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Endogenous product compatibility choice under Cournot competition with a network externality

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Endogenous product compatibility choice under Cournot competition with a network externality

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We provide a simple model of endogenous product compatibility choice under Cournot competition with a network externality. Using the model, we consider how the degree of a network externality and product substitutability affects the choice regarding product compatibility. In particular, if the degree of the network externality is larger than that of the product substitutability, there exist multiple equilibria, involving imperfect, partial, and perfect compatibility. However, if another assumption formula regarding a spillover effect, which is a component of network size, is made, i.e., the converter case, there is a unique equilibrium, i.e., perfect compatibility, irrespective of the degree of the network effect versus product substitutability. Furthermore, we show that a perfectly compatible product standard is socially optimal and analyze, therefore, whether a social dilemma arises in the network products market.

Keywords: product compatibility; network externality; fulfilled expectation; Cournot duopoly; horizontally differentiated product

JEL classification: D21, D43, D62, L15

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1. Introduction

With the progress of information and communication technologies, the last two decades have witnessed a proliferation of consumer electronic products that exhibit network externalities, e.g., hardware and software of video games, computers, smartphones, iPads, and others. In such network product markets, problems of compatibility and standardization are very important. In particular, the choice regarding compatibility of products is significant for both the providers and for the (potential) users of the products. Compatibility between products tends to enhance the utility of a product for a user, because compatibility is a characteristic of products that interact with other products in order to create enhanced performance for users. However, compatibility between products tends to be a choice made by providers rather than by users.

Since the seminal paper by Katz and Shapiro (1985), many researchers have examined the social and private incentives to achieve product compatibility; that is, the trade-off between compatibility and standardization (or between incompatibility and perfect compatibility) in network products markets.[1] In this paper, following the framework of Economides (1996), we provide a simple model of endogenous product compatibility choice under Cournot competition with a network externality.[2] Using the model, we consider how the degree of the network externality and of product

[1] Regarding previous research on compatibility choice and network externalities, see, for example, Katz and Shapiro (1994), Matutes and Regibeau (1996), Shapiro and Varian (1999), Gandal (2002), Farrell and Klemperer (2007), Shy (2011), and others.

[2] Although Katz and Shapiro (1985) assume a homogenous product market, we deal with a horizontally differentiated product market, in which firms provide their products associated with a network externality. Furthermore, Baake and Boom (2001) consider price and quality competition in a vertically differentiated products market with network externalities.

substitutability affects the endogenous product compatibility choice.

Chen and Chen (2011), in a study closely related to ours, analyze product compatibility under differentiated duopoly with network externalities. However, based on their specific assumption regarding the parameters, i.e., product substitutability and a network externality, they consider the case of a strategic substitute relationship. As a result, they conclude that the firm's optimal compatibility choice is to set different product standards in the market, whereas socially optimal compatibility involves setting one product standard. In their case, a social dilemma arises.

In this paper, by relaxing the assumption of Chen and Chen (2011), we show that if the degree of the network externality is larger than that of product substitutability, there exist multiple equilibria, involving an imperfectly, a partially, and a perfectly compatible product. Thus, in our case, a social dilemma may not always arise.

Furthermore, Chen and Chen (2011) assume that the network size, i.e., the network externality, of one firm is composed of the *own effect* and the *spillover effect*, which depends on the degree of product compatibility chosen by another firm. In other words, one firm will design its product for the users of another firm's product. However, based on the formula for network size presented by Shy (1995, 2001), we assume that the *spillover effect* depends on the degree of product compatibility chosen by the firm itself. Related to this point, as analyzed in Farrell and Saloner (1992), the effects of incompatibility can be mitigated by using an adapter to connect incompatible products. There are various types of adapters such as an interface, emulator, converter, or gateway. Converters may provide compatibility without constraining product variety or technological innovation. Following this view, the progress in emulation technology will make standardization unnecessary because, using converters, compatibility can be achieved.

