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Central Bank Financial Strength and Credibility: A Simple Dynamic Optimization Model

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Abstract

In this paper, we develop a simple dynamic optimization model of a central bank, in which the bank's profit affects its balance sheet. The model derives the transversality condition that is necessary for a central bank to be sustainable and to conduct an optimal monetary policy. In this sense, the transversality condition needs to be satisfied to maintain central bank credibility. We discuss some factors affecting the transversality condition and show that what is important to satisfy the condition and thus to maintain central bank credibility is not capital alone but the financial strength that generates no sustained loss.

Keywords: central bank, capital, financial strength, credibility, monetary policy
JEL classification: E5

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1. Introduction

In this paper, we develop a simple dynamic optimization model of a central bank, in which the bank's profit affects its balance sheet. With this model, we show that weaker financial strength creates difficulty in conducting appropriate monetary policy and thus jeopardizes the bank's credibility.

Since the "Lehman shock" in 2008, many central banks in the industrialized countries have been using the unconventional measures of monetary easing policy. One of the main measures is quantitative easing, where a central bank expands the quantity of fund supply beyond the zero interest rate point. Another one is credit easing, where a central bank purchases some risky assets.

Many have studied whether such unconventional monetary easing measures are effective or not, and it still remains controversial. Whether they are effective or not, it is better to use such measures so long as they have no possibility of creating any problem. However, some argue that they jeopardize central bank credibility. A central bank needs credibility that it is sustainable and maintains its ability to perform its functions, especially to stabilize prices.[1] If the unconventional measures damage a central bank's balance sheet, then the central bank loses such credibility. Therefore, we need to examine the effectiveness of unconventional measures on one hand, and to assess the cost or risk of damaging the credibility on the other.

Though it is an important issue, surprisingly few studies, as compared with those on policy effectiveness, have analyzed central bank credibility in this context. Among few are Stella (1997, 2003), Bindseil *et al*. (2004), Ize (2005), Klüh and Stella (2008), Cincibuch *et al*. (2009), Adler *et al*. (2012) and Tanaka (2013), and they contend that low/negative profit or weak financial strength of central bank creates difficulty in conducting appropriate future monetary policy, which leads to jeopardizing central bank credibility.

These precedent studies have discussed the influence of financial strength on future monetary policy without showing any model. Bindseil *et al*. (2004), Ize (2005), and Cincibuch *et al*. (2009) are exceptions, but their models are not derived from any optimizing behavior of a central bank.[2]

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[1] Note that the credibility here is not the one that a central bank keeps a promise to fight against inflation when it has the ability to do so, as Barro and Gordon (1983) has examined.

[2] Berriel and Bhattarai (2009) is probably the only study to set up a model of central bank optimizing behavior with its balance sheet and profit constraints. They do not, however, derive any explicit optimal solution but run a simulation instead. With the simulation, they examine not central bank credibility but the monetary policy where a

In this paper, we develop a formal central bank model from its dynamic optimizing behavior. Our model incorporates the central bank balance sheet and profit on infinite time horizon and shows their influence on monetary policy. From this model, we derive the condition that a central bank must satisfy in order to conduct appropriate monetary policy. If the bank's financial strength is weak, the condition is not satisfied, and the bank has difficulty in conducting appropriate monetary policy and thus loses central bank credibility.

This paper is organized as follows. In Section 2, we discuss what is considered to jeopardize central bank credibility, and we argue that the financial strength is important. Then, we develop a dynamic optimization model of a central bank in Section 3, and analyze financial strength, monetary policy, and credibility in Section 4. Section 5 summarizes the analysis in this paper.

2. Central Bank Credibility

2.1. Capital, Profit, and Credibility

Bank credibility depends on the possibility that a bank is sustainable and will not fail. Capital is considered to be important for any bank, but private banks do not fail immediately when capital becomes negative. With negative capital, sooner or later they face the difficulty in raising necessary liquidity, and liquidity shortage leads them to bankruptcy. Like private banks, central banks do not fail with negative capital. Unlike private banks, central banks do not fall short of liquidity, because they can create liquidity for themselves.

