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in Overlapping Generations Version of an
R&D-based Growth Model**

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Rising Longevity, Human Capital and Fertility in Overlapping Generations Version of an R&D-based Growth Model *

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Abstract

This paper constructs a simple, overlapping generations version of an R&D-based growth model à la Diamond (1965) and Jones (1995), and examines how an increase in old-age survival probability impacts purposeful R&D investment and long-run growth by affecting fertility and education decisions. We demonstrate that under certain conditions, old-age survival probability, when relatively low (high), positively (negatively) affect economic growth. This study also compares the growth implications of child education subsidies and child rearing subsidies and demonstrates that although child education subsidies always foster economic growth, child rearing subsidies may negatively impact economic growth in particular situations. Finally, we briefly consider the effects of a child education subsidy on welfare levels.

Keywords: R&D, Fertility, Human Capital, Child Education Subsidy, Child Rearing Subsidy.

JEL classification: J13, J24, O10, O30, O40

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1 Introduction

During the previous several decades, substantial demographic changes have been taken place in industrialized countries. Table 1 depicts the observed changes in the total fertility rate (TFR), life expectancy at birth, and population growth rate among the G7 countries. All of the G7 countries have exhibited substantial decreases in TFR and increases in life expectancy. These developments have provided the main forces that have driven the aging of the populations of these nations. While increasing life expectancies have allowed individuals to gain additional years of life, declining fertility rates have slowed population growth rates. For example, in Japan, the TFR has declined from 2.05 in 1960-65 to 1.34 in 2005-10, whereas life expectancy at birth has increased from 67.7 years in 1960 to 81.9 years in 2005. Consequently, in Japan, the population growth rate has declined from 1.26 % in 1960-65 to -0.04 % in 2005-10, and the old age dependency ratio has increased from 9.0% in 1960 to 29.9% in 2005; in fact, Japan is now experiencing rapid population decline and population aging.¹ In addition to these changes in demographics, investments in education, particularly higher education, have greatly increased in industrialized countries over the past several decades. Table 2 indicates the changes in educational levels that have occurred in advanced countries during this time period. In these countries, the share of workers who have obtained a higher education has increased from 8.0 % in 1975 to 16.6 % in 2010.

In this paper, we address how these demographic changes and increases in education have affected the growth of per capita output over a long time horizon. Because technological progress has been identified as the main driving force for modern economic growth (e.g., Romer, 1990), we are particularly interested in the effects of demographic changes on research and development (R&D) investment. Therefore, this paper constructs a simple overlapping generations version of an R&D-based growth model à la Diamond (1965) and Jones (1995). This model is then used to examine how an increase in old age survival probability impacts purposeful R&D investment and long-run growth by affecting fertility and education decisions.

Many existing studies (e.g., Boucekkine et al. 2002, 2003, de la Croix and Licandro 1999, 2012, Kalemli-Ozcan 2002, 2003, Zhang and Zhang 2005) examine the effect of rising longevity on long-run growth by assessing the influ-

¹According to the UN (2011), the TFR is now below replacement level in more than 80 countries around the world; moreover, in Europe, Asia and Latin America, the TFR is predicted to remain far lower than replacement level over the course of the entire 21st century.

ence of this longevity on fertility, education and saving decisions.² However, most of these studies are based on a model with the following fundamental engines of growth: the accumulation of human capital; the accumulation of physical capital; technological progress via learning by doing; or knowledge spillovers that occur during production. Therefore, these studies cannot analyze the effects of rising longevity and the resulting demographic changes on purposeful R&D investments, which play a crucial role in modern technological development.

Conventional semi-endogenous growth models involving purposeful R&D investment (e.g., Jones 1995, Kortrum 1997, Segerstrom 1998) state that long-run per capita output growth rate is linearly related to the population growth rate. A fundamental feature of semi-endogenous growth models is the idea of decreasing returns from existing scientific knowledge with respect to the production of new knowledge. This notion of decreasing returns implies that an increasing supply of researchers is required to maintain a particular pace of technological progress because the creation of novel scientific knowledge becomes increasingly complex with as technological frontiers expand. If a constant fraction of population is engaged in the research sector (a condition that must be fulfilled in a long-run equilibrium), this increasing supply of researchers can only be achieved through positive population growth. A declining population would lead to the stagnation of both productivity and income per capita.³

However, the empirical evidence during the 20th century does not support the predictions of semi-endogenous growth models. In particular, many empirical studies have not identified a simple positive association between population growth and per capita income growth but have instead demonstrated the existence of a negative association between these two types of growth (e.g., Brander and Dowrick 1994, Kelley and Schmidt 1995, Ahituv 2001, and Herzer et al. 2012).⁴ To reconcile these discrepancies between

²Higher life expectancy increases the return on education and saving, which accelerates human and physical capital accumulation. The resultant increase in wage and technology raises the opportunity cost of child bearing, which decreases fertility.

³Early endogenous growth models (e.g., Romer 1990, Grossman and Helpman 1991, Aghion and Howitt 1992) imply that long-run per capita output growth rate is linearly related to population size. Thus a declining population will imply vanishing growth of per capita income. Peretto (1998), Young (1998) and others integrate features of quality- and variety-based R&D. Assuming that no knowledge spillovers exist between these two types of R&D, they predict that only variety-based R&D will be associated with population growth rate. However, as discussed in Li (2000), if knowledge spillovers between the quality- and variety-based types of R&D are permitted, the positive association between population growth and per capita income growth is re-established.

⁴Furthermore, using data for 67 countries between 1940 and 2000, Struik et al. (2012)

theoretical predictions and empirical findings, Dalgard and Kreiner (2001), Struulik (2005) and others incorporate human capital accumulation into conventional semi-endogenous growth models and argue that the creation of new scientific knowledge is dependent on an economy's aggregate human capital rather than simply the number of workers. These authors demonstrate that the rapid accumulation of per capita human capital can sustain positive per capita output growth even under conditions in which overall population growth stagnates or declines.

Although these studies elucidate the role of aggregate human capital in economic development by endogenizing individuals' education decisions, these investigations assume that the population growth rate is exogenously determined and ignore the micro-level trade-offs that individuals face with respect to fertility and education decisions. In particular, the following two trade-offs exist: (1) the trade-off between the number of children that individuals have and the amount of education that they can afford for each child (i.e., a Beckerian quality-quantity trade-off), and (2) the trade-off between the number of children that individuals have and the amount of time that they can allocate for their own education. However, in the literature that addresses demographic transitions, these two types of trade-offs are considered to be relevant for explaining observed demographic changes and shifts in educational levels. Therefore, this paper constructs a model in which these two types of trade-offs are explicitly considered and examines how a rise in old age survival probability impacts purposeful R&D investment and long-run growth by affecting fertility and education decisions.⁵

In the model presented here, we show that under certain parameter conditions, the effect of old-age survival probability on growth is positive in economies in which this probability is relatively low but it could be negative in economies in which this probability is relatively high. This result is explained as follows. In economies in which old-age survival probability is sufficiently low, an increase in old-age survival probability motivates individuals to invest more in their own education, accelerating the accumulation of

find a negative correlation between TFP growth rate and population growth rate. They confirm that this negative correlation holds even after controlling for country- and time-specific fixed effects and/or confining their samples to OECD countries (i.e., countries that push the world technology frontier).

⁵Jones (2003), Connolly and Peretto (2003) and others account for endogenous fertility in the conventional R&D growth model. However, these researchers do not simultaneously consider endogenous human capital accumulation. To the best of our knowledge, only Tournemaine and Luangaram (2012) and Chu et al. (2013) provide exceptional R&D growth models that incorporate both endogenous fertility and endogenous human capital accumulation simultaneously. However, these models do not consider the growth implications of increases in longevity and the resulting demographic changes.

per capita human capital and thereby enhancing the long-run growth rate of the economy (i.e., a growth-enhancing quality effect). However, in economies in which old-age survival probability is sufficiently high, an increase in old-age survival probability will lead to greater declines in population growth rates, retard the rising supply of researchers and thereby lower the long-run growth rate of the economy (i.e., a growth-impeding quantity effect). These theoretical results produce somewhat pessimistic predictions regarding future economic growth. If an economy reaches the stage of an aging society in which the old-age survival probability is relatively high and the fertility rate is relatively low, then the aforementioned growth-impeding quantity effect is likely to dominate the growth-enhancing quality effect. Therefore, increases in old-age survival probability and their accompanying declines in fertility (i.e., population aging) may cause the long-run growth rate of the economy to deteriorate.

Furthermore, we examine the growth implications of child education subsidy policies and child rearing subsidy policies. A child education subsidy policy is designed to enhance the accumulation of human capital among children by reducing the opportunity costs that parents must pay to educate their children, whereas a child rearing subsidy policy is designed to promote fertility increases by reducing the opportunity cost that parents incur from child rearing and bearing (i.e., child rearing subsidy policies are pro-natal policies). This study demonstrates that a child education subsidy always fosters long-run growth, whereas a child rearing subsidy may oppose long-run growth under certain conditions. These results indicate that even under the framework of an R&D-based growth model, a child rearing subsidy (i.e., a pro-natal policy) may not be justified as a growth-enhancing policy. A child rearing subsidy promotes fertility increases by reducing parents' opportunity costs for child bearing; however, these increases in fertility rates increase parents' opportunity costs for child education and thereby lower parental investments in educating their children. Therefore, the net effect of a child rearing subsidy on an economy's aggregate human capital is ambiguous. By contrast, a child education subsidy lowers the parents' opportunity costs for both child education and child bearing and thereby positively affects the aggregate human capital of an economy.

During the course of writing this manuscript, we found two recent interesting studies that are closely related to our paper: Prettnner (2011) and Strulik et al. (2012). Prettnner (2011) introduces Blanchard (1985)-type realistic demographic structures into conventional endogenous and semi-endogenous growth models and examines how a rise in longevity and a decline in fertility affect the long-run growth rate of an economy. Prettnner (2011) demonstrates that a rise in longevity positively affects per capita output growth,

whereas a decline in fertility negatively affects it. This work is quite interesting because it succeeds in introducing realistic demographic structures into an R&D-based growth model. However, Prettner (2011) does not explicitly consider micro-level interactions between fertility and the accumulation of human capital.

Strulik et al. (2012) introduce R&D-based innovation into a unified growth framework that includes micro-level fertility and schooling behaviors and attempt to explain the historical emergence of R&D-based growth and the subsequent emergence of mass education and accompanying demographic transitions. Because they employ an overlapping generations version of the R&D-based growth model à la Diamond (1965) and Jones (1995), their work is closely related to our research. However, Strulik et al. (2012) do not consider the effects of rising longevity and its accompanying demographic changes but instead focus on the effects of demographic changes that have been induced by technological progress. Furthermore, to simultaneously explain several historical observed facts, they incorporate various factors (e.g., capital accumulation, technological progress through learning by doing) into their model and primarily argue their numerical simulation results in the transition process. They also employ a simplified one-period patent model to avoid mathematical complications.⁶ By contrast, this paper constructs an analytically solvable overlapping generations model with infinite patent protection assumptions and focuses on the long-run growth implications of rising longevity and its accompanying demographic changes. Although we share many research interests with Strulik et al. (2012), our research provides several original contributions that complement their analyses.

This paper is organized as follows. Section 2 presents the basic model. In Section 3, we demonstrate how an increase in old-age survival probability impacts long-run growth by affecting fertility and education decisions. Section 4 considers child education and child rearing subsidy policies and the influences of these policies on long-run growth rates and welfare. Section 5 provides several concluding remarks.

⁶Strulik et al. (2012) include two restrictive assumptions within their conventional one-period patent model. First, they assume that even after a patent has expired, intermediate good firms can continue to maintain their monopoly prices. Second, they assume that an R&D producer in period t that invents a blueprint for a new variety of intermediate goods that become available from period $t + 1$ and onwards obtains the monopoly profits of an intermediate good firm in period t . Thus, the reward that this R&D producer in period t will receive will not relate to the profits that the producer's invented goods will generate from period $t + 1$ and onwards.

2 The model

2.1 Environments

Consider a three-period overlapping generations economy in which economic activity extends over infinite discrete time denoted by $t = 0, 1, \dots$. Individuals live for a maximum of three periods: childhood, adulthood, and old age. During childhood, individuals do not make any decisions and are reared and educated by their parents. During adulthood, individuals invest in their own education, raise and educate their children, supply labor to the market, and consume goods. During old age, individuals retire and only consume goods. An individual dies at the beginning of old age with a probability of $1 - \pi \in [0, 1)$, and lives through old age with a probability of $\pi \in (0, 1]$. The cohort born in period $t - 1$ becomes active workers in period t . Thus, we call this cohort as generation t , and use N_t to represent the number of young adults who exist in period t . Let n_t denote the number of births for each young adult. The relationship between the sizes of the young adult population during any two consecutive periods can therefore be expressed as $N_{t+1} = n_t N_t$.

