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The Effects of a Production Subsidy in the Model of Differentiated Goods and Monopolistic Competition

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1. Introduction

The standard theory of international trade tells us that free trade is efficient in the sense that one nation cannot get better off without other nations getting worse off. This basic principle is, however, not necessarily true in the presence of economies of scale within firms to produce a variety of differentiated goods. As shown by Dixit-Stiglitz (1977) for a closed economy, firms with economies of scale tend to produce more in volume and fewer in varieties of their products at the market equilibrium than the socially optimum values, so that a production subsidy to the firms is expected to raise the level of world welfare, defined as a sum of welfare levels of all nations, to a higher level than the one realized under free trade by reducing their outputs and increasing varieties of their products.

The purpose of the present paper is twofold. Constructing a two-country, two-sector, two-factor general equilibrium model with differentiated goods, economies of scale and monopolistic competition, it shows first that either a production subsidy granted by one of the two countries to its differentiated-goods sector with a greater degree of scale economies relative to the other sector or a production tax levied on its differentiated-goods sector with a smaller degree of scale economies increases the level of world welfare to a higher level than under free trade. This is due to a variety effect of such a policy that the subsidy expands the variety of differentiated goods in the subsidized sector and the tax expands the total variety of differentiated goods in a *laissez faire* sector. Secondly, it shows that the welfare effect of the policy on an individual nation depends upon not only the variety effect but the terms of trade effect of the policy: a production subsidy, for instance, that raises the level of world welfare improves either the welfare of a labor-abundant country provided that the subsidized sector is capital intensive and satisfies some conditions for production technology or that of a capital-abundant country provided that the subsidized sector is labor intensive and satisfies other conditions for technology.

A few studies have so far been done in this field. Flam-Helpman (1987) has examined the resource allocation and welfare effects of a production subsidy and other trade policy measures in a two-country, two-sector, and multi-factor model under a restrictive assumption that a policy-making country is small in the sense that the price and the number of vaieties of differentiated goods produced in the partner country are given. It has, nevertheless, succeeded in getting only ambiguous results on the policy effects. Wong (1995) has studied the resource allocation effects of a production subsidy in a special case of the two-sector and two-factor model in that one of the two sectors produces a homogeneous good by a constant-returns-to-scale production function in the perfectly competitive market and the other of them produces a variety of differentiated goods with a homothetic production function in the monopolistically competitive market. It has gotten a result that the volume of a firm's output in the subsidized differentiated-goods sector coincides with the *laissez faire* market equilibrium value but has failed to derive any result on the policy effect on the number of varieties in this sector. The purpose of the present paper is, therefore, to fill the gap by deriving unambiguous conclusions about the policy effects on income distribution, resource allocation, and welfare in a more general model in which no countries are small, all the sectors produce their particular kinds of differentiated goods in monopolistically competitive markets, and production functions are non-homothetic in at least one of the two sectors.

The configuration of this paper is as what follows. Section 2 presents the two-country, two-sector, two-factor model. Section 3 is devoted to the development of this model using the elasticities originally defined by Horn (1983) for the cost function of a firm with respect to its output and factor prices and the consideration of the relationships between commodity prices and factor rewards including the magnification effects of the Stolper-Samuelson theorem. Section 4 explores the effects of the policy on the size and number of firms in the two sectors, and Section 5 clarifies the effects of the policy on the welfare levels of an individual country and the world. Section 6 deals with the policy effects in a special case where firms in one of the two sectors produce a homogeneous good with a uniform constant-returns-to-scale production function in a perfectly competitive market.

2. The Model

There are two countries, home and foreign countries, in the world which are engaged in free international trade in goods. In each country there are two sectors, first and second sectors, in each of which a variety of differentiated goods are produced by monopolistically competitive firms using two factors of production, labor and capital, in production functions which exhibit increasing returns to scale. For the sake of distinction, a variety of the differentiated goods produced in the jth sector of the home country are numbered from 1 to a positive natural number n_j $(j=1,2)$, and a variety of the differentiated goods produced in the jth sector of the foreign country are from n_j to a greater positive natural number N_j $(j=1,2)$. A variety belonging to a set of natural numbers from 1 to n_j is different from each other as well as from the varieties belonging to a set of natural numbers from n_j+1 to N_j.

Consumers in each country have identical and homothetic preference and consume all the varieties of differentiated goods produced in the two countries. Suppose that the community welfare function of the home country is of the CES type and represented by

$$U=[\sum_{i=1}^{n_1} c_{H1}(i)^{\beta_1}+\sum_{i=n_1+1}^{N_1} c_{F1}(i)^{\beta_1}]^{\alpha_1/\beta_1}[\sum_{i=1}^{n_2} c_{H2}(i)^{\beta_2}+\sum_{i=n_2+1}^{N_2} c_{F2}(i)^{\beta_2}]^{\alpha_2/\beta_2} \qquad 0<\alpha_1,\alpha_2,\beta_1,\beta_2<1,$$
$$\alpha_1+\alpha_2=1 \tag{1}$$

where U denotes the level of the home country's community welfare, $c_{Hj}(i)$ and $c_{Fj}(i)$ its consumptions of the ith variety of the differentiated good produced in the jth sectors of the home and foreign country respectively, and α_j and β_j are constants $(j=1,2)$. The consumers as a whole maximize their community welfare under their budget constraint. By solving this maximization problem the demand functions for domestically produced and imported differentiated goods can be obtained. In the symmetric case the appearance of which is secured by the assumption of identical welfare functions of the CES type on the consumers' side and that of identical production functions of firms in a sector, which will be made below, they are represented respectively by:

$$c_{Hj}=\frac{\alpha_j I p_j^{1/(\beta_j-1)}}{P_j} \qquad c_{Fj}=\frac{\alpha_j I p_j^{*1/(\beta_j-1)}}{P_j} \qquad j=1,2$$

where the foreign labor is taken as a numeraire, p_j and p_j^* denote the consumers' prices of each variety of differentiated goods produced in the jth sectors of the home and foreign countries respectively $(j=1,2)$, I the home country's disposable income in terms of the numeraire, and P_j the consumers' price index for differentiated goods of the jth sector in both countries,

$$P_j = \sum_{i=1}^{n_j} p_j(i)^{\beta_j/(\beta_j-1)} + \sum_{i=n_j+1}^{N_j} p_j^*(i)^{\beta_j/(\beta_j-1)}$$
$$= n_j p_j^{\beta_j/(\beta_j-1)} + n_j^* p_j^{*\beta_j/(\beta_j-1)} \qquad j=1,2.$$

It should be noted that this price index is an increasing function in the number of varieties of differentiated goods produced in each country and a decreasing function in their prices. If N_j is very large the elasticity of P_j with respect to p_j or p_j^* will be negligible, and hence the price elasticity of the demand for differentiated goods of the jth sector, ε_j, can be derived from (2) as a constant which is common to all the varieties in this sector:

$$\varepsilon_j = -\frac{p_j}{c_{Hj}}\frac{\partial c_{Hj}}{\partial p_j} = -\frac{p^*_j}{c_{Fj}}\frac{\partial c_{Fj}}{\partial p^*_j} = \frac{1}{1-\beta_j} \qquad j=1,2.$$

