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THE ROLE OF FACTOR INTENSITIES, INTER-FACTOR COOPERATION, AND PRODUCTIVITY IN A GENERAL EQUILIBRIUM MODEL

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THE ROLE OF FACTOR INTENSITIES, INTER-FACTOR COOPERATION, AND PRODUCTIVITY IN A GENERAL EQUILIBRIUM MODEL

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The purpose of this paper is to explore thoroughly the effects of the change in factor supplies on outputs with commodity prices held constant and to disentangle the intricate involvement between the roles of the factor intensities and factor substitution played in determining such Rybczynski effects in the three-factor, two-good general equilibrium model. It turns out clearly that in place of the factor substitution relations in the cost function, the introduction of cooperative relations between different factors in the production function is helpful for this purpose.

1. Introduction

The properties of the general equilibrium production model with three factors and two goods have recently been clarified to some extent by comparing those discovered by Jones (1971) in the specific-factors model. For the relation between factor prices and endowments, it has been made clear by Batra and Casas (1976), Ruffin (1981), and Suzuki (1981) that, as in the specific-factors case, if some factor-intensity condition is properly assumed to classify three factors into two groups, say, extreme and middle factors, the increase in the supply of one factor, at constant commodity prices, affects unfavorably the rewards of the factors belonging to the same group as the growing factor belongs to and favorably the rewards of the other factors.

For the relation between commodity outputs and factor endowments, or its reciprocity relation, that between factor and commodity prices, it has been shown by Suzuki (1981) and Jones and Easton (1983) respectively that some conditions associated jointly with the factor intensities and the Allen substitute-complement relations between factors are sufficient conditions to account for the fact that the increase in the supply of the extreme factor, at constant commodity prices, expands the output of the sector which uses the growing factor relatively intensively and decreases the output of the other sector,[1] or that the rise in a relative price with factor supplies held constant raises the real reward of the factor which is used relatively intensively in the production of the price-rised good and lowers the real reward of the other extreme factor with the change in the reward of the middle factor trapped between those of the extreme factors. They have also brought out that the role of the substitute-complement relations played in determining the relation of outputs to factor supplies or that of factor prices to commodity prices is so involved with that of the factor intensities that the former cannot be disentangled from the latter except in some special cases like the one with perfectly complementary factors.

The purpose of the present paper is to disentangle the intricate relations between the roles of the factor intensities and factor substitution in the general case and to make a more thorough search for the effects of the change in factor supplies on commodity outputs in order to answer the questions as to what kind of factor substitution relation is sufficient for those effects to be unambiguously predicted. In order to do so, it is helpful to define the cooperative relation between factors in the production function, in place of the factor substitution relations in the cost function, such that a factor is cooperative with another if its marginal productivity is increased by an extra input of the other factor.

Section 2 presents a model in which the usual neoclassical properties of the production function and the familiar factor intensity conditions are assumed, and in which the assumption is made that different factors are cooperative with each other in every industry. In Section 3, it is demonstrated that this assumption on the inter-factor cooperation is a sufficient condition for the favorable output effect of the growth in an extreme factor's supply to be predicted. Section 4 is devoted to deriving the fact that the responsiveness to the change in the factor intensities of the marginal rates of substitution between the relevant factors is crucial for the determination of the output effects of a factor-supply change. It is clarified, in Section 5, that the responsiveness of the marginal rates of substitution depends on the specification of the relative strength of the inter-factor cooperation and that some specification is a sufficient condition for the unfavorable effects of the growth in an extreme factor's supply to be predicted. Moreover, in this section, a search is made for the role the middle factor's productivity played in determining the output effects of the growth in its supply. Section 6 summarizes the results obtained through the analysis.

2. The model

Consider a perfectly competitive economy producing two goods, goods 1 and 2, with three factors of production: capital, labor, and land. The production function of each good is assumed to be continuously twice differentiable, increasing in each argument, strictly quasi-concave, and linearly homogeneous. Let X_j be the output of good j, K_j, L_j, and N_j respectively the inputs of capital, labor, and land in the jth industry producing good j alone. Then the production function of the jth industry is

$$X_j = F^j(K_j, N_j, L_j), \qquad j = 1, 2. \tag{1}$$

Let $x_j = X_j/L_j$, $k_j = K_j/L_j$, and $n_j = N_j/L_j$ for any positive value of L_j, then the linearly homogeneous function (1) can be reduced to

$$x_j = f^j(k_j, n_j), \qquad j = 1, 2.$$

Let $F^j_I = \partial F^j/\partial I_j$ for I = K, L, and N, and $f^j_i = \partial f^j/\partial i_j$ for i = k and n, then the positive marginal product of each factor employed in the jth sector is denoted by

$$f^j_k = F^j_K, \; f^j_n = F^j_N, \text{ and } f^j_l \equiv F^j_L = f^j - k_j f^j_k - n_j f^j_n, \; j = 1, 2. \qquad (2)$$

The restriction of positive marginal productivity excludes from our consideration the specific-factors model in which the marginal productivity of a specific factor to a particular industry is regarded to be zero in the other industries. Concerning each factor's marginal productivity, we assume that it is diminishing in every industry. Then, letting $F^j_{SI} = \partial F^j_I/\partial S_j$ for S = K, L, and N and $f^j_{si} = \partial f^j_i/\partial s_j$ for s = k and n, we have

$$f^j_{kk} = L_j F^j_{KK} < 0,$$

$$f^j_{nn} = L_j F^j_{NN} < 0, \qquad j = 1, 2, \; (3)$$

$$f^j_{ll} = L_j F^j_{LL} = (k_j)^2 f^j_{kk} + 2k_j n_j f^j_{nk} + (n_j)^2 f^j_{nn} < 0.$$

Following Morishima (1953-54), we define:

Factors S and I are cooperative in the production of good j if $F^j_{SI} \geqq 0$ for $S \neq I$.

