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## **Price and quantity competition in a differentiated duopoly with network compatibility effects**

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# Price and quantity competition in a differentiated duopoly with network compatibility effects

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## **Abstract**

We consider endogenous choice of the strategic variables, price and quantity, in a horizontally differentiated duopoly market, assuming network effects and product compatibility (hereafter, network compatibility effects). We demonstrate the following. If the degree of network compatibility effects of the other rival firm is smaller (larger) than the degree of product substitutability, then choosing quantity (price) is a dominant strategy for the firm. In this case, the Cournot (Bertrand) equilibrium arises. If there are asymmetric network compatibility effects between the firms, the firm with larger (smaller) network compatibility effects than the degree of product substitutability chooses quantity (price). In this case, the Cournot–Bertrand equilibrium arises.

*Keywords:* Bertrand equilibrium; Cournot equilibrium; Cournot–Bertrand equilibrium; product compatibility; network effect; fulfilled expectation; horizontally differentiated duopoly

*JEL Classifications:* C72; D01; D43; L13

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## 1. Introduction

The analysis of endogenous strategy choice, i.e., price and quantity, in market competition is an important problem in the field of industrial organization. Based on a differentiated duopoly model, Singh and Vives (1984) consider endogenous strategy choice in a two-stage game, in which the firms each choose either a quantity contract or a price contract in the first stage and then they compete on the chosen strategy in the second stage. They demonstrate that choosing a quantity (price) contract is a dominant strategy if the products are substitutes (complements). Thus, the Cournot (Bertrand) equilibrium arises. See also Cheng (1985).

Since the seminal article of Singh and Vives (1984), several studies have appeared that related to ours. For example, Tanaka (2001a) demonstrates that choosing a quantity strategy is dominant for both firms in the case of a vertically differentiated products market. Furthermore, based on a differentiated oligopoly model, Tanaka (2001b) shows that a quantity strategy is the best response for each firm when all other firms choose a price strategy. Using the model of Zanchettin (2006), in which asymmetric costs are assumed, Wang (2008) considers a two-stage game with endogenous strategy choice. He shows that the two-stage game has a weakly-dominant strategy equilibrium in which both firms choose a quantity contract in the first stage, and if both firms produce a positive output level, then the Bertrand equilibrium does not hold.

Recently, Chirco and Scrimatore (2013) consider strategy choice, focusing on the role of delegation under network effects. They show that both firms choose a quantity contract in the case of no-delegation, whereas a price contract is the unique equilibrium in the case of delegation if certain conditions regarding the parameters of the network

effect and product differentiation are satisfied. Furthermore, using the demand function of Chirco and Scrimatore (2013), but not assuming delegation, Pal (2014), which is closely related to our paper, demonstrates that choosing a quantity contract is the dominant strategy regardless of the strength of network effects, and that the profit in the Bertrand equilibrium is larger than that in the Cournot equilibrium unless network effects are weak.

We reconsider endogenous choice of the strategic variables, price and quantity, introducing product compatibility into the network effects models of Katz and Shapiro (1985) and Economides (1996). In this paper, denoting the combination of network effects and product compatibility as network compatibility effects, we consider the role of asymmetric network compatibility effects between firms.

We demonstrate the following:

(i) Singh and Vives case. If the degree of network compatibility effects of both firms is smaller (larger) than the degree of product substitutability, choosing quantity is a dominate strategy for both firms. In this case, the Cournot (Bertrand) equilibrium arises.

(ii) Asymmetric network compatibility effects case. When there is asymmetry between the network compatibility effects of the products produced by firms, the firm producing the product with degree of network compatibility effects that are larger (smaller) than the degree of product substitutability chooses quantity (price). In this case, the Cournot–Bertrand equilibrium arises.

With respect to the Cournot–Bertrand equilibrium, e.g., Tremblay and Tremblay (2011) and Tremblay et al (2011) present the example that Scion (and Saturn) dealers behave as a Bertrand-type firm whereas Honda (and Subaru) dealers behave as a Cournot-type firm.

