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Credit constraints, firm selection, and growth: The international transmission of a credit crunch via trade*

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Abstract

This study develops a two-country, heterogeneous-firm model of trade and growth with country-specific credit constraints to examine firm selection as a channel through which a credit crunch is internationally transmitted. Under mild conditions, a credit crunch in one country reduces growth rates in both countries in the short and the long run. Even the country not directly hit by the shock experiences a growth slowdown because the shock reduces its export profitability, inducing firms to shift from exporting to domestic sales. This change in firm selection hinders asset accumulation in that country. We further compare the effects of the credit crunch under financial autarky and financial integration and find that trade alone can generate cross-border transmission of financial shocks, even in the absence of international financial flows.

JEL classification: F12; F43; O16; O41

Keywords: Credit crunch; Trade-induced spillovers; Firm selection; Financial frictions; Endogenous growth

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1 Introduction

Since the 2007–2009 financial crisis, many developed countries have experienced prolonged recessions.[1] It is well recognized that not all countries that experienced prolonged recessions faced significant disruptions in their domestic financial markets after the crisis. For instance, IMF (2009) estimated the potential valuation loss for the U.S. financial sector from 2007 to 2010 to be \$2.712 trillion. In contrast, although Japan experienced a substantial decline in its GDP after the financial crisis, the potential valuation loss for the Japanese financial sector was estimated at \$149 billion, only approximately 5% of the U.S. loss.[2] This episode raises a central question: whether a financial disruption in one country can generate persistent growth effects abroad, even in economies that do not experience severe domestic financial distress.

As discussed below, a growing body of research has incorporated endogenous growth mechanisms into business cycle models to study how adverse financial shocks lead to secular stagnation. Although these findings are insightful, most studies have focused on closed economies. Therefore, to provide a theoretical explanation for an economy's slow recovery, models must rely on the occurrence of adverse financial shocks within that economy. Another strand of research examines the international transmission of financial shocks through trade or financial linkages, but typically assumes that long-run growth is exogenous. Consequently, while it is well established that financial crises propagate internationally in the short run, less is known about whether and how they affect long-run growth across countries.

This study addresses this gap by investigating how a country-specific credit crunch influences global economic growth through international trade, more specifically, through firm-selection dynamics in the tradable goods sector. We develop a two-country endogenous growth model that integrates heterogeneous firms, international trade, and financial frictions. The model builds on the lab-equipment variety-expansion model of endogenous growth in Rivera-Batiz and Romer (1991). We incorporate two layers of heterogeneity into the model. First, firms in the tradable goods sector are heterogeneous in productivity, and they face fixed costs to enter markets. Second, the fixed inputs are produced by entrepreneurs who are heterogeneous in productivity and credit-constrained. Thus, our framework combines two strands of standard modeling approaches: firm

[1]The average growth rate in the U.S. for the decade 1997–2007 was 2.06%, while that for the following decade was only 0.66%. Even when we exclude the two years of the Great Recession (2007 and 2008) and change the sample decade to 2009–2019, the average growth rate was only 1.51%. In the 20 countries in the Euro area, the average growth rates were 1.95% and 1.15% for 1997–2007 and 2009–2019, respectively. The data are from OECD Statistics.

[2]IMF also estimated total financial sector write-downs of \$1.193 trillion for the countries comprising the Euro area and the U.K. Even taking the number of countries into account, the loss is relatively small compared to that of the U.S. See Table 1.3 in IMF (2009, p.35) for more details.

heterogeneity in trade following Melitz (2003) and heterogeneous entrepreneurs with financial frictions following Moll (2014).

Despite these multiple sources of heterogeneity, the framework remains tractable, allowing us to obtain clear analytical insights into the mechanism linking credit conditions, trade reallocation, and growth. The analysis yields two main results regarding both the short- and long-run effects of a permanent credit crunch in one country. First, under mild conditions, the growth rates in both countries decline at all points in time. Second, the mechanism operates through asymmetric changes in productivity cutoffs in the tradable goods sector. In the crisis-hit country, tighter credit accelerates firm exit by raising the cost of entry. In the partner country, reduced export demand lowers the profitability of foreign sales, raising the export cutoff while lowering the domestic cutoff. This inefficient reallocation toward less productive domestically oriented firms reduces entrepreneurs' asset accumulation, thereby reducing growth.

We further ask whether international financial integration amplifies or dampens such trade-induced spillovers. Another contribution of this study is to compare international spillovers under financial autarky and financial integration. After analytically establishing that the above mechanism operates in both regimes, we conduct numerical simulations and show that both the short- and long-run effects are remarkably similar across the two regimes. This reveals a channel through which credit shocks can propagate internationally, even in the absence of cross-border financial transactions. We also examine a temporary credit crunch and obtain qualitatively similar results across the two regimes.

While financial globalization is often viewed as the primary channel of international shock transmission, our results indicate that trade alone can generate substantial cross-border growth effects. These findings are consistent with Japan's experience during and after the financial crisis. For example, Forbes (2004) and Claessens et al. (2012) document that trade connections played a central role in the international propagation of the global financial crisis. In addition, Hosono et al. (2016) show that Japanese firms were affected mainly through trade-related channels. Moreover, the "Great Trade Collapse"—the sharp and widespread contraction in global trade during the financial crisis—remains a vivid example. This episode is in line with our findings, which emphasize international trade as a powerful channel of business-cycle transmission in the short and the long run.[3]

1.1 Related literature

Several business-cycle studies incorporate endogenous growth mechanisms into DSGE frameworks to examine financial shocks. Bianchi et al. (2019), Guerron-Quintana and Jinnai (2019), Ikeda

[3]For the Great Trade Collapse, see, for example, Bems et al. (2013).

and Kurozumi (2019), Queralto (2020), and Ohdoi (2024) are examples of such studies. They assume closed or small open economies, thus abstracting from the international transmission of financial shocks. Devereux and Sutherland (2011), Kollmann et al. (2011), Dedola and Lombardo (2012), Perri and Quadrini (2018), Ohdoi (2018), and Yao (2019) construct two-country models to elucidate the mechanism of international transmission of financial shocks. Their open-economy models assume that the long-run growth rate is exogenous. Therefore, the implications for the long-run growth rate are outside the scope of their analysis.

Although their research objectives differ from ours, the following two strands of literature are closely related to this study. The first strand incorporates the Melitz model into an endogenous growth framework. Examples include Baldwin and Robert-Nicoud (2008), Ourens (2016), Sampson (2016), Haruyama and Zhao (2017), and Naito (2017a,b; 2019; 2021), among others. These studies primarily examine the effects of trade liberalization on economic growth and welfare across countries, and are therefore clearly distinct from the focus of the present paper. The second strand introduces financial frictions into the Melitz model, including Manova (2013), Feenstra et al. (2014), Chaney (2016), Kohn et al. (2016, 2020), and Leibovici (2021). These studies are either static in nature or, even when formulated in a dynamic setting, do not analyze the growth effects of a credit crunch as in our framework. Hence, our analysis differs fundamentally from this line of research.

1.2 Organization of the paper

The remainder of this paper is organized as follows. Section 2 sets up the model. Section 3 characterizes the equilibrium and examines the effects of a credit crunch under financial autarky and financial integration. Section 4 presents numerical analysis. Section 5 discusses robustness and extensions. Section 6 concludes. Proofs are provided in the Appendix.

2 Model

Time is continuous and indexed by $t \in [0, \infty)$. The world economy consists of two countries: country 1 and country 2. We attach an asterisk "$*$" to variables in country 2. Each country has a non-tradable final good sector, a tradable intermediate goods sector, and a non-tradable knowledge creation sector. There are two primary factors, labor and capital, neither of which is internationally mobile. We consider two polar cases for international asset markets: financial autarky and financial integration.

2.1 Final good sector

We omit the time subscript unless doing so would cause confusion. The representative firm produces the final good according to

$$Y = \frac{1}{\alpha} L^{1-\alpha} \left(\int_{\omega \in \Omega} x(\omega)^{\alpha} d\omega \right),$$

where Y is output, L is labor input, Ω is the set of available varieties of intermediate goods, $x(\omega)$ is the demand for variety $\omega \in \Omega$, and $\alpha \in (0,1)$ is the cost share of intermediate goods.[4] Let P, w, and $p(\omega)$ respectively denote the price of the final good, the wage rate, and the demand price of variety ω. Profit maximization under perfect competition yields

$$w = (1-\alpha)PY/L, \quad p(\omega) = PL^{1-\alpha} x(\omega)^{\alpha-1} \text{ for } \omega \in \Omega.$$

In country 2,

$$w^* = (1-\alpha)P^*Y^*/L^*, \quad p^*(\omega) = P^* L^{*1-\alpha} x^*(\omega)^{\alpha-1} \text{ for } \omega \in \Omega^*.$$

2.2 Intermediate goods sector

We focus on firm behavior in country 1. Firms operate under monopolistic competition. Following a standard variety-expansion framework, the intermediate goods are produced using the final good as the variable input. Producing one unit of an intermediate good requires $1/\varphi$ units of the final good. Firms are heterogeneous in φ.

Upon observing φ, a firm decides whether to exit, serve only the domestic market (D), or serve both the domestic and export markets (X). The index $j(= D, X)$ denotes the destination market. Hereafter, we suppress ω because it is sufficient to identify firms by their productivity φ. Let $y_j(\varphi)$ and $p_j(\varphi)$ denote the output and producer price in market j. Then, $x(\varphi) = y_D(\varphi)$, $x^*(\varphi) = y_X(\varphi)/\tau$, $p(\varphi) = p_D(\varphi)$, and $p^*(\varphi) = \tau p_X(\varphi)$, where $\tau \geq 1$ denotes iceberg trade costs.

As in Melitz (2003), firms incur fixed costs both upon entry and when serving market j. In endogenous growth theory, entry typically requires a fixed input of a specialized good, variously referred to as a patent, blueprint, or idea. We adopt the same assumption and refer to this fixed input as a "knowledge good" following Baldwin and Robert-Nicoud (2008). We assume that it is produced by entrepreneurs, as discussed below.

The price of the knowledge good is denoted by P_K. Serving market j requires a fixed input $f_j > 0$ in terms of the knowledge good. The profit from market j is

$$\pi_j(\varphi) \equiv (p_j(\varphi) - P/\varphi)\, y_j(\varphi) - P_K f_j.$$

[4]We include the term $1/\alpha$ in the production function for notational simplicity. See, for example, Acemoglu (2009, Ch.13).

Profit maximization yields the producer price $p_j(\varphi) = P/(\alpha\varphi)$, and the zero-cutoff profit condition determines the productivity cutoff, denoted by φ_j: $\pi_j(\varphi_j) = 0$.

For analytical tractability, firms pay the entry cost and draw productivity at each point in time.[5] Let $G(\varphi)$ denote the cumulative distribution function. Entry requires $f_E > 0$ units of the knowledge good. The free-entry condition is therefore $P_K f_E = \sum_{j=D,X} \int_{\varphi \geq \varphi_j} \pi_j(\varphi) dG(\varphi)$. Appendix A.1 shows that this can be rewritten as

$$f_E = \sum_{j=D,X} f_j \Pi(\varphi_j); \quad \Pi(\varphi_j) \equiv \int_{\varphi \geq \varphi_j} [(\varphi/\varphi_j)^{\alpha/(1-\alpha)} - 1] dG(\varphi). \tag{1}$$

Since entry and exit occur at each point in time, aggregate profits in this sector are zero. Let N^e denote the endogenously determined mass of entrants. The number of firms serving market j is $(1 - G(\varphi_j))N^e$.

Fixed inputs (f_E, f_D, f_X) and the productivity distribution G are identical across countries. Accordingly, in country 2, the producer price, zero-cutoff profit condition, and free-entry condition are $p_j^*(\varphi) = P^*/(\alpha\varphi)$, $\pi_j^*(\varphi_j^*) = 0$, and $f_E = \sum_{j=D,X} f_j \Pi(\varphi_j^*)$, respectively.

2.3 Entrepreneurs

Entrepreneurs are indexed by $i \in [0, 1]$ and have preferences $\mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log C_{it}^e dt\right]$, where C_i^e is consumption and $\rho > 0$ is the subjective discount rate. Each entrepreneur produces the knowledge good using capital under constant returns to scale technology and perfect competition:

$$Y_{K,it} = z_{it} K_{it},$$

where $Y_{K,i}$ is the output of the knowledge good, K_i is capital, and z_i is the marginal productivity of capital. At each point in time, z_i is drawn from a stationary distribution, where the cumulative distribution function is given by $F(z)$. Productivity is independently and identically distributed (iid) across agents and over time.

