DISCUSSION PAPER SERIES

Discussion paper No. 156

Asset Bubbles, Unemployment, and a Financial Crisis

Takuma Kunieda
(School of Economics, Kwansei Gakuin University)
Ken-ichi Hashimoto
(Graduate School of Economics, Kobe University)
Ryonghun Im
(Graduate School of Economics, Kobe University)

February 2017

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

Asset Bubbles, Unemployment, and a Financial Crisis*

Ken-ichi Hashimoto† Ryonghun Im‡ Takuma Kunieda§

February 8, 2017

Abstract

A tractable model with asset bubbles is presented to demonstrate that a financial crisis caused by a bubble bursting increases unemployment rates. A bubbly asset has a positive market value because purchasing the asset is the sole saving method for agents who draw insufficient productivity, whereas selling the asset is a fund-raising method to initiate an investment project. The presence of bubbles corrects allocative inefficiency, relocating investment resources from low-productivity agents to high-productivity agents. Accordingly, the presence of bubbles can promote capital accumulation and reduce unemployment rates. However, a self-fulfilling financial crisis would result in high unemployment rates.

Keywords: Overlapping generations, Labor market friction, Borrowing constraints, Asset bubbles, Unemployment.
JEL Classification Numbers: J64, O41, O42.

---

*This research is financially supported by JSPS Grant-in-Aid for Scientific Research (Nos. 16H02026, 16K03624, and 16K03685) and Grant-in-Aid for JSPS Fellows (No. 16J04950). The authors would like to express thanks to Hiroki Arato, Ryoji Hiraguchi, Takeo Hori, Takashi Kamihigashi, Koji Kitaura, Akira Momota, Kazuo Nishimura, Ryoji Ohdoi, Yoshiyasu Ono, and Akihisa Shibata for their invaluable comments and suggestions. The authors are also grateful to the seminar participants at Kyoto University and Yamaguchi University and the participants at the Economic Theory and Policy Workshop held at Hosei University and the 22nd International Conference on Difference Equations and Applications held at Osaka Prefecture University for their comments. All remaining errors, if any, are ours.

†Address: Graduate School of Economics, Kobe University, Rokko-dai 2-1, Kobe 657-8501, Japan; Fax: +81 78 803 7293; E-mail: hashimoto@econ.kobe-u.ac.jp

‡Address: Graduate School of Economics, Kobe University, 2-1 Rokko-dai, Nada, Kobe 657-8501, JAPAN; E-mail: ryonghunim@gmail.com

§Address: School of Economics, Kwansei Gakuin University, 1-155 Uegahara Ochiban-cho, Nishinomiya 662-8501, Hyogo, Japan; Fax: +81 798 51 0944; E-mail: tkunieda@kwansei.ac.jp

1 Introduction

A bubble on an asset is defined as the deviation of the asset's market value from its fundamental value. Economic history has repeatedly witnessed severe financial crises accompanied by the collapse of asset prices in modern monetary and financial systems. Before a financial crisis, asset prices often deviate upward from their fundamental values, and possibly lead to higher output, stimulating employment. When asset prices collapse, however, output suddenly declines, and the economy goes into a depression and unemployment increases.[1] As such, a bubble bursting arguably causes a higher unemployment rate. For example, as the Japanese asset price bubble collapsed in 1991, the unemployment rate in Japan increased from 2.1% in 1991 to 4.8% in 2000, and as the US subprime loan crisis occurred in 2009, the unemployment rate in the US increased from 4.7% in 2007 to 9.7% in 2010 (World Bank, 2016). Despite the historical observations related to the collapse of asset prices and depressions, the impact of a bubble bursting on unemployment has not been fully theoretically investigated in macroeconomics. In this paper, we present a tractable overlapping-generations model with asset bubbles to demonstrate that a financial crisis triggered by a bubble bursting depresses an economy and increases unemployment.

In our model, a bubbly asset has a positive market value because selling the asset is a fund-raising method for those who draw sufficiently high productivity to initiate an investment project, and purchasing the asset is the sole saving method for those who draw insufficient productivity to initiate a project. Our model is closely related to that of Martin and Ventura (2012), who develop a tool to investigate how the occurrence of asset bubbles promotes capital accumulation and the bursting of bubbles causes depressions. As in Martin and Ventura's model, the young generation can issue new bubbly assets to raise funds. Once they sell the new bubbly assets in the asset market, they do not have to purchase them back from the market. Accordingly, the young generation always has incentives to issue new bubbly assets and obtain more funds that cannot be otherwise acquired because of borrowing constraints.

Although the central role of asset bubbles in our model is similar to that of Martin and Ventura (2012), our model departs from theirs in some respects.

[1] Empirical studies such as Phelps (1999) and Fitoussi et al. (2000) provide evidence showing that a reduction in unemployment rates is accompanied by increasing asset prices.

First, we employ a continuous distribution with respect to idiosyncratic productivity shocks, whereas Martin and Ventura apply a binary distribution. The use of a continuous productivity distribution significantly simplifies the analysis. In particular, one can derive the productivity cutoff that divides agents into bubbly-asset holders and investors. Those who draw productivity shocks smaller than the cutoff purchase bubbly assets, and those who draw productivity shocks greater than the cutoff become investors. Our model obtains a simple two-dimensional dynamical system with respect to capital and the cutoff, by which one can easily investigate the dynamic behavior of the system. Second, we introduce labor market frictions. The investigation of the relationship between a bubble bursting and unemployment is a main theme in this paper. By introducing labor market matching frictions in our tractable model following Bean and Pissarides (1993), we can demonstrate that a bubble bursting increases unemployment under mild parameter conditions, which is a novel result in the literature that addresses asset bubbles á la Tirole (1985).[2]

The presence of asset bubbles corrects allocative inefficiency, relocating investment resources from low-productivity agents to high-productivity agents, and promotes capital accumulation if the bubbles' crowding-out effect á la Tirole (1985) is relatively weak. As capital accumulates and output increases, the number of vacant positions increases because each firm acquires more funds to cover a fixed search cost. As a result, the unemployment rate decreases.[3] However, extrinsic uncertainty may burst asset bubbles and cause a self-fulfilling financial crisis, which is followed by increased unemployment. The bubbly asset plays a financial intermediation role, as noted by Mitsui and Watanabe (1989). As previously stated, however, the bubbly asset that is newly issued in each period is never withdrawn from the economy and investors never repay the funds raised by issuing the bubbly asset as in the model of Martin and Ventura (2012). This Ponzi game can be played because financial market imperfections render the market interest rate less than the economic growth rate in equilibrium when the bubbly asset is not present (see Theorem 3.3 in Santos and Woodford, 1997).

[2]For traditional models that address the relationship between economic growth and unemployment with labor market imperfections, see Aghion and Howitt (1994), Eriksson (1997), Caballero and Hammour (1996), and Haruyama and Leith (2010). See also Pissarides (2000) for an introduction to search friction models.

[3]This outcome accords with many empirical studies that show a negative relationship between unemployment and economic growth (Ball and Moffitt, 2001; Muscatelli and Tirelli, 2001; Staiger et al., 2001; Tripier, 2006; Pissarides and Vallanti, 2007).

The literature on asset bubbles and economic growth has recently experienced a resurgence and argues that the presence of asset bubbles promotes capital accumulation and economic growth.[4] In this stream of literature, financial market imperfections and the productivity differences across agents are key factors in producing a situation in which asset bubbles á la Tirole (1985) enhance capital accumulation. Farhi and Tirole (2012), Martin and Ventura (2012), Carvalho et al. (2012), Kunieda (2014), and Ikeda and Phan (2015) create such a situation by applying the overlapping generations framework of Samuelson (1958), Tirole (1985), or Blanchard (1985). To produce the same situation, Aoki and Nikolov (2015), Hirano et al. (2015), Hirano and Yanagawa (2016), and Kunieda and Shibata (2016) develop dynamic general equilibrium models in which asset bubbles occur in equilibrium despite the assumption of infinitely lived agents and the presence of bubbles promotes economic growth through essentially the same mechanism as that initially found by Mitsui and Watanabe (1989). Although all these studies obtain the result that the presence of asset bubbles promote capital accumulation as in the current model, they do not investigate how unemployment rates are affected by the presence and bursting of asset bubbles. Miao et al. (2016) investigate the relationship between unemployment and stock market bubbles in an economy with labor and financial market frictions. However, their definition of bubbles is totally different from ours: they essentially consider indeterminacy of a firm's fundamental value. Although Kocherlakota (2011) investigates the impact of the occurrence of asset bubbles on unemployment, he does not consider capital accumulation. Hashimoto and Im (2016a,b) also study the relationship between bubbles and unemployment. In contrast to our paper, however, they do not consider financial frictions to obtain the bubbly steady state.

The remainder of this paper is organized as follows. Section 2 develops the model and section 3 investigates the dynamic behavior in equilibrium and derives the relationship between the unemployment rate and capital accumulation. In section 4, the growth-promoting effect of asset bubbles is analyzed

[4] Researchers in the traditional literature on asset bubbles and economic growth have long discussed the growth effects of bubbles by applying the overlapping generations model. See Tirole (1985), Weil (1987), Grossman and Yanagawa (1993), King and Ferguson (1993), Futagami and Shibata (2000), Kunieda (2008), Mino (2008) and Matsuoka and Shibata (2012), among others. Regrettably, their results cannot explain the historical events in which severe economic depressions arguably follow the collapse of asset bubbles.

and section 5 derives a self-fulfilling financial crisis as a rational expectations equilibrium. Section 6 concludes the paper. The proofs of propositions and lemmata are collected in the online appendix.

2 The model

The economy is represented in discrete time, ranging from time $t = 0$ to $t = \infty$, and it consists of overlapping generations: young and old agents. Each agent lives for two periods. The population of each generation is constant, which is given by L. Only young agents have an opportunity to work, matching with a firm, and thus, L is also the size of the total labor force supplied in each period.

2.1 Final goods sector

In the final goods sector, many identical firms produce final goods with the same production technology. In addition to capital, one worker is necessary for a firm to produce the final goods. More concretely, workers and firms with vacant positions search for one another in the labor market. Firms that successfully match with a worker can operate their business. Firm i produces final goods, $y_{i,t}$, at time t with a Cobb-Douglas production technology: $y_{i,t} = Az_{i,t}^{\alpha} l_{i,t}^{1-\alpha}$, where $\alpha \in (0, 1)$ is the capital share of output, $z_{i,t}$ is capital, which depreciates in one period, $l_{i,t}$ is the labor employed by firm i, and A is the productivity of the technology. Because an operating firm hires only one worker, eventually it holds that $l_{i,t} = 1$, and the production function is condensed as follows:

$$y_{i,t} = Az_{i,t}^{\alpha}. \tag{1}$$

Because the capital market is competitive, capital is paid its marginal product:

$$q_t = \alpha Az_{i,t}^{\alpha-1}, \tag{2}$$

where q_t is the capital price. Then, the remainder of output to be allotted between firm i and its worker is given by

$$\pi_t := y_{i,t} - q_t z_{i,t} = (1 - \alpha)Az_t^{\alpha}. \tag{3}$$

The firm-specific index i is dropped because each firm employs the same amount of capital, facing the common capital price.

2.2 Agents

An agent born at time t exclusively derives her utility from consumption in old age, which is denoted by c_{t+1}^{ι}. Note that ι represents an agent's employment status: $\iota = e$ if employed and $\iota = u$ if unemployed, which is an outcome of job search in youth. Because she does not consume in the first period of her lifetime, she turns over all her income in youth to maximize her lifetime utility given by $U_t^{\iota} = c_{t+1}^{\iota}$. In the first period, she is endowed with one unit of labor. A successful match with a firm enables her to work for the firm and earn a wage income, w_t. Otherwise, she receives an unemployment benefit, γ_t, from the government. Because the government imposes a lump-sum tax, τ_t, on the young agents to cover the unemployment benefit, the agent's net income in the first period is given by $\omega_t^{\iota} - \tau_t$ where $\omega_t^e = w_t$ if employed and $\omega_t^u = \gamma_t$ if unemployed.

Following Martin and Ventura (2012) and Ikeda and Phan (2015), it is assumed that agents can issue new bubbly assets, which are intrinsically useless, to obtain additional funds in the first period, although they cannot borrow, as they face borrowing constraints. The agent's total funds available for saving are given by

$$s_t^{\iota} := \omega_t^{\iota} - \tau_t + b_t^N, \tag{4}$$

where b_t^N represents new bubbly assets issued by the agent at time t.[5] To derive an equilibrium in which bubbly assets exist, we limit the ceiling of bubbly assets in the amount that each agent can issue as follows:

$$b_t^N \leq \mu \tilde{b}_t \quad \mu \in (0, 1), \tag{5}$$

[5] To understand the bubbly assets that are newly issued by agents but never redeemed, one can imagine the securitization of commercial loans. Recent financial innovation allows for the securitization of commercial loans, and an asset backed by the loans can be purchased and sold in the primary and secondary markets. In the process of securitization, asset holders may be unable to identify the fundamental value of the asset. In such a case, although the fundamental value of the asset is actually zero, such a worthless asset would be traded in the financial market to the extent that participants in the market believe in the market value of the asset.

where $\tilde{b}_t$ is the average amount of bubbly assets per young agent that exist at the end of time t.[6] More concretely, $\tilde{b}_t$ satisfies $B_t = \tilde{b}_t L$, where B_t is the real value of the total bubbly asset at time t, which includes the newly issued bubbly asset at time t.[7] Agents are willing to raise the greatest amount of new bubbly assets as possible because once they obtain additional funds by issuing the assets, they do not have to repay these funds. Therefore, the equality holds in inequality (5) in equilibrium.