Suppose that no costs for a firm in the jth sector to transfer its production from one variety to another are incurred, then each firm in this sector can specialize in the production of one variety which is different from the other in this sector. This implies that a firm in each sector can be identified with the variety which it produces. Suppose also that it has no monopsony powers in any factor market and that the production functions of firms in one sector are identical, twice differentiable, increasing in each argument, and strictly quasi-concave. In the symmetric case, the cost function of each firm in the home jth sector can be represented by $G^j(w,r,x_j)$, where w and r denote the wage and rental rates, measured with the numeraire, in the home country respectively and x_j the size of a firm, measured with output, in the home jth sector. Let $G^j_W(\cdot) = G^j_W(w,r,x_j) = \partial G^j(w,r,x_j)/\partial w$, $G^j_R(\cdot) = G^j_R(w,r,x_j) = \partial G^j(w,r,x_j)/\partial r$ and $G^j_X(\cdot) = G^j_X(w,r,x_j) = \partial G^j(w,r,x_j)/\partial x_j$ for $j=1,2$. Then $G^j_X(\cdot)$ denotes the marginal cost of each firm, and, according to the Shepherd's lemma, $G^j_W(\cdot)$ and $G^j_R(\cdot)$ are equal to the equilibrium amounts of labor and capital, respectively, required to produce x_j. If the production function of each firm in the jth sector is homothetic as assumed in Dixit-Norman (1980), its cost function is represented by

$$G^j(w,r,x_j) = \tilde{G}^j(w,r)m_j(x_j), \quad 0 < m'_j(x_j) < m_j(x_j)/x_j.$$

Some of the properties of the cost function in its general form could be captured with the following elasticities for it. Define its output elasticity as

$$\theta_j(\circ) = \theta_j(w,r,x_j) = \frac{x_j G_X^j(w,r,x_j)}{G^j(w,r,x_j)} \qquad j = 1,2;$$

the output elasticity of the marginal cost as

$$\tau_j(\circ) = \tau_j(w,r,x_j) = \frac{x_j G_{XX}^j(w,r,x_j)}{G_X^j(w,r,x_j)} \qquad j = 1,2;$$

and the output elasticity of $\theta_j(\circ)$ as

$$\eta_j(\circ) = \eta_j(w,r,x_j) = \frac{x_j}{\theta_j(\circ)}\frac{\partial\theta_j(\circ)}{\partial x_j} = 1 - \theta_j(\circ) + \tau_j(\circ) \qquad j = 1,2.$$

It should be noted that $1/\theta_j(\circ)$ measures the degree of scale economies because $1/\theta_j(\circ)>1$ when the production function in the jth sector exhibits increasing returns to scale while $1/\theta_j(\circ)=1$ when it exhibits constant returns to scale (j=1, 2). Although $\tau_j(\circ)$ is not positive for the jth sector with increasing-returns-to-scale technology $\eta_j(w,r,x_j)$ is assumed to be positive for any set of positive $w, r,$ and x_j through the following analysis.

The government of the home country grants a uniform ad valorem subsidy, the rate of which is denoted by a positive number s, to each firm in home sector 2 or levies a tax on it, the rate of which is denoted by a negative number s, per one unit of output evaluated by the consumers' price. Denoting the producers' price of each variety of differentiated goods in the home sector j by q_j, they are represented by:

$$q_1 = p_1 \qquad q_2 = (1+s)p_2.$$

The government expects the partner country to keep the *laissez faire* policy and its own policy measure to make the home country better off through its effects on the total number of varieties and the output of differentiated goods in sector 2. The total payment of the subsidy or revenue of the tax, $S = sp_2n_2x_2$, is assumed to be evenly raised from or distributed to domestic consumers in a lump sum fashion

A firm in every sector is a monopolist in the production of its particular variety of differentiated goods so that its profit maximization condition requires its marginal cost to equal its marginal revenue. In the home country the profit of each firm in the jth sector is $q_j x_j - G^j(w, r, x_j)$ so that its profit maximization condition is represented by:

$$G_X^j(w, r, x_j) = \beta_j q_j \qquad j = 1, 2. \tag{2}$$

Suppose that entry barriers to the market of each variety of differentiated goods in every sector is not so high that new firms can enter the production of this variety as long as existing firms earn positive profits from it. Then the condition for an entry into the production of this variety to stop must be the one for their profits to be zero:

$$G^j(w, r, x_j) = q_j x_j \qquad j = 1, 2. \tag{3}$$

Combining (2) and (3) for each sector gives:

$$\theta_j(w, r, x_j) = \beta_j \qquad j = 1, 2, \tag{4}$$

which implies that the size of a firm in every sector can be determined by w, r, and β_j all of which are common to all the varieties in this sector.

Free trade in differentiated goods and no implementation of a trade policy measure on sector 1 in either country ensure $p_1 = p_1^*$, so that the demand functions of home consumers for each variety of differentiated goods in home and foreign sectors 1 are reduced to:

$$c_{H1} = c_{F1} = \frac{\alpha_1 I p_1^{*-\varepsilon_1}}{P_1} \tag{5}$$

where the consumers' price index for these differentiated goods is reduced to $P_1 = p_1^{*1-\varepsilon_1} N_1$. The consumers' price of differentiated goods produced in home sector 2 will not be necessarily equal to that of differentiated goods produced in foreign sector 2 because the subsidy (tax) by the home country will decrease (increase) the output of the former to create the excess demand for (supply of) it under the *ceteris paribus* condition. Thus the demand functions for each variety of differentiated goods produced in home and foreign sectors 2 are respectively represented by:

$$c_{H2} = \left(\frac{p_2^*}{p_2}\right)^{\varepsilon_2} c_{F2}, \qquad c_{F2} = \frac{\alpha_2 I p_2^{*-\varepsilon_2}}{P_2}. \tag{6}$$

Suppose that foreign consumers as a whole have the same community welfare function as home consumers do and maximize their welfare under their budget constraint. Then their demand functions for each variety of differentiated goods produced in home and foreign sectors 1 and 2 take respectively similar forms to the counterparts of the home country,

$$c_{F1}^* = c_{H1}^* = \frac{\alpha_1 I^* p_1^{*-\varepsilon_1}}{P_1} \tag{7}$$

and

$$c_{H2}^* = \left(\frac{p_2^*}{p_2}\right)^{\varepsilon_2} c_{F2}^*, \qquad c_{F2}^* = \frac{\alpha_2 I^* p_2^{*-\varepsilon_2}}{P_2}, \tag{8}$$

where an asterisk denotes the foreign country's variable. Firms in each sector of the foreign country have the same cost function as home firms do and produce their own particular varieties of differentiated goods in monopolistically competitive markets with no foreign government interventions. Combining their profit maximization conditions and free-entry-and-exit conditions gives the foreign relationships between the firm size and factor prices in the symmetric case:

$$\theta_j(w^*, r^*, x_j^*) = \beta_j \qquad j = 1, 2. \tag{9}$$

Since firms in each differentiated-goods sector can control the levels of demand for their products from consumers in both countries, they can determine the supply just equal to the world demand for them: $c_{Hj} + c_{Hj}^* = x_j$ and $c_{Fj} + c_{Fj}^* = x_j^*$ for $j = 1, 2$. Substituting equations (5)-(8) into these identities gives the international supply-demand identities for differentiated goods produced in sectors 1 and 2 respectively:

$$\begin{aligned} &\frac{\alpha_1 I^W p_1^{*-\varepsilon_1}}{P_1} = x_1 = x_1^*, \\ &x_2 = \left(\frac{p_2^*}{p_2}\right)^{\varepsilon_2} x_2^*, \\ &\frac{\alpha_2 I^W p_2^{*-\varepsilon_2}}{P_2} = x_2^*, \end{aligned} \tag{10}$$

where I^W denotes the world income, $I+I^*$. Obviously, $x_2 <(>) x_2^*$ when $p_2 >(<) p_2^*$. Substituting (5)-(8) into (1) and its foreign counterpart gives the levels of community welfare of the home and foreign countries respectively in terms of disposable income and the consumers' price indexes for differentiated goods in sectors 1 and 2:

$$U = AIP_1^{\alpha_1/(\varepsilon_1-1)} P_2^{\alpha_2/(\varepsilon_2-1)}, \qquad U^* = (I^*/I)U \tag{11}$$

where $A = \alpha^{\alpha_1}\alpha^{\alpha_2} > 0$.