On the production side of the economy, we essentially follow the approach of Romer (1990) and Jones (1995). In this approach, the economy consists of three sectors: a final goods sector, an intermediate goods sector, and an R&D sector. In accordance with Grossman and Helpman (1991), to simplify the model, we regard labor as the primary production factor in all three of these sectors and do not consider the role of raw capital. R&D firms invent blueprints of intermediate goods and conduct the market launches of these goods. Each intermediate good is produced by a single monopoly firm, using labor as an input; by contrast, each final good is produced by competitive firms, using labor and a variety of imperfectly substitutable intermediate goods as inputs.

2.2 Preferences and optimization

Individuals derive utility from $c_{1,t}$, their own consumption during their youth; $c_{2,t+1}$, their own consumption during old age; n_t , the number of children that they have; and e_t , their investment in education for each child. The lifetime expected utility of individuals in generation t is expressed as

$$u^t = \gamma [\ln n_t + \phi \ln e_t] + (1 - \gamma) [\ln c_{1,t} + \pi \ln c_{2,t+1}], \quad \gamma, \phi \in [0, 1), \quad (1)$$

where γ measures the extent to which parents care about their children relative to their own lifetime consumption and π expresses the old-age survival

probability. Parents derive utility from e_t , their investments in education for each of their children. This utility, captured by the $\gamma\phi \ln e_t$ component, could originate from either the warm glow of giving (Andreoni, 1989) or a preference for having higher-quality children (Becker, 1960). Here, ϕ measures the strength of this utility factor relative to an individual's number of children. Our condition that $\phi \in [0, 1)$ implies that having a family must be more important than investments in the education of one's children.⁷

After being raised and educated by their parents, during their second period of life, individuals are endowed with one unit of time, which is devoted to working in the labor market (ℓ_t), rearing n_t identical children, educating each child (e_t) and investing in the individual's own education (m_t). Individuals also divide their income $w_t h_t \ell_t$ between consumption $c_{1,t}$ and saving s_t for their old age. Here, h_t represents the consequent level of human capital as a result of own education m_t . We discuss this point rigorously later. For simplicity, we assume that insurance companies are risk-neutral and that the private annuities market is competitive. Insurance companies promise individuals a payment $(R_{t+1}/\pi)s_t$, in exchange for which the estate s_t accrues to the companies, where π is the average probability of surviving and R_{t+1} represents the gross rate of interest. In the absence of a bequest motive, individuals are willing to invest their assets in such insurance. Finally, during the third period of life, survivors are retired and spend their savings on their old age consumption $c_{2,t+1}$. Thus, the budget and time constraints for individuals in generation t are expressed as follows:

$$c_{1,t} + s_t = w_t h_t \ell_t, \quad (2)$$

$$c_{2,t+1} = \frac{R_{t+1}}{\pi} s_t, \quad (3)$$

$$\ell_t + (z + e_t)n_t + m_t = 1. \quad (4)$$

Following Becker (1965) and others, we assume that a fixed amount of time z is required to bear and raise a child. The time constraint defined in (4) identifies two crucial trade-offs that parents face. First, parents face a trade-off between the number of children they have and the amount of education that they can afford for each child (i.e., a Beckerian quality-quantity trade-off). If other variables are held constant, we note from (4) that a larger investment in education for each child e_t is associated with a smaller number of children n_t . Second, parents face a trade-off in allocating their time between rearing and educating their children and investing in their own education. If other

⁷This assumption ensures the existence of a consistent solution and is common in the unified growth literature. For instance, see Strulik (2004, 2008).

variables are held constant, we observe from (4) that a larger investment in one's own education m_t is accompanied by either a smaller number of children n_t or a smaller investment in education for each child e_t .⁸

In accordance with the specifications that have been provided by Kalemli-Ozcan (2002, 2003), the production function of human capital h_t is given by the following expression:

$$h_t = E h_{t-1} e_{t-1}^{\sigma_e} m_t^{\sigma_m}, \quad E > 0, \sigma_e \geq 0, \sigma_m \geq 0, \sigma_e + \sigma_m \leq 1 \quad (5)$$

where E , σ_e and σ_m are parameters; h_{t-1} indicates the human capital stock of parents; e_{t-1} denotes the parental investment in education for each child during period $t - 1$; and m_t represents investments in one's own education during period t .⁹ As in de la Croix and Licandro (2012), parental investment in education for each child e_{t-1} can be interpreted as investment in basic education (i.e., early childhood education, primary education and early secondary education) because parents must devote a great deal of time and money to their children's basic education. By contrast, investment in one's own education m_t can be interpreted as investment in higher education (i.e., upper secondary education and tertiary education) because children face high opportunity costs for obtaining a higher education and typically are responsible for paying for a substantial proportion of these costs.

By maximizing (1), subject to (2)-(5), we obtain the following solution:

$$s_t = \frac{(1 - \gamma)\pi}{(1 - \gamma)(1 + \sigma_m)(1 + \pi) + \gamma} w_t h_t, \quad (6)$$

⁸The first trade-off is stressed by Becker et al. (1990), Galor and Weil (2000) and other studies in the unified growth literature. The second trade-off is stressed by Kimura and Yasui (2007), Soares and Falcão (2008) and de la Croix and Licandro (2012), among others.

⁹In accordance with the approaches of Lucas (1988), Kalemli-Ozcan (2002, 2003), Strulik (2005) and others, we assume that the human capital production function is linear with respect to human capital level in the previous period h_{t-1} so as to be compatible with endogenous growth. This linearity assumption may be somewhat relaxed by explicitly considering externality effects generated by economy-wide human capital stocks. For example, as discussed by Yakita (2010), the production function of human capital h_t in (5) may be written as follows: $h_t = E \bar{h}_{t-1}^{1-\sigma_e-\sigma_m} (h_{t-1} e_{t-1})^{\sigma_e} (h_{t-1} m_t)^{\sigma_m}$ where h_{t-1} is human capital stock of their parents, and \bar{h}_{t-1} represents the average human capital stock of generation $t - 1$. The term $\bar{h}_{t-1}^{1-\sigma_e-\sigma_m}$ represents the spillovers from society, reflecting the fact that learning is more productive if an individual interacts with more knowledgeable people. By contrast, the terms $(h_{t-1} e_{t-1})^{\sigma_e}$ and $(h_{t-1} m_t)^{\sigma_m}$ represent peer effects within a family, reflecting the fact that learning is more productive if one's parents are well educated. Because no individual heterogeneities exist within a generation, the relation $h_{t-1} = \bar{h}_{t-1}$ holds at equilibrium. Thus, this human capital production function that is homogeneous of degree one is compatible with endogenous growth.

$$n_t = \frac{1}{z} \frac{\gamma(1 - \phi)}{(1 - \gamma)(1 + \sigma_m)(1 + \pi) + \gamma} \equiv n, \quad (7)$$

$$e_t = \frac{\phi z}{1 - \phi} \equiv e, \quad (8)$$

$$m_t = \frac{\sigma_m(1 - \gamma)(1 + \pi)}{(1 - \gamma)(1 + \sigma_m)(1 + \pi) + \gamma} \equiv m, \quad (9)$$

$$h_t = Ee^{\sigma_e} m^{\sigma_m} h_{t-1}. \quad (10)$$

From (6)-(10), saving (s_t) increases with wages (w_t); by contrast, fertility (n_t), investments in education for each child (e_t), and investments in one's own education (m_t) are constant with respect to w_t because positive income and negative substitution effects cancel each other.

According to (7) and (9), fertility n_t decreases with π , the old-age survival probability (i.e., $\frac{\partial n_t}{\partial \pi} < 0$), whereas investment in one's own education m_t increases with π (i.e., $\frac{\partial m_t}{\partial \pi} > 0$). An increase in π , the old-age survival probability, stimulates a demand for consumption relative to the demand for children because this increase causes individuals to anticipate a need for consuming goods over a longer period of time. In response to this change, individuals shift their time from child rearing to work but maintain a constant level of education for each child (e_t). The resulting increases in working time raise the returns from an individual's own education, which increases the individual's investment in this education. This positive relationship between life expectancy and investment in one's own education is the well-known Ben-Porath mechanism; higher life expectancy affects education because individuals anticipate having longer active lives.¹⁰

¹⁰Boucekkine et al. (2002, 2003), Soares and Falcão (2008), de la Croix and Licandro (2012) and others convincingly argue that the Ben-Porath mechanism plays an active role in explaining the increases in secondary educational attainments and demographic transitions that have been observed over time. However, the Ben-Porath mechanism has recently been subject to criticism from Hazan (2009). Hazan (2009) reveals that the lifetime labor input of American men who were born from 1840-1970 declined despite dramatic gains in life expectancy. Hazan further argues that a rise in the lifetime labor supply is a necessary implication of the Ben-Porath model; this implication would create concerns regarding the ability of this type of model to explain increases in education that have occurred in various nations. Cervellati and Sunde (2009) demonstrate that Hazan's critique is only valid under specific assumptions and is much less general than Hazan claims. Hazan (2009) himself also notes that if the labor supply of women is explicitly considered, estimates of the decline in lifetime labor supply across cohorts are much smaller in magnitude compared with the corresponding labor supply estimates for men. In our paper, due to the explicit consideration of fertility choices, the labor supply of woman is a relevant issue. Taking into accounts of limitations of the labor supply data for women, Hazan's results should be interpreted with caution. In fact, Soares and Falcão (2008)

These theoretical results are partially consistent with recent empirical findings. For example, Soares and Falcão (2008) find that the female labor supply is positively associated with adult survival probability through time both across and within countries. Furthermore, Lehr (2009) and Becker et al. (2011) reveal that higher levels of parental education are causally related to negative effects on fertility. A cross-country analysis by Zhang and Zhang (2005) reveals that life expectancy is positively correlated with saving rate and secondary school enrollment ratio but negatively related to fertility rate.

2.3 Final goods sector

A final good Y_t is produced by competitive firms. This production uses a composite of intermediate goods, X_t , and labor, which may be expressed in terms of efficiency units, $H_{Y,t}$:

$$Y_t = H_{Y,t}^{1-\alpha} X_t^\alpha, \quad 0 < \alpha < 1. \quad (11)$$

The composite factor X_t is a CES aggregate of quantities $x_{i,t}$ of intermediate goods:

$$X_t = \left(\int_0^{A_t} x_{i,t}^\varepsilon di \right)^{1/\varepsilon}, \quad 0 < \varepsilon < 1. \quad (12)$$

A_t denotes the variety of intermediate goods or the level of technological knowledge in an economy, which grows through R&D. The existing intermediate goods exhibit a constant elasticity of substitution that is expressed as follows: $\frac{1}{1-\varepsilon}$. In this expression, a higher ε indicates the existence of greater substitutability between the intermediate inputs.

Let w_t and $p_{i,t}$ represent the wage of workers and the price of intermediate good i , respectively. Using final goods as our *numeraire*, the conditions for profit maximization in the competitive final goods sector produce the following equations:

$$w_t = (1 - \alpha) H_{Y,t}^{-\alpha} X_t^\alpha = (1 - \alpha) \frac{Y_t}{H_{Y,t}}, \quad (13)$$

$$p_{i,t} = \alpha H_{Y,t}^{1-\alpha} X_t^{\alpha-\varepsilon} x_{i,t}^{\varepsilon-1}. \quad (14)$$

From (11) and (14), given Y_t , we can express the demand for the intermediate good i as follows:

$$x_{i,t} = \frac{p_{i,t}^{-1/(1-\varepsilon)}}{\int_0^{A_t} p_{i,t}^{-\varepsilon/(1-\varepsilon)} di} \alpha Y_t. \quad (15)$$

find that the female labor supply tends to be positively associated with adult survival probability through time both across and within countries.

