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Which Should the Government Subsidize: Child Care or Elderly Care?

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Which Should the Government Subsidize: Child Care or Elderly Care?*

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Abstract

This paper presents an examination of how a government should provide subsidies for households. An elderly care subsidy for services can increase the purchase of elderly care services. Therefore, it can reduce the need for elderly care provision by adult children. This decrease implies an increase in the labor supply. The consequent increase in the labor supply raises the household income, making increased fertility affordable: this paper presents derivation that the child care subsidy service cannot increase the fertility and labor supply if the productivity of elderly care services is high. However, the subsidy for older people has the effect of preventing a decrease in the number of children. Therefore, the elderly care subsidy can alleviate social problems caused by fewer children.

JEL Classification: H51, H21, J14

Keywords: Child care, Elderly care, Fertility

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1 Introduction

Some papers have presented examination of an increase in the demand for elderly care, as shown by Cremer and Pestieau (2012), Pestieau and Ponthiere (2012), and Colombo et al. (2011). Colombo et al. (2011) shows that average public spending on long-term care in 2008 reached 1.5% of gross domestic product (GDP) in the study of long-term care in OECD countries. Most elderly care is provided by informal family care. Actually, 1.3% of the total labor supply is engaged in the formal elderly care service.[1]

In Japan, an aging society with fewer children is progressing, creating numerous and daunting social issues. This paper presents examination of the effects of a subsidy policy for elderly care and child care. The household can raise the labor supply and can afford to have more children because of an increase in household income if informal family care decreases thanks to the policy for elderly care.

If socially relevant difficulties associated with families having fewer children cannot be resolved, then the future labor supply can be expected to decrease. Therefore, the government must increase the fertility rate through appropriate child care policies. By virtue of administration of child care policies, the respective fertility rates in Sweden and France have recovered to a certain degree. In Japan, subsidies for child care services have increased. Thereby the fertility rate is expected to increase.

Policies for older people are important, too. Especially, elderly care presents a severe social problem. To provide informal family care, some people of the working generation must decrease their working time, thereby reducing household income. They have insufficient income to care for children because they must care for older people using informal family care. The number of people who have stopped working to provide informal family care in all is 486.9 thousand in Japan (2007–2012).[2] The shares of caregivers younger than 50 years old among total informal elderly caregivers were 12.7% (men) and 10.6% (women) in Japan in 2010.[3] With elderly care service provided by the market, children whose parents need elderly care can supply labor. Without elderly care services because of a lack of elderly care services, however, children are compelled to provide time to care for their parents. Therefore, labor cannot be supplied. Even if the problem of fewer children is resolved, the labor supply cannot be expected to increase later

[1] Hashimoto and Tabata (2010) show that workers with formal elderly care increase with progressing age.
[2] Data: Ministry of Internal Affairs and Communications in Japan, Employment Status Survey.
[3] Data: Ministry of Health, Labour and Welfare in Japan, Comprehensive Survey of Living Conditions

because of a lack of elderly care services.

Working-generation people take care not only of their parents but also of their children. Therefore, a balance between elderly care and child care should be considered.

Results derived from this study are the following. If the productivity of formal elderly care service in the market is high, then elderly care subsidy services can raise the purchase of elderly care services supplied by the market and can raise the fertility rate. Because of an increase in elderly care services, children can reduce informal elderly care. Thereby, the labor supply increases. However, child care subsidy services cannot always increase the fertility rate because of the tax burden. If children consider the quality of elderly care is low, then they provide care time to raise elderly care quality. An increase in care time necessitates a decrease in the labor supply. Therefore, the household income decreases and the household becomes unable to afford to have children. Consequently, elderly care subsidies can raise fertility. The subsidy policy for elderly care might be better than the child care policy for younger people. It is an interesting result that such policies for older people improve the quality of life not only for older people but also for younger people.

The remainder of this paper consists of the following. Section 2 explains the related literature about elderly care and child care policy to clarify the results obtained from the analyses explained in this paper. Section 3 sets the model with endogenous fertility and elderly care. Section 4 derives the equilibrium. Section 5 examines the effects of the two subsidy policies on fertility, elderly care services, and labor supply. Section 6 analyzes the level of welfare. The final section concludes this paper.

2 Related Literature

Many studies have examined elderly care policies. First, insurance for elderly care was examined. The study of elderly care was started by Pauly (1990). Pauly (1990) reports that publicly provided long-term care insurance is better than privately provided insurance. This result is obtained also by and Miyazawa et al. (2000). Although this long-term care insurance entails a beneficial risk-pooling effect reported by Smith and Witter (2004), a moral hazard effect that poses a disincentive against long-term care makes it inefficient, as explained by Richter and Ritzberger (1995). Tabata (2005) and Mizushima (2009) examine the elderly care given not only by the older people themselves but also by their children

and examine the effects of subsidies for elderly care on income per capita and welfare. Tabata (2005) derives that the elderly care subsidy service increases the welfare of older people. However, the welfare levels of younger people and future people decrease because of a decrease in income growth with tax burden: intergenerational conflict occurs. Lundholm and Ohlsson (1998) and Mizushima (2009) examine the optimal elderly care subsidy. Cremer and Pestieau (2011) and Cremer and Roeder (2013) consider a model including both private elderly care and public elderly care and derive that a subsidy for private elderly care is desirable.