Let q_t denote the real price of the knowledge good, i.e., the relative price of the knowledge good to the final good:

$$q_t \equiv P_{K,t}/P_t.$$

The real budget constraint, expressed in units of the final good, is $(q_t z_{it} K_{it} - r_t^b B_{it})dt + dB_{it} = (C_{it}^e + I_{it})dt$, where B_i is the real debt in terms of the domestic final good, r^b is the real interest rate, and I_i is the capital investment. The level of capital changes according to $dK_{it} = (I_{it} - \delta K_{it})dt$, where δ is the depreciation rate.[6] Entrepreneurs are subject to the following credit constraint:

$$B_{it} \leq (1 - 1/\theta) K_{it}; \quad \theta \geq 1.$$

[5] We follow Naito (2017a) in assuming that firms pay the entry cost and draw productivity at each point in time.

[6] Since debt and capital may jump over time at the individual level, we do not write them in differential form.

Capital can only be partially financed by external funds, and the value of θ governs the degree of financial frictions.[7]

Let A_i denote an entrepreneur's net worth: $A_i \equiv K_i - B_i$. Following Moll (2014) and Itskhoki and Moll (2019), we assume that at each point in time, entrepreneurs can determine the amount of investment after observing their productivity. The problem is therefore basically the same as theirs. As shown in Appendix A.1, the problem can be solved in two stages. First, at each point in time, the entrepreneur determines K_i and B_i to maximize the net profit, $(qz_i - \delta - r^b)K_i$, subject to the credit constraint while taking (A_i, z_i) as given. The optimal choices are

$$K_{it} = \begin{cases} 0 & \text{if } z_{it} < \widetilde{z}_t, \\ \theta A_{it} & \text{if } z_{it} \geq \widetilde{z}_t, \end{cases} \quad B_{it} = \begin{cases} -A_{it} & \text{if } z_{it} < \widetilde{z}_t, \\ (\theta - 1)A_{it} & \text{if } z_{it} \geq \widetilde{z}_t, \end{cases}$$

where $\widetilde{z}$ denotes the productivity cutoff for entrepreneurs to survive and produce the knowledge good:

$$\widetilde{z}_t \equiv \frac{r_t^b + \delta}{q_t}. \tag{2}$$

Thus, the borrowing constraint always binds when entrepreneurs are active.[8] Second, the entrepreneur chooses the time paths of consumption and net worth to maximize the lifetime utility subject to the budget constraint. Since instantaneous utility is logarithmic, this problem yields the following results: $dA_{it} = (r_t^b - \rho)A_{it}dt$ if $z_{it} < \widetilde{z}_t$; $dA_{it} = [r_t^b + \theta q_t (z_{it} - \widetilde{z}_t) - \rho]A_{it}dt$ if $z_{it} \geq \widetilde{z}_t$; and $C_{it}^e = \rho A_{it}$.

Let A denote the aggregate net worth: $A \equiv \int_0^1 A_i di$. Since net worth is predetermined at each date, A_i is independent of z_i. It then follows that the aggregate net worth A evolves deterministically (see Appendix A.1). Hereafter, let a dot over a variable denote the time derivative of this variable; for example, $\dot{A}_t \equiv dA_t/dt$. Using the entrepreneurs' cutoff condition (2), we can obtain the dynamic equation of A as follows:

$$\dot{A}_t/A_t = g_t \equiv q_t\widetilde{z}_t\left(1 + \theta\Psi(\widetilde{z}_t)\right) - \delta - \rho, \tag{3}$$

[7]The case of $\theta = 1$ results in the entrepreneurs' self-financing. The case of $\theta \to \infty$ implies that entrepreneurs can finance all their capital investment through external financing.

[8]As previously mentioned, knowledge creation can be interpreted as an R&D activity in the real economy. Several empirical studies have suggested that credit constraints significantly limit R&D activities. Himmelberg and Petersen (1994), Harhoff (1998), and Bond et al. (2005) find a significant positive relationship between R&D investment and cash flow; that is, internal funds. Following Kaplan and Zingales's (2000) critique that the cash flow sensitivity of R&D investment is not an appropriate measure of credit constraints, more recent studies have directly examined firms' access to external finance. Examples include Czarnitzki and Hottenrott (2011) and Hottenrott and Peters (2012). Bakhtiari et al. (2020) provide a review of the literature on financial constraints and the performance of small and medium-sized firms.

where $\Psi(\widetilde{z})$ is defined as

$$\Psi(\widetilde{z}) \equiv \int_{z\geq\widetilde{z}} (z/\widetilde{z}-1)dF(z); \quad \Psi'(\widetilde{z}) = -\frac{\Psi(\widetilde{z})+1-F(\widetilde{z})}{\widetilde{z}} < 0.$$

Given A, other aggregate variables are accordingly obtained: $B_t = [\theta(1-F(\widetilde{z}_t))-1]A_t$, $K_t = \theta(1-F(\widetilde{z}_t))A_t$, $Y_{K,t} = \theta A_t \int_{z\geq\widetilde{z}} zdF(z)$, and $C_t^e = \rho A_t$.

In this study, we focus on the international asymmetry in the degree of financial frictions: θ^* generally differs from θ. Thus, in country 2, the following Euler equation holds:

$$\dot{A}_t^*/A_t^* = g_t^* \equiv q_t^* \widetilde{z}_t^* \left(1+\theta^*\Psi(\widetilde{z}_t^*)\right) - \delta - \rho. \tag{4}$$

2.4 Workers

There is a continuum of homogeneous workers of mass $L > 0$ who do not save. Each worker chooses consumption c^w and labor supply l to solve the following optimization problem at each point in time: $\max_{c_t^w, l_t} \log c_t^w - l_t^{1+\nu}/(1+\nu)$ subject to the budget constraint $w_t l_t = P_t c_t^w$, where $\nu > 0$ is the inverse of the Frisch elasticity of labor supply. The optimal choice yields $l_t = 1$ and $c_t^w = w_t/P_t$ for all t. Then, the aggregate supply of labor always equals the population L, and the aggregate consumption of workers is given by $C_t^w \equiv w_t L/P_t$.

3 Equilibrium

Throughout this study, we choose country 2's final good as the numeraire: $P_t^* = 1\ \forall t$. We assume that the labor endowment and trade costs are symmetric across the two countries: $L^* = L$ and $\tau^* = \tau$. Note that P_t may differ from unity because θ^* generally differs from θ.

3.1 Definition of the equilibrium

From this point onward, to avoid repetition, we use the superscript "(*)" so that $A^{(*)}$ denotes either A or A^*, and the same convention applies to all other variables. The equilibrium is defined as follows:

Definition 1. *The equilibrium is a time path of (i) allocations* $\left\{y_{D,t}^{(*)}(\varphi),\ y_{X,t}^{(*)}(\varphi),\ N_t^{e(*)},\ A_t^{(*)},\ K_t^{(*)},\ B_t^{(*)},\ Y_{K,t}^{(*)},\ C_t^{e(*)},\ C_t^{w(*)}\right\}$, *(ii) prices* $\left\{P_t,\ q_t^{(*)},\ r_t^{b(*)},\ w_t^{(*)}\right\}$, *and (iii) productivity cutoffs* $\left\{\varphi_{D,t}^{(*)},\ \varphi_{X,t}^{(*)},\ \widetilde{z}_t^{(*)}\right\}$ *such that:*

- *Final good firms' profit maximization leads to their factor demand;*
- *Intermediate-good firms' behavior satisfies the mark-up pricing, the zero-cutoff profit condition, and the free entry condition;*

- *Workers' behavior leads to their labor supply and consumption;*

- *Entrepreneurs' behavior leads to their cutoff condition, aggregate debt, aggregate capital, aggregate supply of the knowledge good, and the Euler equation;*

- *The market for the final good clears in each country,* $Y_t^{(*)} = C_t^{e(*)} + C_t^{w(*)} + \dot{K}_t^{(*)} + \delta K_t^{(*)} + N_t^{e(*)} \sum_{j=D,X} \int_{\varphi \geq \varphi_{j,t}^{(*)}} y_{j,t}^{(*)}(\varphi)/\varphi dG(\varphi)$;

- *The market for the knowledge good clears in each country,* $Y_{K,t}^{(*)} = N_t^{e(*)} \left[f_E + \sum_{j=D,X} f_j (1 - G(\varphi_{j,t}^{(*)})) \right]$;

- *In each country, the balance of payments is given by*[9]

$$\dot{B}_t^{(*)} = r_t^{b(*)} B_t^{(*)} - \frac{EX_t^{(*)} - IM_t^{(*)}}{P_t^{(*)}}, \tag{5}$$

  where $EX^{(*)}$ *and* $IM^{(*)}$ *are country 1's (2's) exports and imports, respectively:*

$$EX^{(*)} \equiv N^{e(*)} \int_{\varphi \geq \varphi_X^{(*)}} p_X^{(*)}(\varphi) y_X^{(*)}(\varphi) dG(\varphi), \quad IM = EX^*, \quad IM^* = EX;$$

- *The bond market clears;*

  1. *In the case of financial autarky,*

$$B_t^{(*)} = 0 \Leftrightarrow \theta^{(*)}(1 - F(\tilde{z}_t^{(*)})) - 1 = 0; \tag{6}$$

  2. *In the case of financial integration,*

$$P_t B_t + B_t^* = 0 \Leftrightarrow P_t A_t [\theta(1 - F(\tilde{z}_t)) - 1] + A_t^* [\theta^*(1 - F(\tilde{z}_t^*)) - 1] = 0, \tag{7}$$

  with the interest rate parity condition,

$$r_t^b + \frac{\dot{P}_t}{P_t} = r_t^{b*}. \tag{8}$$

In Appendix A.2, we characterize the equilibrium when the two countries are completely closed, and demonstrate that each country grows at a constant rate through variety expansion of intermediate goods from the initial time.

3.2 Intermediate-good firms' cutoffs given P

Following Melitz (2003), we assume the following inequality: $T \equiv \tau \left(f_X / f_D \right)^{(1-\alpha)/\alpha} > 1$, which helps ensure that the productivity cutoff for exporting exceeds that for domestic operations.

[9] Recall that $B^{(*)}$ represents real debt.

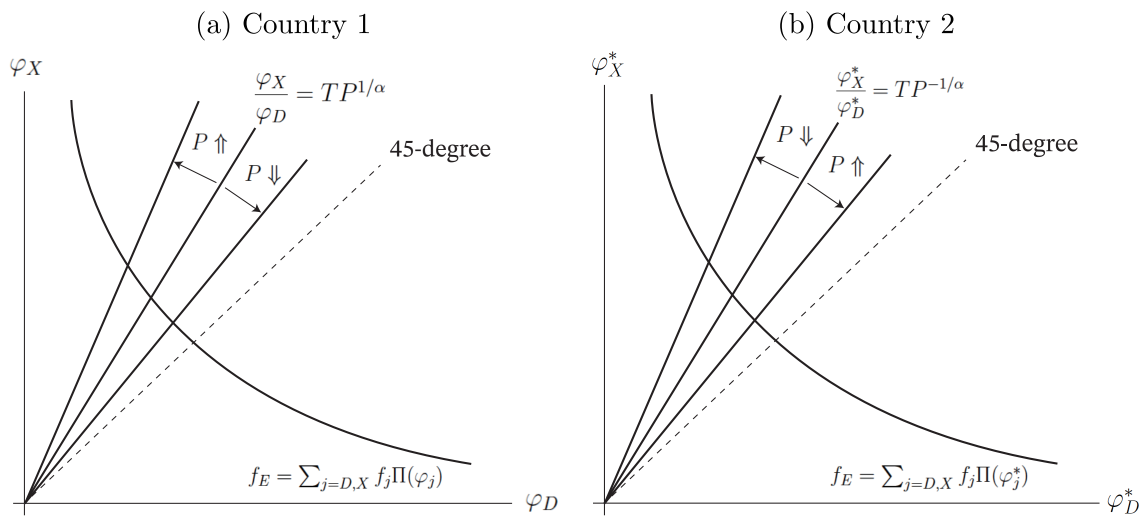

Figure 1: Intermediate-good firms' cutoffs given P

As is apparent from the demand functions for intermediate goods, the final-good price in each country acts as a demand shifter. Since P^* is normalized to unity, the price in country 1, P, represents country 1's relative market size for the intermediate goods. Thus, the final-good price P affects the intermediate-good firms' productivity cutoffs. In fact, $\varphi_D^{(*)}$ and $\varphi_X^{(*)}$ are related as

$$\frac{\varphi_{X,t}}{\varphi_{D,t}} = TP_t^{1/\alpha}, \quad \frac{\varphi_{X,t}^*}{\varphi_{D,t}^*} = TP_t^{-1/\alpha}. \tag{9}$$

Derivation of (9) is given in Appendix A.2. The higher the price of the final good in country 1, the greater the demand for intermediate goods in that country. For intermediate-good firms in country 1 (2), this implies a relatively greater demand in the domestic (foreign) market, in turn leading to an increase (decrease) in φ_X/φ_D (φ_X^*/φ_D^*).

