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Tsuyoshi TOSHIMITSU

School of Economics, Kwansei Gakuin University

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SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

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Tsuyoshi TOSHIMITSU[☆]

School of Economics, Kwansei Gakuin University

Abstract

Based on a differentiated duopoly model, we consider the efficiency of Bertrand and Cournot equilibrium in the presence of network effects and product compatibility. In particular, we demonstrate that if an asymmetric product compatibility with a strong network effect between the firms arises, give certain conditions, Cournot equilibrium is more efficient than Bertrand equilibrium in terms of greater consumer, producer, and social surplus.

Keywords: Bertrand equilibrium; Cournot equilibrium; product compatibility; network effect; fulfilled expectation; horizontally differentiated duopoly

JEL classifications: L13, L32, L33

[☆] School of Economics, Kwansei Gakuin University, 1-155, Nishinomiya, Japan, 662-8501, Tel: +81 798 54 6440, Fax: +81 798 51 0944, Email: ttsutomu@kwansei.ac.jp

1. Introduction

Many studies have analyzed the efficiency of Cournot (quantity) and Bertrand (price) competition (e.g., Singh and Vives, 1984; Vives, 1985; Häckner, 2000; Amir and Jin, 2001). These studies consider how the difference in the mode of competition affects the market outcomes and social welfare. There is a conventional view that price competition is more efficient than quantity competition in terms of greater consumer surplus and social welfare.

Using a differentiated duopoly model, Singh and Vives (1984) demonstrate that when products are substitutes, consumer and total surpluses are larger and profits smaller in Bertrand competition than in Cournot competition. Using a differentiated oligopoly model, Vives (1985) shows that price competition is more efficient than quantity competition. Furthermore, he finds that as the number of firms goes to infinity, the prices in the Bertrand equilibrium and the Cournot equilibrium go to marginal cost. Similarly, Häckner (2000) reconsiders the results in Singh and Vives (1984). In particular, he introduces product quality measure in a quadratic utility function. In this case, he demonstrates the following. If quality differences are large and products are complements, prices in the Bertrand equilibrium are higher than in the Cournot equilibrium; and if products are substitutes, high-quality firms may earn higher profits in the Bertrand equilibrium than in the Cournot equilibrium. However, he points out that it is not evident which mode of competition is more efficient in the n -firm specification. Hsu and Wang (2005) examine this issue and find that consumer and total surpluses are larger in the Bertrand equilibrium than in the Cournot equilibrium, regardless of the mode of competition.

In a differentiated duopoly with linear demand and cost functions, Zanchettin (2006) allows for a wider range of demand and cost asymmetry between firms, i.e., product quality differences as in Häckner (2000) and shows that if asymmetry is strong and/or products are weakly differential, profits are higher in the Bertrand equilibrium than in the Cournot equilibrium.

Recently, using a differentiated oligopoly model with symmetric product compatibility and network effects, Pal (2014) examines the efficiency of price and quantity competition. He demonstrates that if there are strong network externalities and imperfectly substitutable products, profit is higher in the Bertrand equilibrium than in the Cournot equilibrium.

In this paper, we reconsider the important results of Singh and Vives (1984) by introducing product compatibility into the network effect models of Katz and Shapiro (1985) and Economides (1996). In particular, focusing on the strength of a network effect relative to product substitutability and the degree of product compatibility between firms, we demonstrate that if asymmetric product compatibility with a strong network effect between firms arises, given certain conditions, the Cournot equilibrium is more efficient than the Bertrand equilibrium in terms of greater consumer, producer, and social surpluses. In fact, firms compete on price-cutting of network products such as the Internet and mobile phone services. However, if asymmetric product compatibility between the firms arises, consumers may lose their benefits and social welfare may decrease.