Second formal elderly care and informal elderly care are examined in some studies. If elderly care services provided by the market are accessible for older people, then informal care that requires time spent by children becomes less necessary. Thereby, the children can increase the labor supply. Korn and Werde (2012) and Mou and Winer (2015) set the model with formal and informal elderly care. However, Korn and Werde (2012) do not consider the endogenous fertility. Although Mou and Winer (2015) consider the endogenous fertility, they do not consider the elderly care subsidy and child care. Our paper presents consideration of both the elderly care and the child care and shows how the informal elderly care give the negative effect on the fertility, which can not be examined by the related earlier studies.

Byrne et al. (2009) and Courbage and Zweifiel (2011) derive how the demand of formal elderly care and informal family care are determined. Substitution between informal family care and formal elderly care is examined using some empirical research. White-Means and Rubin (2004) derive the tradeoff between formal home health care and informal family caregiving. Bonsang (2009) uses data from Europe to derive substitution between informal family care and formal care. van Houtven and Norton (2004) present the substitution that occurs between informal care and home health care. López-Anuarbe (2013) and Park (2014) specifically examine the fact that the children care for their parents as the informal elderly care and examine how the level of informal elderly care is determined.

Our paper presents consideration of endogenous fertility. Subsidies for child care services have been examined in many studies. van Groezen et al. (2003) and Yasuoka and Goto (2015) derive that a child allowance can raise the fertility rate. However, these studies do not include consideration of the labor supply or child care time. Apps and Lees (2004) and Yasuoka and Miyake (2010) set models with the child care sector resources provided by the market. By virtue of child care services, the parents can care

for children without loss of labor supply, with a positive relation between the fertility rate and female labor participation rate, as described also by Ahn and Mira (2002) and Sleebos (2003).

Our paper presents examination of how the informal care is decreased by the subsidy policy for elderly care and how the fertility and labor supply are determined. To examine the elderly care and child care simultaneously, this paper presents consideration of a three-sector model with a final goods sector, an elderly care sector, and a child care sector. Lundholm and Ohlsson (1998) and Hashimoto and Tabata (2010) consider a two-sector model with a final goods sector and an elderly care services sector. Yasuoka and Miyake (2010) consider a two-sector model with a final goods sector and a child care services sector. No report of the related literature describes a three-sector model including a final goods sector, an elderly care sector, and a child care sector simultaneously.

3 Model

This model economy is a two-period overlapping generations with endogenous fertility and two subsidy policies. Our paper assumes a small open economy. This model economy has agents of three types: households, firms, and the government.

3.1 Household

The individuals in the household live in two periods: young and old. In any t period, the younger people and the older people exist simultaneously. The younger people supply labor by $1 - l_t$ units, where l_t denotes informal elderly care for parents ($0 < l_t < 1$). Households pay for child care services, consumption in the young period, and formal elderly care services for themselves in the old period. The lifetime budget constraint is

$$(1 - \epsilon_t)p_t n_t + \frac{(1 - \delta_{t+1})z_{t+1}e_{t+1}}{1 + r_{t+1}} + c_t = (1 - l_t)w_t - T_t, \tag{1}$$

where p_t and z_{t+1} respectively denote the price of child care services per child and the price of elderly care services. Also, n_t and e_{t+1} denote the number of children (fertility) and the purchase for formal elderly care services provided by the market, which is produced by the elderly care sector. The government provides the child care subsidy service at the subsidy rate ϵ_t and formal elderly care service at the

subsidy rate δ_{t+1}. The subsidy rates ϵ_t and δ_{t+1} are in the range of [0, 1].[4] Households supply labor time to obtain labor income, where w_t denotes the wage rate to allocate child care, consumption c_t in the young period, and formal elderly care in the old period. Households save for the old period and can obtain capital income during the old period, where $1 + r_{t+1}$ denotes the interest rate. The government collects tax revenue T_t to provide child care and elderly care subsidies.

The utility function of household u_t is assumed by the following function.[5]

$$u_t = \alpha \ln n_t + \beta \ln m_t + \gamma \ln c_t + (1 - \alpha - \beta - \gamma) \ln e_{t+1}, 0 < \alpha, \beta, \gamma < 1,\ \alpha + \beta + \gamma < 1. \tag{2}$$

Children care about the level of elderly care $m_t = l_t + e_t + \bar{L}_t$, where $\bar{L}_t$ denotes the informal elderly care provided by other children (sibling). If the children think that the purchase for elderly care services bought by their parents is a small amount, then the children increase the level of informal elderly care with their time.[6]

The optimal allocations decided by the household are obtainable, considering the maximization of utility (2) subject to the budget constraint (1) as[7]

$$c_t = \gamma \left[(1 + e_t + \bar{L}_t)w_t - T_t\right], \tag{3}$$

$$n_t = \frac{\alpha}{(1 - \epsilon_t)p_t} \left[(1 + e_t + \bar{L}_t)w_t - T_t\right], \tag{4}$$

$$e_{t+1} = \frac{(1 - \alpha - \beta - \gamma)(1 + r_{t+1})}{(1 - \delta_{t+1})z_{t+1}} \left[(1 + e_t + \bar{L}_t)w_t - T_t\right]. \tag{5}$$

We obtain $l_t = \min\left[0, \beta\left(1 - \frac{T_t}{w_t}\right) - (1 - \beta)\left(e_t + \bar{L}_t\right)\right]$. That is,

$$l_t = \beta\left(1 - \frac{T_t}{w_t}\right) - (1 - \beta)\left(e_t + \bar{L}_t\right), \text{ if } e_t + \bar{L}_t < e_t^*, \tag{6}$$

$$l_t = 0, \text{ if } e_t + \bar{L}_t > e_t^*. \tag{7}$$

Therein, $e_t^* \equiv \frac{\beta}{1-\beta}\left(1 - \frac{T_t}{w_t}\right)$. We can regard e_t^* as the cutoff point at which the children do not need informal elderly care because of a sufficient level of elderly care. If the purchase of elderly care by parents

[4] Our paper presents consideration of the subsidy policy for both child care and elderly care. These two policies are provided in some OECD countries. Child care services such as kindergarten and others are subsidized by the government. In Japan, elderly people use care services by paying ten percent of the price of elderly care services.

