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Population Aging, Unfunded Social Security and Economic Growth

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Abstract

This paper examines how population aging caused by a decline in the birth rate or a reduction in the mortality rate affects economic growth in an overlapping generations model with a general demographic structure and a sizable unfunded social security system. Through numerical simulations, we show that a decline in the birth rate has non-monotonic effects on economic growth, yielding a hump-shaped relationship between the population growth rate and the economic growth rate, whereas a reduction in the mortality rate has a monotonic positive effect on economic growth, yielding a monotonic positive relationship between the population growth rate and the economic growth rate. We also use our model to study how predicted and occurring demographic changes in Japan affect that country's economic growth rate. We show that the growth effect of the predicted demographic changes in Japan is initially positive but it may turn out to be negative from the mid 2030s forward. This paper also examines the growth and welfare effects of a reduction in pension payments or an extension of the retirement age, and shows that the pension payment reduction policy is better than the retirement extension policy for both growth and welfare in response to population aging.

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1 Introduction

During the last half century, the demographic structure of all developed nations changed dramatically. Due to the decline in both birth and mortality rates, the relative share of elderly persons in society increased rapidly, and this trend is predicted to persist for years to come. Based on estimates by the United Nations, Table 1 depicts the observed and predicted trend in life expectancy at birth, the crude birth rate, the population growth rate, and the old-age dependency ratio (the ratio of the population over age 65 to the population aged 20-64) among the G7 countries. Because of the combined effects of the decline in birth and mortality rates, the population growth rate will decrease and the old-age dependency ratio will increase in all G7 countries. Such rapid population aging and the resulting increases in social security expenses place enormous pressure on the sustainability of social security systems in these nations (OECD, 2015). Japan is now confronting the most severe such situation in the world. According to government projections by the National Institute of Population and Social Security Research, the old age dependency ratio in Japan is predicted to increase from 39% in 2010 to 84.4% in 2060. Oguro (2014) argues that the recent growth in the government debt-GDP ratio in Japan is attributable primarily to the rapidly swelling social security costs due to population aging.[1] The rapid population aging and its consequences for economic growth, as well as the solvency of Pay-As-You-Go (PAYG) social security systems, is a primary concern for both policy makers and researchers in most developed countries.

In this paper, we address how population aging caused by a decline in the birth rate or a reduction in the mortality rate affects economic growth in an overlapping generations model with a general demographic structure and a sizable unfunded social security system. To do so, we employ the dynamic overlapping generations framework developed by Bruce and Turnovsky (2013 a,b) and Mierau and Turnovsky (2014 a,b). We augment their analyses by examining the impact of aging on economic growth under a sizable unfunded social security system.

In the model presented here, we use numerical simulations to demonstrate that a decline in the birth rate has non-monotonic effects on economic growth, yielding a hump-shaped relationship between the population growth rate and the economic growth rate, whereas a reduction in the mortality rate has a monotonic positive effect on economic growth, yielding a monotonic positive relationship between the population growth rate and the economic growth rate. We also use our model to study how the demographic changes occurring and predicted in Japan affect the economic growth rate. We show that the growth effect of the predicted demo-

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[1]According to the OECD economic outlook for 2015, Japan's gross government debt reached 230% of GDP on a gross basis, the highest among all OECD countries.

graphic changes in Japan is initially positive but it may turn out to be negative from the mid 2030s forward. These results obtained in this paper are sharp contrast to those of Mierau and Turnovsky (2014a). Employing a model without unfunded social security, Mierau and Turnovsky (2014a) show that the relationship between the population growth rate and the economic growth rate is monotonic, regardless of the source of demographic change. When the source of the demographic change is a birth rate, they show that there is a monotonic negative relationship between the population growth rate and the economic growth rate, whereas the relationship is monotonically positive if the source of the demographic change is a change in the mortality rate. They also calibrate their model to actual US data and conclude that the decline in both birth and mortality rates over the last half of the twentieth century in the US have led to a steady increase in the long run economic growth rate.

The difference between our results and those of Mierau and Turnovsky (2014a) comes from our explicit considerations of unfunded social security systems and the resulting non-monotonic relationship between the population growth rate and the economic growth rate, when the source of the demographic change is a change in the birth rate. Under a sizable unfunded social security system, the decline in the birth rate and the resulting decrease in population growth rate provide the two competing influences on economic growth. On the one hand, the decline in the population growth rate mitigates the dilution of aggregate capital to the larger population, which enhances the accumulation of the aggregate capital and thereby positively affects economic growth. We denote this positive growth effect of a decline in the birth rate as the "anti-dilution effect." On the other hand, as argued by Bruce and Turnovsky (2013b), the existence of unfunded social security leads to an increase in the aggregate consumption-capital ratio and thereby negatively affects the accumulation of the aggregate capital as well as the economic growth rate. We denote this negative growth effect of the social security as the "social security burden effect." The decline in the birth rate and the resulting decrease in the population growth rate positively affect the social security tax rate. Because the negative growth effect of social security becomes more serious as the social security tax rate becomes higher, the negative "social security burden effect" is more likely to dominate (be dominated by) the positive "anti-dilution effect," when the population growth rate is sufficiently low (high) or the birth rate is sufficiently low (high). Therefore, under a sizable unfunded social security system, we can show that there is a hump-shaped relationship between the population growth rate and the economic growth rate, when the source of the demographic change is a change in birth rate. Due to these properties, suppose that the current social security system remains unchanged; in this case, the growth effect of the predicted demographic changes in Japan is initially positive but it may turn out to be negative from the mid 2030s.

3

Our numerical simulation results are partly consistent with recent empirical findings. A growing empirical literature on the economic consequences of demographic change, including studies by Bloom and Canning (2004), An and Jeon (2006), and Prskawetz et al. (2007), uses the age structure of the population (e.g., the share of population over age 65) as a proxy for demographic changes. Given that the age structure changes owing to the combined effect of the fertility rate and life expectancy, this proxy provides much richer information on demographic changes for a determination of economic performance than the fertility rate and life expectancy alone. These recent empirical studies observe a hump-shaped relationship between the age structure and the per capita output growth rate, based on the positive effect of a larger share of working age individuals in the population on growth and the negative effect of a larger share of elderly individuals in the population on growth. For example, using panel data from OECD countries over the period from 1960 to 2000, An and Jeon (2006) find a hump-shaped relationship between the old-age dependency ratio and economic growth based on the both cross-country regression and non-parametric kernel estimation. Further, using data from the EU-15 countries over the period from 1950 to 2005, Prskawetz et al. (2007) observe a negative correlation between the initial population shares of elderly people and growth in the following period but a positive correlation between growth and the working-age population. Based on their estimates and an extensive literature review, Prskawetz et al. (2007) conclude that an increase in the working-age population positively contributes to economic growth, whereas an increase in either the old-age population or the young population negatively affects economic growth.

Further, in the model presented here, we compare the growth and the welfare implications of the pension payment reduction policy and the retirement extension policy. At present, many OECD countries are introducing these types of policies to maintain the solvency of PAYG social security systems. For example, the 2004 pension reform in Japan shifted away from the standard practice of increasing employee pension contributions to guarantee a 59% benefit level and instead capped future contributions at 18.3%. In addition, Japan introduced a demographically modified indexation program to ensure that the size of pension benefits was consistent with the new contribution cap (i.e., the pension payment reduction policy).[2] Moreover, Denmark, Greece, Hungary, Italy, Korea and Turkey have each linked future increases in pension ages to changes in life expectancy (i.e., the retirement extension policy). [3] This paper demonstrates that the both pension payment reduction policy and retirement extension policy positively affect economic growth

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[2]The new pension level established by the 2004 pension reform was 50%. For further information on the 2004 pension reform in Japan, see, for example, Komamura (2007).

[3]See OECD (2015) for additional details.

and lifetime utility. However, the growth and welfare enhancing effects of the pension payment reduction policy are larger than those of the retirement extension policy. Therefore, the pension payment reduction policy is better than the retirement extension policy for both growth and welfare in response to population aging.

This paper relates to recent theoretical studies that attempt to explain the observed hump-shaped relationship between demographic change and economic growth (e.g., de la Croix and Licandro, 1999; Fuster, 1999; Boucekkine, de la Croix and Licandro, 2002; Miyazawa, 2006; Ito and Tabata, 2008; Tabata, 2015; Hashimoto and Tabata, 2016).[4] Among these studies, those by Ito and Tabata (2008) and Tabata (2015) are closely related to our contributions because they emphasize the design of the social security system as a factor to explain the observed hump-shaped relationship between the age structure and economic growth. Using a two period overlapping generations model with unfunded social security, Ito and Tabata (2008) show that there is a hump-shaped relationship between the population growth rate and economic growth. In economies in which the population growth rate is sufficiently low (e.g., developed countries), the decline in the population growth rate increases the dependency ratio, increases the social security tax burden of young agents, decreases the aggregate saving rate, and thus negatively affects economic growth. In economies in which the population growth rate is sufficiently high (e.g., developing countries), however, the decline in the population growth rate mitigates a dilution of savings, increases the accumulation of the aggregate capital, strengthens the learning-by-doing effect, and thus positively affects economic growth. Although our paper shares numerous research interests with these analyses, our research differs from them because we employ the continuous time overlapping generations model with more general demographic structures. Our mechanism to generate the negative growth effect of unfunded social security is different from the one proposed in the existing two period overlapping generations framework. In this sense, our research succeeds in providing much richer implications regarding the effect of demographic changes on economic growth.

This paper also relates to recent theoretical studies that attempt to incorporate more realistic demographic structures into the continuous time overlapping generations model (e.g., de la Croix and Licandro, 1999; Boucekkine, de la Croix

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[4]For example, de la Croix and Licandro (1999) and Boucekkine, de la Croix and Licandro (2002) focus on the vintage nature of human capital and find a hump-shaped relationship between population growth (or life expectancy) and per capita output growth. Moreover, Fuster (1999) and Miyazawa (2006) focus on the role of accidental bequests and find a hump-shaped relationship between life expectancy and per capita output growth. Furthermore, Hashimoto and Tabata (2016) focus on the interactions between demographic changes and R&D-based innovations and find a hump-shaped relationship between demographic change and per capita output growth.

and Licandro, 2002; d'Albis and Augeraud-Véron, 2009 ; Heijdra and Mireau, 2012). [5] In particular, our paper is greatly indebted to a series of contributions by Stephan Turnovsky and his co-authors (e.g., Bruce and Turnovsky, 2013 a,b ; Mierau and Turnovsky, 2014 a,b; Mierau and Turnovsky, 2016). Among them, Bruce and Turnovsky (2013b) and Mierau and Turnovsky (2015) are closely related to our contributions because these analyses rigorously examine the impact of unfunded social security reform on growth and welfare. [6] The basic structure of the model presented in this paper is almost same as that of Bruce and Turnovsky (2013b) except for some minor modifications. Our research differs from theirs, however, because we are primarily concerned with the growth effect of population aging under a sizable unfunded social security system. We also examine how unfunded social security reform affects the relationship between the age structure and economic growth. To the best of my knowledge, these issues have yet to be examined rigorously in the literature. In this sense, this paper complements the analyses conducted by Bruce and Turnovsky (2013b) and Mierau and Turnovsky (2014a).

This paper is organized as follows. Section 2 presents the basic model. Section 3 discusses its macroeconomic equilibrium properties. Section 4 reports the numerical simulation results for the benchmark equilibrium. Section 5 demonstrates how population aging caused by a decline in the birth rate or a reduction in the mortality rate affects economic growth. Section 6 calibrates our model to actual and predicted demographic data in Japan. Section 7 compares the growth and welfare implications of a pension payment reduction policy and the retirement extension policy. Section 8 concludes, while the Appendix contains technical details regarding macroeconomic equilibrium properties.

2   The model

We consider the single sector endogenous growth model of overlapping generations with a general demographic structure and an unfunded social security system as set out by Bruce and Turnovsky (2013 a,b) and Mierau and Turnovsky (2014 a,b). We augment their analyses by examining the impact of aging on economic growth under a sizable unfunded social security system.[7]

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[5]See, for example, the Introduction of Mierau and Turnovsky (2014 a) for a more complete literature survey.

[6]Bruce and Turnovsky (2013b) examine this issue using the endogenous growth framework of Barro (1990), whereas Mierau and Turnovsky (2015) use the neoclassical growth framework.

[7]Because we share numerous modeling strategies with Bruce and Turnovsky (2013 a,b) and Mierau and Turnovsky (2014 a,b), we try to keep the model explanations simple by placing the technical details in the Appendix. Further, we try to follow their model explanations for ease of comparison.

2.1 Production

On the production side of the economy, we essentially follow the approach of Romer (1986). The production sector comprises many individual firms that exert productive externalities on each other so that, in equilibrium, the aggregate economy sustains endogenous growth.

There are $L(t)$ symmetric firms in this economy. Firms face perfectly competitive markets and maximize their profits. The production function of firm i at time t is represented as $Y_i(t) = Z(t)K_i(t)^\alpha L_i(t)^{1-\alpha}$, where $Y_i(t)$ is individual output, $K_i(t)$ is individual capital, $L_i(t)$ is individual labor demand, $Z(t)$ is the aggregate level of technology in the economy, and α is the capital share of output. In per worker terms, the production function can be expressed as: $\hat{y}_i(t) = Z(t)\hat{k}_i(t)^\alpha$, where $\hat{y}_i(t) \equiv \frac{Y_i(t)}{L_i(t)}$ and $\hat{k}_i(t) \equiv \frac{K_i(t)}{L_i(t)}$, respectively.

The firm hires capital and labor, paying each their respective marginal products:

$$r(t) = \alpha Z(t)\hat{k}_i(t)^{\alpha-1} - \delta, \tag{1}$$

$$w(t) = (1 - \alpha)Z(t)\hat{k}_i(t)^\alpha, \tag{2}$$

where $r(t)$ is the return to capital (interest rate), $w(t)$ is the wage rate and δ denotes the depreciation of capital. In accordance with Romer (1986), the interfirm productive externality is given by $Z(t) \equiv Z\hat{k}(t)^{1-\alpha}$ so that the aggregate per worker production function is of the AK-type, $\hat{y}(t) \equiv Z\hat{k}(t)$, where Z is the technology index and $\hat{k}(t) \equiv \frac{K(t)}{L(t)}$ is the economy wide capital-labor ratio. Accounting for the aggregate production externality, equilibrium factor prices are:

$$r = \alpha Z - \delta, \tag{3}$$

$$w(t) = (1 - \alpha)Z\hat{k}(t). \tag{4}$$

The interest rate remains constant over time, whereas the wage rate is proportional to the economy wide capital-labor ratio, $\hat{k}(t)$.

2.2 Individual behavior

Consider an individual born at time v. The probability that this agent lives to become $t - v$ years old is governed by the survival function $S(t - v)$, where $S'(t - v) < 0$, decreases with age. Given this function, the hazard rate, or instantaneous probability of death, is given by:

$$\mu(t - v) = -\frac{S'(t - v)}{S(t - v)}. \tag{5}$$

The probability that an individual dies before reaching age $t - v$ is described by the cumulative mortality rate:

$$M(t - v) = \int_0^{t-v} \mu(\tau)d\tau. \tag{6}$$

Combining equations (5) and (6), the survival function can be related to the mortality function by:

$$S(t - v) = e^{-M(t-v)}, \tag{7}$$

where $S(0) = e^{-M(0)} = 1$ and $S(D) = e^{-M(D)} = 0$, so that D defines the maximum age that individuals can attain.