2. The Model

2.1 Quantity competition with network compatibility effects

We consider quantity competition in a horizontally differentiated products market with network compatibility effects. Following the model of Economides (1996), the linear inverse demand function of product i is given by:

$$p_i = A - q_i - \theta q_j + f(S_i^e), \quad i, j = 1, 2, i \neq j, \quad (1)$$

where A is the intrinsic market size, q_i (q_j) is the quantity of firm i (j), and $\theta \in (0, 1)$ represents the degree of product substitutability. The network externality function is given by $f(S_i^e)$, where S_i^e represents the expected network size of firm i . Based on the concept of fulfilled rational expectations, we assume that $S_i^e = S_i$, where S_i is the real network size of firm i . We assume a linear network effect function; $f(S_i) = aS_i$, where $a \in (0, 1)$ is a network effect parameter with network size. Furthermore, using equation (3.15) in Shy (2001, p. 62), the real network size of firm i is given by:

$$S_i = q_i + \alpha_i q_j, \quad i, j = 1, 2, i \neq j, \quad (2)$$

where $\alpha_i \in [0, 1]$, $i = 1, 2$, denotes the degree of product i 's compatibility with product j . Equation (2) implies that firm i will provide a compatible product with which the rival firm's product j can operate. If $\alpha_i = 1$ (0), $i = 1, 2$, a user of product i operates (does not operate) perfectly with product j . q_i ($\alpha_i q_j$), $i, j = 1, 2, i \neq j$, in equation (2) represents the *own (incoming) effect* on network size.

Based on equations (1) and (2), the inverse demand function of firm i can be

expressed by:

$$p_i = A - (1-a)q_i - (\theta - a\alpha_i)q_j, \quad i, j = 1, 2, i \neq j, \quad (3)$$

where we assume that the own-price effect exceeds the cross-price effect, i.e.,

$$\left| \frac{dp_i}{dq_i} \right| > \left| \frac{dp_i}{dq_j} \right|, \quad i, j = 1, 2, i \neq j. \quad \text{In this case, it follows that } 1-a > |\theta - a\alpha_i| > 0, \quad i = 1, 2.$$

To simplify the analysis, we assume that production costs are zero. Thus, the profit of firm i is expressed by: $\pi_i = p_i q_i, i = 1, 2$. Using the first-order profit-maximizing

condition, $\frac{\partial \pi_i}{\partial q_i} = p_i - (1-a)q_i = 0$, and equation (3), we derive the reaction function

for firm i as follows:

$$q_i = \frac{A}{2(1-a)} - \frac{\theta - a\alpha_i}{2(1-a)} q_j, \quad i, j = 1, 2, i \neq j. \quad (4)$$

From equation (4), the strategic relationship between the firms depends on the degree of product substitutability and the network compatibility effects:

$$\frac{\partial q_i}{\partial q_j} < (>) 0 \Leftrightarrow \theta > (<) a\alpha_i, \quad i, j = 1, 2, i \neq j. \quad (5)$$

Equation (5) implies that a strategic substitute (complement) relationship between the firms holds if the degree of product substitutability is larger (smaller) than that of network compatibility.

Furthermore, using the first-order profit-maximizing condition, the profit function is represented by $\pi_i = (1-a)(q_i)^2, i = 1, 2$. Thus, we derive the external effect of an increase in the quantity of firm j on the profit of firm i as follows:

$$\frac{\partial \pi_i}{\partial q_j} = 2(1-a)q_i \frac{\partial q_i}{\partial q_j} < (>) 0 \Leftrightarrow \theta > (<) a\alpha_i, \quad i, j = 1, 2, i \neq j. \quad (6)$$

For the following analysis, without the loss of generality, we assume asymmetric product compatibility between firms as follows:

Assumption 1: $1 \geq \alpha_1 > \alpha_2 \geq 0$.

Given equation (4), we derive the following Cournot–Nash equilibrium:

$$q_i^c = \frac{A\{2(1-a) - (\theta - a\alpha_i)\}}{D}, \quad i = 1, 2, \quad (7)$$

where $D \equiv 4(1-a)^2 - (\theta - a\alpha_1)(\theta - a\alpha_2) > 0$ and $2(1-a) - (\theta - a\alpha_i) > 0$, $i = 1, 2$.

Both of these conditions are satisfied because the own-price effect exceeds the cross-price effect. Superscript *C* denotes the Cournot–Nash equilibrium.