[5] We can assume $u_t = \alpha \ln n_t + \beta \ln m_t + \gamma \ln c_t + (1 - \alpha - \beta - \gamma) \ln m_{t+1}$ as another formation. However, it is too complicated to derive the equilibrium in this setting. Considering this setting, the children provide informal family care in the purchase of elderly care service that is purchased by older people. Therefore, we use utility function (2) for simplicity.

[6] Our paper assumes m_t as substitutive care, which is supported by some empirical studies as shown in section 1. Mou and Winer (2015) set substitutive care model between the formal elderly care bought by the parents and that by children. We can assume that $m_t = \phi_1 l_t + \phi_2 e_t + \phi_3 \bar{L}_t$, $0 < \phi_j$ for $j = 1, 2, 3$ instead of $\phi_1 = \phi_2 = \phi_3 = 1$. However, this assumption does not affect the main results.

[7] See Appendix for a detail proof.

is $e_t + \bar{L}_t < e_t^*$, then the children regard the level of elderly care as low. Therefore, the children provide informal elderly care for their parents with their time to pull up the level of elderly care. Otherwise, the children do not provide elderly care time and can provide full time as the labor supply.

Considering symmetric individuals, we can consider $\bar{L}_t = (n_{t-1} - 1)l_t$. The parents have the number of n_{t-1} children. $n_{t-1} - 1$ represents the number of siblings, except for the person himself. The informal care provided by the siblings except for the person himself is $(n_{t-1} - 1)l_t$ if the person himself provides informal care l_t. Then, informal elderly care time l_t, fertility n_t, and the demand for formal elderly care service e_{t+1} with $l_t > 0$ are[8]

$$l_t = \frac{\beta\left(1 - \frac{T_t}{w_t}\right) - (1-\beta)e_t}{1 + (1-\beta)(n_{t-1} - 1)}, \tag{8}$$

$$n_t = \frac{\alpha}{p_t(1-\epsilon_t)\beta}(e_t + n_{t-1}l_t), \tag{9}$$

$$e_{t+1} = \frac{(1 + r_{t+1})(1 - \alpha - \beta - \gamma)}{\alpha} \frac{p_t(1-\epsilon_t)}{z_{t+1}(1-\delta_{t+1})} n_t. \tag{10}$$

3.2 Firms

This model includes firms of three types: the child care service sector, the elderly care sector, and the final goods sector.

Child care sector The child care service production function is given as $Y_t^c = \frac{1}{\rho} L_t^c$ $(0 < \rho)$, where Y_t^c and L_t^c denote the aggregate child care service and labor input for the sector. Defining the profit function of child care sector π_t^c as $\pi_t^c = \frac{p_t L_t^c}{\rho} - w_t^c L_t^c$, where w_t^c denotes the wage rate of the child care service sector. With a competitive market, the following equation is obtained as a result of profit maximization:

$$w_t^c = \frac{p_t}{\rho}. \tag{11}$$

Elderly care sector The elderly care service production function is given as $Y_t^e = \frac{1}{\sigma} L_t^e$ $(0 < \sigma)$, where Y_t^e and L_t^e denote the aggregate elderly care service and labor input for the sector. Defining the profit function of the elderly care sector π_t^e as $\pi_t^e = \frac{z_t L_t^e}{\sigma} - w_t^e L_t^e$, where w_t^e represents the wage rate prevailing in the elderly care service sector. With a competitive market, the following equation is obtained as a

[8] If $l_t = 0$, then we obtain $n_t = \frac{\alpha}{(1-\beta)p_t} \frac{1 - \frac{T_t}{w_t}}{1-\epsilon_t}$, $c_t = \frac{\gamma}{\alpha}(1-\epsilon_t)p_t n_t$ and $e_{t+1} = (1 + r_{t+1}) \frac{1-\alpha-\beta-\gamma}{\alpha} \frac{1-\epsilon_t}{1-\delta_{t+1}} \frac{p_t}{z_{t+1}} n_t$.

result of profit maximization.

$$w_t^e = \frac{z_t}{\sigma} \tag{12}$$

Final goods sector The final goods sector has the production function of $y_t = f(k_t)$, where y_t denotes the final goods per unit of labor and where k_t denotes the capital labor ratio. This function is assumed as $f' < 0 < f''$ as a neoclassical economic model. Profit maximization implies the following factor price in the competitive market:

$$w_t^g = f(k_t) - f'(k_t)k_t, \tag{13}$$

$$1 + r_t = f'(k_t). \tag{14}$$

In that equation above, w_t^g denotes the wage rate of the final goods sector. The capital stock is assumed to be fully depreciated in a period.