Given P, the intermediate-good firms' cutoffs are determined from (1) and (9). In Figure 1, the downward-sloping curve represents the free entry condition (1), while the upward-sloping line represents (9). The following lemma follows directly from the figure:

Lemma 1. *1. Given the final-good price, P, the productivity cutoffs for the intermediate-good firms are uniquely determined;*

2. $\varphi_D'(P) < 0$, $\varphi_X'(P) > 0$, ${\varphi_D^*}'(P) > 0$, ${\varphi_X^*}'(P) < 0$; and

3. If $P = 1$, $\varphi_j = \varphi_j^*$ ($j = D, X$).

Thus, φ_D and φ_X^* respond in the same direction to shocks. Demidova (2008) shows this co-movement in a static model analyzing the effects of a unilateral increase in labor productivity on productivity cutoffs. In contrast, P is endogenous in our model, evolving in a dynamic environment.

Following the literature building on heterogeneous-firm trade models, we focus on the situation where not all operating firms can export: $\varphi_X^{(*)} > \varphi_D^{(*)}$. We therefore assume that T is sufficiently large such that $T^{-\alpha} \leq P \leq T^{\alpha}$ holds in equilibrium. Using Lemma 1, we can express the real price of the knowledge good as a function of P (see Appendix A.2 for derivation):

$$q_t^{(*)} = q^{(*)}(P_t) \equiv \frac{\alpha^{\alpha/(1-\alpha)}(1-\alpha)L}{f_D} \left(\varphi_D^{(*)}(P_t)\right)^{\alpha/(1-\alpha)}; \quad q'(P) < 0, \quad q^{*\prime}(P) > 0, \tag{10}$$

3.3 Equilibrium with financial autarky

When global financial linkages are absent, the bond market equilibrium is given by (6). From this equation, the entrepreneurs' productivity cutoff is given by

$$\bar{z}_t^{(*)} = \bar{z}(\theta^{(*)}) \equiv F^{-1}(1 - 1/\theta^{(*)}); \quad \bar{z}'(\cdot) = \frac{1 - F(\bar{z})}{\theta^{(*)}F'(\bar{z})} > 0. \tag{11}$$

Thus, in the absence of international financial transactions, cross-country differences in the productivity cutoffs arise solely from differences in the degree of financial friction.

We derive the dynamic system of the economy. Let a denote the relative wealth of country 1 to country 2:

$$a_t \equiv A_t / A_t^*.$$

Substituting (10) and (11) into the Euler equations (3) and (4), the dynamic equation for relative assets is given by

$$\frac{\dot{a}_t}{a_t} = q(P_t)\bar{z}(\theta)\Big(1 + \theta\Psi(\bar{z}(\theta))\Big) - q^*(P_t)\bar{z}(\theta^*)\Big(1 + \theta^*\Psi(\bar{z}(\theta^*))\Big). \tag{12}$$

As (5) and (6) show, international trade is always balanced under financial autarky. In Appendix A.2, we show that $EX = IM$ can be rewritten as

$$a = \Gamma(P) \equiv \frac{\mu^*(P)q^*(P)}{\mu(P)q(P)P} \frac{\theta^* \int_{z \geq \bar{z}(\theta^*)} z dF(z)}{\theta \int_{z \geq \bar{z}(\theta)} z dF(z)}, \tag{13}$$

where $\mu^{(*)}(P)$ denotes the share of export sales in total sales for country 1's (2's) intermediate-good firms (see Appendix A.2 for derivation):

$$\mu^{(*)}(P) \equiv \frac{f_X H(\varphi_X^{(*)}(P))}{\sum_{j=D,X} f_j H(\varphi_j^{(*)}(P))}; \quad \mu'(P) < 0, \ \mu^{*\prime}(P) > 0,$$
$$H(\varphi_j) \equiv \int_{\varphi \geq \varphi_j} (\varphi/\varphi_j)^{\alpha/(1-\alpha)} dG(\varphi),$$

The relative wealth $a \equiv A/A^*$ is the state variable, whose initial value is historically given. Under financial autarky, at each point in time, the final-good price P is determined by (13) given a, which in turn governs the dynamics of a through (12). Equations (12) and (13) therefore jointly

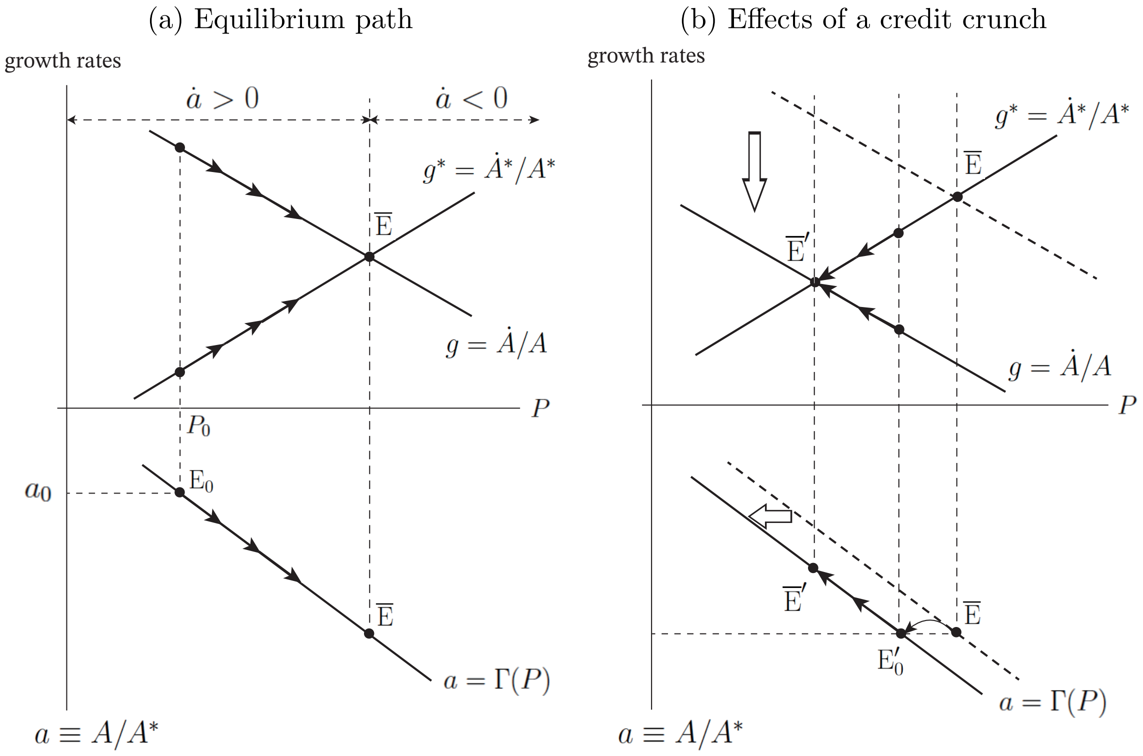

Figure 2: Dynamics under financial autarky

constitute an autonomous dynamic system. Once the time paths of a_t and P_t are determined, the paths of all other variables are accordingly determined.

Panel (a) of Figure 2 depicts the dynamic behavior of the economy, where the sign of the slope of $\dot{A}^{(*)}/A^{(*)}$ follows from $q^{(*)'}(P) < (>) 0$. Given the initial condition a_0, the final-good price P_0 jumps to point E_0 so as to satisfy (13). As the figure shows, there uniquely exists the equilibrium path converging to the steady state (point $\overline{\mathrm{E}}$) if and only if the function Γ is increasing. Accordingly, we proceed under the assumption that $\Gamma'(P) > 0$. In Appendix A.2, we show that this condition indeed holds when φ follows a Pareto distribution.

Now we examine how a credit crunch in country 1 affects international trade and economic growth in both countries. Following Buera and Moll (2015), we model the credit crunch as the tightening of the credit constraint. We focus on a permanent decrease in θ, while we discuss the case of transitory shocks in Section 4.2. We can analyze both the short- and long-run effects of a permanent credit crunch using Panel (b) of Figure 2. As θ decreases, the $\dot{A}_t/A_t$ curve in the top quadrant shifts downward, and the Γ curve in the (a, P) plane also shifts downward.[10]

Suppose that the world economy is initially in the old steady state (point $\overline{\mathrm{E}}$) and that θ falls permanently. As an initial response, the following two phenomena occur in country 1. First, a reduction in the borrowing limit directly lowers the rate of return on knowledge-good production

[10] These shifts are shown in Appendix A.2.

for active entrepreneurs, which in turn hinders their asset accumulation. This is the direct effect, reflected in the downward shift of the $\dot{A}_t/A_t$ curve. Second, the credit crunch also hampers knowledge-good production through an inefficient reallocation of financial resources from more productive to less productive entrepreneurs.

To see the second phenomenon more clearly, consider knowledge-good production at the instant the credit crunch occurs:

$$Y_{K,0} = \underbrace{A_0}_{\text{fixed}} \underbrace{\theta(1 - F(\widetilde{z}_0))}_{=1} \underbrace{\frac{\int_{z \geq \widetilde{z}_0} z dF(z)}{1 - F(\widetilde{z}_0)}}_{\text{Average productivity}},$$

where $\widetilde{z}_0 = \overline{z}(\theta)$. The first term, A_0, is predetermined. The second term captures aggregate leverage, since each entrepreneur leverages up to θ and the measure of operating entrepreneurs is $1 - F(\widetilde{z})$. This term is also fixed at unity by the asset market equilibrium (6). Finally, the third term captures average productivity in knowledge-good production, which declines because the tightening of credit constraints lowers the productivity cutoff and allows less efficient entrepreneurs to operate.

These phenomena in country 1 affect the subsequent growth performance of both countries via international trade in the following way. Let us focus on the bottom quadrant of Panel (b). As discussed above, since knowledge-good production stagnates in country 1, entry of intermediate-good firms is also dampened in that country, leading to a decline in its export volume. However, as implied by (13), trade must remain balanced, and hence exports from country 2 must also decrease. Consequently, the final-good price P, which serves as a demand-shifter for the intermediate goods, jumps to point E_0' to reduce the profitability of the export market for country 2. This intuition explains the shift in the Γ curve and the initial decline in the final-good price.

The economy evolves along the path as indicated by the arrows in the figure and eventually converges to a new steady state (point $\overline{\mathrm{E}}'$). Country 1's growth rate gradually recovers over time, although it remains below its initial level. The intuition for this recovery is as follows. Since knowledge-good production stagnates in country 1, this good becomes increasingly scarce, which drives up its real price, q. Note that while the decline in θ and the associated fall in the cutoff $\widetilde{z}$ occur instantaneously and remain unchanged thereafter, the increase in q takes place gradually over time.

In contrast, country 2's growth rate continues to decline over time. The reason is that international trade induces inefficient firm selection in its intermediate goods sector, as explained below. As shown in the figure, along the transition path, P continues to fall. This reduces the profitability of the export market for country 2, thereby raising the export cutoff, φ_X^*. At the same time, however, it lowers the operating cutoff, φ_D^* (by the free-entry condition (1)), allowing less efficient firms to remain active. By the zero-cutoff profit condition, the fixed cost $q^* f_D$ must adjust downward for inefficient firms to operate.[11] Since fixed input requirements are exogenously given, this adjustment occurs through a decline in the real price of the knowledge good, q^* (see (10)). This lowers the rate of return on knowledge-good production and thereby slows down asset accumulation in country 2. This process continues until the economy reaches the new steady state.

Panel (b) of Figure 2 illustrates the case where the initial jump of P is not very large. Therefore, in this case, the following proposition holds based on the foregoing analysis.