Therefore, we deal with other tools to achieve compatibility between products than do Chen and Chen; that is, we consider the case of converters. One firm will attach a converter with its product, enabling the users of the product to also use the product provided by the rival firm. In this case, as shown below, the perfect compatibility equilibrium only exists, irrespective of the degree of the network externality and product substitutability.

The remainder of this paper is organized as follows. In Section 2, we present a simple model and then consider the Nash equilibrium in the product compatibility decision game under Cournot duopoly with a network externality. In particular, we show that there are multiple Nash equilibria. In Section 3, first, we establish that the full compatibility standard is socially optimal. Second, with respect to the case of multiple Nash equilibria, we consider the equilibrium selection and then show that the perfect compatibility equilibrium is chosen if firms follow the payoff dominant standard. Third, we analyze another case of compatibility choice, i.e., the converter case, and then show that the perfect compatibility equilibrium exists and, thus, a social dilemma does not arise. In Section 4, we summarize the results and present remaining issues.

2. The model

2.1 Setup

We deal with Cournot competition in a horizontally differentiated products market associated with network externalities and consider how firms endogenously choose the degree of compatibility of their products.

Based on the framework of Economides (1996), we assume the following inverse demand function of product *i*:

$$P_i = A - (q_i + \theta q_j) + f(S_i^e), \tag{1}$$

where A is the potential market size, q_i (q_j) is the output of firm *i* (*j*), and $\theta \in (0,1)$ represents product substitutability. The network-externality function is given by: $f(S^e{}_i)$, where S_i^e represents the expected network-size of product *i*. Following the concept of a fulfilled expectations equilibrium presented by Katz and Shapiro (1985), we assume that $S_i^e = S_i$. Furthermore, following Chen and Chen (2011), we assume a linear network-externality function; that is, $f(S_i) = aS_i$, where $a \in (0,1)$ is a network-externality parameter for network size. In particular, the network size of product *i* is given by:

$$S_i = q_i + \alpha_j q_j, \tag{2}$$

where $\alpha_j \in [0,1]$ is the degree of product *j*'s compatibility with product *i*. If $\alpha_j = 1$ (0), product *j* operates perfectly (does not operate) with product *i*. q_i $(\alpha_j q_j)$ represents the *own (spillover) effect* in equation (2). In particular, the *spillover effect* associated with the network size of product *i* depends on the degree of product compatibility of the rival firm's product *j*. In other words, firm *j* chooses the degree of product compatibility of product *j* in terms of operating product *i* and thus affects the network size of product *i*.

Based on equations (1) and (2), the inverse demand function of firm *i* can be expressed by:

$$P_i = A - (1-a)q_i - (\theta - a\alpha_j)q_j. \tag{3}$$

Regarding equation (3), we assume that the own-price effect exceeds the cross-price effect; i.e., $\left|\frac{dP_i}{dq_i}\right| = 1-a > \left|\frac{dP_i}{dq_j}\right| = |\theta - a\alpha_j|$. For simplicity, we assume that production costs are zero.

2.2 The Cournot–Nash equilibrium under a fixed degree of product compatibility

We derive the following reaction function for firm *i*:

$$q_i = \frac{A}{2(1-a)} - \frac{\theta - a\alpha_j}{2(1-a)} q_j. \tag{4}$$

Given equation (4), under Cournot competition, the strategic relationship between the firms depends on the degree of product substitutability and the degree of product compatibility associated with a network externality, i.e., $\frac{\partial q_i}{\partial q_j} < (>) 0 \Leftrightarrow \theta > (<) a\alpha_j$. This condition implies that, under Cournot competition, a strategic substitute (complement) relationship between the firms holds if the degree of product substitutability is higher (lower) than the degree of product compatibility associated with the network externality.[3] In particular, even though the two products are substitutable, a relationship of strategic complementarity is sustained under Cournot competition if the degree of product compatibility associated with the network externality is sufficiently high, i.e., $\theta < a\alpha_j$.

Taking equation (4) into account, we categorize the Cournot–Nash equilibrium into three cases.