However, this does not necessarily mean that central banks do not need any capital to maintain credibility. A central bank with less capital tends to generate less profit. If it continues to make losses, it continues to create liquidity to finance them. Since supplying more liquidity means monetary easing policy, it puts an obstacle to conducting monetary tightening policy when necessary. This possibly leads inflation to be out of control and, in this sense, it might jeopardize central bank credibility.

Thus, the important factor for credibility is whether or not a central bank does not make sustained losses. It depends not only on capital but also on other financial elements that affect present and future profits. Central banks need the financial condition good enough to generate no sustained loss even when they suffer shocks from large financial fluctuations, domestic or overseas. Stella (2003) argues the importance of such condition and calls it "financial strength."

Ueda (2004) and Ize (2005) have examined several cases of troubled central banks.

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central bank targets its own capital together with inflation and output gap.

For example, the central banks in Venezuela, Jamaica, and Costa Rica had negative capital due to foreign exchange losses or the cost to deal with domestic financial crisis. This caused the rise in the interest rate on borrowings by central banks and expanded the losses. The banks had difficulty in stopping monetary easing policy due to the expansion in interest payment, which accelerated inflation.

2.2. The Case of the Bank of Japan

For more than a decade, the Bank of Japan (BOJ) has been taking aggressive monetary easing policy, including some unconventional measures. While effectiveness of such unconventional measures is a controversial issue, some are concerned about the risk that such measures might damage the BOJ's credibility in the future. In this subsection, we discuss the recent situation of the BOJ and see what might be a problem.

The BOJ has taken unconventional measures in three periods: the periods of zero interest rate policy (February 1999 to August 2000), quantitative easing (March 2001 to March 2006), and the policy against Lehman shock (from September 2008). In these periods, the policy rate was set nearly at zero, and the BOJ kept supplying more funds to the private sector.

Figure 1 shows that the monetary base was expanded drastically in the above three periods. Not only in the quantitative easing period, but also in the other two periods, the BOJ was taking the quantitative easing measure. The main measure of the monetary base expansion was outright purchases of Japanese Government Bonds (JGBs) as the figure shows. The BOJ also took credit easing by purchasing some risky assets, especially in the past few years, and total holdings of such risky assets amount to 8.0 trillion yen as shown in Table 1.

Insert Figure 1 and Table 1.

There are two main concerns about the BOJ's credibility. One is the risk with risky assets and foreign assets. Fortunately, the holdings of these assets are not large up to now, but they are increasing. The other is the risk with the huge holdings of JGBs. Their price fluctuates, and the BOJ bears a large capital loss if the price falls. A fall in price can be caused by Japanese government's loss in credibility or a rise in the interest rate when the economy is exiting a slump. The BOJ experienced the latter, so we examine its experience in the next subsection.

2.3. Exit Strategy

It is an important issue how to absorb a large amount of liquidity at the exit of quantitative easing, which is called exit strategy. The BOJ accomplished such absorption when it ended the quantitative easing in March 2006.

The BOJ's exit strategy can be examined by Table 2. The table shows the changes in the balance sheet components corresponding to the decrease in monetary base during the period of half a year and the period of two years and a half after ending the quantitative easing. The BOJ decreased the monetary base by more than 20 trillion yen within half a year. Though it used mainly JGB purchases to expand the monetary base as seen in Figure 1, it used two measures to shrink it. In the first half a year, it used the measure of the short-term operations such as funds-supplying operations against pooled collateral, RAs, and bills. It should be noted that the BOJ decreased the funds-supplying operations, not increasing the funds-absorbing operations. Two years and a half after, it used the other measure, which is a decrease in the JGB holdings. It should be noted that it never sold any JGBs but waited them to be redeemed.

Insert Table 2.

At the exit of quantitative easing, the interest rates were rising and asset prices including that of JGBs were falling. To avoid any capital loss, the BOJ did not sell any JGBs. It waited till they were redeemed, and for the meantime it absorbed the liquidity by shrinking the funds-supplying operations.

The BOJ succeeded in exit strategy without bearing any capital loss, but such an exit strategy is not always feasible. Since most JGBs are long-term bonds, the BOJ seemed to have prepared for the exit by purchasing those with shorter maturity period in order to have many of them redeemed within a couple of years. It decreased the funds-supplying operations, but to decrease them it needed to have them expanded before the exit. Hence, it seemed that the BOJ prepared for the exit carefully.

