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One Share-One Vote, Market for Control and Corporate Democracy

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One Share-One Vote, Market for Control and Corporate Democracy[1]

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Abstract

This paper attempts to provide a framework for the formal analysis of the institution of voting in a corporation. I assume that there is a market for control, and examine whether corporate democracy is attained through voting by shareholders. It is shown that whether corporate democracy is attained or not depends on the managers' preferences over the risk of the projects. It is also shown that, while higher reward makes managers more risk tolerant and hence brings the competitive outcome closer to the democratic solution, it also tends to induce collusion between the rival managers and then to realize a collusive outcome that is far from the democratic solution.

1. Introduction

One of the major characteristics of modern corporations is the separation of the ownership of the firm from the control of the firm. In large corporations, in particular, the ownership is often widespread over a large number of shareholders. Economic analysis on these aspects of modern corporations was first made by Berle and Means (1932). In this seminal work, Berle and Means insisted that the power of control had been necessarily shifted away from the owners to the managers as the result of the diffusion of ownership.

In spite of the arguments by Berle and Means, however, shareholders today still have voting rights on some important corporate issues. For instance, Japanese corporate rules allow shareholders to elect and dismiss executives, to decide the reward to executives, to change corporate charters, and to admit the merger of the firm. Taking account of the fact that it costs for the firm to maintain the institution of voting, natural questions arise why shareholders still have the voting rights and what the shareholders' voting rights are for. So far, there are two types of work that attempted to answer these questions.

The first type of work is based on the monitoring approach, which follows the conventional argument that the vote is attached to the ownership because of the status of the owners as the residual claimants. That is, it is economically efficient to give the decision rights to the shareholders because shareholders have the right incentives to monitor the managers as the residual claimants. According to this idea, Easterbrook and Fischel (1983) examined the actual state corporate rules in the United States and concluded that the security-voting contributes more or less to the shareholders' interests.

The second strand of work focuses on the takeover bids competition by rival managers, which includes Grossman and Hart (1986) and Harris and Raviv (1986). Grossman and Hart considered competition for control between rival managers through tendering offers of stocks. In their model, shareholders are assumed completely ignorant about involving corporate decisions through directly exercising their voting rights but conscious about selling their stocks at a preferable term. Harris and Raviv (1986) made a similar analysis to Grossman and Hart in a more sophisticated mathematical model of the firm. The essence of their results is that the institution of voting, in particular the one share-one vote, brings shareholders in some benefits not through the direct exercise of voting rights by shareholders but through selling their voting rights to the managers.

Although the approaches are different, the two types of work mentioned in the previous paragraph commonly assumed the homogeneity of shareholders, implicitly and explicitly. This setting of the model contributes to a great extent to simplify the complicated reality of the corporations and enables us to derive clear-cup results about the benefit of voting. In particular, under the homogeneity assumption, we can treat shareholders as if they were a single agent and therefore can avoid dealing with conflicts among shareholders. Hence, we can evaluate the institution of voting based on the monetary benefit the vote brings shareholders in as a whole.

In the present paper I construct a model of the corporation with shareholders who have heterogeneous economic profiles. Especially, in the model firm's investment is made under uncertainty and shareholders have different assessment on several investment options. One difficulty arising from introducing the heterogeneity of shareholders is how to evaluate the benefit of the voting. That is, the question is which shareholders' benefit should be taken into account, and in what extent. This problem can be settled somehow if monetary transfer is possible among shareholders. But allowing for the fact that a great number of shareholders are involved in a corporation and they are usually unorganized, it is unrealistic to assume that there is a system of redistribution among shareholders. In the present paper I adopt the concept of democracy as a normative criteria in evaluating the institution of voting in a corporation. It might be debatable to let corporate democracy be the normative criteria of the analysis. Roughly speaking, democracy is a collective decision-making rule which attaches more importance on the "average" shareholders' interests than on the "extreme" shareholders'. Overall, for most shareholders, saving the "average" shareholders' interests are considered more preferable than protecting the "extreme" shareholders' interests. By this reason, I consider that the concept of democracy is a meaningful criteria, if not the only one, of course, to evaluate the institution of voting. I assume the existence of the market for control, and examine whether the corporate democracy is attained by the institution of voting.

The major results obtained in the present paper are as follows. First, whether corporate democracy is attained or not depends on the degree of risk tolerance of the agents competing for the manager. Only when they are risk tolerant enough, corporate democracy is attained. Second, under an economic circumstance in which agents competing for the manager are more risk tolerant, they are more likely to collude and choose the project that is far from the democratic solution.

The remainder of the paper is constructed as follows. In the next section, the model of a production subeconomy is described. In section 3, I examine whether and in what extent the democratic solution is chosen through voting when there is a market for control. In section 4, I examine the circumstances in which collusion occurs between rival managers and consider the implication of collusion on the corporate democracy. Section 5 concludes the paper with some remarks.

2. The model

Consider a subeconomy $\mathcal{E} = (N, A)$, where N is the set of agents and A is the set of assets for production.

Agent $i \in N$ has a preference $\succeq_i$ over a lottery space $L = \{[q, y; 1-q, \bar{y}] : 0 \leq q \leq 1, y, \bar{y} \in R\}$. Let $u_i : L \to R$ be the expected utility function that represents the preference $\succeq_i$ over L. I abuse the notation such that $u_i : R \to R$ with $u_i(y) = u_i([1, y; 0, \bar{y}])$, where it is assumed that $u_i' > 0$ and $u_i" < 0$.

Agents are classified by two characteristics: the ownership of the assets for production (qualification for the shareholder), and the ability to manage the firm (qualification for the manager). Let $S \subset N$ be the set of agents with some ownership of the assets, and $M \subset N$ be the set of agents with ability to manage the firm. The ownership of the assets is represented by the stocks. [1] Let e_i be the number of stocks $i \in S$ owns, and $E = \sum_{i \in S} e_i$ be the total number of stocks.

Assumption 2.1: Each e_i for $i \in S$ is a natural number, and E is an odd number.

Let $\theta_i = e_i/E$ be agent i's ownership share.

Lemma 2.1: Under Assumption 2.1, for any partition $\{S_1, S_2\}$ of S, $\sum_{i \in S_1} \theta_i \neq \sum_{i \in S_2} \theta_i$.

(Proof) Suppose that, for some partition $\{S_1, S_2\}$ of S, $\sum_{i \in S_1} \theta_i = \sum_{i \in S_2} \theta_i$. Since $E \neq 0$, this implies that $\sum_{i \in S_1} e_i = \sum_{i \in S_2} e_i$. Add $\sum_{i \in S_2} e_i$ to both sides of the equation to get $E = 2 \sum_{i \in S_2} e_i$, where the LHS is odd by assumption, while the RHS is even. This is a contradiction. □

For simplicity, I assume that S and M constitute a partition of N (i.e., $S \cup M = N$ and $S \cap M = \emptyset$) and $\sharp M = 2$. [2] I also assume that the number of shareholders, $\sharp S$, is so large that cooperation among shareholders is impossible.

Let $W_i \in R$ be the initial non-stock wealth of agent $i \in N$.