## 2. The Model

### 2.1 The Cournot equilibrium

We consider Cournot quantity competition in horizontally differentiated products with network compatibility effects. Using the framework of Economides (1996), the linear inverse demand function of product  $i$  is given as:

$$p_i = A - q_i - \theta q_j + f(S_i^e), \quad i, j = 1, 2, i \neq j, \quad (1)$$

where  $A$  is the intrinsic market size,  $q_i$  ( $q_j$ ) is the quantity of firm  $i$  ( $j$ ), and  $\theta \in (0, 1)$  is the degree of product substitutability. The network externality function is given by  $f(S_i^e)$ , where  $S_i^e$  is the expected network size of firm  $i$ . Based on the concept of fulfilled expectations, we assume that  $S_i^e = S_i$ , where  $S_i$  is the real network size of firm  $i$ . Using equation (3.15) in Shy (2001, p. 62), the real network size of firm  $i$  is given by:

$$S_i = q_i + \alpha_i q_j, \quad i, j = 1, 2, i \neq j, \quad (2)$$

where  $\alpha_i \in [0, 1]$ ,  $i = 1, 2$ , denotes the degree of product  $i$ 's compatibility with product  $j$ . Equation (2) implies that firm  $i$  will provide a compatible product with which the rival firm's product  $j$  can operate. If  $\alpha_i = 1$  ( $0$ ),  $i = 1, 2$ , a user of product  $i$  operates (does not operate) perfectly with product  $j$ .  $q_i$  ( $\alpha_i q_j$ ),  $i, j = 1, 2, i \neq j$ , represents the *own* (*incoming*) effect on the network size. Furthermore, we assume a linear network effect function;  $f(S_i) = aS_i$ , where  $a \in (0, 1)$  is the network effect parameter of network

size.

Based on equations (1) and (2), the inverse demand function of firm  $i$  can be expressed as:

$$p_i = A - (1 - a)q_i - (\theta - a\alpha_i)q_j, \quad i, j = 1, 2, i \neq j. \quad (3)$$

For equation (3), we assume that the own-price effect exceeds the cross-price effect,

i.e.,  $\left| \frac{dp_i}{dq_i} \right| > \left| \frac{dp_i}{dq_j} \right|, i, j = 1, 2, i \neq j.$  Thus, it follows that  $1 - a > |\theta - a\alpha_i| > 0, i = 1, 2.$

Furthermore, regarding the network effect parameter, we assume the following:

*Assumption 1:*  $a > \theta.$

That is, the network effects are stronger than the product substitutability effects. Otherwise, as  $\theta > a,$  a strategic substitutes relationship arises, because  $1 \geq \alpha_i, i = 1, 2.$  In the case of price competition, a strategic complements relationship arises.

To simplify the analysis, we assume that production costs and fixed costs are both zero. In other words, this assumption implies that there are no capacity constraints. Thus, the profit of firm  $i$  is expressed as  $\pi_i = p_i q_i, i = 1, 2.$

As shown below, although the products are horizontally differentiated,  $1 > \theta > 0,$  the products are complements (substitutes) if the degree of product compatibility with network effects is larger (smaller) than the degree of product substitutability,  $a\alpha_i > (<) \theta, i = 1, 2.$  This nature of the products determines the strategic relationships between the firms and the external effects on their profits, and thus, in turn, affects the choice of strategic variables. Hereafter, we denote the combination of product

compatibility and network effects,  $a\alpha_i$ ,  $i = 1, 2$ , as network compatibility effects.

Considering equation (3), we derive the reaction function for firm  $i$  as follows:

$$q_i = \frac{A}{2(1-a)} - \frac{\theta - a\alpha_i}{2(1-a)} q_j, \quad i, j = 1, 2, i \neq j. \quad (4)$$

Given equation (4), the strategic relationship between the firms depends on the degree of product substitutability and the degree of network compatibility effects:

$$\frac{\partial q_i}{\partial q_j} < (>) 0 \Leftrightarrow \theta > (<) a\alpha_i, \quad i, j = 1, 2, i \neq j. \quad (5)$$

Equation (5) implies that a strategic complements (substitutes) relationship between the firms holds if the degree of network compatibility effects is larger (smaller) than that of product substitutability.