2.4 Intermediate goods sector

Each intermediate good i is produced by monopolistically competitive firms that hold a blueprint of the intermediate good i . One efficiency unit of labor is required to produce one unit of an intermediate good, and the operating profit of each intermediate good producer $d_{i,t}$ is expressed as follows: $d_{i,t} = (p_{i,t} - w_t)x_{i,t}$. Under monopolistic competition, each firm maximizes its profit given a demand of (15) by establishing a price that is equal to a constant markup over unit cost:

$$p_{i,t} = p_t = \frac{1}{\varepsilon}w_t, \quad (16)$$

Thus, the firm-specific index in the intermediate goods sector can be dropped, and profits may therefore be expressed as follows:

$$d_t = (1 - \varepsilon)p_t x_t. \quad (17)$$

2.5 R&D sector

The development of R&D technology requires labor as its only private input, and the existing stock of knowledge can have an external effect on the productivity of the R&D sector. Between periods t and $t + 1$, competitive R&D firms employ $H_{A,t}$ efficiency units of labor as researchers, develop $A_{t+1} - A_t$ new blueprints, and sell these blueprints to intermediate good firms at their market values of V_t . Thus, given a research productivity of δ_t , output is expressed as follows:

$$A_{t+1} - A_t = \delta_t H_{A,t}. \quad (18)$$

In accordance with Jones (1995), research productivity is given to each single firm but depends on the aggregate level, positively on the number of already existing ideas ($0 < \psi < 1$ *the standing-on-shoulders effect*) and possibly negatively on the size of the researchers ($0 \leq \nu < 1$, *the stepping-on-toes effect*);

$$\delta_t = \bar{\delta} A_t^\psi (H_{A,t})^{-\nu}, \quad \bar{\delta} > 0. \quad (19)$$

The standing-on-shoulders effect may arise because part of the output of R&D is knowledge that contributes to the capacity to innovate. The stepping-on-toes effect may arise out of patent races in which multiple firms run parallel research programs in the hope of being the first to succeed at creating and patenting a new good or process. If all other factors are held constant, an increase in R&D efforts will induce increased duplication of research efforts that reduces the average productivity of R&D in the economy.

Assuming free entry in R&D sector, the expected gain of $V_t\delta_t H_{A,t}$ from R&D must not exceed the cost of $w_t H_{A,t}$ for a finite size of R&D activities at equilibrium. Thus we have the following conditions:

$$V_t\delta_t \begin{cases} = w_t, & \text{then } H_{A,t} > 0, \quad A_{t+1} > A_t, \\ < w_t, & \text{then } H_{A,t} = 0, \quad A_{t+1} = A_t. \end{cases} \quad (20)$$

We next consider non-arbitrage conditions. The market value of intermediate good firms V_t (i.e., the market value of blueprints) is related to the risk-free interest rate R_t . Shareholders of intermediate good firms that bought these shares during period t obtain dividends of d_{t+1} during period $t+1$ and can sell these shares to the subsequent generation at the value of V_{t+1} . In the financial market, the total returns from holding the stock of a particular intermediate firm must be equal to the returns on the risk-free asset $R_{t+1}V_t$, which implies the following no-arbitrage condition:

$$R_{t+1} = \frac{d_{t+1} + V_{t+1}}{V_t}. \quad (21)$$

2.6 Market-clearing conditions

The market clearing condition for labor is expressed as follows:

$$H_{Y,t} + A_t x_t + H_{A,t} = H_t, \quad (22)$$

where $H_t \equiv h_t \ell_t N_t$. The sum of $H_{Y,t}$, the labor demands for the final goods sector; $A_t x_t$, the labor demands for the intermediate goods sector; and $H_{A,t}$, the labor demands for the R&D sector must be equal to the total supply of efficiency units of labor by young adults H_t .

Furthermore, using the final goods market equilibrium condition of $Y_t = c_{1,t}N_t + c_{2,t}\pi N_{t-1}$, we can obtain the following asset market equilibrium condition:¹¹

$$V_t A_{t+1} = s_t N_t, \quad \text{for } V_t \delta_t = w_t. \quad (23)$$

The derivation of (23) is provided in Appendix A. This condition states that the savings of young adults in period t must be used for investments in new inventions ($V_t(A_{t+1} - A_t)$) or for the purchase of existing stocks that were owned by the preceding generations ($V_t A_t$). Given the externality effects in the R&D sector specified in (19), as demonstrated in Appendix B, the case in which the R&D sector does not operate (i.e., $H_{A,t} = 0$) never occurs at equilibrium.

¹¹An analogous asset market equilibrium condition is also presented by Tanaka and Iwaisako (2011).

3 Equilibrium

3.1 Dynamics

This subsection examines the dynamic properties of the economy. First, we consider the dynamics of the aggregate effective labor supplies ($H_t \equiv h_t \ell_t N_t$), which are determined by the per capita human capital (h_t), labor supply (ℓ_t) and population size (N_t) for young adults.

From (10), the dynamics of the per capita human capital are determined by the following equation: $h_t = Ee^{\sigma_e} m^{\sigma_m} h_{t-1}$. In addition, from (4), (7), (8) and (9), the labor supply of each young adult is constant (i.e., $\ell_t = \frac{(1-\gamma)(1+\pi)}{(1-\gamma)(1+\sigma_m)(1+\pi)+\gamma} \equiv \ell$). Thus, using $N_{t+1} = n_t N_t$ and (7), the dynamics of the aggregate effective labor supply ($H_t \equiv h_t \ell_t N_t$) are described by the following expression:

$$\frac{H_{t+1}}{H_t} = Ee^{\sigma_e} m^{\sigma_m} n. \quad (24)$$

We then consider the dynamics of product variety A_t . Because the relationship $V_t \delta_t = w_t$ holds at equilibrium, from (23) with (6), (19) and $H_t \equiv h_t \ell_t N_t$, we can obtain the following equation:

$$A_{t+1} = \frac{\pi}{1+\pi} \bar{\delta} A_t^\psi H_{A,t}^{-\nu} H_t. \quad (25)$$

In addition, using (18) and (19), the labor engaged in R&D sector $H_{A,t}$ is represented as follows:

$$H_{A,t} = \left(\frac{1}{\bar{\delta}}\right)^{\frac{1}{1-\nu}} A_t^{\frac{1-\psi}{1-\nu}} \left(\frac{A_{t+1}}{A_t} - 1\right)^{\frac{1}{1-\nu}}. \quad (26)$$

Hence, by substituting (26) into (25), the dynamics of A_t is expressed in the following manner:

$$\frac{A_{t+2}}{A_{t+1}} = \left(\frac{A_{t+1}}{A_t}\right)^{\frac{\psi-\nu}{1-\nu}} \left(\frac{A_{t+2}/A_{t+1} - 1}{A_{t+1}/A_t - 1}\right)^{\frac{-\nu}{1-\nu}} \frac{H_{t+1}}{H_t}. \quad (27)$$

Now, let us define the growth rate of product variety as $g_{A,t} \equiv (A_{t+1} - A_t) / A_t$. By substituting (24) into (27) and rearranging them, we obtain the following autonomous dynamic system of $g_{A,t}$:

$$\Phi_L(g_{A,t+1}) = \Phi_R(g_{A,t}), \quad (28)$$

where

$$\Phi_L(g_{A,t+1}) \equiv (1 + g_{A,t+1}) (g_{A,t+1})^{\frac{\nu}{1-\nu}},$$

$$\Phi_R(g_{A,t}) \equiv Ee^{\sigma_e} m^{\sigma_m} n (1 + g_{A,t})^{\frac{\psi-\nu}{1-\psi}} (g_{A,t})^{\frac{\nu}{1-\psi}}.$$

We define the state of the economy at which the growth rate of product variety A_t does not change (i.e., $g_{A,t+1} = g_{A,t} = g_A$) as the balanced growth path (BGP). From (28), the growth rate of product variety g_A in the balanced growth path (BGP) satisfies the following condition:

$$1 + g_A = (Ee^{\sigma_e} m^{\sigma_m} n)^{\frac{1-\nu}{1-\psi}}. \quad (29)$$

From (24) and (29), similarly to conventional R&D-based growth models with human capital accumulation, the growth rate of product variety g_A in the balanced growth path is linearly related to the growth rate of aggregate effective labor supplies (H_t). Furthermore, given that $\psi < 1$, the following relationship holds around the steady-state equilibrium:

$$\frac{dg_{A,t+1}}{dg_{A,t}} \Big|_{g_{A,t+1}=g_{A,t}=g_A} = \frac{\Phi'_R(g_A)}{\Phi'_L(g_A)} = \frac{(\psi - \nu) + \nu(1 + g_A)g_A^{-1}}{(1 - \nu) + \nu(1 + g_A)g_A^{-1}} \in (0, 1). \quad (30)$$

Given initial values of A_0 , N_0 , h_{-1} and e_{-1} , the value of $g_{A,0}$ is derived automatically from (24) and (27).¹² Thus, Equation (30) indicates that the steady-state equilibrium that is characterized by the balanced growth path (i.e., $g_{A,t+1} = g_{A,t} = g_A$) is locally stable.

Finally, we consider the growth rate of per capita GDP y_t , which is defined by $y_t \equiv \frac{Y_t}{N_t}$. According to (11), the GDP in this economy is given by the following expression:¹³

$$Y_t = H_{Y,t}^{1-\alpha} \left(A_t^{\frac{1}{\varepsilon}} x_t \right)^\alpha = A_t^{\frac{1-\varepsilon}{\varepsilon}\alpha} H_{Y,t}^{1-\alpha} (A_t x_t)^\alpha. \quad (31)$$

Based on this equation, as shown in Appendix C, we can derive per capita GDP y_t as follows:

$$y_t \equiv \frac{Y_t}{N_t} = \xi A_t^{\frac{1-\varepsilon}{\varepsilon}\alpha} \left(1 - \frac{\pi}{1 + \pi} \frac{g_{A,t}}{1 + g_{A,t}} \right) \frac{H_t}{N_t}, \quad (32)$$

¹²Note that h_{-1} and e_{-1} are per capita human capital and parental investment in education for each child of generation -1 . Given A_0 , N_0 , h_{-1} and e_{-1} , ℓ_0 and m_0 are defined to be the optimal choices of generation 0. Thus $H_0 \equiv N_0 h_0 \ell_0$ is uniquely determined. By substituting A_0 and H_0 into equations (24) and (27), the values of A_1 and H_1 are explicitly calculated. Therefore, the value of $g_{A,0} \equiv (A_1 - A_0)/A_0$ is automatically derived.

¹³As specified by (16), all intermediate goods hold the same price; therefore, final good producers utilize the same quantity of each intermediate good (i.e., $x_{i,t} = x_t$). As a result, (12) can be rewritten as follows: $X_t = A_t^{1/\varepsilon} x_t$.

where $\xi \equiv \varepsilon^\alpha \alpha^\alpha (1 - \alpha)^{1 - \alpha} / (1 - \alpha + \alpha \varepsilon)$. Thus, in the balanced growth path (BGP) in which $g_{A,t}$ becomes constant, the per capita GDP growth rate $g_{y,t}$ must satisfy the following condition:

$$1 + g_{y,t} \equiv \frac{y_{t+1}}{y_t} = \left(\frac{A_{t+1}}{A_t} \right)^{\frac{1-\varepsilon}{\varepsilon} \alpha} \left(\frac{H_{t+1}}{H_t} \right) \left(\frac{N_{t+1}}{N_t} \right)^{-1}. \quad (33)$$

Substituting (24), (29) and $N_{t+1} = nN_t$ into (33), the per capita GDP growth rate g_y in the balanced growth path (BGP) is expressed as follows:

$$1 + g_y = (Ee^{\sigma_e} m^{\sigma_m})^{1 + \frac{1-\nu}{1-\psi} \frac{1-\varepsilon}{\varepsilon} \alpha} (n)^{\frac{1-\nu}{1-\psi} \frac{1-\varepsilon}{\varepsilon} \alpha}. \quad (34)$$

This equation (i.e., (34)) appears to suggest that per capita GDP growth g_y and population growth n are positively correlated; this positive correlation would be consistent with the results of conventional R&D-based growth models. Furthermore, in accordance with the findings from conventional R&D-based growth models that consider human capital accumulation, sufficiently high levels of investment in education (i.e., $e, m > 0$) appear to ensure sustainable growth even for situations in which population size is constant (i.e., $\frac{N_{t+1}}{N_t} = n = 1$) or decreasing (i.e., $\frac{N_{t+1}}{N_t} = n < 1$). However, these macro-level superficial examinations disregard the micro-level interactions between human capital accumulation and fertility rate. Based on these micro-level interactions, we have derived that investment in education and fertility are endogenous and inversely correlated via quantity-quality trade-off. In particular, in an environment in which adult longevity increases (e.g., as a result of improvements in medical knowledge and health-related infrastructure), increases in individuals' investments in their own education and declines in fertility will simultaneously occur, as shown in Section 2-2. To rigorously investigate this issue, in the following subsection, we examine how an increase in old-age survival probability impacts per capita GDP growth rate g_y by affecting fertility and education decisions.