There is no storage technology for the final goods, which are perishable in one period. Instead, agents have two saving methods: one is initiating an investment project and the other is purchasing bubbly assets. Agents that purchase one unit of bubbly assets at time t earn (gross) return r_{t+1} at time $t+1$, whereas agents that invest one unit of funds in a project at time t create Φ units of capital goods and sell them to firms at price q_{t+1} at time $t+1$; namely, they earn $q_{t+1}\Phi$ as a return on the investment of one unit of funds. Φ is the productivity of capital production and varies across agents. When an agent is born, she receives an individual-specific shock, Φ. The support of Φ is $[0, \eta]$, where $\eta > 0$, and its cumulative distribution function is given by $G(\Phi)$, which is time-invariant and continuously differentiable on the support. Although Φ is an idiosyncratic shock, the realization of low productivity cannot be insured against because there is no insurance market for it. Φ is independent of employment status. When agents invest in a project, the shocks are already realized. Knowing their own productivity, they make a portfolio choice between investing in a project and purchasing bubbly assets to maximize their lifetime utility. As such, the individual-specific return is deterministic when they make a portfolio choice, which is given by $R_{t+1} = \max\{q_{t+1}\Phi, r_{t+1}\}$, and thus, an agent's lifetime utility is given by

$$U_t^\iota = R_{t+1} s_t^\iota. \tag{6}$$

Define $\phi_t := r_{t+1}/q_{t+1}$. Then, a portfolio choice of an agent who draws produc-

[6] The assumption regarding the limitation on the new issuance of bubbly assets is also imposed in Martin and Ventura (2012) and Ikeda and Phan (2015). In any case, one must impose an upper limit of the new issuance of bubbly assets; otherwise, the market for bubbly assets cannot be sustainable. One may consider that the new issuance is regulated institutionally.

[7] In section 2.5, the formal definition of B_t is provided.

tivity Φ is given by

$$k_t^\iota = \begin{cases} 0 & \text{if } \Phi \leq \phi_t \\ s_t^\iota & \text{if } \Phi > \phi_t, \end{cases} \tag{7}$$

and

$$b_t^\iota = \begin{cases} s_t^\iota & \text{if } \Phi \leq \phi_t \\ 0 & \text{if } \Phi > \phi_t, \end{cases} \tag{8}$$

where k_t^ι is the investment in a project and b_t^ι represents the bubbly assets. As seen in Eqs. (7) and (8), agents who draw productivity smaller than ϕ_t purchase bubbly assets, and agents who draw productivity greater than ϕ_t invest in a project.[8] Note that ϕ_t is a productivity cutoff that divides agents into investors and bubbly asset holders. The population of investors is $(1-G(\phi_t))L$, and that of bubbly asset holders is $G(\phi_t)L$.

2.3 Government

The government runs a balanced budget to provide unemployment benefits such that

$$\tau_t L = \gamma_t u_t L, \tag{9}$$

where u_t is the unemployment rate. The left-hand side of Eq. (9) denotes the aggregate tax revenue, and the right-hand side represents the total payments for unemployment benefits.

2.4 Labor market

We introduce labor-market matching frictions in the model following Bean and Pissarides (1993). Although the matching mechanism follows from the standard unemployment model (e.g., Diamond, 1982; Mortensen and Pissarides, 1999; Petrongolo and Pissarides, 1990), there is no time lag between a match of parties and the start of business operations in the current model.

[8] In the current model, agents can issue new bubbly assets before the portfolio choice and the realization of individual-specific productivity shocks, as presented in Eq. (4). For the trade timing in such market circumstances, we implicitly assume that a market maker is present in the asset market.

2.4.1 Matching mechanism

Because workers and firms face matching frictions, unemployment occurs in equilibrium, although each agent is born endowed with one unit of labor that she supplies inelastically in youth. The number of successful matches is given by $F(L, \upsilon_t)$, which is a function of the population of workers, L, and the number of firms with a vacancy, υ_t, where $0 \leq F(L, \upsilon_t) \leq \min\{L, \upsilon_t\}$ for $L \in [0, \infty)$ and $\upsilon_t \in [0, \infty)$, and $F(0, \upsilon_t) = 0$ and $F(L, 0) = 0$. The matching function, $F(L, \upsilon_t)$, is continuously differentiable, concave, homogeneous of degree one, and increasing with respect to both L and υ_t. The tightness of the labor market is expressed by $\theta_t := \upsilon_t/L \in (0, \infty)$, which is regarded as the jobs-to-applicants ratio, and the probability that a firm with a vacancy matches with a worker is given by $F(L, \upsilon_t)/\upsilon_t = F(1/\theta_t, 1) =: f(\theta_t)$. It is assumed that $f(\theta_t)$ is continuously differentiable in $(0, \infty)$ where $f'(\theta_t) < 0$ for $\theta_t \in (0, \infty)$, $\lim_{\theta_t \to 0} f(\theta_t) = 1$, and $\lim_{\theta_t \to \infty} f(\theta_t) = 0$. Because the number of employment is equal to the number of successful matches, it follows that $(1 - u_t)L = F(L, \upsilon_t)$, which is rewritten as

$$1 - u_t = \theta_t f(\theta_t). \tag{10}$$

Eq. (10) yields the unemployment rate, u_t, as a function of θ_t such that $u_t = u(\theta_t)$ where $u'(\theta_t) < 0$ because $\partial[\theta_t f(\theta_t)]/\partial\theta_t = \partial F(1, \theta_t)/\partial\theta_t > 0$. Therefore, Eq. (10) derives a negative relationship between the unemployment rate and the jobs-to-applicants ratio, which is the so-called Beveridge curve.

A successful match enables a firm to produces the final goods. Because $f(\theta_t)$ is the probability that a firm matches with a worker at time t, the firm's expected profits, V_t, are given by

$$V_t = f(\theta_t)(\pi_t - w_t) - h, \tag{11}$$

where h is the search cost in the labor market that the firm incurs when searching for a worker.[9] Because the ceiling of $f(\theta_t)$ is 1, if the actual revenue, $\pi_t - w_t$, is less than h, no firms operate because the expected profits are negative. In other words, only if $\pi_t - w_t \geq h$, successful matches occur between workers and firms. In our investigation, we examine the case in which $\pi_t - w_t \geq h$ unless otherwise

[9] The search cost covers the recruitment activities such as job interviews and the evaluation of reference letters, which are performed using the firm's resources. One can regard the search cost associated with these activities as an implicit opportunity cost that the firm incurs.

stated. The free-entry condition for the final goods sector leads to zero profits for each firm. Accordingly, it follows that $V_t = 0$, or equivalently,

$$\pi_t - w_t = \frac{h}{f(\theta_t)}. \tag{12}$$

2.4.2 Nash bargaining

The remainder of output after payments to capital is allotted between the firm and its worker. The shares received by each are determined by maximizing the following Nash product with respect to the wage:

$$w_t = \arg\max_{w_t} (U_t^e - U_t^u)^\beta (\pi_t - w_t)^{1-\beta} = \{R_{t+1}(w_t - \gamma_t)\}^\beta (\pi_t - w_t)^{1-\beta},$$

where $\beta \in (0, 1)$ is the worker's bargaining power and R_{t+1} is the return on saving, which was derived in section 2.2. From the Nash bargaining solution, it follows that

$$w_t = (1 - \beta)\gamma_t + \beta\pi_t. \tag{13}$$

The government policy regarding unemployment determines the unemployment benefits paid to unemployed workers in such a way that $\gamma_t = \gamma w_t$ where $\gamma \in [0, 1)$. Note that when the firm and its worker are bargaining, they do not have concrete information about the government's unemployment policy, and thus, the Nash product is maximized with γ_t being given. Inserting Eq. (3) and $\gamma_t = \gamma w_t$ into Eq. (13) yields

$$w_t = \Omega(1 - \alpha)Az_t^\alpha, \tag{14}$$

where $\Omega := \beta/\{1 - (1 - \beta)\gamma\} \in (0, 1)$ is the worker's share of π_t. Note from $\Omega = \beta/\{1 - (1 - \beta)\gamma\}$ that a larger outside option, γw_t, and a larger Nash bargaining power, β, lead to a greater share for the worker. Substituting Eqs. (3) and (14) into Eq. (12) yields

$$(1 - \Omega)(1 - \alpha)Az_t^\alpha = \frac{h}{f(\theta_t)}. \tag{15}$$

Thus far, we have investigated the model under the assumption that there are always operating firms. However, as noted from Eq. (15), given the parameter values, if z_t is very small, firms cannot cover the search cost, h, because the upper limit of $f(\theta_t)$ is 1.

Proposition 1 *Define* $\bar{z} := [h/\{(1-\Omega)(1-\alpha)A\}]^{1/\alpha}$.

- *If* $z_t \leq \bar{z}$, *then there are no operating firms in the economy at time* t.
- *If* $z_t > \bar{z}$, *then there are operating firms in the economy at time* t.

Proof: See the Appendix.

In the first case in Proposition 1, the economy certainly breaks down at time t because no firms produce final goods at that time. In this case, no agents initiate an investment project at time $t-1$, anticipating the breakdown. Moreover, no young agents at time t can purchase the bubbly asset because they do not earn labor income at that time, and thus, the bubbly asset has no value at time t. Anticipating this, no young agents purchase the bubbly asset at time $t-1$. Accordingly, backward induction shows that the bubbly asset has no value even at time zero. Additionally, young agents at time $t-1$ anticipate that they cannot obtain returns from investment projects at time t. Given their expectations, we can reasonably assume that young agents at time $t-1$ do not supply labor at time $t-1$ because they do not consume in the first period of their lifetime and are not necessarily benevolent. As a result, the economy breaks down at time $t-1$. Backward induction, again, shows that the economy breaks down at time zero. In summary, if z_t becomes less than $\bar{z}$ at a certain point in time, it is highly likely that the economy is unsustainable for all $t \geq 0$ without the occurrence of production. In what follows, we assume away the first case in Proposition 1 and focus on the case in which $z_t > \bar{z}$ for all $t \geq 0$.

In the second case in Proposition 1, it follows from Eq. (15) that $\theta_t = v_t/L > 0$, which implies that there exist firms with a vacancy at time t and the unemployment rate is less than one.

2.5 Bubbly Asset

The bubbly asset is intrinsically useless. It is assumed that at time 0, there are identical old agents who hold the bubbly asset, M_{-1}, in total. Additionally, in each period, young agents issue new units of the bubbly asset. Formally, for $t \geq 0$, we have a dynamic equation with respect to the nominal amount of the bubbly asset as follows:

$$M_t = M_{t-1} + M_t^N, \tag{16}$$

where M_t is the total nominal supply of the bubbly asset and M_t^N is the asset that is newly issued by the young agents at time t. Multiplying both sides of Eq. (16) by the bubbly asset's price, p_t, and defining the real value of the bubbly asset as $B_t = p_t M_t$ and $B_t^N = p_t M_t^N$, we obtain the dynamic equation for the real value of the bubbly asset as $B_t = (p_t/p_{t-1})B_{t-1} + B_t^N$, or equivalently,

$$B_t = r_t B_{t-1} + B_t^N, \tag{17}$$

where $r_t := p_t/p_{t-1}$, which is the return from holding the bubbly asset.

3 Equilibrium

The equilibrium is characterized by the optimization conditions of the agents and firms, the outcome of the Nash bargaining in the labor market, and the market clearing conditions for the bubbly asset and capital.

3.1 Market clearing conditions

B_t and B_t^N are the aggregations of $\tilde{b}_t$ and b_t^N over all agents, namely $B_t = \tilde{b}_t L$ and $B_t^N = b_t^N L$. Because the equality in inequality (5) holds in equilibrium, it follows that

$$B_t^N = \mu B_t. \tag{18}$$

Inserting Eq. (18) into Eq. (17) yields

$$B_t = \frac{r_t}{1-\mu} B_{t-1}. \tag{19}$$

In each period, the bubbly asset is purchased by less-productive agents, regardless of their employment status. Because the population of less-productive agents who drew productivity smaller than the cutoff, ϕ_t, and purchase the bubbly assets is $G(\phi_t)L$, the demand for the bubbly asset is given by

$$B_t^d := G(\phi_t)L \left[(1-u_t)(w_t - \tau_t + b_t^N) + u_t(\gamma_t - \tau_t + b_t^N) \right]. \tag{20}$$

It follows that $B_t = B_t^d$ in equilibrium, and thus, the use of Eqs. (9), (14), and

(18) rewrites Eq. (20) as follows:

$$B_t = \frac{\Omega(1-\alpha)Az_t^\alpha(1-u_t)G(\phi_t)L}{1-\mu G(\phi_t)}. \tag{21}$$

From Eqs. (2), (19), (21), and $\phi_{t-1} = r_t/q_t$, we obtain

$$\frac{G(\phi_t)}{1-\mu G(\phi_t)} z_t(1-u_t) = \frac{\alpha A\phi_{t-1}G(\phi_{t-1})}{(1-\mu)(1-\mu G(\phi_{t-1}))} z_{t-1}^\alpha(1-u_{t-1}). \tag{22}$$

Capital at time t is produced by the agents who draw such high productivity that $\Phi > \phi_{t-1}$. Therefore, aggregate capital is given by

$$Z_t := \int_{\phi_{t-1}}^{\eta} \Phi \left(k_{t-1}^e(1-u_{t-1})L + k_{t-1}^u u_{t-1}L\right) dG(\Phi). \tag{23}$$

The number of firms that successfully match with a worker at time t is $(1-u_t)L$, and thus, capital per operating firm (or per worker), z_t, is given by $z_t = Z_t/\{(1-u_t)L\}$. The use of Eqs. (4), (7), (9), (14), (18), and (21) rewrites Eq. (23) as follows:

$$z_t(1-u_t) = \frac{(1-\alpha)\Omega AH(\phi_{t-1})}{1-\mu G(\phi_{t-1})} z_{t-1}^\alpha(1-u_{t-1}), \tag{24}$$

where $H(\phi_{t-1}) := \int_{\phi_{t-1}}^{\eta} \Phi dG(\Phi)$.