However, we cannot expect that a central bank can always prepare for the exit beforehand. The exit may not be predictable, or a sudden external shock, such as an oil shock, may hit the economy so that the central bank needs to absorb liquidity immediately. In these cases, such an exit strategy as the one by the BOJ in 2006 is not possible.

Bernanke (2009) proposes two measures in these cases. One is using reverse repos, and the other is to pay high interest on private banks' balances at a central bank to have the balances increased. Both measures are funds-absorbing measures. They decrease the monetary base by expanding the central bank liabilities, not by shrinking the central bank assets as the BOJ did. Since the interest rates should rise after the exit, large expansion in interest bearing liabilities might impose losses on the central bank, which might damage its balance sheet and credibility.

Thus, it should be emphasized that the funds-absorbing measures are different from the funds-supplying measures. Though both change the monetary base, the former measures change the size of liabilities on a central bank's balance sheet, while the latter measures change the size of assets. The BOJ was successful by using the latter, and Bernanke proposes the former when the latter measures are not available. Our model in the next section takes into account the difference between funds-supplying and funds-absorbing measures. With the model, we examine the relation of capital, balance sheet, and profit with monetary policy and credibility, and we show that the relation differs depending on the funds-supplying or funds-absorbing measures.

3. A Simple Dynamic Optimization Model of a Central Bank

3.1. Model Setting

In this section, we develop a simple dynamic optimization model of a central bank. We incorporate the central bank balance sheet and profit on infinite time horizon into the central bank behavior.

A central bank is assumed to minimize the following quadratic loss function by changing the monetary base:

$$\min_{\Delta H_t} \quad L = \sum_{t=1}^{\infty} \beta^t \left[\frac{1}{2} \left(\Delta P_t - \Delta P^* \right)^2 \right]. \qquad (1)$$

ΔP_t is a change in the general prices at t, ΔP^* is its target level, ΔH_t is a change in monetary base, and β is a discount factor. With uncertainty, we need an expectation operator, but we assume certainty for simplicity, since introducing uncertainty does not change any important implications discussed in this paper.

The prices are assumed to be determined as follows:

$$\Delta P_t = \alpha_1 \Delta H_t + \alpha_2 \eta_t . \qquad (2)$$

η_t is a vector of other factors affecting the price change, such as output, government spending, foreign exchange rate, and so on. Given equations (1) and (2) and the initial condition ΔH_0, the central bank sets ΔH_t so that it minimizes the loss function. The optimal policy is,

$$\Delta H_t = \frac{1}{\alpha_1} \Delta P^* - \frac{1}{\alpha_1} \alpha_2 \eta_t \qquad \text{for } t=1, \ldots, \infty. \qquad (3)$$

This is a traditional policy reaction function that appeared in many precedent studies.

In the rest of this paper, we assume for simplicity,

$$\Delta P^* = 0, \alpha_1 = 1, \alpha_2 = 0. \tag{4}$$

Then, the equations (1) and (3) reduce to,

$$\min_{\Delta H_t} \quad L = \sum_{t=1}^{\infty} \beta^t \left(\frac{1}{2} \Delta H_t^2 \right), \tag{1'}$$

$$\Delta H_t = 0 \qquad \text{for } t\text{=1, …, ∞}. \tag{3'}$$

In any case, the central bank hits the target ΔP^* is at t=1, …, ∞ regardless of the initial condition ΔH_0.

The traditional policy reaction functions (3) and (3') presuppose that the central bank can set ΔH_t freely. It is not the case, however, if we take into account the central bank's profit and balance sheet, which impose constraint on the central bank behavior.

The central bank's balance sheet in our model is Table 3, where A_t is the assets with interest rate r_{At}, H_t is the monetary base, B_t is the other liabilities that are all assumed to bear interest at the rate r_{Bt}, and K_t is the capital. Its profit π_t is,

$$\pi_t = r_{At} A_t - r_{Bt} B_t - O, \tag{5}$$

where O is the central bank operation expenditures, and the profit is added to the capital in the next period:

$$K_t = K_{t-1} + \pi_{t-1}. \tag{6}$$

K_t and ΔH_t can be negative, while A_t and B_t should be non-negative.

Insert Table 3.