2.2 Price competition with network compatibility effects

Taking equation (3) into account, we derive the direct demand function of firm i as follows:

$$q_i = \frac{\{(1-a) - (\theta - a\alpha_i)\}A - (1-a)p_i + (\theta - a\alpha_i)p_j}{\Delta}, \quad i, j = 1, 2, i \neq j, \quad (13)$$

where $\Delta \equiv (1-a)^2 - (\theta - a\alpha_1)(\theta - a\alpha_2) > 0$. Based on the first-order

profit-maximization condition, i.e., $\frac{\partial \pi_i}{\partial p_i} = q_i - \frac{1-a}{\Delta} p_i = 0$, and equation (13), the

reaction function for firm i is:

$$p_i = \frac{\{(1-a) - (\theta - a\alpha_i)\}A}{2(1-a)} + \frac{\theta - a\alpha_i}{2(1-a)} p_j, \quad i, j = 1, 2, i \neq j. \quad (14)$$

Thus, the strategic relationship between the firms depends on the degree of product substitutability and network compatibility:

$$\frac{\partial p_i}{\partial p_j} > (<)0 \Leftrightarrow \theta > (<)a\alpha_i, i, j = 1, 2, i \neq j. \quad (15)$$

Equation (15) implies that a strategic complement (substitute) relationship between the firms holds if the degree of product substitutability is higher (lower) than that of network compatibility.

Furthermore, using the first-order profit-maximization condition, the profit function is $\pi_i = \frac{1-a}{\Delta}(p_i)^2, i = 1, 2$. Thus, we derive the external effect of an increase in firm j on

the profit of firm i as follows:

$$\frac{\partial \pi_i}{\partial p_j} = \frac{2(1-a)}{\Delta} p_i \frac{\partial p_i}{\partial p_j} > (<)0 \Leftrightarrow \theta > (<)a\alpha_i, i, j = 1, 2, i \neq j. \quad (16)$$

Given equation (14), we derive the following Bertrand–Nash equilibrium:

$$p_i^B = \frac{A\{2(1-a)^2 - (1-a)(\theta - a\alpha_i) - (\theta - a\alpha_1)(\theta - a\alpha_2)\}}{D}, i = 1, 2, \quad (17)$$

where $2(1-a)^2 - (1-a)(\theta - a\alpha_i) - (\theta - a\alpha_1)(\theta - a\alpha_2) > 0, i = 1, 2$. This condition is satisfied because the own-price effect exceeds the cross-price effect. Superscript B denotes the Bertrand–Nash equilibrium.

3. Comparison: Cournot equilibrium vs. Bertrand equilibrium in the presence of asymmetric network compatibility effects

3.1 Prices, quantities, and profits

Using the first-order profit-maximizing conditions in the case of quantity and price competition, price and profit in the Cournot equilibrium are expressed as

$p_i^C = (1-a)q_i^C$ and $\pi_i^C = (1-a)(q_i^C)^2$, $i=1,2$. Similarly, for the quantity and profit in the Bertrand equilibrium, we have $q_i^B = \frac{1-a}{\Delta} p_i^B$ and $\pi_i^B = \frac{1-a}{\Delta} (p_i^B)^2$, $i=1,2$.

For the following analysis, with respect to the parameters of network effect and product substitutability, we make the following assumption.

Assumption 2: $a > \theta$.

Assumption 2 implies that the network effect is larger than the substitutability effect between the products. Taking equations (7) and (17) into account, given Assumption 1, we directly obtain the following results.

Lemma 1

(i) $q_i^C < (>) q_i^B$, $p_i^C > (<) p_i^B \Leftrightarrow (\theta - a\alpha_1)(\theta - a\alpha_2) > (<) 0$, $i=1,2$,

(ii) $\pi_i^C > (<) \pi_i^B \Leftrightarrow \theta > (<) a\alpha_j$, $i, j=1,2, i \neq j$.