3.2 Old-age survival probability and per capita GDP growth

In this subsection, we examine how π , the old-age survival probability, impacts long-run per capita GDP growth rate by affecting fertility and education decisions. As discussed in section 2-2, the old-age survival probability (π) positively affects individuals' investments in their own education (i.e., $\frac{\partial m}{\partial \pi} > 0$), whereas it negatively affects fertility (i.e., $\frac{\partial n}{\partial \pi} < 0$). Thus, from (34), π , the old-age survival probability, has two competing effects on g_y , the long-run per capita GDP growth rate. These effects include a growth-enhancing

quality effect caused by increases in individuals' investments in their own education (m) and a growth-impeding quantity effect that is caused by declines in fertility (n). On the one hand, an increase in the old-age survival probability (π) enhances individuals' investments in their own educations (m_t), accelerates the accumulation of human capital by each researcher in an economy (h_t) and thereby increases the per capita GDP growth rate (i.e., a growth-enhancing quality effect). On the other hand, an increase in the old-age survival probability (π) lowers the population growth rate (n_t), retards the growth in the supply of available researchers, and thereby reduces the per capita GDP growth rate (i.e., a growth-impeding quantity effect).

By differentiating (34) with respect to π , given (7) and (9), we obtain the following expression:

$$\frac{\partial g_y}{\partial \pi} = B [\gamma - \Gamma(\pi; \nu)], \quad (35)$$

where

$$B \equiv \frac{(1 + g_y)\sigma_m \left[1 + \frac{1-\nu}{1-\psi} \frac{1-\varepsilon}{\varepsilon} \alpha \left(1 + \frac{(1+\sigma_m)(1+\pi)}{\sigma_m} \right) \right]}{(1 + \pi) [(1 - \gamma)(1 + \sigma_m)(1 + \pi) + \gamma]} > 0,$$

$$\Gamma(\pi; \nu) \equiv \frac{1}{\frac{1}{1+\pi} \frac{\sigma_m}{1+\sigma_m} \left(1 + \frac{1-\psi}{1-\nu} \frac{\varepsilon-1}{\varepsilon} \alpha \right) + 1} \in [0, 1],$$

$\frac{\partial \Gamma(\pi; \nu)}{\partial \pi} > 0$, $\frac{\Gamma(\pi; \nu)}{\partial \nu} < 0$, $\frac{\partial}{\partial \nu} \left[\frac{\partial \Gamma(\pi; \nu)}{\partial \nu} \right] < 0$, $\lim_{\nu \rightarrow 0} \Gamma(\pi; \nu) < 1$ and $\lim_{\nu \rightarrow 1} \Gamma(\pi; \nu) = 0$. See Appendix D for the derivation of (35). Given that $B > 0$, the sign of $\frac{\partial g_y}{\partial \pi}$ depends on the sign of $[\gamma - \Gamma(\pi; \nu)]$. To understand the properties of $[\gamma - \Gamma(\pi; \nu)]$, we consider the following two parameters: γ , the relative utility weight that is allocated to children, and ν , the strength of the stepping-on-toes effect. We rigorously examine how these two parameters affect the sign of $\frac{\partial g_y}{\partial \pi}$. To emphasize our concern, the two parameters of γ and ν are highlighted in the above equations.

From (35) and the fact that $\Gamma(0; \nu) < \Gamma(1; \nu)$, we obtain the following proposition.

Proposition 1 *In the balanced growth path (BGP), suppose we define $\hat{\pi}$ as $\gamma = \Gamma(\hat{\pi}; \nu)$, then following statements hold.*

(1) *If $\gamma < \Gamma(0; \nu)$,*

$$\frac{\partial g_y}{\partial \pi} < 0 \quad \text{for } \pi \in (0, 1],$$

(2) If $\gamma > \Gamma(1; \nu)$,

$$\frac{\partial g_y}{\partial \pi} > 0 \text{ for } \pi \in (0, 1],$$

(3) If $\Gamma(0; \nu) \leq \gamma \leq \Gamma(1; \nu)$,

$$\frac{\partial g_y}{\partial \pi} \begin{cases} \geq 0, & \text{for } \pi \in (0, \hat{\pi}], \\ < 0, & \text{for } \pi \in (\hat{\pi}, 1]. \end{cases}$$

Figure 1 represents the parameter regions in which proposition 1-(1), 1-(2) and 1-(3) hold. Proposition 1-(1) and 1-(2) indicate that in an economy in which the relative utility weight given to children γ is sufficiently small (resp. large) to satisfy $\gamma < \Gamma(0; \nu)$ (resp. $\gamma > \Gamma(1; \nu)$), the growth-impeding quantity effect dominates (resp. is dominated by) the growth-enhancing quality effect, and an increase in π , the old-age survival probability, negatively (resp. positively) affects g_y , the per capita GDP growth rate.

Intuitively, the relative strengths of the growth-impeding quantity effect and the growth-enhancing quality effect depend on the elasticity of fertility (n_t) with respect to π (i.e., $\frac{\pi}{n_t} \frac{\partial n_t}{\partial \pi} = -\frac{\pi(1-\gamma)(1+\sigma_m)}{(1-\gamma)(1+\sigma_m)(1+\pi)+\gamma} < 0$) and the elasticity of one's own education (m_t) with respect to π (i.e., $\frac{\pi}{m_t} \frac{\partial m_t}{\partial \pi} = \frac{\pi}{[(1-\gamma)(1+\sigma_m)(1+\pi)+\gamma]} \frac{\gamma}{1+\pi} > 0$). From these two elasticities, a small (resp. large) value of γ , the relative utility weight that is allocated to children, produces the following inequality: $|\frac{\pi}{n_t} \frac{\partial n_t}{\partial \pi}| > |\frac{\pi}{m_t} \frac{\partial m_t}{\partial \pi}|$ (resp. $|\frac{\pi}{n_t} \frac{\partial n_t}{\partial \pi}| < |\frac{\pi}{m_t} \frac{\partial m_t}{\partial \pi}|$). Since parents evaluate less (resp. more) about their children, a small (resp. large) value of γ leads to low (resp. high) fertility (n_t), and high (resp. low) one's own education (m_t). Thus, given a sufficiently small (resp. large) value of γ and correspondingly low (resp. high) values for fertility (n_t) and high (resp. low) values for a one's own education (m_t), a one-unit increase in old-age survival probability (π) will produce a relatively small (resp. large) increase in individuals' own education m_t and large (resp. small) decreases in fertility n_t . Consequently, if the relative utility weight that is allocated to children (γ) is sufficiently small (resp. large) to satisfy $\gamma < \Gamma(0; \nu)$ (resp. $\gamma > \Gamma(1; \nu)$), then $|\frac{\pi}{n_t} \frac{\partial n_t}{\partial \pi}| > |\frac{\pi}{m_t} \frac{\partial m_t}{\partial \pi}|$ (resp. $|\frac{\pi}{n_t} \frac{\partial n_t}{\partial \pi}| < |\frac{\pi}{m_t} \frac{\partial m_t}{\partial \pi}|$), and the growth-impeding quantity effect dominates (resp. is dominated by) the growth-enhancing quality effect, and the old-age survival probability therefore has a negative (resp. positive) overall effect on the per capita GDP growth rate.

Furthermore, Proposition 1-(3) indicates that as shown in Figure 2-1, if the relative utility weight given to children (γ) is at intermediate values that satisfy $\Gamma(0; \nu) \leq \gamma \leq \Gamma(1; \nu)$, there is a hump-shaped relationship between old-age survival probability (π) and the per capita GDP growth rate (g_y).

Figure 2-1 presents a numerical example of the relationship between old-age survival probability (π) and per capita GDP growth rate (g_y), if the value of γ satisfies $\Gamma(0; \nu) \leq \gamma \leq \Gamma(1; \nu)$. Figure 2-2 and 2-3 illustrate the corresponding relationships between old-age survival probability (π) and population growth rate (n), and between old-age survival probability (π) and an individuals' investments in their own education (m), respectively. Moreover, Figure 2-4 depicts the relationship between the per capita GDP growth rate and the population growth rate (n) as the value of the old-age survival probability π is increased from 0 to 1. We discuss Figure 2-4 rigorously in the next subsection. The parameters used in the base-case simulation are described in footnote 14, and explanations of these parameters are provided in Appendix E.^{14 15}

In economies in which the old-age survival probability is sufficiently low, the growth-enhancing quality effect dominates the growth-impeding quantity effect. Therefore, increases in old-age survival probability (π) will enhance individuals' investments in their own education (m_t), accelerate researchers' accumulation of human capital (h_t), and thereby raise the per capita GDP growth rate. However, in economies in which old-age survival probability is sufficiently high, the growth-impeding quantity effect dominates the growth-enhancing quality effect. Therefore, increases in old-age survival probability (π) will lower population growth rates (n_t), retard the expansion of the supply of researchers and thereby decrease rates of per capita GDP growth.

The results of proposition 2-(3) are intuitively explained as follows. Analogously to the case of γ , a small value of π , the old-age survival probability, produces the following inequality: $|\frac{\pi}{n_t} \frac{\partial n_t}{\partial \pi}| < |\frac{\pi}{m_t} \frac{\partial m_t}{\partial \pi}|$ (resp. $|\frac{\pi}{n_t} \frac{\partial n_t}{\partial \pi}| > |\frac{\pi}{m_t} \frac{\partial m_t}{\partial \pi}|$). From (7) and (9), a small (resp. large) value of π results in a high (resp. low) fertility rate n_t and a low (resp. high) level of a one's own education m_t . Thus, given a sufficiently small (resp. large) value of π and the resulting high (resp. low) level of fertility (n_t) and low (resp. high) level of one's own education (m_t), a one-unit increase in old-age survival probability (π) generates a relatively large (resp. small) increase in one's own education (m_t) and a small (resp. large) decrease in fertility (n_t). Consequently, if the old-age survival probability (π) is sufficiently low (resp. high), then $|\frac{\pi}{n_t} \frac{\partial n_t}{\partial \pi}| < |\frac{\pi}{m_t} \frac{\partial m_t}{\partial \pi}|$ (resp. $|\frac{\pi}{n_t} \frac{\partial n_t}{\partial \pi}| > |\frac{\pi}{m_t} \frac{\partial m_t}{\partial \pi}|$); thus, the growth-enhancing qual-

¹⁴In the base-case simulation, we set $\gamma = 0.3$, $\phi = 0.7$, $E = 6.0675$, $\sigma_e = 0.25$, $\sigma_m = 0.75$, $z = \frac{\gamma(1-\phi)}{2(1-\gamma)(1+\sigma_m)+\gamma}$, $\psi = 0.5$, $\nu = 0.2$, $\bar{\delta} = 10$, $\epsilon = 0.83$, $\alpha = 0.4$, $\pi = 1$, $\theta_e = 0$, $\theta_n = 0$. To investigate the effect of an increase in old-age survival probability, we increased the value of π from 0.1 to 1 in increments of 0.1.

¹⁵The objective of this numerical analysis is to supplement the qualitative results of Proposition 1. Although the authors chose the values of the parameters carefully, these quantitative results should be interpreted with caution.

ity effect dominates (resp. is dominated by) the growth-impeding quantity effect, and the old-age survival probability has a positive (resp. negative) effect on the rate of per capita GDP growth.

Finally, as described in Figure 1, a stronger stepping-on-toes effect (i.e., a higher value of ν) expands the parameter regions in which the old-age survival probability positively affects the per capita GDP growth rate (i.e., $\frac{\partial g_y}{\partial \pi} > 0$).¹⁶ When the stepping-on-toes effect is strong, the size of workers engaged in the R&D sector has little effect on the new variety expansions because the benefits from R&D efforts from a large numbers of researchers are offset by the increased duplication of research efforts. In particular, from (34), if the stepping-on-toes effect is extremely strong (i.e., if $\nu = 1$), the growth rate of both the overall population and of researchers in particular (n) has no effect on the per capita GDP growth rate, and the growth-impeding quantity effect becomes negligible. Therefore, if the stepping-on-toes effect is strong, then the growth-enhancing quality effect is more likely to dominate the growth-impeding quantity effect; thus, the old-age survival probability (π) is likely to have a positive effect on the per capita GDP growth rate.