3.2 Dynamical system

From Eqs. (10) and (15), we obtain the following equation:

$$1-u_t = \frac{h}{(1-\Omega)(1-\alpha)Az_t^\alpha} f^{-1}\left(\frac{h}{(1-\Omega)(1-\alpha)Az_t^\alpha}\right) =: \Psi(z_t). \tag{25}$$

Inserting this equation into Eq. (24) yields the dynamic equation with respect to z_t as follows:

$$z_t\Psi(z_t) = \frac{(1-\alpha)\Omega AH(\phi_{t-1})}{1-\mu G(\phi_{t-1})} z_{t-1}^\alpha \Psi(z_{t-1}). \tag{26}$$

Eqs. (22) and (24) yield the dynamic equation with respect to the cutoff, ϕ_t:

$$\frac{(1-\alpha)\Omega G(\phi_t)}{1-\mu G(\phi_t)} = \frac{\alpha\phi_{t-1}G(\phi_{t-1})}{(1-\mu)H(\phi_{t-1})}. \tag{27}$$

Eqs. (26) and (27) can be used to derive an autonomous dynamical system with respect to z_t and ϕ_t. Eq. (27) is solely an autonomous difference equation with respect to ϕ_t. Because the cutoff, ϕ_t, is in $[0, \eta]$ and because we focus on the case in which $z_t > \bar{z}$ for all $t \geq 0$ as discussed in the previous section, the domain of the dynamical system consisting of Eqs. (26) and (27) is given by $(\bar{z}, \infty) \times [0, \eta]$.

We assume that the initial total capital, Z_0, already exists at time 0. Then, the initial capital per worker, z_0, the initial labor-market tightness, θ_0, and the initial unemployment rate, u_0, are simultaneously determined by Eqs. (10), (15), and $z_0(1 - u_0)L = Z_0$, which means that all three variables are pre-determined at time 0. In contrast, the initial real value of the bubbly asset, B_0, is not pre-determined because its price, p_0, can jump depending upon agents' self-fulfilling expectations. Accordingly, ϕ_0 is not pre-determined either because ϕ_t has a one-to-one relationship with B_t, as seen in Eq. (21), given z_t and u_t. This means that ϕ_0 is also affected by agents' self-fulfilling expectations. With the economy starting with $\{z_0, u_0, \theta_0, B_0\}$, the equilibrium sequences, $\{z_t, u_t, \theta_t, B_t, \phi_t\}_{t=0}^{\infty}$, are produced from Eqs. (10), (15), (21), (26), and (27), where $(z_t, \phi_t) \in (\bar{z}, \infty) \times [0, \eta]$ for all $t \geq 0$.

3.3 Steady states and stability

Proposition 2 *In the dynamical system consisting of Eqs. (26) and (27), there exist two (non-trivial) steady states: (z^*, ϕ^*) and (z^{**}, ϕ^{**}) such that*

$$z^* = Q(\phi^*)^{\frac{1}{1-\alpha}}, \tag{28}$$

$$\frac{H(\phi^*)}{1 - \mu G(\phi^*)} = \frac{\alpha \phi^*}{(1-\alpha)(1-\mu)\Omega}, \tag{29}$$

$$z^{**} = Q(0)^{\frac{1}{1-\alpha}}, \tag{30}$$

and

$$\phi^{**} = 0, \tag{31}$$

where $Q(x) = (1-\alpha)\Omega A H(x)/(1 - \mu G(x))$.

Proof: See the Appendix.

Because $\phi^* > 0$ and the unemployment rate is always less than one, Eq. (21)

implies that the bubbly asset has a market value in the steady state given by (z^*, ϕ^*). Thus, we call this steady state a bubbly steady state. In contrast, in the steady state given by (z^{**}, ϕ^{**}), the bubbly asset has no market value, and we call this steady state a bubbleless steady state. The linear approximation of the dynamical system around a steady state is computed from Eqs. (26) and (27) as follows:

$$\begin{pmatrix} z_t - \hat{z} \\ \phi_t - \hat{\phi} \end{pmatrix} = \begin{pmatrix} \kappa_1(\hat{z}) & \frac{Q'(\hat{\phi})\hat{z}^{\alpha}\Psi(\hat{z})}{\Psi(\hat{z})+\hat{z}\Psi'(\hat{z})} \\ 0 & \kappa_2(\hat{\phi}) \end{pmatrix} \begin{pmatrix} z_{t-1} - \hat{z} \\ \phi_{t-1} - \hat{\phi} \end{pmatrix}, \tag{32}$$

where $(\hat{z}, \hat{\phi})$=(z^*, ϕ^*) or (z^{**}, ϕ^{**}). Note that $\kappa_1(\hat{z})$ and $\kappa_2(\hat{\phi})$ are the eigenvalues of the local dynamical system associated with Eq. (32).

Lemma 1 *The eigenvalues of the local dynamical system associated with Eq. (32) around the bubbly steady state, (z^*, ϕ^*), are given by*

$$\kappa_1(z^*) = \frac{\alpha\Psi(z^*) + z^*\Psi'(z^*)}{\Psi(z^*) + z^*\Psi'(z^*)},$$

and

$$\kappa_2(\phi^*) = \frac{G(\phi^*)(1-\mu G(\phi^*))}{\phi^* G'(\phi^*)}\left(1 + \frac{\phi^* G'(\phi^*)}{G(\phi^*)} + \frac{(\phi^*)^2 G'(\phi^*)}{H(\phi^*)}\right).$$

The eigenvalues of the local dynamical system associated with Eq. (32) around the bubbleless steady state, (z^{**}, ϕ^{**}), are given by

$$\kappa_1(z^{**}) = \frac{\alpha\Psi(z^{**}) + z^{**}\Psi'(z^{**})}{\Psi(z^{**}) + z^{**}\Psi'(z^{**})},$$

and

$$\kappa_2(\phi^{**}) = 0.$$

Proof: See the Appendix.

Proposition 3 *In the dynamical system consisting of Eqs. (26) and (27), the bubbly steady state, (z^*, ϕ^*), is a saddle point and the bubbleless steady state, (z^{**}, ϕ^{**}), is totally stable.*

Proof: See the Appendix.

[Figure 1 around here]

Figure 1 provides a phase diagram that illustrates the dynamic behavior of the economy. Because ϕ_0 can jump and the bubbly steady state is a saddle point, the bubbly steady state is locally determinate. However, the bubbleless steady state is totally stable, and thus, any sequence of $\{z_t, \phi_t\}_{t=0}^{\infty}$ with $(z_0, \phi_0) \in (\bar{z}, \infty) \times (0, \phi^*)$ that converges to $(z^{**}, 0)$ is an equilibrium. This means that equilibrium is globally indeterminate. Because of indeterminacy of equilibrium, self-fulfilling financial crises are caused by extrinsic uncertainty as investigated in section 5. Note that any sequence of $\{z_t, \phi_t\}_{t=0}^{\infty}$ with $(z_0, \phi_0) \in (\bar{z}, \infty) \times (\phi^*, \eta]$ cannot be an equilibrium because ϕ_t becomes greater than η or z_t becomes less than $\bar{z}$ in finite time.

3.4 Beveridge curve and capital accumulation

Eq. (10) can be rewritten as follows:

$$u_t = 1 - \theta_t f(\theta_t), \tag{33}$$

where $\partial u_t / \partial \theta_t < 0$. Eq. (33) is the Beveridge curve as stated in section 2.4. From Eq. (15), it follows that

$$\theta_t = f^{-1}\left(\frac{h}{(1-\Omega)(1-\alpha)Az_t^{\alpha}}\right), \tag{34}$$

which we call the job-creation condition following Pissarides (2000). From Eq. (34), it is straightforward to show that $\partial \theta_t / \partial z_t > 0$ because $f^{-1}(.)$ is a decreasing function. This means that capital accumulation promotes employment, rendering the labor market tighter. As capital accumulates, an economy moves down along the Beveridge curve from point A to point B in Figure 2 and the unemployment rate decreases.

[Figure 2 around here]

4 Capital accumulation, asset bubbles, and unemployment

The Beveridge curve given by Eq. (33) and the job-creation condition given by Eq. (34) demonstrate that capital accumulation decreases the unemployment rate. This means that if the presence of asset bubbles promotes capital accumulation, it decreases the unemployment rate. In this section, we investigate the effects that asset bubbles have on capital accumulation and the unemployment rate.

4.1 Comparison between bubbly and bubbleless steady states

Because the bubbly steady state is a saddle point, and because the initial cutoff, ϕ_0, is non-predetermined and initial capital is predetermined, the equilibrium in the neighborhood of the bubbly steady state is locally determinate. On the stable saddle path that converges to the bubbly steady state, the cutoff is constant, which is given by $\phi_t = \phi^*$, as illustrated in Figure 1, and the rational expectations equilibrium in the neighborhood of the bubbly steady state is given by the following equations:

$$\phi_t = \phi^* \tag{35}$$

and

$$z_t \Psi(z_t) = \frac{(1-\alpha)\Omega A H(\phi^*)}{1-\mu G(\phi^*)} z_{t-1}^{\alpha} \Psi(z_{t-1}). \tag{36}$$

By contrast, because the bubbleless steady state is totally stable, the equilibrium in the neighborhood of the bubbleless steady state is indeterminate, and there exist an uncountably infinite number of equilibrium trajectories around the bubbleless steady state. Under these circumstances, for the sake of the investigation of the growth-promoting effects of asset bubbles, we consider a particular rational expectations equilibrium in which agents anticipate no presence of asset bubbles for all $t \geq 0$, which is given by the following equations:

$$\phi_t = \phi^{**} (= 0) \tag{37}$$

and

$$z_t\Psi(z_t) = \frac{(1-\alpha)\Omega A H(\phi^{**})}{1-\mu G(\phi^{**})} z_{t-1}^{\alpha}\Psi(z_{t-1}). \tag{38}$$

Note from the right-hand sides of Eqs. (36) and (38) that the presence of asset bubbles promotes (impedes) capital accumulation if $H(\phi^*)/[1-\mu G(\phi^*)]$ is greater (less) than $H(\phi^{**})/[1-\mu G(\phi^{**})]$. To investigate whether $H(\phi^*)/[1-\mu G(\phi^*)]$ is greater or less than $H(\phi^{**})/[1-\mu G(\phi^{**})]$, consider the following function:

$$\Lambda(\phi) = \frac{H(\phi)}{1-\mu G(\phi)}, \tag{39}$$

which is used in the proof of Proposition 2 (in the Appendix). The first derivative of $\Lambda(\phi)$ is given by

$$\Lambda'(\phi) = G'(\phi)J(\phi)/[1-\mu G(\phi)]^2, \tag{40}$$

where $J(\phi) = \mu(1-\mu G(\phi))[\Lambda(\phi)-\phi/\mu]$. As shown in the proof of Proposition 2, there exists $\phi = \bar{\phi} \in (0,\eta)$ such that for $\phi \in [0,\bar{\phi})$, we have $J(\phi) > 0$, and for $\phi \in (\bar{\phi},\eta]$, we have $J(\phi) < 0$. From Eq. (39), it follows that $\Lambda(0) = H(0) > 0$, and $\Lambda(\eta) = 0$. Then, the configuration of $\Lambda(\phi)$ is obtained as in Figure 3. Note that $\bar{\phi}$ is given by the intersection of $\Lambda(\phi)$ and ϕ/μ. Moreover, note from Proposition 2 that ϕ^* is given by the intersection of $\Lambda(\phi)$ and $\Gamma(\phi) := \alpha\phi/[(1-\alpha)(1-\mu)\Omega]$. Figure 3 illustrates $\Lambda(\phi)$ and $\Gamma(\phi)$. Because $\Lambda(\phi)$ is inverted-U shaped, there exist a solution, $\tilde{\phi} > 0$, for $\Lambda(\phi) = H(0)$.

Proposition 4 *With all parameter values being fixed, if* $\alpha/[(1-\alpha)(1-\mu)\Omega] >$ $(<)$ $H(0)/\tilde{\phi}$*, then more (less) capital accumulates in the bubbly steady state than in the bubbleless steady state.*

Proof. See the Appendix.

As can be seen in Figure 3, as the upper limit of the issuance of new bubbly assets is more relaxed, i.e., as μ increases, $\Lambda(\phi)$ shifts upward and $\Gamma(\phi)$ rotates counterclockwise. Therefore, as the upper limit of the issuance of new bubbly assets is more relaxed, it is more likely that more capital accumulates in the bubbly steady state than in the bubbleless steady state. In such a case, substantial issuance of the bubbly asset increases the market interest rate, r_t, and, thus, excludes a larger number of less-productive agents from production activities. Accordingly, more-productive agents intensively use more production resources.

As a result, they produce the final goods to a larger extent, and thus, capital accumulation is promoted.

Remark 1 below immediately follows from Proposition 4 because capital accumulation reduces the unemployment rate.

Remark 1 *With all parameter values being fixed, if* $\alpha/[(1-\alpha)(1-\mu)\Omega] > (<) H(0)/\tilde{\phi}$*, the unemployment rate in the bubbly steady state,* u^**, is lower (higher) than in the bubbleless steady state,* u^{**}*.*

[Figure 3 around here]

4.2 Numerical analysis

In this section, we numerically investigate the effects that labor market conditions, such as workers' Nash bargaining power, β, the unemployment benefit ratio, γ, and the search cost, h, have on macroeconomic variables such as capital accumulation per worker, consumption per capita, unemployment rates, and labor market tightness (the jobs-to-applicants ratio) in both bubbly and bubbleless steady states.