Since the twice continuous differentiability of the production function implies $F^j_{SI} = F^j_{IS}$, this type of cooperation between factors S and I implies that an additional input of factor S does not decrease the marginal productivity of factor I and vice versa. Using this definition, we shall make the following assumption on inter-factor cooperation.

Assumption. Every factor is cooperative with the other factors in each industry. In symbols,

$$f^j_{kn} = L_j F^j_{KN} \geqq 0,$$

$$f^j_{kl} = L_j F^j_{KL} = -(k_j f^j_{kk} + n_j f^j_{nk}) \geqq 0, \qquad j = 1, 2, \qquad (4)$$

$$f^j_{nl} = L_j F^j_{NL} = -(k_j f^j_{kn} + n_j f^j_{nn}) \geqq 0.$$

It may be noted that at least two of the inequalities in (4) must hold in a strict form.

The factor intensities of the two industries are assumed to be

$$K_1/K_2 > L_1/L_2 > N_1/N_2, \qquad (5)$$

and also no factor-intensity reversals are assumed to occur in the range of factor prices considered here. The conditions (5) are equivalent to $k_1/k_2 > 1 > n_1/n_2$[2] and mean that good 1 needs more capital per one unit of labor than good 2 does in production while good 2 needs more land per one unit of labor than good 1 does in production. We hereafter refer to capital as an extreme factor of sector 1, land as an extreme factor of sector 2, labor as a middle factor, good 1 as a capital-intensive, and good 2 as a land-intensive good, respectively.

The basic structure of our model consists of the full employment and profit-maximizing conditions. Let K, L, and N be the endowments of capital, labor, and land, respectively and let $k = K/L$, $n = N/L$, and $l_j = L_j/L$. Then the full employment conditions are

$$l_1k_1+l_2k_2 = k,$$

$$l_1n_1+l_2n_2 = n, \qquad (6)$$

$$l_1+l_2 = 1.$$

Let p_j stand for the price of commodity j, and r, w, and t for the prices of capital, labor, and land respectively. Then the profit-maximizing conditions of competitive firms are

$$r = p_1f_k^1 = p_2f_k^2,$$

$$t = p_1 f_n^1 = p_2 f_n^2, \tag{7}$$

$$w = p_1 f_l^1 = p_2 f_l^2.$$

3. The role of the factor intensities and the inter-factor cooperation

Taking logarithmic differentiation of (6) and the right part of (7) with commodity prices held constant, we get

$$(l_1 k_1/k)(\hat{l}_1+\hat{k}_1)+(l_2 k_2/k)(\hat{l}_2+\hat{k}_2) = \hat{k},$$

$$(l_1 n_1/n)(\hat{l}_1+\hat{n}_1)+(l_2 n_2/n)(\hat{l}_2+\hat{n}_2) = \hat{n}, \tag{8}$$

$$\phi\hat{l}_1+\hat{l}_2 = 0,$$

where $\hat{k} = dk/k$, for instance, and $\phi = l_1/l_2$, and

$$p_1 k_1 f_{kk}^1 \hat{k}_1 + p_1 n_1 f_{nk}^1 \hat{n}_1 - p_2 k_2 f_{kk}^2 \hat{k}_2 - p_2 n_2 f_{nk}^2 \hat{n}_2 = 0,$$

$$p_1 k_1 f_{kn}^1 \hat{k}_1 + p_1 n_1 f_{nn}^1 \hat{n}_1 - p_2 k_2 f_{kn}^2 \hat{k}_2 - p_2 n_2 f_{nn}^2 \hat{n}_2 = 0, \tag{9}$$

$$p_1 k_1 f_{kl}^1 \hat{k}_1 + p_1 n_1 f_{nl}^1 \hat{n}_1 - p_2 k_2 f_{kl}^2 \hat{k}_2 - p_2 n_2 f_{nl}^2 \hat{n}_2 = 0.$$

Denote the weighted sum of f_{ks}^j and f_{ns}^j by

$$\Gamma_s^j = (k_1-k_2)f_{ks}^j+(n_1-n_2)f_{ns}^j, \qquad s = k,\ l,\ n, \text{ and } j = 1,\ 2.$$