At the beginning of t=1, the central bank sets ΔH_t for t=1, …, ∞ given the initial conditions and exogenous variables. The initial conditions are A_0, B_0, ΔH_0, K_0, and π_0 that is determined by the other initial variables. The exogenous variables are r_{At}, r_{Bt}, and O. To control H_t, the central bank needs to change B_t or A_t, and we discuss the case of changing each of them in the subsequent subsections.

3.2. The Model with Changes in Liabilities

In this subsection, we discuss the case where the central bank changes the interest bearing liabilities B_t to control the monetary base. As discussed in the Subsection 2.3, the measures to change the assets are not always available, especially when exit strategy requires quick monetary tightening policy. If not available, the central bank needs to expand the liabilities such as reverse repos for tightening policy.

We assume $A_t = \overline{A}$. With this and equation (6), the following balance sheet constraint applies:

$$H_t + B_t = H_{t-1} + B_{t-1} - \pi_{t-1}.$$

Using equation (5), this constraint becomes,

$$B_t + \Delta H_t = (1 + r_{Bt-1})B_{t-1} - r_{At-1}\overline{A} + O\,; \qquad \Delta H_t = H_t - H_{t-1}. \tag{7}$$

We also need another constraint, $B_t \geq 0$, but we neglect it to simplify the model handling and restrict our discussion to the case of non-negative B_t.[3] Thus, the central bank minimizes the loss function (1') subject to (7) with respect to ΔH_t and B_t. ΔH_t is a control variable, and B_t is a state variable.

We set the Lagrangian V, where λ_t is the Lagrangian multiplier.

$$V = \sum_{t=1}^{\infty} \beta^t \left[\frac{1}{2} \Delta H_t^2 + \lambda_t \left\{ (1 + r_{Bt-1}) B_{t-1} - r_{At-1} \overline{A} + O - B_t - \Delta H_t \right\} \right]. \tag{8}$$

The first order conditions are as follows.[4]

$$\partial V / \partial \Delta H_t = \beta^t (\Delta H_t - \lambda_t) = 0, \tag{9a}$$

$$\partial V / \partial \Delta B_t = -\beta^t \lambda_t + \beta^{t+1} \lambda_{t+1} (1 + r_{Bt}) = 0, \tag{9b}$$

$$\partial V / \partial \lambda_t = \beta^t \left[(1 + r_{Bt-1}) B_{t-1} - r_{At-1} \overline{A} + O - B_t - \Delta H_t \right] = 0. \tag{9c}$$

From equations (9a) and (9b),

$$\beta (1 + r_{Bt}) \Delta H_{t+1} = \Delta H_t. \tag{10}$$

The following transversality condition must be satisfied:

$$\lim_{T \to \infty} \beta^T \lambda_T B_T = 0. \tag{11}$$

Equation (11) can be rewritten,

$$\lim_{T \to \infty} \beta^T \lambda_T B_T = \lim_{T \to \infty} \beta^T \lambda_T \prod_{t=1}^{T} (1 + r_{Bt-1}) \frac{B_T}{\prod_{t=1}^{T} (1 + r_{Bt-1})}.$$

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[3] We can modify our model to have the non-negative constraint on B_t by using Kuhn-Tucker theorem. In that case, when the non-negative constraint is binding, ΔH_t cannot be controlled by the central bank but determined by equation (7).

[4] To check the second order condition, it is easier to calculate by substituting (7) into (1') to eliminate ΔH_t:

$$\min_{B_t} \quad L = \sum_{t=1}^{\infty} \beta^t \left[\frac{1}{2} \left\{ (1 + r_{Bt-1}) B_{t-1} - r_{At-1} \overline{A} + O - B_t \right\}^2 \right].$$

The second order condition is always satisfied as follows:

$$d^2 L / dB_t^2 = \beta^t + \beta^{t+1} (1 + r_{Bt})^2 > 0.$$

From equation (9b), $\beta^T \lambda_T \prod_{t=1}^{T}(1+r_{Bt-1})$ is constant at any T, so the transversality condition reduces to,

$$\lim_{T\to\infty} \frac{B_T}{\prod_{t=1}^{T}(1+r_{Bt-1})} = 0 . \qquad (11')$$

This condition implies that B_T should not grow faster than its interest payment.