4.2.1 Specification and parameterization

In performing the numerical analysis, the matching function is specified as $F(L, v_t) = Lv_t(L^\sigma + v_t^\sigma)^{-1/\sigma}$, following Den Haan et al. (2000). This matching function appropriately satisfies the conditions imposed in section 2.4. It is assumed that the individual-specific productivity shock, Φ, is uniformly distributed in $[0, \eta]$. Under these assumptions, each variable can be computed as in the following. In the bubbleless steady state, we obtain

$$z^{**} = \left(\frac{A\eta(1-\alpha)\Omega}{2}\right)^{\frac{1}{1-\alpha}},$$

$$u^{**} = 1 - \left(1 - \frac{1}{(A(1-\Omega)(1-\alpha)(z^{**})^\alpha/h)^\sigma}\right)^{\frac{1}{\sigma}},$$

$$c^{**} = \alpha A(z^{**})^\alpha(1-u^{**}),$$

and

$$\theta^{**} = \left(\left(\frac{A(1-\Omega)(1-\alpha)(z^{**})^{\alpha}}{h} \right)^{\sigma} - 1 \right)^{\frac{1}{\sigma}},$$

where u^{**}, c^{**} and θ^{**} are the unemployment rate, consumption per capita, and labor market tightness (the jobs-to-applicants ratio) in the bubbleless steady state, respectively. Similarly, in the bubbly steady state, we obtain

$$z^{*} = \left(\frac{A\alpha\phi^{*}}{1-\mu} \right)^{\frac{1}{1-\alpha}},$$

$$u^{*} = 1 - \left(1 - \frac{1}{(A(1-\Omega)(1-\alpha)(z^{*})^{\alpha}/h)^{\sigma}} \right)^{\frac{1}{\sigma}},$$

$$c^{*} = \alpha A(z^{*})^{\alpha}(1-u^{*}) \left(\frac{\eta^{2} + (\phi^{*})^{2}}{\eta^{2} - (\phi^{*})^{2}} \right)$$

and

$$\theta^{*} = \left(\left(\frac{A(1-\Omega)(1-\alpha)(z^{*})^{\alpha}}{h} \right)^{\sigma} - 1 \right)^{\frac{1}{\sigma}},$$

where u^{*}, c^{*} and θ^{*} are the unemployment rate, consumption per capita, and labor market tightness (the jobs-to-applicants ratio) in the bubbly steady state, respectively. Moreover, ϕ^{*} is computed from Eq. (29) as

$$\phi^{*} = \frac{\eta \left(-\alpha + \sqrt{\alpha^{2} + (1-\alpha)(1-\mu)\Omega[(1-\alpha)(1-\mu)\Omega - 2\alpha\mu]} \right)}{(1-\alpha)(1-\mu)\Omega - 2\alpha\mu}.$$

[Table 1 around here]

The parameters applied in the analysis are given in Table 1. Following Den Haan et al. (2000), we set $\alpha = 0.36$. We examine the effects of β (workers' Nash bargaining power), γ (the unemployment benefit ratio) and h (the search cost) by varying these variables. If β and/or γ are close to 1, Ω is also close to 1. In this case, the economy becomes infeasible because production never occurs as clarified in Proposition 1 (in section 2) and the discussion following the proposition. Therefore, we must impose a ceiling on β and γ. Den Haan et al. (2000) examine the case in which the firm's Nash bargaining power is 0.50, and thus, we vary β from 0.40 to 0.60 when examining the effect of β. Regarding the unemployment benefit, the 45%-80% of the average wage for the past six months when employed is paid to the unemployed in Japan as the unemployment benefit, and thus, we vary γ from 0.60 to 0.87 when examining the effect of γ. As in the case of β and γ, if the search cost, h, is too high, the economy becomes infeasible. Then, we set h as a relatively low value and vary h from 0.08 to 0.179. We fix $\beta = 0.5$, $\gamma = 0.8$ and $h = 0.11$ when the other variables are examined. We set η at a relatively high value, $\eta = 4$. This is because if η is small, the cutoff in the bubbly steady state, ϕ^*, is close to 0, and as a result, there appear only small differences between the bubbly and bubbleless steady states in capital per worker, z, consumption per capita, the unemployment rate, u, and the jobs-to-applicants ratio, θ. However, in the Great Recession of 2009, the difference in the unemployment rate before and after the crisis was approximately 5% in the United States. To yield such a significant difference in the unemployment rate following the bubble bursting, a relatively high value of η is necessary. Regarding μ, we assume an aggressive issuance of the new bubbly asset by private agents and set $\mu = 0.7$. Regarding the remaining parameter values, A and σ, we set $A = 1.5$ and $\sigma = 4$ such that the average unemployment rate is approximately 10% in the bubbleless steady state and approximately 2% in the bubbly steady state and the average jobs-to-applicants ratio is approximately 1.44 in the bubbleless steady state and 2.05 in the bubbly steady state when varying β.

Under these parameter values, the economy reflects the case in which the presence of asset bubbles promotes capital accumulation, which we focus on in the current analysis.

4.2.2 Labor market conditions and macroeconomic variables

As seen in Figure 4, the unemployment rate in the bubbly steady state, u^*, is always lower than that in the bubbleless steady state, u^{**}. In particular, when $\beta = 0.54$, the unemployment rates in the bubbly and bubbleless steady states are 2.1% and 9.9%, respectively. Although the difference in the unemployment rate between these two steady states when $\beta = 0.54$ is larger than the difference actually observed in 2007 and 2010 in the United States (4.7% and 9.7% respectively, as stated in the introduction), we can perceive the impact of the presence of asset bubbles from Figure 4. The jobs-to-applicants ratio in the bubbly steady state, θ^*, is also greater than that in the bubbleless steady state,

θ^{**}, and consumption per capita in the bubbly steady state, c^*, is also greater than that in the bubbleless steady state, c^{**}. These outcomes are obtained because we focus on the case in which the presence of asset bubbles promotes capital accumulation.

When varying β, which is the worker's bargaining power, from 0.40 to 0.60, capital, z, increases in both bubbly and bubbleless steady states. This outcome is not surprising. As β increases, Ω increases, and thus, the savings of young agents increase. Accordingly, more capital accumulates as β increases. Moreover, as β changes from 0.40 to 0.60, the jobs-to-applicants ratio decreases and the unemployment rate increases. These outcomes are not obvious because the workers' output share, Ω, has non-linear effects on these variables. As Ω increases, capital accumulation is promoted through agents savings, whereas the firms' output share decreases and the decrease in the firms' output share exerts downward pressure on the number of vacant positions. The effect of Ω on the unemployment rate (the jobs-to-applicants ratio) in both steady states can be proven to be U-shaped (inverted U-shaped). In both steady states, the minimum unemployment rate is achieved at approximately $\beta = 0.1$, which is unrealistically low bargaining power for advanced countries. When β changes from 0.40 to 0.60, the undesirable effect of Ω on the unemployment rate dominates the desirable effect and the unemployment rate increases. In that case, the marginal impact of β on the unemployment rate in the bubbleless steady state is always greater than that in the bubbly steady state. Therefore, as β increases, the difference in the unemployment rate between the bubbly and bubbleless steady states widens, although the difference in capital accumulation is relatively stable. Without asset bubbles, as β increases, the undesirable effect of Ω on the unemployment rate is accelerated. However, the presence of asset bubbles mitigates the acceleration of the undesirable effect. Finally, note from the descriptions of c^* and c^{**} that when both capital and the unemployment rate increase, they have conflicting impacts on consumption and the effect of β on consumption per capita in both bubbly and bubbleless steady states is inverted U-shaped. In particular, c^* is maximized at $\beta = 0.57$ and c^{**} is maximized at $\beta = 0.45$.

As seen in Figure 5, the effects of the unemployment benefit ratio, γ, on the macroeconomic variables are similar to those of the worker's bargaining power because both γ and β affect the macroeconomic variables through the worker's output share, Ω. Like β, γ also has non-linear effects on the unemployment rate and the jobs-to-applicants ratio in both steady states. However, assuming realistic values from 0.60 to 0.87 for γ, the undesirable effect of Ω on the unemployment rate dominates the desirable effect and the unemployment rates in both steady states increase. If the government implements a policy that increases the unemployment benefit ratio, γ, to a larger extent without asset bubbles, the unemployment rate increases significantly. In particular, if $\gamma = 0.87$, the unemployment rate in the bubbleless steady state is greater than 40%. However, the presence of asset bubbles lessens this undesirable outcome, and the unemployment rate is only approximately 6% when $\gamma = 0.87$. As in the case of β, the effects of γ on consumption in both steady states are inverted U-shaped: c^* is maximized at $\beta = 0.85$ and c^{**} is maximized at $\beta = 0.76$.

The search cost is also a labor market condition. Because the determination of capital in both steady states is independent of the search cost, z^* and z^{**} are constant in Figure 6 when varying h, although the minimum requirement of capital, $\bar{z}$, for the economy to be feasible increases as h increases. As expected, capital per worker, consumption per capita, and the jobs-to-applicants ratio in the bubbly steady state are greater than in the bubbleless steady state. When h changes from 0.08 to 0.179, the marginal impact of h on the unemployment rate is always greater in the bubbleless steady state than in the bubbly steady state, and the difference in the unemployment rate between these two steady states widens as h increases. In particular, when $h = 0.179$, the unemployment rate in the bubbleless steady state is more than 40%, whereas that in the bubbly steady state is approximately 6%, and the presence of asset bubbles mitigates the negative effect of the search cost to a larger extent.

[Figure 4 around here]

[Figure 5 around here]

[Figure 6 around here]

5 Self-fulfilling financial crisis

We consider a sunspot variable, ϵ_t, that follows a two-state Markov process, with support $\{0,1\}$, and transition probabilities are given by $Pr(\epsilon_t = 1|\epsilon_{t-1} = 1) = \pi^a$ and $Pr(\epsilon_t = 0|\epsilon_{t-1} = 0) = \pi^b$, where π^a and $\pi^b \in (0,1]$. Denote the history of sunspot events until time t by $\epsilon^t = \{\epsilon_0, \epsilon_1, ..., \epsilon_t\}$. The sunspot events are common across agents in each generation, being independent of idiosyncratic productivity shocks. The market price of the bubbly asset is subject to the sunspot variable, and thus, we denote $p_t = p_t(\epsilon_t)$. When determining the cutoff, ϕ_{t-1}, agents have rational expectations regarding future sunspot events given the sunspot event, ϵ_{t-1}, at time $t-1$, and thus, we denote $\phi_{t-1} = \phi_{t-1}(\epsilon_{t-1})$. Note that $\phi_{t-1}(\epsilon_{t-1})$ becomes a deterministic variable when ϵ_{t-1} is realized although it is a stochastic variable before the realization of ϵ_{t-1}.

5.1 Cutoffs in the stationary states

The cutoff, $\phi_{t-1}(\epsilon_{t-1})$, is no longer equal to r_t/q_t because the individual-specific return is a random variable. The market price of the bubbly asset is affected by the sunspot variable, and thus, the individual-specific return, R_{t+1}, is a function of ϵ_{t+1}, given ϵ^t. Then, R_{t+1} is denoted by $R_{t+1}(\epsilon_{t+1})$ and obtained as follows:

$$R_{t+1}(\epsilon_{t+1}) = \begin{cases} p_{t+1}(\epsilon_{t+1})/p_t(\epsilon_t) & \text{if } \Phi \leq \phi_t(\epsilon_t) \\ q_{t+1}(\epsilon^t)\Phi & \text{if } \Phi > \phi_t(\epsilon_t), \end{cases}$$

Note that $q_{t+1} = \alpha z_{t+1}^{\alpha-1}$ depends upon the sunspot history, ϵ^t, because capital at time $t+1$ is determined at time t. Given the sunspot event, ϵ_t, an agent at time t chooses a portfolio to maximize her expected lifetime utility:

$$E_t\left[U_t^\iota|\epsilon_t\right] = s_t^\iota E_t\left[R_{t+1}(\epsilon_{t+1})|\epsilon_t\right].$$

$E_t\left[R_{t+1}(\epsilon_{t+1})|\epsilon_t\right]$ is given by

$$E_t\left[R_{t+1}(\epsilon_{t+1})|\epsilon_t = 1\right] = \begin{cases} \rho_{t+1}^a & \text{if } \Phi \leq \phi_t(\epsilon_t = 1) \\ q_{t+1}(\epsilon_t = 1, \epsilon^{t-1})\Phi & \text{if } \Phi > \phi_t(\epsilon_t = 1), \end{cases}$$

and

$$E_t\left[R_{t+1}(\epsilon_{t+1})|\epsilon_t=0\right]=\begin{cases}\rho^b_{t+1} & \text{if } \Phi\le\phi_t(\epsilon_t=0)\\ q_{t+1}(\epsilon_t=0,\epsilon^{t-1})\Phi & \text{if } \Phi>\phi_t(\epsilon_t=0),\end{cases}$$

where $\rho^a_{t+1}=\{\pi^a p_{t+1}(1)+(1-\pi^a)p_{t+1}(0)\}/p_t(1)$ and $\rho^b_{t+1}=\{\pi^b p_{t+1}(0)+(1-\pi^b)p_{t+1}(1)\}/p_t(0)$. From these two equations, we obtain the cutoffs depending upon the sunspot realizations as $\phi^a_t:=\phi_t(\epsilon_t=1)=\rho^a_{t+1}/q_{t+1}(\epsilon_t=1,\epsilon^{t-1})$ and $\phi^b_t:=\phi_t(\epsilon_t=0)=\rho^b_{t+1}/q_{t+1}(\epsilon_t=0,\epsilon^{t-1})$.