If the differences in the factor intensities between goods 1 and 2 are sufficiently small, this can be regarded as the total differential of f_s^j when $dk_j = k_1 - k_2$ and $dn_j = n_1 - n_2$. It is clear from the factor-intensity conditions and the assumption on the inter-factor cooperation that $\Gamma_k^j < 0$ and $\Gamma_n^j > 0$ $(j = 1, 2)$. Solving (9) for $\hat{k}_j$, $\hat{n}_j$, and $\hat{l}_j$ after substituting (8) into it, we get

$$\begin{pmatrix} k_j(p_1 f_{kk}^1 + p_2\phi f_{kk}^2) & n_j(p_1 f_{nk}^1 + p_2\phi f_{nk}^2) & (-p_1\Gamma_k^1)^{j-1}(p_2\phi\Gamma_k^2)^{i-1} \\ k_j(p_1 f_{kn}^1 + p_2\phi f_{kn}^2) & n_j(p_1 f_{nn}^1 + p_2\phi f_{nn}^2) & (-p_1\Gamma_n^1)^{j-1}(p_2\phi\Gamma_n^2)^{i-1} \\ k_j(p_1 f_{kl}^1 + p_2\phi f_{kl}^2) & n_j(p_1 f_{nl}^1 + p_2\phi f_{nl}^2) & (-p_1\Gamma_l^1)^{j-1}(p_2\phi\Gamma_l^2)^{i-1} \end{pmatrix} \times$$

$$\begin{pmatrix} \hat{k}_j \\ \hat{n}_j \\ \hat{l}_j \end{pmatrix} = \frac{1}{l_2}\begin{pmatrix} p_i k f_{kk}^i \hat{k} + p_i n f_{nk}^i \hat{n} \\ p_i k f_{kn}^i \hat{k} + p_i n f_{nn}^i \hat{n} \\ p_i k f_{kl}^i \hat{k} + p_i n f_{nl}^i \hat{n} \end{pmatrix} \qquad i, j = 1, 2, \text{ and } i \neq j. \qquad (10)$$

Let $D_j = (k_1-k_2)\Gamma_k^j + (n_1-n_2)\Gamma_n^j$, $\Delta_j = f_{kk}^j f_{nn}^j - (f_{nk}^j)^2$, and $\Delta = p_1\Delta_1 D_2 + p_2\phi\Delta_2 D_1$ $(j = 1, 2)$. Then, obviously, $D_j < 0$; $\Delta_j > 0$ because the function $f^j(k_j, n_j)$ is strictly concave under the law of diminishing returns; and hence $\Delta < 0$. A look at the profit-maximizing conditions (7) would reveal that the factor intensities of each commodity depend only on factor prices. Since the factor prices in turn are affected by the change in a relative factor endowment here in contrast with the 2×2 Heckscher-Ohlin model,[3] the factor intensities change as the supplies of factors change. We can denote the elasticity of the factor intensity in response to the change in the relative factor endowment at constant commodity prices by

$$\alpha^j_{is} = \frac{s}{i_j}\frac{\partial i_j}{\partial s} \qquad i = k,\ n,\ l,\ s = k,\ n, \text{ and } j = 1,\ 2.$$

Since the determinant of the coefficient matrix on the left-hand side of (10) does not vanish,[4] the solution to (10) can be obtained in terms of α's by Cramer's rule:

$$\begin{pmatrix} \hat{k}_j \\ \hat{n}_j \\ \hat{l}_j \end{pmatrix} = \begin{pmatrix} \alpha^j_{kk} & \alpha^j_{kn} \\ \alpha^j_{nk} & \alpha^j_{nn} \\ \alpha^j_{lk} & \alpha^j_{ln} \end{pmatrix} \begin{pmatrix} \hat{k} \\ \hat{n} \end{pmatrix} \qquad j = 1,\ 2, \tag{11}$$

where, for i, j = 1, 2, and i ≠ j,[5]

$$\begin{aligned} \alpha^j_{kk} &= p_i(n_1-n_2)k\Gamma^j_n\Delta_i/l_2k_j\Delta, \\ \alpha^j_{kn} &= -n(k_1-k_2)\alpha^j_{kk}/k(n_1-n_2), \\ \alpha^j_{nn} &= p_i(k_1-k_2)n\Gamma^j_k\Delta_i/l_2n_j\Delta, \\ \alpha^j_{nk} &= -k(n_1-n_2)\alpha^j_{nn}/n(k_1-k_2), \\ \alpha^1_{lk} &= -\alpha^2_{lk}/\phi = k\textstyle\sum_{i=1}^2 l_in_i\alpha^i_{nn}/nl_1(k_1-k_2), \\ \alpha^1_{ln} &= -\alpha^2_{ln}/\phi = n\textstyle\sum_{i=1}^2 l_ik_i\alpha^i_{kk}/kl_1(n_1-n_2). \end{aligned} \tag{12}$$

The factor-intensity and inter-factor cooperation conditions assure that $\alpha^j_{is} > 0$ (i, s = k, n, and j = 1, 2), $\alpha^1_{lk} > 0$, and $\alpha^1_{ln} < 0$.

We assume that the transformation curve between the two goods is strictly concave to the origin.[6] Let $\theta_{Ij} = p_I I_j / p_j F^j$ (I = K, L, N), the distributive share of factor I in the production of good j, and $\gamma_{jS} = (S/X_j)(\partial X_j/\partial S)$ (S = K, N, L); the elasticity of the jth industry's output with respect to the change in factor S's endowment at constant commodity prices, or the Rybczynski elasticity of good j. Then the efficient production point is unique for a given set of commodity prices, and its shift due to the change in the changes in the relative factor endowments at constant commodity prices can be derived from the differentiated form of the production function into which (11) is substituted,

$$\hat{X}_j = \gamma_{jK}\hat{K}+\gamma_{jN}\hat{N}+\gamma_{jL}\hat{L}, \tag{13}$$

where

$$\gamma_{jK} = \theta_{Kj}\alpha^j_{kk}+\theta_{Nj}\alpha^j_{nk}+\alpha^j_{lk},$$

$$\gamma_{jN} = \theta_{Kj}\alpha^j_{kn}+\theta_{Nj}\alpha^j_{nn}+\alpha^j_{ln}, \tag{14}$$

$$\gamma_{jL} = 1-(\gamma_{jK}+\gamma_{jN}).$$

The last expression of (14) reflects the fact that the output of good j is the homogeneous function of degree one in K, N, and L. Since $\gamma_{1K} > 0$ and $\gamma_{2N} > 0$, we can conclude:

Theorem 1. If every factor is cooperative with the other factors in each industry, then the increase in the endowment of one of the extreme factors, at constant commodity prices, expands the output of the good which uses intensively the increasing factor in production, and vice versa.

