DISCUSSION PAPER SERIES

Discussion paper No. 125

Population Aging and Growth: the Effect of PAYG Pension Reform

Ken Tabata

School of Economics, Kwansei Gakuin University

January, 2015

MASTERY FOR SERVICE

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho

Nishinomiya 662-8501, Japan

Population Aging and Growth: the Effect of PAYG Pension Reform

Ken Tabata *

January 6, 2015

Abstract

This paper examines how pay-as-you-go (PAYG) pension reform from a defined-benefit scheme to a defined-contribution scheme affects economic growth in an overlapping generations model with endogenous growth. We show that in economies in which the old-age dependency ratio is relatively high and the size of pension benefits under a defined-benefit scheme is relatively large, PAYG pension reform from a defined-benefit scheme to a defined-contribution scheme mitigates the negative growth effect of population aging caused by a decline in the population growth rate or an increase in life expectancy.

Keywords: Population aging, PAYG pensions, Defined-benefit schemes, Defined-contribution schemes
JEL classification: D91, H55, O41

*Address: School of Economics, Kwansei Gakuin University, 1-155 Uegahara Ichiban-cho, Nishinomiya 662-8501, Hyogo, Japan; Fax: +81 798 51 0944, Tel: +81 798 54 4655, E-mail: tabataken@kwansei.ac.jp

1 Introduction

Many developed countries that employ pay-as-you-go (PAYG) pension systems maintain elderly living standards by adjusting pension benefits to maintain the ratio of pensions to average wages of working generations at a constant value (i.e., wage indexation). Under this type of defined-benefit PAYG pension scheme, declining birth rates and aging populations have necessitated increases in the pension contributions of younger generations. However, because of the rapid pace at which populations are aging, the conventional approach (i.e., increasing contributions to maintain pension benefit levels) imposes an excessive social security tax burden on younger generations and endangers the sustainability of PAYG pension systems.

Faced with these challenges, policy makers in several developed countries have abandoned the traditional practice of continuously increasing contributions. Instead, they employ a demographically modified indexation program. Under a demographically modified indexation program, if the number of workers supporting the pension system decreases, then the wage indexation rate for pension benefits also decreases even if the average wages of the working generations increase. Hence, elderly pensions are adjusted automatically based on changes in demographic conditions. The introduction of a demographically modified indexation program paves the way to a cap on future contribution increases or to a fixed-contribution PAYG pension scheme. For example, the 2004 pension reform in Japan moved away from the standard practice of increasing employee pension contributions to guarantee a 59% benefit level and instead capped future contributions at 18.3% (i.e., a fixed-contribution program). In addition, Japan introduced a demographically modified indexation program to ensure that the size of pension benefits was consistent with the new contribution cap.[1] Today, many other OECD countries are introducing automatic pension benefit adjustment mechanisms and modified indexation systems to prevent unrestricted future contribution increases.[2] From a theoretical perspective, these recent PAYG pension reforms signify a transition from defined-benefit schemes to defined-contribution schemes.

This paper develops a two-period OLG model of endogenous growth with a PAYG pension scheme and examines how PAYG pension reform from a defined-benefit scheme to a defined-contribution scheme affects economic growth. We show that in economies in which the old-age dependency ratio is relatively high and the size of pensions under a defined benefit scheme is relatively large, PAYG

[1] The new pension level established by the 2004 pension reform was 50%. For more information regarding the 2004 pension reform in Japan, see, for example, Komamura (2004).

[2] For example, Denmark, Greece, Hungary, Italy, Korea and Turkey each have linked future increases in pension ages to changes in life expectancy. In addition, Sweden introduced a defined-contribution PAYG pension scheme in 1999. See OECD (2013) for additional details.

pension reform from a defined-benefit scheme to a defined-contribution scheme positively affects economic growth. Furthermore, we consider how PAYG pension reform from a defined-benefit scheme to a defined-contribution scheme affects the relationship between population aging and economic growth. We show that in economies in which the old-age dependency ratio is relatively high and the size of pensions under a defined benefit scheme is relatively large, this type of pension reform mitigates the negative growth effect of population aging caused by a decline in the population growth rate or by an increase in life expectancy.

This paper is related to the vast body of literature that examines the effects of population aging on the sustainability of a PAYG pension system and on economic growth. Relevant studies include those by Pecchenino and Pollard (1997), Wigger (1999), Yakita (2001), Zhang et al. (2001, 2003, 2005), Gradstein and Kaganovich (2004) and, more recently, Fanti and Gori (2012) and Cipriani (2014). Using various stylized models with different engines of growth, these studies rigorously examine how economic growth is affected by changes in the size of PAYG pensions (e.g., Wigger, 1999), by the transition from an unfunded to a funded pension system (e.g., Pecchenino and Pollard, 1997), and by population aging (e.g., Yakita, 2001; Zhang et al., 2001, 2003, 2005). Furthermore, Fanti and Gori (2012) and Cipriani (2014) debate the effects of population aging on the sustainability of PAYG pension systems. However, to the best of our knowledge, there are few theoretical studies that examine the impact of a transition from a defined-benefit scheme to a defined-contribution scheme on economic growth, despite the fact that this type of PAYG pension reform has recently become common among OECD countries. Thus, this paper seeks to fill the gap between theoretical research and policy debates in the political arena.

During the course of writing this paper, I found one exceptional study by Artige et al. (2014) that compares the respective macroeconomic performances of a defined-benefit scheme and a defined-contribution scheme in an aging society.[3] Using a two-period OLG model of neoclassical growth with general utility and production functions, Artige et al. (2014) study how a decline in the fertility rate and an increase in longevity affect capital accumulation and welfare under different PAYG pension schemes. Artige et al. (2014) find that the effect of a decline in the fertility rate or an increase in longevity on capital accumulation is always positive under a defined-contribution scheme, whereas the effect is ambiguous under a defined-benefit scheme. Then, they show that if two economies are initially dynamically efficient and are identical in all respects except pension schemes, the result of identical declines in fertility rates and identical increases in longevity is that the welfare level of the defined-benefit economy is lower than that of the defined-contribution economy. In Artige et al. (2014), the descrip-

[3]I am grateful that one of the referees apprised me of this related work.

tion of these two economies as initially identical means that the initial size of PAYG pensions or the demographic conditions in each of the two economies are adjusted to ensure that all initial endogenous economic outcomes are identical. Based on the theoretical results, Artige et al. (2014) conclude that in an aging society, the defined-contribution scheme dominates the defined-benefit scheme in terms of both income per capita and welfare.

Given that Artige et al. (2014) offer a comprehensive analysis with general utility and production functions, there are many common research themes and theoretical implications between the present study and the study of Artige et al. (2014). However, this paper provides several original contributions that complement the analysis of Artige et al. (2014) and differs from Artige et al. (2014) in at least the two respects. First, to facilitate the comparison of our results with the results obtained in previous studies on population aging and growth, we develop a two-period OLG model of endogenous growth à la Romer (1986). Studies such as Yakita (2001), Gradstein and Kaganovich (2004) and Zhang et al. (2001, 2003b, 2005), among others, examine how population aging caused by a decline in the fertility rate or an increase in longevity affects economic growth in an OLG model of endogenous growth with social security. In particular, Tabata (2005), Ito and Tabata (2008) and Mizushima (2009) propose various growth models that explain the empirically observed hump-shaped relationship between the old-age dependency ratio and economic growth; these growth models emphasize the design of the social security system.[4] However, to the best of our knowledge, no study to date has examined how the transition from a defined-benefit scheme to a defined-contribution scheme affects existing theoretical predictions regarding the effect of population aging on growth. Therefore, this paper provides new insight into the relationship between population aging and economic growth.