Because the return from holding one unit of the bubbly asset is given by $p_t(\epsilon_t)/p_{t-1}(\epsilon_{t-1})$, Eq. (19) is rewritten as

$$B_t(\epsilon_t)=\frac{p_t(\epsilon_t)}{(1-\mu)p_{t-1}(\epsilon_{t-1})}B_{t-1}(\epsilon_{t-1}), \tag{41}$$

where $B_t(\epsilon_t)=p_t(\epsilon_t)M_t$. Given the sunspot event, ϵ_{t-1}, taking the expectation of both sides of Eq. (41) yields

$$E_{t-1}\left[B_t(\epsilon_t)|\epsilon_{t-1}\right]=\frac{E_{t-1}\left[p_t(\epsilon_t)|\epsilon_{t-1}\right]}{(1-\mu)p_{t-1}(\epsilon_{t-1})}B_{t-1}(\epsilon_{t-1}).$$

Depending upon the realization of ϵ_{t-1}, this equation can be rewritten as

$$\pi^a B_t(1)+(1-\pi^a)B_t(0)=\frac{\rho^a_t}{1-\mu}B_{t-1}(1), \tag{42}$$

and

$$\pi^b B_t(0)+(1-\pi^b)B_t(1)=\frac{\rho^b_t}{1-\mu}B_{t-1}(0). \tag{43}$$

Inserting Eq. (21) into Eqs. (42) and (43) yields

$$\begin{aligned}Az^\alpha_t(1-u_t)&\left[\pi^a\frac{G(\phi^a_t)}{1-\mu G(\phi^a_t)}+(1-\pi^a)\frac{G(\phi^b_t)}{1-\mu G(\phi^b_t)}\right]\\ &=\frac{\rho^a_t}{1-\mu}\times\frac{Az^\alpha_{t-1}(1-u_{t-1})G(\phi^a_{t-1})}{1-\mu G(\phi^a_{t-1})}.\end{aligned} \tag{44}$$

and

$$Az_t^{\alpha}(1-u_t)\left[\pi^b \frac{G(\phi_t^b)}{1-\mu G(\phi_t^b)} + (1-\pi^b)\frac{G(\phi_t^a)}{1-\mu G(\phi_t^a)}\right] = \frac{\rho_t^b}{1-\mu} \times \frac{Az_{t-1}^{\alpha}(1-u_{t-1})G(\phi_{t-1}^b)}{1-\mu G(\phi_{t-1}^b)}, \tag{45}$$

respectively. In what follows, our analysis focuses on a stationary rational expectations equilibrium with sunspots, such that $\phi^a := \phi_t^a = \phi_{t-1}^a$ and $\phi^b := \phi_t^b = \phi_{t-1}^b$.[10] By using Eqs. (24), (44) and (45) with $\phi^a = \rho_t^a/q_t(\epsilon^{t-1})$ and $\phi^b = \rho_t^b/q_t(\epsilon^{t-1})$, we obtain the following two equations:

$$\pi^a \frac{G(\phi^a)}{1-\mu G(\phi^a)} + (1-\pi^a)\frac{G(\phi^b)}{1-\mu G(\phi^b)} = \frac{\alpha\phi^a G(\phi^a)}{(1-\mu)(1-\alpha)\Omega H(\phi^a)} \tag{46}$$

and

$$\pi^b \frac{G(\phi^b)}{1-\mu G(\phi^b)} + (1-\pi^b)\frac{G(\phi^a)}{1-\mu G(\phi^a)} = \frac{\alpha\phi^b G(\phi^b)}{(1-\mu)(1-\alpha)\Omega H(\phi^b)}. \tag{47}$$

We assume that $\phi^a > \phi^b$ without loss of generality. Because the bubbly asset is freely disposable, $B_t \geq 0$, and thus, $\phi_t \geq 0$ for all $t \geq 0$ from Eq. (21).

Lemma 2 *Suppose that $\pi^a \in (0,1)$ and $0 \leq \phi^b < \phi^a$. Then, if there exists a rational expectations equilibrium with the two-state sunspot variable that satisfies Eqs. (46) and (47), it must follow that $\phi^b = 0$ and $\pi^b = 1$.*

Proof. See the Appendix.

Proposition 5 *There exists a rational expectations equilibrium with the two-state sunspot variable that satisfies Eqs. (46) and (47) such that $\phi^b = 0$ with $\pi^b = 1$ and $\phi^a \in (0, \phi^*)$ with $\pi^a \in (0,1)$.*

Proof. See the Appendix.

The state given by ϕ^a is bubbly whereas the state given by ϕ^b is bubbleless. Proposition 5 implies that once asset bubbles burst due to self-fulfilling expectations, the bubbly asset never has a market value after the bubble bursts. This

[10] To be accurate, z_t and u_t in Eqs. (44) and (45) depend on the sunspot history, ϵ^{t-1}. Therefore, we should have explicitly written $z_t(\epsilon_{t-1} = 1, \epsilon^{t-2})$ and $u_t(\epsilon_{t-1} = 1, \epsilon^{t-2})$ in Eq. (44) and $z_t(\epsilon_{t-1} = 0, \epsilon^{t-2})$ and $u_t(\epsilon_{t-1} = 0, \epsilon^{t-2})$ in Eq. (45); however, we use simple notations to save space.

outcome is obtained because the bubbly asset is freely disposable and because the steady state, ϕ^*, is unstable and the steady state, $\phi^{**} = 0$, is stable in the dynamical system (27), as demonstrated in the previous section. As noted from Eq. (26), capital accumulation in each state is given by

$$z_t \Psi(z_t) = \frac{(1-\alpha)\Omega A H(\phi^a)}{1-\mu G(\phi^a)} z_{t-1}^{\alpha} \Psi(z_{t-1}) \tag{48}$$

and

$$z_t \Psi(z_t) = (1-\alpha)\Omega A H(0) z_{t-1}^{\alpha} \Psi(z_{t-1}), \tag{49}$$

respectively. Figure 7 illustrates the case in which the presence of asset bubbles promotes capital accumulation, i.e., $H(\phi^a)/[1-\mu G(\phi^a)] > H(0)$. In this case, Eq. (48) is located in the upper place relative to Eq. (49).

[Figure 7 around here]

Now, we assume that $\epsilon_0 = 1$, meaning that asset bubbles are present at time 0. In this case, capital accumulates over time if $z_0 < z^a$, where $z^a = Q(\phi^a)^{1/(1-\alpha)}$, as seen in Figure 7. However, once asset bubbles burst at a certain time, say, $t = \hat{t}$, capital begins to decrease if $z_{\hat{t}} > z^b$, where $z^b = Q(\phi^b)^{1/(1-\alpha)}$, and accordingly, the unemployment rate begins to increase following Eq. (26).

6 Conclusion

An overlapping-generations model is presented in which the presence of asset bubbles á la Tirole (1985) can promote capital accumulation under mild parameter conditions. In a financially constrained economy, although the presence of asset bubbles corrects allocative inefficiency by excluding less-productive agents from production activities, only the second-best outcome can be attained, as clarified by Bewley (1980). This consequence can be easily verified in our model by observing that not only agents who draw the highest productivity shock but also agents who draw relatively low productivity shocks engage in capital production when asset bubbles are present. Therefore, the unemployment rate when asset bubbles occur is not the lowest relative to that in the first-best outcome, which means that government policy is necessary for the economy to be Pareto-improved despite that the presence of asset bubbles reduces the unemployment rate. The analysis of such government policy is beyond the scope of the current paper and left for future research.

Appendix

Proof of Proposition 1

If $z_t \leq \bar{z}$, no firms with a vacancy can cover the search cost, h, at time t, despite that the matching probability is equal to one. Therefore, no firms can operate at time t. If $z_t > \bar{z}$, given the matching probability $f(\theta_t) \in (0,1)$, firms with a vacancy will cover the search cost, h, and firms that successfully match with a worker operate their business at time t. $\square$

Proof of Proposition 2

From Eq. (25), $\phi^{**} = 0$ is obviously a steady state of the dynamical system because $G(\phi^{**}) = 0$. It is noted from Eq. (25) that ϕ^* is a candidate for another steady state. Therefore, all we must show is that ϕ^* is uniquely determined. Define $\Lambda(\phi) := H(\phi)/(1-\mu G(\phi))$ and $\Gamma(\phi) := \phi\alpha/[(1-\alpha)(1-\mu)\Omega]$. Note that $\Lambda(\phi)$ is the left-hand side of Eq. (28) and $\Gamma(\phi)$ is the right-hand side. $\Gamma(\phi)$ is linear with respect to ϕ with a positive slope and passes through the origin. To investigate the configuration of $\Lambda(\phi)$, define a function such that $J(\phi) := \mu H(\phi) - \phi(1-\mu G(\phi))$. Because $J'(\phi) = \mu G(\phi) - 1 < 0$, $J(\phi)$ is monotonically decreasing. Additionally, $J(0) = \mu H(0) > 0$ and $J(\eta) = -h(1-\mu) < 0$. Therefore, $J(\phi) = 0$ has a unique solution $\phi = \bar{\phi} \in (0,\eta)$ such that for $\phi \in [0,\bar{\phi})$, it follows that $J(\phi) > 0$ and for $\phi \in (\bar{\phi},\eta]$, it follows that $J(\phi) < 0$. Because $\Lambda'(\phi) = G'(\phi)J(\phi)/[1-\mu G(\phi)]^2$, $\Lambda'(\phi)$ is increasing in $\phi \in [0,\bar{\phi})$ and decreasing in $\phi \in (\bar{\phi},\eta]$. Moreover, $\Lambda(0) = H(0) > 0$ and $\Lambda(\eta) = 0$. As such, the configurations of $\Gamma(\phi)$ and $\Lambda(\phi)$ confirm the uniqueness of ϕ^* in Eq. (27). $\square$

Proof of Lemma1

Because $(1-\alpha)\Omega AH(\hat{\phi})\hat{z}^{\alpha-1}/(1-\mu G(\hat{\phi})) = 1$, the linearization of Eq. (26) around the steady state, $(\hat{z}, \hat{\phi})$, yields

$$z_t - \hat{z} = \frac{\alpha\Psi(\hat{z}) + \hat{z}\Psi'(\hat{z})}{\Psi(\hat{z}) + \hat{z}\Psi'(\hat{z})}(z_{t-1} - \hat{z}) + \frac{\hat{z}^{\alpha}\Psi(\hat{z})Q'(\hat{\phi})}{\Psi(\hat{z}) + \hat{z}\Psi'(\hat{z})}(\phi_{t-1} - \hat{\phi}), \tag{A.1}$$

where $Q(x) = (1-\alpha)\Omega H(x)/(1-\mu G(x))$. Eq. (A.1) yields $\kappa_1(\hat{z})$ as follows:

$$\kappa_1(\hat{z}) = \frac{\alpha\Psi(\hat{z}) + \hat{z}\Psi'(\hat{z})}{\Psi(\hat{z}) + \hat{z}\Psi'(\hat{z})},$$

where $\hat{z} = z^*$ or z^{**}. The linearization of Eq. (25) around the steady state yields

$$\phi_t - \hat{\phi} = \left(\frac{\alpha(1-\mu G(\hat{\phi}))}{(1-\mu)(1-\alpha)\Omega H(\hat{\phi})}\right)\left(\frac{G(\hat{\phi})(1-\mu G(\hat{\phi}))}{G'(\hat{\phi})}\right) \times \left[1 + \frac{\hat{\phi}G'(\hat{\phi})}{G(\phi)} + \frac{\hat{\phi}^2 G'(\hat{\phi})}{H(\hat{\phi})}\right](\phi_{t-1} - \hat{\phi}). \tag{A.2}$$

When $\hat{\phi} = \phi^*$, Eq. (A.2) can be rewritten as

$$\phi_t - \phi^* = \left(\frac{G(\phi^*)(1-\mu G(\phi^*))}{\phi^* G'(\phi^*)}\right)\left[1 + \frac{\phi^* G'(\phi^*)}{G(\phi^*)} + \frac{(\phi^*)^2 G'(\phi^*)}{H(\phi^*)}\right](\phi_{t-1} - \phi^*). \tag{A.3}$$

because $\alpha(1-\mu G(\phi^*))/[(1-\mu)(1-\alpha)\Omega H(\phi^*)] = 1/\phi^*$. Therefore, we have

$$\kappa_2(\phi^*) = \frac{G(\phi^*)(1-\mu G(\phi^*)}{\phi^* G'(\phi^*)}\left(1 + \frac{\phi^* G'(\phi^*)}{G(\phi^*)} + \frac{(\phi^*)^2 G'(\phi^*)}{H(\phi^*)}\right).$$

When $\hat{\phi} = \phi^{**} = 0$, Eq. (A.2) can be rewritten as

$$\phi_t - \phi^{**} = 0(\phi_{t-1} - \phi^{**}). \tag{A.4}$$

Therefore, we have

$$\kappa_2(\phi^{**}) = 0. \quad \square$$

Proof of Proposition 3

Obviously, it follows that $|\kappa_1(z^{**})| < 1$ and $|\kappa_2(\phi^{**})| < 1$. Therefore, the bubbleless steady state, (z^{**}, ϕ^{**}), is totally stable. Because $|\kappa_1(z^*)| < 1$, all we must show is that $|\kappa_2(\phi^*)| > 1$. To show this, define

$$\Theta(\phi) := \Gamma(\phi) - \Lambda(\phi),$$

where $\Gamma(\phi)$ and $\Lambda(\phi)$ are defined in the proof of Proposition 2. As shown in the proof of Proposition 2, $\Theta(\phi) = 0$ has a unique solution, which is $\phi = \phi^*$. Therefore, the fact that $\Theta(0) < 0$ and $\Theta(\eta) > 0$ implies that $\Theta'(\phi^*) > 0$, or equivalently,

$$\frac{\alpha}{(1-\alpha)(1-\mu)\Omega} - \frac{G'(\phi^*)[\mu H(\phi^*) - \phi^*(1-\mu G(\phi^*))]}{(1-\mu G(\phi^*))^2} > 0. \tag{B.1}$$