First, as in Lemma 1 (i), if the degree of network compatibility effects of both products is either lower or higher than that of product substitutability, it follows that $(\theta - a\alpha_1)(\theta - a\alpha_2) > 0$. In this case, we have the same results as those of Singh and Vives (1984).¹ That is, the quantity (price) is lower (higher) in the Cournot equilibrium than in the Bertrand equilibrium.

¹ If either Assumption 1 does not hold (i.e., $\theta > a$), symmetric product compatibility (i.e., $0 \leq \alpha_1 = \alpha_2 = \alpha \leq 1$) or no network effects (i.e., $a = 0$), we have the same results as those of Singh and Vives (1984).

However, if there are asymmetric network compatibility effects between the firms, i.e., $a\alpha_1 > \theta > a\alpha_2$, it follows that $(\theta - a\alpha_1)(\theta - a\alpha_2) < 0$. In this case, we derive the opposite results: quantity (price) is higher (lower) in the Cournot equilibrium than in the Bertrand equilibrium.

Second, as in Lemma 1 (ii), the amount of profits in the Cournot equilibrium and in the Bertrand equilibrium depend on the degree of the network compatibility effects of the rival firm's product. If the degree of network compatibility effects of both products is lower than that of product substitutability, we have the same results as those of Singh and Vives (1984).² That is, the profit is higher in the Cournot equilibrium than that in the Bertrand equilibrium. However, if the degree of network compatibility effects of both firms is higher than that of product substitutability, profit is higher in the Bertrand equilibrium than that in the Cournot equilibrium. This result is the same as that in Pal (2014, Proposition 1).

Furthermore, if there are asymmetric network compatibility effects between the firms, i.e., $a\alpha_1 > \theta > a\alpha_2$, we derive the following results: the profit of firm 1 (2) producing the product with larger (smaller) network compatibility effects than a certain level of product substitutability is lower (higher) in the Cournot equilibrium than in the Bertrand equilibrium. In this case, following Singh and Vives (1984), because firm 2's product is a substitute good as a result of the smaller network compatibility effects, the profit of firm 1 is higher in the Cournot equilibrium than in the Bertrand equilibrium. Conversely, because the nature of firm 1's product is a complement good as a result of the larger network compatibility effects, the profit of firm 2 is higher in the Bertrand

² Under the same conditions presented in footnote 1, the profit is higher (lower) in the Cournot equilibrium than that in the Bertrand equilibrium, if products are substitutes (complements).

equilibrium than in the Cournot equilibrium.

3.2 Consumer surplus, producer surplus, and social surplus

First, taking equation (1) into account, consumer surplus is given by:

$$CS^k = \frac{(q_1^k)^2 + 2\theta q_1^k q_2^k + (q_2^k)^2}{2}, k = C, B. \quad (18)$$

From equation (18), we can express consumer surplus as $CS^k = CS(q_1^k, q_2^k)$, $k = C, B$.

Based on Lemma 1 (i), we derive the following relationship:

$$CS^C > (<) CS^B \Leftrightarrow (\theta - a\alpha_1)(\theta - a\alpha_2) < (>) 0 \Leftrightarrow \left(\frac{\theta}{a} - \alpha_1\right) \left(\frac{\theta}{a} - \alpha_2\right) < (>) 0. \quad (19)$$

In view of equation (19), we summarize the results as follows.

Lemma 2

(i) *If it holds that either $\alpha_1 > \alpha_2 > \frac{\theta}{a}$ or $\frac{\theta}{a} > \alpha_1 > \alpha_2$, it follows that $CS^B > CS^C$.*

(ii) *If it holds that $\alpha_1 > \frac{\theta}{a} > \alpha_2$, it follows that $CS^C > CS^B$.*

As in Lemma 2 (i), if the degree of network compatibility effects of both products is either lower or higher than that of product substitutability, it follows that $(\theta - a\alpha_1)(\theta - a\alpha_2) > 0$. In this case, as in Singh and Vives (1984), consumer surplus is larger in the Bertrand equilibrium than in the Cournot equilibrium. Conversely, as shown in Lemma 2 (ii), if there are asymmetric network compatibility effects between the firms, i.e., $\alpha_1 > \frac{\theta}{a} > \alpha_2$, it follows that $(\theta - a\alpha_1)(\theta - a\alpha_2) < 0$. In this case,

consumer surplus is higher in the Cournot equilibrium than in the Bertrand equilibrium.