The mechanism which leads to the results in Theorem 1 can be easily illustrated: Suppose that the supply of capital increases with other factor endowments kept constant. Then the output of good 1 will expand while that of good 2 will shrink to keep land and capital in full employment at an initial set of factor prices. These changes in outputs create the excess demand for labor which lowers the relative factor prices, r/w and t/w. The substitution effects of these factor price changes reinforced by the inter-factor cooperation increase the capital- and land-intensities of both goods, k_j and n_j. Therefore, N_1, L_1, and K_1 increase while N_2 and L_2 decrease, implying that the output of good 1 expands but that of good 2 is ambiguous as to the direction of its change.

4. The Rybczynski elasticity and the marginal rate of substitution between factors

The argument in the previous section suggests that the Rybczynski elasticities, γ_{1N} and γ_{2K}, may take negative or positive values. In fact, it turns out to be clear that not only their values but also those of γ_{1L} and γ_{2L} depend upon the responsiveness of the marginal rates of substitution (MRS's) between capital and labor and between land and labor to the change in the factor intensities.

The elasticities of the MRS's between capital and labor and between land and labor with respect to the change in the capital-intensity or land-intensity of the jth good can be represented in terms of the partial derivatives of the production function:

$$\begin{pmatrix} \hat{f}^j_k - \hat{f}^j_l \\ \hat{f}^j_n - \hat{f}^j_l \end{pmatrix} = \frac{1}{(f^j_l)^2} \begin{pmatrix} k_j(f^j_l f^j_{kk} - f^j_k f^j_{kl}) & n_j(f^j_l f^j_{nk} - f^j_k f^j_{nl}) \\ k_j(f^j_l f^j_{kn} - f^j_n f^j_{kl}) & n_j(f^j_l f^j_{nn} - f^j_n f^j_{nl}) \end{pmatrix} \begin{pmatrix} \hat{k}_j \\ \hat{n}_j \end{pmatrix}. \quad (15)$$

Denote the weighted sum of the elasticities of the MRS between capital and labor in the jth industry by

$$\psi^j_K = \frac{k_1-k_2}{k_j}\left(-\frac{k_j}{f^j_k/f^j_l}\frac{\partial(f^j_k/f^j_l)}{\partial k_j}\right) + \frac{n_2-n_1}{n_j}\left(\frac{n_j}{f^j_k/f^j_l}\frac{\partial(f^j_k/f^j_l)}{\partial n_j}\right),$$

and the counterpart between land and labor by

$$\psi^j_N = \frac{k_1-k_2}{k_j}\left(\frac{k_j}{f^j_n/f^j_l}\frac{\partial(f^j_n/f^j_l)}{\partial k_j}\right) + \frac{n_2-n_1}{n_j}\left(-\frac{n_j}{f^j_n/f^j_l}\frac{\partial(f^j_n/f^j_l)}{\partial n_j}\right).$$

If k_1-k_2 and n_2-n_1 are sufficiently small, Ψ_K^j (Ψ_N^j) is equal to the rate of change in the MRS between capital (land) and labor when $dk_j = k_1-k_2$ and $dn_j = n_2-n_1$. Using (15), Ψ_K^j and Ψ_N^j can be expressed as

$$\Psi_K^j = \frac{1}{(f_l^j)^2}\{(k_1-k_2)(f_k^j f_{kl}^j - f_l^j f_{kk}^j)+(n_2-n_1)(f_l^j f_{nk}^j - f_k^j f_{nl}^j)\},$$

$$\Psi_N^j = \frac{1}{(f_l^j)^2}\{(k_1-k_2)(f_l^j f_{kn}^j - f_n^j f_{kl}^j)+(n_2-n_1)(f_n^j f_{nl}^j - f_l^j f_{nn}^j)\}. \qquad (16)$$

Denote the weighted sum of Ψ_I^1 and Ψ_I^2 by

$$\Psi_I = \sum_{\substack{j=1 \\ i\neq j}}^{2} l_j p_i^2 \Delta_i \Psi_I^j / p_j f^j, \qquad j = 1, 2, \text{ and } I = K, N.$$

Then the Rybczynski elasticities can be expressed in terms of these elasticities with the aid of (12), (14), and (16):

$$\gamma_{1K} = -kf_l^2\{n_2 f_k^1 \Psi_N + (f_l^1 + n_2 f_n^1)\Psi_K\}/l_1 l_2 \Delta,$$

$$\gamma_{1N} = nf_l^2\{(f_l^1 + k_2 f_k^1)\Psi_N + k_2 f_n^1 \Psi_K\}/l_1 l_2 \Delta,$$

$$\gamma_{1L} = -f_l^1 f_l^2 (n_2 \Psi_N - k_2 \Psi_K)/l_1 l_2 \Delta, \qquad (17)$$

$$\gamma_{2K} = kf_l^2\{n_1 f_k^1 \Psi_N + (f_l^1 + n_1 f_n^1)\Psi_K\}/(l_2)^2 \Delta,$$

$$\gamma_{2N} = -nf_l^2\{(f_l^1+k_1f_k^1)\Psi_N+k_1f_n^1\Psi_K\}/(l_2)^2\Delta,$$

$$\gamma_{2L} = f_l^1f_l^2(n_1\Psi_N-k_1\Psi_K)/(l_2)^2\Delta.$$ [7]