The use of Eq. (29) rewrites Eq. (B.1) as

$$\frac{H(\phi^*)}{\phi^*(1-\mu G(\phi^*))} - \frac{G'(\phi^*)[\mu H(\phi^*) - \phi^*(1-\mu G(\phi^*))]}{(1-\mu G(\phi^*))^2} > 0. \tag{B.2}$$

Furthermore, Eq. (B.2) can be computed as

$$(1-\mu G(\phi^*))\left(1 + \frac{\phi^* G'(\phi^*)}{H(\phi^*)}\right) - \phi^* \mu G'(\phi^*) > 0,$$

which can be transformed into

$$\frac{G(\phi^*)(1-\mu G(\phi^*))}{\phi^* G'(\phi^*)}\left(1 + \frac{\phi^* G'(\phi^*)}{G(\phi^*)} + \frac{(\phi^*)^2 G'(\phi^*)}{H(\phi^*)}\right) > 1. \tag{B.3}$$

The left-hand side of Eq. (B.3) is $\kappa_2(\phi^*)$, and thus, the bubbly steady state is a saddle point. □

Proof of Proposition 4

From Figure 4, if $\alpha/[(1-\alpha)(1-\mu)\Omega] > (<)\ H(0)/\tilde{\phi}$, it follows that $\Lambda(\phi^*) > (<)\ \Lambda(0) = H(0)$, and thus, $Q(\phi^*) > (<)\ Q(0)$. From the last and Proposition 2, we obtain the desired conclusion. □

Proof of lemma 2

The proof is done by contradiction. Suppose that $\pi^b \in (0,1)$. From Eq. (47), we have

$$\pi^b \left[-\frac{G(\phi^b)}{1-\mu G(\phi^b)} + \frac{G(\phi^a)}{1-\mu G(\phi^a)} \right] = -\frac{\alpha\phi^b G(\phi^b)}{(1-\mu)(1-\alpha)\Omega H(\phi^b)} + \frac{G(\phi^a)}{1-\mu G(\phi^a)}. \tag{C.1}$$

Because $\pi^b \in (0,1)$ and $\phi^b < \phi^a$, it follows from Eq. (C.1) that

$$\frac{G(\phi^b)}{1-\mu G(\phi^b)} < \frac{\alpha\phi^b G(\phi^b)}{(1-\mu)(1-\alpha)\Omega H(\phi^b)}. \tag{C.2}$$

Similarly, from Eq. (46), it follows that

$$\frac{G(\phi^a)}{1-\mu G(\phi^a)} > \frac{\alpha\phi^a G(\phi^a)}{(1-\mu)(1-\alpha)\Omega H(\phi^a)}. \tag{C.3}$$

From the configurations of $\Lambda(\phi)$ and $\Gamma(\phi)$ (which are defined in the proof of Proposition 2), for $\phi \in (\phi^*, \eta]$, it holds that $\Lambda(\phi) < \Gamma(\phi)$, and for $\phi \in [0, \phi^*)$, it holds that $\Lambda(\phi) > \Gamma(\phi)$. Therefore, we obtain $\phi^* < \phi^b$. Because $\phi^b < \phi^a$, we have $\Lambda(\phi^a) < \Gamma(\phi^a)$, or equivalently,

$$\frac{G(\phi^a)}{1-\mu G(\phi^a)} < \frac{\alpha\phi^a G(\phi^a)}{(1-\mu)(1-\alpha)\Omega H(\phi^a)}. \tag{C.4}$$

Eq. (C.4) contradicts Eq. (C.3). Therefore, it must follow that $\pi^b = 1$. When $\pi^b = 1$, Eq. (47) yields $\phi^b = \phi^*$ or $\phi^b = 0$. If $\phi^b = \phi^*$, we have $\phi^* < \phi^a$. However, $\phi^* < \phi^a$ again leads a contradiction. Hence, $\phi^b = 0$. □

Proof of Proposition 5

From Lemma 2, it must hold that $\pi^b = 1$ and $\phi^b = 0$. In this case, Eq. (46) can be rewritten as

$$\pi^a \frac{G(\phi^a)}{1-\mu G(\phi^a)} = \frac{\alpha\phi^a G(\phi^a)}{(1-\mu)(1-\alpha)\Omega H(\phi^a)}. \tag{D.1}$$

From Eq. (D.1), we obtain $\Lambda(\phi^a) > \Gamma(\phi^a)$ for $\pi^a \in (0,1)$, and thus, $\phi^a \in (0, \phi^*)$ with $\pi^a \in (0,1)$. □

References

[1] Aghion, Philippe and Peter Howitt (1994) Growth and unemployment. *Review of Economic Studies*, 61 (3), pp. 477-94.

[2] Aoki, Kosuke and Kalin Nikolov (2015) Bubbles, banks and financial stability. *Journal of Monetary Economics*, 74, pp. 33-51.

[3] Ball, Laurence and Robert Moffitt (2001) Productivity growth and the philips curve. In: Krueger, A. B., Solow, R. (Eds.) *The Roaring Nineties: Can Full Employment Be Sustained?*, Russell Sage Foundation, New York, pp. 61-90.

[4] Bean, Charles and Christopher A. Pissarides (1993) Unemployment, consumption and growth. *European Economic Review*, 37 (4), pp. 837-54.

[5] Bewley, Truman F. (1980) The optimum quantity of money. In: Kareken, J.H., Wallace, N. (Eds.), *Models of Monetary Economies*, Federal Reserve Bank of Minneapolis, Minneapolis, pp. 169-210.

[6] Blanchard, Olivier J. (1985) Debt, deficits, and finite horizons. *Journal Political Economy*, 93 (2), pp. 223-247.

[7] Caballero, Ricardo J. and Mohamad L. Hammour (1996) On the timing and efficiency of creative destruction. *Quarterly Journal of Economics*, 111 (3), pp. 805-852.

[8] Carvalho, Vasco M., Alberto Martin, and Jaume Ventura (2012) Understanding bubbly episodes. *American Economic Review: Papers & Proceedings*, 102 (3), pp. 95-100.

[9] Den Haan, Wouter J., Garey Ramey, and Joel Watson (2000) Job Destruction and Propagation of Shocks. *American Economic Review*, 90 (3), pp. 482-498.

[10] Diamond, Peter A (1982) Wage determination and efficiency in search equilibrium. *Review of Economic Studies*, 49 (2), pp. 217-227.

[11] Eriksson, Clas (1997) Is there a trade-off between employment and growth? *Oxford Economic Papers*, 49 (1), pp. 77-88.

[12] Farhi, Emmanuel and Jean Tirole (2012) Bubbly liquidity. *Review of Economic Studies*, 79 (2), pp. 678-706.

[13] Fitoussi, Jean-Paul, David Jestaz, Edmund S. Phelps, and Gylfi Zoega (2000) Roots of the Recent Recoveries: Labor Reforms or Private Sector Forces? *Brookings Papers on Economic Activity*, 31 (1), pp. 237-311.

[14] Futagami, Koichi and Akihisa Shibata (2000) Growth effects of bubbles in an endogenous growth model. *Japanese Economic Review*, 51 (2), pp. 221-35.

[15] Grossman, Gene M. and Yanagawa Noriyuki (1993) Asset bubbles and endogenous growth. *Journal of Monetary Economics*, 31 (1), pp. 3-19.

[16] Haruyama, Tetsugen and Campbell Leith (2010) Unemployment and the productivity slowdown: An efficiency wage perspective. *Japanese Economic Review*, 61 (3), pp. 301-319.

[17] Hashimoto, Ken-ichi and Ryonghum Im (2016a) Bubbles and unemployment in an endogenous growth model. *Oxford Economic Papers*, 68 (4), pp. 1084-1106.

[18] Hashimoto, Ken-ichi and Ryonghum Im (2016b) Asset bubbles, labor market frictions, and R&D-based growth. Mimeograph, *Kobe University.*

[19] Hirano, Tomohiro, Masaru Inaba, and Noriyuki Yanagawa (2015) Asset bubbles and bailouts. *Journal of Monetary Economics*, 76, S71-S89.

[20] Hirano, Tomohiro and Noriyuki Yanagawa (2016) Asset bubbles, endogenous growth, and financial frictions. *Review of Economic Studies*, forthcoming.

[21] Ikeda, Daisuke and Toan Phan (2015) Asset bubbles and global imbalances. Mimeograph, *University of North Carolina at Chapel Hill.*

[22] King, Ian and Don Ferguson (1993) Dynamic inefficiency, endogenous growth, and ponzi games. *Journal of Monetary Economics*, 32, pp. 79-104.

[23] Kocherlakota, Narayana R. (2011) Bubbles and unemployment. Mimeograph, *Federal Reserve Bank of Minneapolis.*

[24] Kunieda, Takuma (2008) Asset bubbles and borrowing constraints. *Journal of Mathematical Economics*, 44 (2), pp. 112-131.

[25] Kunieda, Takuma (2014) A Note on the crowd-in effect of asset bubbles in the perpetual youth model. *Mathematical Social Sciences*, 72 (C), pp. 50-54.

[26] Kunieda, Takuma and Akihisa Shibata (2016) Asset bubbles, economic growth, and a self-fulfilling financial crisis. *Journal of Monetary Economics*, 82, pp. 70-84.

[27] Martin, Alberto and Jaume Ventura (2012) Economic growth with bubbles. *American Economic Review*, 102 (6), pp. 3033-58.

[28] Matsuoka, Tarishi and Akihisa Shibata (2012) Asset bubbles, credit market imperfections, and technology choice. *Economics Letters*, 116 (1), pp. 52-55.

[29] Miao, Jianjun, Pengfei Wang and Lifang Xu (2016) Stock market bubbles and unemployment. *Economic Theory*, 61 (2), pp. 273-307.

[30] Mino, Kazuo (2008) Growth and bubbles with consumption externalities. *Japanese Economic Review*, 59 (1), pp. 33-53.

[31] Mitsui, Toshihide and Shinichi Watanabe (1989) Monetary growth in a Turnpike Environment. *Journal of Monetary Economics*, 24 (2), pp. 123-137.

[32] Mortensen, Dale T. and Christpher A. Pissarides (1999) Job reallocation and employment fluctuations. In: Woodford, M., Taylor, J. B. (Eds.), *Handbook of Macroeconomics, Vol. 1.* Elsevier Science, Amsterdam, pp. 1171-227.

[33] Muscatelli, Anton V. and Patrizio Tirelli (2001) Unemployment and growth: Some empirical evidence from structural time series models. *Applied Economics*, 33 (8), pp. 1083-88.

[34] Petrongolo, Barbara and Christopher A. Pissarides (2001) Looking into the black box: A survey of the matching function. *Journal of Economic Literature*, 39 (2), pp. 390-431.

[35] Phelps, Edmund S. (1999) Behind this structural boom: The role of asset valuations. *American Economic Review*, 89 (2), pp. 63-68.

[36] Pissarides, Christopher A. (2000) *Equilibrium Unemployment Theory 2nd edition*, MIT Press, Cambridge, MA.

[37] Pissarides, Christopher A. and Giovanna Vallanti (2007) The impact of TFP growth on steady-state unemployment. *International Economic Review*, 48 (2), pp. 607-640.

[38] Samuelson, Paul A. (1958) An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of Political Economy*, 66 (6), 467-482.

[39] Santos, Manuel S. and Michael Woodford (1997) Rational asset pricing bubbles. *Econometrica*, 65 (1), 19-57.

[40] Staiger, Douglas, James H. Stock, and Mark W. Watson (2001) Prices, wages and the U.S. NARIU in the 1990s. In: Krueger, A. B., Solow, R. (Eds.), *The Roaring Nineties: Can Full Employment Be Sustained?*, Russell Sage Foundation, New York, pp. 3-64.

[41] Tirole, Jean (1985) Asset bubbles and overlapping generations. *Econometrica*, 53 (6), pp. 1499-528.

[42] Tripier, Fabien (2006) Sticky prices, fair wages, and the co-movements of unemployment and labor productivity growth. *Journal of Economic Dynamics and Control*, 30 (12), pp. 2749-2774.

[43] Weil, Philippe (1987) Confidence and the real value of money in an overlapping generations economy. *Quarterly Journal of Economics*, 102 (1), pp. 1-22.

[44] World Bank (2016) *World Development Indicators.* Washington, DC: World Bank.