Agent $i \in N$ is characterized by his expected utility function $u_i : L \to R$, his qualification either as a shareholder with his stockholding e_i or a manager, and his initial non-stock wealth W_i.

Let $P = \{1, ..., p\}$ be the set of project options available when the assets in A are operated by the manager. Project $x \in P$ is characterized by a pair of real numbers $(r(x), V(x))$, where $r(x)$ is the risk of project x (i.e., $r(x)$ is the probability project x fails, while $1 - r(x)$ is the probability it succeeds),

[1] Throughout the paper, I ignore the capital market.

[2] $\sharp M$ shows the cardinality of the set M.

and $V(x)$ is the return of project x when it succeeds. If the project fails, the return is assumed zero. Let $\tilde{V} : [0,1] \to R_{++}$ be a twice differentiable function with $\tilde{V}(0) > 0$, $\tilde{V}' > 0$ and $\tilde{V}'' < 0$. [3] We assume that $V(x) = \tilde{V}(r(x))$ for all $x \in P$. Without loss of generality, I order projects $1, ..., p$ such that $r(1) < r(2) < ... < r(p)$. [4]

The shareholders exercise their voting rights based on their preferences. Under one share-one vote rule, agent $i \in S$ with e_i stocks has e_i votes in total E votes. For $i \in S$ define a function $U_i : [0,1] \to R$ such that

$$U_i(r) = ru_i(W_i) + (1-r)u_i(W_i + \theta_i \tilde{V}(r)). \tag{1}$$

Let $x_j \in P$ be the project presented by $j \in M$ to the shareholders for voting. For any pair $x_1, x_2 \in P$ put forward by agents in M, agent $i \in S$ votes for x_1 if $U_i(r(x_1)) > U_i(r(x_2))$, and for x_2 if $U_i(r(x_2)) > U_i(r(x_1))$. I here emphasize that shareholders make their voting decisions solely based on their preferences, and hence it is projects put forward, not candidates for the manager, that shareholders vote for.

Assumption 2.2: Suppose that $U_i(r(x_1)) = U_i(r(x_2))$ for $i \in S$.

(1) If $x_1 \neq x_2$, agent i votes for either project deterministically. For convenience, I represent agent i's voting behavior over all pairs of indifferent projects by the symbol Γ_i. [5] [6]

(2) If $x_1 = x_2 = x$, agent i votes for x (for no choice).

As for Assumption 2.2.(1), an example of Γ_i is the voting behavior of agent i such that whenever two projects are indifferent for agent i he votes for $\min\{x_1, x_2\}$, i.e., when indifferent agent i votes for the less risky project. As for Assumption 2.2(2) refer to Assumption 2.3 below.

It is easily shown that U_i is strictly concave in r for all $i \in S$. [7] The following property of the shareholder's preference ensures the existence of the voting equilibrium.

Lemma 2.2: U_i is single-peaked with respect to $\{r(1), ..., r(p)\}$ for all $i \in S$.

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[3]The assumption $\tilde{V}(0) > 0$ ensures that all shareholders do not throw their stocks away.

[4]If $r(x) = r(\bar{x})$ and hence $V(x) = V(\bar{x})$, projects x and $\bar{x}$ can be regarded as the same project and labeled as x. Then, I can order the risk of the projects with strict inequality.

[5]In the analysis of the text, I assumed for simplicity that, when agent i votes for either project, he votes all his shares for the project. Essentially it will not affect at all to the analysis if I allow the agent to divide his shares and vote for two projects as long as the divided shares are natural numbers.

[6]In fact, it suffices for the following analysis that this assumption holds only for the "median" voter.

[7]From equation (1), for each $i \in S$ we have $U_i''(r) = -2u_i'(W_i + \theta_i \tilde{V}(r))\theta_i \tilde{V}'(r) + (1-r)u_i''(W_i + \theta_i \tilde{V}(r))(\theta_i \tilde{V}'(r))^2 + (1-r)u_i'(W_i + \theta_i \tilde{V}(r))\theta_i \tilde{V}''(r) < 0$.

(Proof) Strict concavity of U_i in r over $[0,1]$ is sufficient for its single-peakedness with respect to $\{r(1), ..., r(p)\}$. □

Let $v : P * P \rightarrow [0,1]$ be the vote function, where, for any pair $(x_1, x_2) \in P * P$ with $x_1 \neq x_2$, $v(x_1, x_2)$ is defined to be the sum of the ownership shares of the agents who vote for x_1 against x_2, and for any pair $(x_1, x_2) \in P * P$ with $x_1 = x_2$ it is defined such that $v(x_1, x_2) = 1/2$. By definition, for any pair $(x_1, x_2) \in P * P$, $v(x_1, x_2) + v(x_2, x_1) = 1$.

In $\mathcal{E}$, a project choice $x \in P$ is said to be the *majority project* [8] iff $v(x, \bar{x}) > v(\bar{x}, x)$ for all $\bar{x} \in P \backslash \{x\}$. Equivalently, a project choice $x \in P$ is the majority project iff $v(x, \bar{x}) > 1/2$ for all $\bar{x} \in P \backslash \{x\}$. The choice of the majority project in E is considered a democratic outcome in the sense that majority shareholders' preferences are reflected in the corporate decisions. [9]

Consider for $i \in S$ a set $\arg\max_{x \in P} U_i(r(x))$. Since U_i is strictly concave in r, the set has either one element or two different elements. If it is singleton, let $\hat{x}_i$ denote the element of the set. If the set has two different elements, select one according to Γ_i in Assumption 2.2(1) and let $\hat{x}_i$ denote the selected element. In either case, I call $\hat{x}_i$ the best project for agent $i \in S$. Without loss of generality order agents in S such that $\hat{x}_1 \leq \hat{x}_2 \leq ... \leq \hat{x}_s$. Let $k \in S$ be an agent such that $\sum_{i=1}^{k-1} \theta_i < 1/2$ and $\sum_{i=1}^{k} \theta_i \geq 1/2$, i.e., agent $k \in S$ is the "median" shareholder. Notice, by Assumption 2.1 and Lemma 2.1, that $\sum_{i=1}^{k} \theta_i \geq 1/2$ is equivalent to $\sum_{i=1}^{k} \theta_i > 1/2$. To avoid heavy notation, let $\hat{x}_k = \kappa$ (kappa), i.e. $\kappa \in P$ is the best project of the median shareholder. It is shown that κ is the only majority project in E

Lemma 2.3 (Black (1958)): $\kappa \in P$ is the unique majority project .

(Proof) That agent $i \in S$ votes for x_1 against x_2 is regarded as that e_i agents, each with one vote, vote for x_1 against x_2. Then, Black's Theorems (Black (1958), p.14 and p.16) ensure that $\kappa \in P$ is the unique majority project. □

When $x_1 \neq x_2$, if $v(x_1, x_2) > 1/2$, 1 wins the competition for control and manages the firm with project x_1, and vice versa.

Assumption 2.3: When $x_1 = x_2$, 1 and 2 in M tie with each other in the competition and will draw a 1/2 win-1/2 lose lottery. The winner of the lottery manages the firm with the project.