The expected lifetime utility of an individual born at time v is given by:

$$E\Lambda(v) = \int_v^{v+D} U(C(v,t))e^{-\rho(t-v)-M(t-v)}, \tag{8}$$

where $C(v, t)$ is the consumption at time t of an individual born at time v, and ρ is the pure rate of time preference. We assume that the utility function is of the iso-elastic form:

$$U(C(v,t)) = \frac{C(v,t)^{1-1/\sigma}}{1 - 1/\sigma}, \tag{9}$$

where σ is the inter-temporal elasticity of substitution, which we shall assume lies in the range $(0, \frac{r-\pi}{r-\rho})$. We shall also assume that individuals are relatively patient and that the economy is dynamic efficient (i.e., $r > \rho > \pi$) so that this assumption allows $\sigma \in (0, \sigma^*)$, where $\sigma^* > 1$. [8] Each individual chooses his/her consumption and savings to maximize his/her discounted life time utility, (8), subject to the budget constraint:

$$A_t(v,t) \equiv \frac{\partial A(v,t)}{\partial t} = [r + \mu(t-v)]A(v,t) + [1 - \tau_s(t)]w(t)L(t-v) + B(v,t) - C(v,t), \tag{10}$$

where $A(v, t)$ are financial assets, $\tau_s(t)$ is the tax rate on labor income used to fund social security, and $B(v, t)$ is the household's age and time-dependent social security benefit, and $L(t - v) \leq 1$ is the exogenously given fraction of the household's unit time endowment supplied as labor. We assume that households reduce the fraction of time spent working as they age. That is $L'(t - v) < 0$. Given the function $L(t - v)$, the rate at which the labor supply fraction decreases with age is given by: $n(t - v) = -\frac{L'(t-v)}{L(t-v)}$. The cumulative of this rate before reaching age $t - v$ is described by $N(t - v) = \int_0^{t-v} n(\tau)d\tau$. Therefore, analogous to the case of the

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[8]While most estimates place σ well within the range (0,1), allowing σ to exceed unity is desirable because it enables us to accommodate some of the more extreme estimates reported in the literature.

survival function $S(t - v)$, we obtain $L(t - v) = e^{-N(t-v)}$, where $L(0) = e^{-N(0)} = 1$ and $L(s) = e^{-N(s)} = 0$ for $s \in [R, D]$. R defines the maximum age that individuals can work and satisfies the following relationship $R \leq D$.

To preserve tractability, as in Bruce and Turnovsky (2013b), we adopt a stylized form of social security benefit, namely $B(v, t) = bw(t)[1 - L(t - v)]$. That is, a household's social security benefit is equal to a time-independent fraction b (the benefit rate) of the earnings foregone as a result of the reduced labor supply due to retirement. In fact, a household cannot receive a social security retirement benefit until it reaches a certain age of eligibility (in Japan, 60 for a reduced benefit). Households can, however, receive social security disability benefits before the usual eligibility age for a social security retirement benefit (in Japan, 20 for a disability benefits). The disability benefit is determined according to a modified version of the usual social security benefit formula. Of course, the above payment scheme is rather abstract and cannot capture the complex structures of recent social security systems in their entirety. Nevertheless, this simple framework improves the tractability of the model greatly without changing the qualitative implications of this paper.

Individuals are born without assets, have no bequest motive, and are not allowed to have debt upon reaching the maximum attainable age, D. Therefore, $A(v, v) = 0$, and individuals fully annuitize all their assets. Annuities are life-insured financial assets that pay, conditional on the survival of the individual. Individuals receive a premium on these annuities equal to their instantaneous probability of death, $\mu(t - v)$, and in return, if an individual dies, his assets flow to the insurance company. Thus, the overall rate of return received by an individual on his/her assets is $r + \mu(t - v)$. Alternatively, an individual may engage in borrowing. In that case, he/she pays a premium of $\mu(t - v)$, and if he/she dies, his/her debts are canceled.

Optimizing (8) subject to (10) with respect to $C(v, t)$ and $A(v, t)$, yields the individual consumption Euler equation:

$$\frac{\partial C(v, t)/\partial t}{C(v, t)} = \sigma(r - \rho). \tag{11}$$

In addition, the agent must satisfy the transversality condition: $A(v, v + D) = 0$[9].

Solving (11) forward from time of birth, v, the individual's consumption at any age $t \geq v$ is linked to consumption at birth by the compounding relationship:

$$C(v, t) = C(v, v)e^{\sigma(r-\rho)(t-v)}. \tag{12}$$

To solve for $C(v, v)$, we integrate the budget constraint (10) forward from time v and impose the transversality condition, $A(v, v + D) = 0$, to yield the individual's

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[9]In the absence of a bequest motive, individuals want to ensure that $A(v, v+D) \leq 0$, and annuity firms want to ensure that $A(v, v + D) \geq 0$. Thus, the only feasible solution is $A(v, v + D) = 0$.

intertemporal budget constraint operative from time v. Substituting (12) into that constraint enables us to derive the following expression for $C(v, v)$:

$$C(v, v) = \frac{\hat{H}(v, v)}{\Delta(v, v)}, \tag{13}$$

where

$$\hat{H}(v, v) \equiv \int_v^{v+D} \{[1 - \tau_s(\tau)]w(\tau)L(\tau - v) + b(\tau)[1 - L(\tau - v)]w(\tau)\}e^{-r(\tau-v)-M(\tau-v)}d\tau, \tag{14}$$

is human wealth (the discounted lifetime income from after-tax wages and social security benefits) and:

$$\Delta(v, v) \equiv \int_v^{v+D} e^{-[(1-\sigma)r+\sigma\rho](\tau-v)-M(\tau-v)}d\tau, \tag{15}$$

is the inverse marginal propensity to consume out of total wealth. From (13) and (14), we observe that social security affects individual capital accumulation and consumption through its impact on human wealth.

For the tractability of the following dynamic stability analysis, we decompose the expression of $\hat{H}(v, v)$ into two parts.

$$\hat{H}(v, v) = H(v, v) + E(v), \tag{16}$$

where

$$H(v, v) \equiv \int_v^{v+D} [1 - b(\tau) - \tau_s(\tau)]w(\tau)L(\tau - v)e^{-r(\tau-v)-M(\tau-v)}d\tau,$$

$$E(v) \equiv \int_v^{v+D} b(\tau)w(\tau)e^{-r(\tau-v)-M(\tau-v)}d\tau.$$

This decomposition greatly improves the tractability of the following stability analysis.

2.3 Aggregate demography

Let $P(t)$ denote the size of the total population at time t. The birth rate, β, is constant, so that at every instant v, a cohort of size $P(v, v) = \beta P(v)$ is born. Given the mortality function, the number of individuals of cohort v still alive at time t is $P(v, t) = \beta P(v)e^{-M(t-v)}$. Similarly, at every instant v, a mass of $\bar{\mu}P(v)$ individuals dies, where $\bar{\mu}$ is the average mortality rate across cohorts: $\bar{\mu} \equiv \int_{t-D}^t \mu(t - v)P(v, t)dv/P(t)$, which, assuming the demographic steady state, defined in (21) below, is constant.

10

In the absence of migration, the growth rate of the population is equal to $\pi = \beta - \bar{\mu}$, which is therefore also constant. Hence, from the perspective of time v, the population at time t is equal to:

$$P(t) = P(v)e^{\pi(t-v)}. \tag{17}$$

The relative weight of a cohort v at time t is:

$$\frac{P(v,t)}{P(t)} = \beta e^{-\pi(t-v)-M(t-v)} \equiv p(t-v), \tag{18}$$

the dynamics of which are as follows:

$$\frac{p_t(t-v)}{p(t-v)} \equiv \frac{\partial p(t-v)}{\partial t} = -[\pi + \mu(t-v)]. \tag{19}$$

Thus, the decline in the relative size of each cohort reflects both the overall population growth rate and its individual mortality rate because of the arrival of newborns and the mortality of the existing cohort.

Aggregating over the surviving cohort members at each point in time, the total population at any time t is equal to:

$$P(t) = \beta \int_{t-D}^{t} P(v)e^{-M(t-v)}dv. \tag{20}$$

Substituting (17) into (20) yields the relationship:

$$\beta \int_{t-D}^{t} e^{-\pi(t-v)-M(t-v)}dv = 1, \tag{21}$$

which defines the demographic steady state (see Lotka, 1998). That is, (21) defines a constraint linking of the birth rate, β, mortality structure, ($M(t-v)$ and D), and the overall population growth rate, π. For example, given the birth rate, β, and mortality structure, ($M(t-v)$ and D), (21) yields the implied population growth rate, π. This relationship is an integral component of any consistently specified aggregate demographic structure.

Here, suppose that the birth rate, β, mortality structure, ($M(t-v)$ and D), and the overall population growth rate, π, are constant (i.e., time invariant demographic structure) and (21) depends only on age $s \equiv t-v$ but not on calendar time t. Therefore, we can rewrite (21) as

$$\beta \int_{0}^{D} e^{-\pi(s)-M(s)}ds = 1. \tag{22}$$

Analogously, from (18), the relative size of a cohort on age $s \equiv t-v$ at any time t is rewritten as

$$p(s) = \beta e^{-\pi s - M(s)}, \tag{23}$$

11

which also depends only on age $s \equiv t - v$. Therefore, suppose that the birth rate, β, mortality structure, ($M(t - v)$ and D), and the overall population growth rate, π, are constant (i.e., time invariant demographic structure), the relative size of a cohort on age $s \equiv t - v$ at any time t is also constant in the demographic steady state.

2.4 Aggregate behavior

Employing the following generic aggregator function, we can obtain the aggregate per capita equivalents of the individual quantities:

$$x(t) \equiv \int_{t-D}^{t} p(t - v)X(v, t)dv = \beta \int_{t-D}^{t} e^{-\pi(t-v)-M(t-v)}X(v, t)dv, \qquad (24)$$

where $x(t)$ is the aggregate per capita value of $X(v, t)$.

The straightforward application of (24) implies that aggregate per capita consumption is given by: $c(t) \equiv \int_{t-D}^{t} p(t - v)C(v, t)dv$. Taking the time derivative of $c(t)$ and substituting (11) and (18) into it, the dynamics of $c(t)$ can be expressed as:

$$\dot{c}(t) = \sigma(r - \rho)c(t) - \Phi(t), \qquad (25)$$

where

$$\Phi(t) \equiv \int_{t-D}^{t} \mu(t - v)p(t - v)C(v, t)dv - \beta C(t, t) + \pi c(t).$$

Here, $\Phi(t)$ is the generational turnover term. It measures the net reduction in the growth rate of aggregate per capita consumption due to the arrival of new individuals without assets, combined with the departure of individuals with positive assets.

Further, applying (24) to assets, we can define aggregate per-capita asset holdings as: $a(t) \equiv \int_{t-D}^{t} p(t - v)A(v, t)dv$. Taking the time derivative of $a(t)$ and substituting (10) , (18), and $B(v, t) = bw(t)[1 - L(t - v)]$ into it, the dynamics of $a(t)$ is given by:

$$\dot{a}(t) = (r - \pi)a(t) + w(t)l(t) - c(t) + \{bw(t)[1 - l(t)] - \tau_s(t)w(t)l(t)\}, \qquad (26)$$

where $l(t) \equiv \int_{t-D}^{t} L(t - v)p(t - v)dv$ is the fraction of the aggregate labor force relative to the overall population (i.e., $l(t) = \frac{L(t)}{P(t)}$). The curly bracket in the right hand side of (26) describes the contribution of social security to wealth accumulation.

Here, suppose that the birth rate, β, mortality structure, ($M(t - v)$ and D), the overall population growth rate, π, and the labor supply structure, ($L(t - v)$ and R), are constant (i.e., time invariant demographic structure), the fraction of the aggregate labor force relative to the overall population at any time t, $l(t)$, depends only on age $s \equiv t - v$ but not on calender time t. Thus, we can rewrite it as $l(t) = \int_0^D L(s)p(s)ds \equiv l$, and therefore the fraction of the aggregate labor force relative to the overall population at any time t is also constant in the demographic steady state.

2.5 The government budget constraint

We assume that the social security program is financed on a Pay-As-You-Go (PAYG) basis and funded by a dedicated tax on the earnings of labor. As outlined above, individuals pay their contribution through a tax on wage income, $\tau_s(t)$, and receive a benefit, $B(v, t)$, based upon the payment scheme specified above (i.e., $B(v, t) = bw(t)[1 - L(t - v)]$). Assuming no other government spending and abstracting from debt, the government faces the following balanced-budget constraint:

$$\tau_s(t) = b\frac{1 - l(t)}{l(t)} = bd(t), \tag{27}$$

where $d(t) \equiv \frac{1-l(t)}{l(t)}$ is the beneficiary-contributor ratio or the dependency ratio.

Here, as explained above, suppose that the birth rate, β, mortality structure, ($M(t - v)$ and D), the overall population growth rate, π, and the labor supply structure, ($L(t-v)$ and R), are constant (i.e., time invariant demographic structure), the relation $l(t) = l$ holds. Thus, we can rewrite equation (27) as $\tau_s(t) = bd \equiv \tau_s$, where $d \equiv \frac{1-l}{l}$, and therefore, the social security rate is also constant in the demographic steady state.

3 Equilibrium

We now derive the economy-wide equilibrium and describe its properties. In equilibrium, both the labor and the capital market must clear. In the absence of government bonds, all non-human wealth must be held in the form of physical capital so that $a(t) = k(t)$ and, therefore, $\dot{a}(t) = \dot{k}(t)$. Labor market clearance is achieved when the total demand for labor is equal to the all supplied labor, namely, $L(t) = l(t)P(t)$.

3.1 Existence

In this subsection, we first assume that the economy is always on its balanced growth path where equilibrium output and capital per capita grows at a constant rate of γ. The local stability analysis in the next subsection provides some justifications for this assumption. Noting the definitions of $k(t) \equiv \frac{K(t)}{P(t)}$ and $\hat{k}(t) \equiv \frac{K(t)}{L(t)}$, and substituting $a(t) = k(t)$, (3), (4) and (27) into (26), the dynamics of the aggregate per capita capital, $k(t)$, is given by

$$
\begin{aligned}
\dot{k}(t) &= Zk(t) - (\delta + \pi)k(t) - c(t), \\
&= (r - \pi)k(t) + (1 - \alpha)Zk(t) - c(t).
\end{aligned}
\tag{28}
$$

Noting that $\frac{c(t)}{k(t)} = \frac{c(t)}{w(t)l(t)} \frac{w(t)l(t)}{k(t)}$, the economy grows at a rate:

$$
\begin{aligned}
\gamma(t) &\equiv \frac{\dot{k}(t)}{k(t)} = Z - (\delta + \pi) - \frac{c(t)}{k(t)}, \\
&= r - \pi + (1 - \alpha)Z[1 - \frac{c(t)}{w(t)l(t)}].
\end{aligned}
\tag{29}
$$

Along the balanced growth path and noting $l(t) = l$, $d(t) = d$ and $\tau_s(t) = \tau_s$ under the time invariant demographic structure, the wage rate will grow at the common growth rate of γ. Hence, we can write aggregate per capita consumption as:

$$
\frac{c(t)}{w(t)} = \int_{t-D}^{t} p(t - v) \frac{C(v, v)}{w(v)} e^{[\sigma(r-\rho)-\gamma](t-v)} dv
\tag{30}
$$

Thus, substituting (13) to (17), (30), $d = \frac{1-l}{l}$ and $\tau_s = bd$ into (29), the growth rate of the economy is described implicitly as:

$$
\gamma = r - \pi + (1-\alpha)Z[1 - \frac{(1 - b - bd)\Omega(r - \gamma) + b\varphi(r - \gamma)}{\varphi((1 - \sigma)r + \sigma\rho)} \frac{\varphi(\gamma + \pi - \sigma(r - \rho))}{\Omega(\pi)}] \equiv f(\gamma),
\tag{31}
$$

where $\varphi(x) \equiv \int_0^D e^{-xs-M(s)} ds$ and $\Omega(x) \equiv \int_0^D L(s)e^{-xs-M(s)} ds$. Further, the demographic steady state (22) is rewritten as

$$
\beta\varphi(\pi) = 1.
\tag{32}
$$

The equilibrium growth rate γ of this economy is completely characterized by equations (31) and (32). For example, given the birth rate, β, mortality structure, ($M(t - v)$ and D) and labor supply structure ($L(t - v)$ and R), the definition of $d(t)$ and (32) yield the implied dependency ratio, d, and the population growth rate, π. By substituting these values into (31), the equilibrium growth rate γ of this economy is determined implicitly as (31).

