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The Economics of Cannibalization: A Duopoly in which Firms Supply Two Vertically Differentiated Products

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The Economics of Cannibalization: A Duopoly in which Firms Supply Two Vertically Differentiated Products

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Abstract

In this paper, we consider and propose a new duopoly model of cannibalization in which firms produce and sell two vertically differentiated products in the same market. We show that each firm produces the high-quality good more (less) than the low-quality good if the upper limit of taste of consumers is sufficiently high(not so high). Further, we find that the increase in the difference in quality between two goods leads to cannibalization, such that the high-quality goods keep out the low-quality goods from the market. Furthermore, we conduct a welfare analysis.

Keywords: Multiproduct firm, Duopoly, Cannibalization

JEL Classification Numbers: D21, D43, L13, L15

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1 Introduction

In the real economy, there are oligopolistic markets in which each firm produces and sells multiple products that are vertically differentiated in the same market. For example, GM sells Chevrolet Cruze and GMC Sierra PU and Toyota sells Camry, Corolla Matrix, and Prius—the hybrid car—in the same segment of the car market. In contrast, Hyundai also sells Elantra, Hybrid Sonata, etc., in the same segment of the United States car market. Since the quality of technology of each firm differs according to the viewpoint of each consumer, each consumer places different value on a high-quality good of each firm. Another example of this type of market is the beer-like beverage market that emerged in Japan in 1994, which comprises beer and *happoshu* or *low-malt beer*. This market emerged as a by-product of the congested Japanese economy during the deflationary recession in the Lost Decade in Japan. *Happoshu* or low-malt beer is a tax category of Japanese liquor that most often refers to a beer-like beverage with less than 67% malt content. In the Japanese alcoholic beverage tax system, lower tax was imposed on low-malt beer than on beer with more than 67% malt content. Consequently, the market price of the former is lower than that of the latter. Therefore, Kirin, Asahi, and Sapporo Breweries sell beer and low-malt beer brands in the same beer-like beverage market. Consumers who prefer beer place higher value on beer than low-malt beer, both of which obviously differ in terms of quality. However, consumers who do not have such high preferences choose to buy low-malt beer as a substitute for beer when they are faced with a choice between low-malt beer and beer brands in liquor shops; this is because the price of the former is lower than that of the latter. This market is not only horizontally but also vertically differentiated.

In such markets, there are more cases of *cannibalization*. Cannibalization is said to occur when a company reduces the sales of one of its products by introducing a similar, competing product in the same market.

In existing literature, a multi-product firm model is used to examine cannibalization. Judd (1985) considers a four-stage entry-exit game with *two distinct* goods. That is, in stage 1 of the game, firm 1 chooses to produce nothing, one good, or two distinct goods and pays a necessary entry cost; in stage 2, firm 2 makes this choice. In stage 3, both firms simultaneously make choices regarding whether to exit and pay the necessary exit cost if they chose to exit. Finally, in stage 4, firms form a duopoly as the final market structure and bear production costs. Judd (1985) shows that credible preemption by a two-product incumbent is impossible unless the costs of exit are high and the competition between the incumbent and its rival for the same good is less intense. In other words, he shows that brand proliferation to deter entry is not credible in the absence of substantial exit costs in a duopoly.

Tabuchi (2012) explores brand proliferation by considering an oligopoly with more than two multiproduct firms in Hotelling's spatial competition. In other words, he considers Hotelling's and Salop's location-then-price competition model and shows that firms proliferate brands (supply multiple products) in an oligopoly with three or more firms but not under a duopoly. However, both Judd (1985) and Tabuchi (2012) do not consider the case in which firms sell multiple products that are differentiated in terms of quality (vertically) in the same market.

In dealing with the case in which cannibalization arises in such a market, we need to employ a model that allows for a multiproduct firm (MPF) that differs in terms of its features or characteristics.[1] As mentioned earlier, there is no previous study that addresses an oligopolistic market with MPFs producing two goods differentiated in terms of quality.[2] This is the unique contribution of this paper since the existing models of

[1]This paper is an extention of the analysis conducted in the master's thesis, Kitamura(2013).

[2]For the sake of simplicity, we focus on a duopoly model.

horizontal and vertical product differentiation assume that each firm produces only one kind of good that differs from that of its rival.

