka and Shibata (2012) introduce a bubbly asset into a version of Matsuyama's (2007) overlapping-generations model and show that asset bubbles always crowd out capital accumulation. Moreover, they show that in their case of two types of technologies, asset bubbles can affect technology choice and may preclude the adoption of high-productivity technology. Thus, in their model, bubbles have negative effects on capital accumulation in two different ways. In contrast to Matsuoka and Shibata (2012), in the present paper's model, infinitely lived agents can be endowed with net worth that includes a bubbly asset in each period. The presence of asset bubbles increases the net worth, which relaxes borrowing constraints, and thereby, the market interest rate increases (the net worth effect).[2] This increase in the interest rate forces entrepreneurs to select high-productivity technologies and reject low-productivity technologies (endogenous technology choice). As a result, capital accumulation is stimulated, and output increases.

Researchers have long debated the macroeconomic effects of asset bubbles. Tirole (1985) and Weil (1987) develop neoclassical growth models with asset bubbles and demonstrate that asset bubbles impede capital accumulation by *crowding out* private investments, although asset bubbles also correct dynamic inefficiency and improve consumer welfare. Similarly, the literature on endogenous growth investigates the *crowd-out* effect of asset bubbles on private investments. For example, Grossman and Yanagawa (1992), King and Ferguson (1993), and Futagami and Shibata (2000) show that asset bubbles slow economic growth. Each of these studies develops overlapping generations models and investigates only the *crowd-out* effect of asset bubbles.[3] These previous models predict that the collapse of asset bubbles stimulates capital accumulation, but that prediction is inconsistent with historical observations.

[2]This net worth effect of bubbles is emphasized by Hirano et al. (2015) and Hirano and Yanagawa (2016), and most of the recent models featuring a crowd-in effect of bubbles contain the effect. Exceptions are Takao (2015) and Hillebrand et al. (2016). In the model of Takao (2015), bubbles affect the size of firms and stimulate the intensity of in-house R&D, leading to higher growth. In the model of Hillebrand et al. (2016), bubbles raise the interest rate and thereby cause a savings glut, which injects greater resources into the economy. When this effect dominates the standard crowd-out effect, bubbles stimulate capital accumulation.

[3]Azariadis and Smith (1996), Boyd and Smith (1998) and Kunieda (2008) also analyze the *crowd-out* effect of asset bubbles in financially constrained economies.

In contrast to traditional approaches, several recent studies on economic growth and bubbles consider not only the *crowd-out* effect but also the *crowd-in* effect of asset bubbles on private investments, thereby overcoming the shortcomings of traditional growth models. Based on the modeling strategy, these recent studies can be categorized into two groups. The first group, which includes Caballero and Krishnamurthy (2006), Farhi and Tirole (2012), Martin and Ventura (2012), Ventura (2012), and Kunieda (2014), uses the overlapping-generations modeling approach in the same way as Samuelson (1958), Tirole (1985), or Blanchard (1985). The second group, which includes Kocherlakota (2009), Kiyotaki and Moore (2012), Aoki et al. (2015), Hirano et al. (2015), Hirano and Yanagawa (2016), and Kunieda and Shibata (2016), uses infinitely lived agent models to derive asset bubbles á la Tirole (1985).[4] Our model belongs to the second group in that we extend Matsuyama's (2007) overlapping-generations model to an infinitely lived agent model and show that asset bubbles can exist.[5] Our analysis shows that bubbles have both *crowd-in* and *crowd-out* effects on capital accumulation. It should be noted here that there are two early studies addressing the same issue. The early studies that show a crowd-in effect of bubbles are Mitsui and Watanabe (1989) and Olivier (2000). Using a continuous-time overlapping-generalizations model, Olivier (2000) shows that bubbles on a productive asset enhance economic growth through the corresponding increase in the value of new R&D firms, that is, the presence of bubbles in the R&D sector gives further incentives for new firms to enter the R&D sector, and thus stimulates growth. Mitsui and Watanabe (1989) construct a growth model with imperfect credit markets and show that the effect of bubbles on the growth rate can be either positive or negative, depending on the elasticity of intertemporal substitution. More detailed explanations are provided below.

An important feature of the two groups is the existence of financial frictions. As Martin and Ventura (2012) and Carvalho et al. (2012) note, asset bubbles can promote capital accumulation in the presence of financial frictions. The presence of asset bubbles increases the market interest rate and thereby eliminates *inefficient* investments. Obviously, when financial frictions exist, asset bubbles appear more easily in an overlapping-generations model than in an infinitely lived agent model.[6] The reason is that the former can exhibit

[4]By explicitly considering the role of banks in a model similar in structure to that of Hirano and Yanagawa (2016), Aoki and Nikolov (2015) also study the bursting of asset bubbles, which may or may not cause serious economic recessions.

[5]There is another strand of infinitely lived agent models that examine the relationship between asset price bubbles and macroeconomic activities. Miao and Wang (2011) consider asset bubbles on productive capital. In their model, firms holding bubbles *directly* weakens credit constraints and stimulates economic activities. It should be noted that in Miao and Wang's (2011) model, asset bubbles can exist even when the total output growth rate is *less than* the interest rate in the bubbleless steady state. In this way, they derive a new class of bubbles that differs from those observed in other studies á la Tirole (1985). The aforementioned second group considers bubbles on intrinsically useless assets and analyzes the effects of the bubbles on resource allocation through *indirect* channels such as changes in returns on savings and the productivity distribution of active investors. In the models in the second group, asset bubbles occur when the total output growth rate is *greater than* the interest rate in the bubbleless steady state.

[6]Kitagawa (1994, 2001a) show that in overlapping-generations models the existence of financial fric-

two conditions–capital over-accumulation and financial market imperfections–that favor the development of asset bubbles.[7] Even if we do not use an overlapping-generations model, the market interest rate can be smaller than the economic growth rate in equilibrium, which is a necessary condition for asset bubbles to appear, as demonstrated by Santos and Woodford (1997). The market interest rate in a borrowing-constrained economy with no asset bubbles will be smaller than the rate in an economy with perfect financial markets, as the demand for borrowing will be lower in the former case. The combination of a lower interest rate and heterogeneous agent productivity will cause the equilibrium interest rate to be less than the economic growth rate. Kocherlakota (2009), Kiyotaki and Moore (2012), Aoki et al. (2015), Hirano et al. (2015), Hirano and Yanagawa (2016), and Kunieda and Shibata (2016) model this situation with infinitely lived agents and show that the presence of financial frictions is the crucial factor for generating asset bubbles. In other words, without financial frictions, their models become like the Ramsey-type models, in which equilibrium is unique and asset bubbles never occur.

An investigation into the structure of the extant models in the second group would show that their modeling strategy derives from Bewley (1980) and Townsend (1980). In the Bewley-Townsend model, infinitely lived agents face borrowing constraints and receive uninsured idiosyncratic income shocks in each period. Under these circumstances, if fiat money is introduced into the economy, high-income agents will buy the fiat money, and low-income agents will sell the money to smooth their consumption. Fiat money cannot achieve the first best outcome, but it will render all agents better off. The structure of Mitsui and Watanabe's (1989) model is similar to those of Bewley (1980) and Townsend (1980). Mitsui and Watanabe (1989) develop a growth model with high- and low-productivity technologies. In their model, there are infinitely many islands, separated from one another. On each island, there are many infinitely lived agents who cannot trade with agents on other islands. Each agent faces a random relocation shock to other islands. Agents remaining at the same islands can directly or indirectly utilize the high-productivity technology, whereas agents who must move to other places at the end of the period cannot reencounter other agents living on their original islands and can utilize only the low-productivity technology. Under this circumstance, Mitsui and Watanabe (1989) prove the existence of a bubbly equilibrium, in which the return from bubbles is higher than that of the low-productivity technology, and show that the presence of bubbles increases the economic growth rate by eliminating the low productivity tech-

tions relaxes the condition for bubbles to exist, that is, in the presence of financial frictions, bubbles can exist even when the interest rate in the bubbleless equilibrium is higher than the economic growth rate. Kitagawa (2001b) investigates the welfare implications of bubbles in this situation and shows that bubbles can reduce the welfare of almost all generations

[7]As noted by Aiyagari and McGrattan (1998), Cozzi (2001), and Farhi and Tirole (2012), the infinitely lived agent model with financial frictions and overlapping-generations models (without financial frictions) share a common feature in the sense that the existence of financial frictions divides the horizons of the infinitely lived agents.

nology. The modeling strategy used in the second group of models is essentially the same as that in Mitsui and Watanabe (1989).[8] Specifically, borrowing-constrained agents in the recent models in the second group face uninsured idiosyncratic productivity shocks in each period, and the agents with lower productivity have an incentive to hold an intrinsically useless asset because they would otherwise receive lower returns on their investment projects.[9] In contrast to the extant models in the second group, our model is not based on Bewley (1980) or Townsend (1980); we directly follow Matsuyama's (2007) model, in which the potential entrepreneurs face random credit rationing instead of uninsured idiosyncratic productivity shocks.