[8] I owe this terminology to Berle and Means (1932)

[9] By the term corporate democracy here I mean democracy among shareholders, not other agents in and around the firm like managers, employees or bondholders. This terminology is a tradition since Berle and Means (1932).

When $j \in M$ manages the firm, he incurs a certain cost of management $c_j \geq 0$. The task of the manager of the firm is to choose and execute the project. If he chooses project x and succeeds, his reward is $b(x) > 0$. If he fails, he earns nothing. I adapt an interpretation that the managerial reward $b(x)$ is due to his increased market value as a talented or hard-working manager, not due to the performance-based compensation set by the shareholders. [10] The reason for this interpretation is that in this paper I think of unorganized small shareholders who are unlikely to set the compensation schemes on the manager. I assume that, for all $x \in P$, $b(x) = \tilde{b}(r(x))$ for some function $\tilde{b} : [0,1] \rightarrow R_{++}$ with $\tilde{b}(0) > \max\{c_1, c_2\}$, $\tilde{b}' \geq 0$ and $\tilde{b}" \leq 0$. [11] For $j \in M$ define a function $U_j : [0,1] \rightarrow R_+$ such that

$$U_j(r) = ru_j(W_j - c_j) + (1-r)u_j(W_j - c_j + \tilde{b}(r)). \tag{2}$$

It is easily shown that U_j is strictly concave in r for $j \in M$. [12]

Lemma 2.4: U_i is single-peaked with respect to $\{r(1), ..., r(p)\}$ for $i \in M$.

(Proof) Strict concavity of U_i in r over $[0,1]$ is sufficient for its single-peakedness with respect to $\{r(1), ..., r(p)\}$. □

Agent $j \in M$ prefers managing the firm with project x to withdrawing if $U_j(r(x)) > u_j(W_j)$, and prefers withdrawing to managing the firm with project x if $U_j(r(x)) < u_j(W_j)$. I assume that $j \in M$ prefers withdrawing to managing the firm with project x if $U_j(r(x)) = u_j(W_i)$. [13]

For $j \in M$, let $x_j^0 \in P$ be a project such that $U_j(r(x_j^0)) > u_j(W_j)$ and $U_j(r(x_j^0 + 1)) \leq u_j(W_j)$. I call x_j^0 the marginal project for $j \in M$ in the sense that it is the riskiest project $j \in M$ can endure. Let $x_j^* \in P$ be a project such that $U_j(r(x_j^*)) \geq U_j(r(x_j))$ for all $x \in P$. I call x_j^* the best project for $j \in M$.

Lemma 2.5: Let $j \in M$.
(1) There exists a unique $x_j^0 \in P$.
(2) There exists $x_j^* \in P$.

(Proof) (1) For each $j \in M$, $U_j(0) = u_j(W_j - c_j + \tilde{b}(0)) > u_j(W_j)$ since

[10] On the human capital of managers, see Milgrom and Roberts (1991), chapter 13, pp. 429-432.

[11] The assumption $\tilde{b}(0) > \max\{c_1, c_2\}$ ensures that there exists a project which is profitable for the manager. Refer to the proof of Lemma 2.4(1) below.

[12] From equation (2), for each $j \in M$ we have $U_j"(r) = -2u_j'(W_j + \tilde{b}(r))\tilde{b}'(r) + (1-r)u_j"(W_j + \tilde{b}(r))(\tilde{b}'(r))^2 + (1-r)u_j'(W_j + \tilde{b}(r))\tilde{b}"(r) < 0$.

[13] Assuming the reverse, i.e., that $j \in M$ prefers managing the firm to withdrawing with project x if $U_j(r(x)) = u_j(W_j)$, will not affect the results of the following analysis. What is necessary is solely that when indifferent $j \in M$ behaves deterministically.

$\tilde{b}(0) > c_j$ by assumption, and $U_j(1) = u_j(W_j - c_j) < u_j(W_j)$. Since U_j is strictly concave in r, there exists a unique r in $[0,1]$ with $U_j(r) = u_j(W_j)$. Let r_j^0 denote that r. Let $x_j^0 = \max\{x \in P : r(x) < r_j^0\}$. Obviously this x_j^0 is unique, and in fact it is the one I am looking for.

(2) Since $\{U_j(r(x)) : x \in P\}$ is a finite subset of R, there exists a maximum element. Let $x_j^* \in \arg\max_{x \in P} U_j(r(x))$. This x_j^* is the one I am looking for. □

Since U_j is strictly concave in r for $j \in M$, the set $\arg\max_{x \in P} U_j(r(x))$ has either one element or two different elements. Throughout the rest of the paper, for simplicity of the analysis I consider the case when the set has only one element. [14]

Assumption 2.4: For $j \in M$ with $x_j^0 \geq 2$,

$$\frac{1}{2}U_j(r(1)) + \frac{1}{2}u_j(W_j) < U_j(r(2))$$

$$\frac{1}{2}U_j(r(x)) + \frac{1}{2}u_j(W_j) < \min\{U_j(r(x-1)), U_j(r(x+1))\} \quad for \quad 1 < x < x_j^0$$

and

$$\frac{1}{2}U_j(r(x_j^0)) + \frac{1}{2}u_j(W_j) < U_j(r(x_j^0 - 1)).$$

Assumption 2.4 requires that project options P be "dense" enough. That is, if with $x_j \leq x_j^0$ agent $j \in M$ ties with the other, j becomes better off by moving to a neighboring project, i.e., $x_j - 1$ or $x_j + 1$, with which he wins the competition.

3. Competition and Corporate Democracy

The competition for control can be formulated as a noncooperative game played by agents 1 and 2 in M, where their strategy space is P and their utility levels are given by

$$(u_1, u_2) = \begin{cases} (U_1(r(x_1)), u_2(W_2)) & \text{if } v(x_1, x_2) > 1/2 \\ (\frac{1}{2}U_1(r(x_1)) + \frac{1}{2}u_1(W_1), \frac{1}{2}U_2(r(x_2)) + \frac{1}{2}u_2(W_2)) & \text{if } x_1 = x_2 \\ (u_1(W_1), U_2(r(x_2))) & \text{if } v(x_1, x_2) < 1/2. \end{cases}$$

[14] If two different elements $\bar{x}_j^*$ and $\check{x}_j^*$ are allowed to be in that set, I can proceed essentially the same analysis by replacing x_j^* in the following section with $\min\{\bar{x}_j, \check{x}_j\}$ or $\max\{\bar{x}_j, \check{x}_j\}$ accordingly.

In what follows as the equilibrium concept I use Nash equilibrium. Throughout the rest of the paper, without loss of generality, I assume that $x_1^0 \le x_2^0$. The following lemma is used to show Lemmas 3.2, 3.3 and 3.4 below.

Lemma 3.1: When $x_1^0 < \kappa$, if (x_1, x_2) is an equilibrium, then $x_1^0 \le x_2 \le x_2^0$.