Second, this paper focuses on the role of wage indexation of pension payments and proposes a tractable PAYG pension scheme that possesses both defined-benefit and defined-contribution aspects (i.e., a mixed payment scheme).[5] With

[4]Recent empirical studies by An and Joen (2006) and Prskawetz et al.(2007) suggest a non-monotonic relationship between population aging and economic growth. Using panel data from OECD countries during the period 1960-2000, An and Joen (2006) demonstrate a hump-shaped relationship between the old-age dependency ratio and economic growth. Furthermore, using data from EU-15 countries for the period 1900-2005, Prskawetz et al. (2007) find that an increase in the working-age population (ages 50-64) positively contributes to economic growth, whereas a large old-age population and a large young-age population each has a negative effect on economic growth. Recent theoretical studies (e.g., de la Croix and Licandro, 1999; Boucekkine et al., 2002; Miyazawa, 2006; and Kunze, 2014) attempt to explain the observed non-monotonic relationship between population aging and economic growth.

[5]Artige et al. (2014) employ a lump-sum tax and a lump-sum pension payment and only compare a pure defined-contribution scheme to a pure defined-benefit scheme. A wage indexation specification might be more realistic and plausible in the context of a growing economy.

the introduction of demographically modified indexation programs, PAYG pension schemes in many OECD countries possess both defined-benefit and defined-contribution attributes. Our simple formulation of a PAYG pension scheme enables us to capture these features of recent PAYG pension schemes in a reduced-form manner. Moreover, combined with our log-utility specifications, this formulation enables us to clarify the economic preconditions under which the transition from a defined-benefit scheme to a defined-contribution scheme will positively (negatively) affect capital accumulation and economic growth.

This paper is organized as follows. Section 2 establishes the basic model. Section 3 discusses the impact of PAYG pension reform from a defined-benefit scheme to a defined-contribution scheme on existing theoretical predictions regarding the effect of population aging on growth. Section 4 concludes the paper.

2 The model

We consider a two-period overlapping generations model with PAYG pensions. In each period, L individuals are born, and each individual is endowed with one unit of labor. The population grows at the constant rate $n \in (-1, \infty)$. Individuals live for a maximum of two periods (youth and old age). An individual dies at the beginning of old age with a probability of $1 - p$ and lives through old age with a probability of $p \in (0, 1]$. In each period t, there exists only two generations: the active working young and the retired old. Therefore, the old-age dependency ratio (i.e., the ratio of old dependents to the young working population) in period t is given by $\frac{pL_{t-1}}{L_t} = \frac{p}{1+n}$. Thus, population aging is triggered if there is either a decrease in the population growth rate n or an increase in the old-age survival probability p.

2.1 Agents

Each individual derives utility from his or her own consumption in both youth and old age. Thus, the lifetime expected utility of an individual born at period t is expressed as

$$u_t = \ln c_t + p \ln d_{t+1}, \tag{1}$$

where c_t is consumption in youth and d_{t+1} is consumption in old age. As in Yakita (2001) and Cipriani (2014), the instantaneous utility function is assumed to be logarithmic for the sake of tractability.

In youth, each individual inelastically supplies one unit of labor, earns wage income w_t, and allocates w_t to his or her own current consumption c_t, saving s_t and the payment of social security taxes $\tau_t w_t$. Following Blanchard (1985), we assume the existence of an actuarially fair insurance for the sake of simplicity.

The insurance company promises each individual a payment $\frac{R_{t+1}}{p}s_t$ in exchange for which the individual's estate s_t accrues to the company, where p is the average survival probability and R_{t+1} is the gross rate of interest. In old age, survivors retire and consume their returns on private annuities $\frac{R_{t+1}}{p}s_t$ and pension benefits b_{t+1}.

Thus, the lifetime budget constraints are

$$c_t + s_t = (1 - \tau_t)w_t, \tag{2}$$

$$d_{t+1} = \frac{R_{t+1}}{p}s_t + b_{t+1}. \tag{3}$$

By maximizing (1), subject to (2) and (3), we obtain

$$s_t = \frac{p}{1+p}[(1 - \tau_t)w_t - \frac{b_{t+1}}{R_{t+1}}]. \tag{4}$$

This savings equation states that a higher old-age survival probability p implies higher savings, whereas both higher pension benefits b_{t+1} and higher tax rates τ_t imply lower savings.

2.2 Production

Each firm has constant returns-to-scale technology, and the aggregate production function is expressed as $Y_t = F(K_t, A_t L_t)$, where Y_t, K_t, and L_t denote the aggregate levels of output, physical capital, and labor input, respectively. A_t represents labor productivity, which is assumed to be driven by a positive spillover from the size of aggregate capital stock to the productivity of workers in the manner suggested by Romer (1986). To ensure the existence of a balanced growth path, as in Grossman and Yanagawa (1993), it is assumed that A_t takes the following form: $A_t = \frac{1}{a}\frac{K_t}{L_t} = \frac{k_t}{a}$, where $k_t = K_t/L_t$ and a is a positive technological parameter. Representing the production function in effective per capita terms, $\hat{y}_t = f(\hat{k}_t)$, where $\hat{y}_t = Y_t/A_t L_t$, $\hat{k}_t = K_t/A_t L_t$, we obtain the optimal condition for a representative firm as follows: $R_t = 1 + f'(a) - \delta = R$, $w_t = [\frac{f(a)-f'(a)a}{a}]k_t = \bar{w}k_t$. The interest rate and the wage rate per labor in efficiency units are constant over time.

To facilitate the comparison of our results with those of the existing literature on population aging and growth, we adopt a simple Romer-type spillover specification that generates an AK-type endogenous growth path without transition. Although this simplification clearly restricts the scope of our analysis, it yields intuitive and manageable results.

2.3 Government

In this economy, the government pursues a social security policy based on a PAYG pension system with a defined-benefit scheme. Let us assume that the government balances its budget in each period and faces the following budget constraints:

$$\tau_t w_t L_t = b_t p L_{t-1}. \tag{5}$$

To formulate the transition from a defined-benefit scheme to a defined-contribution scheme in the simplest manner, we focus on the role of wage indexation. More specifically, the pension payment for old agents in generation $t-1$, b_t, is adjusted according to changes in the next generation's wages w_t:

$$b_t = [\eta \frac{L_t}{pL_{t-1}} \psi + (1-\eta)\phi] w_t = [\eta \frac{1+n}{p} \psi + (1-\eta)\phi] w_t, \tag{6}$$

where $\eta \frac{1+n}{p} \psi + (1-\eta)\phi$ denotes the replacement ratio for the next generation's wages w_t. The replacement ratio is defined as a linear combination of a demographic adjustment component (i.e., $\frac{1+n}{p}\psi$) and a fixed coefficient component (i.e., ϕ). The index $\eta \in [0,1]$ measures the weight given to the demographic adjustment component. A demographic adjustment component (i.e., $\frac{1+n}{p}\psi$) characterizes a demographically modified indexation in which the wage indexation rate for pension benefits decreases automatically as the size of the old-age population increases relative to the size of the working population. In addition, the index $\psi \in [0,1)$ measures the size of pension payments in the demographic adjustment component, whereas $\phi \in [0,1)$ measures the size of pension payments in the fixed coefficient component.