In case i) (case ii)), there is strong (weak) product compatibility associated with a

[3] As mentioned above, Chen and Chen (2011) assume the strategic substitute relationship, i.e., $\theta > a\alpha_j$, holds.

network externality: $\theta < (>) a\alpha_j$. When there is strong (weak) product compatibility associated with a network externality, because of the strategic complementarity (substitutability) relationship under Cournot competition, the reaction curves of both firms are upward (downward) sloping.

In case iii), there is asymmetric product compatibility: e.g., $a\alpha_i > \theta > a\alpha_j$. In this case, the reaction curve of firm *j* (*i*) is upward (downward) sloping.

Therefore, we derive the following Cournot–Nash equilibrium:

$$q_i = \frac{A\{2(1-a) - (\theta - a\alpha_j)\}}{D}, \tag{5}$$

where $D \equiv 4(1-a)^2 - (\theta - a\alpha_i)(\theta - a\alpha_j) > 0$ and $2(1-a) - (\theta - a\alpha_j) > 0$. Both of these conditions are satisfied because the own-price effect exceeds the cross-price effect.

Given equation (5), the effects of an increase in the degree of product compatibility on output levels are given by:

$$\frac{\partial q_i}{\partial \alpha_i} = -\frac{a(\theta - a\alpha_j)A\{2(1-a) - (\theta - a\alpha_j)\}}{D^2} > (<) 0 \Leftrightarrow \frac{\theta}{a} < (>) \alpha_j, \tag{6}$$

$$\frac{\partial q_i}{\partial \alpha_j} = \frac{2a(1-a)A\{2(1-a) - (\theta - a\alpha_i)\}}{D^2} > 0. \tag{7}$$

Equation (6) implies that the effect of an increase in the degree of own-product compatibility depends on the degree of product substitutability in comparison to the network-externality parameter for network size, i.e., $\frac{\theta}{a}$, and the degree of the rival firm's product compatibility, i.e., α_j. Furthermore, equation (7) implies that an increase in the degree of the rival firm's product compatibility increases the output of firm *i*. This is because an increase in the degree of product compatibility of firm *j* enhances the network size of firm *i* and thus increases the willingness to pay for product *i*, e.g., equations (1) and (2).

2.3 Endogenous product compatibility choice

The profit function of firm *i* is given by: $\pi_i = P_i q_i = (1-a)q_i^2$. Based on equations (6) and (7), we have the following:

$$\frac{\partial \pi_i}{\partial \alpha_i} = 2(1-a)q_i \frac{\partial q_i}{\partial \alpha_i} > (\le) 0 \Leftrightarrow \frac{\theta}{a} < (\ge) \alpha_j, \tag{8}$$

$$\frac{\partial \pi_i}{\partial \alpha_j} = 2(1-a)q_i \frac{\partial q_i}{\partial \alpha_j} > 0. \tag{9}$$

Taking equation (8) into account, we can easily derive the Nash equilibria of the endogenous product compatibility choice as follows.

Proposition 1 The following Nash equilibria exist:

(i) If $\frac{\theta}{a} > 1$, *then* $\alpha_i^* = \alpha_j^* = 0.$

(ii) If $\frac{\theta}{a} = 1$, *then* $\alpha_i^* = \alpha_j^* = 0$, *and* $\alpha_i^* = \alpha_j^* = 1.$

(iii) If $\frac{\theta}{a} < 1$, *then* $\alpha_i^* = \alpha_j^* = 0$, $\alpha_i^* = \alpha_j^* = \frac{\theta}{a}$, *and* $\alpha_i^* = \alpha_j^* = 1.$

In regard to *(i)*, if the degree of the network externality is smaller than the degree of the product substitutability, both firms provide an incompatible product. In regard to *(ii)*, if the degree of the network externality is just equal to the degree of the product substitutability, there are two Nash equilibria; both firms provide either an incompatible or a perfectly compatible product. In regard to *(iii)*, if the degree of the network externality is larger than the degree of the product substitutability, there are three Nash equilibria; both firms provide either an incompatible, or a perfectly compatible, or a partially compatible product (see Figure 1).