Proposition 1. *Suppose that a permanent credit crunch occurs in country 1. Then,*

1. *The balanced growth rate declines;*

2. *The growth rates of both countries decline at all points in time, if the required adjustment in P is sufficiently small.*

This result is partly consistent with Japan's experience during the 2007–2009 financial crisis. Kawai and Takagi (2011) argue that the collapse of Japanese exports primarily reflected the sharp contraction in global demand after the Lehman shock.

Under what circumstances does the international spillover effect become stronger? Recall that, under financial autarky, only the change in q^* matters for country 2's growth rate. Appendix A.2 shows that when α is large (the elasticity of substitution is high), q^* responds more strongly to the change in P.[12] In theory, the initial jump in P may overshoot $\overline{\mathrm{E}}'$. Note that even in this case, the growth rate in country 2, which does not experience a credit crunch, is lower at every point in time than in the old steady state. This is because the $\dot{A}^*/A^*$ curve remains unchanged, and when P overshoots $\overline{\mathrm{E}}'$, the economy moves toward a new steady state from the left of that point.

Since our primary interest lies in international spillovers through trade, we focus on the case in which a credit crunch occurs only in country 1. However, using Figure 2, it is straightforward to analyze the case in which both countries experience a credit crunch. When the credit crunch occurs in both countries, the $\dot{A}^*/A^*$ curve also shifts downward. As a result, the decline in the growth rate becomes stronger. At the same time, the adjustment in P is weaker, implying that the reduction in growth in both countries is driven more by the direct effects of the credit crunch than by spillovers through trade.

[11] Recall that $P^* = 1$ and hence $P_K^* = q^*$

[12] Although this may appear obvious from equation (10), it is in fact not trivial, since φ_D^* also depends on α.

3.4 Equilibrium with financial integration

As in the case of financial autarky, let a denote the relative wealth of country 1 to country 2. In addition, let $b^{(*)}$ denote debt relative to aggregate assets:

$$b_t^{(*)} \equiv B_t^{(*)}/A_t^{(*)}.$$

From the definition of $B^{(*)}$, the entrepreneurs' cutoff satisfies

$$\widetilde{z}_t^{(*)} = F^{-1}[1-(1+b_t^{(*)})/\theta^{(*)}]. \tag{14}$$

The bond market equilibrium under financial integration (7) becomes

$$b_t^* = -P_t a_t b_t. \tag{15}$$

The autonomous dynamic system is therefore characterized by (14)–(15) and

$$\dot{a}_t/a_t = q(P_t)\widetilde{z}_t\Big(1+\theta\Psi(\widetilde{z}_t)\Big) - q^*(P_t)\widetilde{z}_t^*\Big(1+\theta^*\Psi(\widetilde{z}_t^*)\Big), \tag{16}$$

$$\begin{aligned}\dot{b}_t &= \Big(\rho - \theta q(P_t)\widetilde{z}_t\Psi(\widetilde{z}_t)\Big)b_t \\ &\quad - \frac{1}{1-\alpha}\left(\theta\mu(P_t)q(P_t)\int_{z\geq\widetilde{z}_t} zdF(z) - \theta^*\frac{\mu^*(P_t)q^*(P_t)}{P_t a_t}\int_{z\geq\widetilde{z}_t^*} zdF(z)\right),\end{aligned} \tag{17}$$

$$\dot{P}_t/P_t = q^*(P_t)\widetilde{z}_t^* - q(P_t)\widetilde{z}_t. \tag{18}$$

Equations (16) and (18) follow from the definition of a and interest rate parity, while the derivation of (17) is given in Appendix A.2. In general, the system (14)–(18) does not admit a closed-form solution. In the symmetric case $\theta^* = \theta \equiv \overline{\theta}$, the steady state is given by $a=1$, $b=0$, $P=1$, and $\widetilde{z}^{(*)} = \overline{z}(\overline{\theta}) \equiv F^{-1}(1-1/\overline{\theta})$.[13]

Suppose the economy is initially at the symmetric steady state and θ falls unilaterally, while θ^* remains unchanged. Unlike under financial autarky, the credit crunch in country 1 now affects the entrepreneurs' cutoff in country 2. From (16) and (18), $\theta\Psi(\widetilde{z}) = \theta^*\Psi(\widetilde{z}^*)$ holds in the steady state. Totally differentiating at the initial steady state $\widetilde{z}^{(*)} = \overline{z}(\overline{\theta})$ yields

$$d\widetilde{z} - d\widetilde{z}^* = -\frac{\Psi(\overline{z})}{\overline{\theta}\Psi'(\overline{z})}d\theta.$$

Thus, the credit crunch generates asymmetric effects on entry across countries. Using (14) and (15) yields:[14]

$$d\widetilde{z} + d\widetilde{z}^* = \overline{z}'(\overline{\theta})d\theta,$$

[13] Substituting $\theta^* = \theta$ and $q^*\widetilde{z}^* = q\widetilde{z}$ (from (18)) into (16) leads to $\widetilde{z}^* = \widetilde{z}$. Then, $b^* = b$ holds in the steady state. For this result to be consistent with (15), $b^* = b = 0$ must be satisfied. The result of $q^* = q$ leads to $P = 1$. Substituting these results into (17), we obtain $a = 1$.

[14] To obtain this, we use the facts that in the old steady state, $a = 1$, $b = 0$, $P = 1$, and $(1 - F(\overline{z}(\overline{\theta}))) = 1/\overline{\theta}$ hold.

Table 1: Choices of parameters

Parameter	Description	Value	Target/Reference
ρ	Discount rate	0.01	Standard
δ	Depreciation rate	0.025	Standard
α	Reciprocal of gross markup	0.75	Balistreri et al. (2011), $(1-\alpha)^{-1}=4$
κ	Shape parameter of $G(\varphi)$	4.5	Balistreri et al. (2011), midpoint of 3.924–5.171
$\overline{\theta}$	Financial frictions (benchmark)	3.226	Buera and Nicolini (2020), $1-1/\overline{\theta}=0.69$
γ	Shape parameter of $F(z)$	3	Exogenously chosen
f_D	Fixed input (D)	10	Normalization (so that $\varphi_D>1$)
f_X	Fixed input (X)	4.197	Bernard et al. (2003), Exporter share = 0.21
f_E	Fixed input (entry)	5.213	$r^b=r^{b*}=0.015$
τ	Iceberg costs	1.889	Import penetration ratio = 0.081
β	Upper bound of z	4.175	$g=g^*=0.02$

where $\overline{z}'>0$ is the cutoff response under financial autarky. Financial integration therefore also induces symmetric effects. A unilateral tightening reduces domestic borrowing demand, raising global excess supply of funds; equilibrium allows the entry of less productive entrepreneurs in both countries. From these equations, it follows that $\widetilde{z}$ necessarily decreases while $\widetilde{z}^*$ also decreases if and only if $\overline{z}'(\overline{\theta})$ is sufficiently large that $\overline{z}'(\overline{\theta})>-\frac{\Psi(\overline{z})}{\overline{\theta}\Psi'(\overline{z})}$.

As in financial autarky, the balanced growth rate is affected through changes in P. Differentiating $q(P)\widetilde{z}=q^*(P)\widetilde{z}^*$ at the old steady state yields $\left(q'(1)-q^{*\prime}(1)\right)\overline{z}dP=-q(1)(d\widetilde{z}-d\widetilde{z}^*)$. Eliminating $d\widetilde{z}-d\widetilde{z}^*$, we obtain

$$\frac{dP}{d\theta}\Big|_{\theta^*=\theta=\overline{\theta}}=\frac{q(1)\Psi(\overline{z})}{\overline{\theta}\overline{z}\Psi'(\overline{z})(q'(1)-q^{*\prime}(1))}>0.$$

Thus, for the change in P, we obtain the same qualitative results as in the financial autarky case. This also means that the impact on the balanced growth rate is qualitatively the same as in the financial autarky case.

Proposition 2. *Consider the steady state with financial integration and $\theta^*=\theta=\overline{\theta}$ initially. The balanced growth rate declines with a permanent credit crunch in country 1.*

4 Numerical analysis

The result in Proposition 2 is analytically derived locally around the symmetric steady state. Moreover, it is important to provide a quantitative assessment of how the magnitude of international spillovers differs between financial autarky and financial integration. Therefore, this section turns to numerical analysis. We specify $G(\varphi)$ as $G(\varphi)\equiv 1-\varphi^{-\kappa}$ with $\varphi\geq 1$ and

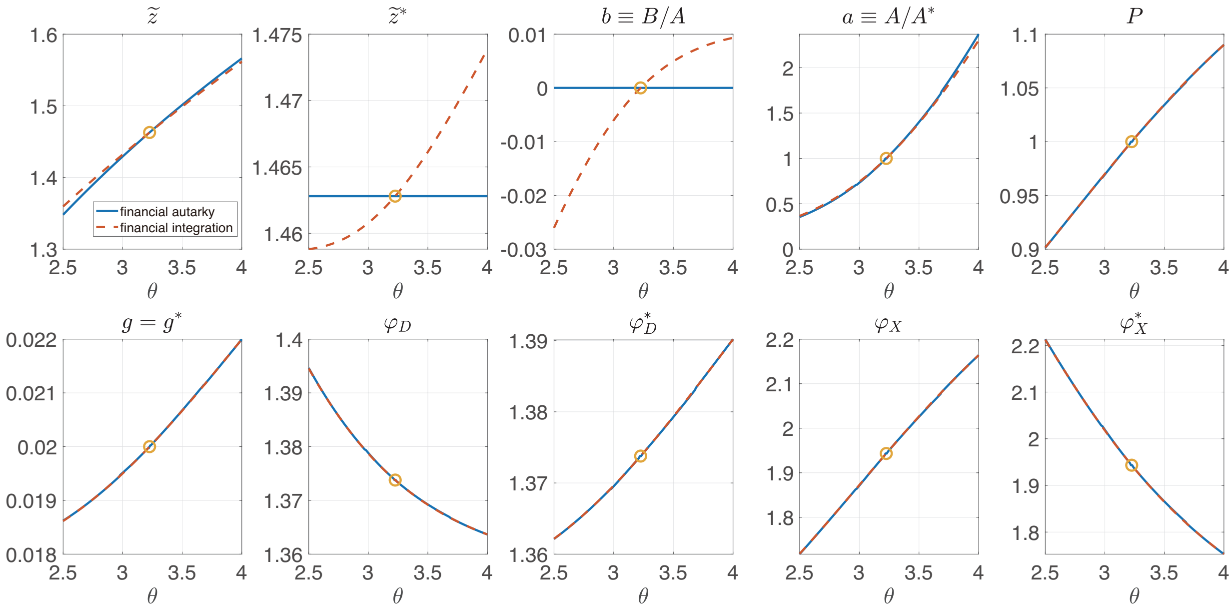

Figure 3: Long-run effects of a permanent decrease in θ on the steady state

$\kappa > \max\{\alpha/(1-\alpha), 1\}$. In addition, we assume that the entrepreneurs' productivity z satisfies $z \in [1, \beta]$ and follows an upper-truncated Pareto distribution: $F(z) \equiv (1-z^{-\gamma})/(1-\beta^{-\gamma})$, where $\gamma > 1$ is the shape parameter.[15] We normalize the population to unity: $L^* = L = 1$. When setting the baseline degree of financial frictions, we assume that they are symmetric between the two countries; that is, $\theta^* = \theta = \overline{\theta}$. Table 1 shows the calibration results, with details given in Appendix A.3. The implied degree of trade barriers, $T \equiv \tau(f_X/f_D)^{(1-\alpha)/\alpha}$, is 1.414.

4.1 Long-run effects

Figure 3 illustrates the long-run effects on the key variables of a unilateral change in θ. In each panel, a circle marker indicates the benchmark value. The blue solid and red dashed lines represent the changes in the variable under financial autarky and financial integration, respectively.

As already confirmed, a qualitative difference in the effects between financial autarky and financial integration is that, in the latter case, credit tightening affects the entrepreneurs' cutoff in the other country through the asset market. As Panel 2 shows, under the calibrated parameters, financial integration induces less productive entrepreneurs to operate in country 2. Accordingly, as shown in Panel 3, country 1 becomes a creditor country due to the increased demand for financial funds in country 2. These effects do not arise under financial autarky, as indicated by the blue solid lines in these panels.

[15]As shown in Appendix A.3, if z follows a standard Pareto distribution ($\beta \to \infty$), b and b^* are always zero, and financial autarky arises although there are international financial transactions. To avoid such a situation, we assume an upper bound for z.