Given (6), suppose that $\eta = 0$, the per capita pension benefit of generation $t-1$, is given simply by $b_t = \phi w_t$ and that the wage replacement ratio is a fixed constant at $\phi \in [0,1)$ irrespective of the value of the old-age dependency ratio $\frac{p}{1+n}$. In this case, the PAYG pension scheme is characterized by a pure defined-benefit scheme with a fixed replacement ratio $\phi \in [0,1)$. Next, suppose $\eta = 1$, the per capita pension benefit of generation $t-1$, is fully adjusted in response to changes in demographic conditions and satisfies the following equality condition: $\psi w_t L_t = b_t p L_{t-1}$. This equality condition is the well-known formulation of government budget constraints under a pure defined-contribution scheme. Therefore, in this case, the PAYG pension scheme is characterized by a pure defined-contribution scheme with a fixed contribution rate $\psi \in [0,1)$. Finally, suppose $\eta \in (0,1)$. In this case, the PAYG pension scheme is characterized by a linear combination of a defined-benefit scheme with a fixed replacement ratio $\phi \in [0,1)$ and a defined-contribution scheme with a fixed contribution rate $\psi \in [0,1)$. We refer to this type of pension system as a mixed payment scheme. Under a mixed payment scheme, as the value of η approaches 1, the weight given to the defined-contribution component increases. Therefore, the transition from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0,1)$ to a defined-contribution scheme with a fixed contribution rate $\psi \in [0,1)$ is described simply by the increase in the value of η from 0 to 1.

As explained in the Introduction, as many OECD countries implement demographically modified indexation programs, their PAYG pension systems possess both defined-benefit and defined-contribution components. Our simple specification enables us to capture this combination of attributes in a reduced-form manner.[6]

By differentiating (6) with respect to η, we obtain

$$\frac{\partial b_t}{\partial \eta} \begin{cases} \leq 0, & \text{for } \frac{p}{1+n} \geq \frac{\psi}{\phi}, \\ > 0, & \text{for } \frac{p}{1+n} < \frac{\psi}{\phi}. \end{cases} \tag{7}$$

Giving a larger weight η to the defined-contribution component implies a lower (resp. higher) per capita pension benefit when (i) the old-age dependency ratio $\frac{p}{1+n}$ is relatively high (resp. low); (ii) the fixed contribution rate ψ under a defined-contribution scheme is relatively small (resp. large); or (iii) the fixed replacement ratio ϕ under a defined-benefit scheme is relatively large (resp. small) to satisfy $\frac{p}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{p}{1+n} < \frac{\psi}{\phi}$).

From equations (5) and (6), the social security tax rate is given by

$$\tau_t = \tau = \eta\psi + (1-\eta)\frac{p}{1+n}\phi \equiv \tau(n, p; \eta). \tag{8}$$

The social security tax rate τ depends on the values of the old-age survival probability p, the population growth rate n and the weight η given to the defined-contribution component. To emphasize these relationships, we describe τ as $\tau(n, p; \eta)$. Because $\tau \in (0,1)$, the parameter condition $\eta\psi + (1-\eta)\frac{p}{1+n}\phi < 1$ must hold. Based on (8), when $\eta = 1$ (i.e., a pure defined-contribution scheme), the social security tax rate is given by $\tau_t = \psi$ and is constant regardless of the value of the old-age dependency ratio $\frac{p}{1+n}$. However, when $\eta \in [0,1)$ (i.e., a pure defined-benefit scheme or a mixed payment scheme), the social security tax rate increases with the old-age dependency ratio $\frac{p}{1+n}$ due to the effects of demographically modified indexation.

By differentiating (8) with respect to η, we obtain

$$\frac{\partial \tau}{\partial \eta} \begin{cases} \leq 0, & \text{for } \frac{p}{1+n} \geq \frac{\psi}{\phi}, \\ > 0, & \text{for } \frac{p}{1+n} < \frac{\psi}{\phi}. \end{cases} \tag{9}$$

[6]Of course, our mixed payment scheme is rather abstract and cannot capture the very complicated structures of recent PAYG pension systems in their entirety. Nevertheless, this simple framework enables us to capture several realistic features of recent pension reforms.

Giving a larger weight η to the defined-contribution component implies a lower (resp. higher) social security tax rate when (i) the old-age dependency ratio $\frac{p}{1+n}$ is relatively high (resp. low); (ii) the fixed contribution rate ψ under a defined-contribution scheme is relatively small (resp. large); or (iii) the fixed replacement ratio ϕ under a defined-benefit scheme is relatively large (resp. small) to satisfy $\frac{p}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{p}{1+n} < \frac{\psi}{\phi}$).

Equations (7) and (9) indicate that in economies in which the old-age dependency ratio is relatively high and the size of pension benefits under a defined-benefit scheme is relatively large, the transition from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0,1)$ to a defined-contribution scheme with a fixed contribution rate $\psi \in [0,1)$ leads to a lower per capita pension benefit as well as a lower social security tax rate. (6) indicates that the social security tax rate increases monotonically with the old-age dependency ratio under a pure defined-benefit scheme but remains constant under a pure defined-contribution scheme. Therefore, when the old-age dependency ratio is relatively high and the size of pension payments under a defined-benefit scheme is relatively large, the transition from a defined-benefit scheme to a defined-contribution scheme negatively affects the overall size of PAYG pension.

2.4 Dynamics

The market-clearing condition for capital is given by $K_{t+1} = s_t L_t$ or $k_{t+1} = \frac{s_t}{1+n}$. Using (8), the per capita pension benefit b_t in (6) is rewritten as $b_t = \frac{1+n}{p}\tau(n,p;\eta)w_t$. By substituting (4), (8), $b_{t+1} = \frac{1+n}{p}\tau(n,p;\eta)w_{t+1}$, $R_t = R$ and $w_t = \bar{w}k_t$ into $k_{t+1} = \frac{s_t}{1+n}$, the gross per capita output growth rate G in this economy is described by

$$\frac{k_{t+1}}{k_t} = \frac{p}{1+n}\frac{1-\tau(n,p;\eta)}{[1+p+\tau(n,p;\eta)\frac{\bar{w}}{R}]}\bar{w} \equiv G(n,p;\eta). \tag{10}$$

We assume that $G > 1$, as in the literature on endogenous growth.[7] From (10), the gross per capita output growth rate G depends on the values of the old age survival probability p, the population growth rate n and the weight η given to the component of a defined-contribution scheme. To stress these relationships, we describe G as $G(n,p;\eta)$.

[7]When $f(\hat{k}) - \hat{k}f'(\hat{k})$ is concave, the relationship $G > 1$ always holds under a sufficiently small value of a because the relationships $\frac{\partial \bar{w}}{\partial a} < 0$ and $\frac{\partial R}{\partial a} < 0$ hold.

3 PAYG pension reform

Our purpose in this paper is to examine how the transition from a defined-benefit scheme to a defined-contribution scheme affects economic growth. We also consider the impact of such a transition on existing theoretical predictions regarding the effect of population aging on economic growth. These issues are rigorously investigated in this section.

3.1 PAYG pension reform and growth

In this subsection, we examine how the transition from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0,1)$ to a defined-contribution scheme with a fixed contribution rate $\psi \in [0,1)$ affects economic growth. In this paper, this type of pension reform is described by the increased weight η given to the defined-contribution component. Because $\frac{\partial G}{\partial \tau} < 0$ from (10), and considering the results of (9), by differentiating (10) with respect to η, we obtain

$$\frac{\partial G}{\partial \eta} = \frac{\partial G}{\partial \tau}\frac{\partial \tau}{\partial \eta} \begin{cases} \geq 0, & \text{for } \frac{p}{1+n} \geq \frac{\psi}{\phi}, , \\ < 0, & \text{for } \frac{p}{1+n} < \frac{\psi}{\phi}. \end{cases} \tag{11}$$

Equation (11) indicates that an increase in the weight η given to the defined-contribution component positively (resp. negatively) affects economic growth when (i) the old-age dependency ratio $\frac{p}{1+n}$ is relatively high (resp. low); (ii) the fixed contribution rate ψ under a defined-contribution scheme is relatively small (resp. large) or (iii) the fixed replacement ratio ϕ under a defined-benefit scheme is relatively large (resp. small) to satisfy $\frac{p}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{p}{1+n} < \frac{\psi}{\phi}$).