Let us focus on the third case among the three Nash equilibria, where the degree of the network externality is larger than the degree of the product substitutability. In this case, a firm's product compatibility strategy depends on the degree of product substitutability in comparison to the network-externality parameter for network size and the rival firm's strategy. In particular, if the firm expects that the rival firm's strategy is smaller (larger) than the degree of product substitutability in comparison to the network-externality parameter for network size, the *spillover effect* on its own network size is small (large). Hence, the firm provides an incompatible (a perfectly compatible) product to maximize its profit. Furthermore, suppose that the firm expects that the rival firm's strategy is equal to the degree of product substitutability in comparison to the network-externality parameter for network size. This means that the rival firm provides a partially compatible product. Thus, in maximizing profit, the firm is indifferent in regard to the degree of product compatibility chosen, in the range from zero to unity. If the rival firm has the same expectations, partial product compatibility equilibrium is sustained.

3. Discussion

3.1 Social optimality

The consumer surplus is $CS = \frac{(1-a)\{q_i^2 + q_j^2\}}{2}$ and the producer surplus is $PS = (1-a)\{q_i^2 + q_j^2\}$. Thus, social welfare is $SW = CS + PS = \frac{3(1-a)\{q_i^2 + q_j^2\}}{2}$.

Taking into account equations (5), (6), and (7), the social welfare effect of an increase in the degree of product *i*'s compatibility is given by:

$$\frac{\partial SW}{\partial \alpha_i} = 3(1-a)\left\{ q_i \frac{\partial q_i}{\partial \alpha_i} + q_j \frac{\partial q_j}{\partial \alpha_i} \right\} = \frac{3a(1-a)A}{D^2} q_i H > 0, \tag{10}$$

where $H \equiv \{2(1-a)[2(1-a) - (\theta - a\alpha_i) - (\theta - a\alpha_j)] + (\theta - a\alpha_j)^2\} > 0$ because the own-price effect exceeds the cross-price effect. Similarly, we obtain a positive effect on social welfare with regard to the degree of compatibility of product *j*. Thus, we present the following result.

Proposition 2 (Chen and Chen, 2011, Proposition 5) *A perfectly compatible standard is socially optimal, i.e.,* $\alpha_i = \alpha_j = 1$.

In view of *Propositions 1* and *2,* if the degree of the network externality is smaller than the degree of product substitutability, i.e., $\frac{\theta}{a} > 1$, a social dilemma always arises.[4] However, otherwise, i.e., when $\frac{\theta}{a} \le 1$, a social dilemma may or may not arise.

Furthermore, with respect to the aggregate profits, i.e., the producer surplus, we easily derive the following:

[4] See *Proposition 4* in Chen and Chen (2011).

$$\frac{\partial PS}{\partial \alpha_i} = 2(1-a)\left\{q_i \frac{\partial q_i}{\partial \alpha_i} + q_j \frac{\partial q_j}{\partial \alpha_i}\right\} = \frac{2a(1-a)A}{D^2} q_i H > 0. \qquad (11)$$

Equation (11) implies that it is optimal for both firms to cooperate in relation to the degree of product compatibility of both products, or, in other words, for each firm to choose a perfectly compatible product standard. Thus, both firms are likely to construct an alliance in relation to the network products (e.g., see Shapiro and Varian, 1999, Ch. 8 and Ch. 9).

3.2 Equilibrium selection

With respect to the third case in *Proposition 1*, we derive the output and profit of firm *i* in each equilibrium as follows:

$$q_i(1,1) > q_i\left(\frac{\theta}{a}, \frac{\theta}{a}\right) > q_i(0,0) \text{ and thus } \pi_i(1,1) > \pi_i\left(\frac{\theta}{a}, \frac{\theta}{a}\right) > \pi_i(0,0). \qquad (12)$$

Regarding firm *j*, we have the same results as those in equation (12). Thus, based on the equilibrium selection standard presented by Harsanyi and Selten (1988), the payoff from the perfectly compatible product equilibrium dominates the others. Therefore, both firms provide the perfectly compatible product if they follow the payoff-dominant standard. In this case, a social dilemma does not arise.