3.3 The relationship between per capita GDP growth and population growth

In this subsection, we examine how changes in per capita GDP growth and population growth are correlated in a balanced growth path in which the old-age survival probability is improved by exogenous factors (e.g., improvements in medical knowledge and in health-related infrastructure). In this study, to avoid lexicographic explanations, we focus our analyses on the case in which the condition $\Gamma(0; \nu) \leq \gamma \leq \Gamma(1; \nu)$ holds. The results for situations in which $\gamma < \Gamma(0; \nu)$ or $\gamma > \Gamma(1; \nu)$ are easily inferred from the following analyses.¹⁷

As shown in Figure 2-1 and 2-2, in the balanced growth path, if the old-age survival probability (π) is sufficiently low, then old-age survival probability (π) produce increases in the per capita GDP growth rate (g_y) and decreases in the population growth rate (n). Therefore, in an economy in which the old-age survival probability (π) is sufficiently low and the population growth

¹⁶From (35), analogous to the effect of ν , a higher elasticity of substitution among intermediate goods (ε) and a lower intensity of intermediate goods in the final goods sector (α) will enlarge the parameter regions in which the relationship $\frac{\partial g_y}{\partial \pi} > 0$ holds. Both a higher ε and a lower α induce smaller monopoly profits for intermediate good firms, reduce the rates of growth in product variety in the R&D sector, and weaken the growth-impeding quantity effect of increases in the old-age survival probability.

¹⁷The issue of which parameter regions are most likely to exist is a purely empirical concern. However, it is difficult to obtain precise estimates of γ and ν .

rate is sufficiently high, changes in the per capita GDP growth rate (g_y) and population growth rate (n) are negatively correlated with increases in the old-age survival probability.¹⁸ However, if the old-age survival probability (π) is sufficiently high, both the per capita GDP growth rate (g_y) and the population growth rate (n) will decrease with increases in the old-age survival probability (π). Thus, in an economy in which the old-age survival probability (π) is sufficiently high and the corresponding population growth is sufficiently low, changes in the per capita GDP growth rate (g_y) and the population growth rate (n) will be positively correlated with increases in the old-age survival probability.¹⁹ Consequently, as shown in Figure 2-4, we observe a negative relationship between the per capita GDP growth rate and the population growth rate in regions in which the population growth rate is high but a positive relationship between the per capita GDP growth rate and the population growth rate in regions in which the population growth rate is low.

A conventional semi-endogenous growth model predicts that the per capita GDP growth rate will be positively correlated with the population growth rate n ; however, this correlation is difficult to observe in the extant empirical data. However, if increases in old-age survival probability are regarded as an exogenous source of demographic transition, then by considering the micro-level interactions between human capital investment and fertility, we can confirm that the per capita GDP growth rate is not necessarily positively correlated with the population growth rate (n).

The predictions of Figures 2-1 to 2-4 produce somewhat gloomy perspectives regarding future economic growth. If the economy reaches the stage of an aging society in which the old-age survival probability is relatively high and the fertility rate is relatively low, the growth-impeding quantity effect of increases in the old-age survival probability is likely to dominate the growth-enhancing quality effect of these increases. Therefore, further increases in old-age survival probability and the accompanying declines in fertility (i.e., population aging) may deteriorate the long-run growth rate of the economy.

¹⁸Analogously, in an economy in which the relative utility weight allocated to children γ is sufficiently large to satisfy $\gamma > \Gamma(1; \nu)$ (i.e., Proposition 1-(2)), changes in the per capita GDP growth rate (g_y) and population growth rate (n) are negatively correlated with increases in the old-age survival probability (π).

¹⁹Analogously, in an economy in which the relative utility weight allocated to children (γ) is sufficiently small to satisfy $\gamma < \Gamma(0; \nu)$ (i.e., Proposition 1-(1)), changes in the per capita GDP growth rate (g_y) and population growth rate (n) are positively correlated with increases in the old-age survival probability (π).

4 Child education and child rearing subsidy policies

All developed countries provide a set of education policies for enhancing the accumulation of human capital among children (for instance, industrialized nations typically feature not only the public provision of primary and secondary education but also subsidies for pre-primary and tertiary education). Broadly speaking, these policies can be interpreted as a child education subsidy policy that encourages parents to increase their investments in the education of their children. Furthermore, many developed countries that face extremely low fertility rates and rapid population aging provide a set of welfare policies for improving the child rearing environment (e.g., maternity/paternity leave, child care facilities, child allowances, and income tax exemptions). One of the main objectives of these child rearing subsidy policies is to prevent further declines in the fertility rate by reducing parents' opportunity costs for child bearing and rearing. For example, in Japan, the government has been adopting various pro-natal policies since the 1990s (e.g., the expansion of the child allowance, the introduction of child care leave, and improvements in child care services). These policies are intended to avoid demographic changes that could produce a crisis of the public pension system, labor shortages, economic stagnation, and a loss of societal vitality.²⁰ In this section, we examine how these two types of subsidy policies (child education subsidies and child rearing subsidies) affect the per capita GDP growth rate and the lifetime utility levels of individuals.

4.1 Growth effects

In this subsection, we examine the growth effects of two types of subsidy policies: child education subsidies and child rearing subsidies. The government levies a tax τ_t on the labor income of all young adult individuals and compensates these individuals for a fraction of their parenting-related opportunity costs, which consist of the periods of time that are spent away from work to raise (zn_t) and educate their children ($e_t n_t$). If individuals in generation t allocated their time to work, they would obtain $(1 - \tau_t)w_t h_t$ per unit of time. Thus, this paper assumes that the government subsidizes a fraction $\theta_e \in (0, 1)$ of $(1 - \tau_t)w_t h_t$ for each unit of time that parents devote to educating their children $e_t n_t$. This subsidy represents the government's child education subsidy policy. Analogously, the government subsidizes a fraction $\theta_n \in (0, 1)$ of $(1 - \tau_t)w_t h_t$ for each unit of time that parents devote

²⁰See, e.g., Atoh and Akachi (2003) and Suzuki (2006).

to bearing and raising their children zn_t . This subsidy represents the child rearing subsidy policy.²¹

Under these two types of subsidy policies, the government's budget constraints at any point of time t is written as follows:

$$\tau_t w_t h_t \ell_t = \theta_n (1 - \tau_t) w_t h_t z n_t + \theta_e (1 - \tau_t) w_t h_t e_t n_t, \quad (36)$$

where $\tau_t w_t h_t \ell_t$ expresses the per capita tax revenue, $\theta_n (1 - \tau_t) w_t h_t z n_t$ expresses the per capita child rearing subsidy payment, and $\theta_e (1 - \tau_t) w_t h_t e_t n_t$ expresses the per capita child education subsidy payment.

The budget constraint of the individual in generation t is represented, together with (3) and (4), as follows:

$$c_{1,t} + s_t = (1 - \tau_t) w_t h_t \{1 - [(1 - \theta_n)z + (1 - \theta_e)e_t] n_t - m_t\}. \quad (37)$$

Thus, an individual in generation t maximizes (1) subject to (3), (4), (5) and (37). Consequently, we obtain the following results:

$$s_t = \frac{(1 - \gamma)\pi}{(1 - \gamma)(1 + \sigma_m)(1 + \pi) + \gamma} (1 - \tau_t) w_t h_t, \quad (38)$$

$$n_t = \frac{1}{(1 - \theta_n)z} \frac{\gamma(1 - \phi)}{(1 - \gamma)(1 + \sigma_m)(1 + \pi) + \gamma} \equiv n(\theta_n), \quad (39)$$

$$e_t = \frac{1 - \theta_n}{1 - \theta_e} \frac{\phi z}{1 - \phi} \equiv e(\theta_e, \theta_n), \quad (40)$$

In addition, we obtain (9), $h_t = E h_{t-1} [e(\theta_e, \theta_n)]^{\sigma_e} m^{\sigma_m}$. From (39), a higher child rearing subsidy (θ_n) leads to a higher fertility rate (n_t) because this subsidy decreases parents' opportunity costs of child bearing. However, from (40), a higher child rearing subsidy (θ_n) produces a lower investment in education for each child (e_t) because the increase in the fertility rate (n_t) that occurs as a result of θ_n increases the opportunity costs that parents incur for providing their children with educations. Thus, a child rearing subsidy θ_n generates a substitutive effect in which quality of children is replaced by quantity of children.

By contrast, according to (39) and (40), a higher child education subsidy (θ_e) produces a higher investment in education for each child e_t but leaves the fertility rate (n_t) unchanged. This result is explained as follows. According to (36), a child education subsidy θ_e decreases the parental opportunity costs

²¹These representations of subsidy policies are abstract and may initially appear to be unrealistic. However, these specifications allow us to clearly capture the difference between child education subsidies and child rearing subsidies.

of both child education and child bearing, positively affecting both child education and fertility. However, because the decline in the opportunity cost of child education is greater than the corresponding decline in the opportunity cost of child bearing, the investment in education for each child e_t increases more elastically. This increase in a parent's investment in education for each child (e_t) increases the opportunity costs of child bearing for parents, offsetting the direct positive effects of θ_e on the fertility rate (n_t). Consequently, the fertility rate (n_t) will remain constant with respect to θ_e . Hence, a child education subsidy θ_e increases the quality of children without affecting the quantity of children. To emphasize these relationships, we denote n_t by $n(\theta_n)$ and e_t by $e(\theta_e, \theta_n)$. Finally, according to (9), an investment in one's own education m_t is neutral with respect to these two types of subsidy policies.

Solving the model in an analogous way to previous sections, the per capita GDP growth rate (g_y) in the balanced growth path is again given by (34). Hence, using (34), $n_t = n(\theta_n)$ and $e_t = e(\theta_e, \theta_n)$, we obtain the following proposition.²²

Proposition 2 *Under child education and child rearing subsidy policies, in the balanced growth path (BGP), the following comparative statics results hold.*

$$(1) \quad \frac{\partial g_y}{\partial \theta_e} > 0,$$

$$(2) \quad \frac{\partial g_y}{\partial \theta_n} < (\geq) 0, \quad \text{if } \sigma_e > (\leq) \Omega(\nu),$$

where $\Omega(\nu) \equiv \frac{(1-\nu)(1-\varepsilon)\alpha}{(1-\nu)(1-\varepsilon)\alpha + (1-\psi)\varepsilon}$, $\frac{\partial \Omega(\nu)}{\partial \nu} < 0$, $\frac{\partial}{\partial \nu} \left[\frac{\partial \Omega(\nu)}{\partial \nu} \right] < 0$, $\lim_{\nu \rightarrow 0} \Omega(\nu) < 1$ and $\lim_{\nu \rightarrow 1} \Omega(\nu) = 0$.

Proposition 2-(1) indicates that a child education subsidy θ_e positively impacts the per capita GDP growth rate by increasing the parental investment in education for each child (e). Using (34) and the fact that $\frac{\partial e}{\partial \theta_e} > 0$, $\frac{\partial n}{\partial \theta_e} = 0$ and $\frac{\partial m}{\partial \theta_e} = 0$, we can easily confirm that the relationship $\frac{\partial g_y}{\partial \theta_e} > 0$ holds.

The results of Proposition 2-(2) are summarized in Figure 3. Proposition 2-(2) indicates that if the intensity of parental educational investment in human capital production σ_e is sufficiently large (resp. small) to satisfy $\sigma_e > \Omega(\nu)$ (resp. $\sigma_e \leq \Omega(\nu)$), child rearing subsidies θ_n have negative (resp. non-negative) effects on the per capita GDP growth rate g_y . As discussed above, a child rearing subsidy θ_n positively affects the fertility rate (i.e., $\frac{\partial n}{\partial \theta_n} > 0$) but negatively affects the parental investment in education for each child (i.e., $\frac{\partial e}{\partial \theta_n} < 0$). Thus, based on (34), a child rearing subsidy θ_n has two

²²See Appendix D for the derivation of Proposition 2-(2).

competing effects on the per capita GDP growth rate g_y . These effects comprise a growth-enhancing quantity effect that reflects an increase in fertility (n) and a growth-impeding quality effect that is mediated by a decline in parental investments in education for each child (e). Thus, a rise in the child rearing subsidy (θ_n) increases the population growth rate (n), contributes to expanding the supply of available researchers, and consequently increases g_y , the per capita GDP growth rate (i.e., a growth-enhancing quantity effect). However, an increase in the child rearing subsidy (θ_n) lowers the parental investment in education for each child (e), retards the accumulation of human capital by researchers, and thereby decreases g_y , the per capita GDP growth rate (i.e., a growth-impeding quality effect).

From (34), the value of σ_e indicates the significance of parental educational investment as a determinant of the per capita GDP growth rate. Therefore, if σ_e , the parental educational investment intensity in the production of human capital, is sufficiently large (resp. small) to satisfy $\sigma_e > \Omega(\nu)$ (resp. $\sigma_e \leq \Omega(\nu)$), then the growth-impeding quality effect dominates (resp. is dominated by) the growth-enhancing quantity effect, and child rearing subsidies θ_n have negative (resp. non-negative) effects on the per capita GDP growth rate.