14

Being nonlinear, (31) suggests the potential existence of more than one equilibrium growth rate, and indeed, inspection of (31) and (32) reveal that $\gamma = r - \pi$ is an equilibrium growth rate, satisfying $\gamma = f(\gamma)$. As in Mierau and Turnovsky (2014a), however, Appendix A shows that this solution is incompatible with the household's intertemporal budget constraint because it violates the transversality condition, and thus we can rule out this solution. Moreover, under some mild assumptions regarding the properties of the labor supply fraction function, $L(t - v)$, Appendix B confirms that there exists a unique consistent equilibrium growth rate γ^* satisfying (31) and (32). For example, in the following Section 4, we numerically solve (31) and (32) given the parameterized demographic functions, and confirm that there exists a unique consistent equilibrium growth rate γ^* satisfying (31) and (32) for plausible parameter values. Point A of Figure 1 in Section 4-2 illustrates the equilibrium growth rate by plotting both the left-hand and right-hand sides of (31) for our benchmark parameter values, defined below.

3.2   Local stability

In general, the macrodynamic equilibrium of this economy is described by the pair of equations of (25) and (28). It implies that we must take into account the dynamics of the intergenerational turnover term $\Phi(t)$. As discussed by Mierau and Turnovsky (2014a), for general mortality structures this term leads to a high order differential difference equation system that is generally intractable. The procedure developed by Mierau and Turnovsky (2014a), however, enables us to obtain a tractable local approximation of the dynamics. In the following analysis, we basically follow the local approximation method developed by Mierau and Turnovsky (2014a).

From (15) and (16), we rewrite $\Delta(t, t)$ and $H(t, t)$ as

$$\Delta(t) \equiv \Delta(t, t) \equiv \int_t^{t+D} e^{-[(1-\sigma)r+\sigma\rho](\tau-t)-M(\tau-t)} d\tau,$$

$$H(t) \equiv H(t, t) \equiv \int_t^{t+D} (1 - b - bd)w(\tau)L(\tau - t)e^{-r(\tau-t)-M(\tau-t)} d\tau.$$

By defining the stationary variables $x(t) \equiv \frac{c(t)}{k(t)}$, $y(t) \equiv \frac{H(t)}{k(t)}$, $e(t) \equiv \frac{E(t)}{k(t)}$ and $\Delta(t)$, we can show that the local dynamics in the neighborhood of the steady state,$(\tilde{x}, \tilde{y}, \tilde{e}, \tilde{\Delta})$, can be expressed as:

$$
\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{e}(t) \\ \dot{\Delta}(t) \end{pmatrix} = \begin{pmatrix} x - \beta\frac{y+e}{\Delta x} & \frac{\beta}{\Delta} & \frac{\beta}{\Delta} & -\beta\frac{y+e}{\Delta^2} \\ y & (1-b-bd)\frac{(1-\alpha)Z}{ly} & 0 & 0 \\ e & 0 & b\frac{(1-\alpha)Z}{le} & 0 \\ 0 & 0 & 0 & \frac{1}{\Delta} \end{pmatrix} \begin{pmatrix} x(t) - \tilde{x} \\ y(t) - \tilde{y} \\ e(t) - \tilde{e} \\ \Delta(t) - \tilde{\Delta} \end{pmatrix} \tag{33}
$$

The derivation of (33) is explained in Appendix C. To establish the stability characteristics of the system (33), we must analyze its four eigenvalues, $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. For example, if we suppose that these eigenvalues are all positive, the system is locally unstable. An inspection of (33) indicates that the relation $\lambda_4 = \frac{1}{\Delta} > 0$ holds; however, it is difficult to confirm the sign of other eigenvalues analytically. To obtain further insight regarding stability properties, in Section 4, we numerically compute the equilibrium values of λ_i, $(i = 1, 2, 3, 4)$. Numerical simulation results for very general demographic functions and a wide variety of underlying parameter values reveal that for any plausible parameter, all four eigenvalues are positive, which indicate that the equilibrium dynamics (33) are locally unstable. Therefore, the only viable equilibrium is for the system to always be on its balanced growth path, as in Romer (1986) and Mierau and Turnovsky (2014a). Below, we focus our analysis on the case where the economy is always on its unique consistent equilibrium balanced growth path.

3.3 Welfare

Substituting (4), (12) and (13) into (8), along the balanced growth path with time invariant demographic structure, the expected lifetime utility of a new-born household at time t is expressed as a multiple u of the wage rate at time t, that is,

$$
\begin{aligned}
E\Lambda(t) &= \frac{C(t,t)^{1-1/\sigma}}{1-1/\sigma} \int_t^{t+D} e^{-[(1-\sigma)r+\sigma\rho](\tau-t)-\sigma M(\tau-t)} d\tau, \\
&= uw(t)^{1-1/\sigma}, \\
&= uw(0)^{1-1/\sigma} e^{(1-1/\sigma)\gamma t},
\end{aligned} \tag{34}
$$

where

$$
u \equiv \frac{1}{1-1/\sigma}[(1-b-bd)\Omega(r-\gamma) + \varphi(r-\gamma)]^{1-1/\sigma}\varphi((1-\sigma)r+\sigma\rho)^{1/\sigma},
$$

and $s = t - v$. Here, note that the relation $u < 0$ (resp. $u > 0$) holds, if $\sigma \in (0, 1)$ (resp. $\sigma > 1$). Equation (34) indicates that given the utility multiplier, u, the higher equilibrium growth rate, γ, implies higher utility (i.e., $\frac{\partial E\Lambda(t)}{\partial \gamma} = (1 -$

$1/\sigma)E\Lambda(t) \geq 0)$. Moreover, suppose that initial wage rate $w(0)$ being compared is the same for an economy that is growing faster than another, and the value of its utility multiplier u is larger, we can conclude that all generations enjoy higher utility. We use these properties of the expected lifetime utility to analyze the welfare effects of social security in the following numerical simulation analysis.

4 Numerical simulations for benchmark equilibrium

To obtain further insights, we resort to numerical simulations of our model. In the first exercise, we examine how population aging caused by a decline in the birth rate or a reduction in the mortality rate affects economic growth under a sizable unfunded social security system. The second analysis addresses how demographic changes occurring and predicted in Japan affect economic growth. The final analysis examines the growth and welfare effects of a reduction in pension payments or an extension of retirement age in response to population aging. Before reporting these simulation results, however, we select functional forms of demographic functions and illustrate some of their basic properties.

4.1 Model parameterization

For the survival function, we adopt the parametric survival function proposed by Boucekkine et al. (2002):

$$S(t - v) = e^{-M(t-v)} = \frac{\mu_0 - e^{\mu_1(t-v)}}{\mu_0 - 1},\tag{35}$$

where μ_0 and μ_1 are parameters governing youth and old age mortality. As we do not consider childhood, we normalize the function so that birth corresponds to age 20. The maximum attainable age, D, for an individual entering the economy at age 20 is determined by $S(t - v) = 0$ and therefore, satisfies $D = [ln(\mu_0)]/\mu_1$. We estimate the two parameters, μ_0 and μ_1, by nonlinear least squares using life tables for Japan in 2010 and obtain a tight fit ($\bar{R}^2 = 0.996$) with highly significant parameter estimates of $\mu_0 = 145.734$ and $\mu_1 = 0.0607205$. These parameter values imply that the oldest survival age is 102.0445 or $D = 82.0445$, the life expectancy at age 20 is 66.1425 and the life expectancy at age 65 is 24.9432, which closely approximates the life expectancy of 66.67 years for a woman of age 20, and 23.80 years for a woman of age 65 reported in the life table for Japan in 2010. [10] [11]

---

[10]Table 4 and Figure 7 in Appendix E shows our estimated results and the resulting survival function.

[11]The longer life expectancy of women may be appropriate for our analysis because the social security benefit in Japan is partially a dual survival annuity, and the majority of beneficiaries are married.

For the labor supply fraction function, following Bruce and Turnovsky (2013b), we adopt the following functional form:

$$L(t - v) = e^{-N(t-v)} = \frac{l_0 - e^{l_1(t-v)}}{l_0 - 1},\qquad(36)$$

where l_0 and l_1 are parameters governing the labor supply fraction. The oldest working age, R, for an individual entering the economy at age 20 is given by $L(t - v) = 0$ and therefore satisfies $R = [ln(l_0)]/l_1$. We assume that the oldest age a worker remains in the labor force is 78. Thus, the oldest working age is set to R=58, and the value of $ł_0$ is adjusted to satisfy $ln(ł_0) = Rł_1$. Then, we set the value of l_1 so that the expected retirement (and claiming) age of an individual, conditional on survival, is 65, and so that the labor force participation rate at age 65 is 64.5%, in accordance with observed values.[12] These values are achieved by setting $l_0 = 65.1049$ and $l_1 = 0.072$.

To satisfy the demographic steady state, (22), we set the birth rate such that the population growth rate becomes almost zero ($\pi = 0.0016$), as is also observed empirically. This leads to a birth rate of 1.6% ($b = 0.016$), which is somewhat higher than the 0.8% that is observed empirically. Further, the social security benefit is set to $b = 0.4$, which is consistent with the current replacement ratio in Japan.[13] These demographic and social security parameter values imply a dependency rate of 49.9% ($d = 0.499$) and social security tax rate ($\tau_s = 0.1996$). [14]

Table 2 summarizes the remaining key structural parameters for the baseline economy. To ease the comparison, we employ the standard parameter values used in Mierau and Turnovsky (2014 a). The elasticity of capital is $\alpha = 0.35$ and the depreciation rate is $\delta = 0.05$. The aggregate level of technology equals $Z = 0.3286$, which yields a real interest rate of 6.7%. With respect to preferences, we set the intertemporal elasticity of substitution to $\sigma = 0.75$ and the rate of time preference at birth to $\rho = 0.035$.

4.2 Benchmark equilibrium

Figure 1 illustrates the equilibrium growth rate by plotting both the left-hand and right-hand sides of (31). Point A is the consistent equilibrium growth rate and point B is the inconsistent equilibrium discussed in Section 3-1. Figure 1 confirms

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[12]According to the Labor Force Survey in Japan, the labor force participation rates of the population aged 60 to 64 was 60.5 % in 2010.

[13]According to the OECD (2015), the net pension replacement in Japan was 40.4% for both men and women.

[14]The old age dependency ratio (the ratio of the population over age 65 to the population aged 20 to 64) in Japan was 39% in 2010, whereas the contribution rate of employees' pensions in Japan was 16.058 % in 2010.

that there exists a unique consistent equilibrium growth rate γ^* for benchmark parameter values. Table 3 indicates that the value of the equilibrium growth rate for the current parametrization equals 2.03%. The parameterization of the model allows us to calculate the values of the eigenvalues determined in (33). Carrying out these calculations, we find $\lambda_1 = 0.2658$, $\lambda_2 = 0.0337$, $\lambda_3 = 0.0534$, and $\lambda_4 = 0.0499$, so that all eigenvalues are indeed positive. Therefore, Table 3 confirms that any transitional path is locally unstable and thus the economy must always be on its balanced growth path, just as in Romer (1986) and Mierau and Turnovsky (2014a). The parametrization also permits us to characterize the magnitude of the μ_i ($i = C, H, E, \Delta$) and the n_H terms in relation to the stationary variables, ($\tilde{x}, \tilde{y}, \tilde{e}, \tilde{\Delta}$). The stationary variables are, respectively, equal to 0.2623, 2.3377, 2.6104 and 20.0359. The implied values of μ_C, μ_H, μ_E and μ_Δ are, respectively, 0.0161, 0.0032, 0.0036, 0.0015 and 0.0076, confirming our comment that the μ_i ($i = C, H, E, \Delta$) and the n_H terms are negligible when compared to the stationary variables.

5   Population aging and growth

This section examines the growth effects of two different channels of the demographic change, namely, a decline in the birth rate and a reduction in the mortality rate.

5.1   A decline in birth rate

Let us first examine how demographic changes caused by a decline in the birth rate, β, affect the economic growth rate. Figures 2-1 to 2-4 show how the decline in β from 0.035 to 0.005 affects the population growth rate (Figure 2-1), the relative share of each cohort (Figure 2-2), the share of population aged 65 and over (Figure 2-3) and the dependency ratio (Figure 2-4), respectively.[15] Under the demographic steady state defined in (22), a decline in the birth rate leads to the lower growth rate of the population, which results in the larger share of the older population. The population growth rate decreases from 2.93% ($\beta = 0.035$) to -2.71% ($\beta = 0.005$), whereas the share of the population aged 65 and over increases from 14.32% ($\beta = 0.035$) to 57.72% ($\beta = 0.005$). Therefore, given the stable labor supply structures (i.e., $L(t - v)$ and R), the dependency ratio also increases from 20.24% ($\beta = 0.035$) to 125.80% ($\beta = 0.005$), as shown in Figure 2-4.

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[15]The National Institute of Population and Social Security Research (IPSS) estimates that the birth rate, β, in Japan will decrease from 0.0193 in 1973 to 0.0056 in 2060. In this subsection, we changed the value of β enough to cover this predicted decline in the birth rate.

Figures 2-5 and 2-6 show how this population aging caused by a decline in the birth rate affects the social security tax rate (Figure 2-5) and the per capita output growth rate (Figure 2-6) under several alternative values of social security benefit rate, b. From (31) (resp. (27)), the per capita output growth rate (social security tax rate) is directly related to the population growth rate. Thus, in Figure 2-6 (Figure 2-5), we depict the relationship between the population growth rate and the per capita output growth rate (the social security tax rate). As show in Figure 2-5, the social security tax rate is negatively related to the population growth rate (birth rate) because the dependency ratio increases along with the decline in population growth rate (birth rate). When $b = 0.4$, for example, the social security tax rate increases from 8.1% ($\beta = 0.035$) to 50.32% ($\beta = 0.005$). Moreover, in Figure 2-6, when the social security benefit rate is relatively high (i.e., $b \geq 0.4$), we find that there is a hump-shaped relationship between the population growth rate (birth rate) and the rate of per capita output growth. When $b = 0.4$, for example, the per capita output growth rate increases from 1.63% to 2.06% in response to the decline in β from 0.035 to 0.012. The further decline in β from 0.012 to 0.005, however, lowers the per capita output growth rate from 2.06% to 1.71%. Therefore, the maximum per capita output growth rate is achieved when $\beta = 0.012$ or when the population growth rate is -0.64% (i.e., $\pi = -0.0064$).