3.3. The Model with Changes in Assets

The other case is where the central bank changes the asset holdings A_t to control the monetary base. We assume $B_t = \overline{B}$, and the balance sheet constraint is,

$$A_t = A_{t-1} + \pi_{t-1} + \Delta H_t .$$

Using equation (5), this constraint becomes,

$$A_t - \Delta H_t = (1+r_{At-1})A_{t-1} - r_{Bt-1}\overline{B} - O . \qquad (12)$$

As in Subsection 3.2, we neglect the non-negative constraint and discuss only the case of $A_t \geq 0$. The central bank minimizes the loss function (1') subject to (12) with respect to ΔH_t and A_t.

We set the Lagrangian V:

$$V = \sum_{t=1}^{\infty} \beta^t \left[\frac{1}{2}\Delta H_t^2 + \lambda_t \left\{ (1+r_{At-1})A_{t-1} - r_{Bt-1}\overline{B} - O - A_t + \Delta H_t \right\} \right]. \qquad (13)$$

The first order conditions are as follows:[5]

$$\partial V / \partial \Delta H_t = \beta^t (\Delta H_t + \lambda_t) = 0 , \qquad (14a)$$

$$\partial V / \partial \Delta B_t = -\beta^t \lambda_t + \beta^{t+1}\lambda_{t+1}(1+r_{At}) = 0 , \qquad (14b)$$

$$\partial V / \partial \lambda_t = \beta^t \left[(1+r_{At-1})A_{t-1} - r_{Bt-1}\overline{B} - O - A_t + \Delta H_t \right] = 0 . \qquad (14c)$$

From equations (14a) and (14b),

$$\beta(1+r_{At})\Delta H_{t+1} = \Delta H_t . \qquad (15)$$

The following transversality condition must be satisfied:

$$\lim_{T\to\infty} \beta^T \lambda_T A_T = 0 . \qquad (16)$$

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[5] The second order condition is always satisfied as follows:

$$d^2 L / dA_t^2 = \beta^t + \beta^{t+1}(1+r_{At})^2 > 0 .$$

Equation (16) reduces to,

$$\lim_{T\to\infty} \frac{A_T}{\prod_{t=1}^{T}\left(1+r_{At-1}\right)} = 0 . \qquad (16')$$

It implies that A_T should not grow faster than its interest revenue.

4. Implications

4.1. Policy Response

The optimal policy derived from our model is shown as equations (10) and (15). They are policy response functions with central bank balance sheet constraint, while equation (3') is the traditional one. The traditional one indicates that the central bank immediately sets ΔH_t at zero, but ours state that it adjusts ΔH_t only gradually.

$$\Delta H_{t+1} = \frac{1}{\beta\left(1+r_{Bt}\right)}\Delta H_t \quad \text{or} \quad \Delta H_{t+1} = \frac{1}{\beta\left(1+r_{At}\right)}\Delta H_t .$$

If $\beta\left(1+r_{it}\right)<1$ (i=A, B), then ΔH_t gradually deviates from zero, and such monetary policy is hard to justify. β is inversely related to the central bank's time preference, and we assume that the time preference is small enough to have $\beta\left(1+r_{it}\right)>1$. Under this assumption, ΔH_t asymptotically approaches to zero. [6]

The gradual adjustment is because changing ΔH_t induces a change in B_t or A_t. In the model with changes in liabilities, if $\Delta H_0>0$, the central bank wants to decrease ΔH_t. However, decreasing ΔH_t means holding more B_t, which reduces the profit. Then, it puts an expansionary pressure on future ΔH_t and B_t, as shown in equation (7). Thus, the central bank needs to determine an optimal point that balances the current and future monetary base adjustment.

4.2. Transversality Condition and Central Bank Credibility

(i) The model with changes in liabilities

The transversality condition in our model with changes in liabilities is equation (11'). It states that B_T should not grow faster than its interest payment. The central bank cannot increase B_T unlimitedly in order to control the monetary base. If it increases B_T beyond the transversality condition, it makes less profits or more losses and faces the pressure to supply more monetary base to finance the losses. To avoid the monetary base expansion, it must absorb the added monetary base by increasing B_T, which causes

[6] Since $\beta\left(1+r_{it}\right)<1$ (i=A, B) is thought to be close to 1, the monetary base adjustment seems to be unrealistically slow. It might be because our model is too much simplified.

more losses. The liabilities become out of control, and the central bank is no longer sustainable. If it reduces or stops piling up liabilities, then it is forced to accelerate the monetary base growth. The monetary policy is not optimal, and we will have inflation. Either case damages the economy due to the central bank problem. Thus, the central bank loses credibility if the transversality condition is not satisfied.