In our model, multiple bubbly steady states can appear because of technology choice. We provide the parameter conditions for multiple bubbly steady states to appear in equilibrium. In particular, two technologies can lead to one bubbleless and two bubbly steady states. One of two bubbly steady states is associated with a low-productivity technology and the other with a high-productivity technology, whereas the bubbleless steady state is associated with a low-productivity technology. Therefore, when asset bubbles in the presence of high-productivity technology collapse, a depression will follow. Although multiple bubbly steady states are also obtained by Gokan (2011), Tanaka (2011) and Matsuoka and Shibata (2012) in the two-period overlapping-generations framework, there are differences between their works and ours. Introducing financial frictions into the standard two-period overlapping-generations model, Gokan (2011) shows that there can exist two bubbly steady states and analyzes their dynamic properties. Matsuoka and Shibata (2012) consider two types of technologies in a manner similar to the present paper and demonstrate the possibility of multiple bubbly steady states. In these models, however, bubbles have only the crowd-out effect, and thus, when asset bubbles collapse, the economy always experiences a boom, not a recession. In contrast, Tanaka (2011) shows that pure bubbles, bubbles on useless assets, always have only the crowd-out effect, which differs from our result. Considering bubbles on a productive asset in a manner similar to that of Olivier (2000), however, Tanaka (2011) demonstrates that in the unstable bubbly steady state, an increase in the size of bubbles stimulates economic growth. Thus, in his model, the bursting of bubbles on a productive asset can cause a recession. In this sense, his result is similar to ours. However, in his model, as he shows, a pure bubble, one on a useless asset, always has only the crowd-out effect, whereas in our model, a pure bubble can have a crowd-in effect. Moreover, Caballero et al. (2006) construct a model of bubbles using an overlapping-generations model with a discontinuous technology. In their model, at a critical level of capital, the production function jumps upward, and thus,

[8] Kocherlakota (1992) also demonstrates that an asset price bubble can occur when agents are infinitely lived but face short-sales constraints.

[9] In Townsend's (1980) model, although the alternation of high and low endowments among agents is deterministic, the mechanism through which the intrinsically useless asset has a positive value is similar to that in the extant models in the second group.

there can be two *bubbleless* steady states, one of which is located below and the other above the critical level. Using this model, they show that transitory bubbles converging to the high bubbleless steady state exhibit a positive correlation with capital accumulation. However, in their model, the amount of capital on a bubbly equilibrium path is always smaller than that on the corresponding bubbleless equilibrium path, meaning the model reveals a crowd-out effect relative to the bubbleless equilibrium.

We use simple phase diagrams to investigate the dynamic behavior of the economy. The phase diagrams that we depict are very similar to that used by Tirole (1985), although we employ an infinitely lived agent model. The phase diagrams help us to see the difference between Tirole's model and our model with respect to the mechanisms through which asset bubbles arise. The phase diagrams also show that financial crises can accompany the bursting of asset bubbles.

The remainder of this paper proceeds as follows. In the next section, we derive an equilibrium dynamical system. In section 3, we discuss how asset bubbles' *crowd-in* effect on private investments is induced by endogenous *technology choice*, and we develop simple phase diagrams to investigate the equilibrium dynamics of the model. In section 4, we provide an example of financial crises accompanying the bursting of asset bubbles. We present our concluding remarks in section 5. The appendix presents the proofs of propositions and lemmata.

2 Model

While the basic structure of our model follows Matsuyama's (2007) overlapping-generations model, we introduce an intrinsically useless asset as one of the saving methods. The economy is represented in discrete time, ranging from time 0 to infinity, and it consists of a measure-one continuum of infinitely lived entrepreneurs and a measure-one continuum of infinitely lived workers. The potential entrepreneurs are ex-ante homogeneous. However, because of the non-convexity of investment projects and agency problems in the financial market, the potential entrepreneurs are ex-post heterogeneous, facing credit rationing in the financial market. In each period, some of them randomly become capital producers, and others become savers, lending their net worth in the financial market or buying the intrinsically useless asset.

As assumed in Weil (1992), Mankiw (2000), Galí et al. (2004, 2007), Kocherlakota (2009), and Kiyotaki and Moore (2012), workers are rule-of-thumb consumers; that is, they consume their current labor income entirely in a "hand-to-mouth" manner and do not save or borrow in the financial market. The presence of rule-of-thumb consumers does not directly affect equilibrium dynamics in our model, but this assumption simplifies our model and enables us to focus on the optimal behavior of entrepreneurs and the crucial role of asset bubbles in determining technology choice.[10]

2.1 Production

A general good, to be consumed or invested in projects, is produced by a production function, $Y_t = F(K_t, L_t)$, where K_t represents capital and L_t labor at time t. The capital depreciates entirely in one period, and $F : [0, \infty) \times [0, \infty) \to [0, \infty)$ is continuous, exhibiting constant returns to scale. The production function exhibits positive and diminishing marginal products with respect to both K_t and L_t. We define $f(k_t) := F(K_t/L_t, 1)$, where $k_t := K_t/L_t$ is the capital-labor ratio. We assume that $F(K_t, L_t)$ is at least twice differentiable. It follows that $f'(k_t) > 0 > f''(k_t)$, and that $f(k_t)$ satisfies $f(0) = 0$ and the Inada conditions $\lim_{k_t \to 0} f'(k_t) = \infty$, and $\lim_{k_t \to \infty} f'(k_t) = 0$.

Because the capital and labor markets are competitive, the production factors are paid their marginal products:

$$\begin{aligned} \rho_t &= f'(k_t) \\ w_t &= f(k_t) - k_t f'(k_t), \end{aligned}$$

where ρ_t is the rental rate and w_t the wage rate.

We impose the following technical assumption to focus on a meaningful economic situation:

Assumption 1 $[k_t f'(k_t)]' > 0$.

Note that Assumption 1 does not contradict the Inada conditions and holds if $F(K_t, L_t)$ is a Cobb-Douglas production function.

2.2 Entrepreneurs

Potential entrepreneurs in our model are not endowed with any labor. Let Ω denote the whole set of potential entrepreneurs. To generate income, they turn over their net worth. Let $a_t(i)$ denote entrepreneur i's net worth at time t, where $i \in \Omega$. As assumed in Matsuyama (2007), to be capital producers, potential entrepreneurs must borrow funds in the financial market, but they face credit rationing in each period, whereby borrowers are randomly selected from among the potential entrepreneurs in each period. The random selection of borrowers is independent over time and across potential entrepreneurs. The potential entrepreneurs who are not selected as borrowers must become savers. Because the financial sector is competitive, the borrowing and saving interest rates are the same. Figure 1 illustrates that an agent at time t becomes a capital producer with probability

[10] Mankiw and Zeldes (1991), Bertaut (1996), King and Leape (1998), and Guiso et al. (2002) provide empirical evidence suggesting the existence of hand-to-mouth consumers.

θ_t and a saver with probability $1 - \theta_t$. As the total population is equal to one, θ_t and $1 - \theta_t$ are the number of capital producers and the number of savers, respectively.

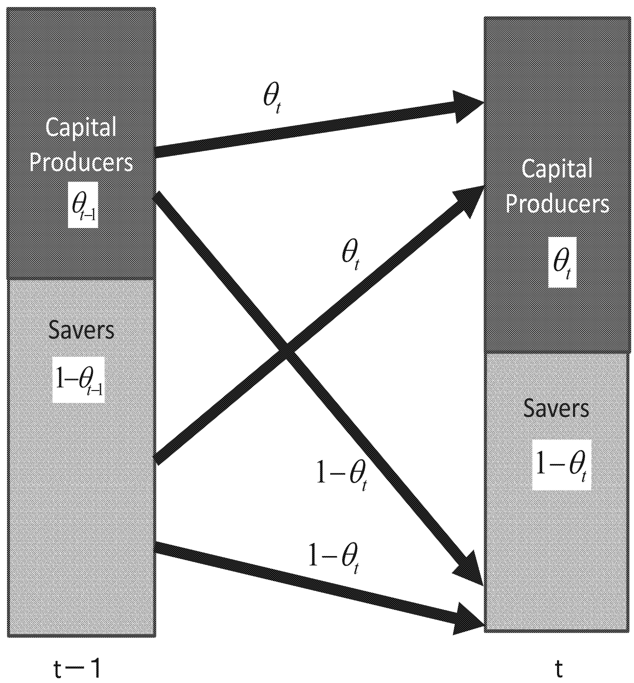

Figure 1: Credit rationing. *Notes:* An agent at time t will become a capital producer with probability θ_t and a saver with probability $1-\theta_t$.

2.2.1 Returns

Entrepreneur i may become a saver or a capital producer in each period. The individual-specific return, $R_{t+1}(i)$, at time $t+1$ is determined by whether the entrepreneur becomes a saver, a constrained capital producer, or an unconstrained capital producer at time t. If entrepreneur i becomes a saver at time t, she lends her net worth, $a_t(i)$, in the financial market or purchases an intrinsically useless asset to receive the gross interest rate, r_{t+1}. If entrepreneur i becomes a capital producer at time t, she chooses only one of the J types of investment projects ($j = 1, 2, ..., J$). Project j transforms one unit of general good at time t into Φ_j units of capital at time $t+1$, but the investment size that entrepreneur i takes on is indivisible and fixed for entrepreneur i and is determined according to her net worth share among all the potential entrepreneurs; that is, the investment size for project j that entrepreneur i engages in is $M_j s_t(i)$ units of the general good, where $s_t(i) := a_t(i)/\int_{i\in\Omega} a_t(i)di$ is entrepreneur i's net worth share. Although our assumption that an entrepreneur's investment size is based on her net worth share differs from Matsuyama (2007), we might consider syndicated investment projects to motivate this assumption.[11]

[11] Under our assumptions, the return from the selected investment project is always higher than that on saving, and any entrepreneur cannot implement the project alone. Thus, all entrepreneurs have incentives to form a syndicate to undertake the investment project.