3.3 Government

The government provides subsidies for child care services and elderly care services. Considering the government balanced budget constraint, we obtain

$$N_t T_t = N_t \epsilon_t p_t n_t + \delta_t N_{t-1} z_t e_t,$$

where N_t denotes the population size of younger people in t period. We notify $n_{t-1} \equiv \frac{N_t}{N_{t-1}}$, where n_{t-1} denotes the fertility in $t-1$ period. The population size of younger people in $t-1$ period is N_{t-1}. They have n_{t-1} of children per capita. Then, the population size of younger people in t period is $N_t = N_{t-1} n_{t-1}$. Therefore, the gross population growth rate is given as n_t for any t period. Assumed as $S_t = \tau w_t$, the budget constraint is given as[9]

$$\tau w_t = \epsilon_t p_t n_t + \frac{\delta_t z_t e_t}{n_{t-1}}. \tag{15}$$

[9]These analyses assume T_t as income-proportional taxation. Even if we consider proportional income taxation in deriving the optimal allocations of household, the optimal allocations do not change. However, if we consider the wage income proportinal taxation in deriving the optimal allocations, then the labor supply might change. Zhang (1997) sets the lump-sum tax in the household decision problem. However, in equilibrium, Zhang (1997) assumes the lump-sum tax equals to proportional wage income taxation that is multiplied by labor supply, i.e., $T_t = \tau(1 - l_t)w$ is assumed as the form of our manuscript. Therefore, this setting does not affect the household decision problem and the setting is no substantial difference as long as the model economy fixes T_t as lump-sum tax in deriving the optimal allocations of household.

4 Equilibrium

The model economy is a small open. Therefore, the wage rate and interest rate are fixed respectively as w^g and $1+r$ because of (13) and (14). Assuming free labor mobility, we obtain $w = w^g = w_t^c = w_t^e$. Considering (8)–(10), the dynamics of fertility n_t is derived as follows:[10]

$$n_t = \frac{\alpha}{\rho(1-\epsilon_t)}\left[1-\tau+\frac{\rho(1+r)(1-\alpha-\beta-\gamma)}{\alpha\sigma}\frac{1-\epsilon_{t-1}}{1-\delta_t}\right]\frac{n_{t-1}}{\beta+(1-\beta)n_{t-1}}. \tag{16}$$

Given n_{t-1}, one obtains n_t, l_t, e_{t+1}, and c_t as equilibrium. Calculating $\frac{dn_t}{dn_{t-1}}$ without providing policies ($\epsilon = \delta = \tau = 0$) and defining the right-hand-side of (16) as $G(n_{t-1})$, we obtain

$$G'(n_{t-1}) = \frac{\alpha}{\rho}\left[1+\frac{\rho(1+r)(1-\alpha-\beta-\gamma)}{\alpha\sigma}\right]\frac{\beta}{(\beta+(1-\beta)n_{t-1})^2} > 0, \tag{17}$$

and $G'' < 0$ for any n_{t-1}. Moreover we obtain $G'(0) = \frac{\alpha}{\rho\beta}\left[1+\frac{\rho(1+r)(1-\alpha-\beta-\gamma)}{\alpha\sigma}\right]$ and $G'(\infty) = 0$. Then, the dynamics of fertility n_t can be depicted as Fig. 1 and the steady state equilibrium is stable.[11]

[Insert Fig. 1 around here.]

The equilibrium values in the steady state with $l > 0$ is given by the following equations. [12]

$$n = \frac{\alpha}{\rho(1-\beta)}\left[\frac{1-\tau}{1-\epsilon}+\frac{(1+r)(1-\alpha-\beta-\gamma)\rho}{\alpha\sigma(1-\delta)}\right]-\frac{\beta}{1-\beta}, \tag{18}$$

$$e = \frac{1-\epsilon}{1-\delta}\frac{\rho(1+r)(1-\alpha-\beta-\gamma)}{\alpha\sigma}n, \tag{19}$$

$$l = \frac{\beta(1-\tau)-(1-\beta)e}{1+(1-\beta)(n-1)}, \tag{20}$$

$$\tau = \epsilon\rho n+\frac{\delta\sigma e}{n}. \tag{21}$$

5 Policy

Our paper presents an examination of how the subsidy policy affects the purchase of elderly care, the purchase of child care and informal care by the children in the steady state. We assess policies of two types: one for the elderly care subsidy and the other for the child care subsidy. This report presents only

[10] See Appendix for a detail proof.

[11] Consideration of steady state fertility dynamics (16) gives $\frac{dn_t}{dn_{t-1}} = \frac{\alpha}{\rho}\left[1+\frac{(1+r)(1-\alpha-\beta-\gamma)}{\alpha\sigma}\right]\frac{\beta}{[\beta+(1-\beta)n]^2} = \frac{\beta}{\beta+(1-\beta)n} < 1$. If $\frac{\alpha}{\rho\beta}\left[1+\frac{\rho(1+r)(1-\alpha-\beta-\gamma)}{\alpha\sigma}\right] > 1$, then we can obtain the unique steady state equilibrium. Our paper assumes this parameter condition. Otherwise, there exists non-zero steady state equilibrium. With high α or low ρ, i.e., the preference for fertility or the productivity of child care sector, this assumption can hold.

[12] With $l = 0$, the fertility is given as follows: no dynamics appears, as shown by $n_t = \frac{\alpha(1-\tau)}{(1-\beta)(1-\epsilon)}$

an examination of the case in which (6) holds, i.e., the children use the time for elderly care for their parents to examine the effect on the equilibrium with informal family care. The inequality condition in the steady state is shown as $e < \frac{\beta(1-\tau)}{1-\beta}$ to hold positive l shown by (20).