Table 1: Parameterization

$\alpha = 0.36$	$\beta = 0.50$ (Fig. 5, Fig.6)	$\gamma = 0.80$ (Fig. 4, Fig. 6)	$\eta = 4$
$\sigma = 4$	$\mu = 0.7$	$A = 1.5$	$h = 0.11$ (Fig. 4, Fig. 5)

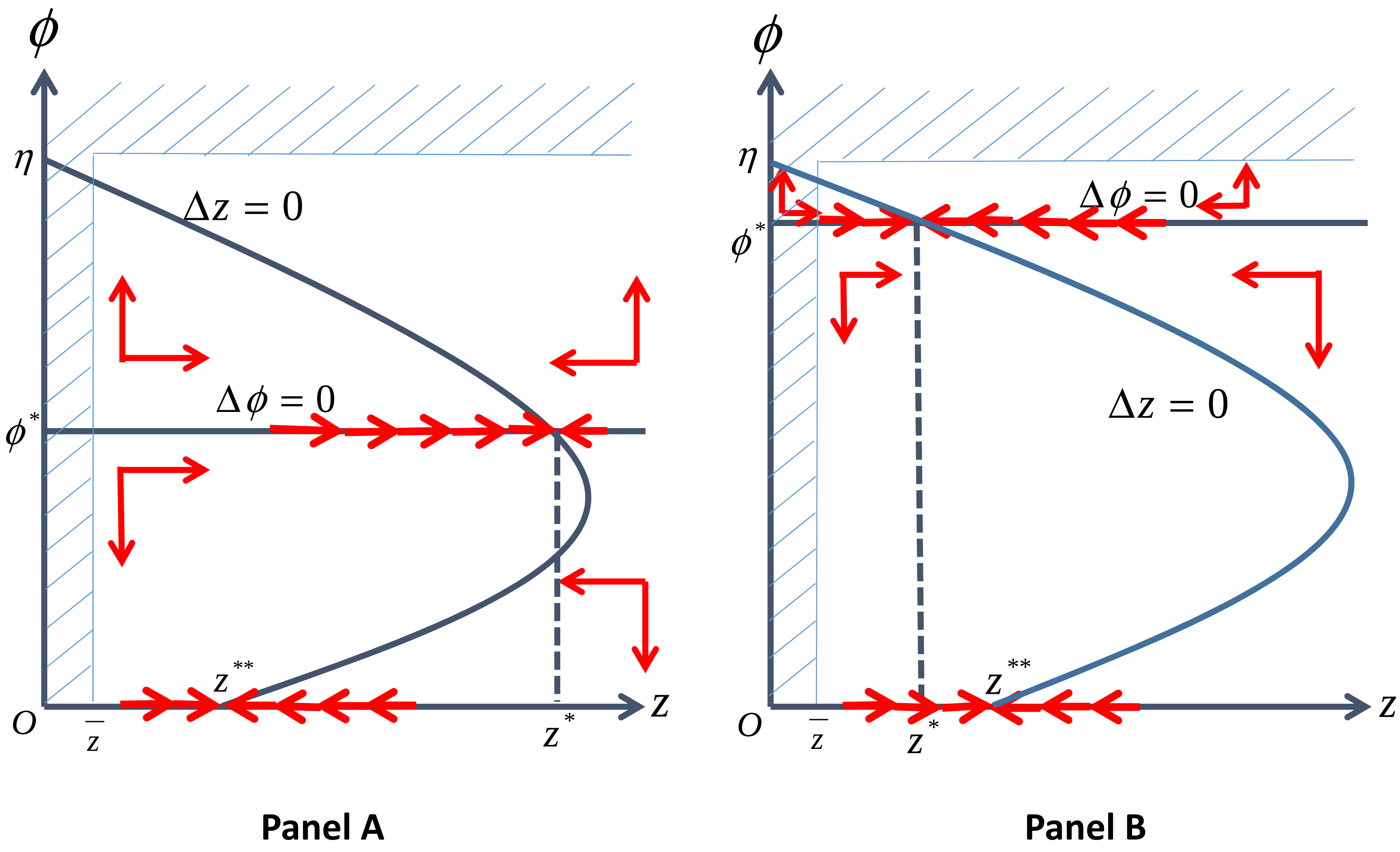

Fig. 1. Phase diagrams. *Notes:* **Panel A** is the case in which the presence of bubbles promotes capital accumulation and **Panel B** is the case in which the presence of bubbles impedes capital accumulation. The shadow area is *not* a domain of the dynamical system.

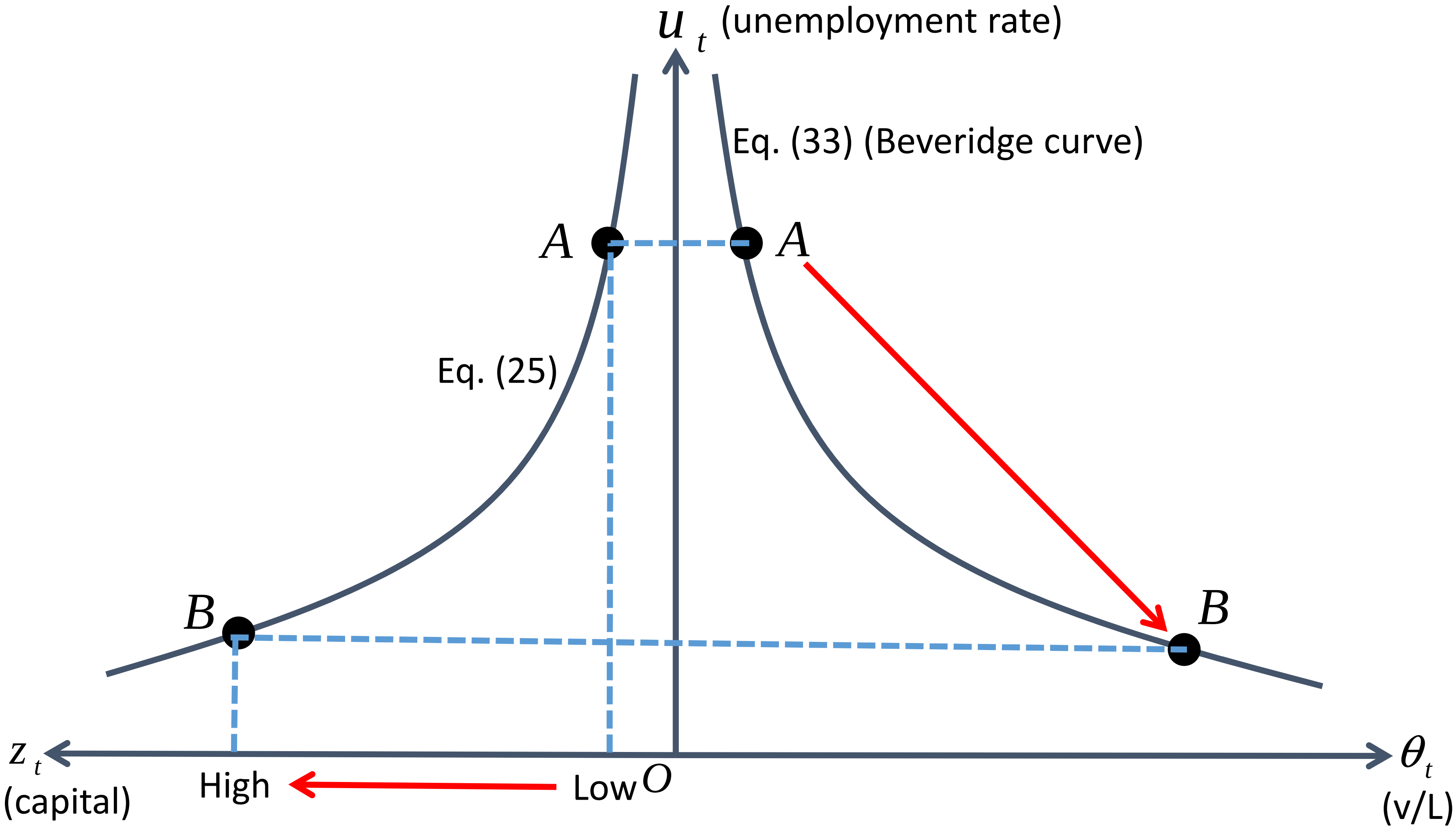

Fig. 2. Beveridge curve and capital accumulation. *Notes:* Capital accumulation is low at point A and high at point B. Capital accumulation promotes employment, rendering the labor market tighter.

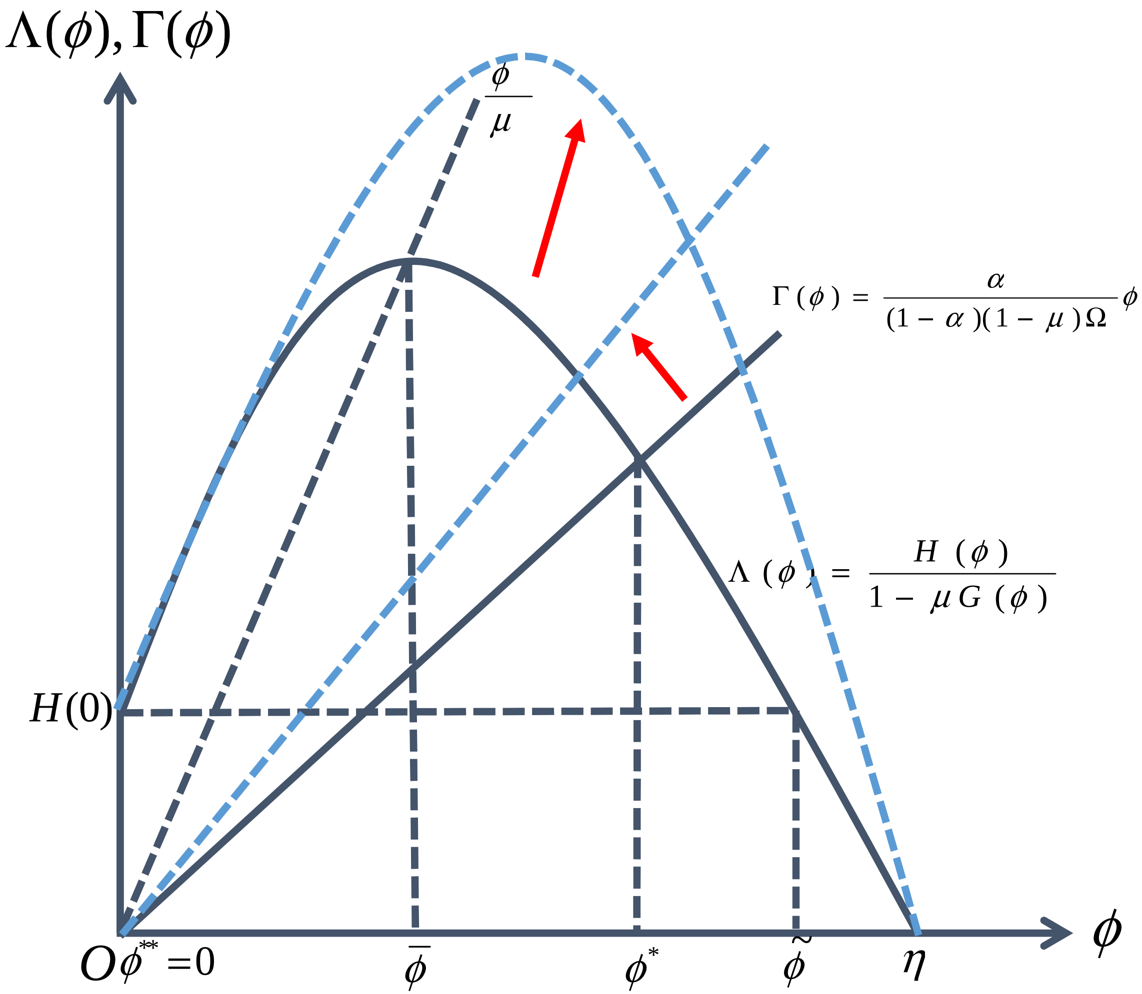

Fig. 3. Comparison between bubbly and bubbleless steady states. *Notes:* If $\Lambda(\phi^*) > H(0)$, capital accumulates more in the bubbly steady state, and if $\Lambda(\phi^*) < H(0)$, capital accumulates less in the bubbly steady state than in the bubbleless steady state. As μ increases, $\Lambda(\phi)$ shifts upward and $\Gamma(\phi)$ rotates counterclockwise.

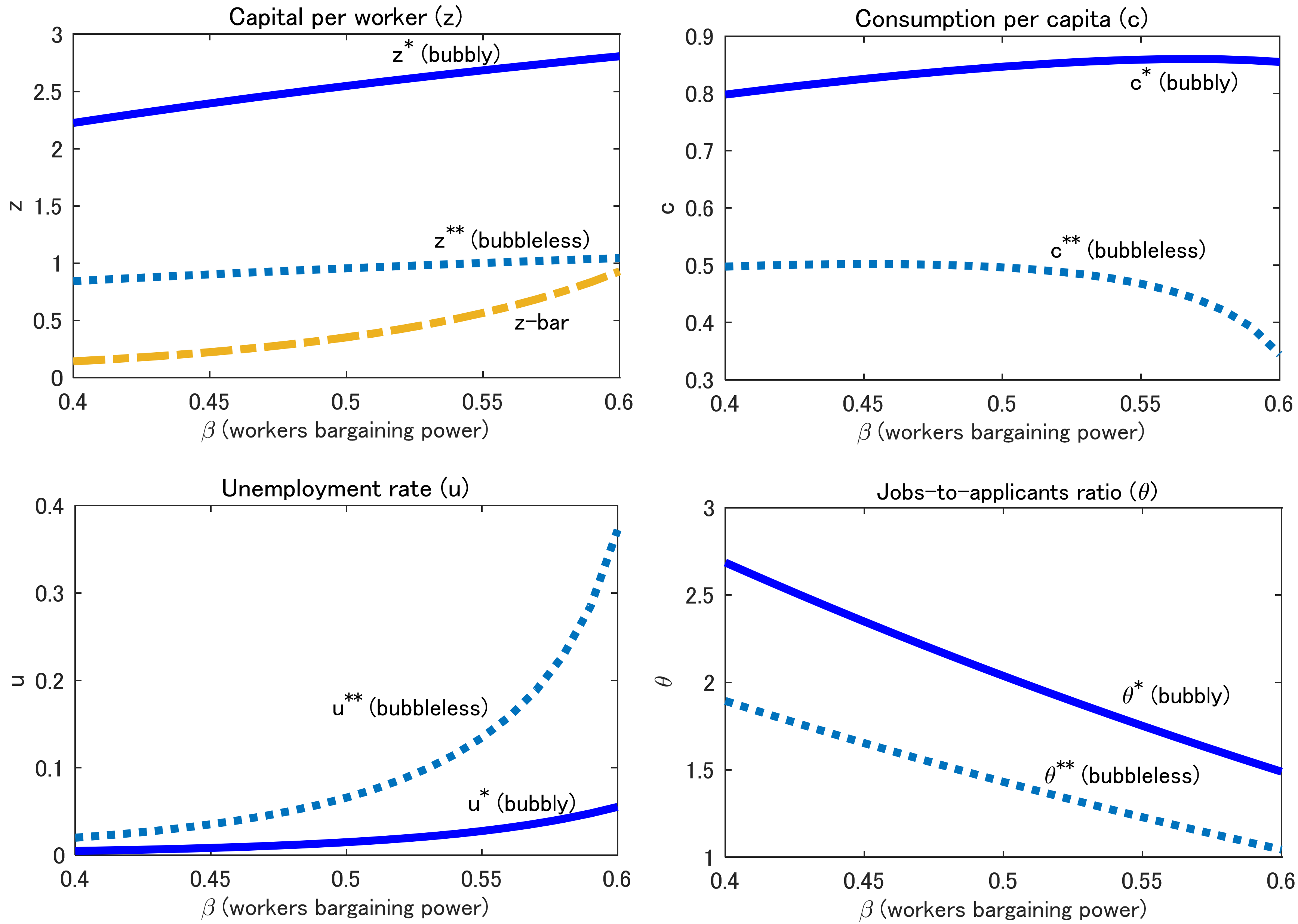

Fig. 4. Effects of workers' bargaining power. *Notes:* z-bar means the lower limit of capital, $\bar{z}$, defined in Proposition 1.