Ellison (2005), the study most closely related to the current one, analyzes a market in which each firm sells a high-end and low-end version of the same product. Although each firm produces two differentiated goods, the two goods are sold in different markets where consumer types are different.[3] In contrast, the current study provides a model in which both firms produce two vertically differentiated products.

The remainder of this paper is organized in the following manner. Section 2 presents the model. Section 3 proves and discusses the main results. Section 4 provides the conclusion.

2 The model

Suppose there are two firms, $(i = 1, 2)$, producing two goods that differ in terms of quality—$(\alpha = A, B)$. Let A and B denote the quality level of the two goods; then, the maximum money value of each good for which consumers are willing to pay is assumed to be $A > B > 0$. Moreover, good $\alpha(= A, B)$ is assumed to be homogenous for any consumer. Further, suppose that each firm has constant returns to scale and that $c_A > c_B$, where c_α is the marginal and α is the average cost of the good; this implies that a high-quality good incurs a higher cost of production than a low-quality good. Without loss of generality, we also assume that $c_B = 0$.[4] Under these assumptions, each firm's profit is defined in the following manner:

$$\pi_i = (p_{iA} - c_A)q_{iA} + p_{iB}q_{iB} \qquad i = 1, 2, \tag{1}$$

[3]His model combines vertical (two distinct qualities) and horizontal differentiation (two firms located at distinct points of a linear city).

[4]In the Appendix, I calculate the equilibrium outcome in the case of $c_B > 0$.

where $p_{i\alpha}$ is the price of good α sold by firm i and $q_{i\alpha}$ is the firm's output. Each firm is supposed to choose quantities to maximize this profit function in Cournot fashion.

Now, we describe the consumers' behavior in our model.

Following the standard specification in the literature–for example, Katz and Shapiro (1985)–we assume that there is a continuum of consumers that is characterized by a taste parameter, θ, which is uniformly distributed between $-\infty$ and $r > 0$ with density 1. It is assumed that a consumer of type $\theta \in (-\infty, r], r > 0$ obtains a net surplus from one unit of good α of firm i at price $p_{i\alpha}$. Thus, the utility (net benefit) of consumer θ who buys good α $(= A, B)$ from firm i $(= 1, 2)$ is given by

$$U_{i\alpha}(\theta) = \alpha\theta - p_{i\alpha} \qquad i = 1, 2 \quad \alpha = A, B. \tag{2}$$

Each consumer determines to buy nothing, or one unit of the good α from firm i to maximize his/her surplus.

Before deriving the inversed demand of each good, we assume that for an arbitrary type-θ_α consumer,

$$U_{1\alpha}(\theta_\alpha) = U_{2\alpha}(\theta_\alpha).. \tag{3}$$

This assumption implies that the net surplus of consumer θ_α from buying a good produced by firm 1 and that from buying a good from firm 2 must be equal as long as the two firms produce the same quality of good α and have positive sales. From (2) and (3), we obtain

$$\begin{aligned} \hat{\theta}_\alpha \alpha - p_{1\alpha} &= \hat{\theta}_\alpha \alpha - p_{2\alpha} \\ \Longleftrightarrow p_{1\alpha} &= p_{2\alpha}, \alpha = A, B. \end{aligned} \tag{4}$$

Further, let

$$p_\alpha \equiv p_{1\alpha} = p_{2\alpha}, \alpha = A, B. \tag{5}$$

Another important assumption regarding consumers in our model is that there exists a consumer who is indifferent between the two goods of the same firm. This type of consumer is denoted by $\hat{\theta}_i$ ($< r$); then, from (5), we obtain

$$U_{iA}(\hat{\theta}_i) = U_{iB}(\hat{\theta}_i) > 0 \tag{6}$$

$$\iff \hat{\theta}_i A - p_A = \hat{\theta}_i B - p_B$$

$$\iff \hat{\theta}_i = \frac{p_A - p_B}{A - B}, i = 1, 2. \tag{7}$$

Because $\hat{\theta}_i$ in (7) does not depend on i, we let

$$\widehat{\theta} \equiv \hat{\theta}_i, i = 1, 2. \tag{8}$$

Furthermore, we assume that there always exists a consumer $\underline{\theta}_B$ who is indifferent between purchasing good B and purchasing nothing. Then, consumer $\underline{\theta}_B$ satisfies the following equation from (3):

$$U_{iB}(\underline{\theta}_B) = 0$$

$$\Leftrightarrow \underline{\theta}_B = \frac{p_B}{B}. \tag{9}$$

Then, from (2), (9), (6) and the increase in θ of $U_{iB}(\hat{\theta})$, it is evident that

$$U_{iA}(\hat{\theta}) = U_{iB}(\widehat{\theta}) > U_{iB}(\underline{\theta}_B) = U_{2B}(\underline{\theta}_B) = 0.$$

Thus, equivalently, we obtain

$$\widehat{\theta} > \underline{\theta}_B. \tag{10}$$

This leads us to the following lemma.

