

# DISCUSSION PAPER SERIES

Discussion paper No.137

## Mixed Oligopoly and Privatization in General Equilibrium

**Kenji Fujiwara**

School of Economics, Kwansai Gakuin University

December, 2015



SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho  
Nishinomiya 662-8501, Japan

# Mixed Oligopoly and Privatization in General Equilibrium

Kenji Fujiwara\*

School of Economics, Kwansai Gakuin University

December 8, 2015

## Abstract

Making use of a general oligopolistic equilibrium model with private and public firms, this paper examines the welfare effects of privatization. We show that in an exogenous market structure privatizing the public firm necessarily reduces welfare, which contrasts with the existing result that some degree of privatization is optimal. In contrast, we find that privatization has no effect on welfare in an endogenous market structure with free entry of private firms.

*Keywords:* Partial privatization, General oligopolistic equilibrium, Exogenous market structure, Endogenous market structure.

*JEL Classifications:* L13, L32, L33.

---

\*School of Economics, Kwansai Gakuin University. Uegahara 1-1-155, Nishinomiya, Hyogo, 662-8501, Japan. Tel: +81-798-54-7066. Fax: +81-798-51-0944. E-mail: kenjifujiwara@kwansai.ac.jp.

# 1 Introduction

Privatization of public or state-owned firms has been as controversial as the other liberalization policies, e.g. competition and trade policies.<sup>1</sup> Reason Foundation (2015) reports the latest cases of successful privatization in the United States in 2014.<sup>2</sup> Moreover, 'Many emerging economies have launched ambitious efforts to privatize their infrastructure industries' (Jiang et al., 2015, p. 294) in order to provide multinational enterprises with investment opportunities.

These facts motivate a large literature on the effects of privatization mainly in the context of a mixed oligopoly.<sup>3</sup> This literature begins with de Fraja and Delbono (1989) who show that moving from full nationalization to full privatization improves welfare. By allowing for the intermediate case between full nationalization and full privatization, Matsumura (1998) finds that partial privatization is optimal (welfare-maximizing). In addition to the effects of privatization alone, White (1996) examines the interplay between privatization and subsidization, demonstrating that privatization has no effect on the optimal production subsidy when the government subsidizes both the state-owned and private firms.<sup>4</sup>

This strand of literature gives rise to a number of useful implications, but they rest on a partial equilibrium model. The purpose of this paper is to examine the welfare effects of privatization by taking into account the general equilibrium effects through the factor market. To this end, we combine Matsumura's (1998) approach with the general oligopolistic equilibrium (GOLE) model of Neary (2003, 2009).<sup>5</sup> We establish two results both of which are comparable to the existing findings. First, privatization necessarily reduces welfare in an exogenous market structure with fixed number of private firms.

---

<sup>1</sup>In this paper, we interchangeably use the terminologies 'public firm' and 'state-owned firm.'

<sup>2</sup>According to this report, 'The U.S. Department of Agriculture announced a plan to privatize poultry inspection in 2014,' and 'The General Services Administration announced in January it is closing its warehouses ... and will no longer buy, ship or store office supplies, tools and other common-use retail items in favor of accepting agency orders.'

<sup>3</sup>For an extensive literature survey, see Matsumura and Shimizu (2010) and Matsumura and Tomaru (2012). Cato and Matsumura (2015) provide a further review by paying special attention to the open economy case.

<sup>4</sup>This 'irrelevance result' or 'Privatization Neutrality Theorem' has been challenged by many works, which are surveyed in Matsumura and Okumura (2013) in detail.

<sup>5</sup>Colacicco (2015) provides a comprehensive survey on the theory and applications of the GOLE model. Beladi et al. (2013) develop a mixed GOLE model, but their focus is on the incentive of cross-border mergers in the presence of a public firm.

Second, privatization has no effect on welfare in an endogenous market structure with free entry among private firms.

This paper is organized as follows. Section 2 presents a basic model of a mixed oligopoly with an exogenous market structure. Then, Section 3 examines the effects of privatization on national income and welfare. Section 4 turns to the case of endogenous market structure. Section 5 concludes.