5.1 Elderly care policy

The government provides an elderly care subsidy for older people. Now the government budget constraint (21) changes to $\tau = \frac{\sigma e \delta}{n}$. With total differentiation with respect to δ and τ, we obtain the following[13]

$$\frac{d\tau}{d\delta}\Big|_{\delta=0} = \frac{\sigma e}{n}. \tag{22}$$

First, we consider the effect of elderly care policy on the fertility. Total differentiation of (18) with respect to n, τ, δ at the steady state with $\tau = \delta = \epsilon = 0$ and substituting (19) and (22), we obtain the following equation:

$$\frac{dn}{d\delta}\Big|_{\delta=0} = \frac{\alpha(1-\sigma)}{(1-\beta)\rho}\frac{e}{n}. \tag{23}$$

With $\sigma < 1$, we obtain $\frac{dn}{d\delta} > 0$. The elderly care subsidy service can raise the fertility rate if σ is smaller than 1, i.e., the elderly care productivity $\frac{1}{\sigma}$ is large. An increase in this subsidy raises the demand for formal elderly care services in the market. Then, the children decrease informal elderly care time by virtue of an increase in the quality of elderly care. By virtue of this subsidy, younger people can increase the labor supply and obtain more household income. Therefore, the household can afford to increase fertility.

Second, we examine the effects on formal elderly care service e. Total differentiation of (19) with respect to e, n and δ and substituting (23) are reduced to

$$\frac{de}{d\delta}\Big|_{\delta=0} = \frac{e}{n}\left(\frac{dn}{d\delta} + n\right) = \frac{e}{n}\left[\frac{(1+r)(1-\alpha-\beta-\gamma)}{1-\beta}\frac{1-\sigma}{\sigma} + n\right]. \tag{24}$$

The first term in the bracket $\frac{(1+r)(1-\alpha-\beta-\gamma)}{1-\beta}\frac{1-\sigma}{\sigma}$ decreases with σ. We define σ^* to hold $\frac{(1+r)(1-\alpha-\beta-\gamma)}{1-\beta}\frac{1-\sigma}{\sigma} + n = 0$. Then, we find $\sigma^* > 1$. The demand for formal elderly care service always increases if the fertility rate increases. An increase in fertility raises the informal elderly care provided by siblings. Moreover, younger people can provide more labor. Then, the household income increases and the household can afford to purchase formal elderly care and child care service in the market because of their increased income.

[13] See the Appendix for a detailed proof.

Third, we examine the effects on the informal elderly care l. Total differentiation of (20) with respect to n, e, l, and δ is reduced to

$$\frac{dl}{d\delta}\Big|_{\delta=0} = -\frac{(1-\beta)\left(\frac{dn}{d\delta}+\frac{de}{d\delta}\right)+\frac{\beta\sigma e}{n}}{\beta+(1-\beta)n}. \tag{25}$$

The sign of this differentiation is ambiguous. If the elderly care subsidy service can raise both the fertility and the demand for formal elderly care services, i.e., $\frac{dn}{d\delta} > 0$ and $\frac{de}{d\delta} > 0$, then one can always obtain $\frac{dl}{d\delta} < 0$, i.e., informal elderly care decreases.

Depending on the level of the productivity of elderly care σ, we can present the sign of $\frac{dn}{d\delta}$, $\frac{de}{d\delta}$, and $\frac{dl}{d\delta}$ as the following table.

[Insert Table 1 around here.]

Therefore, the following proposition can be established.

Proposition 1 With $\sigma < 1$, i.e., the elderly care sector productivity is high, the fertility and the demand for formal elderly care increases, thanks to the elderly care subsidy. Informal care per child decreases.

This proposition demonstrates that the elderly care policy can have a positive effect on the fertility. With the elderly care subsidy services, the individuals can provide more labor and can increase the number of children, which represents a positive correlation between the fertility rate and female labor participation, as shown by Apps and Rees (2004) and others.

The first term in the numerator of (25) signifies the effect that an increase in the fertility raises the number of brothers or sisters who provide informal family care.

By virtue of the elderly care subsidy, informal family care is replaced by formal elderly care in high productivity of elderly care sector. In high-productivity elderly care, the productivity of formal elderly care is higher than that of informal family care. Therefore, elderly care quality increases and the children consider that the parents do not need additional family care. Consequently, the children can increase the labor supply and can afford to have more children.

However, if the productivity of the elderly care sector is small, then the children consider that the parents can not obtain a sufficient level of elderly care and the children continue provide informal family

care to hold a sufficient level of elderly care. Then, the children can not increase the labor supply to any great degree. Because of the tax burden, household incomes decrease and the fertility decreases. A decrease in fertility makes the family care increase because the family care provided by siblings decreases, leaving an insufficient level of elderly care.

The next subsection presents analyses of the effects of child care subsidy services. Although many recent reports have described analyses examining the effects of a child care policy, we would like to examine the model with informal family care.

5.2 Child care policy

We examine child care subsidy services as another policy. If the government provides a child care subsidy services, then the government budget constraint (21) is given as $\tau = \epsilon pn$. Total differentiation with respect to τ and ϵ is reduced to

$$\frac{d\tau}{d\epsilon}\Big|_{\epsilon=0} = \rho n. \tag{26}$$

Total differentiation of (18) with respect to n, and ϵ at the steady state with $\epsilon = \tau = 0$ and substituting (26) are reduced to[14]

$$\frac{dn}{d\epsilon}\Big|_{\epsilon=0} = \frac{\alpha}{1-\beta}\left(\frac{1}{\rho} - n\right). \tag{27}$$

The first term of the bracket $\frac{1}{\rho}$ shows the direct effect of child care subsidy services. The second term shows the tax burden effect. The tax burden decreases the household disposable income and the fertility rate. From (18), fertility in the steady state depends on the level of σ. Then we show $n(\sigma)$ as the fertility in the steady state. Then, we define σ^{**} to hold $\frac{dn(\sigma)}{d\epsilon} = 0$. σ^{**} is shown as

$$\sigma^{**} = \frac{(1+r)(1-\alpha-\beta-\gamma)\rho}{1-\alpha-\beta+\rho\beta} > 0.$$

With $\sigma > \sigma^{**}$, we can obtain $\frac{dn}{d\epsilon} > 0$.