The presentation of the model of two open economies is completed by introducing the equilibrium conditions for factor markets in each country. Let K and K^* denote the endowments of capital, and L and L^* those of labor in the home and foreign countries respectively. Then the full employment conditions for capital and labor in the home country are represented respectively by

$$\begin{aligned} n_1 G_R^1(w,r,x_1) + n_2 G_R^2(w,r,x_2) &= K \\ n_1 G_W^1(w,r,x_1) + n_2 G_W^2(w,r,x_2) &= L \end{aligned} \tag{12}$$

and those in the foreign country are represented by

$$\begin{aligned} n_1^* G_R^1(w^*,r^*,x_1^*) + n_2^* G_R^2(w^*,r^*,x_2^*) &= K^* \\ n_1^* G_W^1(w^*,r^*,x_1^*) + n_2^* G_W^2(w^*,r^*,x_2^*) &= L^*. \end{aligned} \tag{13}$$

One of the equilibrium conditions for factor markets in (12) and (13) is not independent of the other conditions of the model. Since no dividends are distributed to consumers by firms which earn zero profits the disposable incomes of the home and foreign countries can be represented respectively by

$$I = wL + rK - S, \qquad I^* = w^*L^* + r^*K^*. \tag{14}$$

The balance of trade condition for a country can be automatically derived using the equations (3), (10), and (12)-(14):

$$p_1^* n_1 c_{H1}^* + p_2 n_2 c_{H2}^* = p_1^* n_1^* c_{F1} + p_2^* n_2^* c_{F2}.$$

3. The Relation between Commodity and Factor Prices

The equilibrium conditions for firms in the two sectors, (2) and (3), are developed in this section on the assumption that the initial situation is a *laissez faire* economy to derive the relationship between commodity and factor prices under the trade policy intoduced in the previous section. For this purpose it is necessary in the model with economies of scale to consider the relation between firm sizes and factor prices as a preliminary step. Define the output elasticities of the demand for labor and capital in the jth sector respectively as

$$\mu_L^j = \frac{x_j G_{WX}^j(\circ)}{G_W^j(\bullet)} \qquad \mu_K^j = \frac{x_j G_{RX}^j(\bullet)}{G_R^j(\bullet)} \qquad j = 1,2$$

where $G_{WX}^j(\circ) = \partial G_W^j(\circ)/\partial x_j$ and $G_{RX}^j(\circ) = \partial G_R^j(\bullet)/\partial x_j$. Linear homogeneity and concavity of the cost function of the jth sector with respect to factor prices ensure that $\mu_L^j \geq 0$ and $\mu_K^j \geq 0$. These elasticities can be related to the degree of economies of scale by

$$\theta_j = \theta_{Lj}\mu_L^j + \theta_{Kj}\mu_K^j$$

where θ_{Lj} and θ_{Kj} denote the shares of labor and capital respectively in the total cost of differentiated goods in the jth sector. If the production function of this sector is homothetic, $\theta_j = \mu_K^j = \mu_L^j$. Otherwise, $\theta_j \neq \mu_K^j$ and $\theta_j \neq \mu_L^j$. Thus $\mu_K^j - \theta_j$ or $\mu_L^j - \theta_j$ is regarded as the measure of non-homotheticity for this sector's technology. If $\mu_K^j - \theta_j > 0$ its non-homothetic bias is, according to Horn (1983), said to be capital-using, and if $\mu_K^j - \theta_j < 0$ its non-homothetic bias is said to be labor-using.

Taking total and logarithmic differentiation of equations (2) and (3) and using the definition of the output elasticities gives respectively

$$\theta_{Lj}\mu_L^j\hat{w} + \theta_{Kj}\mu_K^j\hat{r} + \theta_j\tau_j\hat{x}_j = \theta_j\hat{q}_j \qquad j = 1,2 \tag{15}$$

and

$$\theta_{Lj}\hat{w} + \theta_{Kj}\hat{r} + (\theta_j - 1)\hat{x}_j = \hat{q}_j \qquad j = 1,2 \tag{16}$$

where a variable with ^ over its head denotes its proportional change. The relationship between a firm size in the jth sector and relative factor price can be obtained by eliminating $\hat{q}_j$ in (15) and (16):

$$\hat{x}_j = E_{Xj}(\hat{r} - \hat{w}) \qquad j = 1,2 \tag{17}$$

where E_{Xj} denotes the elasticity of a shift in the isoquant of a firm in the jth sector corresponding to a change in the relative price of capital:

$$E_{Xj} = \frac{r/w}{x_j}\frac{\partial x_j}{\partial r/w} = -\frac{\theta_{Kj}(\mu_K^j - \theta_j)}{\theta_j \eta_j} \qquad j = 1, 2.$$

If the jth sector's technology has a capital-using (or labor-using) non-homothetic bias a rise in the relative price of capital will shift inward (or outward) the isoquant of a firm in this sector. If its technology is homothetic there will be no shifts in the isoquant due to a relative factor-price change.

An expression for the relationship between factor and producers' commodity prices in terms of a proportional change can be derived by substituting (17) into (15):

$$\Theta_{Lj}\hat{w} + \Theta_{Kj}\hat{r} = \hat{q}_j \qquad j = 1, 2 \tag{18}$$

where Θ_{Lj} and Θ_{Kj} denote the comprehensive cost shares of labor and capital in the jth sector respectively:

$$\Theta_{Lj} = \theta_{Lj} + (1-\theta_j)E_{Xj} \qquad \Theta_{Kj} = \theta_{Kj} - (1-\theta_j)E_{Xj} \qquad j = 1, 2. \tag{19}$$

These cost shares reflect not only substitution effects represented by the first term on the right-hand sides of (19), the usual cost shares of labor and capital, but also output effects to shift the isoquant, represented by the second terms, of a change in the relative factor price. Obviously, they satisfy $\Theta_{Lj} + \Theta_{Kj} = 1$ $(j = 1, 2)$. If the jth sector's technology is homothetic the comprehensive cost share of a factor used in this sector coincides with its usual cost share. Otherwise one of the comprehensive cost shares of a sector might be negative. Denote a matrix of coefficients on the left-hand sides of (18) by Θ and its determinant by $|\Theta|$. Then $|\Theta| = \Theta_{K2} - \Theta_{K1} = |\theta| + (1-\theta_1)E_{X1} - (1-\theta_2)E_{X2}$, where $|\theta| = \theta_{K2} - \theta_{K1}$.