Using the first-order profit-maximization condition, the profit function is  $\pi_i = (1-a)(q_i)^2$ ,  $i = 1, 2$ . Thus, in view of equation (5), we derive the external effect of an increase in the output of firm  $j$  on the profit of firm  $i$  as follows:

$$\frac{\partial \pi_i}{\partial q_j} = 2(1-a)q_i \frac{\partial q_i}{\partial q_j} < (>) 0 \Leftrightarrow \theta > (<) a\alpha_i, \quad i, j = 1, 2, i \neq j. \quad (6)$$

For the following analysis, regarding the degree of product compatibility, we assume the following:

*Assumption 2:*  $1 \geq \alpha_1 > \alpha_2 \geq 0$ .

Given equation (4), we derive the following Cournot equilibrium:

$$q_i^c = \frac{A\{2(1-a) - (\theta - a\alpha_i)\}}{D}, \quad (7)$$

where  $D \equiv 4(1-a)^2 - (\theta - a\alpha_1)(\theta - a\alpha_2) > 0$  and  $2(1-a) - (\theta - a\alpha_i) > 0$ ,  $i = 1, 2$ .

Both of these conditions are satisfied because the own-price effect exceeds the cross-price effect. Superscript  $C$  denotes the Cournot equilibrium.

## 2.2 The Bertrand equilibrium

Taking equation (3) into account, we derive the direct demand function of firm  $i$  as follows:

$$q_i = \frac{\{(1-a) - (\theta - a\alpha_i)\}A - (1-a)p_i + (\theta - a\alpha_i)p_j}{\Delta}, \quad i, j = 1, 2, i \neq j, \quad (8)$$

where  $\Delta \equiv (1-a)^2 - (\theta - a\alpha_1)(\theta - a\alpha_2) > 0$ . Based on equation (8), the reaction function for firm  $i$  is given by:

$$p_i = \frac{\{(1-a) - (\theta - a\alpha_i)\}A}{2(1-a)} + \frac{\theta - a\alpha_i}{2(1-a)} p_j, \quad i, j = 1, 2, i \neq j. \quad (9)$$

Thus, the strategic relationship between the firms depends on the degree of product substitutability and the degree of network compatibility effects:

$$\frac{\partial p_i}{\partial p_j} > (<) 0 \Leftrightarrow \theta > (<) a\alpha_i, \quad i, j = 1, 2, i \neq j. \quad (10)$$

Equation (10) implies that a strategic complements (substitutes) relationship between the firms exists if the degree of product substitutability is higher (lower) than that of network compatibility. Furthermore, we derive the external effect of an increase in the output of firm  $j$  on the profit of firm  $i$  as follows:

$$\frac{\partial \pi_i}{\partial p_j} > (<) 0 \Leftrightarrow \theta > (<) a\alpha_i, \quad i, j = 1, 2, i \neq j. \quad (11)$$

Given equation (10), we derive the following Bertrand equilibrium:

$$p_i^B = \frac{A\{2(1-a)^2 - (1-a)(\theta - a\alpha_i) - (\theta - a\alpha_1)(\theta - a\alpha_2)\}}{D}, \quad (12)$$

where  $2(1-a)^2 - (1-a)(\theta - a\alpha_i) - (\theta - a\alpha_1)(\theta - a\alpha_2) > 0$   $i=1,2$ . This condition is satisfied because the own-price effect exceeds the cross-price effect. Superscript  $B$  denotes the Bertrand equilibrium.

### 2.3 Quantity–price mixed duopoly and the Cournot–Bertrand equilibrium

Before considering the endogenous choice of strategic variables, we derive the Cournot–Bertrand equilibrium. Without losing of generality, we assume that firm  $i$  ( $j$ ) chooses quantity (price) as the strategic variable in the market competition. Taking equations (3) and (8) into account, we derive the following:

$$p_i = \frac{\{(1-a) - (\theta - a\alpha_i)\}A - \Delta q_i + (\theta - a\alpha_i)p_j}{1-a}, \quad (13)$$

$$q_j = \frac{A - p_j - (\theta - a\alpha_j)q_i}{1-a}, \quad (14)$$

where  $i, j=1,2, i \neq j$ . Based on equations (13) and (14), from the first-order profit maximization conditions of each firm, we derive the following reaction functions:

$$q_i = \frac{\{(1-a) - (\theta - a\alpha_i)\}A + (\theta - a\alpha_i)p_j}{2\Delta}, \quad (15)$$

$$p_j = \frac{A - (\theta - a\alpha_j)q_i}{2}, \quad (16)$$

where  $i, j=1,2, i \neq j$ . Thus, regarding the strategic relationships, it follows that