This hump-shaped relationship between the population growth rate (birth rate) and the per capita output growth rate is intuitively explained as follows. From (29), the decline in the birth rate and the resulting decrease in the population growth rate, π, provides the two competing influences on economic growth. On the one hand, the positive growth effect of the decline in π is the "anti-dilution effect," which is reflected in the second term in the first line of the RHS of (29). From (29), the decline in population growth rate mitigates the dilution of aggregate capital to the larger population, which enhances the productivity of all employed workers. This "anti-dilution effect" enhances the capital accumulation and thereby positively affects economic growth. On the other hand, the negative growth effect of the decline in π is the "social security burden effect," which is reflected in the third term in the first line of RHS of (29). As inferred from (29) and (31), the net impact of the social security on growth operates entirely through its effect on the aggregate consumption-capital ratio, $\frac{c(t)}{k(t)}$. With social security, individuals know that they will have to support retirees and will thus have fewer resources for their own consumption over that portion of their lifespan. They also know that if they live to retirement, they, themselves, will be beneficiaries. However, they discount these future benefits at a rate greater than the biological rate offered by the unfunded social security system, and on balance the initial human wealth is reduced. The overall effect is to reduce their rate of asset accumulation more than does consumption. As a result, aggregating overall agents, this leads to an increase in the aggregate consumption-capital ratio, $\frac{c(t)}{k(t)}$, sustaining a decline

in the equilibrium growth rate in accordance with equation (29). This negative growth effect of social security becomes more serious as the social security tax rate becomes higher. Therefore, noting the results of Figure 2-5, the negative "social security burden effect" is more likely to dominate (be dominated by) the positive "anti-dilution effect," when the population growth rate is sufficiently low (high) or the birth rate is sufficiently low (high).

Finally, Figure 2-7 shows how population aging caused by a decline in the birth rate affects the utility multiplier defined in (34). With no social security ($b = 0$), as shown in Figures 2-6 and 2-7, both the utility multiplier and the per capita output growth rate increase with the decline in the population growth rate (birth rate). These results imply that the expected lifetime utility of all existing and future generations will be improved by the decline in population growth rate (birth rate). When the social security benefit rate is relatively high, however (i.e., $b \geq 0.4$), the utility multiplier is positively related to the population growth rate, whereas the relationship between the population growth rate and the per capita output growth rate is hump-shaped. These results imply that when the population growth rate (birth rate) is already sufficiently low, the further decline in the population growth rate (birth rate) may deteriorate the expected lifetime utility of all existing and future generations. These welfare implications of the decline in the birth rate are sufficiently interesting to deserve consideration. Nevertheless, these results should be interpreted with caution. In our specifications, people do not gain any utility from the number of children they have (e.g., Becker and Barro, 1988) or consider the existence of social planners who care about the potential number of children and their utility (e.g., Michel and Wigniolle, 2007). Explicit considerations of these factors might alter the welfare implications of our simulations.

5.2 A decline in mortality rate

Let us next examine how demographic changes caused by a decline in the mortality rate affect the economic growth rate. The demographic changes running through mortality can either be driven by a change in old age, μ_1, or youth mortality, μ_0, defined in (35). Because the two changes give almost the same effect, however, we focus on the old age mortality, μ_1, in our analysis. Figures 3-1 to 3-4 show how the decline in μ_1 from 0.0859 to 0.0459 affects the population growth rate (Figure 3-1), the relative share of each cohort (Figure 3-2), the share of the population aged 65 and over (Figure 3-3) and the dependency ratio (Figure 3-4), respectively.[16] Under the demographic steady state defined in (22), a decline in

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[16]The decline in μ_1 from 0.0859 to 0.0459 increases the life expectancy at age 20 from 46.7583 years to 87.5127 years. The National Institute of Population and Social Security Research (IPSS)

21

old age mortality leads to the higher growth rate of the population and a larger share of the older population. The population growth rate increases from -1.12% ($\mu_1 = 0.0859$) to 0.78% ($\mu_1 = 0.0459$), and the share of the population aged 65 and over increases from 14.56% ($\mu_1 = 0.0859$) to 40.21% ($\mu_1 = 0.0459$). Therefore, given the stable labor supply structures (i.e., $L(t - v)$ and R), the dependency ratio also increases from 20.89% ($\mu_1 = 0.0859$) to 69.48% ($\mu_1 = 0.0459$), as shown in Figure 3-4.

Figures 3-5 and 3-6 show how this population aging caused by a decline in the old age mortality rate affects the social security tax rate (Figure 3-5) and the per capita output growth rate (Figure 3-6) under several alternative values of social security benefit rate, b. As shown in Figure 3-5, the social security tax rate is positively (negatively) related to the population growth rate (old age mortality rate) because the dependency ratio increases along with the increase (decrease) in the population growth rate (old age mortality rate). When $b = 0.4$, for example, the social security tax rate increases from 8.036% ($\mu_1 = 0.0859$) to 27.79% ($\mu_1 = 0.0459$). Moreover, in Figure 3-6, we find that the per capita output growth rate is positively (negatively) related with the population growth rate (old age mortality rate). When $b = 0.4$, for example, the per capita output growth rate increases from 0.94% ($\mu_1 = 0.0859$) to 2.32% ($\mu_1 = 0.0459$).

This positive relationship between the population growth rate and the per capita output growth rate (negative relationship between the old age mortality and the per capita output growth rate) is intuitively explained as follows. From (29), the decline in the old age mortality rate and the resulting increase in the population growth rate, π, provide two negative effects and one positive effect on economic growth. The first negative effect is the "social security burden effect," which is reflected in the third term in the first line of RHS of (29). The presence of unfunded social security leads to an increase in the aggregate consumption-capital ratio, $\frac{c(t)}{k(t)}$, which lowers the equilibrium growth rate. This negative growth effect of social security becomes more serious as the social security tax rate becomes higher. Therefore, noting the results of Figure 3-5, the negative "social security burden effect" becomes more serious as the old age mortality rate declines. The second negative effect is the "dilution effect," which is reflected in the second term in the first line of the RHS of (29). From (29), the decline in old age mortality and the resulting increase in the population growth rate enhances the dilution of aggregate capital to the larger population, which lowers the productivity of all employed workers. This "dilution effect" retards capital accumulation and thereby negatively affects economic growth. One positive effect is the "life-span effect,"

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estimates that the life expectancy for a woman of age 20 will increase from 53.39 years in 1960 to 71.10 years in 2060. In this subsection, we changed the value of μ_1 enough to cover this predicted increase in the life expectancy.

which is also reflected in the third term in the first line of RHS of (29). A decline in the old age mortality rate motivates individuals to save more for their old age, which increases their rate of asset accumulation more than consumption does. As a result, after aggregating overall agents, this leads to a decrease in the aggregate consumption-capital ratio, $\frac{c(t)}{k(t)}$ and thereby positively affects economic growth. The numerical simulation result in Figure 3-6 indicates that the positive "life span effect" dominates the two aforementioned negative effects under benchmark parameter values. Therefore, the positive relationship between the population growth rate and the per capita output growth rate (negative relationship between the old age mortality and the per capita output growth rate) holds irrespective of the values of the social security benefit rate. The positive growth effect of the decline in μ_1 grows smaller as the value of b increases, however, because the social security burden effect becomes more serious with the rise in the social security benefit rate.

Finally, Figure 3-7 shows how population aging caused by a decline in the old-age mortality rate affects the utility multiplier defined in (34). Despite the fact that the per capita output growth rate and the population growth rate are positively related (the per capita output growth rate and the old-age mortality rate are negatively related), the utility index decreases along with the increase in the population growth rate (the decline in the old age mortality rate). Therefore, the effect of the decline in the old-age mortality rate on the expected lifetime utility of all existing and future generations is generally ambiguous, while it is easily expected that future generations will obtain larger utility gains from the higher per capita output growth rate. However, these welfare implications of the decline in old-age mortality rate are heavily dependent upon our utility specifications and should therefore be interpreted with caution. In our baseline simulation, the equilibrium value of the individual flow utility, $u(C(v,t))$, is negative, which a priori implies that the individual prefers a shorter life given the lifetime utility specification defined in (8). Our somewhat counterintuitive result that the utility index decreases along with the decline in the old age mortality rate is derived directly from this utility specification.[17]

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[17]In fact, to avoid this problem, the optimal longevity literature (e.g., Hall and Jones, 2007, Dalgard and Strulik, 2014) employs the utility specifications that guarantee positive utility, although it complicates the analysis to some extent. The precise evaluation of the welfare effect of longevity is not the main concern of this paper. Therefore, to avoid this complication, we do not employ this type of utility specification.

6   Changes in the demographic structures in Japan

In this section, we use the calibrated model to study how the demographic changes occurring and predicted in Japan affect the economic growth rate. To perform this exercise, we first estimate the parameters of our survival function, μ_0 and μ_1, for every 5 years from 2000 to 2060, using past and future life tables for Japan estimated by the National Institute of Population and Social Security Research (IPSS).[18] The total (solid) line in Figure 4-1 shows the evolution of the life expectancy at age 20 calculated by the estimated values of μ_0 and μ_1. The life expectancy at age 20 is predicted to increase from 64.4227 years in 2000 to 70.6572 years in 2060.[19] Next, we trace out the birth rate, β, implied by our demographic model using actual and estimated data for the population growth rate, π, as the input for the demographic steady state in (22). The total (solid) line in Figure 4-2 shows this implied birth rate, which is predicted to decrease from 1.66 % in 2000 to 0.88 % in 2060. Although this implied birth rate is persistently higher than the actual data or other estimates, all of these values follow the same trend and the correlations among them are therefore quite high.[20] Further, the total (solid) line in Figure 4-3 shows the actual and estimated data for the population growth rate, which is used to calculate the birth rate in Figure 4-2. The population growth rate is predicted to decrease from 0.2% in 2000 to -1.19% in 2060.

To analyze the development of economic growth, we use the demographic parameters underlying Figures 4-1 to 4-3 to calculate the equilibrium value of the dependency ratio (Figure 4-4), social security tax rate (Figure 4-5) and per capita output growth rate (Figure 4-6), respectively. To isolate the purely demographic effects, all other exogenous parameters are kept unchanged. Noting $b = 0.4$ under the benchmark parameter values, the total (solid) line in Figure 4-4 shows that the social security tax rate increased from 18.82% in 2000 to 36.02 % in 2060 along with the rise in the dependency ratio from 47.06% in 2000 to 90.06% in 2060, as shown in Figure 4-5 . This rapid rise in the social security tax rate strengthens the negative growth effect of social security. In fact, as shown in the total (solid) line in Figure 4-6, the per capita output growth rate, which initially rises, starts to

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[18]Based on the latest results from the Population Census, the National Institute of Population and Social Security Research (IPSS) released the new population projections and future life tables for Japan in January 2012. We used these projected future life tables and estimated data for the population growth rate from 2010 to 2060 for our numerical simulation analyses. As for the past life tables and population growth rate, we used the Japanese Mortality Database and the results of Population Census for each year.

[19]The National Institute of Population and Social Security Research (IPSS) estimates that the life expectancy of a woman of age 20 will increase from 65.08 years in 2000 to 71.10 years in 2016.

[20]The National Institute of Population and Social Security Research (IPSS) estimates that the crude birth rate in Japan will decrease from 0.95% in 2000 to 0.56% in 2060.

decline in 2035. Intuitively, the negative "social security burden effect" dominates the aforementioned positive growth effects, namely, the "anti-dilution effect" and "life-extension effect" from the mid 2030s.

To understand the mechanism behind this decline in the per capita output growth rate from 2035, we decompose the growth effects of these demographic changes into two parts, namely, the effect of demographic changes caused purely by a decline in the birth rate and those caused purely by a reduction in the mortality rate. The μ fixed (dashed) lines in Figures 4-3 to 4-6 isolate the effect of demographic changes caused purely by a decline in the birth rate on the population growth rate (Figure 4-3), dependency ratio (Figure 4-4), social security tax rate (Figure 4-5), and per capita output growth rate (Figure 4-6), respectively, by holding the mortality rate and thus the life expectancy at age 20 constant at its value of 2010 (i.e., 66.1425 years) as shown in the μ fixed line in Figure 4-1. Conversely, the β fixed (dot-dash) lines in Figures 4-3 to 4-6 isolate the effect of demographic changes caused purely by a decline in the mortality rate on these variables by holding the birth rate constant at its value of 2010 (i.e., 1.52%), as shown in the β fixed line in Figure 4-2. [21] [22]

From Figure 4-6, the β fixed (dot-dash) line increases steadily with time, whereas the μ fixed (dashed) line initially increases but then starts to decline in 2030. These results imply that the effect of demographic changes caused purely by a decline in the mortality rate on growth is always positive, whereas the effect of demographic changes caused purely by a decline in the birth rate on growth is initially positive, but then becomes negative in 2030. The total effect of demographic changes on growth, which is represented by the total (solid) line in Figure 4-6, is heavily influenced by the effect of demographic changes caused purely by a decline in birth rate. [23] Therefore, we can confirm that the growth effect of

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[21] More concretely, the total case, the μ fixed case and the β fixed case are computed, respectively as follows. In the total case, it is computed by using the estimated parameter values of (μ_0, μ_1) and actual and estimated data for population growth rate, π; the birth rate, β, is computed endogenously to satisfy the demographic steady state in (22). The total (solid) line in Figure 4-2 shows this implied birth rate. In the μ fixed case, it is computed by holding the parameter values of (μ_0, μ_1) constant at their values of 2010 and using the computed values of the birth rate, β. In the total case, the population growth rate, π, is computed endogenously to satisfy (22). The μ fixed (dashed) line in Figure 4-3 shows this implied population growth rate. Finally, in the β fixed case, it is computed by holding the parameter values of β constant at its 2010 value and using the estimated parameter values of (μ_0, μ_1); the population growth rate, π, is computed endogenously to satisfy (22). The β fixed (dot-dash) line in Figure 4-3 shows this implied population growth rate.

[22] As shown in the β fixed line (the μ fixed line) in Figure 4-3, by holding the birth rate (mortality rate) constant, the decline in the mortality rate (birth rate) positively (negatively) affects population growth rate. Therefore, the β fixed line lies below the total line.

[23] From Figures 4-4 and 4-5, both the μ fixed (dashed) line and β fixed (dot-dash) line increase with time. These results imply that both the decline in the birth rate and the reduction in the mortality rate positively affect the dependency ratio and thereby the social security tax rate. Because the

the predicted demographic changes in Japan is initially positive but may become negative in the mid 2030s.

7   Policies maintaining the social security tax rate

Population aging caused by the decline in both the birth and mortality rates alters the solvency of the unfunded social security system by increasing the dependency ratio. The analyses in the previous sections suggest that this rise in the dependency ratio and the resulting increase in the social security tax rate may negatively affect economic growth. To address these challenges, this section considers the two different types of policies that maintain the solvency of unfunded social security systems. The first policy is to maintain a constant social security tax rate by reducing the pension benefit rate. We denote this type of policy as the pension payment reduction policy. The second policy is to maintain the dependency ratio and thus the social security tax rate constant by increasing the average claimant age for pension benefits or the average retirement age. We denote this type of policy as the retirement extension policy. We consider these policy experiments based upon the calibrated model developed in Section 6 and focus our analyses on their effect on the steady state equilibrium growth rate and lifetime utility. Figures 5-1 to 5-6 show how these policy experiments affect the evolutions of the pension benefit rate (Figure 5-1), the average retirement age (Figure 5-2), the dependency ratio (Figure 5-3), the social security tax rate (Figure 5-4), the per capita output growth rate (Figure 5-5), and the utility multiplier (Figure 5-6), respectively. Further, the no-policy (solid) lines in Figures 5-1 to 5-6 describe the case without any policy interventions, the benefit policy (dashed) lines describe the case where the pension payment reduction policy is introduced in 2020, and the retirement policy (dot-dash) lines describe the case where the retirement extension policy is introduced in 2020, respectively.