Moreover, as shown in Figure 3, a stronger stepping-on-toes effect (i.e., a higher value of ν) expands the parameter regions in which the child rearing subsidy policy θ_n negatively affects the per capita GDP growth rate (i.e., $\frac{\partial y}{\partial \theta_n} < 0$).²³ If the stepping-on-toes effect is strong, the size of workers engaged in the R&D sector has little effect on new variety expansion because the benefit of R&D efforts by large numbers of researchers is offset by the increased duplication of research efforts. In particular, from (34), if the stepping-on-toes effect is extremely strong (i.e., $\nu = 1$), the growth rates of either the population as a whole or the number of researchers in particular (n) will have no positive effect on the per capita GDP growth rate, and the growth-enhancing quantity effect will become negligible. Therefore, if the stepping-on-toes effect is strong, the growth-impeding quality effect will dominate the growth-enhancing quantity effect, and a child rearing subsidy θ_n will negatively affect the per capita GDP growth rate.

These results indicate that the effect of a child education subsidy on growth is always positive, whereas the effect of a child rearing subsidy on

²³From the definition of $\Omega(\nu)$, analogous to the effect of ν , a higher elasticity of substitution among intermediate goods (ε) and a lower intensity of intermediate goods in the final goods sector (α) will enlarge the parameter regions in which the relationship $\frac{\partial y}{\partial \theta_n} < 0$ holds. Both higher ε and lower α induce smaller monopoly profits for intermediate good firms, reduce the product variety growth rate in the R&D sector, and weaken the growth-enhancing quantity effect of child rearing subsidy policies.

growth can be negative in an environment in which either the strength of the stepping-on-toes effect is large or parental educational investment is critical for the accumulation of human capital. Thus, even in an R&D-based growth model in which population growth positively affects long-run growth through its quantity effect, a child rearing subsidy policy (i.e., a pro-natal policy) cannot be justified as a growth-enhancing policy.

4.2 Welfare effects

In this subsection, we examine the welfare effect of a child education subsidy policy. To avoid lexicographic explanations, we focus our analysis on the child education subsidy policy (i.e., $\theta_e > 0$ and $\theta_n = 0$). We consider the following policy experiment. Initially, we assume that the economy is in a steady-state equilibrium with no child education subsidy policy (i.e., $\theta_{e,t} = 0$ for all $t < k$). Then, in period k , a child education subsidy policy is introduced for young adult individuals in generation k and subsequent generations $t \geq k$. For simplicity, we assume that the government establishes the subsidy rate $\theta_{e,t}$ as a fixed value θ_e in period k and that this value remains constant at θ_e for all $t \geq k$ (i.e., $\theta_{e,t} = \theta_e$ for all $t \geq k$).

Using (1), (3), (4), (5), (36)-(40), and $\theta_n = 0$, the lifetime utility level of individuals in generation t , u^t , is written as follows:

$$\begin{aligned} u^t(w_t, R_{t+1}, h_{t-1}, \theta_{e,t}) &\equiv (1 - \gamma) \ln c_{1,t}(w_t, h_{t-1}, \theta_{e,t}) \\ &\quad + (1 - \gamma)\pi \ln c_{2,t+1}(w_t, R_{t+1}, h_{t-1}, \theta_{e,t}) \quad (41) \\ &\quad + \gamma [\phi \ln e(\theta_{e,t}) + \ln n], \end{aligned}$$

where

$$\begin{aligned} c_{1,t}(w_t, h_{t-1}, \theta_{e,t}) &\equiv \frac{1 - \gamma}{(1 - \gamma)(1 + \sigma_m)(1 + \pi) + \gamma} [1 - \tau(\theta_{e,t})] w_t h(h_{t-1}, \theta_{e,t}), \\ c_{2,t+1}(w_t, R_{t+1}, h_{t-1}, \theta_{e,t}) &\equiv \frac{1 - \gamma}{(1 - \gamma)(1 + \sigma_m)(1 + \pi) + \gamma} R_{t+1} [1 - \tau(\theta_{e,t})] w_t h(h_{t-1}, \theta_{e,t}), \\ \tau(\theta_{e,t}) &\equiv \frac{\gamma \phi}{(1 - \gamma)(1 + \pi)} \frac{\theta_{e,t}}{1 - \theta_{e,t}}, \\ e(\theta_{e,t}) &\equiv \frac{1}{1 - \theta_{e,t}} \frac{\phi z}{1 - \phi}, \\ h(h_{t-1}, \theta_{e,t}) &\equiv E h_{t-1} [e(\theta_{e,t})]^{\sigma_e} m^{\sigma_m}, \\ n &\equiv \frac{1}{z} \frac{\gamma(1 - \phi)}{(1 - \gamma)(1 + \sigma_m)(1 + \pi) + \gamma}, \\ m &\equiv \frac{\sigma_m(1 - \gamma)(1 + \pi)}{(1 - \gamma)(1 + \sigma_m)(1 + \pi) + \gamma}. \end{aligned}$$

Furthermore, as detailed in Appendix F, the equilibrium wage rate w_t and the interest rate R_t are determined by the following equations:

$$w_t = \Upsilon A_t^{\frac{1-\varepsilon}{\varepsilon}\alpha}, \quad (42)$$

$$R_{t+1} = \frac{d_{t+1} + V_{t+1}}{V_t}, \quad (43)$$

where

$$d_t = \frac{\alpha(1-\varepsilon)\Upsilon}{1-\alpha+\varepsilon\alpha} A_t^{\frac{1-\varepsilon}{\varepsilon}\alpha-1} \left(1 - \frac{\pi}{1+\pi} \frac{g_{A,t}}{1+g_{A,t}} \right) H_t,$$

$$V_t = \frac{s_t N_t}{A_{t+1}} = \frac{A_t^{\frac{1-\varepsilon}{\varepsilon}\alpha-1}}{1+g_{A,t}} \frac{\pi}{1+\pi} \Upsilon H_t,$$

$\Upsilon \equiv \varepsilon^\alpha \alpha^\alpha (1-\alpha)^{1-\alpha}$, $H_t \equiv h_t \ell_t N_t$ and $g_{A,t} \equiv \frac{A_{t+1}-A_t}{A_t}$.

A child education subsidy policy affects the lifetime utility level of individuals through three channels: the “tax-subsidy distortion effect”, the “human capital accumulation effect” and the “general equilibrium effect”. First, the tax-subsidy distortion effect is represented by the increase in $\theta_{e,t}$ in (41). Given wage w_t , interest rate R_{t+1} and parental human capital h_{t-1} , an increase in $\theta_{e,t}$ increases the parental investment in education for each child e_t , decreases the labor supply ℓ_t , and increases the tax burden τ_t . Because of the well-known tax subsidy distortion effect, the latter two negative effects always offset the initial (positive) effect. Therefore, the tax-subsidy distortion effect represents a negative influence on the lifetime utility levels of individuals. Second, the human capital accumulation effect is represented by the changes in $h(\theta_{e,t}, h_{t-1})$ that occur in (41). A child education subsidy policy enhances parental investments in education for each child (e_t) and increases the human capital level of all subsequent generations. Therefore, the human capital accumulation effect represents a positive influence on the lifetime utility levels of individuals. Finally, the general equilibrium effect is represented by the changes in the wage rate (w_t) and the interest rate (R_t) that appear in (41). Unfortunately, it is difficult to analytically investigate the influence of the general equilibrium effect. As a result, this subsection provides only numerical examples.

Figure 4-1 illustrates the net lifetime utility gains of individuals who belong to generations 17–25 (9 generations) and describes how the lifetime utility levels of these individuals are affected by the child education subsidy policy implemented from period 20. Thus, we set $k = 20$ in our numerical analysis. The parameters used in the base-case simulation are described in

footnote 14 and explained in Appendix E. The solid line represents $\theta_e = 0$, and the broken and dotted lines show the differences between $\theta_e = 0.03$, 0.06 and 0 , respectively.

As shown in Figure 4-1, a child education subsidy policy lowers the lifetime utility levels of individuals in generation 19 but raises the lifetime utility levels of individuals in all subsequent generations for $t \geq 20$. This result implies that there is an intergenerational conflict between current and future generations with respect to child education subsidy policies. Figures 4-2 to 4-4 help us understand the intuition underlying this result. Figure 4-2 reveals the net differences in the human capital levels of individuals who belong to generations 17 to 25, whereas Figures 4-3 and 4-4 illustrate the net differences in wage rate (w_t) and interest rate (R_t) from period 17 to period 25. Thus, Figure 4-2 represents the human capital accumulation effect, whereas Figures 4-3 and 4-4 represent the general equilibrium effect.

The decline in interest rate R_t that occurs during period 20 (see Figure 4-4) reduces the lifetime utility level of the individuals in generation 19 because it decreases their interest income during their old age.²⁴ However, the increase in the interest rate during period 21 (see Figure 4-4) improves the lifetime utility level of individuals in generation 20 by offsetting the negative tax-subsidy distortion effect of the child education subsidy policy.²⁵

The child education subsidy policy implemented from period 20 leads to the higher human capital level of individuals in generation $t \geq 21$ (see Figure 4-2). Although the decline in wage rate in period 21 (see Figure 4-3) and the tax-subsidy distortion effect negatively affect the lifetime utility levels of individuals in generation 21, the increase in the interest rate during period 22 and the positive human capital accumulation effect offset these negative effects and improve the lifetime utility levels of individuals in generation 21.

As shown in Figure 4-2, the influence of the human capital accumulation effect becomes larger over time because higher human capital levels of previous generations result in higher human capital levels for subsequent generations (i.e., intergenerational spillover effects via human capital production functions). Furthermore, based on (24), (27) and (42), the acceleration of

²⁴Figures 4-3 and 4-4 indicate that the child education subsidy policy implemented from period 20 induces short-run fluctuations in the wage rate (w_t) and interest rate (R_t). The mechanisms underlying this short-run fluctuation are slightly complicated and are difficult to briefly explain. Therefore, we delegate this explanation to Appendix G for readers who are concerned with this issue.

²⁵The child education subsidy policy implemented from period 20 affects neither the human capital levels nor the wage rates of individuals in generations 19 and 20. In addition, individuals in generation 19 do not suffer from the tax-subsidy distortion effect, because the child education subsidy policy has not yet been implemented at the time that these individuals are young adults.

human capital accumulation from period 20 leads to the higher growth rate of aggregate human capital H_t , product variety A_t and wage rate w_t in period 21 and subsequent periods. Therefore, in the long run, the positive human capital accumulation effect, and accompanying rapid increases in wages offset other negative effects, and improve the lifetime utility levels of individuals in generations $t \geq 22$.²⁶

The above numerical simulation results suggest that a child education subsidy policy greatly improves the welfare of future generations, whereas the benefits that current generations obtain are relatively small. In particular, child education subsidy policy may produce immediate reductions in the welfare of old individuals when the policy is implemented. These results suggest that the real-world expansion of a child education subsidy may be politically challenging because it impairs the welfare of elderly individuals or individuals who possess great political power at the time that this policy goes into effect. Therefore, the expansion of a child education subsidy policy may be politically unpopular and difficult for politicians to implement.

5 Concluding remarks

This paper constructed a simple overlapping generations version of an R&D-based growth model à la Diamond (1965) and Jones (1995). This model was used to examine how an increase in old-age survival probability impacts purposeful R&D investment and long-run growth by affecting fertility and education decisions. We demonstrated that under conditions involving relatively low (high) old-age survival probabilities, an increase in this probability can positively (negatively) affect economic growth. This paper also compared the growth implications of child education subsidy policies and child rearing subsidy policies and demonstrated that although child education subsidies always foster economic growth, child rearing subsidies may reduce economic growth under certain conditions. Finally, we considered the effects of a child education subsidy on welfare levels. We found that there exists an inter-generational conflict between current and future generations with respect to public policies that govern child education.

²⁶In the long run, the interest rate R_t gradually converges to a value that is lower than its original steady-state value. Thus, this decline in the interest rate (R_t) and the tax subsidy-distortion effect negatively affect the lifetime utility levels of individuals in generations $t \geq 22$.