## 2 Model with Exogenous Market Structure

Suppose a continuum of industries in a closed interval  $[0, 1]$ , each consisting of one semi-public firm and  $n \geq 1$  private firms. Letting  $c_i$  be consumption of good  $i \in [0, 1]$ , the representative consumer solves the following utility maximization problem.<sup>6</sup>

$$\max_{c_i} \int_0^1 \left( ac_i - \frac{c_i^2}{2} \right) di \quad \text{subject to} \quad \int_0^1 p_i c_i di = I,$$

where  $I$  is national income. Then, the first-order condition yields  $a - c_i = \lambda p_i$ , where  $\lambda$  is the Lagrangean multiplier and stands for marginal utility of income. Following Neary (2003, 2009), we assume that oligopolistic firms have market power in their product market, but do not in the whole economy. And, we can set  $\lambda$  to unity by taking utility as numeraire. Then, indirect utility or welfare  $W$  has a simple form:

$$W = \frac{a^2 - \sigma_p^2}{2}, \quad \text{where} \quad \sigma_p^2 \equiv \int_0^1 p_i^2 di. \quad (1)$$

We now define the profit of each firm. In industry  $i$ , firms use labor under the fixed labor coefficient  $\alpha_i$ , which gives the following definitions of profits:

$$\pi_{i0} \equiv p_i x_{i0} - w \alpha_i x_{i0}$$

$$\pi_{ij} \equiv p_i x_{ij} - w \alpha_i x_{ij},$$

where the inverse demand function is  $p_i = a - x_{i0} - \sum_{j=1}^n x_{ij}$ ,  $w$  is a wage rate, and  $x_{i0}$  and  $x_{ij}$ ,  $j = 1, \dots, n$  are the output of the semi-public firm (firm 0) and a private firm  $j$ , respectively. Following the standard approach since Matsumura (1998), the semi-public firm maximizes the weighted sum of welfare and its own profit  $\theta W + (1 - \theta)\pi_{i0}$ ,  $\theta \in [0, 1]$

---

<sup>6</sup>We use a simpler notation  $c_i$  instead of  $c(i)$ .

whereas the private firms maximize their profits. Then, denoting by  $x_{i1}$  the common output of the  $n$  private firms, the first-order conditions for the Cournot equilibrium are

$$\begin{aligned} a - \theta w \alpha_i - (1 + \theta)x_{i0} - nx_{i1} &= 0 \\ a - w \alpha_i - x_{i0} - (n + 1)x_{i1} &= 0, \end{aligned}$$

which yields the equilibrium outputs:

$$x_{i0} = \frac{a - w \alpha_i [(n + 1)\theta - n]}{1 + (n + 1)\theta}, \quad x_{i1} = \frac{\theta a - w \alpha_i}{1 + (n + 1)\theta}. \quad (2)$$

The model is closed by introducing the labor market-clearing condition:

$$\int_0^1 \alpha_i (x_{i0} + nx_{i1}) di = \frac{(1 + n\theta)a\mu - (n + 1)\theta\sigma^2 w}{1 + (n + 1)\theta} = l, \quad (3)$$

where  $\mu \equiv \int_0^1 \alpha_i di$ ,  $\sigma^2 \equiv \int_0^1 \alpha_i^2 di$ , and  $l$  is the labor endowment. Solving this equation for  $w$ , the equilibrium wage rate is obtained as

$$w = \frac{(1 + n\theta)a\mu - [1 + (n + 1)\theta]l}{(n + 1)\theta\sigma^2}, \quad (4)$$

which immediately leads to:

**Proposition 1.** *Privatization decreases the wage.*

*Proof.* Differentiating (4) with respect to  $\theta$  gives

$$\frac{\partial w}{\partial \theta} = -\frac{a\mu - l}{(n + 1)\theta^2\sigma^2} < 0.$$

||

The intuition underlying this result is as follows. If  $\theta$  rises, output of the public firm decreases, output of the private firms increases, and total output of each industry decreases.<sup>7</sup> Thus, the wage rate declines as a result of privatization since privatization