Second, we can express producer surplus as $PS^k = \pi_1^k + \pi_2^k$, $k = C, B$. Thus, taking Assumptions 1 and 2, and Lemma 1 (ii) into account, we obtain the following results directly.

$$PS^C > PS^B \Leftrightarrow \theta > a\alpha_1 > a\alpha_2 \Leftrightarrow 1 > \frac{\theta}{a} > \alpha_1 > \alpha_2, \quad (20)$$

$$PS^C < PS^B \Leftrightarrow a\alpha_1 > a\alpha_2 > \theta \Leftrightarrow 1 \geq \alpha_1 > \alpha_2 > \frac{\theta}{a}, \quad (21)$$

The results shown in equations (20) and (21) are the same as in Singh and Vives (1984).

However, in the case of asymmetric network compatibility effects, i.e., $a\alpha_1 > \theta > a\alpha_2 \Leftrightarrow 1 \geq \alpha_1 > \frac{\theta}{a} > \alpha_2 \geq 0$, the comparison of producer surplus is not determined uniquely. That is, we can derive the following relationship:

$$PS^C > (<)PS^B \Leftrightarrow (\theta - a\alpha_1) + (\theta - a\alpha_2) < (>)0. \quad (22)$$

Equation (22) implies that if the degree of network compatibility effects of firm I 's product is sufficiently large, producer surplus is larger in the Cournot equilibrium than in the Bertrand equilibrium. For example, whenever product I (2) is perfectly compatible (incompatible), i.e., $\alpha_1 = 1$ and $\alpha_2 = 0$ in the one-sided compatibility case, this result arises. Otherwise, producer surplus is smaller in the Cournot equilibrium than in the Bertrand equilibrium.

Equation (22) can be rewritten as follows:

$$PS^C > (<)PS^B \Leftrightarrow \frac{2\theta}{a} < (>)\alpha_1 + \alpha_2. \quad (23)$$

Given Assumption 2, we have the following two cases according to the size of the

parameters of network effects and product substitutability.

$$\text{Case (1): } \frac{2\theta}{a} > 1 > \frac{\theta}{a}$$

(i) If the parameters of product compatibility of both firms, i.e., (α_1, α_2) , fall into the following set,

$$\Gamma_1(\alpha_1, \alpha_2) \equiv \left\{ (\alpha_1, \alpha_2) \left| 1 \geq \alpha_1 > \frac{\theta}{a} > \alpha_2 \geq 0, \alpha_1 + \alpha_2 < \frac{2\theta}{a} - 1 \right. \right\},$$

then it follows that $PS^C < PS^B$.

(ii) If the parameters of product compatibility of both firms, i.e., (α_1, α_2) , fall into the following set,

$$\Gamma_2(\alpha_1, \alpha_2) \equiv \left\{ (\alpha_1, \alpha_2) \left| 1 \geq \alpha_1 > \frac{\theta}{a} > \alpha_2 \geq 0, \alpha_1 + \alpha_2 > \frac{2\theta}{a} - 1 \right. \right\},$$

then it follows that $PS^C > PS^B$.

$$\text{Case (2): } 1 > \frac{2\theta}{a} > \frac{\theta}{a}$$

(i) If the parameters of product compatibility of the firms, i.e., (α_1, α_2) , fall into the following set,

$$\Psi_1(\alpha_1, \alpha_2) \equiv \left\{ (\alpha_1, \alpha_2) \left| \frac{2\theta}{a} \geq \alpha_1 > \frac{\theta}{a} > \alpha_2 \geq 0, \alpha_1 + \alpha_2 < \frac{2\theta}{a} \right. \right\},$$

then it follows that $PS^C < PS^B$.