The relationship between factor rewards and producers' commodity prices can be obtained by solving (18) as follows:

$$\begin{aligned} \hat{w} &= -(\Theta_{K1}\hat{q}_2 - \Theta_{K2}\hat{q}_1)/|\Theta| \\ \hat{r} &= (\Theta_{L1}\hat{q}_2 - \Theta_{L2}\hat{q}_1)/|\Theta|. \end{aligned} \tag{20}$$

These results indicate how factor prices are influenced by a change in the relative producers' price which is caused by the production subsidy or tax implemented by the home government and that it depends on the signs of the comprehensive cost shares and the factor intensities of the two sectors. There are four types of possible relationships between factor and commodity prices. First, as in the model of external economies developed by Jones (1968), the magnification effect of the Stolper-Samuelson theorem,

$$\hat{r} > \hat{q}_j > \hat{q}_i > \hat{w} \quad or \quad \hat{w} > \hat{q}_i > \hat{q}_j > \hat{r} \qquad when \quad \Theta_{Kj} > \Theta_{Ki} \quad i \neq j, \quad i,j = 1,2 \tag{21}$$

holds provided that every comprehensive cost share is positive and a factor-intensity condition,

$$|\Theta||\theta| > 0 \tag{22}$$

is satisfied. In this case the coefficient matrix Θ is a probability matrix. Secondly, if the comprehensive cost shares of both labor and capital are positive in one of the two sectors but that of either labor or capital is not in the other sector under factor-intensity condition (22), then a rise in producers' commodity prices at different rates causes an alternation effect on factor rewards in the sense that the reward of a factor used intensively in a sector with the fastest rising price is increased at a greater rate than either commodity, and the reward of the other factor is raised at a rate less than the fastest but not less than a slower rising price:

$$\begin{aligned} &\hat{w} > \hat{q}_j > \hat{r} \geq \hat{q}_i \quad or \quad \hat{q}_i \geq \hat{r} > \hat{q}_j > \hat{w} \qquad when \quad \Theta_{Lj} > 0, \ \Theta_{Kj} > 0, \ but \ \Theta_{Li} \leq 0 \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad i \neq j, \quad i,j = 1,2 \\ &\hat{r} > \hat{q}_j > \hat{w} \geq \hat{q}_i \quad or \quad \hat{q}_i \geq \hat{w} > \hat{q}_j > \hat{r} \qquad when \quad \Theta_{Lj} > 0, \ \Theta_{Kj} > 0, \ but \ \Theta_{Ki} \leq 0. \end{aligned} \tag{23}$$

Thirdly, if both the comprehensive cost share of labor in one of the two sectors and that of capital in the other sector are not positive under factor-intensity condition (22), then a rise in producers' commodity prices at different rates increases the reward of a factor intensively used in a sector with the fastest rising price at a rate not greater than this price but greater than the reward of the other factor, which is also raised at a rate not less than a slower rising price:

$$\hat{q}_j \geq \hat{r} > \hat{w} \geq \hat{q}_i \quad or \quad \hat{q}_i \geq \hat{r} > \hat{w} \geq \hat{q}_j \quad when \quad \Theta_{Lj} \leq 0 \ and \ \Theta_{Kj} \leq 0 \quad i \neq j, \quad i,j = 1,2. \tag{24}$$

This relation could be called a shrinkage effect of a relative change in commodity prices on factor rewards. Fourthly, if the comprehensive cost shares of one of the two factors are negative in both sectors under factor-intensity condition (22), then a rise in producers' commodity prices at different rates increases the reward of a factor intensively used in a sector with the fastest rising price at a rate greater than that of the other factor, which is also raised at a rate not less than either commodity:

$$\begin{aligned} &\hat{r} > \hat{w} \geq \hat{q}_j > \hat{q}_i \quad or \quad \hat{q}_i > \hat{q}_j \geq \hat{w} > \hat{r} \quad when \quad 0 > \Theta_{Kj} > \Theta_{Ki} \\ & \hspace{20em} i \neq j, \quad i, j = 1, 2 \qquad (25) \\ &\hat{w} > \hat{r} \geq \hat{q}_j > \hat{q}_i \quad or \quad \hat{q}_i > \hat{q}_j \geq \hat{r} > \hat{w} \quad when \quad 0 > \Theta_{Lj} > \Theta_{Li}. \end{aligned}$$

This relation could be called a one-sided magnification effect of a relative change in commodity prices on factor rewards. These results can be summarized into the following proposition.

Proposition 1. *When* $|\Theta||\theta| > 0$ *a rise in producers' commodity prices at different rates causes on factor rewards*
(a) the magnification effect of the Stolper-Samuelson theorem, (21), provided that every comprehensive cost share is positive;
(b) the alternation effect, (23), provided that the comprehensive cost shares of both labor and capital are positive in one of the two sectors but that of either labor or capital is not in the other sector;
(c) the shrinkage effect, (24), provided that both the comprehensive cost share of labor in one of the two sectors and that of capital in the other sector are not positive; and
(d) the one-sided magnification effect, (25), provided that the comprehensive cost shares of one of the two factors are negative in both sectors.

If production functions in all sectors are homothetic the magnification effect of the Stolper-Samuelson theorem (Proposition 1(a)) holds but the other effects in the Proposition do not because each comprehensive cost share coincides with the corresponding usual cost share. If the production function in one of the two sectors is homothetic but that in the other sector is not the magnification effect and the alternation effect (Propositions 1(a) and 1(b)) hold but the other effects in the Proposition do not.

4. Resource Allocation Effects

The relation between factor rewards and producers' prices of differentiated goods which was clarified in the previous section is applicable to the home and foreign countries so that it is possible in this section to explore the effects on the size and number of firms in each sector of a production subsidy granted to or a production tax levied on a differentiated-goods sector by the home country. The exploration is proceeded on the assumption that in the world economy constituted by the two countries whose production and consumption structures were presented in section 2 an equilibrium is present, unique, and stable. If the home country does not implement any trade policy and leaves the economy *laissez-faire* together with its trade partner, then factor prices are equalized between them, as shown by Helpman (1981), and their economies are integrated into one. The existence, uniqueness, and stability of the equilibrium of this integrated world can be demonstrated by showing that the world excess demand for capital and that for labor are continuous and zeroth degree homogeneous functions in factor prices, and the former is a monotonically decreasing function in the rental-wage ratio. This fact seems to strengthen the plausibility of the basic assumption made above.

In order to simplify the following analysis, three additional assumptions are made: first, within the range of factor prices and firm sizes considered here, sector 2 is capital intensive relative to sector 1, i.e., $|\theta|>0$; secondly, firms in sector 1 have uniformly a homothetic production function while those in sector 2 uniformly have a production function whose non-homothetic bias is not labor-using , i.e., $E_{X1}=0$, $E_{X2}\leq 0$, and $|\Theta|>0$; thirdly, as assumed in the previous section, the initial situation is a *laissez faire* economy with the foreign labor as a numeraire. The last assumption implies that factor prices were initially equalized between the home and foreign countries, and this and the assumption of identical production and welfare functions between them imply that each parameter for the foreign country evaluated at the initial situation coincides with the counterpart for the home country.

The production subsidy or tax implemented by the home country indirecly influences factor and commodity prices and hence firm-sizes in the foreign country through the price mechanism. Substituting $\hat{w}^*=0$, $\hat{q}_1=\hat{p}_1^*$, and $\hat{q}_2=\hat{p}_2^*$ into (20) gives the relationships between those variables and the price of differentiated goods produced in foreign sector 2:

$$\hat{r}^* = \hat{p}_2^*/\Theta_{K2}, \qquad \hat{p}_1^* = (\theta_{K1}/\Theta_{K2})\hat{p}_2^*, \qquad \hat{x}_1^* = 0, \qquad \hat{x}_2^* = (E_{X2}/\Theta_{K2})\hat{p}_2^*. \tag{26}$$

Factor prices and firm-sizes of the two sectors in the home country, on the other hand, are directly affected by the policy. Substituting $\hat{q}_1 = \hat{p}_1^*$ and $\hat{q}_2 = \hat{p}_2 + ds$ into (20) gives a change in these variables:

$$\hat{r} - \hat{w} = \frac{1}{|\Theta|}\left(\hat{p}_2 + ds - \frac{\theta_{K1}\hat{p}_2^*}{\Theta_{K2}}\right), \qquad \hat{x}_1 = 0, \qquad \hat{x}_2 = \frac{E_{X2}}{|\Theta|}\left(\hat{p}_2 + ds - \frac{\theta_{K1}\hat{p}_2^*}{\Theta_{K2}}\right). \tag{27}$$

It should be noted that ds denotes the rate of subsidy granted if it is positive and the rate of tax levied if it is negative.