<sup>7</sup>The effect of privatization on outputs is given by

$$\begin{aligned} \frac{\partial x_{i0}}{\partial \theta} &= -\frac{(n + 1)[a + (n + 1)w\alpha_i]}{[1 + (n + 1)\theta]^2} < 0 \\ \frac{\partial x_{i1}}{\partial \theta} &= \frac{a + (n + 1)w\alpha_i}{[1 + (n + 1)\theta]^2} > 0 \\ \frac{\partial (x_{i0} + nx_{i1})}{\partial \theta} &= -\frac{a + (n + 1)w\alpha_i}{[1 + (n + 1)\theta]^2} < 0. \end{aligned}$$

reduces labor demand in all industries. This effect of privatization on the wage, which is overlooked in the existing literature, will play an important role behind the effects of privatization on income distribution and welfare.

### 3 Privatization, National Income and Welfare

This section examines the effects of privatization on national income and welfare. We begin with the effect on national income, and then proceed to the welfare effect.

Substituting the equilibrium outputs (2) into the inverse demand function, we have

$$p_i = a - x_{i0} - nx_{i1} = \frac{\theta a + (n+1)\theta w\alpha_i}{1 + (n+1)\theta}.$$

Then, the aggregate profits in the whole economy, which is denoted by  $\Pi$ , are computed as

$$\begin{aligned} \Pi &\equiv \int_0^1 (\pi_{i0} + n\pi_{i1}) di \\ &= \int_0^1 (p_i - w\alpha_i)(x_{i0} + nx_{i1}) di \\ &= \int_0^1 \left\{ \frac{(\theta a - w\alpha_i)[(1+n\theta)a - (n+1)\theta w\alpha_i]}{[1 + (n+1)\theta]^2} \right\} di \\ &= \int_0^1 \left\{ \frac{(1+n\theta)\theta a^2 - [(n+1)\theta^2 + n\theta + 1]aw\alpha_i + (n+1)\theta w^2\alpha_i^2}{[1 + (n+1)\theta]^2} \right\} di \\ &= \frac{(1+n\theta)\theta a^2 - [(n+1)\theta^2 + n\theta + 1]aw\mu + (n+1)\theta w^2\sigma^2}{[1 + (n+1)\theta]^2}. \end{aligned} \quad (5)$$

Thus, substituting (4) into (5) and making a lengthy manipulation lead to:

$$\begin{aligned} \Pi &= \frac{\Delta}{(n+1)\theta[1 + (n+1)\theta]^2\sigma^2} \\ \Delta &\equiv (\sigma^2 - \mu^2)(1+n)(1+n\theta)\theta^2 a^2 + [1 + (n+1)\theta] \left\{ [(n+1)\theta^2 - n\theta - 1]a\mu + [1 + (n+1)\theta]l \right\} l. \end{aligned} \quad (6)$$

The effect of privatization on the aggregate profit is summarizes as follows.

**Proposition 2.** *Privatization strictly increases the aggregate profits.*

The proof of this result will be postponed after addressing the effect on national income since straightforward differentiation of (6) involves a quite complicated manipulation.

The reason why privatization increases the aggregate profit is explained as follows. As shown in Proposition 1, privatization reduces the wage rate. The resulting decline in wage rate leads to a decline in marginal cost in each industry, and hence all firms have an incentive to produce more. Therefore, private firms can make more profits since they increase output as a result of the first-order effect of privatization, and their marginal cost decrease as the second-order effect. While the effect on the profit of the public firm is ambiguous, the positive effect on the private firms' profits plays a dominant role in the industry-wide profit, and thereby the aggregate profit increases after privatization.

The national income  $I$  consists of labor income  $wl$  and the aggregate profit  $\Pi$ . By making use of (4) and (6),  $I$  is computed as

$$I = wl + \Pi = \frac{(\sigma^2 - \mu^2)(1 + n\theta)\theta a^2}{[1 + (n + 1)\theta]^2 \sigma^2} + \frac{(a\mu - l)l}{\sigma^2}. \quad (7)$$

Eq. (7) allows us to claim that:

**Proposition 3.** *Privatization increases the national income, strictly so if  $\sigma^2 - \mu^2 > 0$ .*

*Proof.* Immediately from

$$\frac{\partial I}{\partial \theta} = \frac{(\sigma^2 - \mu^2)[1 + (n - 1)\theta]a^2}{[1 + (n + 1)\theta]^3 \sigma^2} \geq 0,$$

with a strict inequality if  $\sigma^2 - \mu^2 > 0$ . ||

Note that Proposition 2 follows from this result; privatization necessarily increases the aggregate profit because it decreases the labor income but increases the national income.