$\frac{\partial q_i}{\partial p_j} > (<)0 \Leftrightarrow \theta > (<)a\alpha_i$  and  $\frac{\partial p_j}{\partial q_i} < (>)0 \Leftrightarrow \theta > (<)a\alpha_j$ ,  $i, j=1,2, i \neq j$ . Furthermore,

regarding the external effect on the profit, it follows that  $\frac{\partial \pi_i}{\partial p_j} > (<)0 \Leftrightarrow \theta > (<)a\alpha_i$  and

$$\frac{\partial \pi_j}{\partial q_i} < (>)0 \Leftrightarrow \theta > (<)a\alpha_j, \quad i, j = 1, 2, i \neq j.$$

Considering equations (15) and (16), we obtain the following Cournot–Bertrand equilibrium:

$$q_i^M = \frac{A\{2(1-a) - (\theta - a\alpha_i)\}}{D - 2(\theta - a\alpha_1)(\theta - a\alpha_2)}, \quad (17)$$

$$p_j^M = \frac{A\{2(1-a)^2 - (1-a)(\theta - a\alpha_j) - (\theta - a\alpha_1)(\theta - a\alpha_2)\}}{D - 2(\theta - a\alpha_1)(\theta - a\alpha_2)}, \quad (18)$$

where  $i, j = 1, 2, i \neq j$ , and superscript  $M$  denotes the Cournot–Bertrand equilibrium.

### 3. Endogenous choice of strategic variables and network compatibility effects

Based on the analysis in Section 2, we express the profit in the Cournot equilibrium as:

$$\pi_i^C[Q_i, Q_j] = (1-a)(q_i^C)^2, \quad i = 1, 2. \quad (19)$$

In equation (19),  $Q_i$  ( $Q_j$ ) is the strategic variable, i.e., quantity, of firm  $i$  ( $j$ ). Similarly,

the profit in the Bertrand equilibrium is expressed as:

$$\pi_i^B[P_i, P_j] = \frac{1-a}{\Delta} (p_i^B)^2, \quad i = 1, 2, \quad (20)$$

where  $P_i$  ( $P_j$ ) is the strategic variable, i.e., price, of firm  $i$  ( $j$ ). Furthermore, regarding the profits in the Cournot–Bertrand equilibrium, where firm  $i$  ( $j$ ) chooses quantity (price), their profits are expressed as:

$$\pi_i^M [Q_i, P_j] = \frac{\Delta}{1-a} (q_i^M)^2, \quad (21)$$

$$\pi_j^M [Q_i, P_j] = \frac{1}{1-a} (p_j^M)^2, \quad (22)$$

where  $i, j = 1, 2, i \neq j$ .

With respect to the endogenous choice of strategic variables, without loss of the generality, we compare the profits of firm  $I$  as follows:

$$\pi_1^C [Q_1, Q_2] > (<) \pi_1^M [P_1, Q_2], \quad (23)$$

$$\pi_1^M [Q_1, P_2] > (<) \pi_1^B [P_1, P_2]. \quad (24)$$

First, regarding equation (23), taking equations (7), (18), (19), and (22), we derive the following relationship.

$$\pi_1^C [Q_1, Q_2] > (<) \pi_1^M [P_1, Q_2] \Leftrightarrow \theta > (<) a\alpha_2. \quad (25)$$

Second, regarding equation (24), taking equations (12), (17), (20), and (21), we derive the following relationship.

$$\pi_1^M [Q_1, P_2] > (<) \pi_1^B [P_1, P_2] \Leftrightarrow \theta > (<) a\alpha_2. \quad (26)$$

The same results apply to firm 2. That is, we have the following relationships.

$$\pi_2^C [Q_1, Q_2] > (<) \pi_2^M [Q_1, P_2] \Leftrightarrow \theta > (<) a\alpha_1, \quad (27)$$

$$\pi_2^M [P_1, Q_2] > (<) \pi_2^B [P_1, P_2] \Leftrightarrow \theta > (<) a\alpha_1. \quad (28)$$