Second, we examine the effect on formal elderly care e. Total differentiation of (19) with respect to n, e, and ϵ at the steady state $\epsilon = \tau = 0$ and substituting (26), and (27) are reduced to

$$\frac{de}{d\epsilon}\Big|_{\epsilon=0} = \frac{e}{n}\frac{dn}{d\epsilon} - e = \frac{e}{n}\frac{\alpha}{1-\beta}\left(\frac{1}{\rho} - n - \frac{1-\beta}{\alpha}n\right). \tag{28}$$

[14]Because this section presents examination of the effect of child care, the level of child care productivity ρ to ascertain the sign of $\frac{dn}{d\epsilon}$ might be derived. However, as shown by (27), an decrease in ρ increases both $\frac{1}{\rho}$ and n. Therefore, the relation between sign of $\frac{dn}{d\epsilon}$ and the level of ρ is ambiguous.

If $n < \frac{1}{\rho}\frac{\alpha}{1+\alpha-\beta}$, then we obtain $\frac{de}{d\epsilon} > 0$. The child care subsidy increases the fertility rate. An increase in fertility raises the informal elderly care provided by siblings. Younger people can provide more labor supply and can obtain a higher income. Therefore, the demand for formal elderly care services in the market increases. Defining σ^{***} to hold $n = \frac{1}{\rho}\frac{\alpha}{1+\alpha-\beta}$, that is $\frac{de}{d\epsilon} = 0$, σ^{***} is shown as

$$\sigma^{***} = \frac{(1+r)(1+\alpha-\beta)(1-\alpha-\beta-\gamma)\rho}{\alpha(1-\beta)+(1+\alpha-\beta)(\rho\beta-\alpha)}.$$

Defining $L = \frac{1}{\rho}$ and $R = \frac{1+\alpha-\beta}{\alpha}n$, the intersect of L and R is shown as the following figure.[15]

[Insert Fig. 2 around here.]

$\sigma > \sigma^{***}$ brings about $\frac{de}{d\epsilon} > 0$ and we can find $\sigma^{**} < \sigma^{***}$.

The effect of the child care subsidy services on the informal elderly care l is given as

$$\frac{dl}{d\epsilon}\Big|_{\epsilon=0} = -\frac{(1-\beta)\left(\frac{dn}{d\epsilon}+\frac{de}{d\epsilon}\right)+\beta\rho n}{\beta+(1-\beta)n}. \tag{29}$$

Depending on the level of the productivity of elderly care σ, one can show the sign of $\frac{dn}{d\epsilon}$, $\frac{de}{d\epsilon}$, and $\frac{dl}{d\epsilon}$ as the following table.

[Insert Table 2 around here.]

Therefore, the following proposition can be established.

Proposition 2 The subsidy for child care services can raise the fertility rate if $\sigma > \sigma^{**}$ and can raise the demand for formal elderly care services if $\sigma > \sigma^{***}$. Informal elderly care always decreases if child care subsidy services can pull up both the fertility rate and the demand for elderly care services.

It is worthwhile deriving the result that the child care subsidy services cannot always raise the fertility rate. The intuitive result is the following. If this subsidy reduces the purchase of elderly care services because of tax burden, then children worry about a decrease in the quality of elderly care and provide informal care time to compensate for the quality of elderly care. However, an increase in the care time reduces the labor supply. Then the household income decreases. Results show that fertility decreases because the household cannot afford to care for the children.

[15] Because L is positive, $n(\sigma)$ is a decreasing function and $n(\sigma)$ is infinity at $\sigma = 0$, σ^{***} is always positive.

Therefore, a positive effect of the elderly care subsidy service on the fertility might not be inferred. The burden of informal elderly care decreases the household income because the households must reduce the labor supply to obtain labor income. However, by virtue of the elderly care subsidy service, the older people purchase a sufficient amount of elderly care services and the children need not provide informal care. An increase in labor supply brings about more labor income. For that reason, the younger people can save more and can purchase a sufficient amount of elderly care service. Because of intergenerational externality, the elderly care subsidy service affects fertility.

6 Welfare Analysis

We examine the welfare level that results from providing a subsidy for formal elderly care services in the steady state. The utility level and the household allocations in the steady state are shown by

$$u = \alpha \ln n + \beta \ln(e + nl) + \gamma \ln c + (1 - \alpha - \beta - \gamma) \ln e, \tag{30}$$

$$c = \frac{\gamma p}{\alpha} n, \tag{31}$$

$$e = \frac{\rho(1+r)(1-\epsilon)(1-\alpha-\beta-\gamma)}{\alpha z(1-\delta)} n, \tag{32}$$

$$nl = \frac{p\beta}{\alpha} n - e. \tag{33}$$

These equations are reduced to the following utility function:

$$\begin{aligned} u = {} & \ln n - (1 - \alpha - \beta - \gamma) \ln(1 - \delta) + (1 - \alpha) \ln(1 - \epsilon) \\ & + \beta \ln \frac{\rho\beta}{\alpha} + \gamma \ln \frac{\rho\gamma}{\alpha} + (1 - \alpha - \beta - \gamma) \ln \frac{\rho(1+r)(1-\alpha-\beta-\gamma)}{\alpha z}. \end{aligned} \tag{34}$$

Total differentiation of (34) with respect to u, n, and δ at $\tau = \epsilon = \delta = 0$, we obtain

$$\frac{du}{d\delta}\Big|_{\delta=0} = \frac{1}{n}\frac{dn}{d\delta} + (1 - \alpha - \beta - \gamma). \tag{35}$$

The welfare at the steady state increases if the elderly care subsidy services can raise the fertility rate. An increase in fertility raises the family care by siblings. This externality effect decreases the informal family care for younger people and promotes the labor supply. An increase in the labor supply raises the household income. Furthermore, the welfare level increases.