Let us first consider the pension payment reduction policy. As shown in the benefit policy (dashed) lines in Figure 5-4, we maintain the social security tax rate constant at its 2020 value (i.e., 25.58 %) by reducing the pension benefit rate, b, from 0.4 in 2020 to 0.284 in 2060 as shown in Figure 5-1. From Figures 5-5 and 5-6, the benefit policy (dashed) lines in Figures 5-5 and 5-6 always lie above the no policy (solid) line, which indicates that the introduction of the pension payment reduction policy positively affects both the per capita output growth rate

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μ fixed (dashed) line in Figure 4-5 always lies above the β fixed (dot-dash) line, the increase in the social security tax rate due to the decline in birth rate is larger than the change due to the decline in mortality rate. Therefore, the total effect of demographic changes on growth, which is represented by the total (solid) line in Figure 4-6, is heavily influenced by the effect of demographic changes caused purely by a decline in the birth rate.

26

and utility multipliers. For example, the per capita output growth rate in 2060 is increased from 2.04 % to 2.60 % by the introduction of the pension payment reduction policy.

Next, let us consider the retirement extension policy and compare it with the pension payment reduction policy. As shown in the retirement policy (dot-dash) lines in 5-3 and 5-4, we maintain the dependency ratio as well as the social security tax rate constant at their 2020 values (i.e., 63.95 % and 25.58 %) by increasing the average claimant age for social security benefits from 65.0159 years in 2020 to 71.1491 years in 2060 as shown in Figures 5-2. From Figures 5-5 and 5-6, the retirement policy (dot-dash) lines in Figures 5-5 and 5-6 also lie above the no policy (solid) lines, which indicates that the retirement extension policy also positively affects both the per capita output growth rate and utility multipliers. For example, the per capita output growth rate in 2060 is increased from 2.04 % to 2.12% by the introduction of the retirement extension policy. Figures 5-5 and 5-6 also show, however, that the retirement policy (dot-dash) lines always lie below the benefit policy (dashed) lines, which indicates that the growth and welfare enhancing effects of the retirement extension policy are smaller than those of the pension payment reduction policy. Therefore, of the two policies considered here for maintaining the solvency of a PAYG social security system, the pension payment reduction policy is better than the retirement extension policy for both growth and welfare in response to population aging. This result may not be surprising, however, because social security reduces growth and welfare in our simple model.

8   Concluding remarks

This paper examined how population aging caused by a decline in the birth rate or a reduction in the mortality rate affect economic growth in an overlapping generations model with a general demographic structure and a sizable unfunded social security system. Through numerical simulations, we showed that a decline in the birth rate has non-monotonic effects on economic growth, yielding a hump-shaped relationship between the population growth rate and the economic growth rate, whereas a reduction in the mortality rate has a monotonic positive effect on economic growth, yielding a monotonic positive relationship between the population growth rate and the economic growth rate. We also used our model to study how demographic changes occurring and predicted in Japan affect the economic growth rate. We showed that the growth effect of the predicted demographic changes in Japan is initially positive but it may become negative from the mid 2030s. This paper also examined the growth and welfare effects of a reduction in social security payments or an extension of the retirement age, and showed

that the pension payment reduction policy is better than the retirement extension policy for both growth and welfare in response to population aging.

Appendix A: The equilibrium for which $\gamma = r - \pi$ is not a consistent (viable) equilibrium

Using the household budget constraint (10) and government budget constraint (27), we can write aggregate consumption as:

$$
\begin{aligned}
c(t) &\equiv \int_{t-D}^{t} p(t-v)C(v,t)dv, \\
&= \int_{t-D}^{t} \{[r+\mu(t-v)]A(v,t) + [1-\tau_s(t)]w(t)L(t-v) + bw(t)[1-L(t-v)] - A_t(v,t)\}p(t-v)dv, \\
&= \int_{t-D}^{t} \{[r+\mu(t-v)]A(v,t) + w(t)L(t-v) - A_t(v,t)\}p(t-v)dv, \\
&= w(t)l(t) + \int_{t-D}^{t} \{[r+\mu(t-v)]A(v,t) - A_t(v,t)\}p(t-v)dv.
\end{aligned}
$$

(37)

Suppose $\gamma = r - \pi$, then (29) implies that $c(t) = w(t)l(t)$; this allows us to write (37) as:

$$
\beta \int_{t-D}^{t} \{[r+\mu(t-v)]A(v,t) - A_t(v,t)\}e^{-\pi(t-v)-M(t-v)}dv = 0. \tag{38}
$$

Integrating an individual agent's budget constraint over his/her life time, recognizing that his/her initial financial wealth is zero, and recalling the transversality condition yields his/her intertemporal budget constraint:

$$
\int_{v}^{v+D} \{[1-\tau_s(\tau)]w(\tau)L(\tau-v) + bw(\tau)[1-L(\tau-v)] - C(v,\tau)\}e^{-r(\tau-v)-M(\tau-v)}d\tau = 0. \tag{39}
$$

Substituting the budget constraint from (10) into (39) gives:

$$
\int_{v}^{v+D} \{[r+\mu(t-v)]A(v,t) - A_t(v,t)\}e^{-r(\tau-v)-M(\tau-v)}d\tau = 0. \tag{40}
$$

Clearly, (38) and (40) can hold simultaneously only if $r = \pi$. As both r and π are set exogenously and independently, there is no reason for this equality to hold. Moreover, setting $r = \pi$ implies $\gamma = 0$, so that the economy has a zero growth rate and no capital accumulation. In addition, it violates the assumption of dynamic efficiency and patience $r > \rho > \pi$ made at the outset. For these reasons, this equilibrium is ruled out.

Appendix B: The sufficient condition for which a unique consistent equilibrium growth rate exists

This section establishes the sufficient conditions for which a unique consistent equilibrium growth rate exists. We first begin by noting that (31) can be written as:

$$\Psi(\gamma) = \Gamma(\gamma), \tag{41}$$

where

$$\Psi(\gamma) \equiv \frac{\gamma - (r - \pi)}{(1 - \alpha)Z} \varphi((1 - \sigma)r + \sigma\rho)\Omega(\pi), \tag{42}$$

and

$$\Gamma(\gamma) \equiv \varphi((1-\sigma)r+\sigma\rho)\Omega(\pi)-[(1-b-bd)\Omega(r-\gamma)+b\varphi(r-\gamma)]\varphi(\gamma+\pi-\sigma(r-\rho)). \tag{43}$$

For $\gamma = r - \pi$, we can easily confirm that the relation $\Psi(r - \pi) = 0$ holds. Further, noting $d = \frac{1-l}{l}$, $l = \beta\Omega(\pi)$ and $1 = \beta\varphi(\pi)$, we can establish that:

$$\begin{aligned}\Gamma(r - \pi) &= \varphi((1 - \sigma)r + \sigma\rho)b[(1 + d)\Omega(\pi) - \varphi(\pi)], \\ &= \varphi((1 - \sigma)r + \sigma\rho)b[\frac{1}{\beta} - \varphi(\pi)], \\ &= 0.\end{aligned} \tag{44}$$

Hence, the equilibrium condition (41) is satisfied when $\gamma = r - \pi$. As shown in Appendix A, however, $\gamma = r - \pi$ violates the transversality condition and can therefore be ignored.

The aim of this section is to establish the sufficient condition for which a unique consistent equilibrium growth rate exists. From the inspections of (41) to 43), suppose that (i) $\Gamma(\gamma)$ is concave, and (ii) the relation $\Gamma'(r - \pi) < 0$ holds, we can confirm that $\gamma = r - \pi$ is not the unique intersection point of $\Psi(\gamma)$ and $\Gamma(\gamma)$, but there is a second, consistent, point of intersection. Therefore, we can ensure the existence of a unique consistent equilibrium growth rate if conditions (i) and (ii) hold simultaneously. In the following, we first derive the parameter conditions that ensure the relation $\Gamma'(r - \pi) < 0$ holds. Then, in the second step, we confirm that $\Gamma(\gamma)$ is concave.

The parameter conditions for $\Gamma'(r - \pi) < 0$

To establish the properties of $\Gamma(\gamma)$ function, we make use of the following properties of the sub-function $\varphi(x)$ and $\Omega(x)$. Specifically, the following relationships hold.

- $\varphi'(x) = -\int_0^D s e^{-xs-M(s)}ds < 0$, $\varphi''(x) = \int_0^D s^2 e^{-xs-M(s)}ds > 0$, $\varphi''(x) > \frac{\varphi'(x)^2}{\varphi(x)} > 0$, and $\frac{d}{dx}[\frac{\varphi'(x)}{\varphi(x)}] = \frac{\varphi''(x)\varphi(x)-\varphi'(x)^2}{\varphi(x)^2} > 0$.[24]

- $\Omega'(x) = -\int_0^D s e^{-xs-M(s)-N(s)}ds < 0$, $\Omega''(x) = \int_0^D s^2 e^{-xs-M(s)-N(s)}ds > 0$, $\Omega''(x) > \frac{\Omega'(x)^2}{\Omega(x)} > 0$, and $\frac{d}{dx}[\frac{\Omega'(x)}{\Omega(x)}] = \frac{\Omega''(x)\Omega(x)-\Omega'(x)^2}{\Omega(x)^2} > 0$.

To proceed further, we study the curvature of the $\Gamma(\gamma)$ function. Its first derivative is given by:

$$
\begin{aligned}
\Gamma'(\gamma) &= (1-b-bd)[\Omega'(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho)) - \Omega(r-\gamma)\varphi'(\gamma+\pi-\sigma(r-\rho))] \\
&\quad + b[\varphi'(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho)) - \varphi(r-\gamma)\varphi'(\gamma+\pi-\sigma(r-\rho))], \\
&= (1-b-bd)\Omega(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho))[\frac{\Omega'(r-\gamma)}{\Omega(r-\gamma)} - \frac{\varphi'(\gamma+\pi-\sigma(r-\rho))}{\varphi(\gamma+\pi-\sigma(r-\rho))}] \\
&\quad + b\varphi(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho))[\frac{\varphi'(r-\gamma)}{\varphi(r-\gamma)} - \frac{\varphi'(\gamma+\pi-\sigma(r-\rho))}{\varphi(\gamma+\pi-\sigma(r-\rho))}].
\end{aligned}
$$
(45)

Evaluating this at $\gamma = r - \pi$, we obtain

$$
\begin{aligned}
\Gamma'(r-\pi) &= (1-b-bd)\Omega(\pi)\varphi((1-\sigma)r+\sigma\rho)[\frac{\Omega'(\pi)}{\Omega(\pi)} - \frac{\varphi'((1-\sigma)r+\sigma\rho)}{\varphi((1-\sigma)r+\sigma\rho)}] \\
&\quad + b\varphi(\pi)\varphi((1-\sigma)r+\sigma\rho)[\frac{\varphi'(\pi)}{\varphi(\pi)} - \frac{\varphi'((1-\sigma)r+\sigma\rho)}{\varphi((1-\sigma)r+\sigma\rho)}].
\end{aligned}
$$
(46)

Because $\sigma \in (0, \frac{r-\pi}{r-\rho})$, the relation $(1-\sigma)r + \sigma\rho > \pi$ holds. Therefore, noting $\frac{d}{dx}[\frac{\varphi'(x)}{\varphi(x)}] > 0$ and $\varphi'(x) < 0$, we can confirm that the following inequalities hold: $\frac{\varphi'(\pi)}{\varphi(\pi)} < \frac{\varphi'((1-\sigma)r+\sigma\rho)}{\varphi((1-\sigma)r+\sigma\rho)} < 0$. It implies that $\frac{\varphi'(\pi)}{\varphi(\pi)} - \frac{\varphi'((1-\sigma)r+\sigma\rho)}{\varphi((1-\sigma)r+\sigma\rho)} < 0$.

Therefore, from (46), the sufficient condition for $\Gamma'(r-\pi) < 0$ is given by

$$
\frac{\Omega'(\pi)}{\Omega(\pi)} \le \frac{\varphi'((1-\sigma)r+\sigma\rho)}{\varphi((1-\sigma)r+\sigma\rho)},
$$
(47)

which is satisfied, for example, when the relation $\frac{\varphi'(\pi)}{\varphi(\pi)} \ge \frac{\Omega'(\pi)}{\Omega(\pi)}$ holds. Therefore, some mild assumptions regarding the properties of labor supply fraction $L(s)$ are necessary to satisfy (47).

---

[24] $\varphi''(x) > \frac{\varphi'(x)^2}{\varphi(x)} > 0$ is proved as follows. We define the characteristic functions as $f(s) = se^{-\frac{1}{2}[xs+M(s)]}$ and $g(s) = e^{-\frac{1}{2}[xs+M(s)]}$. The Cauchy-Schwartz inequality implies that $\int_0^D f^2(s)ds \int_0^D g^2(s)ds \ge [\int_0^D f(s)g(s)ds]^2$. Substituting these two characteristic functions into the inequality, we can confirm that the relation $\varphi''(x) > \frac{\varphi'(x)^2}{\varphi(x)} > 0$ holds. The property $\Omega''(x) > \frac{\Omega'(x)^2}{\Omega(x)} > 0$ is also proven in an analogous way.

30

$\Gamma(\gamma)$ **is concave**

The second derivative of $\Gamma(\gamma)$ is given by

$$
\begin{aligned}
\Gamma''(\gamma) = &-(1-b-bd)[\Omega''(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho)) - 2\Omega'(r-\gamma)\varphi'(\gamma+\pi-\sigma(r-\rho)) \\
&+ \Omega(r-\gamma)\varphi''(\gamma+\pi-\sigma(r-\rho))] \\
&- b[\varphi''(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho)) - 2\varphi'(r-\gamma)\varphi'(\gamma+\pi-\sigma(r-\rho)) \\
&+ \varphi(r-\gamma)\varphi''(\gamma+\pi-\sigma(r-\rho))], \\
= &-\frac{(1-b-bd)}{\varphi(\gamma+\pi-\sigma(r-\rho))\Omega(r-\gamma)}[\Omega''(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho))^2\Omega(r-\gamma) \\
&- 2\Omega'(r-\gamma)\varphi'(\gamma+\pi-\sigma(r-\rho))\varphi(\gamma+\pi-\sigma(r-\rho))\Omega(r-\gamma) \\
&+ \Omega(r-\gamma)^2\varphi(\gamma+\pi-\sigma(r-\rho))\varphi''(\gamma+\pi-\sigma(r-\rho))] \\
&- \frac{b}{\varphi(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho))}[\varphi''(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho))^2\varphi(r-\gamma) \\
&- 2\varphi'(r-\gamma)\varphi'(\gamma+\pi-\sigma(r-\rho))\varphi(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho)) \\
&+ \varphi(r-\gamma)^2\varphi(\gamma+\pi-\sigma(r-\rho))\varphi''(\gamma+\pi-\sigma(r-\rho))].
\end{aligned}
\tag{48}
$$

Because $\varphi''(x) > \frac{\varphi'(x)^2}{\varphi(x)}$ and $\Omega''(x) > \frac{\Omega'(x)^2}{\Omega(x)}$, we can establish

$$
\begin{aligned}
\Gamma''(\gamma) \leq &-\frac{(1-b-bd)}{\varphi(\gamma+\pi-\sigma(r-\rho))\Omega(r-\gamma)}[\Omega'(r-\gamma)^2\varphi(\gamma+\pi-\sigma(r-\rho))^2 \\
&- 2\Omega'(r-\gamma)\varphi'(\gamma+\pi-\sigma(r-\rho))\varphi(\gamma+\pi-\sigma(r-\rho))\Omega(r-\gamma) \\
&+ \Omega(r-\gamma)^2\varphi'(\gamma+\pi-\sigma(r-\rho))^2] \\
&- \frac{b}{\varphi(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho))}[\varphi'(r-\gamma)^2\varphi(\gamma+\pi-\sigma(r-\rho))^2 \\
&- 2\varphi'(r-\gamma)\varphi'(\gamma+\pi-\sigma(r-\rho))\varphi(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho)) \\
&+ \varphi(r-\gamma)^2\varphi'(\gamma+\pi-\sigma(r-\rho))^2], \\
= &-\frac{(1-b-bd)}{\varphi(\gamma+\pi-\sigma(r-\rho))\Omega(r-\gamma)}[\Omega'(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho)) \\
&- \Omega(r-\gamma)\varphi'(\gamma+\pi-\sigma(r-\rho))]^2 \\
&- \frac{b}{\varphi(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho))}[\varphi'(r-\gamma)\varphi(\gamma+\pi-\sigma(r-\rho)) \\
&- \varphi(r-\gamma)\varphi'(\gamma+\pi-\sigma(r-\rho))]^2, \\
< &\ 0.
\end{aligned}
\tag{49}
$$

Thus (49) implies that $\Gamma(\gamma)$ is concave. Therefore, suppose that the relation $\frac{\Omega'(\pi)}{\Omega(\pi)} < \frac{\varphi'((1-\sigma)r+\sigma\rho)}{\varphi((1-\sigma)r+\sigma\rho)}$ holds, there exists an unique consistent equilibrium growth rate γ^* that satisfies $\Psi(\gamma^*) = \Gamma(\gamma^*)$.