Since $\gamma_{1K} > 0$ and $\gamma_{2N} > 0$ by Theorem 1, Ψ_K and Ψ_N must lie in the region

$$\Psi_K > -\{n_2f_k^1/(f_l^1+n_2f_n^1)\}\Psi_N \text{ and } \Psi_K > -\{(f_l^1+k_1f_k^1)/k_1f_n^1\}\Psi_N. \quad (18)$$

Furthermore, the requirement that D_1 and D_2 must be negative implies that

$$\Psi_K > \frac{(n_1-n_2)f_l^1-(k_1n_2-k_2n_1)f_k^1}{(k_1-k_2)f_l^1+(k_1n_2-k_2n_1)f_n^1}\Psi_N. \quad (19)$$ [8]

It follows from (18) and (19) that

Lemma 1. $\gamma_{1K} > 0$ *and* $\gamma_{2N} > 0$, *if and only if*

$$\Psi_K > -\{n_2f_k^1/(f_l^1+n_2f_n^1)\}\Psi_N \text{ for } \Psi_N \geqq 0, \text{ and}$$

$$\Psi_K > -\{(f_l^1+k_1f_k^1)/k_1f_n^1\}\Psi_N \text{ for } \Psi_N < 0.$$ [9]

Since $\gamma_{1K} > 0$, $\gamma_{2N} > 0$, and $D_j < 0$ ($j = 1, 2$) as long as the factor-intensities and inter-factor cooperation conditions hold, it is the feasible region of Ψ_K and Ψ_N that Lemma 1 shows.

Within the feasible region, the necessary and sufficient conditions regarding the signs of γ_{1N} and γ_{2K} and regarding those of γ_{1L} and γ_{2L} can be derived as what follows.

Lemma 2. a) $\gamma_{1N} < 0$ *and* $\gamma_{2K} < 0$, *if and only if*

$$\Psi_K > -\{n_1 f_k^1/(f_l^1+n_1 f_n^1)\}\Psi_N \text{ for } \Psi_N \geqq 0, \text{ and}$$

$$\Psi_K > -\{(f_l^1+k_2 f_k^1)/k_2 f_n^1\}\Psi_N \text{ for } \Psi_N < 0.$$ [10]

b) $\gamma_{1N} < 0$ *and* $\gamma_{2K} \geqq 0$, *if and only if*

$$\Psi_N > 0 \text{ and } -\{n_1 f_k^1/(f_l^1+n_1 f_n^1)\}\Psi_N \geqq \Psi_K > -\{n_2 f_k^1/(f_l^1+n_2 f_n^1)\}\Psi_N.$$

c) $\gamma_{1N} \geqq 0$ *and* $\gamma_{2K} < 0$, *if and only if*

$$\Psi_N < 0 \text{ and } -\{(f_l^1+k_2 f_k^1)/k_2 f_n^1\}\Psi_N \geqq \Psi_K > -\{(f_l^1+k_1 f_k^1)/k_1 f_n^1\}\Psi_N.$$

Lemma 3. a) $\gamma_{1L} > 0$ *and* $\gamma_{2L} < 0$, *if and only if*

$$(n_1/k_1)\Psi_N > \Psi_K > -\{n_2 f_k^1/(f_l^1+n_2 f_n^1)\}\Psi_N \text{ for } \Psi_N > 0.$$

b) $\gamma_{1L} \geqq 0$ *and* $\gamma_{2L} \geqq 0$, *if and only if*

$$(n_2/k_2)\Psi_N \geqq \Psi_K \geqq (n_1/k_1)\Psi_N \text{ for } \Psi_N > 0.$$

c) $\gamma_{1L} < 0$ *and* $\gamma_{2L} > 0$, *if and only if*

$$(k_2/n_2)\Psi_K > \Psi_N > -\{k_1 f_n^1/(f_l^1+k_1 f_k^1)\}\Psi_K \text{ for } \Psi_K > 0.$$

Lemma 2 shows that if Ψ_K and Ψ_N are positive then $\gamma_{1N} < 0$ and $\gamma_{2K} < 0$, implying that the Rybczynski theorem holds for the extreme factor's endowment change. But then the signs of γ_{1L} and γ_{2L} are indeterminate, according to Lemma 3.

5. The inter-factor comparison of cooperation and the inter-industry comparison of labor productivity

It has been made clear in the previous section that the responsiveness of MRS to the change in the factor intensities is crucial in determining the values of the Rybczynski elasticities. In this section we explore the relation of it to the characteristics of the production function represented by the inter-factor cooperation and labor productivity to obtain the sufficient conditions for both γ_{1N} and γ_{2K} to be negative and for γ_{1L} or γ_{2L} to be positive.