(Proof) Suppose that $x_2 < x_1^0$. To that x_2, 1 is better off at x_1 with $x_2 < x_1 \le x_1^0$ than at x_1 with $x_1 \le x_2$, since in the former 1 wins with net benefit while in the latter 1 at most ties with 2. Hence, x_1 must be such that $x_2 < x_1 \le x_1^0$. To that x_1, on the other hand, 2 is better off at x_2 with $x_1 \le x_2 \le x_2^0$ than at x_2 with $x_2 < x_1$ or $x_2^0 < x_2$, since in the former 2 at least ties with 1 with net benefit while in the latter 2 either loses or wins with net loss. Hence, x_2 must be such that $x_1 \le x_2 \le x_2^0$. This is a contradiction.

Next, suppose that $x_2 > x_2^0$. If 2 wins at that x_2, 2 incurs net loss. Hence, in order for (x_1, x_2) to be an equilibrium, x_1 must be such that $v(x_2, x_1) < 1/2$ or equivalently $v(x_1, x_2) > 1/2$. Then, for x_1 to be the best response to x_2, it must be that $x_1 \le x_1^0$, since otherwise 1 incurs net loss. But to that x_1, 2 is better off at, for instance, x_2^0 than at x_2 with $x_2 > x_2^0$, since in the former 2 at least ties with 1 with net benefit while in the latter 2 either loses or wins with net loss. This is a contradiction. □

Recall that the marginal project x_j^0 is the riskiest project agent $j \in M$ can endure. Then, it can be said that the larger the x_j^0 is, the more risk tolerant agent $j \in M$ is. In what follows, I examine how the degree of risk tolerance of the agents in M influences the competitive outcome of the project choice.

Before I proceed to the analysis I introduce a function $T : P \to P$ such that

$$T(x) = \begin{cases} \max\{\bar{x} \in P : v(\bar{x}, x) > 1/2\} & \text{if } x < \kappa \\ x & \text{if } x = \kappa \\ \min\{\bar{x} \in P : v(\bar{x}, x) > 1/2\} & \text{if } x > \kappa. \end{cases}$$

That is, for $x \ne \kappa$, x and $T(x)$ are on the opposite side of κ such that $T(x)$ just break x in the competition. In Figure 1(1), $v(T(x), x) > 1/2$ and $v(T(x) + 1, x) < 1/2$. In Figure 1(2), $v(T(x), x) > 1/2$ and $v(T(x) - 1, x) < 1/2$.

I begin with the case of $x_1^0 \le x_2^0 < \kappa$.

Lemma 3.2: Consider the case of $x_1^0 < x_2^0 < \kappa$.

(1) When $x_2^* \le x_1^0$, (x_1, x_2) is a Nash equilibrium iff x_2 is such that $x_1^0 < x_2 \le x_2^0$, and, to that x_2, x_1 is such that $x_1 = x_2 - 1$ or $T(x_2) < x_1 \le T(x_2 - 1)$.

(2) When $x_1^0 < x_2^*$, (x_1, x_2) is a Nash equilibrium iff: (a) x_2 is such that $x_2 = x_2^*$, and, to that x_2, x_1 is such that $x_1 < x_2$ or $T(x_2) < x_1$, or (b) x_2

is such that $x_2^* < x_2 \leq x_2^0$, and, to that x_2, x_1 is such that $x_1 = x_2 - 1$ or $T(x_2) < x_1 \leq T(x_2 - 1)$.

In both cases, 2 wins.

(Proof) See Appendix A. □

Lemma 3.2(1) is illustrated in Figure 2. Ignoring x_1, with x_2 such that $x_1^0 < x_2 \leq x_2^0$, 2 is better off at $x_2 - 1$ than at x_2. (See Lemma 2.4 and recall the assumption that x_j^* is unique for $j \in M$.) When x_1 is such that $T(x_2) < x_1 \leq T(x_2 - 1)$, however, 2 loses with $x_2 - 1$, so 2 will not move from x_2 to $x_2 - 1$. Obviously 1 has no incentive to move.

Lemma 3.3: Consider the case of $x_1^0 = x_2^0 < \kappa$. (x_1, x_2) is an equilibrium iff $x_1 = x_1^0$ and $x_2 = x_2^0$. In this case, each agent wins with probability 1/2.

(Proof) Lemma 3.1 implies that, for (x_1, x_2) to be an equilibrium, it must be that $x_2 = x_2^0$. If $x_1 < x_2$ where 1 loses, 1 will move to x_1^0 where 1 ties with 2 with net benefit, so not an equilibrium. If $x_1 = x_2$, neither 1 nor 2 can be better off by moving, so an equilibrium. If $x_1 > x_2$ where 1 either wins with net loss or loses, 1 will move to x_1^0 where 1 ties with 2 with net benefit, so not an equilibrium. □

Next consider the case of $x_1^0 < \kappa \leq x_2^0$.

Lemma 3.4: Consider the case of $x_1^0 < \kappa \leq x_2^0$.

(1) When $x_2^* \leq x_1^0$, (x_1, x_2) is an equilibrium iff x_2 is such that $x_1^0 < x_2 \leq \kappa$, and, to that x_2, x_1 is such that $x_1 = x_2 - 1$ or $T(x_2) < x_1 \leq T(x_2 - 1)$.

(2) When $x_1^0 < x_2^* < \kappa$, (x_1, x_2) is an equilibrium iff: (a) x_2 is such that $x_2 = x_2^*$, and, to that x_2, x_1 is such that $x_1 < x_2$ or $T(x_2) < x_1$, or (b) x_2 is such that $x_2^* < x_2 \leq \kappa$, and, to that x_2, x_1 is such that $x_1 = x_2 - 1$ or $T(x_2) < x_1 \leq T(x_2 - 1)$.

(3) When $x_2^* = \kappa$, (x_1, x_2) is an equilibrium iff $x_2 = \kappa$ and $x_1 \neq x_2$.

(4) When $\kappa < x_2^* \leq T(x_1^0)$, (x_1, x_2) is an equilibrium iff: (a) x_2 is such that $\kappa \leq x_2 < x_2^*$, and, to that x_2, x_1 is such that $T(x_2 + 1) \leq x_1 < T(x_2)$ or $x_1 = x_2 + 1$, or (b) x_2 is such that $x_2 = x_2^*$, and, to that x_2, x_1 is such that $x_1 \leq T(x_2)$ or $x_1 > x_2$.

(5) When $T(x_1^0) < x_2^*$, (x_1, x_2) is an equilibrium iff x_2 is such that $\kappa \leq x_2 < T(x_1^0)$, and, to that x_2, x_1 is such that $T(x_2 + 1) \leq x_1 < T(x_2)$ or $x_1 = x_2 + 1$.

In all cases, 2 wins.

(Proof) See Appendix B. □

The last case to be considered is when $\kappa \leq x_1^0 \leq x_2^0$.

Lemma 3.5: Consider the case of $\kappa \leq x_1^0 \leq x_2^0$. (x_1, x_2) is an equilibrium iff $x_1 = x_2 = \kappa$. In this case, each agent wins with probability 1/2.

(Proof) Suppose that $x_2 < \kappa$. If $x_1 \leq x_2$ where 1 loses or ties with 2 with net benefit, 1 will move, e.g., to $x_2 + 1$ where 1 wins with greater net benefit, so not an equilibrium. If $x_2 < x_1 \leq T(x_2)$ where 2 loses, 2 will move, e.g., to κ where 2 wins or ties with 1 with net benefit, so not an equilibrium. If $T(x_2) < x_1$ where 1 loses, 1 moves, e.g., to κ where 1 wins with net benefit, so not an equilibrium.