Based on (7) and (9), when the condition $\frac{p}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{p}{1+n} < \frac{\psi}{\phi}$) holds, an increase in the weight η given to the defined-contribution component negatively (resp. positively) affects both the per capita pension benefits and the social security tax rate. According to (4), the lower (resp. higher) social security tax rate τ_t and per capita pension benefits b_{t+1} motivate young individuals to save more (resp. less) for their old age, which positively (resp. negatively) affects capital accumulation and economic growth. These results suggest that in economies in which the old-age dependency ratio is relatively high and the size of pension benefits under a defined-benefit scheme is relatively large, the transition from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0,1)$ to a defined-contribution scheme with a fixed contribution rate $\psi \in [0,1)$ negatively affects the overall size of PAYG pension and thus positively affects economic growth.

In our simple model, as inferred from (10), any policy that reduces the size of PAYG pension leads to higher rates of capital accumulation and economic growth.

Therefore, ceteris paribus, a reduction in the fixed contribution rate ψ or fixed replacement ratio ϕ positively affects economic growth. Unlike these straightforward pension reduction policies, as inferred from (10), the effect of a transition from a defined-benefit scheme to a defined-contribution scheme on the overall size of PAYG pension and economic growth depends on the extent of population aging. These features suggest that this type of PAYG pension reform may have a non-trivial impact on the relationship between population aging and economic growth. The following subsection investigates this issue more rigorously.

3.2 PAYG pension reform, population aging and growth

In this subsection, we examine how the transition from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0, 1)$ to a defined-contribution scheme with a fixed contribution rate $\psi \in [0, 1)$ affects the existing theoretical predictions regarding the effect of population aging on economic growth. In this paper, population aging is triggered by either a decline in the population growth rate n or an increase in the old-age survival probability p. Let us first consider the growth effect of population aging caused by a decline in the population growth rate. Based on (10), which concerns the effect of the population growth rate n on gross per capita output growth G, we obtain the following proposition.

Proposition 1 .

1. *Suppose that a PAYG pension is financed by a pure defined-benefit scheme (i.e.,* $\eta = 0$*) or a mixed payment scheme (i.e.,* $\eta \in (0, 1)$*); then, there exists a unique* $\hat{n} \in (-[1 - \frac{p(1-\eta)\phi}{1-\eta\psi}], \infty)$ *such that* $G(\hat{n}, p; \eta) > G(n, p; \eta)$ $\forall n \in (-[1 - \frac{p(1-\eta)\phi}{1-\eta\psi}], \infty)$, $\frac{\partial G(n,p;\eta)}{\partial n} > 0$ $\forall n \in (-[1 - \frac{p(1-\eta)\phi}{1-\eta\psi}], \hat{n})$, $\frac{\partial G(n,p;\eta)}{\partial n} < 0$ $\forall n \in (\hat{n}, \infty)$.

2. *Suppose that a PAYG pension is financed by a pure defined-contribution scheme (i.e.,* $\eta = 1$*); then* $G(n, p; 1)$ *satisfies* $\frac{\partial G(n,p;1)}{\partial n} < 0$ $\forall n \in (-1, \infty)$.

The proof of Proposition 1 is provided in Appendix A. Proposition 1 indicates that when a PAYG pension is financed by a pure defined-benefit scheme (i.e., $\eta = 0$) or a mixed payment scheme (i.e., $\eta \in (0, 1)$), there is a hump-shaped relationship between the population growth rate and the per capita output growth rate. However, when a PAYG pension is financed by a pure defined-contribution scheme (i.e., $\eta = 1$), the relationship between the population growth rate and the per capita output growth rate is always negative. The existence of a hump-shaped relationship between population growth and per capita output growth under a pure defined-benefit scheme has been recognized in the existing literature on population aging and growth (e.g., Ito and Tabata, 2008). This paper confirms that an analogous prediction holds under a mixed payment scheme.

Figure 1 shows numerical examples of the relationship between population growth and per capita output growth under alternative weight values η given to the defined-contribution component (i.e., $\eta = 0, 0.3, 0.6$ and 1). In the simulation, we assume that the production function is given by $Y_t = (K_t)^{\alpha}(A_t L_t)^{1-\alpha}$. The parameters used in the baseline simulations are given as follows: $\alpha = 0.3$, $p = 0.75$, $1 + n = 0.75$, $\phi = 0.5$, $\psi = 0.5$, $\delta = 1$ and $a = 0.02$.[8] Note that the objective of these numerical examples is not to calibrate our simple model to actual data but to supplement the qualitative results. The quantitative results obtained in this paper should be interpreted with caution. Consistent with Proposition 1, when $\eta \in [0, 1)$ (i.e., under a pure defined-benefit scheme or a mixed payment scheme), there is a hump-shaped relationship between the population growth rate and the per capita output growth rate. However, when $\eta = 1$ (i.e., under a pure defined-contribution scheme), the per capita output growth rate increases monotonically as the population growth rate declines.

Equation (10) implies that a decline in the population growth rate n exerts two competing influences on capital accumulation. Under a pure defined-benefit scheme (i.e., $\eta = 0$) or a mixed payment scheme (i.e., $\eta \in (0, 1)$), a decline in the population growth rate n increases the old-age dependency ratio $\frac{p}{1+n}$, which positively affects the social security tax rate τ. This higher social security tax rate τ leads to a lower rate of saving by young individuals, which negatively affects capital accumulation. We denote this negative effect of a decline in n on capital accumulation as the "tax burden effect". Under a pure defined-contribution scheme (i.e., $\eta = 1$), however, the social security tax rate is fixed at ψ irrespective of the value of the old-age dependency ratio. Therefore, the negative growth effect of a decline in n due to the tax burden effect does not exist under a pure defined-contribution scheme (i.e., $\eta = 1$). Conversely, a decline in the population growth rate n mitigates the dilution of savings $\frac{s_t}{1+n}$ and thus positively affects capital accumulation. We denote this positive effect of a decline in n on capital accumulation as the "anti-dilution effect". The positive growth effect of a decline in n because of the anti-dilution effect always exists irrespective of the design of the pension scheme. Therefore, a decline in the population growth rate always increases the per capita output growth rate under a pure defined-contribution scheme

[8] Based on estimates of the capital labor share, we set the value of α as 0.3. The life tables for the United States developed by Bell (1992) indicate that in 2000, the probability of a man surviving to age 65 (conditional on his reaching age 15) is 0.79. Thus, the benchmark value of old-age survival probability p is set to 0.75. In addition, to achieve a TFR of 1.5 per couple, the value of the gross population growth rate $1 + n$ is set to 0.75. To achieve a 50 % benefit level under a pure defined-benefit scheme, the value of the fixed replacement ratio ϕ is set to 0.5. Analogously, to achieve a 50% fixed contribution rate under a pure defined-contribution scheme, the value of the fixed contribution rate ψ is set to 0.5. Finally, to achieve a 2% per capita GDP growth rate at benchmark values, the value of a was adjusted to 0.02.

(i.e., $\eta = 1$).