Lemma 1. *For any consumer* $\theta \in (-\infty, \underline{\theta}_B)$ *who buys nothing, consumer* $\theta \in (\underline{\theta}_B, \widehat{\theta})$ $(\theta \in [\widehat{\theta}, r])$ buys *good B(good A).*

Proof. By equations (2), (4), and (9), for an arbitrary type-$\theta > \hat{\theta}$ consumer,

$$\begin{aligned} U_{iA}(\theta) - U_{iB}(\theta) &= \theta A - p_A - \theta B + p_B \\ &= \theta(A - B) - p_A + p_B \\ &> \hat{\theta}(A - B) - p_A + p_B \\ &= 0. \end{aligned}$$

From (2) and (10), for arbitrary type $\theta \in (\underline{\theta}_B, \widehat{\theta})$,, we obtain

$$\begin{aligned} U_B(\widehat{\theta}) - U_B(\underline{\theta}_B) &= \hat{\theta}B - p_B - (\underline{\theta}_B B - p_B) \\ &= (\hat{\theta} - \underline{\theta}_B)B > 0. \end{aligned}$$

■

The demand for good A or good B can be illustrated by a line segment depicted in Figure 1.

Insert Figure 1 here.

2.1 Derivation of Equilibrium

From Lemma 1, we immediately obtain the following system of equations:

$$\begin{cases} r - \hat{\theta} = Q_A \\ r - \underline{\theta}_B = Q_A + Q_B \equiv q_{1A} + q_{2A} + q_{1B} + q_{2B}, \end{cases} \tag{11}$$

where $Q_\alpha = q_{1\alpha} + q_{2\alpha}, \alpha = 1, 2.$

Substituting (6) and (9) into these equations and solving them for p_A and p_B, the inverse demand functions are obtained in the following manner:

$$\begin{cases} p_A = A(r - Q_A) - BQ_B \\ p_B = B(r - Q_A - Q_B). \end{cases} \tag{12}$$

To maximize profit function (1), each firm determines the quantity of its goods, q_{iA} and q_{iB}, in the following manner:

$$\max_{q_{iA}, q_{iB}} \pi_i.$$

The first-order conditions for profit maximization are

$$\begin{cases} \frac{\partial \pi_i}{\partial q_{iA}} = -Aq_{iA} + Ar - AQ_A - BQ_B - c_A - Bq_{iB} = 0 \\ \frac{\partial \pi_i}{\partial q_{iB}} = -Bq_{iB} + Br - BQ_A - BQ_B - Bq_{iA} = 0. \end{cases}$$

By the symmetry of firms, we can set $q^*_\alpha = q^*_{1\alpha} = q^*_{2\alpha}, \alpha = A, B$. Then, we can rewrite the above first-order conditions in the following manner:

$$\begin{cases} A(r-3q_A^*)-3Bq_B^*-c_A=0 \\ B(r-3q_A^*-3q_B^*)=0. \end{cases}$$

Solving this system, we obtain the following Nash equilibrium quantities:[5]

$$q_A^*=q_{iA}^*=\frac{1}{3}\left(r-\frac{c_A}{A-B}\right) \text{ and } q_B^*=q_{iB}^*=\frac{c_A}{3(A-B)}, i=1,2\ . \tag{13}$$

For q_A^* to be positive, we assume that

$$r>\frac{c_A}{A-B}. \tag{14}$$

Hence, the total equilibrium output Q^* becomes constant:

$$Q^*=Q_1^*+Q_2^*=2(q_A^*+q_B^*)=Q_A^*+Q_B^*=\frac{2}{3}r. \tag{15}$$

From (12) and (15), we obtain the following equilibrium prices of the goods:

$$p_A^*=\frac{1}{3}(Ar+2c_A),\ p_B^*=\frac{1}{3}Br. \tag{16}$$

Then, from (8), (9), (15), and (16), we obtain

$$\underline{\theta_B^*}=\frac{p_B^*}{B}=\frac{1}{3}r,\ \widehat{\theta}^*=\frac{1}{A-B}(p_A^*-\ p_B^*)=\frac{1}{3}(r+\frac{2c_A}{A-B})=\underline{\theta_B^*}\ +Q_B^*. \tag{17}$$

We also have the equilibrium profit of firm i from (1), (13), and(16):[6]

[5]We can easily check if the second-order condition is satisfied; see the Appendix.