Propositions 1 and 2 suggest an income distribution effect of privatization such that privatization has a negative effect on labor income but a positive effect on the aggregate profit. However, according to Proposition 3, privatization leads to an increase in national income since the positive effect on the aggregate profit is larger than the negative effect on the labor income.

Let us move on to the welfare effect of privatization. Applying the preceding arguments to Eq. (1),  $\sigma_p^2$  becomes

$$\sigma_p^2 = \int_0^1 \left[ \frac{\theta a + (n + 1)\theta w \alpha_i}{1 + (n + 1)\theta} \right]^2 di$$

$$\begin{aligned}
&= \int_0^1 \left\{ \frac{\theta^2 a^2 + 2(n+1)\theta^2 a w \alpha_i + (n+1)^2 \theta^2 w^2 \alpha_i^2}{[1 + (n+1)\theta]^2} \right\} di \\
&= \frac{\theta^2 a^2 + 2(n+1)\theta^2 a w \mu + (n+1)^2 \theta^2 w^2 \sigma^2}{[1 + (n+1)\theta]^2} \\
&= \frac{1}{\sigma^2} \left\{ \frac{(\sigma^2 - \mu^2) \theta^2 a^2}{[1 + (n+1)\theta]^2} + (a\mu - l)^2 \right\} \\
&= \frac{1}{\sigma^2} \left\{ \frac{(\sigma^2 - \mu^2) a^2}{(1/\theta + n + 1)^2} + (a\mu - l)^2 \right\}. \tag{8}
\end{aligned}$$

Hence, we easily find that  $\sigma_p^2$  is increasing in  $\theta$ , and arrive at:

**Proposition 4.** *Privatization decreases welfare, strictly so if  $\sigma^2 - \mu^2 > 0$ .*

This result is intuitively interpreted as follows. As mentioned earlier, privatization leads total output in each industry to decline, and hence the good price rises. Meanwhile, privatization has a second-order effect such that the wage rate and marginal cost of all firms decline. This induces all firms to produce more, and the upward pressure for good prices is mitigated. But, because the first-order effect on good prices is stronger than the second-order effect, privatization ends up raising good prices and reducing welfare.

What Proposition 4 suggests is that full nationalization is the best policy from the welfare point of view. However, we must carefully recognize that this extreme outcome rests on a number of simplifying assumptions. Among others, we guess that the assumption of homogeneous products (perfect substitutes) is crucial for this finding. If product differentiation within/across industries is allowed, some degree of privatization may be optimal as is confirmed in the existing literature, e.g. Fujiwara (2007).

## 4 Endogenous Market Structure

Thus far, we have focused on the case of exogenous market structure, i.e. the number of (private) firms is exogenously given. Relaxing this assumption, this section turns to the case of endogenous market structure in which the number of firms is endogenously determined by the zero profit condition. Following Neary (2003), we simply assume that all the private firms have to incur a fixed cost  $f > 0$  in entering the market. Then, the

maximized profit of private firms is

$$\pi_{i1} = x_{i1}^2 - f = \left[ \frac{\theta a - w\alpha_i}{1 + (n+1)\theta} \right]^2 - f,$$

where the last equation comes from (2). Solving this equation for  $n$  yields

$$n = \frac{\theta a - w\alpha_i}{\theta\sqrt{f}} - \frac{1}{\theta} - 1,$$

which is shown to be increasing in  $\theta$ , that is, privatization increases the number of firms.<sup>8</sup>