As was expected in section 2, the consumers' price of differentiated goods produced in home sector 2 is increased to a higher level by the subsidy or decreased to a lower level by the tax than p_2^*. This result can be derived by using a differentiated form of the second equation of supply-demand identity, (10), into which (26) and (27) are substituted and recalling the definition of the demand elasticity, $\varepsilon_j = 1/(1-\theta_j)$:

$$\frac{\hat{p}_2 - \hat{p}_2^*}{ds} = -\frac{(1-\theta_2)E_{X2}}{|\theta|} \geq 0. \tag{28}$$

Since p_2 is the price of exportable and p_2^* is that of importable differentiated goods in sector 2 for the home country this result means that its terms of trade measured with products in sector 2 is improved by the subsidy or worsened by the tax. It is important to obtain the effect of the policy on p_2^*, a core effect, because all the resource allocation and welfare effects of the policy can be related to this effect in a functional form. The first step to this task is to get the relationship of consumers' price index ratio between sectors 1 and 2 to p_2^* using differentiated forms of the rest of (10):

$$\hat{I}^W - \hat{P}_1 = \frac{\theta_{K1}}{1-\theta_1}\frac{\hat{p}_2^*}{\Theta_{K2}}, \qquad \hat{I}^W - \hat{P}_2 = \frac{\theta_{K2}}{1-\theta_2}\frac{\hat{p}_2^*}{\Theta_{K2}}. \tag{29}$$

Subtracting the first from the second equation in (29) gives:

$$\hat{P}_1 - \hat{P}_2 = \left(\frac{\theta_{K2}}{1-\theta_2} - \frac{\theta_{K1}}{1-\theta_1}\right)\frac{\hat{p}_2^*}{\Theta_{K2}}, \tag{30}$$

which shows that the price index ratio is only indirectly affected by the policy. Substituting (28) into (27) rewrites the effect of the policy on the rental-wage ratio of the home country in terms of p_2^*:

$$\hat{r} - \hat{w} = \frac{\hat{p}_2^*}{\Theta_{K2}} + \frac{ds}{|\theta|}. \tag{31}$$

Another element to determine the core effect of the production subsidy or tax is the definition of the consumers' price index for each sector. In order to use it effectively it is necessary to know in advance the effects of the policy on the numbers of varieties of differentiated goods in both countries. As will be shown below, they are directly affected by it in the home country but only indirectly affected in the foreign country. These effects for the home country can be derived from differentiated forms of its full employment conditions for capital and labor in (12):

$$\begin{aligned} \theta_{K1} S_1 \hat{n}_1 + \theta_{K2} S_2 \hat{n}_2 &= E_{KK} \left(\frac{\hat{p}_2^*}{\Theta_{K2}} + \frac{ds}{|\theta|} \right) \\ \theta_{L1} S_1 \hat{n}_1 + \theta_{L2} S_2 \hat{n}_2 &= -E_{LK} \left(\frac{\hat{p}_2^*}{\Theta_{K2}} + \frac{ds}{|\theta|} \right) \end{aligned} \tag{32}$$

where S_j denotes the share of the output of sector j in the total output of the home country under the initial *laissez faire* situation, $S_j = n_j G^j / I$, E_{KK} and E_{LK} denote the elasticities of derived demands for capital and labor with respect to the rental-wage ratio for the home country at the initial situation:

$$E_{KK} = -\frac{r/w}{\sum_{j=1}^{2} n_j G_R^j} \frac{d\left(\sum_{j=1}^{2} n_j G_R^j\right)}{d(r/w)} = \sum_{j=1}^{2} \theta_{Kj} S_j (\sigma_j \theta_{Lj} - \mu_K^j E_{Xj}) > 0,$$

$$E_{LK} = \frac{r/w}{\sum_{j=1}^{2} n_j G_W^j} \frac{d\left(\sum_{j=1}^{2} n_j G_W^j\right)}{d(r/w)} = \sum_{j=1}^{2} \theta_{Lj} S_j (\sigma_j \theta_{Kj} + \mu_L^j E_{Xj}),$$

and σ_j denotes the elasticity of substitution between capital and labor in the jth sector at a given level of output:

$$\sigma_j = \frac{r/w}{G_W^j / G_R^j} \frac{\partial\left(G_W^j / G_R^j\right)}{\partial(r/w)} = \frac{G^j(\circ) G_{RW}^j(\circ)}{G_R^j(\circ) G_W^j(\circ)} > 0 \qquad j = 1,2.$$

Let

$$\pi_j = \theta_{Kj}\theta_{Lj}\left(\sigma_j - \frac{(\mu_K^j - \theta_j)E_{Xj}}{\theta_{Lj}}\right) > 0 \qquad j = 1, 2,$$

then the weighted sum of π_j over the two sectors with S_j as a weight equals that of the elasticities of factor demands with the distributive shares in sector 2 as a weight:

$$\sum_{j=1}^{2} S_j\pi_j = \theta_{L2}E_{KK} + \theta_{K2}E_{LK}.$$

Solving (32) in terms of these notations gives the policy effect on the numbers of firms in home sectors 1 and 2:

$$\hat{n}_1 = -\frac{\sum_{j=1}^{2} S_j\pi_j}{S_1|\theta|}\left(\frac{\hat{p}_2^*}{\Theta_{K2}} + \frac{ds}{|\theta|}\right)$$

$$\hat{n}_2 = \left(-\theta_2 E_{X2} + \frac{\sum_{j=1}^{2} S_j\pi_j}{S_2|\theta|}\right)\left(\frac{\hat{p}_2^*}{\Theta_{K2}} + \frac{ds}{|\theta|}\right). \tag{33}$$

Solving similarly differentiated forms of the foreign country's full employment conditions for capital and labor into which (26) is substituted gives the indirect effect of the policy on the numbers of firms in foreign sectors 1 and 2:

$$\hat{n}_1^* = -\frac{\sum_{j=1}^{2} S_j^*\pi_j}{S_1^*|\theta|}\frac{\hat{p}_2^*}{\Theta_{K2}}$$

$$\hat{n}_2^* = \left(-\theta_2 E_{X2} + \frac{\sum_{j=1}^{2} S_j^*\pi_j}{S_2^*|\theta|}\right)\frac{\hat{p}_2^*}{\Theta_{K2}} \tag{34}$$

where S_j^* denotes the share of the output of sector j in the total output of the foreign country, which is not necessarily equal to the counterpart of the home country even at the *laissez faire* situation. The effects of the production subsidy or tax on the total number of varieties in each sector can be derived by substituting (33) and (34) into a differentiated form of its definition:

$$\hat{N}_1 = -\frac{\sum_{j=1}^{2}\alpha_j\pi_j}{\alpha_1|\theta|}\frac{\hat{p}_2^*}{\Theta_{K2}} - \frac{S_H\sum_{j=1}^{2}S_j\pi_j}{\alpha_1}\frac{ds}{|\theta|^2}$$

$$\hat{N}_2 = \left(-\theta_2 E_{X2} + \frac{\sum_{j=1}^{2}\alpha_j\pi_j}{\alpha_2|\theta|}\right)\frac{\hat{p}_2^*}{\Theta_{K2}} + \left(-\frac{n_2\theta_2 E_{X2}}{N_2} + \frac{S_H\sum_{j=1}^{2}S_j\pi_j}{\alpha_2|\theta|}\right)\frac{ds}{|\theta|} \tag{35}$$

where a use is made of the relations, $S_j = \alpha_j n_j/(S_H N_j)$ and $S_j^* = \alpha_j n_j^*/(S_F N_j)$ for $j = 1, 2$, and S_H and S_F denote the shares of the home and foreign national incomes in the world income respectively. These results show that the subsidy (tax) increases (decreases) the total number of varieties in sector 2 but decreases (increases) the one in the other sector at the initial price level.