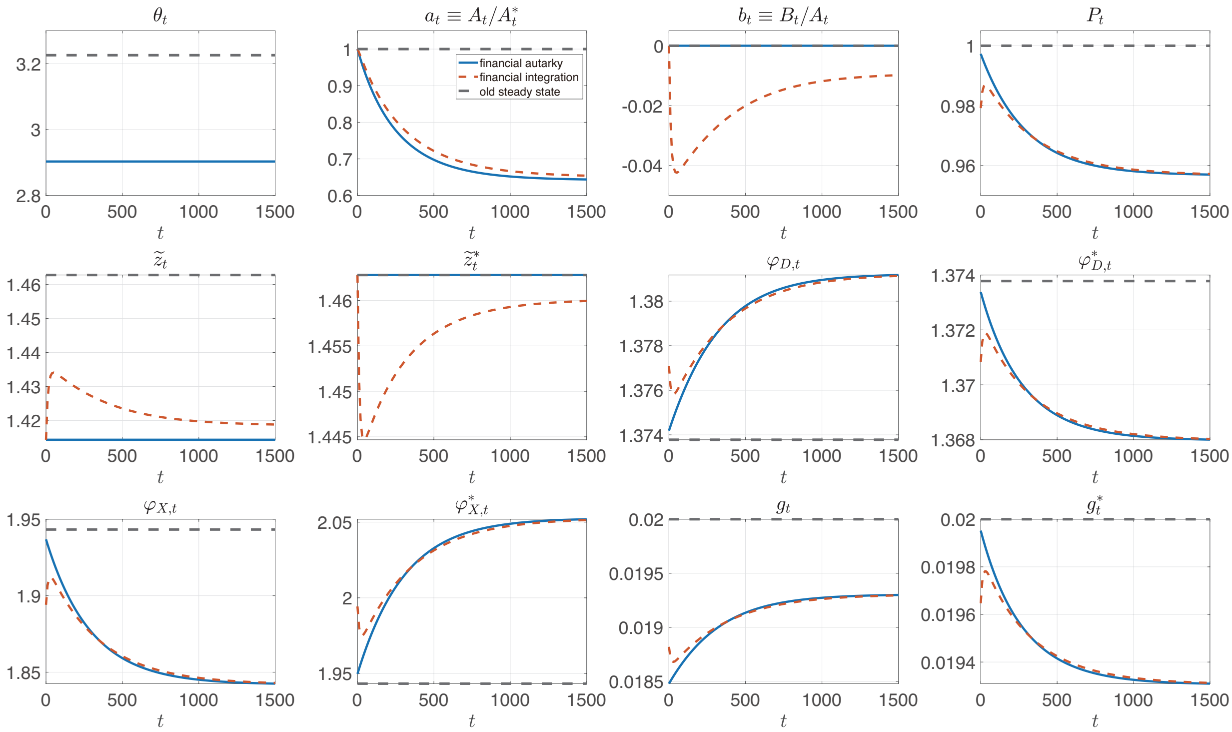

Figure 4: Dynamics to the new steady state following a permanent decrease in θ

However, Panel 2 also shows that the change in $\widetilde{z}^*$ is not very large even under financial integration. Therefore, as shown in Panels 4 and 5, the changes in relative assets a and the final-good price P are remarkably similar in the two cases. Consequently, the change in the long-run growth rate shown in Panel 6, as well as the changes in the productivity cutoffs of intermediate-good firms shown in Panels 7–10, are almost identical. International asset transactions are typically viewed as a key channel for the global propagation of financial shocks. However, the results obtained here imply that even if the two countries cease international financial transactions, trade in goods still facilitates sufficiently strong international propagation of a country-specific financial shock.

4.2 Transitional dynamics

Figure 4 plots the transition paths of the key variables toward the new steady state when θ permanently falls by 10% from $\overline{\theta}$. We compute the transition dynamics using the relaxation algorithm of Trimborn et al. (2008). In all panels, the black dashed line represents the initial value at the symmetric steady state. The behavior of relative assets a_t is quite similar between the two cases, although the decline is slightly milder under financial integration. Country 1's debt-to-assets ratio b_t declines at all points in time under financial integration, and the direction of this change is consistent with its long-run effect. However, this adjustment in the external asset position is not large enough to generate a substantial change in the entrepreneurial cutoff $\widetilde{z}_t^{(*)}$. Moreover, the final-good price P_t falls in both cases and converges to a similar level. Although

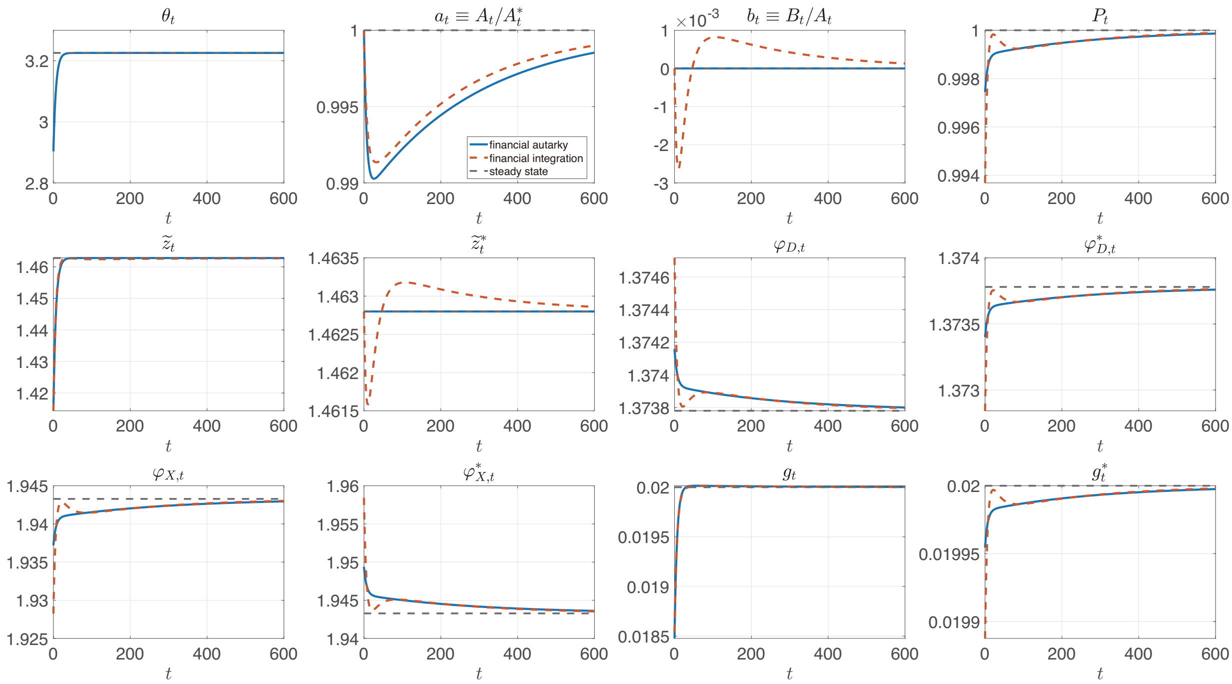

Figure 5: Responses to a temporary decrease in θ

there is a slight difference in the initial phase under financial integration, this gap quickly narrows and remains small thereafter. Taken together, these results imply that the movements of the intermediate-good firms' cutoffs and the growth rates are very similar under financial autarky and financial integration, except during the initial transition period. The last two panels confirm that the dynamics of the growth rates are consistent with those depicted in Panel (b) of Figure 2.

Many studies that incorporate endogenous growth mechanisms into closed-economy business cycle models assume temporary financial shocks. Accordingly, we make a similar assumption here. Specifically, we assume that θ_0 unanticipatedly decreases by 10% from $\overline{\theta}$ and that θ_t gradually returns to its baseline value according to

$$\dot{\theta}_t = -\varrho(\theta_t - \overline{\theta}),$$

where $\varrho > 0$ is a parameter specifying the persistence of the shock. We set $\varrho = 0.15$. Figure 5 presents the responses to a temporary shock.[16] As in the case of a permanent shock, financial integration generates movements in b_t, which in turn affect $\widetilde{z}_t^*$, as shown in Panel 6. However, the magnitude of this effect remains quantitatively small. Consequently, the similarity between financial autarky and financial integration documented under a permanent shock persists even when the shock is temporary.

[16]We calculate the responses by using linear approximations of the dynamic system.

5 Discussion

5.1 On the non-tradability of the final and knowledge goods

In this study, we focus on trade in intermediate goods, abstracting from trade in the final and knowledge goods. In Appendix A.4, we show that if either the final good or the knowledge good is additionally tradable, the two-country economy fails to admit a steady-state equilibrium unless the symmetric condition $\theta^* = \theta = \overline{\theta}$ is always imposed. Thus, the non-tradability assumption of these goods is not just for simplicity. This structure is the minimal structure required to sustain cross-country heterogeneity in financial conditions in a growing equilibrium.

Nevertheless, it is important to examine how this assumption affects our main results. As discussed in Section 3.3, inefficient firm selection arises in country 2's intermediate goods sector, precisely because the final good is non-tradable and its relative price P adjusts endogenously away from 1. If P were fixed, the relative market size of intermediate goods would remain unchanged after the credit crunch, and nothing would occur in country 2. In this sense, the non-tradability of the final good is important for the main results.

Moreover, the assumption that the knowledge good is non-tradable is crucial for obtaining the result that distorted firm selection in country 2 lowers its growth rate. As shown in Section 3.3, allowing less productive intermediate-good firms to operate in country 2 implies a decline in the fixed input cost. This, in turn, reduces the price of the knowledge good. However, if the knowledge good were also importable from country 1, such a decline in its price would not necessarily occur. The non-tradability of the knowledge good can be interpreted as capturing the local nature of R&D activities. Knowledge creation is often embedded in language, institutions, legal systems, and financial relationships, making it far less internationally mobile than manufacturing intermediates. Our assumption therefore reflects limited international integration in intangible capital markets rather than a purely technical restriction.

5.2 Effects under alternative calibration scenarios

In the previous section, we examined the transmission of financial shocks through international trade—more specifically, through firm selection in the tradable goods sector—and their impact on the growth rates of both countries. The following three parameters are particularly important for the sensitivity analysis. The first parameter is α, which governs the elasticity of substitution, or equivalently, the reciprocal of the gross markup. As discussed in Section 3.3, a large value of α can amplify the international spillovers through trade. Conversely, when α is small, this channel becomes weaker, potentially making the effects of financial market integration relatively more important. To confirm this possibility, in Appendix A.4 we recalibrate the model by lowering

α to 0.5, thereby reducing the elasticity of substitution from 4 to 2. We then examine how this alternative calibration affects the long-run consequences of the credit crunch. Although the negative growth effect of the credit crunch is somewhat attenuated, the results indicate that the international transmission remains quantitatively similar under financial autarky and financial integration.

The second key parameter is κ, which determines the shape of the productivity distribution of the intermediate-good firms, G. As this parameter increases, intermediate-good firms become more homogeneous, weakening the selection effect. In that case, the trade channel may become less operative, potentially increasing the relative importance of financial market integration. In Appendix A.4, we recalibrate the model by changing the value of κ from 4.5 to 8. A similar pattern emerges under this alternative calibration: although the decline in growth is mitigated, the magnitude of international transmission remains quantitatively similar across financial autarky and financial integration.

The third key parameter is γ, which determines the shape of the productivity distribution of entrepreneurs, F. Since productivity dispersion is likely to be larger among entrepreneurs than among intermediate-good firms, we set the benchmark value of this parameter to 3. As discussed earlier, under financial integration, a credit crunch in country 1 affects the entrepreneurs' cutoff in country 2. The magnitude of this effect is expected to be larger when γ is smaller, reflecting greater dispersion in entrepreneurial productivity. We recalibrate the model by setting $\gamma = 1.5$, thereby generating a thicker right tail of the distribution, and reassess the international transmission. Under this third scenario, the responses of the entrepreneurial cutoff $\widetilde{z}^{(*)}$ and country 1's external debt position b are amplified. However, as in the first two scenarios, the magnitude of international transmission remains quantitatively similar across financial autarky and financial integration.

In each alternative calibration, we alter one parameter at a time while keeping all other parameters at their benchmark values.