[6]Let $f(r)\equiv A(A-B)r^2-2(A-B)rc_A+c_A^2$ be a $(A-B)c_A^2$ function of r, denote by D, the discriminant of $f(r)$. Then, we have $D/4=(A-B)^2c_A^2-A(A-B)c_A^2=,\ (A-B)c_A^2(A-B-A)=-B(A-B)c_A^2<0$,

$$\pi_i^* = \frac{1}{9}\left\{(Ar - c_A)\left(r - \frac{c_A}{A-B}\right) + \frac{Brc_A}{A-B}\right\}$$
$$= \frac{1}{9(A-B)^2}\left\{A(A-B)r^2 - 2(A-B)rc_A + c_A^2\right\} > 0, i = 1, 2. \quad (18)$$

From (13) and (16), we can immediately obtain the next proposition without proof.

Proposition 1 *Each firm produces high-quality good A in greater (lesser) quantity than low-quality good B, and total output of good A is larger (smaller) than that of good B at the equilibrium if and only if* $r \geq \frac{2c_A}{A-B}$ $(\frac{c_A}{A-B} < r < \frac{2c_A}{A-B})$. *The equilibrium price of good A is higher than that of good B. Formally, we obtain* $q_A^* \gtreqless q_B^*$ and $Q_A^* \gtreqless Q_B^*$ *if and only if* $r \geq \frac{2c_A}{A-B}$ $(\frac{c_A}{A-B} < r < \frac{2c_A}{A-B})$, *where* $Q_\alpha^* = q_{1\alpha}^* + q_{2\alpha}^* = 2q_\alpha^*$, $\alpha = A, B$. *We also have* $p_A^* > p_B^*$.

Proof: From (13) and (17),

$q_A^* - q_B^* = \frac{1}{3}\left(r - \frac{c_A}{A-B}\right) - \frac{c_A}{3(A-B)} = \frac{1}{3}\left(r - \frac{2c_A}{A-B}\right) \Leftrightarrow r - \frac{2c_A}{A-B} \gtreqless 0$ and $2q_A^* = Q_A^* \gtreqless Q_B^* = 2q_B^*$ if and only if $r \gtreqless \frac{2c_A}{A-B}$. However, $r - \widehat{\theta}^* = r - \frac{1}{3}(r + \frac{2c_A}{A-B}) = \frac{2}{3}\left(r - \frac{c_A}{A-B}\right) > 0$, since we assume that there exists $\widehat{\theta}$ in $(\underline{\theta}_B, \overline{\theta})$, and the result follows. We also have from (16) that

$p_A^* - p_B^* = \frac{1}{3}(Ar + 2c_A) - \frac{1}{3}Br = \frac{1}{3}((A-B)r + 2c_A) > 0$, since $A > B$ and $c_A > 0$.

■

Furthermore, from (13), we can easily establish the following proposition.

Proposition 2 *An increase in the difference in the quality of the two goods* $(A - B)$ *brings about cannibalization, such that the high-quality good A keeps the low-quality good*

and $A(A-B) > 0$. Thus, we can see that $f(r) = A(A-B)r^2 - 2(A-B)rc_A + c_A^2 > 0, \forall r$.

B out of the market. Similarly, a decrease in the difference in the quality of the two goods $(A - B)$ causes cannibalization, such that the low-quality good B drives the high-quality good A out of the market.