To join a syndicated investment group associated with project j, each entrepreneur should take up the responsibility of investing according to her net worth share. As in Matsuyama (2007), we assume that $\int_{i\in\Omega} a_t(i)di < M_j$, meaning that there are always savers and borrowers in the economy. This assumption holds if the production function is not too productive and M_j is very large.[12] Because entrepreneur i's investment size, $M_j s_t(i)$, is greater than her net worth, $a_t(i)$, she must borrow $M_j s_t(i) - a_t(i)$ of the general good at the interest rate r_{t+1} to obtain revenue $\Phi_j M_j s_t(i) f'(k_{t+1})$. M_j is the average investment size over capital producers because $\theta_t = \int_{i\in\Theta_t} s_t(i)di$ is the number of capital producers (and the probability that entrepreneur i becomes a capital producer), where Θ_t denotes the set of entrepreneurs at time t. As M_j becomes large, each capital producer must borrow more. If entrepreneur i becomes a capital producer, she acquires the following return from her investment project: $R^u_{t+1} := r_{t+1} + [\Phi_j M_j f'(k_{t+1}) - r_{t+1}M_j]/\int_{i\in\Omega} a_t(i)di$.

As in Matsuyama's (2007) model, to complete project j, both the *profitability* and *borrowing* constraints must be satisfied. The profitability constraint is given by $[\Phi_j M_j s_t(i) f'(k_{t+1}) - r_{t+1}(M_j s_t(i) - a_t(i))]/a_t(i) \geq r_{t+1}$, or equivalently,

$$\Phi_j f'(k_{t+1}) \geq r_{t+1}. \tag{1}$$

Due to agency problems in the financial market, borrowers face borrowing constraints. An entrepreneur's borrowing limit is associated with the pledgeability of her project revenue. Because only a proportion of the revenue, $\lambda_j \Phi_j M_j s_t(i) f'(k_{t+1})$, can be pledged, an entrepreneur i who engages in project j can borrow at most $\lambda_j \Phi_j M_j s_t(i) f'(k_{t+1})/r_{t+1}$; that is, the borrowing constraint is

$$\lambda_j \Phi_j M_j s_t(i) f'(k_{t+1}) \geq r_{t+1}(M_j s_t(i) - a_t(i)), \tag{2}$$

where $\lambda_j \in [0,1]$ can be considered the degree of borrowing constraints, with larger values indicating more relaxed borrowing constraints. If entrepreneur i faces a binding borrowing constraint, her return, R^u_{t+1}, on the investment project becomes $R^c_{t+1} := (1-\lambda_j)\Phi_j M_j f'(k_{t+1})/\int_{i\in\Omega} a_t(i)di$. If her borrowing constraint is not binding, her return remains R^u_{t+1}. However, it follows from $s_t(i) = a_t(i)/\int_{i\in\Omega} a_t(i)di$ that the borrowing constraint (2) is essentially identical across all capital producers. This implies that if entrepreneur i's borrowing constraint is not binding, all other capital producers' borrowing constraints are also not binding. In this case, the profitability constraint (1) holds with *equality*, as will be seen below. Therefore, the return for unconstrained capital producers is actually equal to r_{t+1}. Given the profitability constraint (1) and supposing that project

[12]See footnote 5 in Matsuyama (2007) for further information on this point.

j is selected, the individual-specific return, $R_{t+1}(i)$, is obtained as follows:

$$R_{t+1}(i) = \begin{cases} r_{t+1} & \text{saver, unconstrained producer} \\ R^c_{t+1} & \text{constrained producer.} \end{cases} \tag{3}$$

The individual-specific return cannot be insured because insurance markets are incomplete in this economy. When the borrowing constraint (2) is binding, the profitability constraint (1) holds with *inequality*. Therefore, it is straightforward to show that $R^c_{t+1} > r_{t+1}$.

2.2.2 Intrinsically Useless Asset

We assume that the total nominal supply of the intrinsically useless asset is constant, given by $\bar{B}$. Let p_t be the price of the intrinsically useless asset at time t. Its real value is $B_t := p_t\bar{B}$, as measured by the general good at time t. The intrinsically useless asset is freely disposable, and therefore, B_t is non-negative. An asset bubble is defined as the difference between an asset's fundamental and market values. Thus, a bubble on the intrinsically useless asset exists if B_t has a strictly positive value.

Holding an intrinsically useless asset is a saving method if the asset has a positive value. For the intrinsically useless asset to have a positive value, the returns from holding the asset must be greater than or equal to the interest rate, namely, $p_{t+1}/p_t \geq r_{t+1}$. Otherwise, no one will buy the asset. Suppose that $p_{t+1}/p_t > r_{t+1}$. Under this condition, no one lends to capital producers. However, this inequality does not hold because $\lim_{k_{t+1}\to 0} f'(k_{t+1}) = \infty$ in such a case. Therefore, in equilibrium, it holds that $p_{t+1}/p_t = r_{t+1}$, and the law of motion of the real value of the intrinsically useless asset is

$$B_{t+1} = r_{t+1}B_t. \tag{4}$$

If entrepreneur i faces the individual-specific return $R_{t+1}(i) = r_{t+1}$, she is willing to purchase the intrinsically useless asset, given that the asset has a positive value. As we will see below in the phase diagram analysis, there are savers and constrained capital producers in the neighborhood of the bubbly steady state, where $R^c_{t+1} > r_{t+1}$. In this case, only savers are willing to purchase the intrinsically useless asset, and constrained capital producers do not hold the asset because the return from the investment project is greater than the return on the asset. Savers practice arbitrage between purchasing the intrinsically useless asset and lending in the financial market.

2.2.3 Utility Maximization

Let $c_s(i)$ denote entrepreneur i's consumption at time s. Entrepreneur $i \in \Omega$, with a discount factor $\beta \in (0,1)$, maximizes her expected lifetime utility

$$\max\ E_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\ln c_s(i)\right],$$

subject to

$$c_s(i) + a_s(i) = R_s(i)a_{s-1}(i) =: I_s(i), \tag{5}$$

for $s \geq t \geq 0$, where $R_s(i)$ is an individual-specific return on net worth given by Eq. (3) and $I_s(i)$ is the income at time s. I_0 is given. $E_t[\cdot]$ is an expectation operator given the information available until time t. Entrepreneur i faces uncertainty regarding her future income due to credit rationing. Eq. (5) is the flow budget constraint. The current utility maximization problem is in reduced form, in which $R_s(i)$ already takes into account the borrowing constraint (2), as presented in Eq. (3).[13]

From the utility maximization problem, entrepreneur i obtains the following Euler equation:

$$\frac{1}{c_t(i)} = \beta E_t\left(\frac{R_{t+1}(i)}{c_{t+1}(i)}\right). \tag{6}$$

The transversality condition is given by

$$\lim_{\tau\to\infty}\beta^{\tau}E_t\left[\frac{a_{t+\tau}(i)}{c_{t+\tau}(i)}\right] = 0.$$

Because the current infinite-horizon maximization problem satisfies the conditions assumed in Kamihigashi (2002), the transversality condition, along with the Euler equation, is necessary and sufficient for optimality.[14]

From Eq. (5), it follows that $E_t[a_{t+1}(i)/c_{t+1}(i)]{=}a_t(i)E_t[R_{t+1}(i)/c_{t+1}(i)] - 1$. The use of Eq.(6) yields $a_t(i)/c_t(i){=}\beta E_t[a_{t+1}(i)/c_{t+1}(i)] + \beta$. This equation and the law of iterated expectations yield $a_t(i)/c_t(i){=}\beta^{\tau}E_t[a_{t+\tau}(i)/c_{t+\tau}(i)]{+}\beta{+}\beta^2{+}...{+}\beta^{\tau}$. Applying the transversality condition, $\lim_{\tau\to\infty}\beta^{\tau}E_t[a_{t+\tau}(i)/c_{t+\tau}(i)] = 0$, we have $a_t(i)/c_t(i) = \beta/(1-\beta)$ for all $t \geq 0$. Substituting this equation into Eq.(5), we obtain the law of motion of net worth as follows:[15]

$$a_t(i) = \beta R_t(i)a_{t-1}(i) = \beta I_t(i). \tag{7}$$

Although entrepreneur i faces uncertainty with respect to her individual-specific return, we obtain a simple law of motion of entrepreneur i's net worth in Eq. (7). This is because

[13] We could consider the Kuhn-Tucker condition associated with the borrowing constraint (2), which yields the same outcome as in the main text, but we take a shortcut because the individual-specific return already incorporates the borrowing constraint.

[14] See also Ekeland and Scheinkman (1986) and Kamihigashi (2000).

[15] We can also apply the envelope condition to derive Eq. (7).

the utility function is log-linear. Provided that entrepreneur i follows Eq. (7), her lifetime budget constraint is binding.

2.3 Workers

The workers in our model are endowed with one unit of labor in each time period and inelastically supply their labor to the production sector to earn wage income. As discussed at the beginning of this section, the workers are hand-to-mouth consumers; that is, they consume their current labor income entirely. Thus, their consumption program can be given as

$$\tilde{c}_t = w_t$$

for all $t \geq 0$, where $\tilde{c}_t$ stands for a worker's consumption.[16]

2.4 Equilibrium

The equilibrium of our economy is characterized by the agents' optimization conditions and the market clearing conditions for the bubbly asset, capital, and financial loans. We first derive an equation that is necessary for technology choice and the determination of the equilibrium interest rate. From the profitability constraint (1) and the borrowing constraint (2), we obtain

$$r_{t+1}/f'(k_{t+1}) \leq \frac{\Phi_j}{\max\{1, [1 - a_t(i)/(M_j s_t(i))]/\lambda_j\}}. \tag{8}$$

Eq. (7) rewrites entrepreneur i's net worth share as $s_t(i) = a_t(i)/(\beta \int_{i\in\Omega} I_t(i)di)$, and thus, Eq. (8) can be rewritten as

$$r_{t+1}/f'(k_{t+1}) \leq \frac{\Phi_j}{\max\{1, [1 - \beta I_t/M_j]/\lambda_j\}}, \tag{9}$$

where $I_t := \int_{i\in\Omega} I_t(i)di$.