(ii) If the parameters of product compatibility of the firms, i.e., (α_1, α_2) , fall into the following set,

$$\Psi_2(\alpha_1, \alpha_2) \equiv \left\{ (\alpha_1, \alpha_2) \left| \frac{2\theta}{a} \geq \alpha_1 > \frac{\theta}{a} > \alpha_2 \geq 0, \alpha_1 + \alpha_2 > \frac{2\theta}{a} \right. \right\},$$

then it follows that $PS^C > PS^B$.

Based on the comparative analysis of producer surplus, i.e., equations (20), (21), and (23), taking Assumption 1 into account, we summarize the results as follows.

Lemma 3

(1) *Regarding the parameters of network effects and product substitutability, it holds*

that $\frac{2\theta}{a} > 1 > \frac{\theta}{a}$. In this case, we have the following.

(i) *If the parameters of product compatibility of the firms, i.e., (α_1, α_2) , fall into either of the following sets, it follows that $PS^B > PS^C$.*

$$\Omega_1(\alpha_1, \alpha_2) \equiv \left\{ (\alpha_1, \alpha_2) \left| 1 \geq \alpha_1 > \alpha_2 > \frac{\theta}{a} \right. \right\} \text{ or}$$

$$\Gamma_1(\alpha_1, \alpha_2) \equiv \left\{ (\alpha_1, \alpha_2) \left| 1 \geq \alpha_1 > \frac{\theta}{a} > \alpha_2 \geq 0, \alpha_1 + \alpha_2 < \frac{2\theta}{a} - 1 \right. \right\},$$

(ii) *If the parameters of product compatibility of the firms, i.e., (α_1, α_2) , fall into either of the following sets, it follows that $PS^C > PS^B$.*

$$\Omega_2(\alpha_1, \alpha_2) \equiv \left\{ (\alpha_1, \alpha_2) \left| \frac{\theta}{a} > \alpha_1 > \alpha_2 \geq 0 \right. \right\} \text{ or}$$

$$\Gamma_2(\alpha_1, \alpha_2) \equiv \left\{ (\alpha_1, \alpha_2) \left| 1 \geq \alpha_1 > \frac{\theta}{a} > \alpha_2 \geq 0, \alpha_1 + \alpha_2 > \frac{2\theta}{a} - 1 \right. \right\},$$

(2) *Regarding the parameters of network effects and product substitutability, it holds*

that $1 > \frac{2\theta}{a} > \frac{\theta}{a}$. In this case, we have the following.

(i) *If the parameters of product compatibility of the firms, i.e., (α_1, α_2) , fall into either of the following sets, it follows that $PS^B > PS^C$.*

$$\Omega_1(\alpha_1, \alpha_2) \equiv \left\{ (\alpha_1, \alpha_2) \left| 1 \geq \alpha_1 > \alpha_2 > \frac{\theta}{a} \right. \right\} \text{ or}$$

$$\Psi_1(\alpha_1, \alpha_2) \equiv \left\{ (\alpha_1, \alpha_2) \left| \frac{2\theta}{a} \geq \alpha_1 > \frac{\theta}{a} > \alpha_2 \geq 0, \alpha_1 + \alpha_2 < \frac{2\theta}{a} \right. \right\}.$$

(ii) If the parameters of product compatibility of the firms, i.e., (α_1, α_2) , fall into either of the following sets, it follows that $PS^C > PS^B$.

$$\Omega_2(\alpha_1, \alpha_2) \equiv \left\{ (\alpha_1, \alpha_2) \left| \frac{\theta}{a} > \alpha_1 > \alpha_2 \geq 0 \right. \right\} \text{ or}$$

$$\Psi_2(\alpha_1, \alpha_2) \equiv \left\{ (\alpha_1, \alpha_2) \left| \frac{2\theta}{a} \geq \alpha_1 > \frac{\theta}{a} > \alpha_2 \geq 0, \alpha_1 + \alpha_2 > \frac{2\theta}{a} \right. \right\}.$$

Therefore, based on Lemmas 2 and 3, we present the main results of this paper as follows.

Proposition 1

(1) When it holds that $1 \geq \alpha_1 > \alpha_2 > \frac{\theta}{a}$, consumer, producer, and social surplus are

larger in the Bertrand equilibrium than in the Cournot equilibrium.