For these purposes, let us define under the assumption on factor cooperation:

Factor S is more cooperative with factor I than with factor H in the jth sector if

$$\frac{S_j}{F_I^j}\frac{\partial F_I^j}{\partial S_j} \geq \frac{S_j}{F_H^j}\frac{\partial F_H^j}{\partial S_j} \qquad S \neq I,\ I \neq H, \text{ and } H \neq S. \tag{20}$$

It may be noted that condition (20) is equivalent to

$$f^j_{si}/f^j_i \geqq f^j_{sh}/f^j_h, \qquad s \neq i,\ i \neq h, \text{ and } h \neq s,$$

or to

$$\frac{\partial}{\partial S_j}\left(\frac{F^j_I}{F^j_H}\right) \geqq 0, \qquad S \neq I,\ I \neq H, \text{ and } H \neq S.$$

It is obvious that if the equalities hold in these relationships factor S is locally functionally separable from factors I and H in the production function F^j (see Leontief (1947)). If condition (20) holds in a strict inequality form, one per cent increase in the input of factor S in the jth industry raises the marginal productivity of factor I by a larger rate than that of factor H.

Using this definition, we can classify six cases of ranking by the strength of inter-factor cooperation in the jth industry,

(i-a) $f^j_l f^j_{kn} \geqq f^j_n f^j_{lk} \geqq f^j_k f^j_{ln}$,

(i-b) $f^j_l f^j_{kn} \geqq f^j_k f^j_{ln} \geqq f^j_n f^j_{lk}$,

(ii-a) $f^j_n f^j_{lk} \geqq f^j_l f^j_{nk} \geqq f^j_k f^j_{nl}$,

(ii-b) $f^j_n f^j_{lk} \geqq f^j_k f^j_{nl} \geqq f^j_l f^j_{nk}$, (iii-a) $f^j_k f^j_{nl} \geqq f^j_l f^j_{kn} \geqq f^j_n f^j_{kl}$,

(iii-b) $f^j_k f^j_{nl} \geqq f^j_n f^j_{kl} \geqq f^j_l f^j_{kn}$.

The meaning of these rankings can be illustrated by cases (i), where one of the extreme factors is more cooperative with the other than with the middle factor, that is, $f^j_l f^j_{kn} \geqq f^j_n f^j_{kl}$ or $f^j_l f^j_{nk} \geqq f^j_k f^j_{nl}$; besides, under the former condition, labor is more cooperative with capital than with land ($f^j_n f^j_{lk} \geqq f^j_k f^j_{ln}$), and under the latter condition, labor is more cooperative with land than with capital ($f^j_k f^j_{ln} \geqq f^j_n f^j_{lk}$). The meaning of the other cases can be similarly explained.

The application of each ranking to (16) shows that $\Psi^j_K > 0$ in cases (i-a), (i-b), and (ii-a) while $\Psi^j_N > 0$ in cases (i-a), (i-b), and (iii-a). According to Lemma 2, $\gamma_{1N} < 0$ and $\gamma_{2K} < 0$ if case (i-a) or (i-b) holds in every industry. Thus,

Theorem 2. If one of the extreme factors is more cooperative with the other than with the middle factor, then the increase in the supply of one of the extreme factors, at constant commodity prices, shrinks the output of the good which does not intensively use the expanding factor, and vice versa.

The conclusions obtained so far can be connected with the substitute-complement relations among factors by replacing the presuppositions of Theorems 1 and 2 for the conditions which are represented in terms of the price elasticities of factor substitution used by Suzuki (1981) and Jones and Easton (1983).[11] Let a_{Hj} be the amount of factor H required for the production of one unit of commodity j and ε^j_{HS} the elasticity of a_{Hj} with respect to the change in the price of factor S. If a_{Hj} satisfies the cost-minimization conditions in a perfectly competitive economy, then

$$\hat{a}_{Hj} = \varepsilon^j_{HK}\hat{r}+\varepsilon^j_{HN}\hat{t}+\varepsilon^j_{HL}\hat{w},$$

where $\varepsilon^j_{HH} < 0$ if the cost function of good j has a regular minimum, and $\varepsilon^j_{HK}+\varepsilon^j_{HN}+\varepsilon^j_{HL} = 0$ for j = 1 and 2, and H = K, L, and N. Noticing that $k_j = a_{Kj}/a_{Lj}$, $n_j = a_{Nj}/a_{Lj}$, $f^j_k/f^j_l = r/w$, and $f^j_n/f^j_l = t/w$, and deriving the inverse of the coefficient matrix on the right-hand side of (15), we can get

$$\varepsilon^j_{KK}-\varepsilon^j_{LK} = (f^j_l f^j_{nn}-f^j_n f^j_{nl})f^j_l/k_j f^j \Delta_j,$$

$$\varepsilon^j_{KN}-\varepsilon^j_{LN} = (f^j_k f^j_{nl}-f^j_l f^j_{nk})f^j_l/k_j f^j \Delta_j,$$

$$\varepsilon^j_{NK}-\varepsilon^j_{LK} = (f^j_n f^j_{kl}-f^j_l f^j_{kn})f^j_l/n_j f^j \Delta_j,$$

$$\varepsilon^j_{NN}-\varepsilon^j_{LN} = (f^j_l f^j_{kk}-f^j_k f^j_{kl})f^j_l/n_j f^j \Delta_j.$$