If project j existed such that $r_{t+1}/f'(k_{t+1}) < \Phi_j/\max\{1, [1 - \beta I_t/M_j]/\lambda_j\}$, every potential entrepreneur who is selected to be a borrower would borrow and invest in project

[16]If the workers' subjective discount factor is too small and they cannot borrow in the financial market, they behave in a hand-to-mouth manner. Suppose that a worker at time t maximizes her lifetime utility given by

$$\max \sum_{s=t}^{\infty} \tilde{\beta}^{s-t} \tilde{c}_s,$$

subject to

$$\tilde{c}_s + \tilde{a}_s = r_s \tilde{a}_{s-1} + w_s, \quad \tilde{a}_s \geq 0$$

for $s \geq t \geq 0$, where $\tilde{c}_s$ and $\tilde{a}_s$ are her consumption and net worth at time s, respectively, w_s is the wage income at time s and $\tilde{\beta}$ is the subjective discount factor. Assuming that $\tilde{\beta}$ is a sufficiently small value that $\tilde{\beta} < 1/r_t$ for all $t \geq 0$ in equilibrium, the worker consumes all her labor income in each time period; that is, $\tilde{c}_t = w_t$ for all $t \geq 0$.

j. In this situation, there would be excess demand for credit and the interest rate r_{t+1} would rise. Thus, for the financial market to clear, it follows that

$$r_{t+1}/f'(k_{t+1}) = \max_j \left\{ \frac{\Phi_j}{\max\{1, [1-\beta I_t/M_j]/\lambda_j\}} \right\}. \tag{10}$$

Next, we consider the financial market clearing condition, which is given by

$$\int_{i\in\Omega\backslash\Theta_t} a_t(i)di = \int_{i\in\Theta_t} [M_j s_t(i) - a_t(i)]di + B_t, \tag{11}$$

where project j is supposed to be selected at time t based on the technology choice achieved on the right-hand side of Eq. (10). Again, Θ_t denotes the set of capital producers at time t. Note that the left-hand side is the total savings, and the first term on the right-hand side is the total loans to capital producers. Entrepreneur i, adopting technology j, produces $\Phi_j M_j s_t(i)$ units of capital at time $t+1$, and thus, the total supply of capital is

$$k_{t+1} = \int_{i\in\Theta_t} \Phi_j M_j s_t(i)di. \tag{12}$$

From Eqs. (4), (11), and (12), we compute the aggregate income of the potential entrepreneurs as follows:

$$\begin{aligned} I_t &= \int_{i\in\Omega} R_t(i)a_{t-1}(i)di \\ &= \int_{i\in\Theta_{t-1}} \Big[\Phi_j M_j s_{t-1}(i)f'(k_t) - r_t(M_j s_{t-1}(i) - a_{t-1}(i))\Big]di \\ &\quad + \int_{i\in\Omega\backslash\Theta_{t-1}} r_t a_{t-1}(i)di \\ &= k_t f'(k_t) + r_t B_{t-1}. \end{aligned} \tag{13}$$

From Eq. (7), the aggregate net worth of the economy is $\beta I_t = \beta(k_t f'(k_t) + r_t B_{t-1})$. Combining Eqs. (4), (11), (12), and (13), we obtain the equilibrium law of motion of capital:

$$k_{t+1} = \Phi_j \beta k_t f'(k_t) - \Phi_j(1-\beta)B_t. \tag{14}$$

Given $k_0 > 0$, the equilibrium dynamics of the economy is described by Eqs.(4), (10), (13), and (14) such that $k_t > 0$ and $B_t \geq 0$ for all $t \geq 0$.

3 Bubbly Steady States and Crowd-in Effects

To make the exposition as simple as possible, we impose a technical assumption.

Assumption 2 *For project j, the following equation with respect to k has a unique solu-*

tion:

$$\lambda_j M_j \Phi_j f'(k) = -\frac{\beta}{1-\beta}\left[kf'(k) - \frac{k}{\Phi_j}\right] + M_j. \tag{15}$$

Eq.(15) solves for the capital in a bubbly steady state associated with project j, as will be clarified in the proof of Proposition 1 given in the appendix. The configurations of the right-hand and left-hand sides of Eq.(15) demonstrate that it has at a minimum one solution under Assumption 1 and that the case of multiple solutions is very rare. In particular, a large value of M_j would guarantee Assumption 2. If Eq.(15) has multiple solutions, the dynamic behavior of the economy is highly complicated. We eliminate this unnecessary situation for our purposes.

3.1 Single Technology

In this section, we investigate a single-technology case as our benchmark. Suppose that entrepreneurs access only one project, $J = 1$. In this case, the $\Delta k_{t+1} := k_{t+1} - k_t = 0$ locus is

$$B_t = \frac{1}{1-\beta}\left(\beta k_t f'(k_t) - \frac{k_t}{\Phi_1}\right).$$

The $\Delta k_{t+1} = 0$ locus has an inverted-U shape in the first quadrant of the $k_t B_t$ plane, starting from the origin and increasing before beginning to decline at the maximum. As shown in Figure 2, the $\Delta k_{t+1} = 0$ locus crosses the horizontal axis at point $A_1({f'}^{-1}(1/(\beta\Phi_1)), 0)$.[17]

From Eqs. (10) and (13), we obtain the interest rate as follows:

$$r_{t+1} = \begin{cases} \frac{\lambda_1 M_1 \Phi_1 f'(k_{t+1})}{M_1 - \beta(k_t f'(k_t) + B_t)} & \text{if } \beta(k_t f'(k_t) + B_t) \leq M_1(1-\lambda_1) \\ \Phi_1 f'(k_{t+1}) & \text{if } M_1(1-\lambda_1) < \beta(k_t f'(k_t) + B_t), \end{cases}$$

where we have used $B_t = r_t B_{t-1}$. From Eq. (4), the $\Delta B_{t+1} := B_{t+1} - B_t = 0$ locus is $B_t = 0$ or $r_{t+1} = 1$. Note that the $r_{t+1} = 1$ locus is divided into two sections,

$$\Phi_1 f'(k_{t+1}) = 1, \tag{R1}$$

when $M_1(1-\lambda_1) < \beta(k_t f'(k_t) + B_t)$, and

$$\lambda_1 M_1 \Phi_1 f'(k_{t+1}) = M_1 - \beta(k_t f'(k_t) + B_t), \tag{R2}$$

when $\beta(k_t f'(k_t) + B_t) \leq M_1(1-\lambda_1)$.

Proposition 1 *Suppose that Assumptions 1 and 2 hold and that the entrepreneurs can access only one project, $J = 1$. Then, two non-trivial steady-state equilibria exist in the*

[17]When depicting Figure 2, from $f''(k_t) < 0$, the Inada conditions, and Assumption 1, it follows that $k_t f'(k_t)$ is strictly concave, $\lim_{k_t \to 0} k_t f'(k_t) = 0$, $\lim_{k_t \to 0}[k_t f'(k_t)]' = \infty$, and $\lim_{k_t \to \infty}[k_t f'(k_t)]' = 0$.

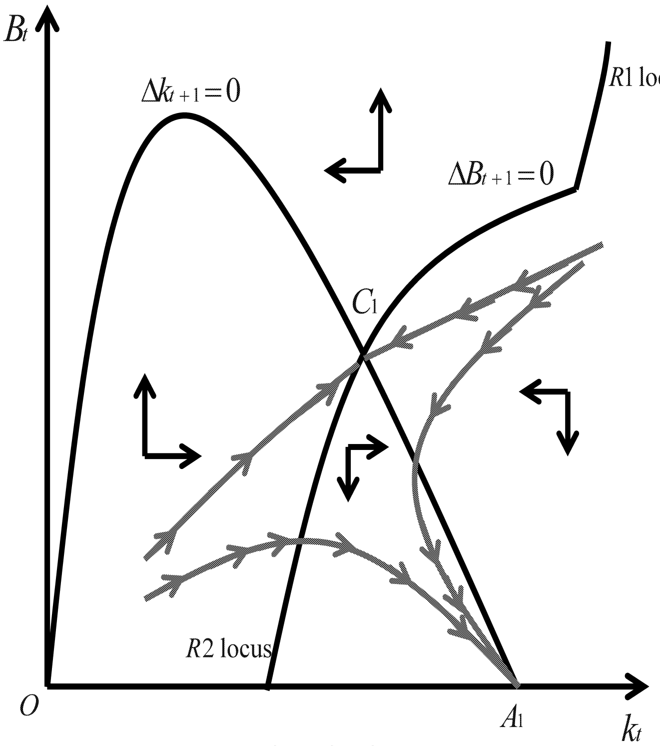

Figure 2: Single technology

economy, one a bubbleless steady state and the other a bubbly steady state, if and only if

$$M_1\left(1-\frac{\lambda_1}{\beta}\right) > f'^{-1}\left(\frac{1}{\beta\Phi_1}\right)\frac{1}{\Phi_1}. \qquad \text{(Condition 1)}$$

Capital accumulation is lower in the bubbly steady state than in the bubbleless steady state, implying that relative to the bubbleless steady state, asset bubbles crowd out private investments in the bubbly steady state.