Combining these results with the equilibrium outputs in (2), we have

$$x_{i0} = \frac{\sqrt{f} + (1-\theta)w\alpha_i}{\theta}, \quad nx_{i1} = \frac{\theta a - w\alpha_i - (1+\theta)\sqrt{f}}{\theta},$$

and the labor market-clearing condition becomes

$$\int_0^1 \alpha_i (x_{i0} + nx_{i1}) di = (a - \sqrt{f})\mu - w\sigma^2 = l.$$

Hence, the equilibrium wage rate in the free entry case is

$$w = \frac{(a - \sqrt{f})\mu - l}{\sigma^2}. \quad (9)$$

Note that the labor market-clearing wage rate does not depend on the degree of privatization  $\theta$ , i.e. privatization has no effect on the labor income. This is a notable result in the free entry case, and the reason behind it is as follows. As noted after Proposition 1, privatization puts downward pressure on the wage rate by decreasing industry output. In addition to this effect, privatization influences the equilibrium wage rate through the change in the number of private firms. Differentiating  $n$  above with respect to  $\theta$ , we find that privatization induces entry. While new entry has a business-stealing effect such that it reduces all firms' output, industry output  $x_{i0} + nx_{i1}$  increases. Therefore, privatization has a positive effect on labor demand as a direct effect, and a negative effect as an indirect effect by inducing entry. In our model, these two conflicting effects just offset, and thereby the equilibrium wage rate remains unchanged before and after privatization.<sup>9</sup>

<sup>8</sup>Note that  $dn/d\theta = (w\alpha_i + \sqrt{f}) / (\theta^2\sqrt{f}) > 0$ .

<sup>9</sup>Algebraically, the overall effect of privatization on industry output is computed as follows.

$$\begin{aligned} \frac{d(x_{i0} + nx_{i1})}{d\theta} &= \frac{\partial(x_{i0} + nx_{i1})}{\partial\theta} + \frac{\partial(x_{i0} + nx_{i1})}{\partial n} \cdot \frac{dn}{d\theta} \\ &= -\frac{a + (n+1)w\alpha_i}{[1 + (n+1)\theta]^2} + \frac{a + (n+1)w\alpha_i}{[1 + (n+1)\theta]^2} = 0. \end{aligned}$$

Making the manipulations parallel to those in the restricted entry case, the aggregate profit becomes

$$\begin{aligned}
\Pi &\equiv \int_0^1 (\pi_{i0} + n\pi_{i1}) di = \int_0^1 \pi_{i0} di \\
&= \int_0^1 (p_i - w\alpha_i)x_{i0} di \\
&= \frac{\sqrt{f} \left\{ \sigma^2 \sqrt{f} + (1 - \theta)\mu \left[ (a - \sqrt{f})\mu - l \right] \right\}}{\theta\sigma^2}.
\end{aligned} \tag{10}$$

Noting that the privatization has no effect on the labor income  $wl$ , the effect of privatization on national income  $I \equiv wl + \Pi$  is the same as that on  $\Pi$ , and given by

$$\frac{\partial I}{\partial \theta} = \frac{\partial \Pi}{\partial \theta} = -\frac{(\sigma^2 - \mu^2)f + \mu\sqrt{f}(a\mu - l)}{\theta^2\sigma^2} < 0.$$

That is, privatization ends up decreasing national income as is the opposite to the case of exogenous market structure. This is because the public firm's output decreases not only as a direct effect of privatization but also as an indirect effect through business-stealing.<sup>10</sup>

Finally, let us address the welfare effect of privatization in the case of endogenous market structure. Since the price of each good becomes

$$p_i = a - x_{i0} - nx_{i1} = \frac{\sigma^2\sqrt{f} + \alpha_i \left[ (a - \sqrt{f}) - l \right]}{\sigma^2},$$

$\sigma_p^2$  in (1) is derived as follows.

$$\sigma_p^2 = \int_0^1 p_i^2 di = \frac{(\sigma^2 - \mu^2)f + (a\mu - l)^2}{\sigma^2}. \tag{11}$$

This equation tells that welfare is not affected by privatization. The underlying intuition is straightforward once noting that privatization has no effect on industry output. Summarizing the preceding arguments, we establish:

**Proposition 5.** *In the endogenous market structure, privatization has no effect on the labor income and welfare while decreasing the aggregate profit and the national income.*

---

<sup>10</sup>This is mathematically proved as follows.

$$\frac{dx_{i0}}{d\theta} = \frac{\partial x_{i0}}{\partial \theta} + \frac{\partial x_{i0}}{\partial n} \cdot \frac{dn}{d\theta} = -\frac{[1 + (n+1)\theta][a + (n+1)w\alpha_i]}{\theta[1 + (n+1)\theta]^2} < 0.$$

## 5 Conclusion

This paper has applied the general oligopolistic equilibrium model of Neary (2003) to the welfare effects of privatization. The main findings are that (i) privatization inevitably worsens welfare in the exogenous market structure (with restricted entry of private firms) and that (ii) privatization has no effect on welfare in the endogenous market structure (with free entry of private firms). Both of these results may shed new light on the ongoing debate over privatization since they are contrasting to the existing results that *partial* privatization is optimal (see Matsumura, 1998 and Matsumura and Kanda, 2005).

However, we admittedly recognize that the above results are so extreme that they are of little applicability to the practical policymaking. This is mainly because we have assumed that all sectors are symmetric except for cost parameter  $\alpha_i$  and that products in each industry are homogeneous (perfect substitute). It is our important research agenda to reexamine the effects of privatization by relaxing these simplifying assumptions.

## References

- [1] Beladi, H., A. Chakrabarti and S. Marjit (2013), 'A Mixed GOLE Model of Cross-Border Mergers and Trade,' Working Paper Series 0002ECO-414-2013, College of Business, University of Texas at San Antonio.
- [2] Cato, S. and T. Matsumura (2015), 'Optimal Privatization and Trade Policies with Endogenous Market Structure,' *Economic Record*, forthcoming.
- [3] Colacicco, R. (2015), 'Ten Years of General Oligopolistic Equilibrium: A Survey,' *Journal of Economic Surveys*, 29, 965-992.
- [4] de Fraja, G. and F. Delbono (1989), 'Alternative Strategies of a Public Enterprise in Oligopoly,' *Oxford Economic Papers*, 41, 302-311.
- [5] Fujiwara, K. (2007), 'Partial Privatization in a Differentiated Mixed Oligopoly,' *Journal of Economics*, 92, 51-65.
- [6] Jiang, Y., M. W. Peng, X. Yang and C. C. Mutlu (2015), 'Privatization, Governance, and Survival: MNE Investments in Private Participation Projects in Emerging Economies,' *Journal of World Business*, 50, 294-301.

- [7] Matsumura, T. (1998), 'Partial Privatization in Mixed Duopoly,' *Journal of Public Economics*, 70(3), 473-483.
- [8] Matsumura, T. and Y. Okumura (2013), 'Privatization Neutrality Theorem Revisited,' *Economics Letters*, 118(2), 324-326.
- [9] Matsumura, T. and O. Kanda (2005), 'Mixed Oligopoly at Free Entry Markets,' *Journal of Economics*, 84(1), 27-48.
- [10] Matsumura, T. and D. Shimizu (2010), 'Privatization Waves,' *Manchester School*, 78(6), 609-625.
- [11] Matsumura, T. and Y. Tomaru (2012), 'Market Structure and Privatization Policy under International Competition,' *Japanese Economic Review*, 63(2), 244-258.
- [12] Neary, J. P. (2003), 'The Road Less Travelled: Oligopoly and Competition Policy in General Equilibrium,' in Arnott, R., B. Greenwald, R. Kanbur and B. Nalebuff (eds.), *Economics for an Imperfect World: Essays in Honor of Joseph E. Stiglitz*, Cambridge: MIT Press, 485-500.
- [13] Neary, J. P. (2009), 'International Trade in General Oligopolistic Equilibrium,' unpublished manuscript, University of Oxford.
- [14] Reason Foundation (2015), *Annual Privatization Report 2015: Federal Government Privatization*.
- [15] White, M. D. (1996), 'Mixed Oligopoly, Privatization and Subsidization,' *Economics Letters*, 53, 189-195.