When B_T is positive and expanding, equation (11') implies,

$$\frac{B_T}{\prod_{t=1}^{T}(1+r_{Bt-1})} - \frac{B_{T-1}}{\prod_{t=1}^{T-1}(1+r_{Bt-1})} = \frac{1}{\prod_{t=1}^{T}(1+r_{Bt-1})}\left[B_T - (1+r_{BT-1})B_{T-1}\right]$$

$$= \frac{1}{\prod_{t=1}^{T}(1+r_{Bt-1})}\left[-\Delta H_T - r_{AT-1}\overline{A} + O\right] < 0.$$

Then,

$$\Delta H_T > -\left(r_{AT-1}\overline{A} - O\right). \tag{11''}$$

Equation (11'') must hold in order to satisfy the transversality condition. Since ΔH_T approaches to zero, equation (11'') is satisfied so long as $r_{AT-1}\overline{A} - O > 0$. However, if the assets cannot earn enough interest to pay for the operation expenditures, ΔH_T cannot be smaller than the positive value, $-\left(r_{AT-1}\overline{A} - O\right) > 0$. This is because the central bank continues to make losses, and it needs to supply more monetary base to finance them. Then, the monetary policy needs to be inflationary, and the central bank loses credibility in the sense that it cannot conduct appropriate future monetary policy.

(ii) The model with changes in assets

In the model with changes in assets, the transversality condition is (16'), stating that A_T should not grow faster than its interest revenue. If this condition is not satisfied, the policy is not optimal, but it does not seem to cause any serious problem. The central bank can slow down the asset accumulation anytime by giving some part of profit to the government.

With similar calculation in (i), when A_T is positive and expanding, equation (16') is satisfied if the following holds:

$$\Delta H_T < r_{BT-1}\overline{B} + O. \tag{16''}$$

Unlike equation (11''), equation (16'') always holds when ΔH_T approaches to zero. The asset holdings do not grow faster than its interest so long as the central bank conducts the optimal monetary policy.

The difference in transversality conditions in the above two models clarifies the difference between using funds-supplying measures or using funds-absorbing measures discussed in Subsection 2.3. In the model, suppose the central bank takes monetary easing policy at t=0, H_0>0. At the beginning of t=1, the bank decides to exit the easing policy and return to the price-stabilizing objective as shown in equation (1'). The BOJ succeeded in exiting the quantitative easing by using the funds-supplying measures, and such exit strategy corresponds to our model with changes in assets. Equation (16") holds, and the transversality condition is always satisfied. On the other hand, if such exit strategy is not available, the bank needs to use funds-absorbing measures, which corresponds to our model with changes in liabilities. Equation (11") does not always hold, and when it does not hold, the bank cannot conduct appropriate monetary policy and loses credibility as discussed above. Our discussion warns that central banks should be cautious in using funds-absorbing measures at exit.[7]

4.3. Financial Strength and Credibility

Our model shows that a central bank might lose credibility when it needs to expand the interest bearing liabilities. When such liabilities are expanding, the condition stated as equation (11") must hold. It is noteworthy that the condition does not depend on the capital. The central bank can be credible even if its capital is small or negative. Figure 2 exhibits a numerical example. The capital becomes negative and decreasing, but the value of transversality condition function (11') is converging to zero. The capital becomes negative, but the central bank can be credible because it can conduct the optimal monetary policy.

Insert Figure 2.

However, the capital does affect the situation of central bank. Less capital K_0 implies less asset holdings $\bar{A}$, which makes the condition (11") more restrictive. A central bank may suffer a large loss due to foreign exchange loss or the cost to deal with domestic financial crisis, as the troubled central banks discussed in Subsection 2.1. Such a loss decreases the central bank's asset holdings and capital. In our model, they correspond to small K_0 and $\bar{A}$. Small $\bar{A}$ incurs sustained loss and makes it difficult to control the monetary base by changing liabilities as shown by equation (11").