It is obvious that $\varepsilon^j_{LK} > \varepsilon^j_{LL}$ and $\varepsilon^j_{LN} > \varepsilon^j_{NN}$ if every factor is cooperative with the other factors in the jth sector. In addition, $\varepsilon^j_{LN} \geqq \varepsilon^j_{KN}$ and $\varepsilon^j_{LK} \geqq \varepsilon^j_{NK}$ if one of the extreme factors is more cooperative with the other than with the middle factor. Therefore, the sufficient condition for the conclusions of Theorems 1 and 2 is:

Corollary. If the extreme and middle factors are substitutes ($\varepsilon^j_{LK} > 0$ and $\varepsilon^j_{LN} > 0$) and if two extreme factors are complements or independent in every industry ($\varepsilon^j_{NK} \leqq 0$), then the increase in the endowment of one of the extreme factors, at constant commodity prices, expands the output of the good which uses intensively the increasing factor in production and shrinks the output of the other good.[12]

Under the factor-intensities condition,

$$\gamma_{1L} > -\frac{f^1_l f^2_l}{l_1 l_2 \Delta}\sum^2_{\substack{j=1\\ i\neq j}} \frac{l_j p^2_i \Delta_i}{p_j f^j}(n_j \Psi^j_N - k_j \Psi^j_K) \text{ if } \Psi^1_K > 0 \text{ and } \Psi^1_N > 0,$$

$$\gamma_{2L} > \frac{f^1_l f^2_l}{(l_2)^2 \Delta}\sum^2_{\substack{j=1\\ i\neq j}} \frac{l_j p^2_i \Delta_i}{p_j f^j}(n_j \Psi^j_N - k_j \Psi^j_K) \text{ if } \Psi^2_K > 0 \text{ and } \Psi^2_N > 0.$$

The consideration of these and the following relations

$$n_j \Psi^j_N - k_j \Psi^j_K \gtreqless \frac{(p_2 f^2 - p_1 f^1) f^j_{kl}}{p_j f^j_k (f^j_l)^2} \text{ as } f^j_k f^j_{nl} \gtreqless f^j_n f^j_{kl},$$

leads to the results:

(i) If $f^j_k f^j_{nl} \geqq f^j_n f^j_{kl}$ $(j = 1, 2)$, and if $p_2 f^2 \geqq p_1 f^1$,[13] then $\gamma_{1L} > 0$ if $\Psi_K^j > 0$, and $\gamma_{1L} > 0$ and $\gamma_{2L} < 0$ if $\Psi_K^j \leqq 0$ $(j = 1, 2)$. Thus, $\gamma_{1L} > 0$ whether Ψ_K^j is positive or negative $(j = 1, 2)$, but the sign of γ_{2L} is ambiguous when $\Psi_K^j > 0$.[14]

(ii) If $f_n^j f_{kl}^j \geqq f_k^j f_{nl}^j$ $(j = 1, 2)$, and if $p_1 f^1 \geqq p_2 f^2$,[15] then $\gamma_{2L} > 0$ if $\Psi_N^j > 0$, and $\gamma_{2L} > 0$ and $\gamma_{1L} < 0$ if $\Psi_N^j \leqq 0$ $(j = 1, 2)$. Thus, $\gamma_{2L} > 0$ regardless of the value of Ψ_N^j $(j = 1, 2)$. However, the sign of γ_{1L} is indeterminate when $\Psi_N^j > 0$.[16]

Since $p_j f^j$ is the labor productivity in the jth sector in value terms, these results are summed up as what follows.[17]

Theorem 3. If the productivity of the middle factor in industry i is not higher than that in industry j and if the middle factor is less cooperative with the extreme factor of good i than with that of good j in every industry, then the increase in the supply of the middle factor, at constant commodity prices, expands the output of good i, and vice versa $(i \neq j,\ i, j = 1, 2)$.

The reasons why the conditions postulated in Theorem 3 are crucial for the result can be conjectured from their effects on the relative factor intensities of both goods, k_j/l_j. Suppose the supply of labor increases. Then, as long as Ψ_K^j and Ψ_N^j $(j = 1, 2)$ are positive, the outputs of both goods probably expand at initial factor prices to create the excess demand for capital and land. Thus, the ratios of rent for capital to wage and rent for land to wage rise, causing the factor intensities of both goods to fall. Which of the capital- and land-intensities of each good falls to a greater extent depends upon the conditions assumed in Theorem 3.[18] If they hold for i = 1 and j = 2, for instance, k_j/n_j-ratios are raised in both industries. This increases the output of good 1 and decreases that of good 2 so that the initial output effects are reinforced in industry 1 and reduced in industry 2. Therefore, $\gamma_{1L} > 0$ but the sign of γ_{2L} is indeterminate as long as ψ_K^j (j = 1, 2) are positive. The role of the conditions in Theorem 3 can be explained for the case of i = 2 and j = 1 in the similar fashion.

6. Conclusions

We have explored the role of the factor intensities, the inter-factor cooperation, and the labor productivity in determining the output effects of an endowment change with commodity prices kept constant. As a result, it has been made clear that each output effect represented by the Rybczynski elasticity depends upon the relative extent of the responsiveness to the change in the factor intensities of the marginal rates of substitution between capital and labor and between land and labor in the whole economy. Since the latter in turn depends upon the properties of the production functions of both industries, an attempt has been made to bring out their influences upon the Rybczynski elasticity, which are summarized in the three theorems.