Suppose that $x_2 = \kappa$. If $x_1 < x_2$ where 1 loses, 1 will move to κ where 1 ties with 2 with net benefit, so not an equilibrium. If $x_1 = x_2$, neither 1 nor 2 can be better off by moving, so an equilibrium. If $x_1 > x_2$ where 1 loses, 1 will move to κ, so not an equilibrium.

Suppose that $x_2 > \kappa$. If $x_1 < T(x_2)$ where 1 loses, 1 will move, e.g., to κ, so not an equilibrium. If $T(x_2) \leq x_1 < x_2$ where 2 loses, 2 will move, e.g., to κ where 2 wins or ties with 1 with net benefit, so not an equilibrium. If $x_2 \leq x_1$ where 1 at most ties with 2 with net benefit, 1 will move, e.g., to $x_2 - 1$ where 1 wins with greater net benefit (by Assumption 2.4), so not an equilibrium. □

Now I summarize Lemmas 3.2, 3.3, 3.4 and 3.5 in a theorem. Let x^N be the project choice that can realize in competition. Here recall that $x_1^0 \leq x_2^0$ is assumed.

Theorem: If $x_1^0 < \kappa$, then $x_1^0 \leq x^N \leq x_2^0$. If $x_1^0 \geq \kappa$, then $x^N = \kappa$.

(Proof) The proof of this theorem is contained in Lemmas 3.2, 3.3, 3.4 and 3.5. □

Notice that when $x_1^0 < \kappa$, x^N can be equal to x_1^0 iff $x_1^0 = x_2^0$. Hence, as a corollary of the first part of Theorem, I obtain a proposition that if $x_1^0 < \kappa$ and $x_1^0 < x_2^0$ then $x_1^0 < x^N \leq x_2^0$.

From Theorem it is observed that in order to attain corporate democracy both competing agents in M need to be risk tolerant enough, i.e., they need to endure the project best preferred by the median shareholder. For 1 and 2 in M, if one agents's marginal project is a little bit below κ while the other agents's marginal project is far above κ, then the competitive outcome may depart far away from the democratic solution.

4. Collusion and Corporate Democracy

In the previous section, I examined how the corporate democracy is or is not attained by means of the voting in a subeconomy with a market for control. When the agents in the market for control are of small number, however, it is always possible that they negotiate not to compete if it pays each of them. In this section, I consider the possibility of collusion between otherwise competing agents. I do not attempt to do a comprehensive analysis, but as an illustration of the problem of collusion I confine my attention to the case of $x_1^0 < x_2^0 < \kappa$.

In order to make the problem manageable, I put the following assumptions throughout the present section.

Assumption 4.1:
(1) $\tilde{b}(r) = \beta$ where β is a constant positive.
(2) $r(1) = 0$.
(3) The cost of collusion for $j \in M$ is $z_j > 0$.

The first assumption ensures that $x_j^* = 1$, i.e., agents in M best prefer the safest project. This reflects the conventional wisdom that in general the risk averse preferences are the intrinsic characteristics of managers in modern corporations. [15] The second assumption says that there is a certain project, i.e., the project with no probability of failure. An example in our real life is the investment on government bonds. The third assumption seems a natural one.

As in the previous section, let x_j^0 be the marginal project for $j \in M$ when the manager's reward is β. In the present case of $x_1^0 < x_2^0 < \kappa$, it follows from Theorem that the competitive outcome x^N will be such that $x_1^0 < x^N \leq x_2^0$. Let $r^N = r(x^N)$ for a possible x^N with $x_1^0 < x^N \leq x_2^0$. Now, when the two agents in M compete, their resulting utility levels are

$$u_1 = u_1(W_1) \tag{3}$$

and

$$u_2 = U_2(r^N) = r^N u_2(W_2 - c_2) + (1 - r^N) u_2(W_2 + \beta). \tag{4}$$

If the two agents collude and agree that 2 manages the firm in return for a transfer t from 2 to 1, their utility levels are

$$u_1 = u_1(W_1 + t - z_1) \tag{5}$$

and

$$u_2 = u_2(W_2 + \beta - t - z_2).^{1617} \tag{6}$$

[15]See Milgrom and Roberts (1992), chapter 13, pp. 429-431.

[16]Recall that $x_2^* = 1$ under constant reward and $r(1) = 0$. See Assumption 4.1(1) and (2).

[17]It is certainly possible the opposite where the two agents collude and agree that 1

From equations (3) to (6), 1 and 2 collude if and only if there exists $t > 0$ such that

$$u_1(W_1 + t - z_1) > u_1(W_1) \tag{7}$$

and

$$u_2(W_2 + \beta - t - z_2) > U_2(r^N). \tag{8}$$

Equivalently, if and only if there does not exist such $t > 0$, the two agents do not reach an agreement to collude and then competition sustains.

Lemma 4.1: Suppose that $x_1^0 < x_2^0 < \kappa$ under a certain β. Then, collusion occurs if and only if

$$u_2(W_2 + \beta - z_1 - z_2) > U_2(r^N). \tag{9}$$

Equivalently, competition sustains if and only if

$$u_2(W_2 + \beta - z_1 - z_2) \leq U_2(r^N). \tag{10}$$

(Proof) Suppose that there exists $t > 0$ with (7) and (8). Notice that, since $u' > 0$, (7) is equivalent to $t > z_1$. Then, by using (8), $u_2(W_2 + \beta - z_1 - z_2) > u_2(W_2 + \beta - t - z_2) > U_2(r^N)$.

Suppose next that $u_2(W_2 + \beta - z_1 - z_2) > U_2(r^N)$. Since u_2 is continuous, there exists $\delta > 0$ with $u_2(W_2 + \beta - (z_1 + \delta) - z_2) > U_2(r^N)$. Let $t = z_1 + \delta$. Then, $u_1(W_1 + t - z_1) = u_1(W_1 + \delta) > u_1(W_1)$ so (7) holds, and obviously $u_2(W_2 + \beta - t - z_2) > U_2(r^N)$ so (8) holds. □

The following observations are immediately obtained from Lemma 4.1.

Observation 1:

(1) The larger the costs of collusion, z_1 and z_2, are, the less likely collusion occurs.

(2) The closer the competitive outcome x^N is to the best project option for the manager $x_2^* = 1$, the less likely collusion occurs.

As for Observation 1(1), if z_1 and z_2 are such that $z_1 + z_2 > \beta + c_2$, collusion never occurs. Indeed, if $z_1 + z_2 > \beta + c_2$,

$$u_2(W_2 + \beta - z_1 - z_2) < u_2(W_2 - c_2) \leq U_2(r^N)$$

---

manages the firm in return for a transfer from 1 to 2. As long as the final payoffs to the two agents are concerned, however, the two patterns of collusion (the one in the text and the one in the present footnote) are equivalent. Let $\hat{t}$ be the transfer from 1 to 2. Then the two agents' utility levels are $u_1 = u_1(W_1 + \beta - \hat{t} - z_1)$ and $u_2 = u_2(W_2 + \hat{t} - z_2)$. By letting $t + \hat{t} = \beta$, the payoffs here are the same as those in the text.

so (8) never holds. Hence, in the comparative statics analysis below I consider the case where $z_1 + z_2 \leq \beta + c_2$. In Observation 1(2), if in particular $x^N = 1$ and hence $r^N = 0$,

$$U_2(r^N) = u_2(W_2 + \beta) > u_2(W_2 + \beta - z_1 - z_2)$$

so (9) does not hold and collusion does not occur.