The intuition of this proposition is explained in the following manner. A larger difference in the quality of the two goods implies that the high-quality good is superior to the low-quality good and this has a positive effect on the utility of the consumer. Thus, when the difference in the quality of the two goods becomes large, both firms have an incentive to increase the supply of the high-quality good; thus, in such a case, cannibalization occurs such that the high-quality good A drives the low-quality good B out of the market and only the high-quality good survives when its demand becomes sufficiently large. In contrast, when there is a decrease in the difference in the quality of the two goods, it implies that the consumer does not valuate the high-quality good over the low-quality good. Thus, both firms have an incentive to expand their production of the low-quality good since both firms incur production costs in producing high-quality goods. Therefore, in this case, cannibalization occurs such that the low-quality good B drives the high-quality good A out of the market and only the low-quality good ultimately survives. Finally, when the difference in quality between the two goods is the range $\in (c_A/r, \infty)$, both firms do not have any incentive to produce only one kind of good and cannibalization does not occur. Thus, unless the market has goods that are extremely differentiated or extremely similar in terms of quality, cannibalization does not occur.

Here, we consider the effects of production quality on profits. From (18), we can show that

$$\frac{\partial \pi_i^*}{\partial B} = \frac{(c_A)^2}{9(A-B)^2} > 0 \text{ and } \frac{\partial \pi_i^*}{\partial A} = \frac{1}{9} r^2 > 0, i = 1, 2.$$

Thus, we obtain following proposition.

Proposition 3 *When the quality of low-quality (high-quality) goods increases, while the quality of high-quality (low-quality) goods remains fixed, the equilibrium profits of both firms increase.*

This proposition is rather natural. That is, the increase in the value of low-quality goods reduces the difference in the qualities of both goods; this also reduces the equilibrium quantity of high-quality goods with positive marginal cost. In contrast, the equilibrium quantity of low-quality goods *with zero marginal cost* increases, since for each firm, an increase in the production of low-quality goods with zero marginal cost is more attractive compared with an increase in the production of high-quality goods with positive marginal cost. As shown in the above derivation, the increase in the marginal cost of high-quality goods enhances the positive effect of quantity of low-quality goods on profit.

3 Welfare Analysis

In this section, we first define social welfare. Then, we present social welfare in the equilibrium derived in the preceding section. We define social welfare W as the social surplus that is the sum of consumer surplus CS and producer surplus PS:

$$W = CS + PS.$$

We define CS and PS as

$$CS \equiv \int_{\underline{\theta}_B}^{\hat{\theta}} (B\theta - p_B)d\theta + \int_{\hat{\theta}}^{r} (A\theta - p_A)d\theta \tag{19}$$

and

$$PS \equiv \pi_1 + \pi_2 = (p_A - c_A)Q_A + p_B Q_B.. \tag{20}$$

Then, from (**??**) and (20), the social surplus is defined by

$$\begin{aligned}
W(\hat{\theta}) &\equiv \int_{\underline{\theta}_B}^{\hat{\theta}} B\theta d\theta + \int_{\hat{\theta}}^{r} (A\theta - c_A) d\theta & (21)\\
&= \frac{B}{2}\left[\theta\right]_{\underline{\theta}_B}^{\hat{\theta}} + \frac{A}{2}\left[\theta^2\right]_{\hat{\theta}}^{r} - c_A \left[\theta\right]_{\hat{\theta}}^{r} & (22)\\
&= -\frac{A-B}{2}\left(\hat{\theta}\right)^2 + c_A\hat{\theta} + \frac{A}{2}r^2 - c_A r - \frac{1}{2}B\underline{\theta}_B^2.
\end{aligned}$$

Therefore, the social surplus in the equilibrium derived in the preceding section is given by

$$\begin{aligned}
W^* &= W(\hat{\theta}^*) = -\frac{A-B}{2}\left(\hat{\theta}^*\right)^2 + c_A\hat{\theta}^* + \frac{A}{2}r^2 - c_A r - \frac{1}{2}B\underline{\theta}_B^{*2}\\
&= -\frac{\hat{\theta}^*}{2}\left(p_A^* - p_B^* - 2c_A\right) + \frac{A}{2}r^2 - c_A r - \frac{1}{18}Br^2\\
&= -\frac{1}{6}(r + \frac{2c_A}{A-B})[\frac{1}{3}\{(A-B)r + 2c_A\} - 2c_A] + \frac{A}{2}r^2 - c_A r - \frac{1}{18}Br^2\\
&= \frac{4}{9(A-B)}[A(A-B)r^2 - 2(A-B)c_A r + c_A^2] \equiv W^*(r) & (23)
\end{aligned}$$

at the equilibrium. In the last line of the above equation, the portion in square brackets is considered a quadratic and convex function of r from $A(A-B) > 0$. Denoted by D^*, its discriminant is

$$D^*/4 = (A-B)^2 c_A^2 - A(A-B)c_A^2 = -B(A-B)c_A^2 < 0.$$

Hence, it is evident that

$$A(A-B)r^2 - 2(A-B)c_A r - c_A^2 > 0,\ W^* > 0.$$

From (23), we have

$$\frac{\partial W^*}{\partial A} = \frac{4\{(A-B)r - c_A\}\{A-B)r + c_A\}}{9(A-B)^2} > 0, \tag{24}$$

since $(A-B)r > c_A$ from (14).