An intergenerational externality exists between parents and children. By virtue of the elderly care subsidy, the parents can increase the demand for elderly care and the children can supply more labor

time. Moreover, an increase in labor supply raises both the household income and fertility because the household can afford to have children. Therefore, the subsidy for the elderly care raises the social welfare.

Even if $\frac{dn}{d\delta} < 0$, welfare might be pulled up. An elderly care subsidy replaces low-productivity informal care with high-productivity formal care. Then, an increase in elderly care quality has a positive effect on welfare.

In some OECD countries, aging societies with fewer children are progressing. Some countries consider raising fertility and the labor supply. A subsidy for older people, not a child care policy, can play an important role as a remedy in an aging society with fewer children.

Next, we consider that the effect of the child care subsidy service can not always raise utility. After total differentiation of (34) with respect to u, n, and ϵ at $\tau = \epsilon = \delta = 0$, we obtain the following:

$$\left.\frac{du}{d\epsilon}\right|_{\epsilon=0} = \frac{1}{n}\frac{dn}{d\epsilon} - (1-\alpha). \tag{36}$$

Even if $\frac{dn}{d\epsilon} > 0$, utility decreases if $0 < \frac{dn}{d\epsilon} < (1-\alpha)n$. This policy has no direct effect of reducing informal family care. An increase in fertility increases informal elderly care by siblings. Then, the informal elderly care for younger people can be decreased and they can supply more labor, which therefore increases the social welfare. If the negative effect of the tax burden on social welfare is dominated by the positive effect of an increase in the fertility on social welfare, then social welfare can be pulled up by the child care subsidy.

Therefore, the elderly care subsidy should be provided to support a policy for fewer children and welfare. In Japan, the burden of informal family care brings about poverty because younger people can not work sufficiently. The elderly care subsidy stimulates the demand for elderly care services provided by the market. Then informal family care might not be needed. Therefore, the poverty problem might vanish.

7 Conclusions

Our paper presents examination of the effects of two policies: child care subsidy services intended for younger people and elderly care subsidy services designed for older people. In an aging society with fewer children, intergenerational conflict presents an important social challenge. Because of an increase in the number and share of older people among the population, the government tends to increase benefits for

older people and decrease those for younger people, such as child care subsidies. However, the arguments presented in this paper demonstrate that the subsidy policy for older people improves the quality of life not only of older people but also of younger people. Moreover, a subsidy for younger people cannot always improve the quality of life for younger people. The results derived through these analyses are interesting, demonstrating how governments should subsidize the needs of both younger and older people. Our paper insists on the intergenerational externality, i.e., one for substitution between formal elderly care and informal elderly care. A subsidy for formal elderly care reduces informal elderly care. An increase in fertility can decrease the availability and provision of informal elderly care for younger people. Because of these effects of the intergenerational externality, younger people can receive benefits even if a subsidy for older people is provided.

Our paper presents consideration of the effect of separate policies: elderly care subsidy or child care. An increase in the elderly care subsidy service must reduce the child care subsidy service as long as tax revenue is constant if we consider two policies simultaneously. Then, the two effects are included. We cannot obtain the results of policy effects clearly. This paper therefore considers separate policies. We would like to examine two policies simultaneously in future research.

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Appendix

Household Decision Problem

We set the following Lagrange function.

$$L = \alpha n_t + \beta \ln(e_t + \bar{L}_t + l_t) + \gamma \ln c_t + (1-\alpha-\beta-\gamma)\ln e_{t+1} + \lambda \left[(1-l_t)w_t - S_t - (1-\epsilon_t)p_t n_t - \frac{(1-\delta_{t+1})z_{t+1}e_{t+1}}{1+r_{t+1}} - c_t \right]. \tag{37}$$

λ denotes Lagrange variables. First-order conditions at $0 < l_t$ are shown below:

$$\frac{\partial L}{\partial n_t} = 0,\ (1-\epsilon_t)p_t n_t = \frac{\alpha}{\lambda}, \tag{38}$$

$$\frac{\partial L}{\partial l_t} = 0,\ (e_t + \bar{L}_t)w_t + l_t w_t = \frac{\beta}{\lambda}, \tag{39}$$

$$\frac{\partial L}{\partial c_t} = 0,\ c_t = \frac{\gamma}{\lambda}, \tag{40}$$

$$\frac{\partial L}{\partial e_{t+1}} = 0,\ \frac{(1-\delta_{t+1})z_{t+1}e_{t+1}}{1+r_{t+1}} = \frac{1-\alpha-\beta-\gamma}{\lambda}. \tag{41}$$

Then, we obtain $\frac{1}{\lambda} = (1 + e_t + \bar{L}_t)w_t - S_t$ and (3)–(6). Considering $\bar{L}_t = (n_{t-1}-1)l_t$, we obtain (8)–(10).