I next proceed to a comparative statics analysis. I consider the two states of subeconomy $\mathcal{E}$, state 1 and state 2. Let $\beta(l)$ be the reward to the manager under state $l = 1, 2$. Without loss of generality, let $\beta(1) < \beta(2)$, i.e., the reward to the manager under state 2 is greater than that under state 1. Let $x_j^0(l)$ be the marginal project of agent $j \in M$ under state l. As in the analysis above in the present section, I consider the case when $x_1^0(l) < x_2^0(l) < \kappa$ under both states $l = 1, 2$. Then, by Theorem, it holds that $x_1^0(l) < x^N(l) \leq x_2^0(l)$ for both $l = 1, 2$. Let $r^N(l) = r(x^N(l))$ for a possible $x^N(l)$ with $x_1^0(l) < x^N(l) \leq x_2^0(l)$ for $l = 1, 2$.

Lemma 4.2: $x_j^0(1) \leq x_j^0(2)$ for $j \in M$.

(Proof) Since $U_j(r)$ for $j \in M$ is nondecreasing in β, by the proof of Lemma 2.3(1), the present lemma is obtained. □

Recall that if $x_1^0(1) < x_2^0(1) < \kappa$ under $\beta(1)$ the competitive outcome $x^N(1)$ will locate between $x_1^0(1)$ and $x_2^0(1)$. Hence, by Lemma 4.2, we are inclined to imagine that, as long as it still holds that $x_1^0(2) < x_2^0(2) < \kappa$ under $\beta(2)$, the outcome $x^N(2)$ possibly comes closer to κ than $x^N(1)$ does. So far as the competitive outcome sustains, this is true, according to the Theorem. The next lemma states that this is not necessarily the case when the possibility of collusion is taken into account.

In what follows, let $x^N(l)$ be the realized competitive outcome under $\beta(l)$ for $l = 1, 2$.

Lemma 4.3: Consider the case of $x_1^0(l) < x_2^0(l) < \kappa$ for $l = 1, 2$ and $z_1 + z_2 \leq \beta(1) + c_2$. Then, if $x^N(2) \geq x^N(1)$,

$$u_2(W_2 + \beta(2) - z_1 - z_2) - U_2(r^N(2)) > u_2(W_2 + \beta(1) - z_1 - z_2) - U_2(r^N(1)).$$

(Proof) For state $l = 1, 2$ notice that

$$\begin{aligned} &u_2(W_2 + \beta(l) - z_1 - z_2) - U_2(r^N(l)) \\ &= r^N(l)(u_2(W_2 + \beta(l) - z_1 - z_2) - u_2(W_2 - c_2)) \\ &\quad - (1 - r^N(l))(u_2(W_2 + \beta(l)) - u_2(W_2 + \beta(l) - z_1 - z_2)). \end{aligned} \tag{11}$$

Note that $z_1+z_2 \leq \beta(1)+c_2$ implies that $u_2(W_2+\beta(1)-z_1-z_2)-u_2(W_2-c_2) > 0$, and also that $u_2(W_2+\beta(2)-z_1-z_2)-u_2(W_2-c_2) > 0$. Then, since $r^N(2) \geq r^N(1)$, $\beta(2) > \beta(1)$ and $u' > 0$,

$$
\begin{aligned}
& r^N(2)(u_2(W_2+\beta(2)-z_1-z_2)-u_2(W_2-c_2)) \\
& > r^N(1)(u_2(W_2+\beta(1)-z_1-z_2)-u_2(W_2-c_2)).
\end{aligned} \tag{12}
$$

Also, since $1-r^N(2) \leq 1-r^N(1)$, $\beta(2) > \beta(1)$, $u' > 0$ and $u" > 0$,

$$
\begin{aligned}
& (1-r^N(2))(u_2(W_2+\beta(2))-u_2(W_2+\beta(2)-z_1-z_2)) \\
& < (1-r^N(1))(u_2(W_2+\beta(1))-u_2(W_2+\beta(1)-z_1-z_2)).
\end{aligned} \tag{13}
$$

Applying (12) and (13) to (11) yields the inequality in the present lemma. □

Lemma 4.3 says that collusion is more likely to occur under state 2, i.e., the state with higher reward to the manager. Suppose that $u_2(W_2+\beta(1)-z_1-z_2)-U_2(r^N(1)) < 0$ holds under $\beta(1)$ and hence competition sustains under state 1. As the economic state improves and β increases, the difference between $u_2(W_2+\beta-z_1-z_2)$ and $U_2(r^N)$ shrinks. Then, at some point $\beta(2)$ the relation between $u_2(W_2+\beta-z_1-z_2)$ and $U_2(r^N)$ can change to $u_2(W_2+\beta(2)-z_1-z_2)-U_2(r^N(2)) \geq 0$ where collusion occurs.

The following property of the model is obtained from Lemmas 4.2 and 4.3.

Observation 2: On the one hand, the larger the reward β is, the closer the competitive outcome x^N is to the democratic project choice κ. But, on the other hand, the larger the reward β is, the more likely collusion occurs. If collusion occurs, the competitive outcome, that is not so far from, if not very close to, the democratic solution, will be replaced with an extreme outcome by the collusion.

5. Conclusion

In this paper I developed a model of a corporation with unorganized heterogeneous shareholders and investigated whether corporate democracy is attained by means of the institution of voting when there is a market for control. I conclude from the analysis that in order to achieve the corporate democracy through voting the agents competing for the manager need to be sufficiently risk tolerant. A further examination attaches a remark to this conclusion that, while the agents in the market for control are more risk tolerant with a better reward, it tends to induce collusion between the agents and consequently to bring in outcomes far from democratic solution.

There are two remarks for the present analysis. First, what can be said in the present analysis is at large whether unorganized small shareholders have a certain power of control over a firm by means of the institution of voting. It is beyond the scope of the paper whether the institution of corporate voting is desirable from the viewpoint of overall economic efficiency. Second, the model developed here is not confined for the use of the present analysis, but is expected to be used for a formal examination of various aspects of voting in corporations.

Appendix

A Proof of Lemma 3.2

(1) The case of $x_2^* \leq x_1^0$.

Suppose that $x_2 = x_1^0$. If $x_1 < x_2$ where 1 loses, 1 will move to x_1^0 where 1 ties with 2 with net benefit, so not an equilibrium. If $x_1 = x_2$ where 2 ties with 1 with net benefit, 2 will move to $x_2 + 1$ where 2 wins with greater net benefit, so not an equilibrium. If $x_2 < x_1 \leq T(x_2)$ where 1 wins with net loss, 1 will move to x_1^0 where 1 ties with 2 with net benefit, so not an equilibrium. If $T(x_2) < x_1$ where 1 loses, 1 will move to x_1^0 where 1 ties with 2 with net benefit, so not an equilibrium.