To clarify the above arguments, we graph the $\Psi(\gamma)$ and $\Gamma(\gamma)$ functions in Figure 6. For our benchmark parameter values, we can easily confirm that (i) $\Gamma(\gamma)$ is concave, and (ii) the relation $\Gamma'(r-\pi) < 0$ holds. Therefore, there exists an unique consistent equilibrium growth rate γ^* that satisfies $\Psi(\gamma^*) = \Gamma(\gamma^*)$.

Appendix C: Stability

From (25) and (28), the dynamics of the macroeconomic equilibrium can be summarized in the following form:

$$\dot{k}(t) = Zk(t) - (\delta + \pi)k(t) - c(t), \tag{50}$$

$$\dot{c}(t) = \sigma(r - \rho)c(t) - \Phi(t), \tag{51}$$

where

$$\Phi(t) \equiv \int_{t-D}^{t} \mu(t-v)p(t-v)C(v,t)dv - \beta C(t,t) + \pi c(t). \tag{52}$$

Using the second mean value theorem, we may write (52) as:[25]

$$\Phi(t) = \mu_C(t-v_1) \int_{t-D}^{t} p(t-v)C(v,t)dv - \beta C(t,t) + \pi c(t), \tag{53}$$

where

$$\mu_C(t-v_1) = \frac{\int_{t-D}^{t} \mu(t-v)p(t-v)C(v,t)dv}{\int_{t-D}^{t} p(t-v)C(v,t)dv}, \quad v_1 \in (t, t-D), \tag{54}$$

is the ratio of the consumption given up by the dying to aggregate per capita consumption. In Appendix D, we show that $\mu_C(t - v_1)$ varies only very slightly over time, enabling us to treat it as essentially constant. Equally important, being a weighted average of mortality rates across cohorts, μ_C is small.

Recalling the definition of $c(t)$, (13), (16) and (27), we can express (53) in a more compact form:

$$\begin{aligned} \Phi(t) &= (\mu_C + \pi)c(t) - \beta C(t,t), \\ &= (\mu_C + \pi)c(t) - \beta \frac{\hat{H}(t,t)}{\Delta(t,t)}, \\ &= (\mu_C + \pi)c(t) - \beta \frac{H(t,t) + E(t)}{\Delta(t,t)}, \end{aligned} \tag{55}$$

---

[25]For any real valued function $f(x)$ on the interval $[a,b]$ and function $g(x)$ that is integrable and does not change sign over the interval (a,b), there exists a value $c \in (a,b)$ such that $\int_a^b f(x)g(x)dx = f(c)\int_a^b g(x)dx$.

where

$$H(t) \equiv H(t,t) \equiv \int_t^{t+D} (1 - b - bd)w(\tau)L(\tau - t)e^{-r(\tau-t)-M(\tau-t)}d\tau, \qquad (56)$$

$$E(t) \equiv \int_t^{t+D} bw(\tau)e^{-r(\tau-t)-M(\tau-t)}d\tau, \qquad (57)$$

$$\Delta(t) \equiv \Delta(t,t) \equiv \int_t^{t+D} e^{-[(1-\sigma)r+\sigma\rho](\tau-t)-M(\tau-t)}d\tau. \qquad (58)$$

The dynamics of (56), (57) and (58) are given by

$$\dot{H}(t) = -(1 - b - bd)w(t) + [r + n_H(\tau_1 - t) + \mu_H(\tau_2 - t)]H(t), \qquad (59)$$

$$\dot{E}(t) = -bw(t) + [r + \mu_E(\tau_3 - t)]E(t), \qquad (60)$$

$$\dot{\Delta}(t) = -1 + [(1 - \sigma)r + \sigma\rho + \mu_\Delta(\tau_4 - t)]\Delta(t), \qquad (61)$$

where n_H, μ_H, μ_E and μ_Δ are defined analogously to μ_C:

$$n_H(\tau_1 - t) = \frac{\int_t^{t+D} n(\tau - t)(1 - b - bd)w(\tau)L(\tau - t)e^{-r(\tau-t)-M(\tau-t)}d\tau}{\int_t^{t+D} (1 - b - bd)w(\tau)L(\tau - t)e^{-r(\tau-t)-M(\tau-t)}d\tau}, \quad \tau_1 \in (t, t + D),$$

$$(62)$$

$$\mu_H(\tau_2 - t) = \frac{\int_t^{t+D} \mu(\tau - t)(1 - b - bd)w(\tau)L(\tau - t)e^{-r(\tau-t)-M(\tau-t)}d\tau}{\int_t^{t+D} (1 - b - bd)w(\tau)L(\tau - t)e^{-r(\tau-t)-M(\tau-t)}d\tau}, \quad \tau_2 \in (t, t + D),$$

$$(63)$$

$$\mu_E(\tau_3 - t) = \frac{\int_t^{t+D} \mu(\tau - t)bw(\tau)e^{-r(\tau-t)-M(\tau-t)}d\tau}{\int_t^{t+D} bw(\tau)e^{-r(\tau-t)-M(\tau-t)}d\tau}, \quad \tau_3 \in (t, t + D), \qquad (64)$$

and

$$\mu_\Delta(\tau_4 - t) = \frac{\int_t^{t+D} \mu(\tau - t)e^{-[(1-\sigma)r+\sigma\rho](\tau-t)-M(\tau-t)}d\tau}{\int_t^{t+D} e^{-[(1-\sigma)r+\sigma\rho](\tau-t)-M(\tau-t)}d\tau}, \quad \tau_4 \in (t, t + D). \qquad (65)$$

Using (55), (59), (60), (61), and assuming μ_i ($i = C, H, E.\Delta$) and n_H are constant, we can write the dynamic system in (50) and (51) as:

$$\dot{k}(t) = (Z - \delta - \pi)k(t) - c(t). \qquad (66)$$

$$\dot{c}(t) = [\sigma(r - \rho) - \mu_C - \pi]c(t) + \beta\frac{H(t) + E(t)}{\Delta(t)}, \qquad (67)$$

$$\dot{H}(t) = -(1 - b - bd)w(t) + (r + n_H + \mu_H)H(t), \qquad (68)$$

33

$$\dot{E}(t) = -bw(t) + (r + \mu_E)E(t), \tag{69}$$

$$\dot{\Delta}(t) = -1 + [(1 - \sigma)r + \sigma\rho + \mu_\Delta]\Delta(t). \tag{70}$$

From here, we can redefine the system in terms of the stationary variables: $x(t) \equiv \frac{c(t)}{k(t)}$, $y(t) \equiv \frac{H(t)}{k(t)}$, $e(t) \equiv \frac{E(t)}{k(t)}$ and $\Delta(t)$. Using (66) to (70) and $w(t) = (1 - \alpha)Z(k(t)/l)$, dynamics of $x(t)$, $y(t)$, $e(t)$ and $\Delta(t)$ can be written as:

$$\frac{\dot{x}(t)}{x(t)} = \sigma(r - \rho) - \mu_C + \beta\frac{y(t) + e(t)}{\Delta(t)x(t)} - (Z - \delta) + x(t), \tag{71}$$

$$\frac{\dot{y}(t)}{y(t)} = -(1 - b - bd)\frac{(1 - \alpha)Z}{y(t)l} + r + n_H + \mu_H - (Z - \delta - \pi) + x(t), \tag{72}$$

$$\frac{\dot{e}(t)}{e(t)} = -b\frac{(1 - \alpha)Z}{e(t)l} + r + \mu_E - (Z - \delta - \pi) + x(t), \tag{73}$$

$$\dot{\Delta}(t) = -1 + [(1 - \sigma)r + \sigma\rho + \mu_\Delta]\Delta(t). \tag{74}$$

These four equations form the basis for the local dynamics of the equilibrium.

Linearizing (71) to (74) around the steady state, $(\tilde{x}, \tilde{y}, \tilde{e}, \tilde{\Delta})$, the local dynamics can be expressed as (33). To study the dynamics further, it is convenient to write the steady-state values of $(\tilde{x}, \tilde{y}, \tilde{e}, \tilde{\Delta})$ in terms of the functions $\varphi(x)$ and $\Omega(x)$. To do this, we use the balanced growth values of (30), (56), (57) and (58) in the definitions of $(\tilde{x}, \tilde{y}, \tilde{e}, \tilde{\Delta})$ resulting in

$$\tilde{x} = \frac{(1 - b - bd)\Omega(r - \gamma) + b\varphi(r - \gamma)}{\varphi((1 - \sigma)r + \sigma\rho)}\frac{\varphi(\gamma + \pi - \sigma(r - \rho))}{\Omega(\pi)}(1 - \alpha)Z, \tag{75}$$

$$\tilde{y} = \frac{(1 - b - bd)(1 - \alpha)Z}{\beta\Omega(\pi)}\Omega(r - \gamma), \tag{76}$$

$$\tilde{e} = \frac{b(1 - \alpha)Z}{\beta\Omega(\pi)}\varphi(r - \gamma), \tag{77}$$

$$\tilde{\Delta} = \varphi((1 - \sigma)r + \sigma\rho), \tag{78}$$

where we have used the demographic steady state $\beta\varphi(\pi) = 1$ and $l = \beta\Omega(\pi)$. (78) indicates that the relation $\lambda_4 = \frac{1}{\Delta} > 0$ holds regardless of the values of the underlying parameters.

Appendix D: Properties of μ_i ($i = C, H, E, \Delta$) and n_H

We first show that the terms μ_i ($i = C, H, E, \Delta$) and n_H are virtually constant over time. Then, in the second step, we compute the equilibrium values of μ_i ($i = C, H, E, \Delta$) and n_H.

8.1 The terms μ_i ($i = C, H, E, \Delta$) and n_H are virtually constant

To show that the terms μ_i ($i = C, H, E, \Delta$) and n_H are virtually constant over time, we focus on the case of μ_C to avoid repetitive explanation. The other cases are analogous. Letting $t - v = s$, (54) may be written as

$$\mu_C(t - v_1) = \frac{\int_0^D \mu(s)p(s)C(t-s,t)ds}{\int_0^D p(s)C(t-s,t)ds}. \tag{79}$$

Recalling (12), we have $C(t-s,t) = C(t-s,t-s)e^{\sigma(r-\rho)s}$. In addition, suppose that consumption were to grow at the time-varying rate $\gamma_C(u)$ over the period $(t-s,t)$. Then, $C(t-s,t-s) = C(t,t)e^{-\int_{t-s}^t \gamma_C(u)du}$ and (79) can be written as

$$\mu_C(t - v_1) = \frac{\int_0^D \mu(s)p(s)e^{-\int_{t-s}^t \gamma_C(u)du + \sigma(r-\rho)s}ds}{\int_0^D p(s)e^{-\int_{t-s}^t \gamma_C(u)du + \sigma(r-\rho)s}ds}. \tag{80}$$

To show that $|d\mu_C|$ is very small, we take the time derivative of (80) to obtain

$$\frac{d\mu_C/dt}{\mu_C} = -\frac{\int_0^D \mu(s)[\gamma_C(t) - \gamma_C(t-s)]f(s,t)ds}{\int_0^D \mu(s)f(s,t)ds} + \frac{\int_0^D [\gamma_C(t) - \gamma_C(t-s)]f(s,t)ds}{\int_0^D f(s,t)ds}, \tag{81}$$

where $f(s,t) \equiv e^{-\int_{t-s}^t \gamma_C(u)du}e^{\sigma(r-\rho)s}p(s) > 0$. Using the second mean value theorem, (81) simplifies to

$$\begin{aligned}
\frac{d\mu_C/dt}{\mu_C} &= -[\gamma_C(t) - \frac{\int_0^D \mu(s)\gamma_C(t-s)f(s,t)ds}{\int_0^D \mu(s)f(s,t)ds}] + [\gamma_C(t) - \frac{\int_0^D \gamma_C(t-s)f(s,t)ds}{\int_0^D f(s,t)ds}], \\
&= \frac{\int_0^D \mu(s)\gamma_C(t-s)f(s,t)ds}{\int_0^D \mu(s)f(s,t)ds} - \frac{\int_0^D \gamma_C(t-s)f(s,t)ds}{\int_0^D f(s,t)ds}, \\
&= \frac{\gamma_C(t-s_1)\int_0^D \mu(s)f(s,t)ds}{\int_0^D \mu(s)f(s,t)ds} - \frac{\gamma_C(t-s_2)\int_0^D f(s,t)ds}{\int_0^D f(s,t)ds}, \\
&= \gamma_C(t-s_1) - \gamma_C(t-s_2), \quad s_i \in (0,D), \ i = 1, 2,
\end{aligned} \tag{82}$$

where

$$\gamma_C(t-s_1) \equiv \frac{\int_0^D \mu(s)\gamma_C(t-s)f(s,t)ds}{\int_0^D \mu(s)f(s,t)ds},$$

$$\gamma_C(t - s_2) \equiv \frac{\int_0^D \gamma_C(t - s) f(s, t) ds}{\int_0^D f(s, t) ds}.$$

We can rewrite (82) as:

$$d\mu_C = \mu_C[\gamma_C(t - s_1) - \gamma_C(t - s_2)]dt. \tag{83}$$

Written in this way, we see that of $|d\mu_C|$ involves the difference of the consumption growth rate $\gamma_C(t - s)$ at two points in time interacting with dt. Thus, it can be observed to be a second-order effect and therefore negligible in the linear approximations describing the local dynamics. Analogous arguments apply to μ_i ($i = H, E, \Delta$) and n_H, thereby enabling us to approximate them all as constants in assessing the potential dynamic adjustment of the aggregate economy.