Appendix A: The market-clearing condition for assets

Due to the perfect competition in the final goods market, the value of final good output is expressed as follows:

$$Y_t = w_t H_{Y,t} + A_t p_t x_t.$$

Thus, using the profits of intermediate good firms $d_t = (p_t - w_t)x_t$, (21) and (22), the above equation can be rewritten as follows:

$$Y_t = w_t (H_t - H_{A,t}) + A_t (V_{t-1} R_t - V_t).$$

Therefore, the market-clearing condition for final goods is expressed in the following manner:

$$w_t (H_t - H_{A,t}) + A_t (V_{t-1} R_t - V_t) = c_{1,t} N_t + c_{2,t} \pi N_{t-1}.$$

In the case of $V_t \delta_t = w_t$, $H_{A,t} > 0$ and $A_{t+1} > A_t$

With respect to (20), consider the case of $V_t \delta_t = w_t$ in which the R&D sector functions; $H_{A,t} > 0$; and $A_{t+1} > A_t$. By substituting (2), (3), (18), $V_t \delta_t = w_t$ and $H_t \equiv h_t \ell_t N_t$ into the market-clearing condition for final goods, we obtain the following expression:

$$s_t N_t - V_t A_{t+1} = R_t (s_{t-1} N_{t-1} - V_{t-1} A_t).$$

Because initial assets are given by $s_{-1} N_{-1} = V_{-1} A_0$, we can obtain (23) for any period $t > 0$.

In the case of $V_t \delta_t < w_t$, $H_{A,t} = 0$ and $A_{t+1} = A_t$

With respect to (20), consider the case of $V_t \delta_t < w_t$ in which R&D sector does not function; $H_{A,t} = 0$; and $A_{t+1} = A_t$. By substituting (2), (3), $H_{A,t} = 0$ and $H_t \equiv h_t \ell_t N_t$ into the market-clearing condition for final goods, we obtain the following expression:

$$s_t N_t - V_t A_t = R_t (s_{t-1} N_{t-1} - V_{t-1} A_t).$$

Because the initial assets are given by $s_{-1} N_{-1} = V_{-1} A_0$, we obtain the following asset market equilibrium condition:

$$V_t A_t = s_t N_t, \quad \text{for } V_t \delta_t < w_t. \quad (44)$$

Appendix B: The non-existence of $H_{A,t} = 0$ at equilibrium

From (20), the case of $H_{A,t} = 0$ occurs only if $V_t \delta_t < w_t$. In this appendix, we demonstrate that the relationships $V_t \delta_t < w_t$ and $H_{A,t} = 0$ cannot simultaneously hold under market equilibrium conditions.

Suppose that $H_{A,t} = 0$; as shown in Appendix A, we then obtain the following asset market equilibrium condition: $V_t A_t = s_t N_t$. Here, note that this condition is derived without explicitly relying on the information that $V_t \delta_t < w_t$. Thus, by substituting (6), $H_t \equiv h_t \ell_t N_t$ and $\ell_t = \frac{(1-\gamma)(1+\pi)}{(1-\gamma)(1+\sigma_m)(1+\pi)+\gamma} \equiv \bar{\ell}$ into $V_t A_t = s_t N_t$, the stock value (V_t) that is consistent with market equilibrium conditions can be rewritten as follows:

$$V_t = \frac{\pi}{1+\pi} \frac{w_t H_t}{A_t}.$$

Hence, using (19) and $V_t = \frac{\pi}{1+\pi} \frac{w_t H_t}{A_t}$, the condition $V_t \delta_t < w_t$ can be rewritten as follows:

$$\frac{\pi}{1+\pi} \bar{\delta} A_t^{\psi-1} H_t < H_{A,t}^\nu.$$

However, because $\frac{\pi}{1+\pi} \bar{\delta} A_t^{\psi-1} H_t > 0$, the above inequality never holds if $H_{A,t} = 0$. Therefore, the relationships of $V_t \delta_t < w_t$ and $H_{A,t} = 0$ cannot simultaneously hold under market equilibrium conditions.

Appendix C: The derivation of per capita GDP in (32)

We derive per capita GDP in (32). By substituting (13), (14) and $X_t = A_t^{1/\varepsilon} x_t$ into (16), we obtain the following equation:

$$\frac{H_{Y,t}}{A_t x_t} = \frac{1-\alpha}{\varepsilon \alpha}. \quad (45)$$

Using (45), GDP in (31) can be expressed as follows:

$$Y_t = A_t^{\frac{1-\varepsilon}{\varepsilon} \alpha} \left(\frac{H_{Y,t}}{A_t x_t} \right)^{1-\alpha} A_t x_t = A_t^{\frac{1-\varepsilon}{\varepsilon} \alpha} \left(\frac{1-\alpha}{\varepsilon \alpha} \right)^{1-\alpha} A_t x_t. \quad (46)$$

Furthermore, by substituting (45) into (22), we obtain the following expression:

$$A_t x_t = \frac{\varepsilon \alpha}{1-\alpha+\varepsilon \alpha} (H_t - H_{A,t}). \quad (47)$$

Thus, by substituting (47) into (46), GDP can be rewritten as follows:

$$Y_t = \frac{\varepsilon^\alpha \alpha^\alpha (1-\alpha)^{1-\alpha}}{1-\alpha+\varepsilon\alpha} A_t^{\frac{1-\varepsilon}{\varepsilon} \alpha} \left(1 - \frac{H_{A,t}}{H_t}\right) H_t. \quad (48)$$

Finally, using (18), (19) and (25), $\frac{H_{A,t}}{H_t}$ can be expressed as follows:

$$\frac{H_{A,t}}{H_t} = \frac{\pi}{1+\pi} \frac{g_{A,t}}{1+g_{A,t}}. \quad (49)$$

Thus, by substituting (49) into (48), and dividing it by the young adult population size (N_t), we obtain (32).

Appendix D: The derivations of (35) and Proposition 2-2

The derivation of (35)

From (34), we obtain the following expression:

$$\ln(1+g_y) = \left(1 + \frac{1-\nu}{1-\psi} \frac{1-\varepsilon}{\varepsilon} \alpha\right) (\ln E e^{\sigma_e} + \sigma_m \ln m) + \frac{1-\nu}{1-\psi} \frac{1-\varepsilon}{\varepsilon} \alpha \ln n. \quad (50)$$

Then, by differentiating (50) with respect to π , we obtain the following result:

$$\frac{1}{1+g_y} \frac{\partial g_y}{\partial \pi} = \left(1 + \frac{1-\nu}{1-\psi} \frac{1-\varepsilon}{\varepsilon} \alpha\right) \frac{\sigma_m}{m} \frac{\partial m}{\partial \pi} + \frac{1-\nu}{1-\psi} \frac{1-\varepsilon}{\varepsilon} \alpha \frac{1}{n} \frac{\partial n}{\partial \pi}. \quad (51)$$

From (7) and (9), the following relationships must hold.

$$\begin{aligned} \frac{1}{m} \frac{dm}{d\pi} &= \frac{1}{[(1-\gamma)(1+\sigma_m)(1+\pi)+\gamma]} \frac{\gamma}{1+\pi}, \\ \frac{1}{n} \frac{\partial n}{\partial \pi} &= -\frac{(1-\gamma)(1+\sigma_m)}{(1-\gamma)(1+\sigma_m)(1+\pi)+\gamma}. \end{aligned}$$

Thus, by substituting the above two equations into (51), we obtain the following equation:

$$\begin{aligned} \frac{\partial g_y}{\partial \pi} &= \frac{1+g_y}{(1-\gamma)(1+\sigma_m)(1+\pi)+\gamma} \times \\ &\quad \left[\left(1 + \frac{1-\nu}{1-\psi} \frac{1-\varepsilon}{\varepsilon} \alpha\right) \frac{\sigma_m \gamma}{1+\pi} - \frac{1-\nu}{1-\psi} \frac{1-\varepsilon}{\varepsilon} \alpha (1-\gamma)(1+\sigma_m) \right]. \end{aligned}$$

By rearranging the above equation, we obtain (35).

The derivation of Proposition 2-2

By differentiating log GDP per capita with respect to θ_n , we obtain the following expression:

$$\frac{1}{1+g_y} \frac{\partial g_y}{\partial \theta_n} = \left(1 + \frac{1-\nu}{1-\psi} \frac{1-\varepsilon}{\varepsilon} \alpha\right) \frac{\sigma_e}{e} \frac{\partial e}{\partial \theta_n} + \frac{1-\nu}{1-\psi} \frac{1-\varepsilon}{\varepsilon} \alpha \frac{1}{n} \frac{\partial n}{\partial \theta_n}. \quad (52)$$

From (39) and (40), the following relationships must hold:

$$\frac{1}{e} \frac{\partial e}{\partial \theta_n} = -\frac{1}{1-\theta_n},$$

$$\frac{1}{n} \frac{\partial n}{\partial \theta_n} = \frac{1}{1-\theta_n}.$$

Thus, by substituting the above two equations into (52) and rearranging the result, we obtain the following expression:

$$\frac{\partial g_y}{\partial \theta_n} = -\frac{1+g_y}{1-\theta_n} \left(1 + \frac{1-\nu}{1-\psi} \frac{1-\varepsilon}{\varepsilon} \alpha\right) \left[\sigma_e - \frac{(1-\nu)(1-\varepsilon)\alpha}{(1-\nu)(1-\varepsilon)\alpha + (1-\psi)\varepsilon} \right].$$

Appendix E: The parameters for the simulation

According to Alvarez-Pelaez and Groth (2005), labor's share in output in US is approximately 0.6, and markup estimates in US industry range between 1.05 and 1.40. Based on these estimates, the share of intermediate good inputs in output (α) is set to 0.4, and the substitution parameter (ε) is set to 0.83. Furthermore, in accordance with the example of Strulik et al. (2011), we set the standing-on-shoulders effect parameter (ψ) to 0.5 and the stepping-on-toes effect parameter (ν) to 0.2. We also normalize the scaling parameter of R&D production ($\bar{\delta}$) to 10.

Because there is little evidence regarding the values of parameters for the relative utility weights that are allocated to children or to the quality of children (γ, ϕ), the human capital production function (E, σ_e, σ_m) and the time cost of child bearing (z), we choose these values in ways that produce plausible per capita GDP growth and population growth. In particular, we chose 0.3 and 0.7 as the values of γ and ϕ , respectively. The values of σ_e and σ_m are established at 0.25 and 0.75, respectively. To achieve a 2 % balanced per capita GDP growth rate at an old-age survival probability (π) of 1, we adjust the value of E to $E = 6.0675$. Further, to achieve a 0 % population

growth rate if the old-age survival probability (π) is 1, we adjusted the value of z to $z = \frac{\gamma(1-\phi)}{2(1-\gamma)(1+\sigma_m)+\gamma}$.

To investigate the effect of an increase in old-age survival probability, we set the value of π , the old-age survival probability, to 1 in the base-case simulation and increase π from 0.1 to 1 in increments of 0.1. In addition, to investigate the effect of a child education subsidy policy, we set the value of θ_e , the child education subsidy rate, at 0 in the base-case simulation and alter it from 0 to 0.06 in increments of 0.03.

Appendix F: The derivations of (42) and (43)

The derivation of the wage rate in (42)

First, we derive the wage rate in (42). By substituting $X_t = A_t^{1/\varepsilon} x_t$ into (13), we obtain the following expression:

$$w_t = (1 - \alpha) H_{Y,t}^{-\alpha} \left(A_t^{1/\varepsilon} x_t \right)^\alpha = (1 - \alpha) \left(\frac{A_t x_t}{H_{Y,t}} \right)^\alpha A_t^{\frac{1-\varepsilon}{\varepsilon} \alpha}. \quad (53)$$

Finally, by substituting (45) into (53), we obtain (42).

The derivation of the interest rate in (43)

Next, we derive the interest rate in (43). By substituting (16), (42), (47), and (49) into (17), we obtain the following expression:

$$d_t = \frac{\alpha(1 - \varepsilon)\Upsilon}{1 - \alpha + \varepsilon\alpha} A_t^{\frac{1-\varepsilon}{\varepsilon}\alpha-1} \left(1 - \frac{\pi}{1 + \pi} \frac{g_{A,t}}{1 + g_{A,t}} \right) H_t. \quad (54)$$

Using (4), (9), (39), (40) and $\tau(\theta_{e,t})$ in (41), the labor supply under a child education subsidy policy is therefore expressed as follows:

$$\ell_t = \frac{(1 - \gamma)(1 + \pi)(1 - \tau_t)}{(1 - \gamma)(1 + \sigma_m)(1 + \pi) + \gamma}. \quad (55)$$

Thus, by substituting (38), (42), (55) and $H_t = h_t \ell_t N_t$ into (23), we obtain the following equation:

$$V_t = \frac{s_t N_t}{A_{t+1}} = \frac{A_t^{\frac{1-\varepsilon}{\varepsilon}\alpha-1}}{1 + g_{A,t}} \frac{\pi}{1 + \pi} \Upsilon H_t. \quad (56)$$

Finally, by substituting (54) and (56) into (21), we obtain (43).