6 Concluding remarks

Our findings reveal a channel through which credit shocks can propagate internationally, even in the absence of cross-border financial transactions, namely, via trade-induced firm selection. One of the advantages of this study over previous research lies in the tractability of the proposed model. Specifically, the model developed here enables us to analytically examine both the short- and long-run effects of country-specific shocks. Extending the model in a more quantitative direction represents a promising avenue for further research. First, in the case of a temporary credit crunch, the decline in the growth rate of country 2, which is not directly affected by the shock, is quantitatively modest. Therefore, to obtain stronger quantitative results, it will be necessary to incorporate mechanisms into the model through which shocks originating abroad are amplified domestically in country 2. Second, it would be beneficial to assume that the productivity of each economic agent exhibits some degree of persistence rather than being determined solely by iid shocks. Third, it is important to examine cases in which not only entrepreneurs but also other economic agents face credit constraints. These extensions are essential for quantitatively assessing the international transmission of financial shocks through trade and financial linkages. Nonetheless, the results of this study are expected to serve as an important benchmark for such work.

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Appendix

A.1 Proof of key results in Section 2

Derivation of (1)

Using the zero-cutoff profit condition: $\pi_j(\varphi_j) = 0$, we can write the output of a firm whose productivity is just the cutoff productivity, φ_j, as

$$y_j(\varphi_j) = \frac{\alpha f_j}{1-\alpha}\varphi_j q. \tag{A.1}$$

Since $y_X(\varphi) = \tau x^*(\varphi)$ and $\tau p_X(\varphi) = p^*(\varphi)$, the demand functions for the intermediate goods imply

$$y_D(\varphi) = p_D(\varphi)^{-1/(1-\alpha)} L P^{1/(1-\alpha)}, \quad \frac{y_X(\varphi)}{\tau} = (\tau p_X(\varphi))^{-1/(1-\alpha)} L^* P^{*1/(1-\alpha)}. \tag{A.2}$$

Using (A.2) and $p_j(\varphi) = P/(\alpha\varphi)$, we can express $y_j(\varphi)$ for any φ as

$$y_j(\varphi) = (\varphi/\varphi_j)^{1/(1-\alpha)} y_j(\varphi_j), \tag{A.3}$$

which is a well-known result of the Melitz model. Using (A.1) and (A.3), $\pi_j(\varphi)$ can be rewritten as

$$\pi_j(\varphi) = P_K f_j \left[(\varphi/\varphi_j)^{\alpha/(1-\alpha)} - 1\right]. \tag{A.4}$$

The free entry condition is $P_K f_E = \sum_{j=D,X} \int_{\varphi \geq \varphi_j} \pi_j(\varphi) dG(\varphi)$. Substituting (A.4) into the right-hand side of the free entry condition, we obtain (1).

A.1.1 Entrepreneurs' optimization

Using the definition of net worth: $A_i \equiv K_i - B_i$, we can rewrite the entrepreneur i's budget and credit constraints as

$$dA_{it} = \left[r_t^b A_{it} + \left(q_t z_{it} - \delta - r_t^b\right) K_{it} - C_{it}^e\right] dt,$$
$$K_{it} \leq \theta A_{it}.$$

The entrepreneur i's optimization problem is decomposed into two stages. In the first stage, at each point in time, the entrepreneur determines K_i to maximize the net profit, $(qz_i - \delta - r^b)K_i$, subject to the credit constraint while taking (A_i, z_i) as given. The optimal choice for K_i is obtained as described in the main body. We can rewrite the budget constraint as $dA_{it} = [R_t(z_{it})A_{it} - C_{it}^e]\, dt$, where

$$R_t(z_i) = \begin{cases} r_t^b & \text{if } z_i < \tilde{z}_t, \\ r_t^b + \theta q_t (z_i - \tilde{z}_t) & \text{if } z_i \geq \tilde{z}_t. \end{cases}$$

In the second stage, the entrepreneur chooses the time paths of consumption and net worth to maximize the lifetime utility subject to the budget constraint. The problem is rewritten as the following Hamilton–Jacobi–Bellman (HJB) equation:

$$\rho U_t(A_i, z_i)dt = \max\left\{(\log C_i^e)dt + \mathbb{E}_t\left[dU_t(A_i, z_i)\right] \;:\; dA_i = \left(R_t(z_i)A_i - C_i^e\right)dt\right\},$$

where U_t is the value function. We can solve this problem using "guess and verify." We guess that the value function takes the form $U_t(A_i, z_i) = \zeta(\phi_t(z_i) + \log A_i)$ with ζ and $\phi_t(z_i)$ being unknown. Using this guess leads to $\mathbb{E}_t[dU_t(A_i, z_i)] = \zeta\left(\mathbb{E}_t[d\phi_t(z_i)] + dA_i/A_i\right)$. The right-hand side of the HJB equation is rewritten as

$$\max_{C_i^e}\ \log C_i^e + \frac{\zeta(R_t(z_i)A_i - C_i^e)}{A_i} + \zeta\frac{1}{dt}\mathbb{E}_t\left[d\phi_t(z_i)\right].$$

Given ζ, the first-order condition is given by $1/C_i^e = \zeta/A_i$. This result must be consistent with the HJB equation. Substituting this result back into the HJB equation, we obtain

$$\rho\zeta(\phi_t(z_i) + \log A_i) = \log A_i - \log\zeta + \zeta R_t(z_i) - 1 + \zeta\frac{1}{dt}\mathbb{E}_t\left[d\phi_t(z_i)\right],$$

the equality of which must be satisfied for all A_i. This implies $\zeta = 1/\rho$, which, in turn, yields $C_i^e = \rho A_i$. The dynamics of the net worth are given by $dA_{it} = \left(R_t(z_{it}) - \rho\right)A_{it}dt$.

A.1.2 Aggregation and Derivation of (3)

We define dA_t as $dA_t \equiv \int dA_{it}di$. Using $dA_{it} = (R_t(z_{it}) - \rho)A_{it}dt$ and the fact that A_i and z_i are independent of each other, we obtain

$$dA_t = A_t\left[r_t^b + \theta q_t\left(\int_{z\geq\widetilde{z}_t}(z - \widetilde{z}_t)dF(z)\right) - \rho\right]dt.$$

The right-hand side of the equation does not contain any stochastic components. Using $r_t^b + \delta = q_t\widetilde{z}_t$, we can obtain the dynamic equation of A as Equation (3).

A.2 Proof of key results in Section 3

A.2.1 Derivations of (9)–(10)

From (A.2) with $\varphi = \varphi_j$,

$$y_D(\varphi_D) = \left(\frac{P}{\alpha\varphi_D}\right)^{-1/(1-\alpha)}LP^{1/(1-\alpha)}, \quad \frac{y_X(\varphi_X)}{\tau} = \left(\frac{\tau P}{\alpha\varphi_X}\right)^{-1/(1-\alpha)}L, \tag{A.5}$$

where we use $L^* = L$ and $P^* = 1$. Substituting (A.1) into (A.5) to eliminate $y_j(\varphi_j)$ and arranging the resulting equations, we obtain

$$q = \frac{\alpha^{\alpha/(1-\alpha)}(1-\alpha)L}{f_D}\varphi_D^{\alpha/(1-\alpha)} = \frac{\alpha^{\alpha/(1-\alpha)}(1-\alpha)L}{f_X}\tau^{-\alpha/(1-\alpha)}P^{-1/(1-\alpha)}\varphi_X^{\alpha/(1-\alpha)}. \tag{A.6}$$

Using the second equation of (A.6), we obtain

$$\varphi_X = \tau \left(\frac{f_X}{f_D} \right)^{(1-\alpha)/\alpha} P^{1/\alpha} \varphi_D,$$

which is the first part of (9).

In country 2, the equations corresponding to (A.5) are given by

$$y_D^*(\varphi_D^*) = \left(\frac{1}{\alpha \varphi_D^*} \right)^{-1/(1-\alpha)} L, \qquad \frac{y_X^*(\varphi_X^*)}{\tau} = \left(\frac{\tau}{\alpha \varphi_X^*} \right)^{-1/(1-\alpha)} L P^{1/(1-\alpha)}. \tag{A.7}$$

Following the same calculation procedure,

$$q^* = \frac{\alpha^{\alpha/(1-\alpha)}(1-\alpha)L}{f_D} {\varphi_D^*}^{\alpha/(1-\alpha)} = \frac{\alpha^{\alpha/(1-\alpha)}(1-\alpha)L}{f_X} \tau^{-\alpha/(1-\alpha)} P^{1/(1-\alpha)} {\varphi_X^*}^{\alpha/(1-\alpha)}. \tag{A.8}$$

Using the second equation of (A.8), we obtain

$$\varphi_X^* = \tau \left(\frac{f_X}{f_D} \right)^{(1-\alpha)/\alpha} P^{-1/\alpha} \varphi_D^*,$$

which is the second part of (9).

Once $\varphi_j^{(*)}$ is given by a function of P, $\varphi_j^{(*)}(P)$, the real price of the knowledge good $q^{(*)}$ is also given by a function of P from the first equation of (A.6) and (A.8):

$$q^{(*)} = q^{(*)}(P) \equiv \frac{\alpha^{\alpha/(1-\alpha)}(1-\alpha)L}{f_D} \varphi_D^{(*)}(P)^{\alpha/(1-\alpha)},$$

which is Equation (10) in Section 3.2.

A.2.2 Autarky equilibrium

We focus on country 1 and choose the final good as the numeraire here: $P_t = 1$ for all t. Without international trade, the free entry condition (1) is replaced by

$$f_E = f_D \Pi(\varphi_{D,t}).$$

Thus, under autarky, the intermediate-good firms' cutoff is always constant, denoted by φ_D^a. The first equality of (A.6) determines the price of the knowledge good, denoted by q^a. In addition, in the absence of international financial transactions, $A = K$ holds, implying that the entrepreneurs' cutoff productivity is given by $\overline{z}(\theta)$ in Section 3.3. Substituting this result into (3), we can show that the growth rate of net worth is always constant:

$$\frac{\dot{A}_t}{A_t} = g^a \equiv q^a \overline{z}(\theta)(1 + \theta \Psi(\overline{z}(\theta))) - \delta - \rho. \tag{A.9}$$

Under autarky, the market-clearing condition of the knowledge good is given by

$$Y_{K,t} = N_t^e \left(f_E + f_D(1 - G(\varphi_D^a)) \right).$$

Therefore, the mass of entrants grows at the same rate as assets from the initial date:

$$N_t^e = \frac{Y_{K,t}}{f_E + f_D(1 - G(\varphi_D^a))} = \frac{\theta A_t \int_{z \geq \bar{z}(\theta)} z dF(z)}{f_E + f_D(1 - G(\varphi_D^a))}.$$

The output of the final good under autarky is given by

$$Y_t = N_t^e \frac{L^{1-\alpha}}{\alpha} \int_{\varphi \geq \varphi_D^a} y_{D,t}(\varphi)^\alpha dG(\varphi) = N_t^e \frac{L^{1-\alpha}}{\alpha} H(\varphi_D^a) \left(y_D(\varphi_D^a)\right)^\alpha,$$

where the function $H(\varphi_j)$ is defined in Section 3.3. Thus, under complete autarky, the economy grows through the expansion of the variety of intermediate goods from the initial date.