The synthetic consideration of these theorems leads to the conclusions that if cases (i) hold in comparison with the inter-factor cooperation, or if the extreme factors are more cooperative with each other than with the middle factor which is in turn more cooperative with land in case (i-b) or with capital in case (i-a) relative to the other extreme factor, then the growth of the supply of one of the extreme factors, at constant commodity prices, increases the output of the industry which uses intensively the growing factor and decreases the output of the other industry. Under the same conditions the growth of the supply of the middle factor increases the output of the industry where its productivity is not higher than in the other industry.

Footnotes

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1. Batra and Casas (1976) has also explored this type of the Rybczynski relation and insisted that the factor intensity condition is a uniquely sufficient condition (the substitute-complement conditions are not necessary) for the fact stated in the text. However, this is not true because one can discover a counterexample to their conclusions (see Suzuki (1983)).

2. The factor intensity conditions assumed in the text are equivalent to those Kemp and Wegge (1969) postulated, $\max_s a_{is}/a_{js} = a_{ii}/a_{ji}$ for i = 1, 2, and j = 1, 2, 3 (a_{is} is the amount of factor s required to produce one unit of good i), if factors 1, 2, and 3 denote capital, land, and labor respectively.

3. For the relations between factor prices and endowments in the three-factor, two-good model, see Batra and Casas (1976), Ruffin (1981), and Suzuki (1981).

4. The determinant of the coefficient matrix on the left-hand side of (10) equals $k_j n_j p_1 p_2 \phi\Delta$ for j = 1, 2.

5. There is a relationship

$$\sum_{j=1}^{2}\{(l_j k_j/k)\alpha_{kk}^{j}+(l_j n_j/n)\alpha_{nn}^{j}\} = 1.$$

It is useful in deriving the relation that the properly weighted sum of γ_{jK}, γ_{jN}, and γ_{jL} equals zero (see footnote 7).

6. The sufficient condition for the transformation curve to be strictly concave to the origin is that the 3×2 input-coefficient matrix has a rank 2, which is met under our factor-intensity conditions, and that the cost function derived from our production function has a regular minimum (see Chang (1979) and Jones and Scheinkman (1977)).

7. Use is made of the relation, $\gamma_{jL} = -(k_i\gamma_{jK}/k+n_i\gamma_{jN}/n)$ (i, j = 1, 2, and i ≠ j) in deriving the expressions for γ_{jL}.

8. $$D_j = [\{(n_1-n_2)f_l^j-(k_1n_2-k_2n_1)f_k^j\}\Psi_N^j-\{(k_1-k_2)f_l^j +(k_1n_2-k_2n_1)f_n^j\}\Psi_K^j]/f^jf_l^j \qquad j = 1,\ 2.$$

9. There are the following relations among the coefficients of Ψ_N on the right-hand sides of (18) and (19),

$$-\frac{f_l^1+k_1f_k^1}{k_1f_n^1} < \frac{(n_1-n_2)f_l^1-(k_1n_2-k_2n_1)f_k^1}{(k_1-k_2)f_l^1+(k_1n_2-k_2n_1)f_n^1} < -\frac{n_2f_k^1}{f_l^1+n_2f_n^1}\ .$$

10. In deriving the conditions in Lemma 2, use is made of the relations,

$$-\frac{f_l^1+k_2f_k^1}{k_2f_n^1} < -\frac{f_l^1+k_1f_k^1}{k_1f_n^1} < -\frac{n_2f_k^1}{f_l^1+n_2f_n^1} < -\frac{n_1f_k^1}{f_l^1+n_1f_n^1}.$$

11. A reference to the connection of the inter-factor cooperation with the substitute-complement relation and to the results summarized in the Corollary has been suggested by Professor Ohyama.

12. For these results, see Suzuki (1981) and Takayama (1982).

13. These conditions ensure that $\Psi_N^j > 0$ for $j = 1, 2$.

14. In the expression for γ_{2L} in (17), $n_1\Psi_N^1-k_1\Psi_K^1 > 0$. But

$$n_1\Psi_N^2-k_1\Psi_K^2 \geqq \frac{1}{f_l^2}\{(k_1-k_2)(n_1f_{nk}^2+k_1f_{kk}^2)-(n_2-n_1)(k_1f_{nk}^2+n_1f_{nn}^2)\}$$

$$+\frac{1}{p_2(f_l^2)^2f_n^2}(n_1f_n^2+k_1f_k^2)(p_2f^2-p_1f^1)f_{nl}^2.$$

The right-hand side of this expression may be positive or negative.

15. These conditions ensure that $\Psi_K^j > 0$ for $j = 1, 2$.

16. In expression for γ_{1L} in (17), $n_2\Psi_N^2 - k_2\Psi_K^2 > 0$. But the sign of $n_2\Psi_N^1 - k_2\Psi_K^1$ is ambiguous by the similar reasoning as in footnote 14.

17. Note that $p_j f^j = w/\theta_{Lj}$. In Jones and Easton's (1983) terminology, sector j is relatively intensive in its use of the middle factor if $\theta_{Lj} > \theta_{Li}$, or $p_j f^j < p_i f^i$ $(i \neq j)$.

18. $$\frac{L}{k_j/n_j}\frac{\partial(k_j/n_j)}{\partial L} = A(n_j\Psi_N^j - k_j\Psi_K^j),$$

where $A \equiv p_i \Delta_i (f_l^j)^2\{n(k_1-k_2)+k(n_2-n_1)\}/(l_2 k_j n_j f^j \Delta) < 0$ for $j = 1, 2$.

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