We also have

$$\frac{\partial W^*}{\partial B} = \frac{4c_A^2}{9(A-B)^2} > 0$$

from (14).

Thus, we obtain the following proposition.

Proposition 4 *The social surplus at the equilibrium increases as the quality of low-quality or high-quality goods increases.*

This proposition is also rather natural. That is, the increase in the value of the low-quality goods leads to a reduction in the difference in the qualities of both goods; therefore, the equilibrium quantity of high-quality goods with positive marginal costs also reduces and the equilibrium quantity of low-quality goods *with zero marginal cost* increases. These effects increase the equilibrium profits of firms from proposition 3. The existence of the marginal cost of high-quality goods enhances the positive effect of the quantity of low-quality goods on profit. Further, the increase in the quality of both types of goods leads to a decrease in the prices of both goods and also an increase in consumer surplus and social surplus.

In the following equation, we assume that the government decides each quantity $(q_{iA}^{**}, q_{iB}^{**})$ in order to maximize social welfare:[7]

$$q_{iA}^{**}, q_{iB}^{**} = \arg\max_{q_{iA}, q_{iB}} W(q_{iA}, q_{iB}).$$

The first-order conditions for social welfare maximization are

$$\begin{cases} 0 = Ar - 2Aq_{iA} - 2Bq_{iB} - c_A \\ 0 = r - 2q_{iA} - 2q_{iB} \end{cases}.$$

This system yields the following Nash equilibrium quantities:[8]

$$q_{iA}^{**} = \frac{1}{2}\left(r - \frac{c_A}{A-B}\right), q_{iB}^{**} = \frac{c_A}{2(A-B)}. \tag{25}$$

From (12) and (25), the first-best prices yield

$$p_A^{**} = c_A, p_B^{**} = 0. \tag{26}$$

From (20), the first-best producer's surplus is

$$PS^{**} \equiv \pi_1^{**} + \pi_2^{**} = (p_A^{**} - c_A)Q_A^{**} + p_B^{**}Q_B^{**} = 0.$$

Let $\widehat{\theta}^{**} = \widehat{\theta}(q_{iA}^{**}, q_{iB}^{**})$ and $\underline{\theta_B^{**}} = \underline{\theta_B}(p_B^{**})$. Then, from (17), (25), and (26) we obtain

$$\underline{\theta_B^{**}} = \frac{p_B^{**}}{B} = 0 \text{ and } \widehat{\theta}^{**} = \frac{1}{A-B}(p_A^{**} - p_B^{**}) = \frac{c_A}{A-B} = Q_B^{**}. \tag{27}$$

Hence, from (21), the first-best social surplus is given by

[7]Note that because we consider symmetric firms, $q_{1\alpha}^{**} = q_{2\alpha}^{**}$.
[8]The second-order condition is satisfied; see the Appendix

$$
\begin{aligned}
W^{**} &= W(\hat{\theta}^{**}) = -\frac{A-B}{2}\left(\hat{\theta}^{**}\right)^2 + c_A\hat{\theta}^{**} + \frac{A}{2}r^2 - c_A r - \frac{1}{2}B\underline{\theta}_B^{**2} \\
&= -\frac{1}{2}(\frac{c_A}{A-B})\,(p_A^{**} - p_B^{**} - 2c_A) + \frac{A}{2}r^2 - c_A r \\
&= \frac{1}{2(A-B)}[A(A-B)r^2 - 2(A-B)c_A r + c_A^2], \qquad (28)
\end{aligned}
$$

where the third and the last equalities hold from (7), (26), and (27). Comparing (23) and (28), it is evident that the expression in both square brackets is the same one and its discriminant is $D^* < 0$ and $W^{**} > 0$.

Then, we obtain the following proposition.