8.2 Computation of the equilibrium values of μ_i ($i = C, H, E, \Delta$) and n_H

Using (12), (13) and the fact that along the balanced growth path $C(v, v)/w(v)$ is independent of v and the wage rate grows at the constant rate γ, we can write (54) as:

$$\mu_C(t - v_1) = \frac{\int_{t-D}^t \mu(t - v) p(t - v) e^{[\sigma(r-\rho)-\gamma](t-v)} dv}{\int_{t-D}^t p(t - v) e^{[\sigma(r-\rho)-\gamma](t-v)} dv}. \tag{84}$$

Recalling the demographic steady-state relationship, we can write (84) in the age domain as:

$$\mu_C = \frac{\int_0^D \mu(s) e^{-[\gamma+\pi-\sigma(r-\rho)]s - M(s)} ds}{\int_0^D e^{-[\gamma+\pi-\sigma(r-\rho)]s - M(s)} ds}. \tag{85}$$

where $s = t - v$ is the age of the agent. Using the $\varphi(x)$ functions and the parameterized survival function of (35), (85) is written as:

$$\mu_C = \frac{\mu_1}{\mu_0 - 1} \frac{1 - e^{-[\gamma+\pi-\sigma(r-\rho)-\mu_1]D}}{\gamma + \pi - \sigma(r - \rho) - \mu_1} \frac{1}{\varphi(\gamma + \pi - \sigma(r - \rho))}, \tag{86}$$

which for our specified parameters yields the value $\mu_C = 0.0161$.

From (63), recalling the demographic steady-state relationship, we obtain

$$\mu_H = \frac{\int_0^D \mu(s) L(s) e^{-(r-\gamma)s - M(s)} ds}{\int_0^D L(s) e^{-(r-\gamma)s - M(s)} ds}. \tag{87}$$

Using the same arguments as above and noting the $\Omega(x)$ functions and the parameterized labor supply fraction function of (36), (87) is written as:

$$\mu_H = \frac{\mu_1}{\mu_0 - 1} \{ \frac{l_0}{l_0 - 1} \frac{1 - e^{-(r-\gamma-\mu_1)R}}{r - \gamma - \mu_1} - \frac{1}{l_0 - 1} \frac{1 - e^{-(r-\gamma-\mu_1-l_1)R}}{r - \gamma - \mu_1 - l_1} \} \frac{1}{\Omega(r - \gamma)}, \tag{88}$$

which for our specified parameters yields the value $\mu_H = 0.0015$.

Similarly, from (62), (64) and (65), we obtain

$$\mu_E = \frac{\mu_1}{\mu_0 - 1} \frac{1 - e^{-(r-\gamma-\mu_1)D}}{r - \gamma - \mu_1} \frac{1}{\varphi(r - \gamma)}, \tag{89}$$

$$\mu_\Delta = \frac{\mu_1}{\mu_0 - 1} \frac{1 - e^{-[(1-\sigma)r+\sigma\rho-\mu_1]D}}{(1 - \sigma)r + \sigma\rho - \mu_1} \frac{1}{\varphi((1 - \sigma)r + \sigma\rho)}, \tag{90}$$

$$n_H = \frac{\int_0^D n(s)L(s)e^{-(r-\gamma)s-M(s)}ds}{\int_0^D L(s)e^{-(r-\gamma)s-M(s)}ds}, \tag{91}$$

which for our specified parameters yields the value $\mu_E = 0.0032$, $\mu_\Delta = 0.0036$ and $n_H = 0.0076$, respectively.

Appendix E: Estimated survival function

We estimate the two parameters μ_0 and μ_1 in (35) by nonlinear least squares, using life tables for Japan in 2010. Our estimated results in Table 4 highlight that we obtain a tight fit with highly significant parameter estimates. The resulting survival function illustrated in Figure 7 confirms that the survival function tracks the actual survival data very well from 20 until 100. Beyond that, the concavity of the function yields a less satisfactory fit.

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Table 1   G7 countries

country	Life expectancy at birth			Crude birth rate (%)			Population growth rate (%)			Old-age dependency ratio 65+/(20-64) (%)		
	1960-65	2010-2015	2050-2055	1960-65	2010-2015	2050-2055	1960-65	2010-2015	2050-2055	1960	2010	2050
Canada	71.27	81.78	87.35	2.48	1.09	0.98	1.90	1.04	0.32	15.1	22.5	49.3
France	70.66	81.84	87.65	1.81	1.24	1.10	1.30	0.45	0.13	20.6	29.1	51.4
Germany	69.98	80.65	86.72	1.76	0.83	0.84	0.75	0.06	-0.43	19.2	34.0	63.7
Italy	69.59	82.84	88.70	1.86	0.86	0.84	0.78	0.07	-0.36	16.4	33.7	73.9
Japan	68.97	83.30	88.68	1.72	0.83	0.80	1.02	-0.12	-0.56	10.6	38.9	77.4
United Kingdom	71.01	80.45	86.20	1.83	1.26	1.10	0.70	0.63	0.27	20.2	27.1	46.4
United States	70.06	78.87	84.58	2.22	1.26	1.17	1.37	0.75	0.37	17.4	21.6	40.9

Table 2. Baseline parameters and implied demographic and social security variables

Baseline Model		
Structural parameters		
Total factor productivity	Z	0.3343
Capital share of output	α	0.35
Depreciation rate	δ	0.05
Intertemporal substitution of elasticity	σ	0.75
Time preference rate	ρ	0.035
Real interest rate	r	0.067
Social security/employment parameters		
Social security benefit rate	b	0.4
Social security tax rate (implied)	τ_s	0.1996
Labor supply fraction parameters	l_0	65.1049
	l_1	0.072
Maximum working age	R	58
Average retirement age		45.0159
Dependency ratio (implied)	d	0.4990
Demographic parameters		
Youth mortality	μ_0	145.734
Old age mortality	μ_1	0.0607205
Maximum attainable age (implied)	D	82.0445
Birth Rate	β	0.016
Life Expectancy at 20 (implied)	L_{20}	66.1425
Life Expectancy at 65 (implied)	L_{65}	24.9432
Population growth rate (implied)	π	0.0016