3.3 The converter case[5]

With respect to the network size, equation (2) implies that firm *j* itself will provide a compatible product that operates compatibly with the rival firm's product *i* for its users. However, in this subsection, we deal with the case of converters. That is, in this case,

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[5] This subsection is based on Toshimitsu (2013).

firm *i* will provide a product with converters that enable users its product *i* compatibly with rival firm's product *j*.

According to this view, using equation (10.11) in Shy (1995, p. 267) or equation (3.15) in Shy (2001, p. 62), let us modify equation (2) as follows:

$$S_i^C = q_i + \beta_i q_j, \tag{13}$$

where superscript *C* stands for the converter case and $\beta_i \in [0,1]$ represents the degree of product compatibility (or convertibility) chosen by firm *i*.

Equation (13) implies that firm *i* will provide a compatible product with which the rival firm's product *j* can operate. Thus, the *spillover effect*, $\beta_i q_j$, depends on the degree of product compatibility of the firm's own product. Alternatively, because firm *i* provides the product *i* attached converter to enable compatibility with product *j*, $\beta_i q_j$ may be denoted as the *convertible effect*, rather than the *spillover effect*. Furthermore, we assume the concept of a fulfilled expectations equilibrium.

Given the modification, we derive the following inverse demand function for firm *i*:

$$P_i = A - (1-a)q_i - (\theta - a\beta_i)q_j, \tag{14}$$

where we assume that the own-price effect exceeds the cross-price effect; i.e., $1 - a > |\theta - a\beta_i|$. In this case, we can derive the following Cournot–Nash equilibrium:

$$q_i = \frac{A\{2(1-a) - (\theta - a\beta_i)\}}{D^C}, \tag{15}$$

where $D^C \equiv 4(1-a)^2 - (\theta - a\beta_i)(\theta - a\beta_j) > 0$. Given equation (15), the effects of an increase in the degree of product compatibility on output levels are:

$$\frac{\partial q_i}{\partial \beta_i} = \frac{2a(1-a)A\{2(1-a) - (\theta - a\beta_j)\}}{(D^C)^2} > 0, \tag{16}$$

$$\frac{\partial q_i}{\partial \beta_j} = -\frac{a(\theta - a\beta_i)A\{2(1-a) - (\theta - a\beta_i)\}}{(D^C)^2} > (<) 0 \Leftrightarrow \frac{\theta}{a} < (>) \beta_i. \tag{17}$$

Based on equations (16) and (17), we derive the following:

$$\frac{\partial \pi_i}{\partial \beta_i} = 2(1-a)q_i \frac{\partial q_i}{\partial \beta_i} > 0, \tag{18}$$

$$\frac{\partial \pi_i}{\partial \beta_j} = 2(1-a)q_i \frac{\partial q_i}{\partial \beta_j} > (<) 0 \Leftrightarrow \frac{\theta}{a} < (>) \beta_i. \tag{19}$$

An increase in the degree of product convertibility of product *i* increases firm *i*'s output and profits. However, the effect of an increase in the degree of product convertibility of product *j* on firm *i*'s output and profits depends on both the degree of product substitutability in comparison to the network-externality parameter for network size and the degree of firm *i*'s product compatibility.

Taking equation (18) into account, we derive the following result.

Proposition 3 There exists such a unique Nash equilibrium as $\beta_i^* = \beta_j^* = 1$.

That is, under system product Cournot competition, the firm's optimal strategy is to provide a perfectly compatible (or convertible) product. Thus, in this case, a social dilemma does not arise.

4. Conclusion

We have considered an endogenous product compatibility decision under Cournot competition in the presence of network externalities. In particular, we have focused on the formula that determines a component of the network size. That is, in the case where the spillover effect of the network size is decided by the degree of product compatibility chosen by the rival firm, there are various product compatibility cases, which vary according to the degree of the network externality and product substitutability. In particular, when the degree of the network externality is larger than the degree of product substitutability, there are multiple product compatibility equilibria: imperfect, partial, and perfect product compatibility. Otherwise, an imperfect product compatibility equilibrium occurs and, in this case, a social dilemma arises. However, in the case where the spillover effect of the network size is decided by the degree of product compatibility chosen by the firm itself, i.e., in the converter case, a perfect product compatibility equilibrium arises. Thus, there is no social dilemma.