Equation (16)

Considering (9), we obtain the following equation.

$$\begin{aligned} n_t &= \frac{\alpha}{\beta\rho(1-\epsilon_t)} \left[\frac{\beta(1-\tau)n_{t-1} - (1-\beta)e_t n_{t-1}}{1+(1-\beta)(n_{t-1}-1)} + \frac{e_t + (1-\beta)(n_{t-1}-1)e_t}{1+(1-\beta)(n_{t-1}-1)} \right] \\ &= \frac{\alpha}{\rho(1-\epsilon_t)} \frac{(1-\tau)n_{t-1} + e_t}{1+(1-\beta)(n_{t-1}-1)}. \end{aligned} \tag{42}$$

We consider the following two equations,

$$e_{t+1} = \frac{(1+r)(1-\alpha-\beta-\gamma)}{(1-\delta_{t+1})\sigma}[1 - -\tau + e_t + l_t(n_{t-1}-1)], \tag{43}$$

$$n_t = \frac{\alpha}{\rho(1-\epsilon_t)}[1 - -\tau + e_t + l_t(n_{t-1}-1)]. \tag{44}$$

Then, we obtain

$$e_t = \frac{(1+r)(1-\alpha-\beta-\gamma)\rho}{\alpha\sigma} \frac{1-\epsilon_{t-1}}{1-\delta_t} n_{t-1}. \tag{45}$$

Therefore, (42) and (45) are reduced to the dynamics of n_t (16).

Total Differentiation of (18)-(21)

Total differentiation of (18)-(21) with respect to $n, e, l, \tau, \epsilon, \delta$ at the approximation of $\tau = 0$, $\epsilon = 0$ and $\delta = 0$.

$$d\tau = \rho n d\epsilon + \frac{\sigma e}{n} d\delta, \tag{46}$$

$$(1-\beta)dn = \frac{\alpha}{\rho}\left[-d\tau + d\epsilon + \frac{(1+r)(1-\alpha-\beta-\gamma)\rho}{\alpha\sigma} d\delta\right], \tag{47}$$

$$\begin{aligned} de = {} & \frac{\rho(1+r)(1-\alpha-\beta-\gamma)}{\alpha\sigma} dn - \frac{\rho(1+r)(1-\alpha-\beta-\gamma)n}{\alpha\sigma} d\epsilon \\ & + \frac{\rho(1+r)(1-\alpha-\beta-\gamma)n}{\alpha\sigma} d\delta, \end{aligned} \tag{48}$$

$$-\beta d\tau - (1-\beta)de = (1 + (1-\beta)(n-1))dl + (1-\beta)l dn. \tag{49}$$

It is noteworthy that the total differentiation of (21) by $n, e, l, \tau, \epsilon, \delta$ reduces to

$$d\tau = \rho n d\epsilon + \frac{\sigma e}{n} d\delta + \left(\epsilon\rho - \frac{\delta\sigma e}{n^2}\right) dn + \frac{\delta\sigma}{n} de. \tag{50}$$

Because of total differentiation at the approximation of $\tau = 0$, $\epsilon = 0$ and $\delta = 0$, substituting $\tau = 0$, $\epsilon = 0$ and $\delta = 0$ are reduced to $d\tau = \rho n d\epsilon + \frac{\sigma e}{n} d\delta$. Other equations (47)–(49) are explained using the same discussion.

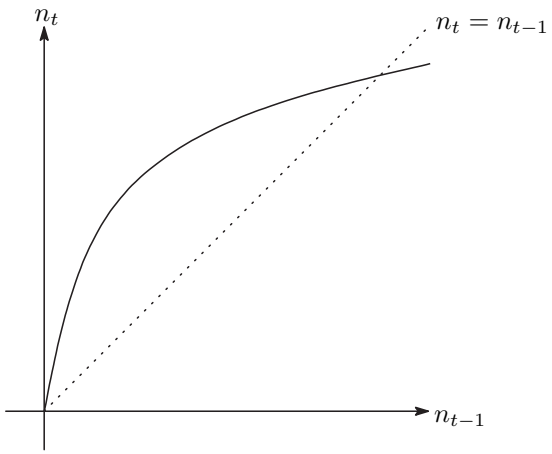

Fig. 1 Dynamics of n_t.

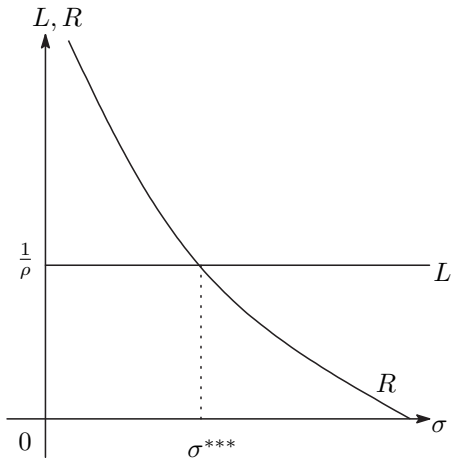

Fig. 2 Level of σ^{***}.

	$\frac{dn}{d\delta}$	$\frac{de}{d\delta}$	$\frac{dl}{d\delta}$
$\sigma < 1$	+	+	-
$1 < \sigma < \sigma^*$	-	+	?
$\sigma^* < \sigma$	-	-	?

Table 1 Effect of elderly care subsidy

	$\frac{dn}{d\epsilon}$	$\frac{de}{d\epsilon}$	$\frac{dl}{d\epsilon}$
$\sigma < \sigma^{**}$	-	-	?
$\sigma^{**} < \sigma < \sigma^{***}$	+	-	?
$\sigma^{***} < \sigma$	+	+	-

Table 2 Effect of child care subsidy