Table 3. Benchmark equilibrium

Baseline Model		
Economic Variables		
Per capita output growth rate	γ	0.0203
Utility multiplier	u	-72.1928
Eigenvalues	λ_1	0.2658
	λ_2	0.0337
	λ_3	0.0534
	λ_4	0.0499
$x(t) = c(t)/k(t)$	$\widetilde{x}$	0.2623
$y(t) = H(t)/k(t)$	$\widetilde{y}$	2.3377
$e(t) = E(t)/k(t)$	$\widetilde{e}$	2.6104
The inverse marginal propensity to consume out of total wealth	$\widetilde{\Delta}$	20.0359
μ_i and n_H terms	μ_C	0.0161
	μ_H	0.0015
	μ_E	0.0032
	μ_Δ	0.0036
	n_H	0.0076
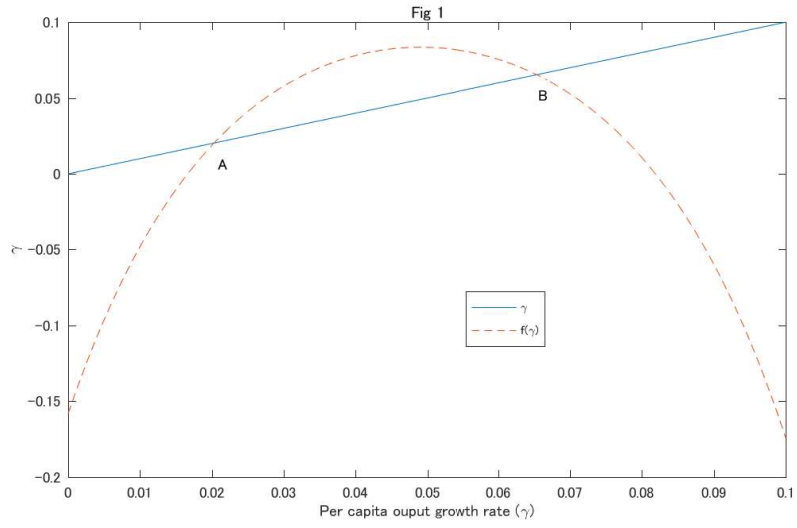
Figure 1: A unique consistent equilibrium growth rate γ^*.
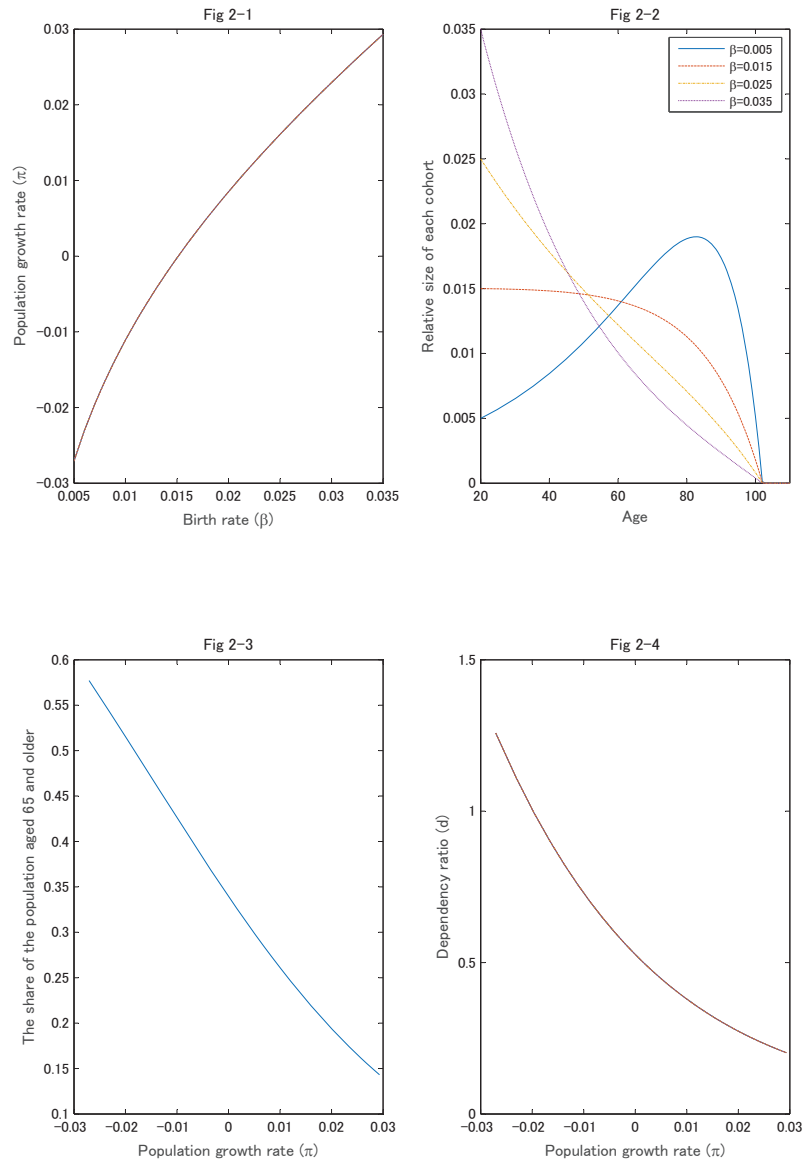
Figure 2: The effects of a decline in birth rate
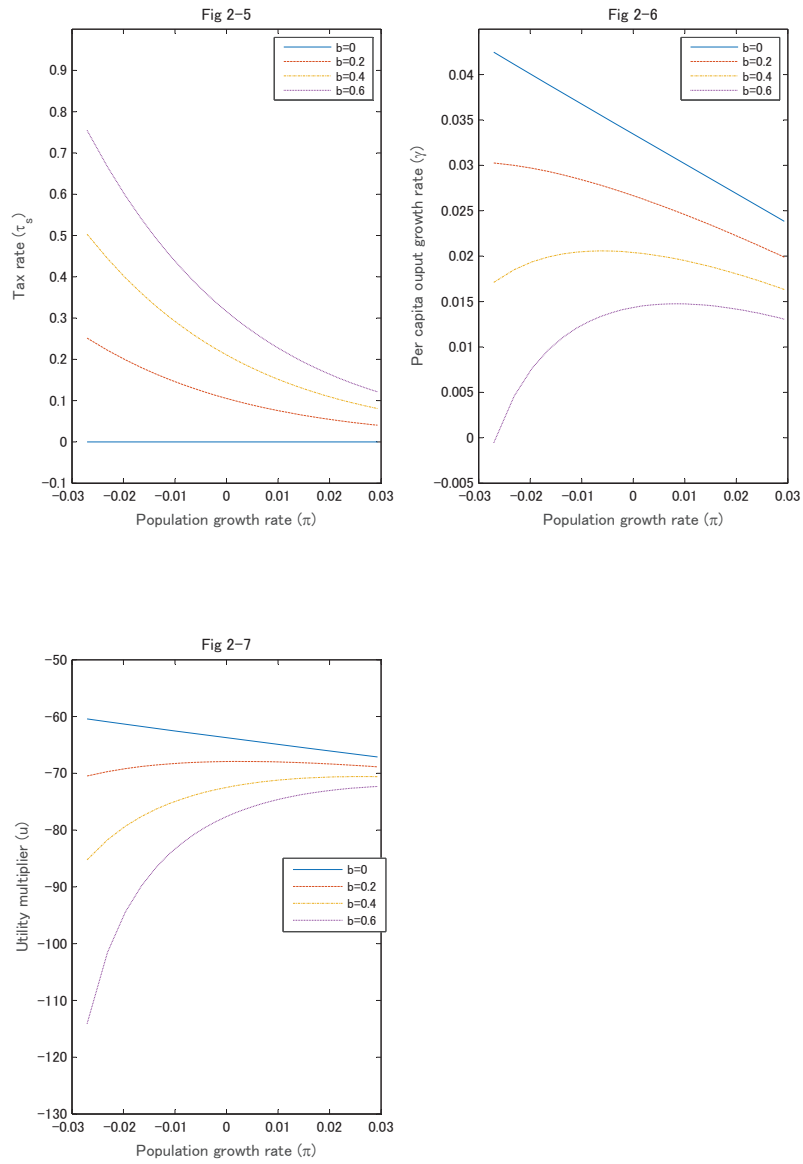
Figure 2: The effects of a decline in birth rate (cont.)
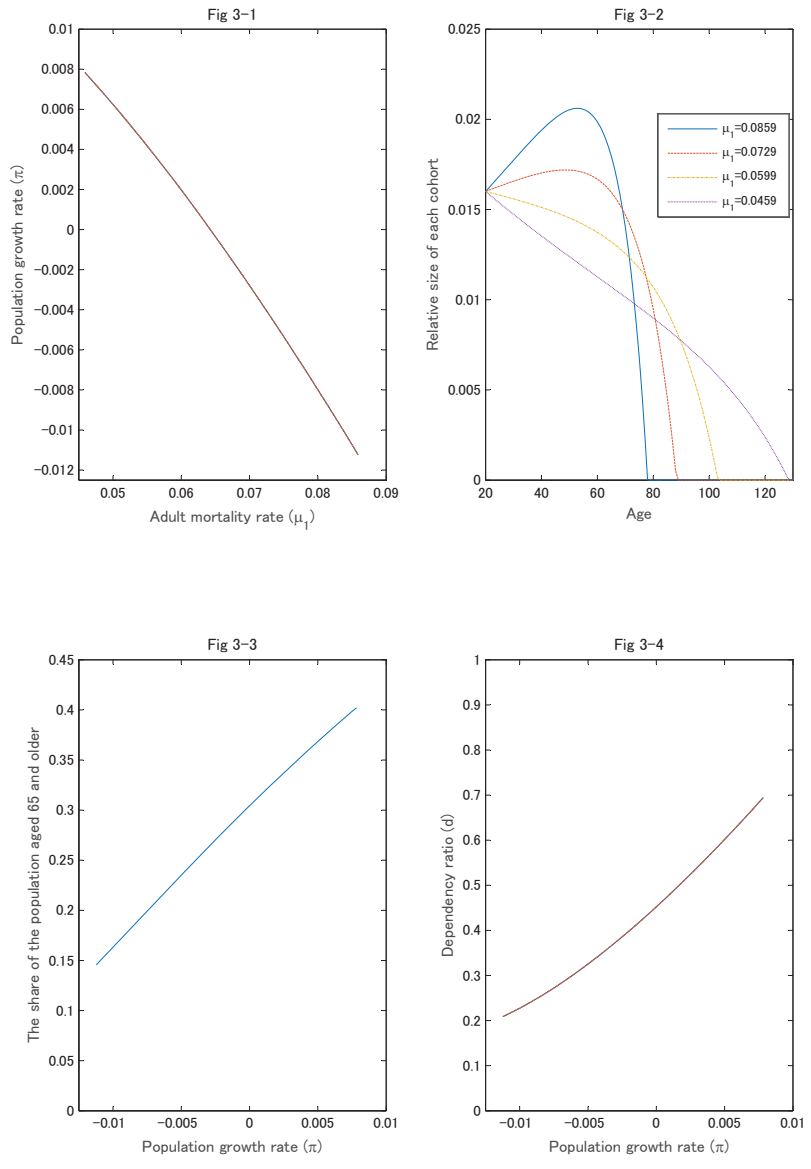
Figure 3: The effects of a decline in mortality rate
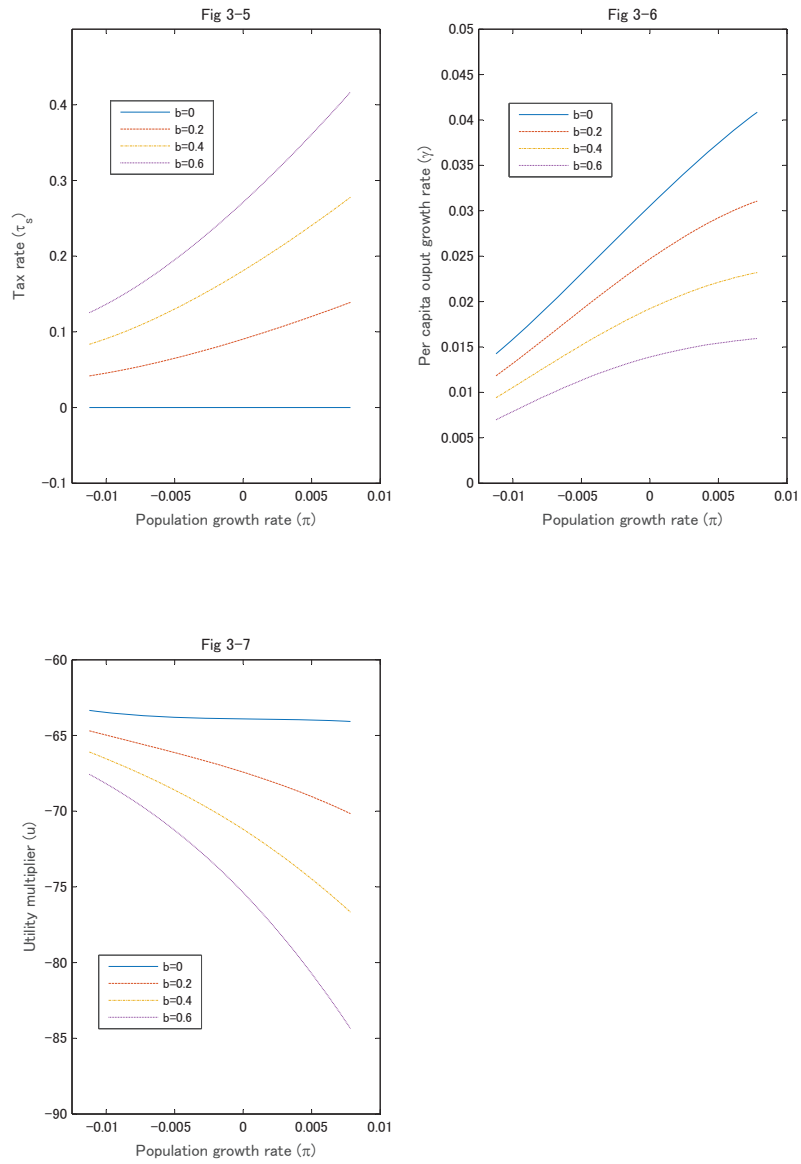
Figure 3: The effects of a decline in mortality rate (cont.)
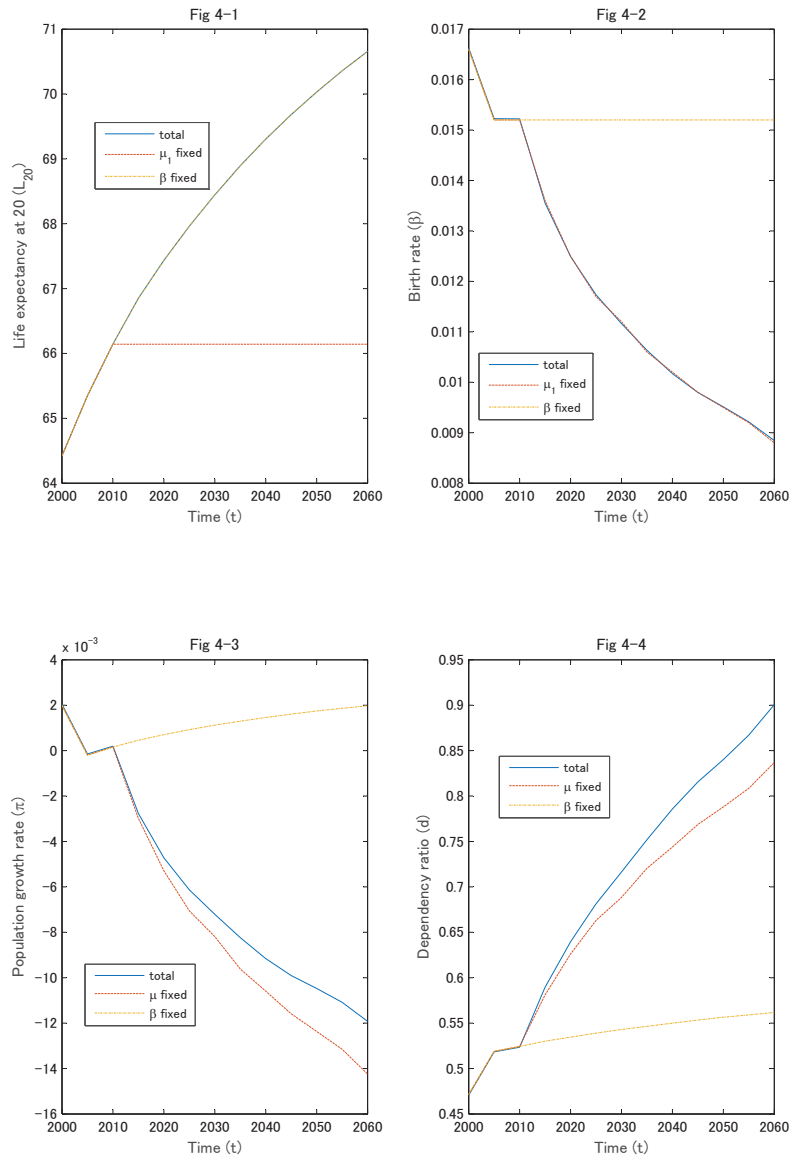
Figure 4: The effects of the predicted demographic changes in Japan
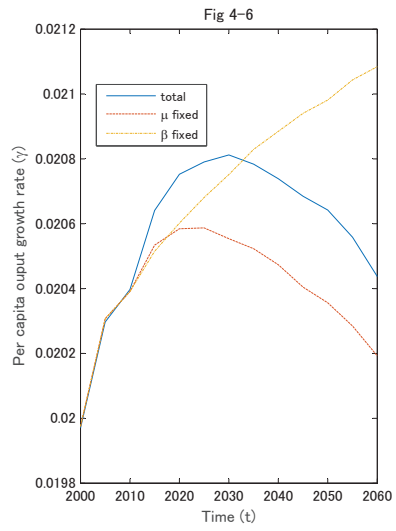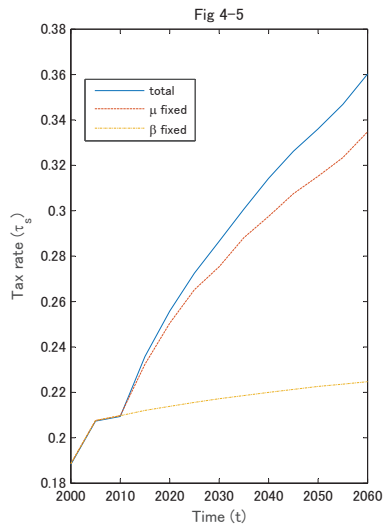
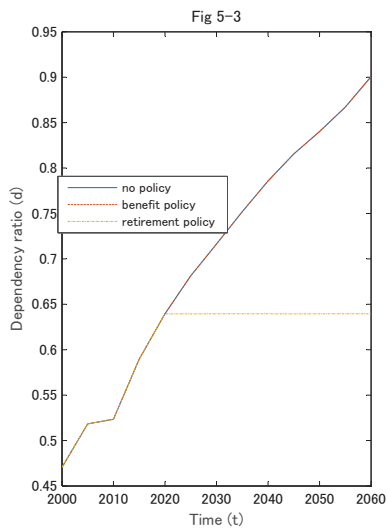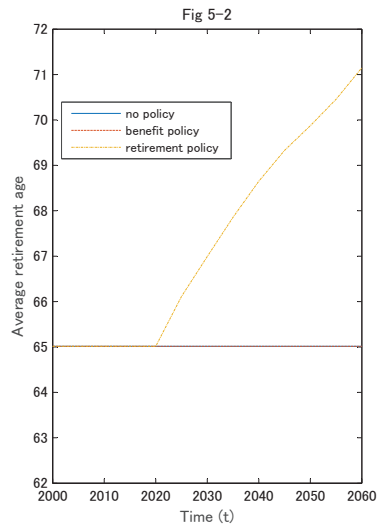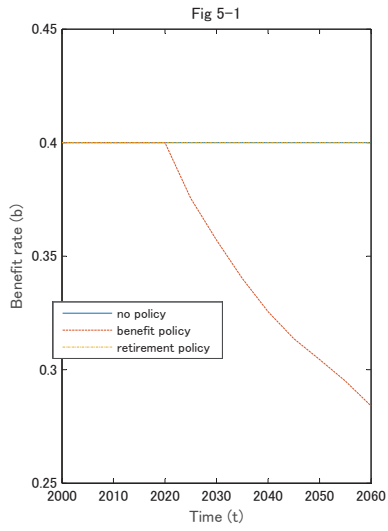Figure 4: The effects of the predicted demographic changes in Japan (cont.)
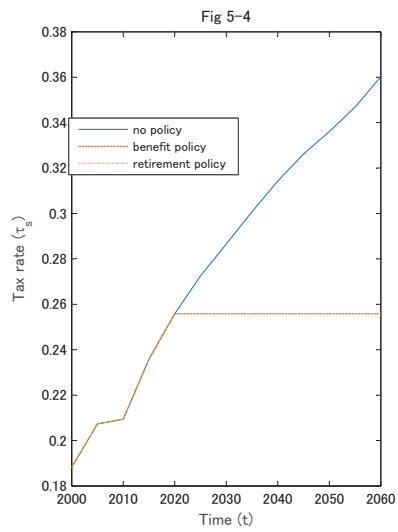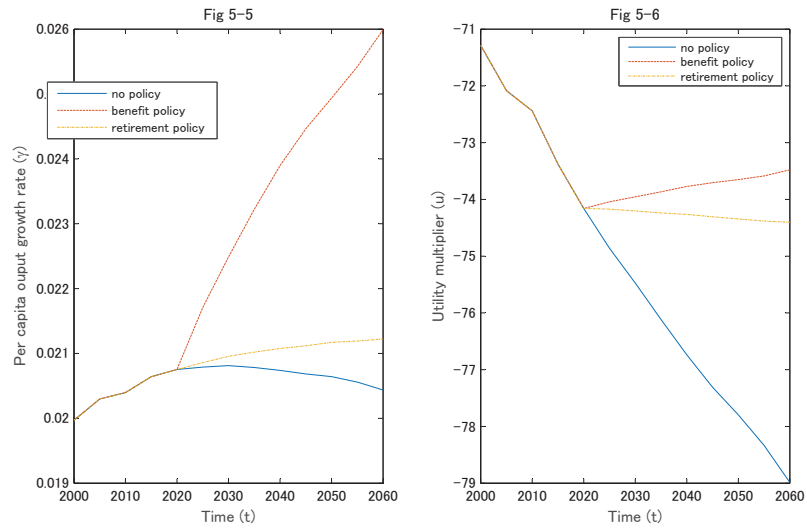
Figure 5: The effects of social security reforms

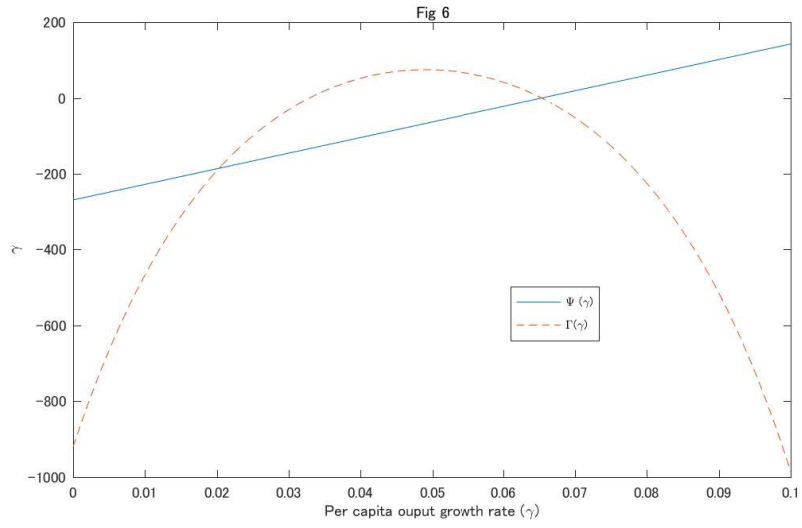Figure 5: The effects of social security reforms (cont.)

Figure 6: A unique consistent equilibrium growth rate γ^*.

Table 4

Estimated survival function in Japan

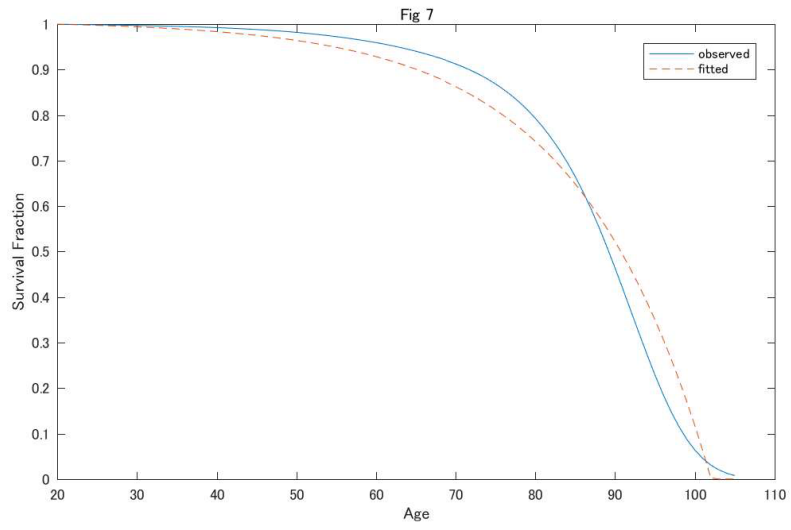| Cohort in Japan (women over 20)	2010
μ_0 [t-value]	145.734 [6.37]
μ_1 [t-value]	0.0607 [30.17]
Adj R^2	0.996

Figure 7: Survival Function