There are various issues in relation to our model, in addition to the specificity of the demand and network functions. For example, we have assumed that there are no costs involved in choosing the degree of product compatibility.[6] Furthermore, in analyzing the decision regarding product compatibility, we have not discussed the issues with intellectual property rights, i.e., the choice between technology patents, a registered design, a registered trademark, and others. Implicitly, we have assumed that a firm can freely exploit the technology and designs of the rival firm to set its own product to be compatible without any costs and legal constraints.

Intuitively, although a perfectly compatible standard is socially optimal, it may be natural for competing firms in network product markets to choose an incompatible

[6] Foros and Hansen (2001) analyze connectivity between competing internet service providers in the case of cost free connectivity.

technology and design. However, in this paper, we have considered how the presence of a network externality affects the compatibility decision of competing firms and have shown that it is possible for the socially preferable product standardization to be sustainable even in a competitive market.

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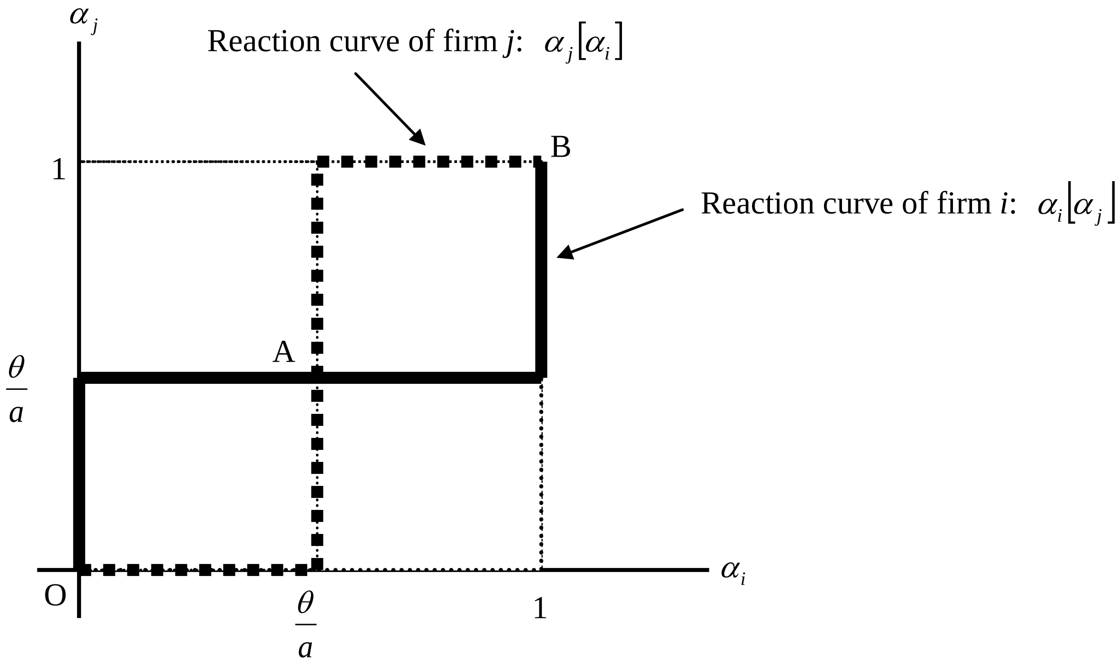

Figure 1

The Nash equilibria under $\frac{\theta}{a} < 1$

Note:

Point O: $(0,0)$ indicates an incompatible equilibrium.

Point A: $\left(\frac{\theta}{a}, \frac{\theta}{a}\right)$ indicates a partially compatible equilibrium.

Point B: $(1,1)$ indicates a perfectly compatible equilibrium.