Proof. See the appendix.

Figure 2 shows the phase diagram embodying Proposition 1. In Figure 2, we focus on the case in which the slope of the R2 locus is positive without loss of generality.[18] This

[18]Under Assumptions 1 and 2 and Condition 1, the bubbly steady state is a saddle point regardless of the slope of the R2 locus. Nevertheless, to draw the R2 locus in Figure 2, we discuss a condition under which the slope of the R2 locus becomes positive for $(B_t, k_t) \in \mathbf{R}^2_+$, where $\beta(k_t f'(k_t) + B_t) \leq M_1(1-\lambda_1)$, in the following. The total differentiation of R2 yields the slope of the R2 locus as follows:

$$\frac{dB_t}{dk_t} = \frac{\beta[k_t f'(k_t)]'[-\lambda_1 M_1(\Phi_1)^2 f''(k_{t+1}) - 1]}{\beta - (1-\beta)\lambda_1 M_1(\Phi_1)^2 f''(k_{t+1})}.$$

The sign of dB_t/dk_t cannot be determined because the sign of $-\lambda_1 M_1(\Phi_1)^2 f''(k_{t+1}) - 1$ is inconclusive. For instance, the R2 locus would be inverted-U shaped if a Cobb-Douglas production function were used. When drawing phase diagrams, we assume that $\lambda_i M_i(\Phi_i)^2 f''(k_{t+1}) < -1$ for $(B_t, k_t) \in \mathbf{R}^2_+$, where $\beta(k_t f'(k_t) + B_t) \leq M_i(1-\lambda_i)$, such that the slope of the R2 locus (and the corresponding locus for the other project) can be positive.

phase diagram is very similar to that depicted in Tirole (1985), although the mechanism through which the bubbly steady state appears in our model is totally different from that of Tirole (1985). In Tirole's overlapping-generations model, the bubbly steady state appears when capital is over-accumulated in the bubbleless steady state relative to the golden rule. Because of capital over-accumulation, the interest rate becomes less than the economic growth rate. This is a necessary condition for asset bubbles to appear in equilibrium. In contrast, capital over-accumulation in the bubbleless steady state never occurs in our model because the R1 locus does not intersect with the $\Delta k_{t+1} = 0$ locus in the first quadrant of the $k_t B_t$ plane. Instead, the R2 locus intersects with the $\Delta k_{t+1} = 0$ locus in the first quadrant, where the borrowing constraint is binding. A bubbly steady state exists in our model because the borrowers face borrowing constraints. Borrowing constraints prevent credit-rationed entrepreneurs from borrowing as much as they want, and as a result, the aggregate demand for borrowing decreases. When there are no asset bubbles in the economy, the decreased demand for borrowing forces the interest rate downward in equilibrium, and the interest rate becomes less than the economic growth rate. This mechanism is reflected in Condition 1. As λ_1 decreases and the borrowing constraint becomes more severe, a bubbly steady state is more likely to appear. Additionally, as M_1 becomes large, the bubbly steady state would arise more easily because the borrowing constraint is more likely to be binding. For a bubbly steady state to arise, β should be greater than λ_1, meaning that for asset bubbles to appear, capital must accumulate to the full extent according to the degree of the borrowing constraint. Note that in a single-technology case, as in Tirole's model, asset bubbles in the bubbly steady state have only a *crowd-out* effect on private investments relative to the bubbleless steady state, as shown in Figure 2.

Finally, we explain the reason that the transversality condition does not rule out bubbles. Consider a path containing a bubble component. On the path, as (4) shows, the bubble grows at the rate r_{t+1}, which is the rate of return to savers. However, in this model, each potential entrepreneur randomly change between being a saver and a capital producer, and the capital producer faces a higher return than r_{t+1} because of credit rationing, meaning that the effective rate at which entrepreneurs discount the future is always not less than r_{t+1} and strictly higher than r_{t+1} when acting as a capital producer. Thus, the transversality condition is satisfied.

3.2 Multiple Technologies

We now investigate a case similar to Figure 2A in Matsuyama (2007); that is, we assume that there are only two technologies, $J = 2$. We also assume that there are trade-offs between productivity and pledgeability in these two technologies, and we impose the following parameter conditions.

Assumption 3 $\Phi_2 > \Phi_1 > \lambda_1\Phi_1 > \lambda_2\Phi_2$, $M_2 > M_1$.

As noted by Matsuyama (2007), some advanced projects associated with leading-edge technologies can be constrained by greater agency problems compared to other, less-advanced projects associated with well-established technologies. The trade-offs between productivity and pledgeability assumed in Assumption 3 are important in such a situation.

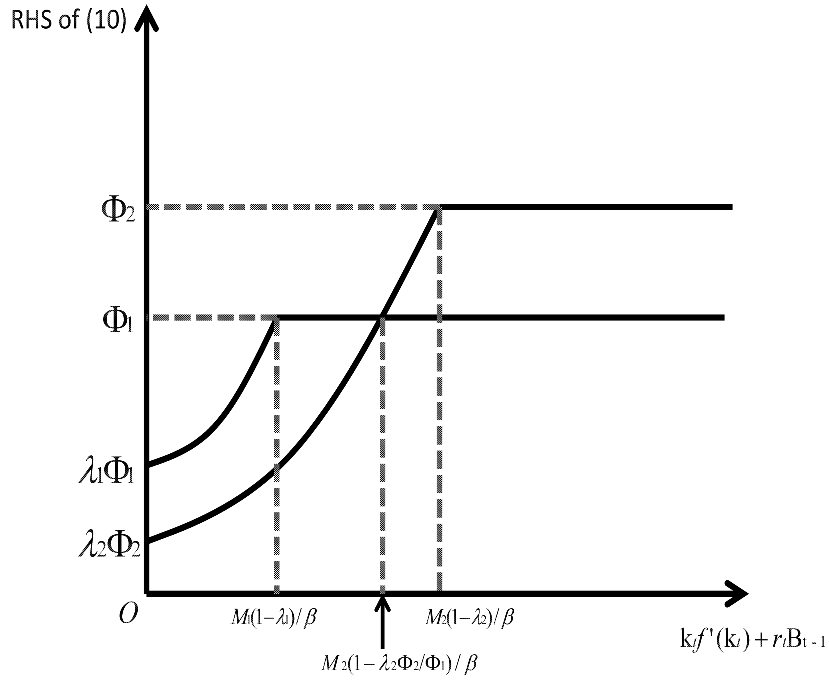

Figure 3: Technology choice I

As shown in Figure 3, one of the two technologies is selected under Assumption 3, and this illustrates the right-hand side of Eq. (10). Figure 3 and Eq. (14) yield the law of motion of capital as follows:

$$k_{t+1} = \begin{cases} \Phi_1\beta k_t f'(k_t) - \Phi_1(1-\beta)B_t & \text{if } \beta(k_t f'(k_t) + B_t) \leq M_2(1-\lambda_2\Phi_2/\Phi_1) \\ \Phi_2\beta k_t f'(k_t) - \Phi_2(1-\beta)B_t & \text{if } M_2(1-\lambda_2\Phi_2/\Phi_1) < \beta(k_t f'(k_t) + B_t). \end{cases} \tag{16}$$

The interest rate is

$$r_{t+1} = \begin{cases} \frac{\lambda_1 M_1 \Phi_1 f'(k_{t+1})}{M_1 - \beta(k_t f'(k_t) + B_t)} & \text{if } \beta(k_t f'(k_t) + B_t) \leq M_1(1-\lambda_1) \\ \Phi_1 f'(k_{t+1}) & \text{if } M_1(1-\lambda_1) < \beta(k_t f'(k_t) + B_t) \leq M_2(1-\lambda_2\Phi_2/\Phi_1) \\ \frac{\lambda_2 M_2 \Phi_2 f'(k_{t+1})}{M_2 - \beta(k_t f'(k_t) + B_t)} & \text{if } M_2(1-\lambda_2\Phi_2/\Phi_1) < \beta(k_t f'(k_t) + B_t) \leq M_2(1-\lambda_2) \\ \Phi_2 f'(k_{t+1}) & \text{if } M_2(1-\lambda_2) < \beta(k_t f'(k_t) + B_t). \end{cases} \tag{17}$$

There are various equilibrium cases with Eqs. (16) and (17). In particular, depending on the scale of investment and pledgeability of output, there can exist a case in which asset bubbles are supportive of the high-productivity, large-scale technology in equilibrium.

The intuition behind the technology choice and the sustainability of asset bubbles is as follows. As the size of asset bubbles grows, both crowding-out and crowding-in effects appear simultaneously. On the one hand, a larger bubble competes with investment and impedes capital accumulation. On the other hand, a larger bubble increases the wealth of entrepreneurs and makes it more appealing for them to invest in the high productivity, large-scale technology. Whether the latter effect gives rise to a new steady state with the high-productivity, large-scale technology depends on the scale of investment and pledgeability associated with the technology: if the investment scale is too high or the pledgeability is too low, the asset bubbles that are required to generate a switch in the technology choice are not sustainable in equilibrium. However, if the investment scale is not too high and the pledgeability is not too low, there appears a bubbly steady state associated with the high-productivity, large-scale technology in equilibrium.