Suppose that $x_1^0 < x_2 \leq x_2^0$. If $x_1 < x_2 - 1$ where 2 wins with net benefit, 2 will move to $x_2 - 1$ where 2 still wins with greater net benefit, so not an equilibrium. If $x_1 = x_2 - 1$ where 2 wins with net benefit, neither 1 nor 2 can be better off by moving, so an equilibrium. If $x_2 \leq x_1 \leq T(x_2)$ where 1 wins or ties with 2 with net loss, 1 will move, e.g., to $x_2 - 1$ where 1 loses, so not an equilibrium. If $T(x_2) < x_1 \leq T(x_2 - 1)$ where 2 wins with net benefit, neither 1 nor 2 can be better off by moving (i.e., 1 cannot win or ties with 2 with net benefit by any move, and 2 cannot win, e.g., with $x_2 - 1$), so an equilibrium. [18] If $T(x_2 - 1) < x_1$ where 2 wins with net benefit, 2 will move, e.g., to $x_2 - 1$ where 2 still wins with greater net benefit, so not an equilibrium.

(2) The case of $x_1^0 < x_2^*$.

Suppose that $x_1^0 \leq x_2 < x_2^*$. If $x_1 \leq x_2$ where 2 wins or ties with 1 with net benefit, 2 will move to x_2^* where 2 wins with greater net benefit, so not an equilibrium. If $x_2 < x_1 \leq T(x_2)$ where 1 wins with net loss, 1 moves, e.g., to x_1^0 where 1 loses or ties with 2 with net benefit, so not an equilibrium. If $T(x_2) < x_1$ where 2 wins with net benefit, 2 will move to x_2^* where 2 still wins with greater net benefit, so not an equilibrium.

Suppose that $x_2 = x_2^*$. If $x_1 < x_2$ where 2 wins with net benefit, neither 1 nor 2 can be better off by moving (i.e., 1 cannot win or ties with 2 with net benefit, and 2 is now enjoying the greatest net benefit), so an equilibrium. If $x_2 \leq x_1 \leq T(x_2)$ where 1 wins or ties with 2 with net loss, 1 will move, e.g., to x_1^0 where 1 loses, so not an equilibrium. If $T(x_2) < x_1$ where 2 wins with net benefit, neither 1 nor 2 can be better off by moving (i.e., 1 cannot win or ties with 2 with net benefit, and 2 is now enjoying the greatest net benefit), so an equilibrium.

Suppose that $x_2^* < x_2 \leq x_2^0$. If $x_1 < x_2 - 1$ where 2 wins with net benefit, 2 will move to $x_2 - 1$ where 2 still wins with greater net benefit, so not an

[18] This case is valid only if $T(x_2) < T(x_2 - 1)$. If $T(x_2) = T(x_2 - 1)$, such x_1 doesn't exist.

equilibrium. If $x_1 = x_2 - 1$ where 2 wins with net benefit, neither 1 nor 2 can be better off by moving (i.e., 1 cannot win or ties with 2 with net benefit by any move, and 2 cannot win, e.g., with $x_2 - 1$), so an equilibrium. If $x_2 \leq x_1 \leq T(x_2)$ where 1 wins or ties with 2 with net loss, 1 will move, e.g., to x_1^0 where 1 loses, so not an equilibrium. If $T(x_2) < x_1 \leq T(x_2 - 1)$ where 2 wins, neither 1 nor 2 can be better off by moving (i.e., 1 cannot win with net benefit by any move, and 2 cannot win, e.g., with $x_2 - 1$), so an equilibrium. [19] If $T(x_2 - 1) < x_1$ where 2 wins with net benefit, 2 will move, e.g., to $x_2 - 1$ where 2 still wins with greater net benefit, so not an equilibrium. □

B Proof of Lemma 3.4

(1) The case of $x_2^* \leq x_1^0$.

Suppose that $x_2 = x_1^0$. The proof of this case is exactly the same as the first case (i.e., the case of $x_2 = x_1^0$) of the proof of Lemma 3.2(1).

Suppose that $x_1^0 < x_2 \leq x_2^0$. The proof of this case is exactly the same as the second case (i.e., the case of $x_1^0 < x_2 \leq x_2^0$) of the proof of Lemma 3.2(1).

Suppose that $\kappa < x_2 \leq x_2^0$. Whatever the value of x_1 and whether 2 wins, ties with 1 or loses with x_2, 2 becomes better off by moving to κ. (Notice that $x_2^* < \kappa$ and κ is the strongest strategy.) Hence, there is no equilibrium in this case.

(2) The case of $x_1^0 < x_2^* < \kappa$.

Suppose that $x_1^0 \leq x_2 < x_2^*$. The proof of this case is exactly the same as the first case (i.e., the case of $x_1^0 \leq x_2 < x_2^*$) of the proof of Lemma 3.2(2).

Suppose that $x_2 = x_2^*$. The proof of this case is exactly the same as the second case (i.e., the case of $x_2 = x_2^*$) of the proof of Lemma 3.2(2).

Suppose that $x_2^* < x_2 \leq \kappa$. The proof of this case is exactly the same as the third case (i.e., the case of $x_2^* < x_2 \leq x_2^0$) of the proof of Lemma 3.2(2).

Suppose that $\kappa < x_2 \leq x_2^0$. The proof of this case is exactly the same as the third case (i.e., the case of $\kappa < x_2 \leq x_2^0$) of the proof of Lemma 3.4(1) above.

(3) The case of $x_2^* = \kappa$.

Suppose that $x_2 \neq x_2^*$. Whatever the value of x_1 and whether 2 wins, ties with 1 or loses with x_2, 2 becomes better off by moving to x_2^*, so there is no equilibrium in this case.

Suppose that $x_2 = x_2^*$. If $x_1 \neq x_2$ where 2 wins, neither 1 nor 2 can be better off by moving (recall that $x_1^0 < \kappa$), so an equilibrium. If $x_1 = x_2$ where 1 ties with 2 with net loss, 1 will move, e.g., to x_1^0, so not an equilibrium.

[19]This case is valid only if $T(x_2) < T(x_2 - 1)$. If $T(x_2) = T(x_2 - 1)$, such x_1 doesn't exist.

(4) The case of $\kappa < x_2^* \le T(x_1^0)$.

Suppose that $x_1^0 \le x_2 < \kappa$. Whatever the value of x_1 and whether 2 wins, ties with 1 or loses, 2 becomes better off by moving to κ. (Notice that $x_2^* > \kappa$ and κ is the strongest strategy.) Hence, there is no equilibrium in this case.

Suppose that $x_2 = \kappa$. If $x_1 < T(x_2+1)$ where 2 wins, 2 will move to x_2+1 where 2 still wins with greater net benefit, so not an equilibrium. If $T(x_2+1) \le x_1 < x_2$ where 2 wins with net benefit, neither 1 nor 2 can be better off by moving (i.e., 1 cannot win with net benefit by any move, and 2 cannot win, e.g., with x_2+1), so an equilibrium. [20] If $x_1 = x_2$ where 1 ties with 2 with net loss, 1 will move, e.g., to x_1^0 where 1 loses, so not an equilibrium. If $x_1 = x_2+1$ where 2 wins with net benefit, neither 1 nor 2 can be better off by moving (i.e., 1 cannot win with net benefit by any move, and 2 cannot win, e.g., with x_2+1), so an equilibrium. If $x_2+1 < x_1$ where 2 wins with net benefit, 2 will move, e.g., to x_2+1 where 2 still wins with greater net benefit, so not an equilibrium.