Based on (10), it is easy to guess that the effect of decreasing n is likely to be negative under a pure defined-benefit scheme (i.e., $\eta = 0$) or a mixed payment scheme (i.e., $\eta \in (0, 1)$), when n is extremely small (i.e., $n \to -1$). Because the ratio of old to young people is large, the positive anti-dilution effect becomes negligible. However, because the social security tax rate is high due to the high dependency ratio, any increase in the social security tax burden stemming from a decline in the population growth rate n causes a considerable negative impact on capital accumulation. Therefore, when n is extremely small, the negative tax burden effect is likely to dominate the positive anti-dilution effect. Conversely, the effect of decreasing n is likely to be positive when n is extremely large (i.e., $n \to \infty$). Because the ratio of old to young in the population is small, the positive anti-dilution effect becomes more serious. However, because the social security tax rate is low due to the low dependency ratio, the negative tax burden effect becomes negligible. Therefore, when n is extremely large, the positive anti-dilution effect is likely to dominate the negative tax burden effect.

Figure 1 also shows how the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) affects the relationship between the population growth rate and the per capita output growth rate. As discussed in Section 3.1, this type of PAYG pension reform positively (resp. negatively) affects economic growth when $\frac{p}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{p}{1+n} < \frac{\psi}{\phi}$), or $1 + n < \frac{p\phi}{\psi}$ (resp. $1 + n > \frac{p\phi}{\psi}$). From (8), the overall social security tax rates remain the same regardless of the value of η when $1+n = \frac{p\phi}{\psi}$. Therefore, based on (10), the per capita output growth rate remains the same regardless of the value of η when $1 + n = \frac{p\phi}{\psi}$, as shown in Figure 1. However, (10) also means that when $1 + n < \frac{p\phi}{\psi}$ (resp. $1 + n > \frac{p\phi}{\psi}$), the overall social security tax rate decreases (resp. increases) with the value of η. Therefore, the per capita output growth rate increases (resp. decreases) with the value of η.

These results suggest that in economies in which the population growth rate is relatively low and the size of pensions under a defined-benefit scheme is relatively large to satisfy $1 + n < \frac{p\phi}{\psi}$, the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) mitigates or overcomes the negative growth effect of population aging caused by a decline in the population growth rate.[9] The intuitions are explained as follows. Under (8), giving a larger

[9]In economies in which the population growth rate is relatively high and the size of pensions under a defined-benefit scheme is relatively small to satisfy $1 + n < \frac{p\phi}{\psi}$, the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) exerts two competing influences on the growth

weight η to the defined-contribution component decreases the impact of changes in the old-age dependency ratio on the social security tax rate, which alleviates the negative growth effect of a decline in n due to the tax burden effect. Furthermore, when $1 + n < \frac{p\phi}{\psi}$, (7) and (9) indicate that an increase in the weight η of the defined-contribution component decreases the size of PAYG pensions and increases savings, which strengthens the positive growth effect of a decline in n due to the anti-dilution effect. These factors mitigate the negative growth effect of population aging caused by a decline in the population growth rate. Therefore, in economies in which the population growth rate is relatively low and the size of pensions under a defined-benefit scheme is relatively large to satisfy $1 + n < \frac{p\phi}{\psi}$, the transition from a defined-benefit scheme with a fixed replacement ratio ϕ to a defined-contribution scheme with a fixed contribution rate ψ mitigates or even overcomes the negative growth effect of population aging caused by a decline in the population growth rate.

Analogous policy predictions hold when we consider population aging caused by an increase in the old-age survival probability rate p. As demonstrated by the numerical examples presented in Figure 2, under plausible parameter conditions, when $\eta \in [0, 1)$ (i.e., under a pure defined-benefit scheme or a mixed payment scheme), there is a hump-shaped relationship between old-age survival probability and the per capita output growth rate.[10] However, when $\eta = 1$ (i.e., under a pure defined-contribution scheme), the per capita output growth rate increases monotonically with the old-age survival probability rate.

Equation (10) implies that the old-age survival probability rate p exerts two competing influences on capital accumulation. Under a pure defined-benefit scheme (i.e., $\eta = 0$) or a mixed payment scheme (i.e., $\eta \in (0, 1)$), an increase in the old-age survival probability rate p increases the old-age dependency ratio $\frac{p}{1+n}$, which positively affects the social security tax rate τ. The higher social security tax rate τ leads to a lower rate of saving by young individuals, which negatively affects capital accumulation. We denote this negative effect of p on capital accumulation as the "tax burden effect". Under a pure defined-contribution scheme (i.e., $\eta = 1$), however, the social security tax rate remains constant at ψ irrespective of the value of the old-age dependency ratio. Therefore, the negative growth effect

effect of population aging caused by a decline in the population growth rate. Based on (8), giving a larger weight η to the defined-contribution component decreases the impact of a change in the old-age dependency ratio on the social security tax rate, which alleviates the negative growth effect of a decline in n due to the tax burden effect. However, when $1 + n > \frac{p\phi}{\psi}$, (7) and (9) imply that giving a larger weight η to the defined-contribution component leads to larger PAYG pensions and smaller savings, which weakens the positive growth effect of a decline in n due to the anti-dilution effect.

[10] The proof of this relationship is provided in Appendix C. Note that Appendix C is for the referees' information only and is not intended for publication).

of p due to the tax burden effect does not exist under a pure defined-contribution scheme (i.e., $\eta = 1$). Conversely, the increase in old-age survival probability p motivates young individuals to save more for old age, which positively affects capital accumulation. We denote this positive effect of p on capital accumulation as the "life-span effect". The positive growth effect of p due to the life-span effect always exists regardless of the type of pension scheme. Therefore, an increase in life expectancy p always increases the per capita output growth rate under a pure defined-contribution scheme (i.e., $\eta = 1$).

Based on (10), it is easy to guess that the effect of increasing p is likely to be negative under a pure defined-benefit scheme (i.e., $\eta = 0$) or a mixed payment scheme (i.e., $\eta \in (0, 1)$), when p is extremely large (i.e., $p \to 1$). Because the old-age dependency ratio is high, the negative tax burden effect becomes more serious. However, the marginal increase in the savings rate due to an increase in p is low. Therefore, when p is extremely large, the negative tax burden effect is likely to dominate the positive life-span effect. Conversely, the effect of increasing p is likely to be positive when p is extremely small (i.e., $p \to 0$). Because the dependency ratio is low, the negative tax burden effect becomes negligible. However, the marginal increase in the savings rate due to an increase in p is high. Therefore, when p is extremely small, the positive life-span effect is likely to dominate the negative tax burden effect.

Figure 2 also shows how the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) affects the relationship between the old-age survival probability rate and the per capita output growth rate. As discussed in Section 3.1, this type of PAYG pension reform positively (resp. negatively) affects economic growth when $\frac{p}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{p}{1+n} < \frac{\psi}{\phi}$) or $p > \frac{\psi(1+n)}{\phi}$ (resp. $p < \frac{\psi(1+n)}{\phi}$). Under (8), when $p = \frac{\psi(1+n)}{\phi}$, the overall social security tax rate remains the same regardless of the value of η. Therefore, under (10), when $p = \frac{\psi(1+n)}{\phi}$, the per capita output growth rate remains the same irrespective of the value of η, as shown in Figure 2. However, (10) indicates that when $p > \frac{\psi(1+n)}{\phi}$ (resp. $p < \frac{\psi(1+n)}{\phi}$), the overall social security tax rate decreases (resp. increases) with the value of η. Therefore, the per capita output growth rate increases (resp. decreases) with the value of η.