Although there are various patterns of dynamic behavior of the economy with Eqs. (16) and (17), we focus on an interesting case with two bubbly steady states and one bubbleless steady state. This case presents a situation consistent with historical episodes featuring the formation and collapse of asset bubbles in which asset bubbles promote capital accumulation, and economic depressions follow the collapse of asset bubbles. In what follows, we derive the conditions to obtain this case. For later reference, we derive the locus, giving a cutoff of the technology choice in Eq.(17), as follows:

$$M_2\left(1-\frac{\lambda_2\Phi_2}{\Phi_1}\right)=\beta(k_tf'(k_t)+B_t). \tag{T1}$$

For a better understanding, refer to Figures 3 and 4 when considering Lemmata 1 to 3 below.

Lemma 1 (The boundary of the $\Delta k_{t+1}=0$ locus of project $j=2$) *Let us define $\tilde{k}$ such that $f'(\tilde{k})+\tilde{k}f''(\tilde{k})=1/\Phi_2$. Additionally, suppose that Assumptions 1 to 3 hold. Then, the $\Delta k_{t+1}=0$ locus of project $j=2$ has two intersections with the T1 locus, point D_2, and point E_2 (the capital value is greater at point D_2 than at point E_2), in the first quadrant of the k_tB_t plane, if and only if*

$$f'^{-1}\left(\frac{1}{\beta\Phi_2}\right)\frac{1}{\Phi_2}<M_2\left(1-\frac{\lambda_2\Phi_2}{\Phi_1}\right)<\frac{1}{1-\beta}\left(f'(\tilde{k})-\frac{1}{\Phi_2}\right)\beta\tilde{k}. \tag{Condition 2}$$

Proof. See the appendix.

In Condition 2, as M_2 becomes large or λ_2 becomes small, the T1 locus shifts upward as the borrowing constraint associated with project $j=2$ becomes severe. When the borrowing constraint becomes very severe, such that the second inequality in Condition 2 fails to hold, the T1 locus no longer intersects with the $\Delta k_{t+1}=0$ locus of project $j=2$. Conversely, as M_2 becomes small or λ_2 becomes large, the T1 locus shifts downward as the borrowing constraint associated with project $j = 2$ becomes relaxed. If the borrowing constraint is relaxed to the extent that the first inequality in Condition 2 fails to hold, a bubbleless steady state with project $j = 2$ appears; however, we do not include this case in our present study. Note that as β approaches one, Condition 2 is more likely to be satisfied.

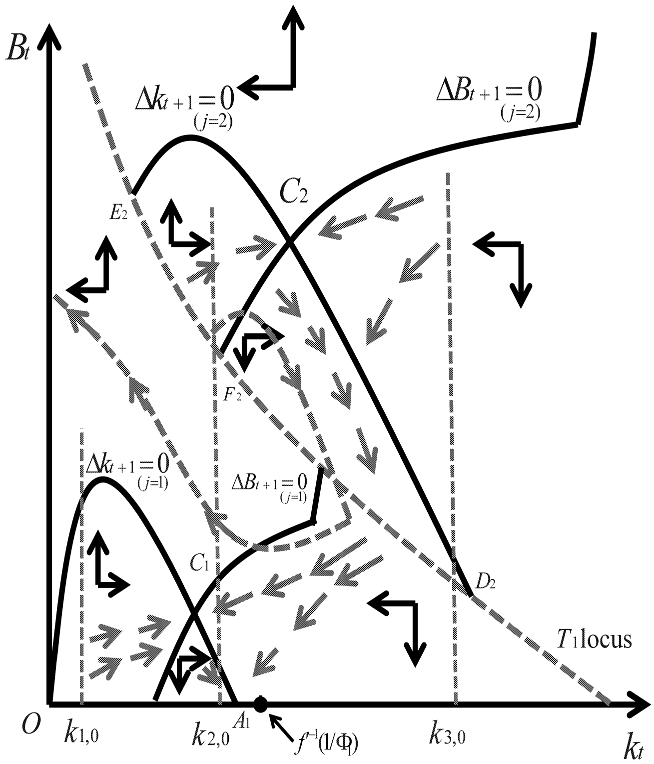

Figure 4: M_1 and M_2 are large.

Lemma 2 (The bubbly steady state with project $j = 2$**)** *Suppose that Assumptions 1 to 3 and Condition 2 hold. Let the capital value at point* D_2 *be* k^d*. If*

$$M_2\left(1 - \frac{\lambda_2\Phi_2}{\Phi_1}\right) < \frac{\beta}{1-\beta} f'^{-1}\left(\frac{1}{\Phi_1}\right)\left(\frac{1}{\Phi_1} - \frac{1}{\Phi_2}\right), \qquad \text{(Condition 3)}$$

then there exists a bubbly steady state with project $j = 2$*, (point* C_2*), the capital value of which is in* $(f'^{-1}(1/\Phi_1), k^d)$*.*[19]

Proof. See the appendix.

Condition 3 is a sufficient condition for the capital stock to be greater in the bubbly steady state with project $j = 2$ than in the bubbleless steady state with project $j = 1$. As Condition 3 shows, the borrowing constraint associated with project $j = 2$ may not be too severe for the capital stock in the bubbly steady state with project $j = 2$ to be

[19]Note that $f'^{-1}(1/\Phi_1)$ is the capital value at the intersection of the $\Delta k_{t+1} = 0$ and R1 loci. See the proof of Proposition 1 in the appendix.

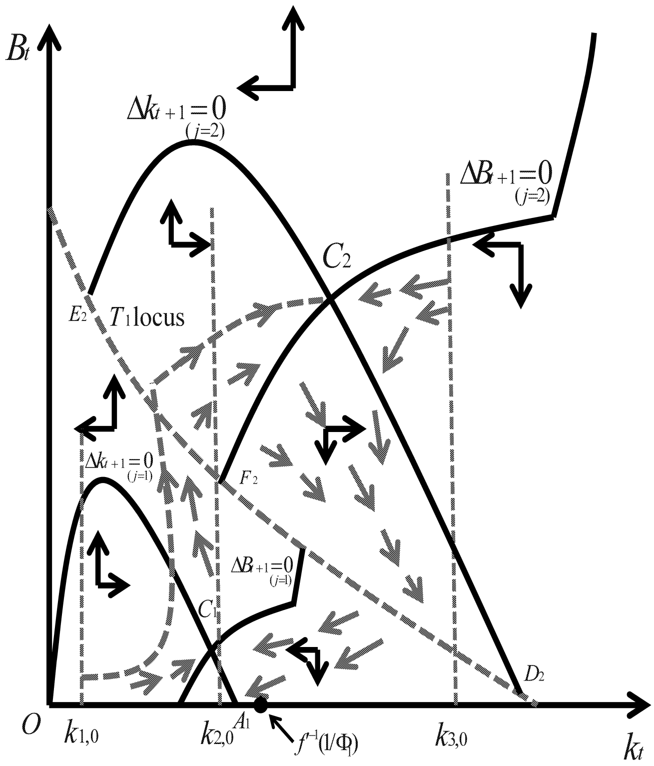

Figure 5: M_1 and M_2 are small.

greater than $f'^{-1}(1/\Phi_1)$. As in Condition 2, Condition 3 is more likely to be satisfied as β approaches one.

Let the intersection point of the $r_{t+1} = 1$ locus of project $j = 2$ with the T1 locus be F_2. We define the locus associated with the borrowing-constrained part of the $\Delta B_{t+1} = 0$ locus of project $j = 2$ as

$$\lambda_2 M_2 \Phi_2 f'(k_{t+1}) = M_2 - \beta(k_t f'(k_t) + B_t) \tag{R3}$$

Then, F_2 is the intersection of the R3 and T1 loci.

Lemma 3 (The boundary of the $r_{t+1} = 1$ locus of project $j = 2$) *Suppose that Assumptions 1 to 3 and conditions 2 and 3 hold. Let the capital value at point F_2 be k^f. Then, it follows that $k^f < f'^{-1}(1/\Phi_1)$.*

Proof. See the appendix.

Suppose that Conditions 1 to 3 hold. The phase diagrams of two typical cases can then be depicted based on Lemmata 1 to 3. In the first case, illustrated in Figure 4, M_1 and M_2 are relatively large; in the second case, illustrated in Figure 5, M_1 and M_2 are relatively small.

Proposition 2 *Suppose that Assumptions 1 to 3 and Conditions 1 to 3 hold. Then, one bubbleless steady state and two bubbly steady states occur. The bubbleless steady state is associated with project $j = 1$, and the two bubbly steady states are associated with project*

$j = 1$ and project $j = 2$. Capital accumulation is larger in the bubbly steady state with project $j = 2$ than in the bubbleless steady state, whereas capital accumulation is smaller in the bubbly steady state with project $j = 1$ than in the bubbleless steady state. This implies that, relative to the bubbleless steady state, asset bubbles crowd in private investments in the bubbly steady state with project $j = 2$ and crowd out private investments in the bubbly steady state with project $j = 1$.

Proof. The claim follows from Lemmata 1 to 3 and Figures 3 and 4. □

Because the value of the intrinsically useless asset is not pre-determined, the equilibrium is indeterminate when asset bubbles appear. For any given initial capital k_0, there exists a continuum of initial values of the intrinsically useless asset, each one consistent with a competitive equilibrium. Specifically, in both Figures 3 and 4, when the initial capital is at an intermediate level such as $k_{2,0}$ in the figures, there are many equilibrium paths depending on B_0. In particular, one equilibrium path converges to the higher bubbly steady state of point C_2, and another converges to the lower bubbly steady state of point C_1. Many other equilibrium paths converge to the bubbleless steady state of point A_1. When the initial capital is at a higher level, such as $k_{3,0}$, an equilibrium path converges to the higher bubbly steady state of point C_2, and many other equilibrium paths converge to the bubbleless steady state of point A_1.