It is now possible to get the effects of the policy on the ratio of consumers' price index for differentiated products between sectors 1 and 2 in a different form than (30) by substituting (35) into differentiated forms of the definitions:

$$\hat{P}_j = \left(-\frac{\theta_j\theta_{Kj}}{1-\theta_j} + (-1)^j\frac{\sum_{i=1}^{2}\alpha_i\pi_i}{\alpha_j|\theta|}\right)\frac{\hat{p}_2^*}{\Theta_{K2}} + (-1)^j\frac{S_H\sum_{i=1}^{2}S_i\pi_i}{\alpha_j}\frac{ds}{|\theta|^2} \qquad j = 1, 2. \tag{36}$$

This shows that the subsidy (tax) increases (decreases) the price index of sector 2 but decreases (increases) that of the other sector at the initial price level. Subtracting the second from the first equation in (36) gives:

$$\hat{P}_1 - \hat{P}_2 = \left(\frac{\theta_2\theta_{K2}}{1-\theta_2} - \frac{\theta_1\theta_{K1}}{1-\theta_1} - \frac{\sum_{j=1}^{2}\alpha_j\pi_j}{\alpha_1\alpha_2|\theta|}\right)\frac{\hat{p}_2^*}{\Theta_{K2}} - \frac{S_H\sum_{j=1}^{2}S_j\pi_j}{\alpha_1\alpha_2}\frac{ds}{|\theta|^2}. \tag{37}$$

Equating (30) and (37) gives the effect of the policy on p_2^*:

$$\frac{1}{\Theta_{K2}}\frac{\hat{p}_2^*}{ds} = -\frac{S_H \sum_{j=1}^{2} S_j \pi_j}{\Omega|\theta|} < 0 \tag{38}$$

where

$$\Omega = \alpha_1 \alpha_2 |\theta|^2 + \sum_{j=1}^{2} \alpha_j \pi_j > 0.$$

The reasons why this result occurs are intuitively explained as what follows. At the initial level of p_2^* the production subsidy to its sector 2 by the home country, for instance, decreases the home and world numbers of varieties in sector 1 and increases the counterparts in sector 2 according to (33)-(35), and hence lowers the consumers' price index of sector 1 and raises that of sector 2 according to (36). These changes in the price indexes in turn increases the demand for differentiated goods produced in home and foreign sectors 1 and decreases that for differentiated goods produced in foreign sector 2 by virtue of (5) and (8). Since the output of each firm in sector 1 is kept constant, the world income must be shrinked to maintain the market equilibrium for this sector. This reduction in the income decreases even more the demand for differentiated goods in foreign sector 2 despite its supply kept at the initial level. Therefore, p_2^* must be lowered to maintain the market equilibrium for this sector. The effects of the production tax on p_2^* can be similarly explained.

The resource allocation effects of the production subsidy or tax are now derivable by bringing the result in (38) back to the equation system developed in this section. Substituting it into (31) gives:

$$\frac{\hat{r} - \hat{w}}{ds} = \frac{1}{\Theta_{K2}}\frac{\hat{p}_2^*}{ds} + \frac{1}{|\theta|}$$

$$= \frac{\alpha_1 \alpha_2 |\theta|^2 + S_F \sum_{j=1}^{2} S_j^* \pi_j}{\Omega|\theta|} > 0. \tag{39}$$

The policy effects on the size of firms in home sector 2 and the numbers of firms in home sectors 1 and 2 can be derived by substituting (39) into (17) and (31):

$$\hat{x}_2/ds \leq 0 \qquad \hat{n}_1/ds < 0 \qquad \hat{n}_2/ds > 0.$$

Substitution of (38) into (26) and (34) gives the effects of the policy on the size of firms in foreign sector 2 and the numbers of firms in foreign sectors 1 and 2:

$$\hat{x}_2^{*}/ds \geq 0 \qquad \hat{n}_1^{*}/ds > 0 \qquad \hat{n}_2^{*}/ds < 0.$$

Similarly, the policy effect on the total number of firms in sector 1 is unambiguously obtained:

$$\frac{\hat{N}_1}{ds} = -\frac{\alpha_2 S_H \sum_{j=1}^{2} S_j \pi_j}{\Omega} < 0.$$

These results can be summarized into the following proposition:

Proposition 2. *If firms in a labor-intensive sector of differentiated goods have uniformly a homothetic production function and firms in a capital-intensive sector of differentiated goods uniformly have a production function whose non-homothetic bias is not labor-using and if the home country grants a production subsidy to or levies a production tax on its capital-intensive sector, then the subsidy (tax) increases (decreases) the numbers of firms both in the subsidized (taxed) sector and in the labor-intensive sector of the partner country, decreases (increases) the numbers of firms both in the labor-intensive sectors of the home country as well as the world and in the capital-intensive sector of the partner country, does not increase (decrease) the size of firms in the subsidized (taxed) sector, and does not decrease (increase) that in the capital-intensive sector of the partner country.*

The effects of the policy on income distribution in the policy-implementing country are also obtainable by utilizing Proposition 1, to which the production conditions postulated specifically in this section and the change in producers' prices in sectors 1 and 2 induced by the policy,

$$\frac{\hat{q}_2 - \hat{q}_1}{ds} = \frac{\left(\alpha_1 \alpha_2 |\theta|^2 + S_F \sum_{j=1}^{2} S_j^{*} \pi_j)\right) |\Theta|}{\Omega |\theta|} > 0,$$

are applied. Since $\Theta_{K1} = \theta_{K1} > 0$, $\Theta_{L1} = \theta_{L1} > 0$, and $\Theta_{K2} > 0$ by the assumption, only Propositions 1(a) and 1(b) are relevant here. Therefore, in the case where $\Theta_{L2} > 0$, the magnification of the Stolper-Samuelson theorem holds. Namely, the production subsidy causes a ranking, $\hat{r} > \hat{q}_2 > \hat{q}_1 > \hat{w}$, while the production tax causes a ranking, $\hat{w} > \hat{q}_1 > \hat{q}_2 > \hat{r}$. In the case where $\Theta_{L2} < 0$, the alternation effect holds. Namely, the production subsidy causes a ranking, $\hat{q}_2 \geq \hat{r} > \hat{q}_1 > \hat{w}$ while the produciton tax causes a ranking, $\hat{w} > \hat{q}_1 > \hat{r} \geq \hat{q}_2$.

5. Welfare Effects

This section considers the effect on an individual country's community welfare of the production subsidy granted to or the production tax levied on firms in home sector 2 and then shows that international trade controlled by the policy can raise the level of world welfare, defined as the sum of welfare levels of the two countries, to a higher level than under free trade. The level of a country's welfare depends upon its disposable income and the consumers' price indexes of differentiated goods in sectors 1 and 2, as shown in (11), and these variables are influenced not only directly but also indirectly through the induced change in p_2^* shown in (38). Thus the results obtained in the previous sections will enable one to get the welfare effects of the policy.