These results suggest that in economies in which the old-age survival probability rate is relatively high and the size of pensions under a defined-benefit scheme is relatively large to satisfy $p > \frac{\psi(1+n)}{\phi}$, the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) mitigates or overcomes the negative growth effect of population aging caused by an increase in old-age survival probability.[11] The intuitions are explained as follows. Under (8), giving a greater weight η to the defined-contribution component decreases the impact of changes in the social security tax rate caused by the old-age dependency ratio, which alleviates the negative growth effect of p due to the tax burden effect. Further, when $p > \frac{\psi(1+n)}{\phi}$, (7) and (9) imply that an increase in the weight η given to the defined-contribution component leads to smaller PAYG pensions and greater savings, which strengthen the positive growth effect of p due to the life-span effect. These factors mitigate the negative growth effect of population aging caused by an increase in old-age survival probability. Therefore, in economies in which old-age survival probability is relatively high and the size of pensions under a defined-benefit scheme is relatively large to satisfy $p > \frac{\psi(1+n)}{\phi}$, the transition from a defined-benefit scheme with a fixed replacement ratio ϕ to a defined-contribution scheme with a fixed contribution rate ψ mitigates or even overcomes the negative growth effect of population aging caused by an increase in life expectancy.

4 Concluding Remarks

This paper examined how the transition from a defined-benefit PAYG pension scheme to a defined-contribution PAYG pension scheme affects economic growth in an OLG model with endogenous growth. We showed that in economies in which the old-age dependency ratio is relatively high and the size of pensions under a defined-benefit scheme is relatively large, this type of pension reform mitigates the negative growth effect of population aging that is caused by a decline in the population growth rate or an increase in life expectancy.

Before concluding this paper, we must describe certain limitations of our analysis and briefly discuss directions for future research. First, we address the interpretation of our numerical simulation results. The numerical simulation results in Figures 1 and 2 appear to suggest that for economies in which the old-age dependency ratio is sufficiently high (e.g., developed countries), the transition from a defined-benefit scheme to a defined-contribution scheme is beneficial for

[11] In economies in which old-age survival probability is relatively low and the size of pensions under a defined-benefit scheme is relatively small to satisfy $p < \frac{\psi(1+n)}{\phi}$, the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) exerts two competing influences on the growth effect of population aging caused by an increase in old-age survival probability. According to (8), an increase in the weight η given to the defined-contribution component decreases the impact of changes in the social security tax rate caused by the old-age dependency ratio, which alleviates the negative growth effect of p due to the tax burden effect. However, when $p > \frac{\psi(1+n)}{\phi}$, (7) and (9) imply that an increase in the weight η given to the defined-contribution component leads to larger PAYG pensions and lower savings, which weakens the positive growth effect of p due to the life-span effect.

economic growth because it mitigates the negative growth effect of population aging caused by a decline in the population growth rate or an increase in life expectancy. However, as inferred from (11), whether the dependency ratio is sufficiently "high" depends on the PAYG pension system parameters, namely, the fixed contribution rate ψ under a defined-contribution scheme and the fixed replacement ratio ϕ under a defined-benefit scheme. Therefore, the relative size of pensions under different pension schemes is also relevant. However, it is difficult to predict the true values of these pension parameters based on actual data because the pension scheme specified in this paper is rather abstract. Moreover, to the best of our knowledge, there is no reliable measure to assess the relative size or relative importance of a defined-contribution scheme. Although we confirm that the qualitative results obtained based on Figures 1 and 2 hold for various combinations of ψ and ϕ, our simulation results should be interpreted with caution when formulating policy predictions.[12]

The second limitation, which is related to the first, is that the scope of this paper is restricted to a simple, analytical and solvable version of an endogenous growth model with rather abstract pension schemes and specified utility and production functions. These simplifications enable us to obtain intuitive and manageable results. However, the application of our simple framework to assess the likely impact of policy reform is obviously limited because our framework omits many important pension scheme design details. For example, although policies that enforce later retirement are commonly used to ensure the sustainability of PAYG pension systems, retirement is not considered in our model. Therefore, the development of a more elaborate numerical version of an endogenous growth model that fully accounts for many important pension scheme design details and the construction of a reliable measure to assess the relative size and relative importance of a defined-benefit scheme are promising directions for future research.

Appendix A: Proof of Proposition 1

Because the proof of Proposition 1-2 is self-evident, this Appendix provides only the proof of Proposition 1-1. Let us denote the old-age dependency ratio by $z \equiv \frac{p}{1+n}$. Under (8) and (10), the gross per capita output growth rate G is rewritten as

$$G = z\frac{1-\tau(z;\eta)}{[1+p+\tau(z;\eta)\frac{\bar{w}}{R}]}\bar{w} \equiv G(z;\eta),$$

$$\tau = \eta\psi + (1-\eta)z\phi \equiv \tau(z;\eta).$$

[12]In Appendix B, we present numerical examples under different values of ψ and ϕ.

Because $\tau \in [0, 1)$, the old-age dependency ratio z must satisfy the following conditions: $z \in (0, \frac{1-\eta\psi}{(1-\eta)\phi})$. By differentiating $G(z;\eta)$ with respect to z, we obtain

$$\frac{1}{G}\frac{\partial G}{\partial z} = \frac{1}{z(1-\tau)(1+p+\tau\frac{\bar{w}}{R})}h(z),$$

where

$$h(z) \equiv [1 - \eta\psi - 2(1-\eta)\phi z](1+p) + \{\eta\psi - [\eta\psi + (1-\eta)z\phi]^2\}\frac{\bar{w}}{R}.$$

Note that $h(z)$ satisfies the following properties: $\frac{\partial h(z)}{\partial z} < 0$, $\lim_{z\to 0} h(z) = (1 - \eta\psi)(1 + p + \eta\psi\frac{\bar{w}}{R}) > 0$, $\lim_{z\to\frac{1-\eta\psi}{(1-\eta)\phi}} h(z) = -(1-\eta\psi)(1+p+\frac{\bar{w}}{R}) < 0$. Because $sign[\frac{\partial G}{\partial z}] = sign[h(z)]$, the above relationships show that there is a unique $\hat{z} \in (0, \frac{1-\eta\psi}{(1-\eta)\phi})$ such that $G(\hat{z};\eta) > G(z;\eta)$ $\forall z \in (0, \frac{1-\eta\psi}{(1-\eta)\phi})$, $\frac{\partial G(z;\eta)}{\partial z} > 0$ $\forall z \in (0, \hat{z})$, $\frac{\partial G(z;\eta)}{\partial z} < 0$ $\forall z \in (\hat{z}, \frac{1-\eta\psi}{(1-\eta)\phi})$. Recalling that $z \equiv \frac{p}{1+n}$ and $sign[\frac{\partial G}{\partial n}] = -sign[\frac{\partial G}{\partial z}]$, we can easily demonstrate that there is a unique $\hat{n} \in (-[1 - \frac{p(1-\eta)\phi}{1-\eta\psi}], \infty)$ such that $G(\hat{n}, p;\eta) > G(n, p;\eta)$ $\forall n \in (-[1-\frac{p(1-\eta)\phi}{1-\eta\psi}], \infty)$, $\frac{\partial G(n,p;\eta)}{\partial n} > 0$ $\forall n \in (-[1-\frac{p(1-\eta)\phi}{1-\eta\psi}], \hat{n})$, $\frac{\partial G(n,p;\eta)}{\partial n} < 0$ $\forall n \in (\hat{n}, \infty)$.

Appendix B: Footnote 12

In this Appendix, we briefly examine the effects of different combinations of PAYG pension parameters ϕ and ψ on the numerical simulation results presented in Figure 2, where $\phi = 0.5$ and $\psi = 0.5$ hold. To avoid unnecessary lexicographic explanations, we provide only two intuitive examples. Figure 3 shows the case in which $\phi = 0.3$ and $\psi = 0.5$ hold, whereas Figure 4 shows the case in which $\phi = 0.5$ and $\psi = 0.3$ hold. The values of the other parameters are the same as those in the baseline simulation case of Figure 2.