Because there is a continuum of equilibria converging to the steady states with low-productivity technology (project $j = 1$), one may wonder whether government policy can prevent these equilibria and lead the economy to a unique equilibrium converging to the higher bubbly steady state of point C_2. Although a detailed discussion of institutional backing of the intrinsically useless asset is beyond the scope of this paper, an appropriate backing of the asset by the government, such as reserve requirements for the asset (e.g., Tirole, 1985) or the government's direct purchase of the asset (e.g., Kunieda and Shibata, 2016), will enable the economy to attain a unique equilibrium converging to the higher bubbly steady state of point C_2.

When the initial capital is very small, such as $k_{1,0}$, the outcomes in Figures 3 and 4 are different. As M_1 and M_2 increase (decrease), the R2, R3, and T1 loci shift upward (downward), with the other loci remaining constant. Therefore, when M_1 and M_2 are large (Figure 4) and the initial capital is very small, such as $k_{1,0}$, no equilibrium path converges to the higher bubbly steady state of point C_2, but almost all equilibrium paths converge to the bubbleless steady state of point A_1, and one equilibrium path converges to the lower bubbly steady state of point C_1. This indicates that when M_1 and M_2 are large and the initial capital is very small, the economy is trapped in underdevelopment. In contrast, when M_1 and M_2 are small, there exists an equilibrium path converging to the higher bubbly steady state of point C_2, as Figure 5 shows. Even if the initial capital is very small, a certain level of initial asset bubbles leads the economy to higher capital accumulation when M_1 and M_2 are small.

Our phase diagram analysis shows that under certain parameter conditions satisfying Assumptions 1 to 3 and Conditions 1 to 3, there must exist competitive endogenous cycles revolving back and forth between the two technological regimes. However, a detailed analysis of such endogenous cycles is beyond the scope of this paper. We also note that, as β approaches one, the right-hand sides of Conditions 2 and 3 go to infinity, with the other parameters remaining constant. This implies that for a sufficiently large β, there exist M_1 and M_2 such that Assumption 3 and Conditions 1 to 3 hold.

4 Financial Crisis with Asset Bubble Bursting

Like Farhi and Tirole (2012), we could consider two types of asset bubble bursts. The first is caused by extrinsic uncertainty. The price of the intrinsically useless asset is not predetermined, and thus, asset bubbles collapse when sunspot variables negatively affect the agents' expectations of their presence. In other words, the agents' expectations of asset bubble bursts are self-fulfilling. The second type is caused by changes in fundamental variables such as productivity, investment conditions, and the tastes of agents. As noted by Brunnermeier and Oehmke (2012), an initial boom in asset prices is often supported by a certain form of innovation, such as technological change or financial innovation. Our analyses with multiple technologies essentially demonstrate that a new technology associated with a new investment condition can create a new bubbly steady state. We therefore focus on asset bubble bursts caused by changes in fundamental variables and investigate both how the bubbles collapse and how depressions follow the burst when the investment conditions for a project change.

Suppose that, as shown in Figure 6, the investment conditions for project $j = 2$ unexpectedly become severe at time $t = \tau$, making M_2 enlarged and/or λ_2 reduced from time τ onward, such that the second inequality in Condition 2 no longer holds. Under these conditions, the borrowing constraint associated with project $j = 2$ becomes more severe, and project $j = 1$ is more likely to be selected. In this case, from time $t = \tau$ onward, both the T1 and R3 loci shift upward, but the $\Delta k_{t+1} = 0$ locus with project $j = 2$ remains unchanged. Because the second inequality of Condition 2 would no longer hold, the T1 locus does not intersect with the $\Delta k_{t+1} = 0$ locus under project $j = 2$, and we do not have the bubbly steady state with project $j = 2$ from time $t = \tau$ onward, as illustrated in Figure 7.

Suppose further that the economy initially starts at point P in Figure 7, which is on the saddle path to the bubbly steady state with project $j = 2$ (point C_2) before a sudden change in the investment condition for project $j = 2$. The economy proceeds toward point C_2 until $t = \tau - 1$. On the transitional path, both the asset bubbles and capital continue to increase synchronously. However, the asset bubbles suddenly burst when the investment condition for project $j = 2$ unexpectedly becomes severe at time $t = \tau$.

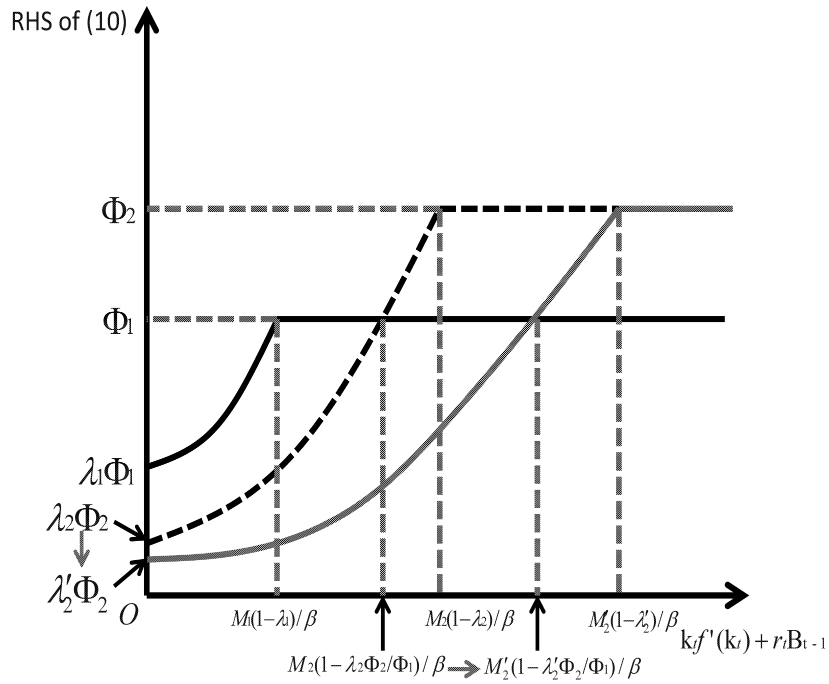

Figure 6: Technology choice II

When asset bubbles burst, three possibilities arise for the economy depending on the entrepreneurs' expectations. In any case, soon after the bubbles burst, capital overshoots in accordance with the dynamical system associated with project $j = 2$. This is because the high productivity technology used was selected one period before the bursting of the asset bubbles, and the *crowd-out* effect of asset bubbles is reduced at the moment when the bubbles burst.

In the first case, the asset bubbles partially crash, and capital overshoots to point P_1 in Figure 7. In this case, the economy converges to a bubbly steady state with project $j = 1$ along the saddle path to the steady state. The second case is also that of a partial crash; here, capital overshoots to point P_2, but the economy converges to a bubbleless steady state with project $j = 1$. The third case is a perfect crash. In this case, the asset bubbles perfectly collapse, and capital overshoots to point P_3, converging to a bubbleless steady state with project $j = 1$. In view of the *crowd-out* effect of asset bubbles, the first case is the worst-case scenario for capital accumulation.

5 Concluding Remarks

The *technology choice* analyzed by Matsuyama (2007) is a key factor for modeling the *crowd-in* effect of asset bubbles in our infinitely lived agent model. In contrast to extant models such as those of Kiyotaki and Moore (2012), Aoki et al. (2015), Hirano et al.

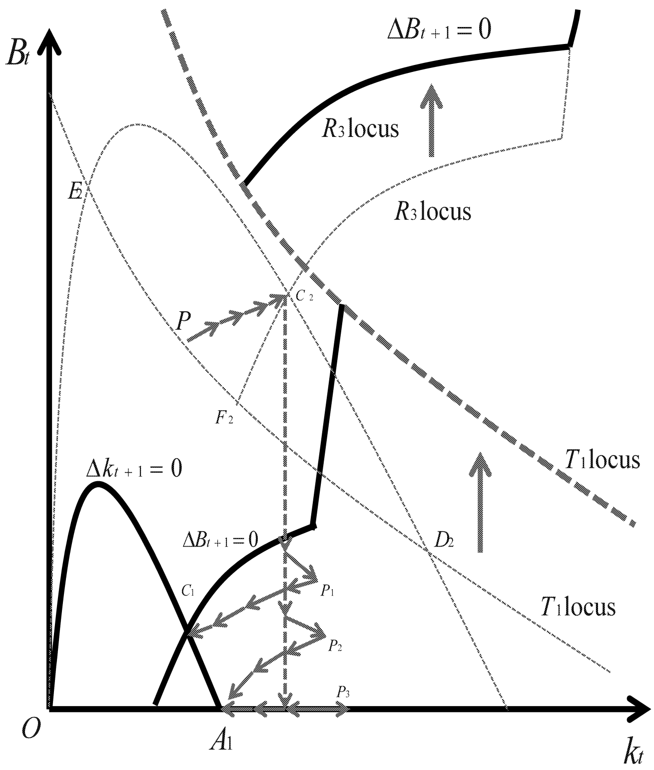

Figure 7: Financial crisis

(2015), Hirano and Yanagawa (2016), and Kunieda and Shibata (2016), the potential entrepreneurs in our model do not receive uninsured idiosyncratic productivity shocks; instead, they face uncertainty with respect to acquiring credit in the financial market. Credit is randomly assigned to the potential entrepreneurs who need it; that is, they are credit-rationed. Although these circumstances of the potential entrepreneurs are the same as in Matsuyama's overlapping-generations model, the extension of Matsuyama's model to our infinitely lived agent model is important for understanding the *technology choice* induced by the presence of asset bubbles.