A.2.3 Derivation of (13)

Using (A.1), (A.3), and $p_j(\varphi) = P/(\alpha\varphi)$, we can express the sales of the firm φ of country 1 in market $j(= D, X)$ as follows:

$$p_j(\varphi) y_j(\varphi) = \frac{f_j}{1-\alpha} \left(\frac{\varphi}{\varphi_j}\right)^{\alpha/(1-\alpha)} Pq. \tag{A.10}$$

Since P^* is normalized to 1,

$$p_j^*(\varphi) y_j^*(\varphi) = \frac{f_j}{1-\alpha} \left(\frac{\varphi}{\varphi_j^*}\right)^{\alpha/(1-\alpha)} q^*. \tag{A.11}$$

Substituting (A.10) and (A.11) into EX and IM given in Section 3.1, respectively, we obtain

$$EX = N^e \frac{P q f_X H(\varphi_X)}{1-\alpha}, \tag{A.12}$$

$$IM = N^{e*} \frac{q^* f_X H(\varphi_X^*)}{1-\alpha}, \tag{A.13}$$

where the function $H(\varphi_j)$ is defined in Section 3. From the market-clearing condition for the knowledge good,

$$\begin{aligned} N^{e(*)} &= \frac{\theta^{(*)} A^{(*)} \int_{z \geq \tilde{z}^{(*)}} z dF(z)}{f_E + \sum_{j=D,X} f_j (1 - G(\varphi_j^{(*)}))} \\ &= \frac{\theta^{(*)} A^{(*)} \int_{z \geq \tilde{z}^{(*)}} z dF(z)}{\sum_{j=D,X} f_j H(\varphi_j^{(*)})}, \end{aligned}$$

where we use the free entry condition (1) and the definition of $H(\varphi_j)$ in Section 3.3 to arrange the denominator. Substituting the resulting $N^{e(*)}$ into (A.12) and (A.13), we obtain

$$EX = \frac{P q(P) \theta A \int_{z \geq \tilde{z}} z dF(z)}{1-\alpha} \mu(P), \tag{A.14}$$

$$IM = \frac{q^*(P) \theta^* A^* \int_{z \geq \tilde{z}^*} z dF(z)}{1-\alpha} \mu^*(P), \tag{A.15}$$

where

$$\mu^{(*)} \equiv \frac{f_X H(\varphi_X^{(*)})}{\sum_{j=D,X} f_j H(\varphi_j^{(*)})} = \frac{\int_{\varphi \geq \varphi_X^{(*)}} p_X^{(*)}(\varphi) y_X^{(*)}(\varphi) dG(\varphi)}{\sum_{j=D,X} \int_{\varphi \geq \varphi_j^{(*)}} p_j^{(*)}(\varphi) y_j^{(*)}(\varphi) dG(\varphi)}.$$

Then, the trade balance $EX = IM$ implies (13).

$\mu'(P)$ is given by

$$\mu'(P) = \frac{f_D f_X \left[H(\varphi_D) H'(\varphi_X) \varphi_X'(P) - H(\varphi_X) H'(\varphi_D) \varphi_D'(P)\right]}{\left(\sum_{j=D,X} f_j H(\varphi_j(P))\right)^2}. \tag{A.16}$$

Since $\varphi_D'(P) < 0$ and $\varphi_X'(P) > 0$ from Lemma 1 and $H'(\varphi_j) < 0$, we can show that $\mu'(P) < 0$. Analogously, since $\varphi_D^{*}{}'(P) > 0$, and $\varphi_X^{*}{}'(P) < 0$ from Lemma 1, we can show that $\mu^{*\prime}(P) > 0$.

A.2.4 Stability of the steady state under financial autarky

We specify G as $G(\varphi) = 1 - \varphi^{-\kappa}$, where $\kappa > \alpha/(1-\alpha)$ is assumed. In this case,

$$\int_{\varphi \geq \varphi_j} (\varphi/\varphi_j)^{\alpha/(1-\alpha)} dG(\varphi) = (h+1)\varphi_j^{-\kappa}; \quad h \equiv \frac{1}{\kappa(1-\alpha)/\alpha - 1} > 0.$$

We can express $\mu^{(*)}$ as

$$\mu = \frac{h f_X}{f_E} \varphi_X^{-\kappa}, \quad \mu^* = \frac{h f_X}{f_E} \varphi_X^{*-\kappa}, \tag{A.17}$$

where we use the fact that $f_D \varphi_D^{-\kappa} + f_X \varphi_X^{-\kappa} = f_E/h$ holds from (1).

Using (A.17), we can rewrite (13) as

$$\begin{aligned} a &= \left(\frac{\varphi_X^*}{\varphi_X}\right)^{-\kappa} \left(\frac{\varphi_D^*}{\varphi_D}\right)^{\alpha/(1-\alpha)} \frac{1}{P} \frac{\theta^* \int_{z \geq \bar{z}(\theta^*)} z dF(z)}{\theta \int_{z \geq \bar{z}(\theta)} z dF(z)} \\ &= \left(\frac{P^{-1/\alpha}\varphi_D^*}{P^{1/\alpha}\varphi_D}\right)^{-\kappa} \left(\frac{\varphi_D^*}{\varphi_D}\right)^{\alpha/(1-\alpha)} \frac{1}{P} \frac{\theta^* \int_{z \geq \bar{z}(\theta^*)} z dF(z)}{\theta \int_{z \geq \bar{z}(\theta)} z dF(z)} \\ &= \left(\frac{\varphi_D^*}{\varphi_D}\right)^{\alpha/(1-\alpha)-\kappa} P^{2\kappa/\alpha - 1} \frac{\theta^* \int_{z \geq \bar{z}(\theta^*)} z dF(z)}{\theta \int_{z \geq \bar{z}(\theta)} z dF(z)}. \end{aligned}$$

Then,

$$\frac{\widehat{a}}{\widehat{P}} = \left(\frac{\alpha}{1-\alpha} - \kappa\right)\left(\frac{\widehat{\varphi}_D^*}{\widehat{P}} - \frac{\widehat{\varphi}_D}{\widehat{P}}\right) + \frac{2\kappa}{\alpha} - 1,$$

where a hat over a variable denotes its rate of change (e.g., $\widehat{\varphi}_D = d\varphi_D/\varphi_D$). Totally differentiating (1) and (9), we obtain

$$\frac{\widehat{\varphi}_D}{\widehat{P}} = -\frac{\mu}{\alpha}, \quad \frac{\widehat{\varphi}_D^*}{\widehat{P}} = \frac{\mu^*}{\alpha}.$$

Using these formulas, we obtain

$$\frac{\widehat{a}}{\widehat{P}} = \frac{\kappa[2 - (\mu + \mu^*)]}{\alpha} + \frac{\mu + \mu^*}{1-\alpha} - 1.$$

Since we assume $\kappa > \alpha/(1-\alpha)$, the following inequality holds:

$$\frac{\widehat{a}}{\widehat{P}} > \frac{2}{1-\alpha} - 1 > 0.$$

Namely, when $G(\varphi) = 1 - \varphi^{-\kappa}$, $da/dP > 0$ is always satisfied. Thus, the steady state with financial autarky is stable when $G(\varphi) = 1 - \varphi^{-\kappa}$.

A.2.5 Shifts in the phase diagram

From (3), we obtain

$$\frac{\partial(\dot{A}/A)}{\partial\theta} = q(P)\Big\{\overline{z}\Psi(\overline{z}) + \big[1 + \theta\Psi(\overline{z}) + \overline{z}\theta\Psi'(\overline{z})\big]\overline{z}'(\theta)\Big\}.$$

As shown in Section 2, $z\Psi'(z) = -\Psi(z) - (1 - F(z))$. In addition, under financial autarky, $\theta(1 - F(z)) - 1 = 0$. Thus, $\partial(\dot{A}/A)/\partial\theta = q(P)\overline{z}\Psi(\overline{z}) > 0$: By a decrease in θ, the curve of $\dot{A}/A$ in Figure 2 shifts downward.

In Equation (13), the term $\theta \int_{z\geq\overline{z}(\theta)} zdF(z)$ satisfies

$$\begin{aligned}
\frac{\partial\theta \int_{z\geq\overline{z}(\theta)} zdF(z)}{\partial\theta} &= \int_{z\geq\overline{z}(\theta)} zdF(z) - \theta\overline{z}F'(\overline{z})\overline{z}'(\theta) \\
&= \int_{z\geq\overline{z}(\theta)} zdF(z) - \overline{z}(1 - F(\overline{z})) \\
&= (1 - F(\overline{z}))\left[\frac{\int_{z\geq\overline{z}(\theta)} zdF(z)}{1 - F(\overline{z})} - \overline{z}\right] > 0.
\end{aligned}$$

Then, the curve of $a = \Gamma(P)$ shifts as indicated in Panel (b) of Figure 2.

A.2.6 Degree of international spillovers

When $G(\varphi) = 1 - \varphi^{-\kappa}$, (1) and (9) explicitly give $\varphi_D^* = [(h/f_E)(f_D + f_X T^{-\kappa} P^{\kappa/\alpha})]^{1/\kappa}$. Substituting this into (10), $q^*(P)$ is expressed as

$$q^*(P) = \frac{\alpha^{\alpha/(1-\alpha)}(1-\alpha)L}{f_D}\left[\frac{h}{f_E}(f_D + f_X T^{-\kappa} P^{\kappa/\alpha})\right]^{\frac{\alpha}{\kappa(1-\alpha)}}.$$

Taking its rate of change,

$$\frac{\widehat{q}^*}{\widehat{P}} = \frac{1}{1-\alpha}\frac{f_X}{f_D T^{\kappa} P^{-\kappa/\alpha} + f_X} > 0.$$

When the initial steady state is symmetric (i.e., $P = 1$), a credit crunch leads to a decline in P below unity. Hence, $P^{-\kappa/\alpha}$ is decreasing in α. Thus, holding other parameters constant, the elasticity is larger for higher values of α.

A.2.7 Derivation of (17)

From $b \equiv B/A$, $\dot{b} = \frac{\dot{B}}{A} - b\frac{\dot{A}}{A}$. Using (3) and (5), we can express $\dot{b}$ as

$$\dot{b} = (\rho - \theta q\widetilde{z}\Psi(\widetilde{z}))b - \frac{EX - IM}{PA}.$$

Substituting (A.14) and (A.15) into this equation yields (17).

A.3 Calibration details

We normalize the population to unity: $L^* = L = 1$. We set $\rho = 0.01$ and $\delta = 0.025$, both of which are standard values in the literature. We choose $\alpha = 0.75$, implying an elasticity of substitution of $1/(1-\alpha) = 4$, following Balistreri et al. (2011). For the shape parameter of the distribution of φ, we set $\kappa = 4.5$, consistent with the estimates in Balistreri et al. (2011), who report values ranging from 3.924 to 5.171.

Buera and Nicolini (2020) report that the average ratio of debt to non-financial assets for the U.S. non-financial business sector over 1997Q3–2007Q3 was 0.69. In our model, the ratio of debt to capital (non-financial assets) for active entrepreneurs (i.e., those with $z \geq \widetilde{z}$) is given by $1 - 1/\theta^{(*)}$. To calibrate the baseline degree of financial frictions, we assume symmetry across the two countries, i.e., $\theta^* = \theta = \overline{\theta}$. Setting the ratio equal to 0.69 implies $\overline{\theta} = 3.226$.[17]

We specify $F(z) = (1 - z^{-\gamma})/(1 - \beta^{-\gamma})$.[18] The balanced growth rate can be expressed as

$$g = (r^b + \delta)\left[1 + \frac{\overline{\theta}}{1-\beta^{-\gamma}}\left(\frac{\widetilde{z}^{-\gamma}}{\gamma - 1} - \frac{\gamma}{\gamma-1}\frac{\beta^{1-\gamma}}{\widetilde{z}} + \beta^{-\gamma}\right)\right] - \delta - \rho, \tag{A.18}$$

where we use the entrepreneurs' cutoff condition: $q\widetilde{z} = r^b + \delta$. In the symmetric steady state, the asset market equilibrium condition becomes

$$\overline{\theta}\frac{\widetilde{z}^{-\gamma} - \beta^{-\gamma}}{1 - \beta^{-\gamma}} = 1. \tag{A.19}$$

Given the parameter values $(\rho, \delta, \theta, \gamma)$ and the target values $g = 0.02$ and $r^b = 0.015$, (A.18) and (A.19) jointly determine β and $\widetilde{z}$.

We normalize f_D to 10 so that $\varphi_D > 1$. We can determine φ_D from (A.6) and $q = (r^b + \delta)/\widetilde{z}$. When $G(\varphi) = 1 - \varphi^{-\kappa}$, it follows that

$$1 - \mu = \frac{f_D H(\varphi_D)}{\sum_{j=D,X} f_j H(\varphi_j)} = \frac{h f_D}{f_E}\varphi_D^{-\kappa}, \tag{A.20}$$

where h is defined in Appendix A.2. The import penetration ratio is one minus the share of expenditure on domestic intermediate goods. Under balanced trade, μ corresponds to the import penetration ratio.[19] We set $\mu = 0.081$, which was borrowed from Sampson (2016, Table 1 on p.351). Since f_D, α, and κ are already fixed, equation (A.20) determines f_E.

[17] The definition of θ in Buera and Nicolini (2020) differs from that in our model; in our notation, it corresponds to $1 - 1/\theta$.