The policy effect on the consumers' price index of differentiated goods in the ith sector is derivable by substituting (38) into (36):

$$\frac{\hat{P}_i}{ds} = \left(\frac{\theta_i \theta_{Ki}}{1-\theta_i} + (-1)^i \alpha_h |\theta|\right) \frac{S_H \sum_{j=1}^{2} S_j \pi_j}{\Omega |\theta|} \qquad i \neq h, \qquad i,h = 1,2. \tag{40}$$

Accordingly, the weighted sum of (40) is:

$$\frac{\alpha_1(1-\theta_1)}{\theta_1}\frac{\hat{P}_1}{ds} + \frac{\alpha_2(1-\theta_2)}{\theta_2}\frac{\hat{P}_2}{ds} = \left(\frac{\alpha_1\alpha_2(\theta_1-\theta_2)|\theta|}{\theta_1\theta_2} + S_K^W\right)\frac{S_H \sum_{j=1}^{2} S_j \pi_j}{\Omega|\theta|} > 0$$

provided that $\theta_1 \geq \theta_2$ (41)

where the share of total capital rewards in the world disposable income, $S_K^W = rK^W / I^W = \alpha_1\theta_{K1} + \alpha_2\theta_{K2}$. The result in (41) shows that if the degree of scale economies in the sector subject to the policy, $1/\theta_2$, is greater than or equal to the one in the other sector, $1/\theta_1$, the subsidy increases or the tax decreases the product of the consumers' price indexes, $P_1^{\alpha_1/(\varepsilon_1-1)} P_2^{\alpha_2/(\varepsilon_2-1)}$, which is an increasing function in $N_1, n_2,$ and n_2^* and a decreasing function in $p_1^*, p_2,$ and p_2^* A proportional change in the foreign disposable income induced by the policy of the home country can be derived by substituting (38) into the differentiated form of its definition (14):

$$\frac{\hat{I}^*}{ds} = -\frac{S_K^* S_H \sum_{j=1}^{2} S_j \pi_j}{\Omega|\theta|} < 0 \tag{42}$$

where S_K^* denotes the share of foreign capital rewards in its disposable income. The result in (42) shows that the subsidy reduces the foreign disposable income because of its deteriorating effect and the tax increases it because of its improving effect on the foreign terms of trade measured by p_2^*. Substituting (40) into (29) gives a relationship between proportional changes in the world and foreign disposable incomes:

$$\frac{\hat{I}^W}{ds} = \frac{S_K^W}{S_K^*}\frac{\hat{I}^*}{ds} < 0. \tag{43}$$

A proportional change in the home disposable income can be obtained from (42) and (43):

$$\frac{\hat{I}}{ds} = \frac{S_K}{S_K^*}\frac{\hat{I}^*}{ds} < 0 \tag{44}$$

where S_K is the counterpart of S_K^* for the home country.

It follows from the results from (41) to (44) that the subsidy or tax influences the level of community welfare in each country negatively and positively at the same time because the former (the latter) decreases (increases) its disposable income but raises (lowers) the weighted sum of the consumers' price indexes provided that the degree of scale economies is greater in the sector subject to the policy. In order to know the net effect of the policy on an individual country's welfare, it is necessary to derive a full expression for it using these results. For the home country it is:

$$\frac{\hat{U}}{ds} = \left(\frac{\alpha_1\alpha_2(\theta_1 - \theta_2)|\theta|}{\theta_1\theta_2} + \frac{r(k^W - k)}{(1+rk^W)(1+rk)}\right)\frac{S_H \sum_{j=1}^{2} S_j \pi_j}{\Omega|\theta|} \tag{45}$$

and for the partner country it is:

$$\frac{\hat{U}^*}{ds} = \left(\frac{\alpha_1 \alpha_2 (\theta_1 - \theta_2)|\theta|}{\theta_1 \theta_2} + \frac{r(k^W - k^*)}{(1 + rk^W)(1 + rk^*)} \right) \frac{S_H \sum_{j=1}^{2} S_j \pi_j}{\Omega |\theta|} \tag{46}$$

where k, k^*, and k^W denote the capital-labor endowment ratios of the home country, partner country, and world, respectively. If the first and second terms on the RHS's of (45) and (46) are nonnegative (nonpositive) and at least one of them is positive (negative) in a country the subsidy (tax) can increase its welfare. The sufficient conditions for the home and partner countries to improve the welfare are obtainable from (45) and (46):

$\frac{\hat{U}}{ds} > 0$ provided that $\theta_2 \le \theta_1$ and $k \le k^W$ with at least one inequality held;
$\frac{\hat{U}}{ds} < 0$ provided that $\theta_2 \ge \theta_1$ and $k \ge k^W$ with at least one inequality held;
$\frac{\hat{U}^*}{ds} > 0$ provided that $\theta_2 \le \theta_1$ and $k^* \le k^W$ with at least one inequality held;
$\frac{\hat{U}^*}{ds} < 0$ provided that $\theta_2 \ge \theta_1$ and $k^* \ge k^W$ with at least one inequality held.

Since $k <(>) k^W$ implies $k <(>) k^*$ which means that the home country is abundant in labor (capital) relative to the foreign country, combining these results together gives the following conclusions:

Proposition 3. (a) *If a country grants a production subsidy to its differentiated-goods sector with a degree of scale economies not smaller than the other sector, and the subsidized sector is capital intensive and has a production function whose non-homothetic bias is not labor-using, then the subsidy increases the community welfare of a labor-abundant country regardless of whether it is a policy-making country.*
(b) *If a country levies a production tax on its differentiated-goods sector with a degree of scale economies not greater than the other sector, and the taxed sector is capital intensive and has a production function whose non-homothetic bias is not labor-using, then the tax increases the community welfare of a capital-abundant country regardless of whether it is a policy-making country.*

If the first two conditions in the symplifying assumptions made in the previous section are altered to: first, within the range of factor prices and firm-sizes considered here sector 2 is labor intensive relative to sector 1, i.e., $|\theta|<0$; and secondly, firms in sector 1 have uniformly a homothetic production function while those in sector 2 have uniformly a production function whose non-homothetic bias is not capital-using, i.e., $E_{X1}=0$, $E_{X2}\geq 0$, then both $\pi_j>0$ for $j=1,2$ and $\Omega>0$ invariably hold, and thus conclusions similar to those in Proposition 3 hold for a production subsidy granted to or a production tax levied on sector 2:

Proposition 4. (a) *If a country grants a production subsidy to its differentiated-goods sector with a degree of scale economies not smaller than the other sector, and the subsidized sector is labor intensive and has a production function whose non-homothetic bias is not capital-using, then the subsidy increases the community welfare of a capital-abundant country regardless of whether it is a policy-making country.*
(b) *If a country levies a production tax on its differentiated-goods sector with a degree of scale economies not greater than the other sector, and the taxed sector is labor intensive and has a production function whose non-homothetic bias is not capital-using, then the tax increases the community welfare of a labor-abundant country regardless of whether it is a policy-making country.*

Considering Propositions 3 and 4 together tells a country a proper choice of trade policy between a production subsidy and tax to raise the level of its welfare to a higher level than under free trade. For a capital-abundant country it is either a production subsidy to a sector with a greater degree of economies of scale provided that it is labor intensive and has a production function whose non-homothetic bias is not capital-using or a production tax on a sector with a smaller degree of economies of scale provided that it is capital intensive and has a production function whose non-homothetic bias is not labor-using. For a labor-abundant country the proper choice is either a production subsidy to a sector with a greater degree of economies of scale provided that it is capital intensive and has a production function whose non-homothetic bias is not labor-using or a production tax on a sector with a smaller degree of economies of scale provided that it is labor intensive and has a production function whose non-homothetic bias is not capital-using.

The implications of Proposition 3 or 4 concerning a production subsidy are that if the home country is relatively abundant in labor (capital) and grants a production subsidy to its differentiated-goods sector which satisfies the technological conditions presented in Proposition 3(a) (4(a)), then this policy certainly makes its nation better off but it is not certain whether it makes the partner nation better off. Those concerning a production tax are that if the home country is relatively abundant in capital (labor) and levies a production tax on its differentiated-goods sector which satisfies the technological conditions presented in Proposition 3(b) (4(b)), then this policy certainly makes its nation better off but it is not certain whether it makes the partner nation better off. Thus, in order for this policy to be accepted by the partner country, it turns out to be necessary to demonstrate that it can at least make the two nations as a whole better off.