It should also be noted that equation (11") does not include r_B. Some

[7] The positive monetary base growth decreases in this setting of our models, while the BOJ actually decreased the monetary base. Our model is too simple to handle the case of negative monetary growth when it is initially positive. Our model needs to be elaborated, it remains for the future study.

experiences of troubled central banks discussed in Subsection 2.1 indicate that a rise in the interest rate on central bank liabilities deteriorated the situation, but our model argues that it does not depend on such an interest rate change whether or not the central bank is sustainable. A rise in the liability interest rate, however, expands the loss, which deteriorates the future balance sheet faster.

Central bank sustainability and credibility depend on the transversality condition in our model. In order to satisfy the condition, the initial conditions $\overline{A}$ and O are important. They are important to avoid losses. An actual central bank has more complicated balance sheet, and one of the important implications from our simple model is that not the capital alone but the financial strength as a whole should be strong enough to generate no sustained loss. It should be so even when a central bank suffers a large economic shock. If the financial strength is weaker, then a central bank may lose credibility, and the situation deteriorates faster.

5. Conclusion

In this paper, we have developed a dynamic optimization model of a central bank and examined the central bank credibility. Unlike the existing literature, we have incorporated the central bank's balance sheet and profit into the model and considered the central bank behavior that minimizes the loss function on infinite time horizon by controlling the monetary base. We also have taken into account the difference between funds-supplying operations and funds-absorbing operations. Our analysis with the model has found the followings.

First, unlike the traditional policy reaction function in the existing literature, our analysis has found it optimal that the central bank only gradually adjusts the monetary base. Second, the central bank credibility is closely related to the transversality condition. If the condition is not satisfied, the central bank is not sustainable or it is forced to conduct inappropriate inflationary policy. Thus, without the condition satisfied, the central bank loses credibility.

Third, the transversality condition differs depending on using either funds-supplying or funds-absorbing operations, and the condition for the case of using funds-absorbing operations is not always satisfied. Central banks may face a sudden necessity for monetary tightening, and funds-supplying operations may not be available. Our analysis, however, warns that central banks should be cautious in using funds-absorbing measures for tightening as the transversality condition is not always satisfied.

Finally, the transversality condition and credibility is not directly related with the central bank capital. They depend on the initial conditions in our model, and we can generalize the initial conditions as the financial strength. Central bank financial strength should be strong enough to generate no sustained loss, even when the central bank suffers a large economic shock.

Our model is a simple one, and many extensions are possible, such as introducing uncertainty, profit transfer to the government, more realistic inflation process, and so on. They remain for the future study.

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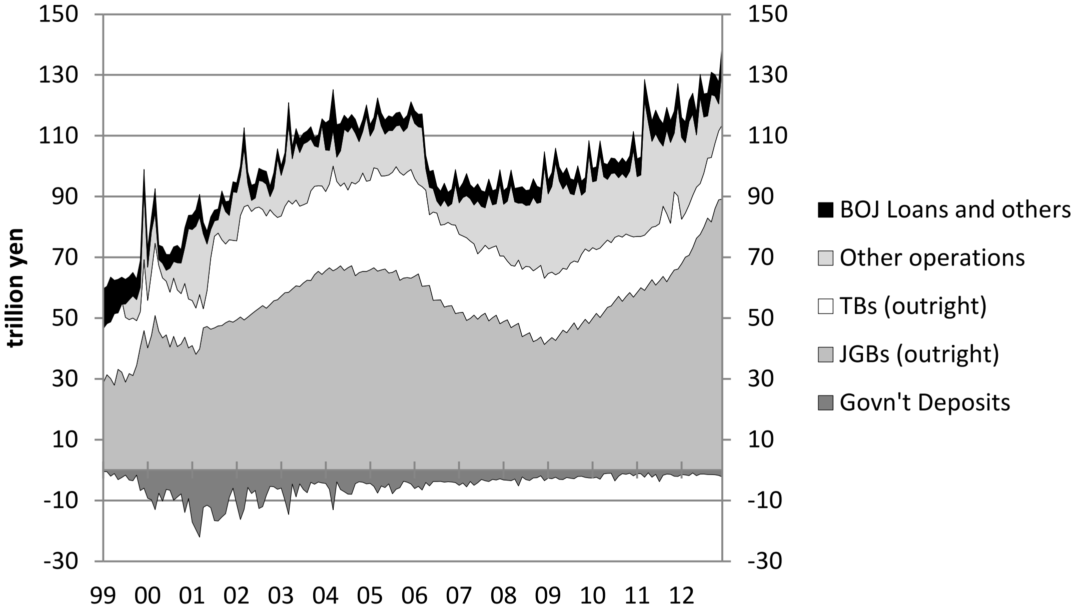
Figure 1. Monetary Base in Japan



Note: TBs are included in JGBs before April 2001 due to data availability.