Taking equations (25), (26), (27), and (28), into account, we derive *Lemma 1* as follows:

*Lemma 1*

*If the degree of network compatibility effects of the rival firm's product is larger*

*(smaller) than that of product substitutability, the firm chooses price (quantity).*

It is important that the optimal choice of the strategies for the firm depends on the nature of the rival firm's product. That is, if the degree of network compatibility effects of the rival firm's product is larger than that of product substitutability, the strategic relationship of the rival firm with respect to the firm is one of complement goods. In other words, the rival firm's product is compatible with the firm's product. Thus, following Singh and Vives (1984), in this case, the firm commits to a price contract, but not a quantity contract. This is because the firm can adjust their quantity in response to a change in the quantity or price of the rival firm, given the contracted price. Suppose that the firm commits to a quantity contract. If the rival firm cuts its price or increases its quantity, the firm cannot increase its quantity. Thus, the quantity contract is not optimal for the firm. Contrarily, if the degree of network compatibility effects of the rival firm's product is smaller than that of product substitutability, the strategic relationship of the rival firm with respect to the firm is one involving substitute goods. In other words, the rival firm's product is less compatible or incompatible with the firm's product. Thus, the firm commits to a quantity contract, but not a price contract. Suppose that the firm commits to a price contract. If the rival firm cuts its price, the firm cannot change the price contract. On the other hand, if the firm commits to a quantity contract, the firm can cut their price in response to the rival firm's price cut, given the contracted quantity.

Therefore, based on *Assumptions 1* and *2*, and *Lemma 1*, we present the main result as follows.

*Proposition 1*

(i) *If it holds that  $\theta > a\alpha_1 > a\alpha_2$ , choosing quantity is a dominant strategy for both firms. In this case, the Cournot equilibrium arises.*

(ii) *If it holds that  $a\alpha_1 > a\alpha_2 > \theta$ , choosing price is a dominant strategy for both firms. In this case, the Bertrand equilibrium arises.*

(iii) *If it holds that  $a\alpha_1 > \theta > a\alpha_2$ , choosing quantity is a dominant strategy for firm 1 and choosing price is a dominant strategy for firm 2. In this case, the Cournot–Bertrand equilibrium arises.*

In the case of *Proposition 1 (i)*, the products of the two firms are substitute because the network compatibility effects of both products are weak. Thus, this corresponds to the case of substitute goods in Proposition 2 of Singh and Vives (1984). On the other hand, in the case of *Proposition 1 (ii)*, the products of both firms are complements because the network compatibility effects of both products are strong, although the products themselves are substitutes in the sense of horizontal product differentiation. Thus, this corresponds to the case of complement goods in Proposition 2 of Singh and Vives (1984).

If we assume either the case of symmetric product compatibility (i.e.,  $\alpha_1 = \alpha_2 = \alpha$ ) or the case of weak network effects (i.e.,  $\theta \geq a$ ), we have the same results as those of Singh and Vives (1984). However, we demonstrate that if there is asymmetric product compatibility with large network effects, i.e., given significantly asymmetric network compatibility effects between the products of the firms, the Cournot-Bertrand equilibrium arises. In other words, although the horizontally differentiated products in

our model are substitutes, if the network compatibility effects are sufficiently strong, the strategic relationship between the firms is one involving complement goods. Furthermore, if there is a substantial asymmetry between the network compatibility effects of the two products, the Cournot-Bertrand equilibrium arises.

#### 4. Conclusion

We introduce network effects and product compatibility, i.e., network compatibility effects, into the model of Singh and Vives (1984), which assumes linear demand and cost functions, no fixed costs, and no capacity constraints. In particular, we demonstrate that if there are asymmetric network compatibility effects between the products, the Cournot-Bertrand equilibrium arises. In this case, choosing quantity (price) is a dominant strategy for the firm producing the product with larger (smaller) network compatibility effects.

We appreciate the shortcomings imposed by the specific assumptions of the model, e.g., linearity of the demand, cost, and network functions. Thus, the issues discussed in this paper should also be examined using more general functions. Furthermore, we analyze the issues in the case of oligopoly. We also assume that the network compatibility effects are exogenous. However, we should consider why asymmetric network compatibility effects between firms arise because product compatibility is an important strategic variable.

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