To answer this question, define the level of world welfare, U^W, as the sum of welfare levels of the two countries: $U^W = U + U^*$. Then the policy effect on it is represented by:

$$\frac{\hat{U}^W}{ds} = S_H \frac{\hat{U}}{ds} + S_F \frac{\hat{U}^*}{ds}$$

$$= \frac{\alpha_1 \alpha_2 (\theta_1 - \theta_2) S_H \sum_{j=1}^{2} S_j \pi_j}{\theta_1 \theta_2 \Omega} \geq (\leq) 0 \qquad \text{according as } \theta_1 \geq (\leq) \theta_2. \qquad (47)$$

This result shows that in determining the level of world welfare the first terms on the RHS's of (45) and (46) remain valid but the second terms are cancelled out. Thus it can be concluded that

Proposition 5. *If the home country grants a production subsidy to firms in the sector with a greater degree of economies of scale or levies a production tax on firms in the sector with a smaller degree of economies of scale, then the policy increases the world welfare to a level higher than that under free trade.*

This proposition shows that free trade is not efficient in the presence of economies of scale in firm to produce differentiated goods and that the home country can make its own nation better off without the partner nation made worse off by a production subsidy or tax and some proper income compensation program for the partner country.

6. A Special Case

Although the model used in the previous sections is symmetric in the sense that firms in every sector produce a variety of differentiated goods with a uniform production function exhibiting increase returns to scale in a monopolistically competitive market, the models used in discussing the issues concerning intra-industry trade have been usually asymmetric in the sense that firms in one of two sectors produce a homogeneous good with an identical production function exhibiting constant returns to scale in a perfectly competitive market and firms in the other sector produce a variety of differentiated goods with an identical production function exhibiting increasing returns to scale in a monopolistically competitive market. The models used by the papers cited in the previous sections, for instance, Dixit-Norman (1980), Helpman (1981), Horn (1983), and Wong (1995), are all included in this group. The model with two asymmetric sectors can be viewed as a special case of the model with two symmetric sectors, as will be shown below, so that it will be interesting to consider the relations between them and examine how the conclusions derived in the previous sections are altered in the special case.

In the model introduced here, three conditions exist and make it a special case of the model with symmetric sectors. The first condition is that the products of home and foreign sectors 1 are a homogeneous good. This is equivalent to saying that $N_1 = 1$. Consequently, the demand functions for the homogeneous good produced in the home and foreign countries of the consumers in both countries, (5) and (7), are respectively altered to:

$$c_{H1} = c_{F1} = \alpha_1 I / p_1^*, \qquad c_{F1}^* = c_{H1}^* = \alpha_1 I^* / p_1^*. \tag{48}$$

The market-clearing condition for the homogeneous good, the first equation in (10), is altered to:

$$\alpha_1 I^W / p_1^* = x_1 + x_1^*. \tag{49}$$

The levels of community welfare of the home and foreign countries, (11), are respectively reduced to:

$$U = AIp_1^{*-\alpha_1} P_2^{\alpha_2/(\varepsilon_2 - 1)}, \qquad U^* = (I^*/I)U. \tag{50}$$

The second condition to make the model special is that firms in sector 1 have uniformly a production function which exhibits constant returns to scale. As a result, the cost function of each firm in home sector 1 can be represented by $\tilde{G}^1(w,r)x_1$. Since the marginal cost equals the average cost, the output elasticity in sector 1 satisfies:

$$\theta_1 = 1.$$

Since the equilibrium amounts of labor and capital required to produce x_1 are represented respectively by $x_1\tilde{G}^1_W(w,r)$ and $x_1\tilde{G}^1_R(w,r)$, the full employment conditions for capital and labor in the home country are altered to:

$$\begin{aligned} &x_1\tilde{G}^1_R(w,r) + n_2G^2_R(w,r,x_2) = K \\ &x_1\tilde{G}^1_W(w,r) + n_2G^2_W(w,r,x_2) = L. \end{aligned} \tag{51}$$

The full employment conditions in the foreign country, (13), also must be altered in the same way.

The third special condition is that perfect competition prevails in the market for the homogeneous good. Then firms in sector 1 are price takers so that the profit maximization condition for firms in home sector 1, the first equation in (2), becomes:

$$\tilde{G}^1(w,r) = q_1 \tag{52}$$

which is equivalent to the zero-profit condition for them. Thus the size of firms in this sector, x_1, cannot be determined in the market. The same things are true in the foreign country.

The consumption, production, and market-clearing conditions for differentiated goods produced in home and foreign sectors 2 as well as the definition of national income of each country remain the same as the ones in the model with symmetric sectors. Since $G^1_{WX}(\bullet) = \tilde{G}^1_W(\bullet)$ and $G^1_{RX}(\bullet) = \tilde{G}^1_R(\bullet)$, the output elasticities of the demand for labor and capital in sector 1 are reduced to:

$$\mu^1_L = \mu^1_K = 1.$$

This immediately leads to zero elasticity of a shift in the isoquant of a firm in sector 1 corresponding to a change in r/w ($E_{X1} = 0$), and the comprehensive cost shares of labor and capital in sector 1 coinciding with their respective usual shares ($\Theta_{L1} = \theta_{L1}$, $\Theta_{K1} = \theta_{K1}$). It follows from these facts that Propositions 1(a) and 1(b) carry over in the model with asymmetric sectors:

Proposition 1'. *When $|\beta||\theta| > 0$ in the model with asymmetric sectors a rise in producers' commodity prices at different rates causes on factor rewards*
(a) *the magnification effect of the Stolper-Samuelson theorem, (21), provided that every comprehensive cost share is positive; and*
(b) *the alternation effect, (23), provided that the comprehensive cost share of either labor or capital is not positive in the differentiated-goods sector.*

Since $\pi_1 = \theta_{K1}\theta_{L1}\sigma_1$ in the model with asymmetric sectors, π_1 is still positive in expressions from (33) to (39) for this case. Therefore, Proposition 2 holds for the sizes and numbers of firms in home and foreign sectors 2:

Proposition 2'. *If in the model with asymmetric sectors the differentiated-goods sector is capital intensive relative to the other sector and firms in the former sector uniformly have a production function whose non-homothetic bias is not labor-using and if the home country grants a production subsidy to or levies a production tax on the differentiated-goods sector, then the subsidy (tax) increases (decreases) the number of firms in the subsidized sector, decreases (increases) the number of firms in the differentiated-goods sector of the partner country, does not increase (decrease) the size of firms in the subsidized sector, and does not decrease (increase) that in the differentiated-goods sector of the partner country.*

Since $\theta_1 - \theta_2 = 1 - \theta_2 > 0$ in expressions from (45) to (47) in the model with asymmetric sectors, it can be obtained that:

when the subsidized sector is capital intensive,
$\hat{U}/ds > 0$ provided that $k \leq k^W$,
$\hat{U}^* / ds > 0$ provided that $k^* \leq k^W$;
when the subsidized sector is labor intensive
$\hat{U}/ds > 0$ provided that $k \geq k^W$,
$\hat{U}^* / ds > 0$ provided that $k^* \geq k^W$; and
Regardless of whether the subsidized sector is capital intensive or labor intensive
$\hat{U}^W / ds > 0$.

Therefore, it can be concluded that Propositions from 3 to 5 hold only for the case of a production subsidy:

Proposition 3'-4'. *If a country grants a production subsidy to its differentiated-goods sector in the model with asymmetric sectors, and the subsidized sector is capital intensive (labor intensive) and has a production function whose non-homothetic bias is not labor-using (capital-using), then the subsidy increases the community welfare of a labor-abundant (capital-abundant) country regardless of whether it is a policy-making country.*

Proposition 5'. *If the home country grants a production subsidy to firms of the differentiated-goods sector in the model with asymmetric sectors, the subsidy raises the level of world welfare to a higher level than under free trade.*

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