Unlike two-period overlapping-generations models such as Matsuoka and Shibata (2012), our model allows each entrepreneur to use the returns from holding the intrinsically useless asset not only for consumption but also for accumulating net worth, as in Kiyotaki and Moore (2012), Aoki et al. (2015), Hirano et al. (2015), Hirano and Yanagawa (2016), and Kunieda and Shibata (2016). The wealth effect of asset bubbles relaxes borrowing constraints and enables the entrepreneurs to more easily select a high productivity project under more severe borrowing conditions. The relaxed borrowing constraints increase the demand for borrowing in the financial market, which raises the market interest rate. As a result, the profitability constraint associated with the low productivity project no longer holds, and the high productivity project is selected. Entrepreneur net worth includes the returns from both the investment project and holding the intrinsically useless asset, and thus, the T1 locus slopes downward, creating a situation in which capital shrinks after a financial crisis in our model.

Two-period overlapping-generations models are sometimes criticized for the two-period lifetimes of the agents. Retaining the important features of Matsuyama's (2007) model, we have extended his model to a dynamic general equilibrium model with infinitely lived agents. Asset bubbles are empirical observations that need to be explained using quantitative methods. Our extension enables us to calibrate Matsuyama's model using actual data, but the calibration exercise is beyond the scope of this paper. We leave this to be addressed in future research.

Appendix

Proof of Proposition 1

See also Figure 2, which helps to explain this proof. The non-trivial intersection of the $B_t = 0$ and $\Delta k_{t+1} = 0$ loci occurs at point $A_1(f'^{-1}(1/(\beta\Phi_1)), 0)$, the bubbleless steady state. The value of capital at the intersection of the $\Delta k_{t+1} = 0$ and R1 loci is computed as $f'^{-1}(1/\Phi_1)$ and is found to be greater than the capital value at point A_1. This implies that the value of B_t at this intersection is negative and cannot represent a bubbly steady state. The capital value at the intersection of the $\Delta k_{t+1} = 0$ and R2 loci, which we call point C_1, satisfies the following condition:

$$\lambda_1 M_1 \Phi_1 f'(k) = -\frac{\beta}{1-\beta}\left[k f'(k) - \frac{k}{\Phi_1}\right] + M_1. \tag{A.1}$$

From Assumption 2, Eq. (A.1) has a unique solution for k. We denote the unique solution by $\hat{k}$. For point C_1 to be a bubbly steady state, $\hat{k}$ must be strictly less than $f'^{-1}(1/(\beta\Phi_1))$; otherwise, B_t becomes negative. It is straightforward to show that when $k < \hat{k}$, the left-hand side of Eq. (A.1) is greater than the right-hand side and that when $k > \hat{k}$, the left-hand side is less than the right-hand side. Therefore, C_1 becomes a bubbly steady state if and only if

$$\begin{aligned}
&\lambda_1 M_1 \Phi_1 f'\left(f'^{-1}\left(\frac{1}{\beta\Phi_1}\right)\right) \\
&< -\frac{\beta}{1-\beta}\left[f'^{-1}\left(\frac{1}{\beta\Phi_1}\right) f'\left(f'^{-1}\left(\frac{1}{\beta\Phi_1}\right)\right) - f'^{-1}\left(\frac{1}{\beta\Phi_1}\right)\frac{1}{\Phi_1}\right] + M_1,
\end{aligned}$$

or equivalently, if and only if $M_1\Phi_1(1 - \lambda_1/\beta) > f'^{-1}(1/(\beta\Phi_1))$. Obviously, capital accumulation is lower in the bubbly steady state than in the bubbleless steady state. □

Proof of Lemma 1

For the claim to hold, the following equation with respect to k must have two solutions in $(0, f'^{-1}(1/(\beta\Phi_2))$:

$$g(k) := \left[k f'(k) - \frac{k}{\Phi_2}\right]\frac{\beta}{1-\beta} = M_2\left(1 - \frac{\lambda_2\Phi_2}{\Phi_1}\right). \tag{B.1}$$

Because $g(k)$ is maximized at $k = \tilde{k}$, Eq. (B.1) has two solutions if $g(\tilde{k}) > M_2(1 - \lambda_2\Phi_2/\Phi_1)$. Therefore, Eq. (B.1) has two solutions in $(0, f'^{-1}(1/(\beta\Phi_2))$ if and only if $g(f'^{-1}(1/(\beta\Phi_2)) < M_2(1-\lambda_2\Phi_2/\Phi_1) < g(\tilde{k})$ or, equivalently, if and only if $f'^{-1}(1/(\beta\Phi_2))/\Phi_2 < M_2(1 - \lambda_2\Phi_2/\Phi_1) < g(\tilde{k})$. □

Proof of Lemma 2

By following the same path as the proof of Proposition 1 and by replacing M_1, Φ_1, and λ_1 in Eq. (A.1) with M_2, Φ_2, and λ_2, we find that the value of capital in the bubbly steady state with project $j = 2$ satisfies the following equation:

$$\lambda_2 M_2 \Phi_2 f'(k) = -\frac{\beta}{1-\beta}\left[kf'(k) - \frac{k}{\Phi_2}\right] + M_2. \tag{C.1}$$

It follows that k^d is a solution for the following equation:

$$\beta k f'(k) = (1-\beta)M_2\left(1 - \frac{\lambda_2\Phi_2}{\Phi_1}\right) + \frac{\beta k}{\Phi_2}. \tag{C.2}$$

From Condition2, we have

$$k^d < f'^{-1}\left(\frac{1}{\beta\Phi_2}\right), \tag{C.3}$$

meaning that if k^d is greater than the solution for Eq.(C.1), the bubbly steady state with project $j = 2$ exists. From the configurations of the left-hand and right-hand sides of Eq. (C.1), k^d is greater than the value of capital in the bubbly steady state with project $j = 2$ if and only if

$$\lambda_2 M_2 \Phi_2 f'(k^d) < -\frac{\beta}{1-\beta}\left[k^d f'(k^d) - \frac{k^d}{\Phi_2}\right] + M_2. \tag{C.4}$$

With Eq. (C.2), Eq. (C.4) can be rewritten as

$$f'^{-1}\left(\frac{1}{\Phi_1}\right) < k^d. \tag{C.5}$$

Because k^d is a greater solution for Eq. (C.2), the sufficient condition for inequality (C.5) to hold is

$$\beta f'^{-1}\left(\frac{1}{\Phi_1}\right) f'\left(f'^{-1}\left(\frac{1}{\Phi_1}\right)\right) > (1-\beta)M_2\left(1 - \frac{\lambda_2\Phi_2}{\Phi_1}\right) + \beta f'^{-1}\left(\frac{1}{\Phi_1}\right)\frac{1}{\Phi_2},$$

or, equivalently,

$$M_2\left(1 - \frac{\lambda_2\Phi_2}{\Phi_1}\right) < f'^{-1}\left(\frac{1}{\Phi_1}\right)\left(\frac{1}{\Phi_1} - \frac{1}{\Phi_2}\right)\frac{\beta}{1-\beta}. \tag{Condition 3}$$

Finally, to show that the solution for Eq. (C.1) is greater than $f'^{-1}(1/\Phi_1)$, it suffices for us to verify that

$$\begin{aligned}&\lambda_2 M_2 \Phi_2 f'\left(f'^{-1}\left(\frac{1}{\Phi_1}\right)\right) \\ &> -\frac{\beta}{1-\beta}\left[f'^{-1}\left(\frac{1}{\Phi_1}\right) f'\left(f'^{-1}\left(\frac{1}{\Phi_1}\right)\right) - f'^{-1}\left(\frac{1}{\Phi_1}\right)\frac{1}{\Phi_2}\right] + M_2.\end{aligned}$$

In fact, this inequality is exactly the same as Condition 3. $\square$

Proof of Lemma 3

The $r_{t+1} = 1$ locus at point F_2 is given by

$$\lambda_2 M_2 \Phi_2 f'(k_{t+1}) = M_2 - \beta(k_t f'(k_t) + B_t), \tag{D.1}$$

where $k_{t+1} = \Phi_2 \beta k_t f'(k_t) - \Phi_2(1-\beta)B_t$. From Eq. (D.1) and the T1 locus, we have $k_{t+1} = f'^{-1}(1/\Phi_1)$. Therefore, k^f is the solution for the following equation:

$$kf'(k) - M_2\left(1 - \frac{\lambda_2\Phi_2}{\Phi_1}\right)\frac{1-\beta}{\beta} = f'^{-1}\left(\frac{1}{\Phi_1}\right)\frac{1}{\Phi_2}. \tag{D.2}$$

From Eq. (D.2), it follows that $k^f < f'^{-1}(1/\Phi_1)$ if

$$f'^{-1}\left(\frac{1}{\Phi_1}\right)\frac{1}{\Phi_1} - M_2\left(1 - \frac{\lambda_2\Phi_2}{\Phi_1}\right)\frac{1-\beta}{\beta} > f'^{-1}\left(\frac{1}{\Phi_1}\right)\frac{1}{\Phi_2}.$$

In fact, this inequality is exactly the same as Condition 3. $\square$

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