Appendix G: Supplementary explanation for Figure 4-3 and 4-4

The effect of child education subsidy on wage rate w_t in Figure 4-3

The effect of a child education subsidy on the wage rate w_t is explained as follows. The child education subsidy policy implemented from period 20 increases parental investment in education for each child e_t in period $t \geq 20$ but decreases the labor supply ℓ_t in period $t \geq 20$. From (27) and $H_t \equiv h_t \ell_t N_t$, the decline in the labor supply ℓ_t during period 20 negatively affects the aggregate human capital (H_t) that is accumulated between periods 19 and 20, lowering the growth rate of product variety (A_t) between periods 20 and 21. Note that the wage rate (w_t) is given by $w_t = \Upsilon A_t^{\frac{1-\varepsilon}{\varepsilon} \alpha}$ from (42). Thus, as shown in Figure 4-3, the wage rate w_t during period 21 decreases due to the child education subsidy policy. However, the increase in parental investment in education for each child (e_t) that occurs during periods $t \geq 20$ positively affects the aggregate human capital (H_t) that is accumulated during periods $t \geq 20$, increasing the growth rate of product variety A_t during periods $t \geq 21$. Consequently, the wage rate (w_t) during periods $t \geq 23$ increases greatly because of the child education subsidy policy.

The effect of a child education subsidy on interest rate R_t in Figure 4-4

By substituting (54) and (56) into (21), we obtain the following expression:

$$R_t = (1 + g_{A,t-1})^{\frac{1-\varepsilon}{\varepsilon} \alpha} \left(\frac{H_t}{H_{t-1}} \right) \frac{\alpha(1-\varepsilon)}{1-\alpha+\varepsilon\alpha} \left[\frac{1}{\pi} + \frac{1}{\alpha(1-\varepsilon)} \frac{1}{1+g_{A,t}} \right]. \quad (57)$$

Thus, we can confirm that interest rate (R_t) is affected by the growth rate of product variety (A_t) and by aggregate human capital (H_t) in several complicated ways. By considering (57), the effect of the child education subsidy on the interest rate R_t may be explained as follows. From (27), (57), and $H_t \equiv h_t \ell_t N_t$, the decline in the labor supply (ℓ_t) during period 20 decreases the growth rate of aggregate human capital (H_t) between periods 19 and 20, which induces a decline in the interest rate (R_t) during period 20. However, the increase in the parental investment in education for each child (e_t) in periods $t \geq 20$ increases the growth rate of aggregate human capital (H_t) between periods 20 and 21, which induces an increase in the interest rate

(R_t) during period 21. After period 21, short-run fluctuation ceases, and the interest rate (R_t) gradually converges to a new steady-state value that is lower than the original steady-state value.

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Table1 Total fertility rate, Life expectancy and population growth for G-7 countries

Country	Total Fertility		Life Expectancy at Birth		Population Growth Rate	
	1960-1965	2005-2010	1960	2005	1960-1965	2005-2010
Canada	3.57	1.64	71.1	80.3	1.95	1.07
France	2.87	1.99	69.9	80.1	1.30	0.62
Germany	2.47	1.36	69.6	78.9	0.84	-0.15
Italy	2.53	1.38	69.1	80.6	0.96	0.65
Japan	2.05	1.34	67.7	81.9	1.26	-0.04
United Kingdom	2.84	1.89	71.1	79.0	0.67	0.65
United States	3.36	2.08	69.8	77.3	1.50	0.92

Source: World Bank (2012)

Table2 Change in level of Education

		Percentage of Adult Population with		
		Average Years of Schooling	Complete Secondary Education	Complete Higher Education
Advanced Countries	1975	8.0	34.9	8.0
	2010	11.0	63.9	16.6

Source: Weil (2012). Table 6-1, and Barro and Lee (2010). Data for population 25⁺

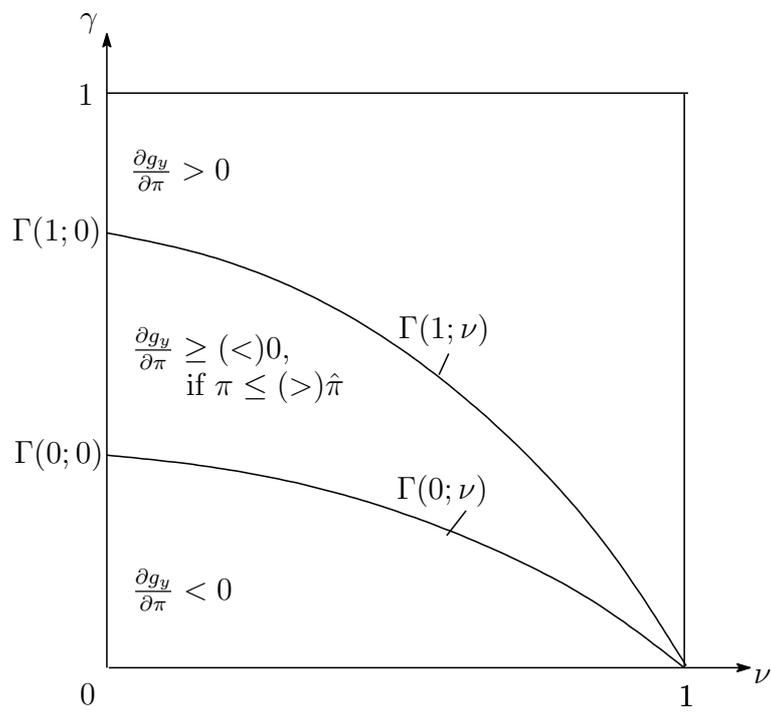


Figure 1: The parameter regions of (ν, γ) where Proposition 1-(1), 1-(2) and 1-(3) hold

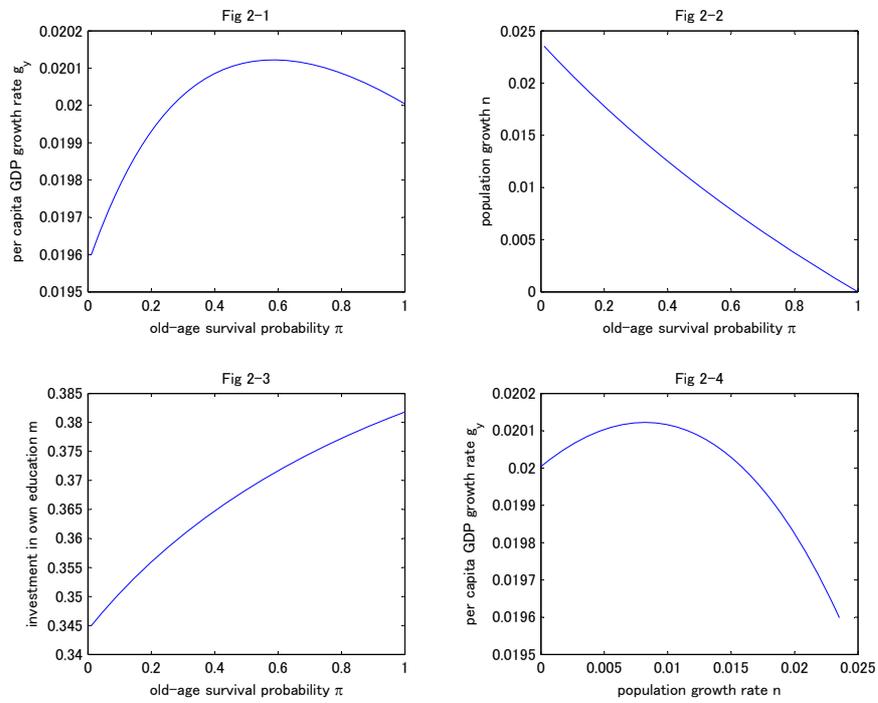


Figure 2: Growth effect of old-age survival probability

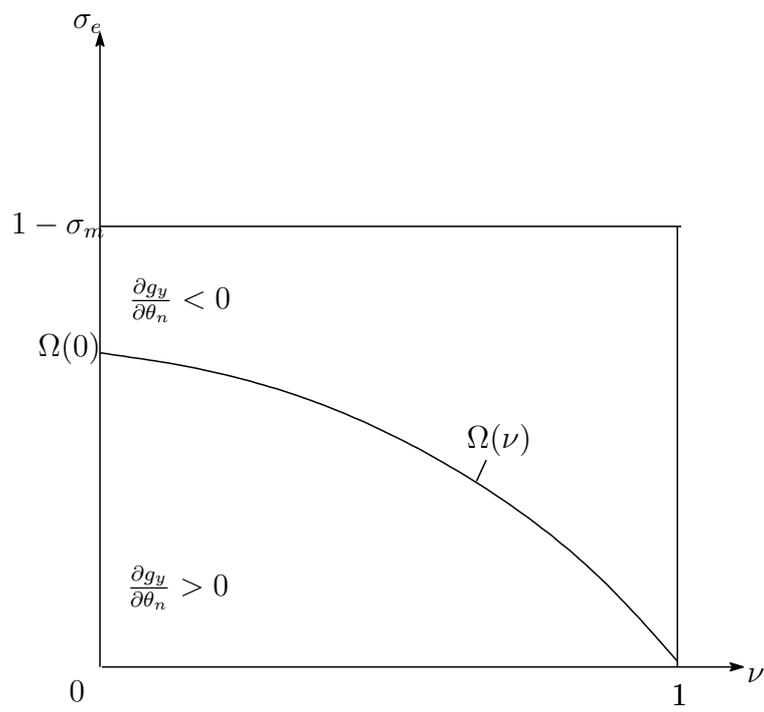


Figure 3: The parameter regions of (ν, σ_e) where Proposition 2-(2) holds

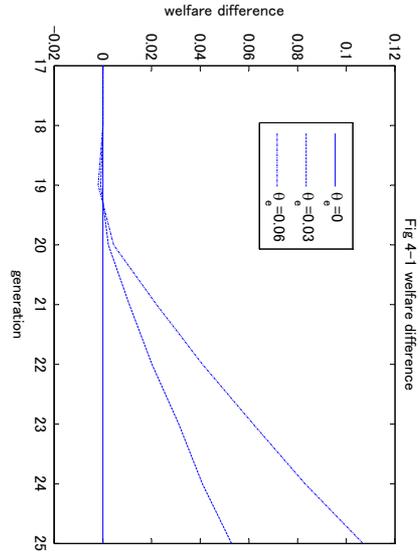


Fig 4-1 welfare difference

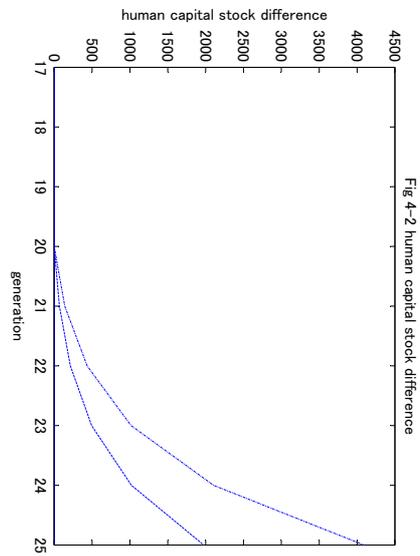


Fig 4-2 human capital stock difference

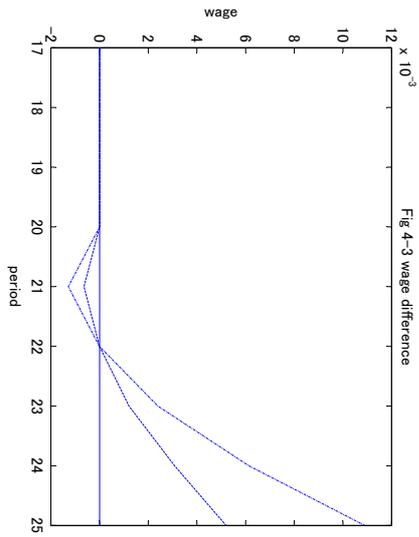


Fig 4-3 wage difference

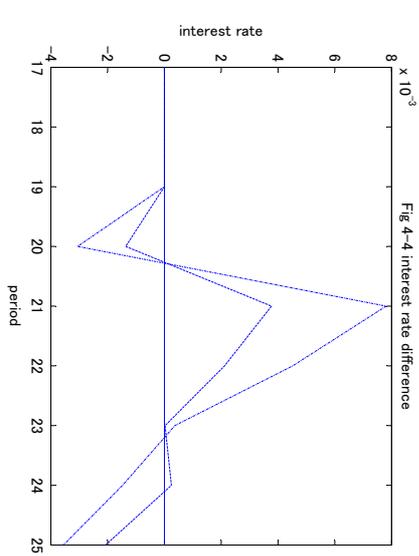


Fig 4-4 interest rate difference

Figure 4: Welfare effect of child education subsidy policy