As discussed in Section 3-2, when $1 + n = \frac{p\phi}{\psi}$, the per capita output growth rate is the same regardless of the value of η. However, when $1 + n < \frac{p\phi}{\psi}$ (resp. $1 + n > \frac{p\phi}{\psi}$), the per capita output growth rate increases (resp. decreases) with the value of η. Because the value of $\frac{p\phi}{\psi}$ in Figure 3 (resp. Figure 4) is smaller (resp. larger) than the value of $\frac{p\phi}{\psi}$ in Figure 2, the threshold value of the population growth rate in Figure 3 becomes smaller (resp. larger) than the threshold value of the population growth rate in Figure 2. Therefore, in Figure 3 (resp. Figure 4), the range of parameter values of the population growth rate under which an increase in η positively affects economic growth becomes smaller (resp. larger) relative to Figure 2.

Appendix C: Footnote 10 (This Appendix is for the referees' review only and is not intended for publication.)

By differentiating $G(z;\eta)$ with respect to p, noting that $z \equiv \frac{p}{1+n}$, we obtain

$$\frac{1+n}{G}\frac{\partial G}{\partial p} = \frac{1}{z(1-\tau)(1+p+\tau\frac{\bar{w}}{R})}q(p),$$

where

$$q(p) \equiv [1-\eta\psi-(1-\eta)\phi z(2+p)] + \{\eta\psi - [\eta\psi + (1-\eta)z\phi]^2\}\frac{\bar{w}}{R}.$$

Note that $q(p)$ satisfies the following properties: $\frac{\partial q(p)}{\partial p} < 0$, $\lim_{p\to 0} q(p) = (1-\eta\psi)(1+\eta\psi\frac{\bar{w}}{R}) > 0$. Therefore, suppose the following parameter conditions hold:

$$q(1) \equiv [1-\eta\psi-(1-\eta)\phi\frac{3}{1+n}] + \{\eta\psi - [\eta\psi + (1-\eta)\frac{\phi}{1+n}]^2\}\frac{\bar{w}}{R} < 0.$$

Because $sign[\frac{\partial G}{\partial p}] = sign[q(p)]$, we can conclude that there is a unique $\hat{p} \in (0,1)$ such that $G(n,\hat{p};\eta) > G(n,p;\eta)\ \forall p \in (0,1)$, $\frac{\partial G(n,p;\eta)}{\partial p} > 0\ \forall p \in (0,\hat{p})$, $\frac{\partial G(n,p;\eta)}{\partial p} < 0\ \forall n \in (\hat{p},1)$. Intuitively, the parameter conditions to assure $q(1) < 0$ hold when the population growth rate n is sufficiently small.

References

[1] An, C-B. and S-H. Jeon, 2006, "Demographic change and economic growth: An inverted-U shape relationship,"*Economics Letters*, 92(3), 447-454.

[2] Artige, L., Cavenaile, L. and P. Pestieau, 2014, "The Macroeconomics of PAYG Pension Schemes in an Aging Society," *CORE Discussion Paper*, 2014/33.

[3] Bell, F. C., A. H. Wade, and S. C. Goss, 1992, Life Tables for the United States Social Security Area 1900-2080. Washington, DC: Government Printing Office.

[4] Blanchard, O. J., 1985, "Debt, Deficits, and Finite Horizons" *Journal of Political Economy* 93(2), 223-247.

[5] Boucekkine, R. and D. de la Croix, 2002,“ Vintage Human Capital, Demographic Trends, and Endogenous Growth” *Journal of Economic Theory* 104(2), 340-375.

[6] Cipriani, G., 2014, “Population aging and PAYG pensions in the OLG model,” *Journal of Population Economics*, 27(1), 251-256.

[7] de la Croix, D. and O. Licandro, 1999,“ Life Expectancy and Endogenous Growth” *Economics Letters* 65(2), 255-263.

[8] Fanti, L. and L. Gori, 2012, “Fertility and PAYG pensions in the overlapping generations model,” *Journal of Population Economics*, 25(3), 955-961.

[9] Gradstein, M. and M. Kaganovich, 2004, “Aging population and education finance,”*Journal of Public Economics*, 88(12), 2469-2485.

[10] Grossman, G. M. and N. Yanagawa, 1993, “Asset bubbles and endogenous growth,” *Journal of Monetary Economics*, 31(1), 3-19.

[11] Ito, H. and K. Tabata, 2008, “Demographic structure and growth: The effect of unfunded social security,”*Economics Letters*, 100(2), 288-291.

[12] Komamura, K., 2007, “The 2004 Pension Reform and the Impact of Rapid Population Aging,”*The Japanese Journal of Social Security Policy*, 6(1), 144-156.

[13] Kunze, L., 2014, “Life expectancy and economic growth,” *Journal of Macroeconomics*, 39(PA), 54-65.

[14] Miyazawa, K., 2006, “Growth and Inequality: a Demographic Explanation” *Journal of Population Economics* 19(3), 559-578.

[15] Mizushima, A., 2009, “Intergenerational transfers of time and public long-term care with an aging population,” *Journal of Macroeconomics*, 31(4), 572-581.

[16] OECD, 2013, Pensions at a Glance 2013: OECD and G20 Indicators, OECD publishing $http://dx.doi.org/10.1787/pension\_glance-2013-en$

[17] Pecchenino, R. and P. Pollard, 1997, “The Effect of Annuities, Bequests and Aging in an Overlapping Generations Model of Endogenous Growth,” *Economic Journal* 107(440): 26-46

[18] Prskawetz, A., T. Fent, W. Barthel, J. Crespo-Cuaresma, T. Lindh, B. Malmberg, M. Halvarsson, 2007,“The Relationship between Demographic Change and Economic Growth in the EU,” Report for Tender VT/2005/035

[19] Romer, P.M., 1986, "Increasing returns and long-run growth," *Journal of Political Economy*, 94(5), 1002-1037.

[20] Tabata, K., 2005, "Population aging, the costs of health care for the elderly and growth," *Journal of Macroeconomics*, 27(3), 472-493.

[21] Wigger, B.U., 1999, "Pay-as-you-go financed public pensions in a model of endogenous growth and fertility," *Journal of Population Economics*, 12(4), 625-640.

[22] Yakita, A., 2001, "Uncertain Lifetime, Fertility and Social Security," *Journal of Population Economics*, 14(4), 635-640

[23] Zhang, J., Zhang, J., and R. Lee, 2001, "Mortality Decline and Long-run Economic Growth," *Journal of Public Economics*, 80(3), 485-507

[24] Zhang, J., Zhang, J., and R. Lee, 2003, "Rising Longevity, Education, Savings, and Growth," *Journal of Development Economics*, 70(1), 83-101

[25] Zhang, J., Zhang, J., and R. Lee, 2005, "The Effect of Life Expectancy on Fertility, Saving, Schooling and Economic Growth: Theory and Evidence," *Scandinavian Journal of Economics*, 107(1), 45-66