(2) When it holds that $\frac{\theta}{a} > \alpha_1 > \alpha_2 \geq 0$, consumer (producer) surplus is larger in the

Bertrand (Cournot) equilibrium than in the Cournot (Bertrand) equilibrium. However, the efficiency of the Bertrand equilibrium and the Cournot equilibrium in terms of social surplus is not determined uniquely.

Under the situation in Proposition 1 (1), i.e., because both firms' network

compatibility effects are smaller than their product substitutability effects, producer surplus is larger in the Bertrand equilibrium than in the Cournot equilibrium. Thus, social surplus is larger in the Bertrand equilibrium than in the Cournot equilibrium. On the contrary, Proposition 1 (2) shows the situation in which both firms' network compatibility effects are smaller than their product substitutability effects. In other words, network effects are negligible under this situation, which corresponds to the case assumed in Singh and Vives (1984). Thus, intuitively, social surplus is larger in the Bertrand equilibrium than in the Cournot equilibrium, although producer surplus is larger in the Cournot equilibrium than in the Bertrand equilibrium.

The results presented in Proposition 1 are virtually identical to those in Singh and Vives (1984). That is, even though network compatibility effects work, if the effects between the firms are symmetric or not sufficiently different, the conventional wisdom presented by Singh and Vives holds. However, if network compatibility effects between the firms are significantly asymmetric, the opposite result is true.

Proposition 2

When it holds that $1 \geq \alpha_1 > \frac{\theta}{a} > \alpha_2 \geq 0$, the following outcomes arise:

(i) If either $\alpha_1 + \alpha_2 > \frac{2\theta}{a} - 1 > 0$ or $\alpha_1 + \alpha_2 > 1 > \frac{2\theta}{a}$ holds, then consumer, producer,

and social surplus are larger in the Cournot equilibrium than in the Bertrand equilibrium.

(ii) If either $\alpha_1 + \alpha_2 < \frac{2\theta}{a} - 1$ or $\alpha_1 + \alpha_2 < \frac{2\theta}{a} < 1$ holds, then consumer (producer)

surplus is larger in the Cournot (Bertrand) equilibrium than in the Bertrand (Cournot)

equilibrium. However, the efficiency of the Bertrand equilibrium and the Cournot equilibrium in terms of social surplus is not determined uniquely.

As shown in Lemma 1 (i), if there are asymmetric network compatibility effects between the firms, in other words, if the degree of product compatibility is sufficiently asymmetric between the firms, quantity is larger in the Cournot equilibrium than in the Bertrand equilibrium. Thus, consumer surplus is larger in the Cournot equilibrium than in the Bertrand equilibrium. Furthermore, as in Proposition 2 (i), if the total value of product compatibility of the firms is larger than a certain value, producer surplus is larger in the Cournot equilibrium than in the Bertrand equilibrium. Thus, social surplus is larger in the Cournot equilibrium than in the Bertrand equilibrium.

On the contrary, as in Proposition 2 (ii), if the total value of product compatibility of the firms is smaller than a certain value, producer surplus is smaller in the Cournot equilibrium than in the Bertrand equilibrium. Thus, the efficiency of the Bertrand equilibrium and the Cournot equilibrium in terms of social surplus is not determined uniquely. The results in this situation are different from those in Proposition 1 (2). Literally, the results are ambiguous.

4. Concluding Remarks

Based on a horizontally differentiated duopoly model, i.e., Singh and Vives (1984), including network effects and product compatibility, we have demonstrated that the Cournot equilibrium is more efficient than the Bertrand equilibrium in terms of

consumer surplus and social surplus, given certain conditions such as sufficient asymmetric network compatibility effects between the firms.

We understand the limitation of our model based on specific assumptions such as the linearity of various functions, duopoly, and Assumption 1, i.e., network effects are larger than product substitutability effects. Thus, unless Assumption 1 holds, we obtain the same results as in Singh and Vives (1984). In other words, the assumption of strong network effects and asymmetric product compatibility follows our main results, Proposition 2 (i).

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