Source: BOJ.

Table 1. Balance Sheet of the Bank of Japan

Domestic Assets	153.1	Monetary Base	133.9
(JGBs	89.2)	Other liabilities	18.7
(Risky Assets	8.0)	Capital	5.8
Foreign Assets	5.3		

Notes: Trillion yen at December 2012.

Capital includes appropriate reserve funds. Risky assets are CPs, corporate bonds, stocks, ETFs, and REITs. Foreign assets include gold.

Source: BOJ.

Table 2. The Bank of Japan’s Exit Strategy

		Mar. 06-Aug. 06	Mar. 06-Aug, 08
Monetary Base		-23.69	-22.98
JGBs		-8.55	-19.35
	Purchases	7.13	36.16
	Redemption	-15.67	-55.50
TBs		-4.88	-7.84
Funds-Supplying Operations against Pooled Collateral, RAs, and Bills		-24.64	-14.39
	Funds-Supplying	-25.44	-15.20
	Funds-Absorbing	0.80	0.80
BOJ Loans and Others		14.38	18.60

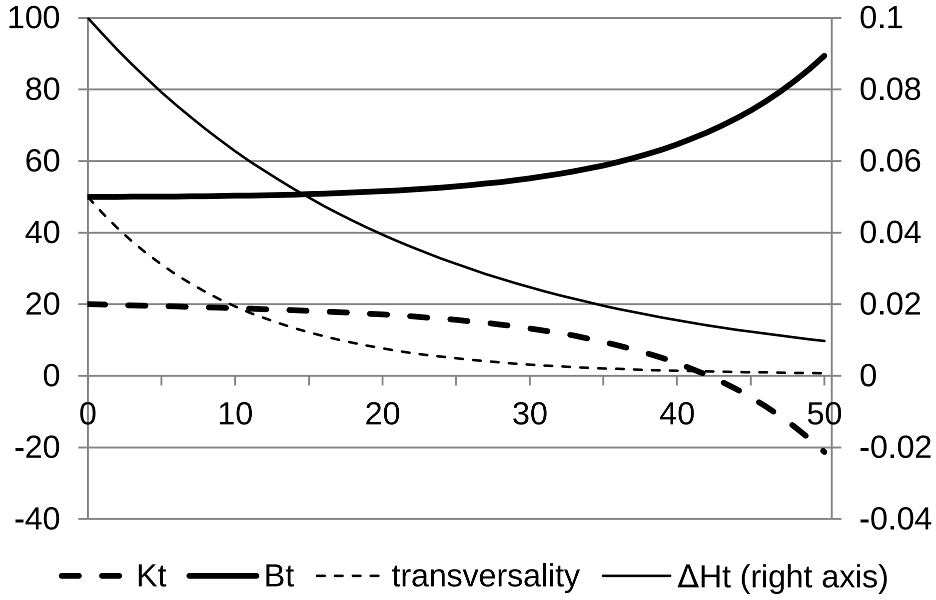
Notes: Flow amounts. Trillion yen.

Source: BOJ.

Table 3. Central Bank Balance Sheet

Assets (A_t)	Monetary Base (H_t) Interest Bearing Liabilities (B_t) Capital (K_t)

Figure 2. A Numerical Example



Notes: The following is assumed.

H_0=30, K_0=20, B_0=50, ΔH_0=0.1, O=5.1, $r_{At} = r_{Bt}$ =0.1, β=0.05

"Transversality" is the value of $B_T \Big/ \prod_{t=1}^{T} (1 + r_{Bt-1})$.

Positive ΔH_t satisfies quation (11") as $-(r_{At}\bar{A} - O) = -4.9$.