Proposition 5 *Both equilibrium quantities q_{iA}^* and q_{iB}^* in the profit-maximization problem are too few as compared with equilibrium quantities q_{iA}^{**} and q_{iB}^{**} in social optimum. Furthermore, $PS^* > PS^{**} = 0, W^{**} > W^*$, and $CS^{**} > CS^*$ hold.*

4 Concluding Remarks

In this paper, we considered and proposed a new duopoly model of cannibalization in which two firms produce and sell two distinct products that are differentiated not only horizontally and but also vertically in the same market. Then, we derived the market equilibrium and showed that each firm produces a greater (lesser) quantity of high-quality good A than low-quality good B; moreover, the total output of high-quality good A is larger (smaller) than that of low-quality good B and the equilibrium price of high-quality good A is higher than that of low-quality good B if the taste of consumers is sufficiently high (not so high).

Further, we presented several comparative statistics and established that an increase in the difference in the quality of the two goods leads to cannibalization, such that the high-quality good drives the low-quality goods out of the market. Similarly, a decrease in the difference in the quality of the two goods causes cannibalization such that low-quality good B drives high-quality good A out of the market. In addition, we conducted a welfare analysis and showed that an increase in the value of the low-quality good reduces the difference in the qualities of both goods and also reduces (increases) the equilibrium quantity of high-quality (low-quality) goods with positive (zero) marginal cost. Thus, we found that the positive marginal cost of the high-quality good enhances the positive effect of the quantity of the low-quality good on profit, and the increase in the qualities of both goods leads to a decrease in the prices of both goods and an increase in consumer surplus.

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Appendices

Appendix A

In the case in which marginal cost of good B C_B is positive, the profit of firm i is

$$\pi_i = (p_{iA} - C_A)q_{iA} + (p_{iB} - C_B)q_{iB}.$$

The first-order conditions and the symmetry assumption between the firms give:

$$\begin{cases} A(r - 3q_{iA}) - 3Bq_{iB} - C_A = 0 \\ B(r - 3q_{iA} - 3q_{iB}) - C_B = 0. \end{cases}$$

By solving these equations, we can get following equilibrium solutions;

$$\begin{cases} q_{iA}^* = \frac{1}{3}\left(r - \frac{C_A - C_B}{A-B}\right) \\ q_{iB}^* = \frac{C_A - C_B}{3(A-B)} - \frac{C_B}{3B}. \end{cases}$$

Appendix B

I now check the second-order condition for profit maximization. The second derivatives of (1) are as follows:

$$\begin{cases} \frac{\partial^2 \pi_i}{\partial q_{iA}^2} = -3A < 0 \\ \frac{\partial^2 \pi_i}{\partial q_{iB}^2} = -3B < 0 \\ \frac{\partial^2 \pi_i}{\partial q_{i\alpha} \partial q_{i\beta}} = -3B, \end{cases}$$

which leads to the Hessian determinant:

$$H_1 \equiv \begin{vmatrix} \frac{\partial^2 \pi_i}{\partial q_{iA}^2} & \frac{\partial^2 \pi_i}{\partial q_{iA} \partial q_{iB}} \\ \frac{\partial^2 \pi_i}{\partial q_{iB}^2} & \frac{\partial^2 \pi_i}{\partial q_{iB} \partial q_{iA}} \end{vmatrix} = 9B(A-B) > 0.$$

Accordingly, we have verified the second-order conditions.

Next, I now check the second-order condition for social welfare maximization. The second derivatives of (12) are as follows:

$$\begin{cases} \frac{\partial^2 W}{\partial q_{iA}^2} = -4A < 0 \\ \frac{\partial^2 W}{\partial q_{iB}^2} = -4B < 0 \\ \frac{\partial^2 W}{\partial q_{iA} \partial q_{iB}} = -4B. \end{cases}$$

Therefore, we obtain the following Hessian matrix H_2:

$$H_2 \equiv \begin{vmatrix} \frac{\partial^2 W}{\partial q_{iA}^2} & \frac{\partial^2 W}{\partial q_{iA} \partial q_{iB}} \\ \frac{\partial^2 W}{\partial q_{iB}^2} & \frac{\partial^2 W}{\partial q_{iB} \partial q_{iA}} \end{vmatrix} = 16B(A-B) > 0.$$

Accordingly, we have verified the second-order conditions.

Figure 1