[18] If $\beta \to \infty$ and hence $F(z) = 1 - z^{-\gamma}$, the balanced growth condition $\theta\Psi(\widetilde{z}) = \theta^*\Psi(\widetilde{z}^*)$ and Equation (15) are rewritten as $\theta\widetilde{z}^{-\gamma} = \theta^*\widetilde{z}^{*-\gamma}$ and $Pa[\theta\widetilde{z}^{-\gamma} - 1] + \left[\theta^*\widetilde{z}^{*-\gamma} - 1\right] = 0$, respectively. From these equations, we obtain $(Pa+1)[\theta\widetilde{z}^{-\gamma} - 1] = 0$. That is, $b = b^* = 0$ always holds. To avoid such a situation, we assume $\beta < \infty$.

[19] Because we impose perfect symmetry between the two countries in the calibration, trade is balanced in equilibrium.

Because $P = 1$, we have $\varphi_X = T\varphi_D$. The share of exporters in this model is therefore

$$\frac{1 - G(\varphi_X)}{1 - G(\varphi_D)} = \left(\frac{\varphi_X}{\varphi_D}\right)^{-\kappa} = T^{-\kappa}.$$

The share is reported by Bernard et al. (2003, p.1270; Table 1). So we set $T = 0.21^{-1/\kappa} \simeq 1.414$. With $G(\varphi) = 1 - \varphi^{-\kappa}$, we also have

$$\mu = \frac{hf_X}{f_E}\varphi_X^{-\kappa},$$

which determines f_X. Finally, the definition $T \equiv \tau(f_X/f_D)^{(1-\alpha)/\alpha}$ pins down τ.

A.4 Details of Section 5

A.4.1 On the non-tradability of the final and knowledge goods

First, consider the case in which the final good is tradable between the two countries, so that $P = 1$ holds for all t. In this case, $\varphi_D^* = \varphi_D$ from Lemma 1, and (A.6) and (A.8) imply $q^*(1) = q(1)$. Then, under financial autarky, setting $\dot{a} = 0$ in (12) yields

$$\overline{z}(\theta)(1 + \theta\Psi(\overline{z}(\theta))) = \overline{z}(\theta^*)(1 + \theta^*\Psi(\overline{z}(\theta^*))),$$

which implies $\theta^* = \theta$. Hence, a steady state exists only if the two countries are perfectly symmetric.

Under financial integration, interest rate parity (8) is now given by $\widetilde{z}^* = \widetilde{z}$ because of $q^*(1) = q(1)$. Then, setting $\dot{a} = 0$ in (16) becomes

$$\theta\Psi(\widetilde{z}) = \theta^*\Psi(\widetilde{z}),$$

which again reduces to $\theta^* = \theta$. Thus, balanced growth requires perfect symmetry.

Next, consider the case in which the knowledge good is tradable between the two countries, so that $P_K^* = P_K$ holds for all t. Then, from the definition of $q^{(*)}$, it follows that $q^*(P) = Pq(P)$. This equation pins down the value of P. $P = 1$ is always the solution, and this is the unique solution if $Pq(P)$ is decreasing. In either case, the fixed P causes the knife-edge situation as in the case of financial autarky: there is no steady state unless $\theta = \theta^*$.

Therefore, the non-tradability of the final and knowledge goods is required to obtain the steady-state equilibrium in general cases where $\theta^* \neq \theta$.

A.4.2 Long-run effects under alternative calibration scenarios

In each alternative calibration, we alter one parameter at a time while keeping all other parameters at their benchmark values.

In the first alternative scenario, we lower the value of α from 0.75 to 0.5 so that the elasticity of substitution decreases from 4 to 2. Under this recalibration, f_E and τ change to 2.077 and

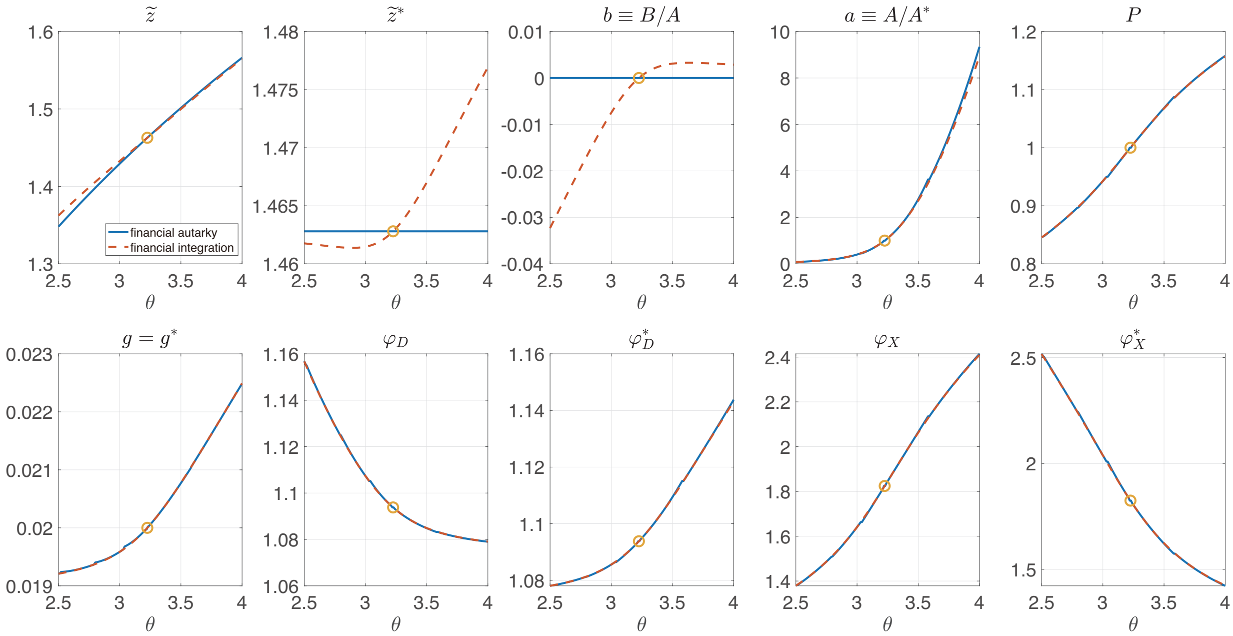

Figure 6: Recalibration case ($\alpha = 0.5$)

3.370, respectively, while the other parameters remain unchanged. Figure 6 shows the long-run effects of a decrease in θ. Compared with Figure 3, which illustrates the benchmark case, the growth-reducing effect of a decline in θ is indeed mitigated. A comparison of Panel 6 across the two figures shows that, in the present case, the growth rate does not fall below 0.019 even when θ declines to 2.5. However, the results indicate that the international transmission remains quantitatively similar under financial autarky and financial integration.

In the second alternative scenario, we recalibrate the model by setting $\kappa = 8$. Under this recalibration, f_E and τ adjust to 0.5146 and 1.623, respectively. Figure 7 shows the corresponding results. A similar pattern emerges under this alternative calibration: although the decline in growth is mitigated, the magnitude of international transmission remains quantitatively similar across financial autarky and financial integration.

Finally, we recalibrate the model by setting $\gamma = 1.5$. Under this specification, β and f_E change to 3.769 and 7.288, respectively. Figure 8 shows the results. Under this third scenario, the responses of the entrepreneurial cutoff $\tilde{z}^{(*)}$ and country 1's external debt position b are amplified. However, as in the first two scenarios, the magnitude of international transmission remains quantitatively similar across financial autarky and financial integration.

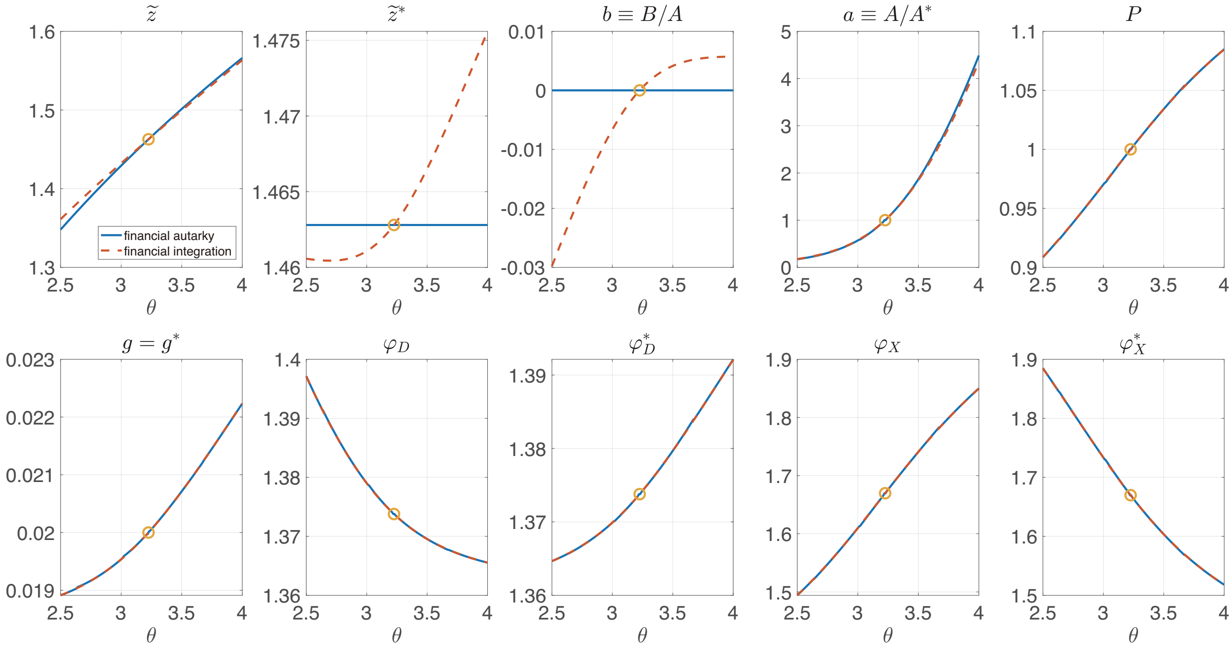

Figure 7: Recalibration case ($\kappa = 8$)

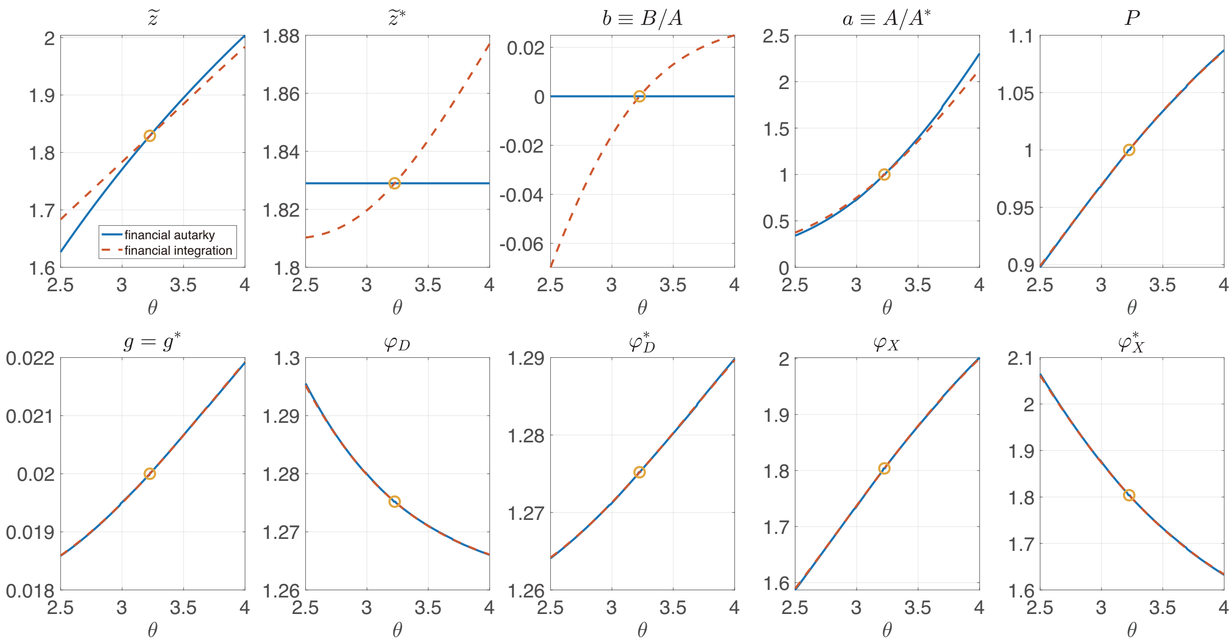

Figure 8: Recalibration case ($\gamma = 1.5$)