Suppose that $\kappa < x_2 < x_2^*$. If $x_1 < T(x_2+1)$ where 2 wins with net benefit, 2 will move, e.g., to x_2+1 where 2 still wins with greater net benefit, so not an equilibrium. If $T(x_2+1) \le x_1 < T(x_2)$ where 2 wins with net benefit, neither 1 nor 2 can be better off by moving (i.e., 1 cannot win with net benefit by any move, and 2 cannot win, e.g., with x_2+1), so an equilibrium. [21] If $T(x_2) \le x_1 < \kappa$ where 2 loses, 2 will move, e.g., to κ where 2 wins with net benefit, so not an equilibrium. [22] If $\kappa \le x_1 \le x_2$ where 1 wins or ties with 2 with net loss, 1 will move, e.g., to x_1^0 where 1 loses, so not an equilibrium. If $x_1 = x_2+1$, neither 1 nor 2 can be better off by moving (i.e., 1 cannot win with net benefit by any move, and 2 cannot win, e.g., with x_2+1), so an equilibrium. If $x_2+1 < x_1$ where 2 wins with net benefit, 2 will move, e.g., to x_2+1 where 2 still wins with greater net benefit, so not an equilibrium.

Suppose that $x_2 = x_2^*$. If $x_1 < T(x_2)$ where 2 wins with net benefit, neither 1 nor 2 can be better off by moving (i.e., 1 cannot win with net benefit by any move, and 2 is now enjoying the greatest net benefit), so an equilibrium. If $T(x_2) \le x_1 < \kappa$ where 2 loses, 2 will move, e.g., to κ where 2 wins with net benefit, so not an equilibrium. If $\kappa \le x_1 \le x_2$ where 1 wins or ties with 2 with net loss, 1 will move, e.g., to x_1^0 where 1 loses, so not an equilibrium. If $x_2 < x_1$ where 2 wins with net benefit, neither 1 nor 2 can be better off by moving (i.e., 1 cannot win with net benefit by any move, and 2 is now enjoying the greatest net benefit), so an equilibrium.

Suppose that $x_2^* < x_2$. If $x_1 < T(x_2-1)$ where 2 wins with net benefit, 2 will move, e.g., to x_2-1 where 2 still wins with greater net benefit, so not an equilibrium. If $T(x_2-1) \le x_1 < \kappa$ where 2 loses, 2 will move, e.g., to κ

[20] This case is valid only if $T(x_2+1) < T(x_2)$. If $T(x_2+1) = T(x_2)$, such x_1 doesn't exist.
[21] This case is valid only if $T(x_2+1) < T(x_2)$. If $T(x_2+1) = T(x_2)$, such x_1 doesn't exist.
[22] This case is valid only if $T(x_2) < \kappa$. If $T(x_2) = \kappa$, such x_1 doesn't exist.

where 2 wins with net benefit, so not an equilibrium. If $\kappa \le x_1 \le x_2$ where 1 wins or ties with 2 with net loss, 1 will move, e.g., to x_1^0 where 1 loses, so not an equilibrium. If $x_2 < x_1$ where 2 wins with net benefit, 2 will move, e.g., to $x_2 - 1$ where 2 still wins with greater net benefit, so not an equilibrium.

(5) The case of $T(x_1^0) < x_2^*$.

Suppose that $x_1^0 \le x_2 < \kappa$. The proof of this case is exactly the same as the first case (i.e., the case of $x_1^0 \le x_2 < \kappa$) of the proof of Lemma 3.4(4) above.

Suppose that $x_2 = \kappa$. The proof of this case is exactly the same as the second case (i.e., the case of $x_2 = \kappa$) of the proof of Lemma 3.4(4) above.

Suppose that $\kappa < x_2 < x_2^*$. The proof of this case is exactly the same as the third case (i.e., the case of $\kappa < x_2 < x_2^*$) of the proof of Lemma 3.4(4) above.

Suppose that $x_2 = x_2^*$. If $x_1 < T(x_2)$ where 1 loses, 1 will move, e.g., to x_1^0 where 1 wins with net benefit, so not an equilibrium. If $T(x_2) \le x_1 < \kappa$ where 2 loses, 2 will move, e.g., to κ where 2 wins with net benefit, so not an equilibrium. If $\kappa \le x_1 \le x_2$ where 1 wins or ties with 2 with net loss, 1 will move, e.g., to x_1^0 where 1 wins with net benefit, so not an equilibrium. If $x_2 < x_1$ where 1 loses, 1 will move, e.g., to x_1^0 where 1 wins with net benefit, so not an equilibrium.

Suppose that $x_2^* < x_2$. If $x_1 < T(x_2 - 1)$ where 2 wins with net benefit, 2 will move, e.g., to $x_2 - 1$ where 2 still wins with greater net benefit, so not an equilibrium. If $T(x_2 - 1) \le x_1 < \kappa$ where 2 loses, 2 will move, e.g., to κ where 2 wins with net benefit, so not an equilibrium. If $\kappa \le x_1 \le x_2$ where 1 wins or ties with 2 with net loss, 1 will move, e.g., to x_1^0 where 1 wins with net benefit, so not an equilibrium. If $x_2 < x_1$ where 2 wins with net benefit, 2 will move, e.g., to $x_2 - 1$ where 2 still wins with greater net benefit, so not an equilibrium. □

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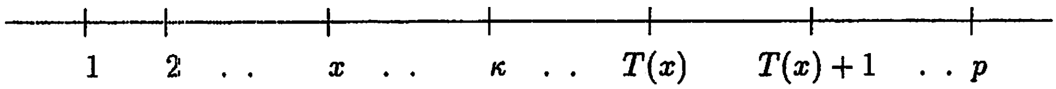
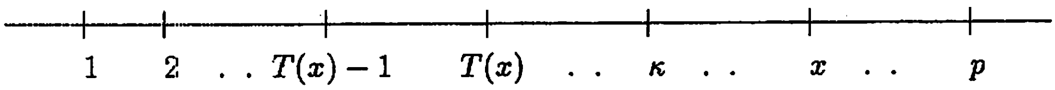


(1) When $x < \kappa$.



(2) When $x > \kappa$.

Figure 1: Function $T : P \rightarrow P$.

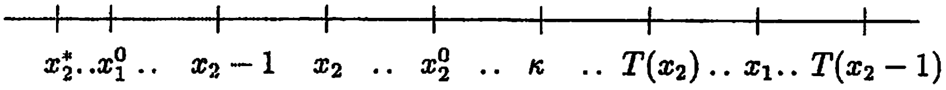


Figure 2: Lemma 3.2(1), when x_1 is such that $T(x_2) < x_1 \leq T(x_2 - 1)$.