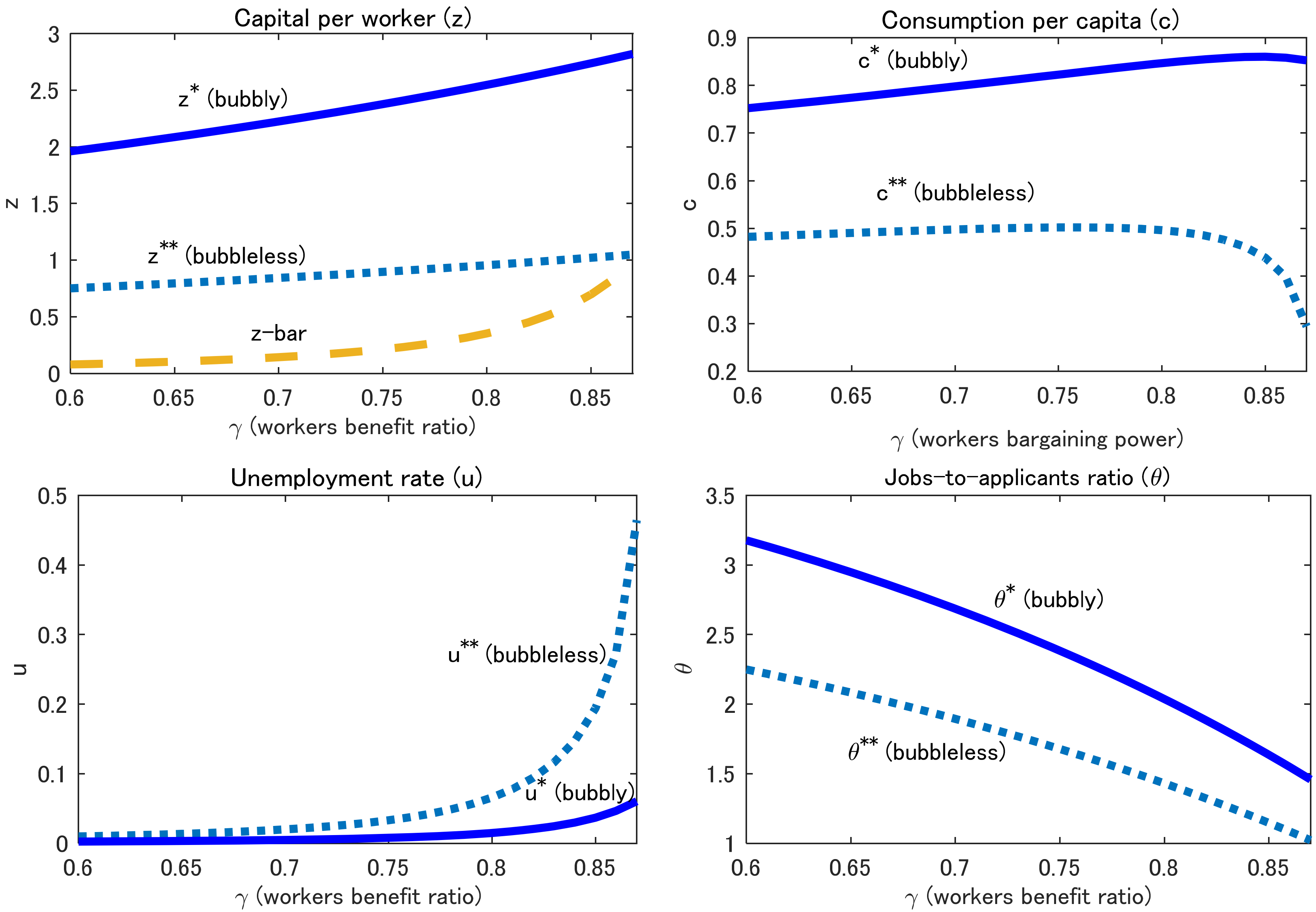

Fig. 5. Effects of the unemployment benefit ratio. *Notes:* z-bar means the lower limit of capital, $\bar{z}$, defined in Proposition 1.

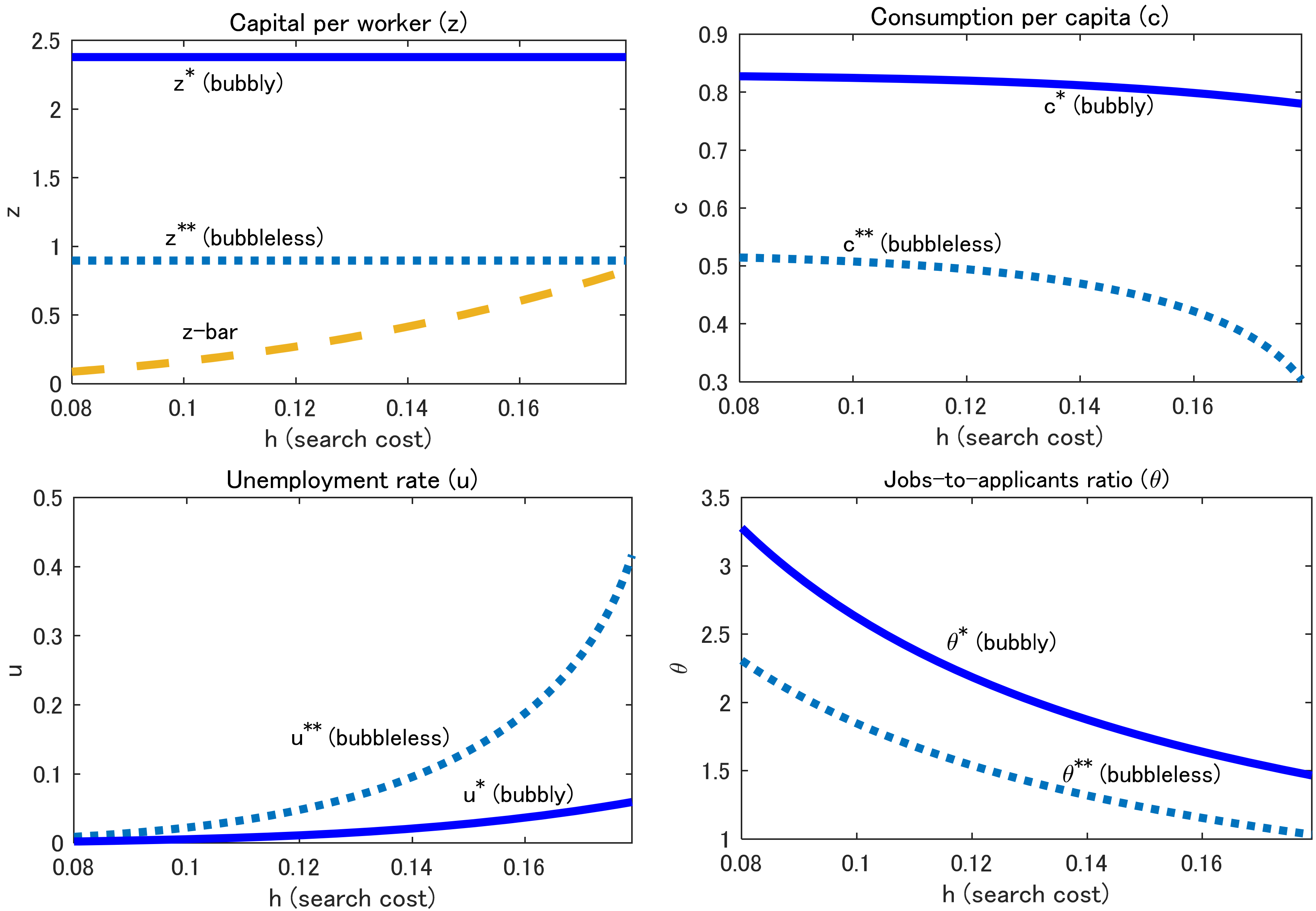

Fig. 6. Effects of the search cost. *Notes:* z-bar means the lower limit of capital, $\bar{z}$, defined in Proposition 1.

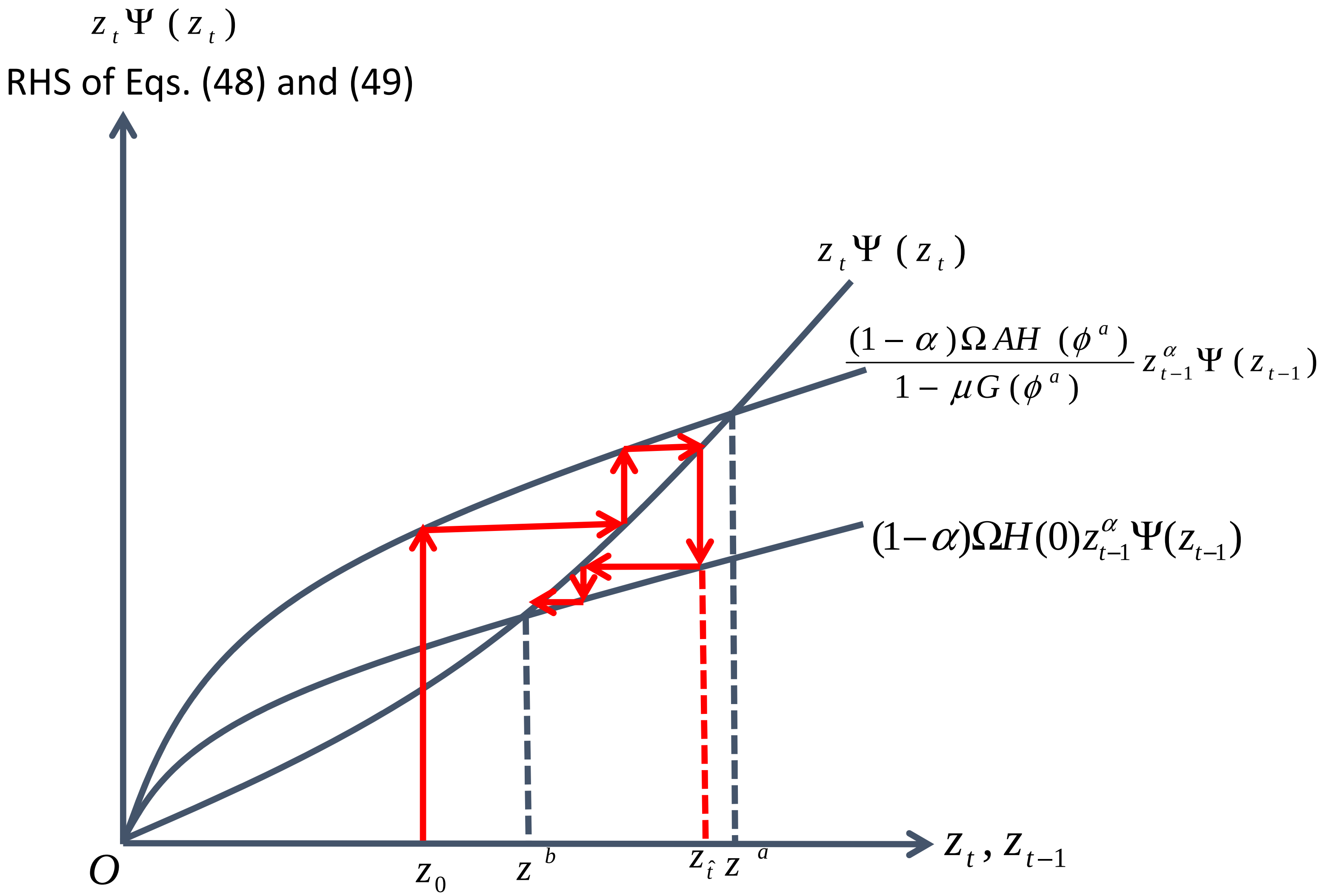

Fig. 7. Bubble bursting and financial crisis. *Notes:* This is the case in which $H(\phi^a)/[1-\mu G(\phi^a)] > H(0)$. Asset bubbles burst at time $\hat{t}$ and capital begins to decrease.