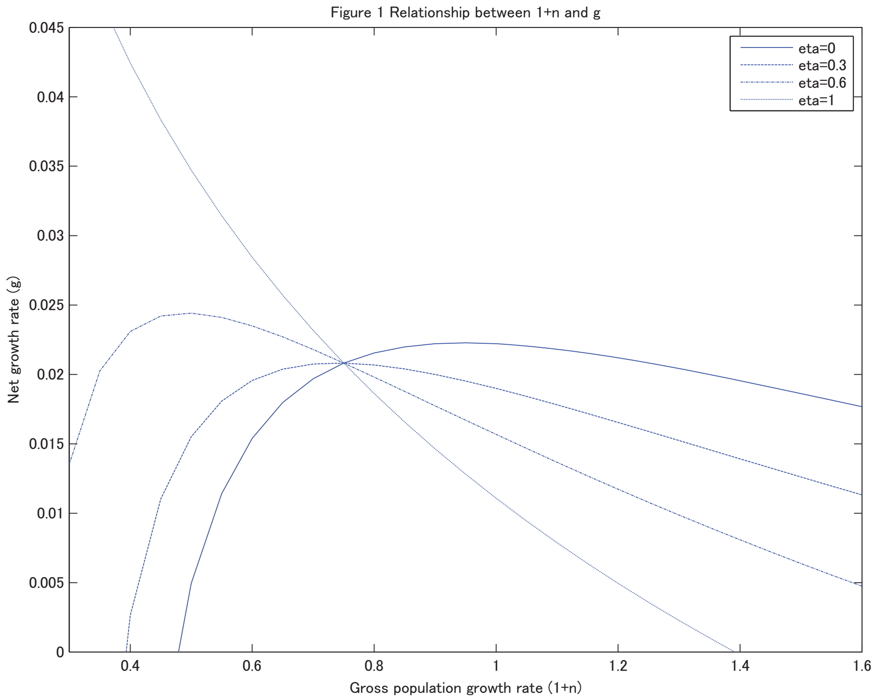

Figure 1: Relationship between gross population growth $1 + n$ and economic growth g under alternative values of η when $\phi = 0.5$ and $\psi = 0.5$

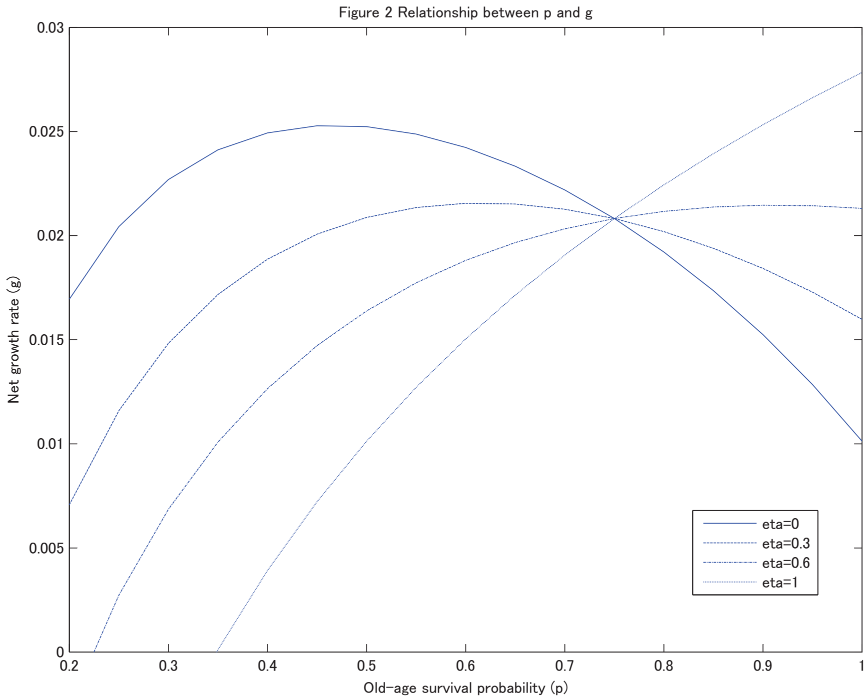

Figure 2: Relationship between old age survival probability p and economic growth g under alternative values of η when $\phi = 0.5$ and $\psi = 0.5$

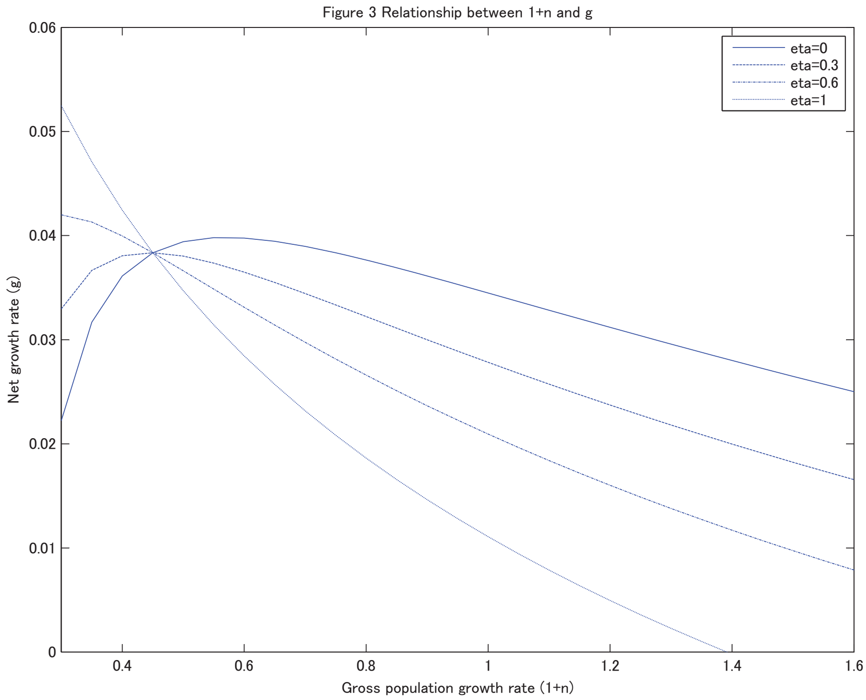

Figure 3: Relationship between gross population growth $1 + n$ and economic growth g under alternative values of η when $\phi = 0.3$ and $\psi = 0.5$

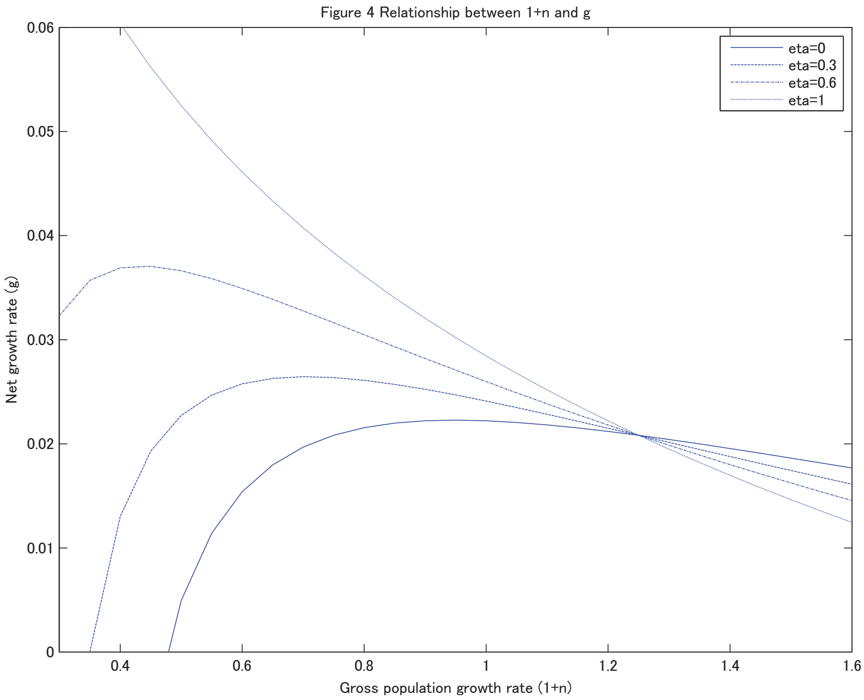

Figure 4: Relationship between gross population growth $1 + n$ and economic growth g under alternative values of η when $\phi = 0.5$ and $\psi = 0.3$